Relation Between Gravitational and Cosmological Constants

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Abstract

Some relations between the quantities and constants in simple physical or "abstract" models are derived with the use of the telegraph operator. It is proposed to introduce the gravitational and cosmological constant into quantum theory by means of the relation $\hbar = A\ (G/\Lambda) = B\ (\Lambda/G)$.

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1. INTRODUCTION

In the article [1] and [2], some relations between the gravitational and cosmological constant were derived. It is possible to continue in these considerations. The telegraph operator is an important operator in physics and mathematics and special forms of this operator (or its generalized form) are used in many areas of physics. It describes the waves, diffusion, potentials, sources of particles and also operators of quantum mechanics, (Schrödinger (1925), Klein-Gordon (1926), Dirac (1928), Proca (1936), etc.). The diffusion description is mainly used in the nuclear reactors, the waves mainly in quantum physics and in electromagnetism and the telegraph processes also in electromagnetism, see e.g. [3 – 40]. The telegraph operator describes the diffusion and wave processes simultaneously; the ratio of both the phenomena depends on the parameters in this operator and in the limit at suitable parameters describes only the wave or diffusion behavior. (The telegraph operator is the special case of the linear differential operator $\Sigma\ a\ (i, j)\ \partial^2/\partial x_i \partial x_j + \Sigma\ b\ (i)\ \partial/\partial x_i + u\ (\mathbf{x})$ or generally of the nonlinear integrodifferential operator $F\ (\mathbf{x}, u\ (\mathbf{x}), \partial/\partial \mathbf{x}, \partial^2/\partial \mathbf{x}^2, \int, \iint)$ in the corresponding N dimensional space or space-time). The term with the first time derivative is in the diffusion, Schrödinger, Dirac and Fokker-Planck equation, the second time derivative is in the „classical wave" equation (or d`Alembert equation), Klein-Gordon and Proca equation and both the time terms are in the telegraph equation. The space operators are „similar" but they differ in „coefficients". These operators and equations belong between the fundamental and corner stones of physics. The following considerations and equations are used only as illustrative and simple model examples and theoretical possibilities. The "space" and „ physical vacuum" in which these phenomena take place are in this paper also only „model or abstract" concepts and theoretical possibilities with special properties. However, under certain assumptions they can be real also. Some things and equations are consciously, deliberately and intentionally and of one's own free will written and interpreted somewhat else and differently than is usual. It is possible that some parts will be for some people hardly acceptable or controversial. The paper contains also several simple and usual philosophical sentences, which can be regarded as "spice" of physical considerations in this paper.

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2. TELEGRAPH OPERATOR

It can be considered and defined the following „generalized telegraph" operator T = T (a, b, c, d, o, e, φ, t) which depends on the parameters a, b, c, d, **o**, e; t is the time and φ is some field.

$$T \equiv e\,\Delta + \mathbf{o}\,\nabla + a + b\,\varphi^{-1} + c\,\partial^2/\partial t^2 + d\,\partial/\partial t$$

Generally: $T f = g$,

for a given set of fields f and „sources" g, but in many applications it is usually $T\varphi = 0$, the source is a zero field and the fields φ are searched (the „material" and mathematical sources can be included in T in this case). In the corresponding „space" then it is $Tf = g$, $f = T^{-1} g$, or $Tf = 0$, $f = T^{-1} (0)$ (formally and under certain assumptions).

This is a special case of the equation y = **A**x, y is the source, x is the field, **A** is an operator. x and y may also represent two states or two fields or a set of fields or states or the state of vacuum and excited state etc. In physics, the notion of fields and sources is usually different, but in algebra both the notions are equivalent, if there is the inversion of the operator A.

2.1. SPECIAL OPERATORS

It is defined (e = 1, **o** = **0**):

$$T_D\,(a, b, 0,d, t,\varphi) \equiv \Delta - \Sigma_a D^{-1} + b\,\varphi^{-1} + S\,(D\,\varphi)^{-1} - (D \langle v\rangle)^{-1}\,\partial/\partial t$$ („diffusion operator")

$$T_G\,(a, b, c, 0,t,\varphi) \equiv \Delta - \Lambda_S + c^2\Lambda\,\varphi^{-1} - 4\pi G M\,\varphi^{-1} - c^{-2}\,\partial^2/\partial t^2$$ („gravitational operator")

$$T_E\,(0,b, c, 0,t,\varphi) \equiv \Delta + Q\,(\varepsilon\varphi)^{-1} - c^{-2}\,\partial^2/\partial t^2$$ („electromagnetic operator")

$$T_P\,(a, b, c, 0,t,\varphi) \equiv \Delta - k + 4\pi Q\,\varphi^{-1} - c^{-2}\,\partial^2/\partial t^2$$ („Proca operator")

$$T_S\,(a, 0,0,d, t) \equiv \Delta - 2m\,V\,\hbar^{-2} + 2i\,m\,\hbar^{-1}\,\partial/\partial t$$ („Schrödinger operator")

$$T_{KG}\,(a, b, c, d, t) \equiv -\hbar^{-2}\,\{(-i\hbar\nabla - e\mathbf{A})^2 - c^{-2}\,(i\hbar\partial/\partial t - eV)^2 + m_0^2\,c^2\}$$ („Klein-Gordon operator", SI)

$$T_{KG}\,(a, b, c, d, t) \equiv -(\hbar c)^{-2}\,\{(-i\hbar c\nabla - e\mathbf{A})^2 - (i\hbar\partial/\partial t - eV)^2 + m_0^2\,c^4\}$$ („Klein-Gordon operator", CGS)

The free Klein-Gordon operator (KGO) is

$$T_{KG} \equiv \Delta - m_0^2\,c^2\,\hbar^{-2} - c^{-2}\,\partial^2/\partial t^2 = \square - m_0^2\,c^2\,\hbar^{-2}$$

$\square$ - is the d`Alembert operator

(It is easily to see that the free KGO is the Proca operator for Q = 0 and $k = m_0^2\,c^2\,\hbar^{-2}$ and also the „gravitational" operator for Λ = 0, M = 0 and $\Lambda_S = m_0^2\,c^2\,\hbar^{-2}$).

For **A** = 0, T_{KG} is in the CGS (as it is known, see e.g. [3])

$$T_{KG} = \Delta - m_0^2\,c^2\,\hbar^{-2} + e^2\,V^2\,\hbar^{-2}\,c^{-2} - c^{-2}\,\partial^2/\partial t^2 - 2i\,e\,\hbar^{-1}\,c^{-2}\,V\,\partial/\partial t - i\,e\,\hbar^{-1}\,c^{-2}\,\partial V/\partial t$$

For V (t) = const and CGS

$$T_{KG} = \Delta - m_0^2\,c^2\,\hbar^{-2} + e^2\,V^2\,(\hbar\,c)^{-2} - c^{-2}\,\partial^2/\partial t^2 - 2i\,e\,\hbar^{-1}\,c^{-2}\,V\,\partial/\partial t$$

From the context and from the literature it is evident what these quantities and operators mean. S, M and Q are sources, masses and charges or their densities, **A** and V are potentials; m, m_0 are masses and e is the charge of the electron, <v> is the mean velocity, k is a constant, ħ is the Planck's constant, **o** = **0**. Some assumptions and simplifications are considered. For the point sources ρ = [S, M, Q] = [S, M, Q] $\delta(\mathbf{r})$, where δ is the Dirac functional, see e.g. [9]. These operators correspond to the usually used operators in certain approximation. For example, in the gravitation, under certain assumptions $M = V (\rho + 3p/c^2)$, where ρ is the mean space density of the mass and p is the mean pressure, V is a volume. Usually it is also $p = f(\rho)$, f is a suitable function, for example $w\rho c^2$. It is often considered p = 0 and the point source $\rho = M\,\delta(\mathbf{r}) \equiv M$.

