Dynamics Under Increments of time

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Abstract:In this paper we establish the Differential form of increments and provide a physical interpretation of them. Also we write the dynamical equation in terms of increments.

1 Introduction

In a previous paper titled On observation of incremental form of time [1], I introduced the increments of time and effects Relativity has on them. In this paper we will treat the increments in the language of calculus. Also we will establish the equations of increments under accelerated frame of reference and provide a physical interpretation of the increments of time. In this paper we will work with Luminous coordinates (coordinates through which light pulses are send).

2 Reformulation

Consider a observer in a Cartesian coordinate system at the origin O. Let it send a light pulse at $t = 0$ to a position vector $\mathbf{r}$. Say at a time t the light pulse reaches the object at $\mathbf{r}$. we also assume that light pulse from the object reaches the observer at time t as well. (I call these pair of coordinates to be Luminous coordinates). Once the light is received by observer the observation starts. Say a time (or increment in time) dt passes.

This suggests us $t + dt$ is the total time observer spends and dt is the total time it could observe the coordinate $\mathbf{r}$.

Equivalently we can write the coordinate as $|\mathbf{r}| = c(t + dt - dt)$. We may ask ourself why the increment dt in the term $t + dt$ and increment dt (with the negative sign) are same? The answer lies in our Claim.

claim: The increments of time are equal as long as the coordinate is stationary.

The coordinate r can be written in terms of observers time and the time it has observed the object which is never less than total time spend by the observer. Name dt_i for the observers clock increment and dt_o as the increment in the clock which measures total time of observation of the coordinate. In a normal circumstances we can completely ignore the subscript i. This gives us the redefined coordinate, $r = c(t+dt-dt_0)$. Note that it represents a stationary case if $dt = dt_0$.

To make the equation more consistent say a increment in position occurs, $r + dr = c(t + dt - dt_0)$ initially $r = ct$ this gives $dr = c(dt - dt_o)$ dividing by increment dt and identifying $dr/dt = v$ we get,

$$v = c\left(1 - \frac{dt_o}{dt}\right) \tag{1}$$

If we would have considered that object is incapable of sending light pulse we would have got a factor of 2 dividing the right hand side of equation (1) (I call such pairs of coordinates Non-luminous). while If the object was capable and observer incapable we would have a factor of 4.

3 Reformulation Under Special relativity

Things change drastically under relativity, and we are required to derive the equation again.

Consider a observer at origin. The observer sends Light pulses so as to observe a coordinate x' which is the coordinate in rest frame. say after a time dt the object moves by a distance $x' + dx'$. By using length contraction $dx' = \gamma dx$. The length traveled by light in time dt is cdt. The difference gives cdt_0 that is $cdt - dx' = cdt_0$.

using the above findings $dx' = c(dt - dt_0)$ and hence $\gamma dx = c(dt - dt_0)$ and dividing by time dt and identifying $dx/dt = v$ we get,

$$\gamma v = c\left(1 - \frac{dt_0}{dt}\right) \tag{2}$$

4 Interpretation

Equation (2) gives us the relation between the rate of change of t_o w.r.t time t. And the rate depends on $1 - \frac{v\gamma}{c}$. We can recognize that $v\gamma$ is the proper velocity and has no limit on its value.

The physical interpretation of t_0: If I was a observer and observing a certain distant object at a particular distance by sending light pulses I can construct two clocks.

One of the clocks will measure the total time I spent. The other clock will measure time spent in observing the object. The two clocks have increments dt and dt_0 respectively.

Its noteworthy that the speed c is not the invariant speed but is the speed of medium which has information about object. In our case it's the Vacuum speed of light. The speed might also be of field or any other wave. The Rate at which the addition of increments in the two change is dependent on velocity of the object. we will safely assume we are in a pure Vacuum and hence the c is vacuum speed of light.

For the rate to be zero, the velocity of object must be $v = \frac{c}{\sqrt{2}}$, for the rate to be one, $v = 0$. As v tends to c the rate $\frac{dt_o}{dt} \to -\infty$. This suggests us that the slope varies from $\theta = 45°$ to $\theta = -90°$. Also under the limit $c \to \infty$ the increments are equal i.e. $dt = dt_0$.

Multiplying the equation (2) with rest mass of object we get the redefined momentum in terms of the increments.

$$P = m_0 c \left(1 - \frac{dt_0}{dt}\right) \tag{3}$$

Note that p is magnitude of the momentum, equivalently the vector $\mathbf{P}$ can be formed by multiplying by the unit vector $\hat{\mathbf{p}}$. write the factor

$$w \equiv 1 - \frac{dt_o}{dt}$$

thus we may write,

$$\mathbf{P} = m_0 c w \hat{\mathbf{p}} \tag{4}$$

we will continue with this under next section.

5 Accelerated Frames

Following the previous sections we have established the Differential version of increments and provided a physical interpretation. The forces can be established by differentiating the equation (4) w.r.t. time and using Relativistic relations we get,

$$\mathbf{a}\gamma = c\left(\frac{dw}{dt}\hat{n} - \frac{\mathbf{v}}{m_0 c^3}(\mathbf{F} \cdot \mathbf{v})\right) \tag{5}$$

Another important result is,

$$\gamma^2 - w^2 = 1 \tag{6}$$

In a sense increment just shifts the dynamics. overall an addition of clock does not change the laws of physics. Does removing one do? The answer might not be yet available.

References

[1] Suraj Deshmukh. "On observation of incremental form of time". In: (). DOI: http://doi.org/10.5281/zenodo.3824448.