If it is considered p = 0, $\Lambda_S = 0$, $\Lambda = 4\pi\rho G/c^2$ then the „standard" equation of the gravitational field is $[\Delta - \Lambda_S + c^2\Lambda\,\varphi^{-1} - 4\pi G(\rho + 3p/c^2)\,\varphi^{-1}]\,\varphi = 0$, i.e. $\Delta\,\varphi = 0$, see [27] and 2nd Friedman eq. gives $(d^2R/dt^2) / R = -4\pi G(\rho + 3p/c^2)/3 + \Lambda\, c^2/3 = 0$.

If it is parameterized $\Lambda = \lambda\, 8\pi\rho G/c^2$ and $H^2 = h^2\, 8\pi\rho G/3$ then from 1st Friedman eq. follows
$kc^2/R^2 = 8\pi\rho G/3 - H^2 + \Lambda\, c^2/3 = 8\pi\rho G/3\,(1 + \lambda - h^2)$, which for $\lambda = 0$ and h = 1 gives the curvature k = 0.
k = 0 also for $\lambda = h^2 - 1$ (or for trivial cases $\rho = 0$ or G = 0).
($H^2 \equiv (dR/dt\, R^{-1})^2$), R is the scale factor or distance (at suitable assumptions).

In this paper it is also assumed that they are suitably chosen formal conditions, i.e. for example the spaces of functions in which these operators are defined, the „boundary" and normalization conditions etc. and also suitable interpretation of these quantities.

2.2. EQUATIONS

Now it is posed

$$T_D\,(a, b, 0, d) + (D\langle v\rangle)^{-1}\,\partial/\partial t = T_G\,(a, b, c, 0) + c^{-2}\,\partial^2/\partial t^2$$

i.e. $\Lambda_S = \Sigma_a D^{-1}$, $-4\pi GM = S\, D^{-1}$, $B = c^2\,\Lambda$

and parameterized

$N = c^2\,\Lambda\,(k\pi GM_1)^{-1}$ (A simple model of the physical vacuum, for purposes of this article it is posed k = 4, k is a coefficient), $\Sigma = N\sigma$

M_1 is a parameter, which can be interpreted as the mass of a scattering point, N is the space density of target points (in some abstract model or in an abstract space or in a model of the vacuum), σ, Σ are the cross sections (microscopic and macroscopic, elastic (index s) and inelastic (index a)), G, Λ, Λ_S is the gravitational, cosmological and Seeliger`s constant, c – the speed of light, D – the diffusion constant, see [2].

Then it is

$$(T_D\,(a, b, 0, d) + (D\langle v\rangle)^{-1}\partial/\partial t)\,\varphi = (T_G\,(a, b, c, 0) + c^{-2}\,\partial^2/\partial t^2)\,\varphi$$

φ represents a potential on the right side of the equation and a flux density on the left side. The scattering parameters are given by the relations for D, Σ, N, σ, which are the relations derived before.

It can be seen and it is evident that the sign of M is negative in the comparison with „conventional" sources, i.e. expressed in the language of philosophers, a part of the „mass and matter" represents the „feminine principle" and it is the „passive suction and attractive force". If the transformation M = -m is performed, then this formalism will correspond to the classical formalism of sources and in this case some quantities change the sign. If the gravitational field is interpreted and understood as a system of gravitons then the energy of the gravitational field may be considered as positive, which is from certain point of view more natural, but in this case some basic concepts must be somewhat changed (and analogically for other fields).

Analogously, if

$$(T_D\,(a, b, 0, d) + (D\langle v\rangle)^{-1}\,\partial/\partial t)\,\varphi = (T_E\,(0, b, c, 0) + c^{-2}\,\partial^2/\partial t^2)\,\varphi$$

for $\Sigma_a = 0$, B=0, $Q\,\varepsilon^{-1} = S\, D^{-1}$
Then φ represents simultaneously the flux density on the left side and the potential on the right side of the equation.

If

$$(T_D\,(a, b, 0,d) + (D\langle v\rangle)^{-1}\,\partial/\partial t)\,\varphi = (T_P\,(a, b, c, 0) + c^{-2}\,\partial^2/\partial t^2)\,\varphi = (T_G\,(a, b, c, 0) + c^{-2}\,\partial^2/\partial t^2)\,\varphi$$

$$B = 0, \quad \Lambda_S = k = \Sigma_a D^{-1}, \quad 4\pi\, Q = S\, D^{-1} = -\,4\pi\, GM$$

then φ is again a flux density in the diffusion equation and some potential in the Proca equation and in the „gravitational“ operator.

Now it is defined

$$(T_D\,(a, b, 0,d) + (D\langle v\rangle)^{-1}\,\partial/\partial t)\,\varphi = (T_S\,(a, 0,0,d) - 2i\, m\, \hbar^{-1}\,\partial/\partial t)\,\varphi = (T_G\,(a, b, c, 0) + c^{-2}\,\partial^2/\partial t^2)\,\varphi$$

$$B = S = M = \Lambda = 0, \qquad \Lambda_S = \Sigma_a D^{-1} = 2mV\,\hbar^{-2}$$

In this equation φ represents the flux density in the diffusion eq. and a part of the wave function in the Schrödinger equation and the potential for the gravitational operator (for corresponding and suitable parameters).

Let

$$(T_D\,(a,b,0,d) + (D\langle v\rangle)^{-1}\,\partial/\partial t)\,\varphi = (T_{KG}\,(a,b,c,d) + c^{-2}\,\partial^2/\partial t^2 + 2i\, e\, \hbar^{-1}\, c^{-2}\, V\,\partial/\partial t)\,\varphi = (T_G\,(a,b,c,0) + c^{-2}\,\partial^2/\partial t^2)\,\varphi$$

$$B = S = M = \Lambda = 0, \qquad \Lambda_S = \Sigma_a D^{-1} = m_0^2\, c^2\, \hbar^{-2} - (eV)^2\,(\hbar c)^{-2}, \qquad \mathbf{A} = 0, \quad V(t) = \text{const},$$

(For a free particle: $V = 0, \quad \Lambda_S = \Sigma_a D^{-1} = m_0^2\, c^2\, \hbar^{-2}$)

i.e. φ on the left side is the flux density and in the middle it is a part of the wave function in the Klein-Gordon equation with the time independent potential and on the right side it is the potential with special parameters. If it is considered the following parameterization

$$\Sigma = N\sigma, \qquad N = c^2\,\Lambda\,(4\pi G M_1)^{-1}$$

where N is some parameter of the „physical vacuum“ or parameter of the „model and abstract“ space, then the gravitational constant G, the cosmological constant Λ and the Seeliger constant Λ_S will be present in the equations of quantum mechanics. From the context it is again clear what all these quantities mean.

Specially, if the gravitational and cosmological constant is introduced into quantum mechanics via the free Klein – Gordon operator and Planck's constant ħ, then:

$$\hbar = (G/\Lambda)(C\,(\sigma_a\sigma_s)^{-1})^{1/2}\,(4\pi M_1/c)\, m_0 \equiv A\,(G/\Lambda) \equiv B\,(\Lambda/G)$$

The quantity (G/Λ) or (Λ/G) can be named „the vacuum constant“.

A, B, C are certain parameters or „constants“. This form can be also used independently on these equations, but it is to see, that it is authorized and it is right in the main.
The value $\Lambda_S = m_0^2\, c^2\, \hbar^{-2}$ and corresponding Λ may be considered as a “reference“ value $\Lambda_S^0\,(m_0^e)$ and $\Lambda^0(m_0^e)$ for the rest mass of the electron, because both the „constants“ may be dependent on the space and vacuum, and on the fields in which they are considered. The time dependence can be also considered.

It is also possible another consideration which seems to be in a better accordance with other considerations in this article.

In quantum physics it is defined and used $\mathbf{p} = -\,i\,\hbar\,\nabla$ (Jordan, 1926, see [20]) and $E = i\,\hbar\,\partial/\partial t$, see [3]. It can be defined the operator

$$T_Q \equiv \Delta + \mathbf{p}^2/\,\hbar^2$$

which for $\mathbf{p}^2 = 2mE$ (nonrelativistic physics) gives T_S $\quad (T_Q = T_S)$
and for $\mathbf{p}^2 = E^2/c^2 - (mc)^2$ (relativistic physics) gives T_{KG} $\quad (T_Q = T_{KG})$

(If $E \to E - e\Phi$ and $cp \to c\mathbf{p} - e\mathbf{A}$, then it gives the operators with the potentials Φ and $\mathbf{A}$ (in CGS), see e.g. [3]).

If it is posed and required $T_Q = T_D = T_G = \Delta + \mathbf{p}^2/\hbar^2 = \Delta + \Sigma_a / D = \Delta - \Lambda_S$ (special diffusion and gravitational processes), then under certain assumptions it will be:

$$- \mathbf{p}^2/\hbar^2 = \Sigma_a / D = \Lambda_S = \Sigma_a \Sigma_s / C = N^2 \sigma_a \sigma_s / C \quad \text{and}$$

$$\hbar = (G/\Lambda)\,(4\pi/c^2)\, M_1\, [i\, (\mathbf{p}^2\, C\, (\mathbf{p})\, (\sigma_a\, (\mathbf{p})\, \sigma_s\, (\mathbf{p}))^{-1})^{1/2}] \equiv A\, (G/\Lambda) \equiv B\, (\Lambda/G)$$

This formula and relation is more general than $\hbar = (G/\Lambda)\,(4\pi/c^2)\, M_1\, m_0\, c\, [C_1\, (\sigma_a^1\, \sigma_s^1)^{-1})^{1/2}]$ (index 1 corresponds to the first consideration). This formally corresponds to the formula: $i\, (\mathbf{p}^2\, C\, (\mathbf{p})\, (\sigma_a\, (\mathbf{p})\, \sigma_s\, (\mathbf{p}))^{-1})^{1/2} = m_0\, c\, [C_1\, (\sigma_a^1\, \sigma_s^1)^{-1})^{1/2}$

It is obvious that if $\hbar$, G, Λ, M_1, 4π, c^2 is real, positive and constant, then also $i\, (\mathbf{p}^2\, C\, (\mathbf{p})\, (\sigma_a\, (\mathbf{p})\, \sigma_s\, (\mathbf{p}))^{-1})^{1/2}$ must be real, positive and constant, i. e. $- \mathbf{p}^2\, C\, (\mathbf{p})\, (\sigma_a\, (\mathbf{p})\, \sigma_s\, (\mathbf{p}))^{-1} = \text{const} > 0$, i. e. $C\, (\mathbf{p})\, (\sigma_a\, (\mathbf{p})\, \sigma_s\, (\mathbf{p}))^{-1}$ must be negative. If Λ_S is negative, then it means (and it can thus be interpreted) that the space and vacuum is not the „absorber" and „brake" of motion but on the contrary the „source" of motion (see further considerations).

Analogously some gravitational, diffusion (or „telegraph") and quantum processes can be compared in further equations, and so it is possible to obtain new relations and interpretation of quantities in these equations.

For more complicated models it is necessary to use more complicated equations, for example the matrix of diffusion coefficients and higher derivatives, the vector of „drift" coefficients etc., or to use the integral or integrodifferential operators also, for example the transport operator. The Dirac equation is an example of the equation that is transformed into first derivatives. However, it is obvious that parallelly and alternatively with the usual explanation, many of these phenomena or processes can be considered and understood as special „telegraph" and „diffusion" or transport processes (at the microscopic or macroscopic or mega level) with suitable parameters. From the mathematical point of view, the telegraph operator creates certain class of "equivalency" or similarity. It can be seen that these different phenomena have much of common. If the mass of a considered object is M » m, where m is the mass of target points and binding forces among these target points are small, then typical diffusion (and Brown's) features are small.

It is necessary to say, that some relations are different from the relations presented in the literature, because in this paper the flux density is used in the difference of often used the space density of particles.

Usually $\varphi = n \langle v \rangle$, where φ is the scalar flux density, n is the density of particles and $\langle v \rangle$ is the mean velocity.

2.3. NUMERICAL EXAMPLES

Some special forms of these operators have under certain assumptions the following approximate numerical values:

$T_{KG}\,(m_e) = \Delta - 7.36\text{ E}54 - 1.11\text{ E-}17\ \partial^2/\partial t^2$ (free particle)
$T_{KG}\,(1\text{kg}) = \Delta - 8.08\text{ E}84 - 1.11\text{ E-}17\ \partial^2/\partial t^2$ (free particle)
$T_S\,(m_e) = \Delta - 1.64\text{ E}38\ V + 1.73\text{ E}4\ i\ \partial/\partial t$
$T_S\,(H) = \Delta + 3.78\text{ E}10\ r^{-1} + 1.73\text{ E}4\ i\ \partial/\partial t$ (atom H)
$T_D(H_2O) = \Delta - 1.23\text{ E}3 - 2.84\text{ E-}1\ \partial/\partial t$
(Neutron field in H_2O at T = 293.4K, without sources)
$T_D\,(D_2O) = \Delta - 0.3460 - 5.04\text{ E-}2\ \partial/\partial t$
(Neutron field in D_2O at T = 293.4K, without sources)
$T_G\,(\text{Earth}) = \Delta - 5.05\text{ E}15\ \delta(\mathbf{r})\ \varphi^{-1} - 1.11265\text{ E-}17\ \partial^2/\partial t^2$
$T_G\,(\text{Sun}) = \Delta - 1.60\text{ E}24\ \delta(\mathbf{r})\ \varphi^{-1} - 1.11265\text{ E-}17\ \partial^2/\partial t^2$

Analogously and similarly it is possible to express some further operators and to write many other numerical examples from classical and quantum equations.
The dimension of these operators is: $[T] = [m^{-2}]$, $[\partial^2/\partial t^2] = [s^{-2}]$, $[\partial/\partial t] = [s^{-1}]$, $[\Delta] = [m^{-2}]$, $[\delta(\mathbf{r})] = [m^{-3}]$, the dimension of the field φ (potential) is $[\varphi] = [m^2\, s^{-2}]$, $[\Delta f] = [[f]\, m^{-2}]$, [f] is the dimension of the field f. (In these numerical examples it is considered the SI system).
For the solar system it is $GM = 1.33\text{ E}20\ [m^3\, s^{-2}] = r\, (dr/dt)^2 = r^2\, (d^2r/dt^2)$, where r is the distance of an object from the Sun. For the Earth, this gives $1.33\text{ E}20 = 1.50\text{ E}11\, (dr/dt)^2 = 2.25\text{ E}22\, (d^2r/dt^2)$.

Many of tasks of quantum mechanics are calculations of characteristic numbers (for example the energy of bound states) and corresponding wave functions. This can be again associated with characteristic numbers of special „diffusion“ or „telegraph“ processes with suitable „diffusion“ and „scattering“ parameters in corresponding space and time scales. More generally, if $A\Psi_n = \lambda_n \Psi_n$ is some mathematical equation without a physical dimension (**A** is an operator) with its characteristic numbers and functions, then it is possible to multiply both sides of this equation by the dimension of energy, i.e. [kg m^2 s^{-2}] and then it is

$\mathbf{A}$ [kg m^2 s^{-2}] $\Psi_n \equiv \mathbf{H}\,\Psi_n = \lambda_n$ [kg m^2 s^{-2}] $\Psi_n \equiv E_n \Psi_n$.

$\mathbf{H} \equiv \mathbf{A}$ [kg m^2 s^{-2}] and $E_n \equiv \lambda_n$ [kg m^2 s^{-2}] represent the “generalized” Hamiltonian and energy, which means that practically “all” mathematical equations can be “quantized” and considered as a generalized version of quantum equations. The dependence of these operators on “variables and parameters” will be generally other than in usual physics. These mathematical operators with a physical dimension represent “generalized physical systems” and usual physical systems are only special cases of these systems and operators. It is clear that this consideration and this generalization can be made. Analogously, if $\partial/\partial t\ \Phi \equiv i\hbar\ \partial/\partial t\ \Psi = \mathbf{A}\ \Phi \equiv \mathbf{H}\ \Psi$ then „any“ mathematical equation with the first time derivative can be converted to the Schroedinger equation with a generalized Hamiltonian. The „normalization“ of these functions and energy can also be performed. For example, if $\lambda_n = k\,E_n$ then $(k^{-1}\,\mathbf{A})\,\Psi_n = E_n \Psi_n$ (Ψ and Φ can be generally considered as complex functions, in special cases as real functions).

It is again assumed that the physical dimension of mathematical equations is [1]. Similarly, the equations with higher time derivatives can be considered as generalized quantum equations with higher powers of energy, for example $\mathbf{E}^2 = -\hbar^2\,\partial^2/\partial t^2$, i.e. $\partial^2/\partial t^2\ \Phi \equiv -\hbar^2\ \partial^2/\partial t^2\ \Psi = \mathbf{A}\ \Phi \equiv \mathbf{E}^2\ \Psi$, ($\Phi \equiv -\hbar^2\ \Psi$). (See also further considerations). Theoretical possibility, which probably is not till the present time used, is also $\mathbf{E}^n = i^n\,\hbar^n\,\partial^n/\partial t^n$ and $\Sigma_n\,\mathbf{E}^n$.

2.4. FURTHER EXAMPLES

a) For the Schrödinger operator and $\mathbf{H}\Psi = E\Psi$ in the approximation $1/D = 3\Sigma_s$ and $\Psi = \Phi$ it is:

$$\Delta\Psi + (2m\,(E - V)/\hbar^2)\,\Psi \equiv \Delta\Phi - (\Sigma_a / D)\,\Phi \equiv \Delta\Phi - 3\Sigma_s\,\Sigma_a\,\Phi = 0$$

$$\Lambda_S = \Sigma_a /D = 3\Sigma_s\,\Sigma_a = 2m\,\hbar^{-2}\,(V-E),$$

which gives $V = E + (\Sigma_a/D)\,\hbar^2\,(2m)^{-1} = E + (3/2)\,\Sigma_a\,\Sigma_s\,\hbar^2 / m = E + (3/2)\,N^2\,\sigma_a\,\sigma_s\,\hbar^2 / m$

The special case is V = 0 (a free particle) or E=0.

b) In the case of the Klein-Gordon eq. with a potential V which is independent of the time it is:

$\Lambda_S = \Sigma_a /D = (m_0\,c/\hbar)^2 - (eV/\hbar c)^2$, $1/D\langle v\rangle = 2ieV/c^2\,\hbar$ (if $c^{-2}\,\partial^2/\partial t^2 \approx 0$)

The stationary Klein-Gordon eq. and the „gravitational vacuum“ give for V=0 (see above):

$\Lambda_S = (m_0\,c / \hbar)^2$, $\Sigma_a\,\Sigma_s = 1/3\,(m_0\,c / \hbar)^2$, $\sigma_a\,\sigma_s = (4\pi\,G\,m_0\,M_1)^2\,(3\,\hbar^2\,\Lambda^2\,c^2)^{-1}$

c) For a free Schrödinger particle ($\Sigma_a = 0$, B = 0, S = 0) can be considered the relation

$1/D\langle v\rangle = -2\,m\,i / \hbar$ and $\Sigma_s = -\,(2/3)\,m\,\langle v\rangle\,i / \hbar$ (for $D = 1/3\Sigma_s$)

and for the „gravitational vacuum“

$\Sigma_s = c^2\,\Lambda\,\sigma_s / (4\pi G M_1) = -\,(2/3)\,m\,\langle v\rangle\,i / \hbar$

And so on .....

Cross sections can be in this interpretation imaginary for some processes, in other words „imaginary Schrödinger‘s time“ can be transformed into the imaginary cross sections.

2.5. HYDROGEN (bound states in the Schrödinger approximation)

„The operator of bound states" may be defined as $\mathbf{T}\,\Psi_n = (\mathbf{H} - E_n)\,\Psi_n = 0$

$$\mathbf{T}_S(H) = (\Delta + 2\,a_B^{-1}\,r^{-1} + 2\,m_e\,E_n\,\hbar^{-2}) = (\Delta + 3.78\text{ E10 }r^{-1} - 3.57\text{ E20 }n^{-2}) \qquad [m^{-2}]$$

For Bohr`s orbits $r_n = 4\pi\varepsilon_0\,\hbar^2(m_e e^2)^{-1} n^2 = a_B\,n^2 = 5.29177\text{ E-11 }n^2$ [m] it is:

$$\mathbf{T} = (\Delta + (a_B\,n)^{-2}) = (\Delta + 3.57\text{ E20 }n^{-2}) \qquad [m^{-2}] \quad ,$$

The energy is: $E_n = -\,(m_e Z^2 e^4)\,(32\pi^2\varepsilon_0^2\,\hbar^2)^{-1}\,n^{-2} = -\,(2\,m_e)^{-1}\,(\hbar/(a_B\,n))^2 = -\,2.17991\text{ E-18 }n^{-2}$ [J] (Z = 1)

$$(\mathbf{H}\,\Psi_n = E_n\Psi_n)$$

(See for example [15]). The dimension of the operator Δ in these equations is m^{-2}, $[\Delta] = [m^{-2}]$. (All terms must have the same dimension).

Corresponding „Seeliger`s constant" for the hydrogen atom and Bohr`s orbits in this „diffusion" interpretation is

$$\Lambda_S\,(H, r_n) \equiv \Sigma_a/D = -\,(a_B\,n)^{-2} = -\,(a_B\,r_n)^{-1} = -\,3.57\text{ E20 }n^{-2} \qquad [m^{-2}] \qquad (T_D \equiv \Delta - \Sigma_a/D \equiv \Delta + (a_B\,n)^{-2})$$

and the „cosmological" constant is $\Lambda = K\,n^{-1}$ for the "space and vacuum" around the proton or hydrogen atom and for the discrete energy levels, where K is still undetermined „constant or parameter" (which may depends on other parameters, and perhaps also on n).
It is used the relation and parameterization $\Lambda = \Lambda_S^{1/2}\,4\pi G M_1 c^{-2}\,(3\sigma_a\sigma_s)^{-1/2}$, in some approximation, i.e. $\Lambda_S = 3\sigma_a\sigma_s\,(c^2\Lambda\,/\,(4\pi G M_1))^2$, see above. The minus sign at Λ_S corresponds to the minus at σ_s. Theoretically and in an abstract sense; it is possible to consider the hypothetical example, „1 m or 1 km" H atom with the corresponding parameters and quantum numbers. (For example, if $r_{n,e} = 1\text{m} = r_{1.375E5}$ then n = 1.37 E5 , $v_{e,n} = v_{1.375E5} = 15.91$ m/s, $\Lambda_S = -\,1.89\text{ E10 m}^{-2}$ and $E_n = E_{1.375E5} = 11.531$ E-29 J. For $r = 1\text{km} = r_{4.3471E6}$ it is n = 4.3471 E6, $E_n = E_{4.3471E6} = 11.531$ E-32 J, $\Lambda_S = -\,1.89\text{ E7 m}^{-2}$ and $v_{4.34716E6} = 0.50316$ m/s). Usual relations were used in the usual sense: $r_n = a_B\,n^2$, $E_n = E_1\,n^{-2} = E_1\,r_1/r_n$, $v_n = (2\,|E_n|\,/\,m)^{1/2}$.

Λ_S is negative, which means that the space and „vacuum" is not the „absorber or brake" of motion but „the source" of motion and it can be alternatively interpreted (in this model) as a „cause" or „reason" why the electrons remain near the nucleus (it may be an „alternative explanation" of the attractive force, similarly as the curvature of space-time or the exchange of virtual particles). Analogously also for planetary systems and free particles (the alternative explanation of inertia). It is obvious that in these considerations Λ_S has the dependence as the energy of particles (the dependence on n^{-2}) and Λ as the velocity (the dependence on n^{-1}) even if Λ is generally considered as the density of the vacuum energy and both the "constants" are the „sources and keepers" of the motion (expressed pictorially and in analogy with classical sources).

2.6. PLANETARY SYSTEMS (in the Schrödinger approximation). (The Sun and Earth, two-point model. Rotating systems with the gravitational potential).

The operator Δ and the potential can be used for any distance and the Schrödinger approximation gives the classical description in the limit (under certain assumptions). The Newton and Coulomb quantum (and also classical) states are similar (beyond constants).
The operator of bound states $\mathbf{T}\,\Psi_n = 0$ in the Schrödinger approximation is

$$\mathbf{T} = \Delta + 2m\hbar^{-2}\,(E_n + GMm/r)$$

For „Kepler`s" or quantum orbits and energy (see [15]) it is

$$\mathbf{T} = \Delta + (G\,m^2 M\,\hbar^{-2})^2\,n^{-2}$$

This operator is invariant under the transformation $(G\,m^2 M\,\hbar^{-2})^2\,n^{-2}) = (G_1\,m_1^2\,M_1\,\hbar_1^{-2})^2\,n_1^{-2})$ which for $G = G_1$, $m = m_1$, $M = M_1$ gives $\hbar^{-4}\,n^{-2} = \hbar_1^{-4}\,n_1^{-2}$, which means that it can be used (under certain assumptions) other constant $\hbar_1$ and quantum numbers n_1, for example, for the planetary systems some considerations are in [34], but some details of the structure may be „lost".

For the system of the Sun and Earth it gives the following numerical values in **T** (see [15]):

$$\mathbf{T} = (\Delta + (G\,M\,m^2 \hbar^{-2})^2\, n^{-2}) = (\Delta + 18.139\text{ E}274\, n^{-2})\,[m^{-2}] = (\Delta + 2.8564\text{ E}126)\,[m^{-2}]$$

$n = 2.52$ E74, $M_Z = 5.974$ E24 kg, $M_S = 1.9889$ E30 kg,
$r_n = r_1 n^2 = 2.35$ E-138 $n^2 = n^2\hbar^2\,(GM_z^2M_s)^{-1} = 1.496$ E11 m
and $E_n = -\,G\,Mm\,/\,(2r_n) = -\,G^2\,M_z^3M_s^2(2\,\hbar^2)^{-1}\,n^{-2} = -1.69$ E182/n^2 [J]

Corresponding „Seeliger`s" constant for the system Sun – Earth is at present

$$\Lambda_S(S\text{-}E) = \Sigma_a / D = -\,(G\,M_z^2M_s\,\hbar^{-2})^2\,n^{-2} = -2.86\text{ E}126\,[m^{-2}]$$

Λ_S is again negative which means that (in this concept) the „space and vacuum" is the „source" of the motion and gravitational attractive forces in planetary systems. Corresponding Yukawa`s mass (1935) m = $(\hbar/c)\ \Lambda_S^{1/2}$ will be imaginary and Yukawa`s potential $\phi = k\ r^{-1}\ \exp(-\Lambda_S^{1/2}\ r)$, k=G for gravitation and $k=(4\pi\varepsilon)^{-1}$ for electromagnetism. Λ_S for the first orbit is –1.81E275 m^{-2}. Both of these quantities will be quantized (i.e. will be dependent on n). The Yukawa mass of the hydrogen atom is: $m_n = (i/n)(\hbar/(ca_B)) = (i/n)\ m_e\ e^2\ (4\pi\varepsilon_0 c\ \hbar)^{-1} = (i/n)$ 6.61E-33 [kg] = i 6.61E-33 [kg] (n=1) = i 7.257E-3 m_e [kg]. The Yukawa mass of the system Sun and Earth is: $m_n = (i/n)\ (GM_SM_E^2)/(\hbar c)$ = i 5.919E20 [kg] = i 9.911E-5 M_E [kg] and for the Sun with the center of the Galaxy it is: $m_n = (i/n)\ (GM_CM_S^2)/(\hbar c)$. These relations and values are not accidental (nothing is accidental, but some things and phenomena in theoretical models can be "accidentally" interpreted, see for example quantum physics or probability and statistics) and they have certain reasons and consequences, but it is an open question what. The number of basic combinations of the fundamental constants and parameters (masses and others) of real physical systems is limited and they have meaning, but sometimes this meaning is not known. It can also mean that simple Yukawa`s model is not valid in this case.
Similar and analogous considerations as above can be made for the cosmological constant Λ.
The quantity in [ ] is the dimension.

In fact, in the quantum operators, which are applied to the macroscopic systems (the Sun – Earth and others), they must be also considered further interaction terms and energies. Energy levels or distances and velocities are in fact quasicontinuous in the usual sense, so as it is considered in quantum and classical physics. For example, if it is absorbed or emitted 1 gamma quantum of the Co^{60} source with E = 1.33 MeV and it is considered that this energy is transformed into the motion of the Earth as a whole and not into the heat or internal energy in any excited state, then ΔE_n is 1.33 MeV = 2.131E-16 J.

It is

$$E_n - E_{n+1} \cong 2E_1/n^3 = 2.11\text{E-}41\ [J]$$
$$R_{n+1} - R_n \cong 2n\ R_1 = 1.19\text{E-}63\ [m]$$

for n » 1. This means that the change of the kinetic (and potential) energy 1.33 MeV gives Δn = 7.59E24 and ΔR_n = 9.02E-39 [m]. It is easily to see that this „big and massive" system is very „sensitive" in these quantum numbers.

The wave functions for the hydrogen and Sun–Earth system are given, formally and under certain assumptions, in the corresponding Hilbert space by the relation:

$$\Psi_n = T^{-1}(0) = (\Delta + 2m\ (E_n - V)\ \hbar^{-2})^{-1}\ (0)$$

where $V = -GMm/r$ for gravitational systems and $V = -(4\pi\varepsilon_0\ r)^{-1}\ e^2Z$ for atoms.

This relation can be considered as some definition of T^{-1} (0) (in this case), because in the usual sense this is not defined.

The wave function for the planetary systems and galaxy or the Metagalaxy and observable Universe will be all objects to „localize" with the probability close to 1 just in the places where they are at any given time, otherwise it can not be. For the planetary system and the system Sun-Earth would not be a problem the wave function explicitly write, in the main it will be usual Ψ function with given quantum numbers n, l, j, m and with gravitational parameters.

The numerical values of T^{-1} for the orbits $r_n = r_1\ n^2$ are

$T^{-1} = (\Delta + b\ n^{-2})^{-1}$ (formally $\Psi_n = (\Delta + b\ n^{-2})^{-1}(0)$),

where $b = 3.571\ E20\ [m^{-2}]$ for the hydrogen atom and $18.139\ E274\ [m^{-2}]$ for the Earth with the Sun. Models with a functional dependence $b=b\ (V,M,m,r,G,\hbar, ...)$, $a_1 \le b \le a_2$ on a large interval $\langle a_1,a_2 \rangle$ (V – are potentials, M, m – are masses or charges, r – distance, G, ħ (and so on) – „constants" and parameters) can be considered and studied. (However, it is necessary to consider corresponding boundary conditions and 1/r or other suitable dependence of the potential).
The quantum numbers for this system for given Ψ_n are formally and again under certain assumptions, given by:

$n^2 = - b\Psi_n\ (\Delta\Psi_n)^{-1}$ and the parameter b is $b = -n^2\ (\Delta\Psi_n)\Psi_n^{-1}$
(The potential energy $V = [((\hbar^2/2m)\ \Delta + E_n)\ \Psi_n]\ \Psi_n^{-1}$)

This equation gives the possibility to determine and define a generalized „potential" energy V probably for „any" system of Ψ and E.
For the „standard" bound (coupled) physical systems with the discrete values E_n will be $V(r) = V_1\ (r) = V_2\ (r) = ... = V_n\ (r)$.
Analogously can be expressed and generalized for „any" physical or abstract mathematical system with V, E_n, Ψ_n, m and ħ: $\hbar^2 / 2m= (V - E_n)\Psi_n/\Delta\Psi_n$, $\hbar^2 = 2m(V - E_n)\Psi_n/\Delta\Psi_n$ and $m = \hbar^2\Delta\Psi_n/2(V - E_n)\Psi_n$
m and ħ can be understood and considered in this sense or in this model as „dual" quantities, generally with space and time dependence. Usual m and ħ that are used in „modern" and present physics are only special cases of these abstract and mathematical quantities or systems and notions.

Elliptical trajectories can be included in certain approximation by the relationship $n = n_r + n_\varphi$ with the eccentricity $e = (1- (n_\varphi/n)^2)^{1/2}$, see [15].

The classical and quantum equations of rotating simple (micro and macro or mega) systems are in the main „the same". Analogously, for vibrating and oscillating or pendulum systems and there is no problem to write the simple quantum (and classical) equations for many of these systems.

It is also possible for these „point" approximations to write the corresponding Klein-Gordon or Dirac equation. For example, for the „point" Sun and Earth will be $(\mathbf{E} – m_{Earth}\ \varphi)^2\ \Psi = [(c\ \mathbf{p})^2 + (m_{Earth}\ c^2)^2]\ \Psi$ or
$\mathbf{E}\ \Psi = (c\ \boldsymbol{\alpha}\ \mathbf{p} + \beta\ m_{Earth}\ c^2 + m_{Earth}\ \varphi)\ \Psi$, where $\mathbf{E} = i\ \hbar\ \partial/\partial t$ is the operator of energy, $\mathbf{p} = -i\ \hbar$ grad is the operator of momentum, and $\varphi = G\ M_{Sun}\ r^{-1}$ is the potential, $\boldsymbol{\alpha}$ and β are the Dirac matrices (with high dimensions).

If it is considered a simple model in which some volume of the observable Universe has approximately 10^{80} protons, neutrons and electrons (which gives the mass m = (1.672E-27 + 1.675E-27 +9.109E-31) E80 kg = (1.672 + 1.675) E53 kg + 9.109E49 kg = 3.35 E53 kg without the binding energy), then it is possible formally to write (in some approximation and under certain assumptions)

$\mathbf{E}\ \Psi = (\Sigma_i\ \mathbf{p}_i^2/2m_i + V(\mathbf{r}_1, ..., \mathbf{r}_N))\ \Psi,$

where i = 1 to N = 3E80 (the Schrödinger eq.) and analogously for the Dirac eq. or Klein-Gordon eq.

$\mathbf{E}\ \Psi = (\Sigma_i\ (c\ \boldsymbol{\alpha}\ \mathbf{p}_i + \beta\ m_i\ c^2) + V(\mathbf{r}_1, ..., \mathbf{r}_N))\ \Psi$;

$\Psi = \Psi(\mathbf{r}_1, ..., \mathbf{r}_N)$, N = 3E80 .

More precisely

$\mathbf{E}\Psi = i\hbar\partial/\partial t\Psi = H\Psi,$

where $H = \Sigma_i\ [(\mathbf{p}_i – e_i\ c^{-1}\ \mathbf{A}_i)^2/2m_i + e_i\ \phi_i\ ] + O_S$ for the Schrödinger eq.
and $H = \Sigma_i\ [(c\ \boldsymbol{\alpha}\ (\mathbf{p}_i – e_i\ c^{-1}\ \mathbf{A}_i) + \beta\ m_i\ c^2) + e_i\ \phi_i\ ] + O_D$ for the Dirac eq.
The Klein Gordon eq. will be $(\mathbf{E} - \Sigma_i\ e_i\ \phi_i)^2 = \Sigma_i\ [(c\ \mathbf{p}_i – e_i\ \mathbf{A}_i)^2 + m_i^2\ c^4)] + O_{KG}$,
for i = 1 to N = 3E80. O_S, O_D, O_{KG} are "remains", i.e. terms with additional operators, because the general description can be more complicated.
Finite velocities of interactions, the Lienard-Wiechert or retarded potentials for gravitation and so-called „gravitomagnetic" fields can also be considered. Spin matrices can have high dimension. It can be also considered the model with dark mass and energy.

2.7. THE „POCKET" UNIVERSE (or Metagalaxy)

The globe is a model of the Earth. Some analogical and similar model or „globe" can be considered also for planetary systems and for the observable Universe. Then in this model it is $R_1 = a\ R$; R_1 , R are distances, a is the coefficient of proportionality. All physical quantities and constants, which contain the distance, can be (in this model) transformed by the same coefficient, i.e. for example the velocity $v_1 = R_1/t = a\ R/t = a\ v$, analogously the acceleration and so on. The Newton constant G_1 then will be $G_1 = a^3\ G$, the Planck constant $\hbar_1 = a^2\ \hbar$ and so on. It is also possible to keep the original values or generally to consider „arbitrary" values of these quantities and constants or to consider these values as the functions of some parameters. It can be also transformed the time and written $t_1 = a\ t$ (then will be $v_1 = v$). Then it is possible to consider „normal, strong and weak“ gravitation, or „normal, fast and slow“ gravitation and other interactions (electromagnetic, weak and strong, considered in the usual sense). For this small model (or „globe“) of the observable Universe can be written for example the Einstein and Friedman equations and also the equations of quantum physics and studied corresponding processes and quantities (cross sections, reaction rates and so on). The rest energy in this model will be $E = m\ (a\ c)^2$ and also the special relativity will be modified by c` = a c.

As an example, it is possible to consider the „R = 10 cm“ „globe“ or model of the Earth, then a = 1.568E-8 and the speed of light c = 4.703 E2 m/s, for the „R_{E-S} = 10 cm“ Sun-Earth system it will be a = 6.689E-13, c = 2.01E-2 cm/s, the radius of the Earth R_E = 4.27 E-6 cm, the radius of the Sun R_S = 4.656 E-2 cm and for the „10 or 20 cm model“ of the observable Universe it is approximately a = 7.545 E-28, c = 2.26 E-17 cm/s, R_E = 4.81 E-19 cm, R_S = 5.25 E-17 cm and the distance between the Sun and Earth is 1.128 E-14 cm, etc. The rest energy of the 1 kg will be 5.12 e-34 J. Analogously all constants can be modified. Similarly, in this pocket universe (and theoretically also in the real Metagalaxy), the parameters of the elementary particles and atomic nuclei and all equations describing micro, macro and mega world (the equations of quantum physics and quantum field theory, strong, weak and electromagnetic interactions, deterministic chaos, nuclear reactors, Einstein‘s and Friedman’s equations, equations of the motion of planets, etc.) can be and will be modified.

It is evident that this model can be only abstract and theoretical or considered as a computer model; material model probably will be never realized or will be realized only approximatelly.

This „pocket“ Universe may be very important from the theoretical point of view. For this „pocket“ Universe it is possible to write (as was said) corresponding Einstein’s and Friedman’s eqs.and also the nonrelativistic and relativistic quantum and classical equations for example for stars and planets. If it will be considered the mass of this „pocket universe“ for example (E52 – E54) kg, then the number of „macro objects“ in this pocket Universe will be approximately (E24 – E26) (under certain assumptions), which is comparable with the number of atoms in „normal“ matter with the mass of 1 kg. For example the number of atoms of Au^{197} is 3.058 E24 kg^{-1}.

The corresponding quantum equations for these objects can be approximately written (in a simple model) for example in the form:

$\mathbf{E}\Psi = i\hbar\partial/\partial t\Psi = H\Psi$,

where $H = \Sigma_i\ [(\mathbf{p}_i - m_i\ c^{-1}\ \mathbf{A}_i)^2/2m_i + m_i\ \phi_i\ ] + O_1$ for the Schrödinger eq.

and $H = \Sigma_i\ [(c\ \boldsymbol{\alpha}\ (\ \mathbf{p}_i - m_i\ c^{-1}\ \mathbf{A}_i\ ) + \beta\ m_i\ c^2) + m_i\ \phi_i\ ] + O_2$ for the Dirac eq.

Klein Gordon eq. will be $(E - \Sigma_i\ m_i\ \phi_i)^2 = \Sigma_i\ [(c\ \mathbf{p}_i - m_i\ \mathbf{A}_i)^2 + m_i^2\ c^4)] + O_3$,

or for some other model (which is not equivalent) $\Sigma_i\ (\mathbf{E}_i - m_i\ \phi_i\ )^2 = \Sigma_i\ [(\ c\ \mathbf{p}_i - m_i\ \mathbf{A}_i\ )^2 + m_i^2\ c^4)] + O_4$, (this form corresponds to the covariant relation of the Klein-Gordon equation, which it is written below). Operator O_4 can create equivalency the both versions of the Klein-Gordon equation.

It is i = 1 to N ~ E25.

m_i – are the masses of these objects

ϕ_i – usual gravitational potentials

$\mathbf{A}_i$ – vector gravitational potentials (as they are sometimes considered)

$\mathbf{P}_i = - i\hbar\nabla_i$ and the operator of energy is $\mathbf{E} = i\hbar\partial/\partial t$

O_1, O_2, O_3 are “remains”, i.e. the terms with another operators, because the reality can be more complicated. For example they may contain further interaction terms. In more general relations will be also replaced $\Sigma_i\ m_i\ \phi_i$ by V ($\mathbf{r}_1$, ... $\mathbf{r}_N$) which will have further terms.

If $H = \Sigma_k\ H_k + W$, where H_k are Hamiltonians of isolated subsystems then for W = 0 it can be written $\Psi = \Psi_E \exp(-i\hbar^{-1}Et) = \Pi_k\ \Psi_k \exp(-i\hbar^{-1}E_k t)$ and $E = \Sigma_k\ E_k$ (additivity of energy), see standard physical literature, for example [4]. For $W \neq 0$ can be used (under certain assumptions) various perturbation methods. Simple gravitational Hamiltonian of a galaxy can be written in the form (if it is neglected the interaction between various planetary systems): $H = \Sigma_i\ H_i$ (a decomposition into planetary systems), H_i is the quantum Hamiltonian for given planetary system, i.e. sum of the kinetic operators and potential energy between central mass and planets and also the potential energy among planets (an analogy of atoms with many electrons with “charges” m_i). Planetary systems (and their central masses) are bound by the gravitational potential to the center of galaxy. (Analogously for the Metagalaxy). Hamiltonian can be also expressed by means of creation and annihilation operators for stars and planets and another objects. For Ψ it is possible to write $\Psi = \mathbf{\Psi}^+ \vert 0 >$ where Ψ is a wave function of the observable Universe and $\vert 0 >$ is a vacuum state. $\mathbf{\Psi}^+$ is corresponding creation operator. Dynamics and fate of the Universe is then determined by $\mathbf{\Psi}^+$ and $\vert 0 >$ and vice versa, $\vert 0 > = \mathbf{\Psi}^- \Psi$ or $\mathbf{\Psi}^+ = \Psi \vert 0 >^{-1}$ (formally and under certain assumptions). Ψ determines the masses, positions, velocities and another characteristics of all objects in the Universe. If it is used $\Psi = A \exp(iW(\mathbf{r},t)/\hbar)$, then for the simple Schrödinger equation $i\hbar\ \partial\Psi/\partial t = (-\hbar^2/2m\ \Delta + V(\mathbf{r})\ )\Psi$ will be $\partial W/\partial t + 1/2m\ (\text{grad } W)^2 + V - i\hbar\ /2m\ \Delta W = 0$. In the limit $\hbar = 0$ it is $\partial W/\partial t + H(\mathbf{r},\mathbf{p}) = 0$, where $\mathbf{p}$ = grad W. For $W(\mathbf{r}, t) = S(\mathbf{r}) - Et$ (Ψ is an energy eigenfunction) it can be written $1/2m\ (\text{grad } S)^2 - (E -$

$V(\mathbf{r})) - i\hbar/2m\,\Delta S = 0$. The WKB method can be used for some approximate solution of S in the series of powers of ħ. The simple Hamiltonian for a planetary system can be written in the form $H = \Sigma_i(-\hbar^2/2m_i\,\Delta_i - G\,Mm_i/r_i) - G\,\Sigma\Sigma_{j>i}\,m_im_j/r_{ji}$ where M is a central mass, m_i are masses of planets, r_i is the distance between M and m_i, r_{ij} is the distance between m_i and m_j. Details (standard quantum methods) are e.g. in [3, 4, 9].

These eqs. can be also written in the covariant form (as is usual) with the covariant derivatives for „point" stars and planets $D_{i\mu} = \partial_{i\mu} + (im_i/\hbar c)\mathbf{A}_{i\mu}$, $\partial_{i\mu} = \partial/\partial x_i^{\mu}$. Then $\Sigma_i(i\gamma^{\mu}D_{i\mu} - k_i)\Psi = 0$ (or $= \mathbf{a}\Psi$) or $\Sigma_i(D_{i\mu}D_i^{\mu} + k_i^2)\Psi = 0$ (or $= \mathbf{b}\Psi$), ($k_i = (m_ic/\hbar)$, k_i^{-1} is the Compton wavelength for the stars and planets). Time can be considered (in the first approximation) the same for all stars and planets. N ~ E25. **a** and **b** are the terms with another operators because the reality can be more complicated, as was said. (The simple classical Dirac or Klein-Gordon equation has **a** and **b** = 0). The spin of stars and planets is approximately (E60 – E80) ħ, which means that γ matrices and ψ in the Dirac equation will have the corresponding dimension, see [10]. If it will be used the logarithmic scale then this dimension can be reduced for example to 100 and the double log log transformation gives the dimension approximately 10, which is not a big number. It is not necessary at present to consider the stars and planets of antimatter (as it is for electrons and positrons). It can be also used the density matrix or description with $\Psi(t) = \mathbf{S}\Psi(0)$ where **S** is corresponding operator. Constants can be used „normal" or „transformed" or „variable" and „arbitrary", which gives several „sub-models". The details of these equations and models will be developed and implemented in the future. With the use of these quantum equations the energy levels and wave functions of this „pocket" universe can be calculated (under certain assumptions as usually) and after rescaling also in the real universe. With this pocket Metagalaxy various abstract „experiments" can be made, for example this object can be compressed, stretched, deformed, heated, cooled, scattering experiments can be considered and so on, so as it is usual in physics. It is possible and probable that some things and conclusions can be generalized and will be valid for the real Metagalaxy also. But it is necessary to say that it is certain problem to write about these things, because as it is known from the history, the reality is usually other than they are some ideas and opinions in given time and in the future in the course of time many of these things fall into oblivion.

As already mentioned, in these models various processes and phenomena can pass much faster or slower than in the real world. For example, if c remains unchanged, then the time 1E10 years will correspond to 7.55E-18 years. (As was written: "Thousand years is as one day and one day is as thousand years"). The quantum and gravitational equations for the 10 cm model of the Metagalaxy will be "trivial" in some abstract level and in some approximation, but detailed numerical calculations can be difficult. More detailed model will consider for example all E80 protons, neutrons, electrons and other elementary constituents (quarks and so on) of the mass and matter. Again the Lienard-Wiechert potentials (A. D. 1898) or retarded potentials can be considered. For this model and every part thereof (e.g. the solar system) it is possible to write a "special action" S = E T= n ħ with the „quantum number" n, where E is the total energy and T is the time. Then n = n(T). Under certain assumptions, this description is equivalent with the „standard" description by means of the equations of quantum physics and has consequences in quantum geometry. This matter will be probably discussed in another article.

2.8. FREE PARTICLE

In the Schrödinger approximation, the operator of „motion" is $\mathbf{T} = (\Delta + 2\,m\,E\,\hbar^{-2})$, i.e. $\mathbf{T}\Psi = 0$, where $E = mv^2/2$ in the nonrelativistic limit (or $\mathbf{E} = i\hbar\,\partial/\partial t$ in the operator description) and

$$\Lambda_S = \Sigma_a/D = -2mE\,\hbar^{-2} = -(p/\hbar)^2$$

It is evident that Λ_S is again „a source" of the motion and energy. The value of energy and momentum is the consequence of the value of Λ_S for this given motion, which means that in the spaces with the different value of Λ_S will be the energy and momentum and also the mass other. For given v, ħ and Λ_S it is:

$$m = (\hbar/v)(-\Lambda_S)^{1/2}, \qquad p = \hbar(-\Lambda_S)^{1/2}, \qquad E = v\,\hbar\,(-\Lambda_S)^{1/2}/2$$

The rest mass will be: $m_0 = \lim(v\to 0)[(\hbar/v)(-\Lambda_S)^{1/2}]$, and analogously the rest energy.

The numerical values for $2m\,\hbar^{-2}$ (in „our" space) are:

The electron:	$2m\,\hbar^{-2}$	= 16.378 E37 [$kg^{-1}\,m^{-4}\,s^2$]
The mass of 1 kg:	$2m\,\hbar^{-2}$	= 1.798 E68 [$kg^{-1}\,m^{-4}\,s^2$]

The mass of the Earth: $2m\,\hbar^{-2} = 10.743\text{ E92 } [kg^{-1}\, m^{-4}\, s^2]$

Similarly for the mass of the Sun or galaxy. It is to see that our space is very strong „source“ of motion. In theoretical models in which it will be $\hbar \gg 0$, then it will be $|\Lambda_S| \ll 1$ and the space will be very weak source of motion.

Analogous considerations as above can be also made for „Einstein's“ Λ and for other equations or operators and some new relations can be obtained.

2.9. SYSTEM OF THE SUN AND „CENTER“ OF THE GALAXY

Similar considerations as above (in some approximation, and under certain assumptions) give for this system the following quantum number $n = [RvM/\hbar] = 1.025E90$ and $M_{C.G.} = v^2 R/G = 1.791E41$kg (the mass of the „attractive center“ of Galaxy). The calculation of E_n, r_n, Λ_S and corresponding operators is trivial and also Λ, which is determined till the factor with the cross sections. (M = 1.99E30 kg, v = 220 km/s, R = 8.0 kpc, see [36]). It can be again determined the Yukawa mass and potential and also "the sensitivity" of quantum numbers for example on the change of the energy 1.33 MeV of γ of Co^{60}.

It is no problem again to write corresponding simple classical and „quantum“ equations for this system in some approximation (the two-point Newton, Schrödinger, Klein-Gordon or "generalized“ Dirac model), but it is only very approximate model, because the reality and real values may be different.

2.10. GENERALIZED TELEGRAPH OPERATORS (recapitulation)

It is possible to consider the generalized and abstract model in which will be defined

$T_D \equiv \Delta - \Sigma_a D^{-1} + b\,\varphi^{-1} + S(D\,\varphi)^{-1} - d\,(D <v>)^{-1}\,\partial/\partial t - a <v>^{-2}\,\partial^2/\partial t^2$ („waves with diffusion“)

$T_G \equiv \Delta - \Lambda_S + c^2\Lambda\,\varphi^{-1} - 4\pi GM\,\varphi^{-1} - d\,(D <v>)^{-1}\,\partial/\partial t - \varepsilon\mu\, c^{-2}\,\partial^2/\partial t^2$ („gravitation with diffusion“)

$T_G \equiv \Delta - \Lambda_S + c^2\Lambda\,\varphi^{-1} - 4\pi GM\,\varphi^{-1} - 4\pi\mu\,\gamma\, c^{-2}\,\partial/\partial t - \varepsilon\mu\, c^{-2}\,\partial^2/\partial t^2$ („telegraph-gravitational operator“)

$T_E \equiv \Delta + 4\pi Q(\varepsilon\varphi)^{-1} - \varepsilon\mu\, c^{-2}\,\partial^2/\partial t^2 - 4\pi\mu\,\gamma\, c^{-2}\,\partial/\partial t$ („electromagnetic telegraph operator“, CGS)

$T_P \equiv \Delta - k + 4\pi Q_P\,\varphi^{-1} - c^{-2}\,\partial^2/\partial t^2 - d\,(D <v>)^{-1}\,\partial/\partial t$ („Proca operator with diffusion“)

$T_P \equiv \Delta - k + 4\pi Q_P\,\varphi^{-1} - c^{-2}\,\partial^2/\partial t^2 - 4\pi\mu\,\gamma\, c^{-2}\,\partial/\partial t$ („telegraph Proca“)

$T_S \equiv \Delta - 2m\,V\,\hbar^{-2} - 2i\, b_1\, m\,\hbar^{-1}\,\partial/\partial t - a <v>^{-2}\,\partial^2/\partial t^2 + Q_S\,\varphi^{-1}$ („Schr. operator with the „wave“ and „source“ term“)

$T_{KG} \equiv -(\hbar c)^{-2}\,\{(-i\hbar c\nabla - e\mathbf{A})^2 - (i\hbar\partial/\partial t - eV)^2 + m_0^2\, c^4\}$ („Klein-Gordon operator“, CGS)

$T_{KG} \equiv \Delta - m_0^2\, c^2\,\hbar^{-2} - c^{-2}\,\partial^2/\partial t^2 - 2\, b_1\, i\, m\,\hbar^{-1}\,\partial/\partial t + Q_S\,\varphi^{-1}$ („K-G with the Schr. and source term“)

It is clear that these operators are „equivalent“ (under certain assumptions) if:

$$\Lambda_S = \Sigma_a D^{-1} = k = 2m\,V\,\hbar^{-2} = m_0^2\, c^2\,\hbar^{-2}$$
$$c^2\Lambda - 4\pi GM = b + S(D)^{-1} = 4\pi Q(\varepsilon)^{-1} = 4\pi Q_P = Q_S$$
$$d\,(D <v>)^{-1} = 4\pi\mu\,\gamma\, c^{-2} = 2\, b_1\, i\, m\,\hbar^{-1}$$
$$a <v>^{-2} = \varepsilon\mu\, c^{-2}$$

Again it is possible to make some further generalizations.

Some operators are not Lorentz or Galileo invariant but in abstract and mathematical models this condition is not necessary. They are invariant only for special values of parameters. It is possible and probable, that generally in the real world, no perfect invariance exists (there is only approximate invariance and symmetry as can be easily seen, because the metric is dependent on the time and space). Perfect invariance and symmetry there is only in simplified physical models or in some abstract and mathematical models. It can be seen that things and phenomena of this world are not invariant in the time and space; they are only partially and approximately invariant. It is obvious that for suitable parameters will be $T_D = T_G = T_E = T_P = T_S = T_{KG}$ which means that all fields in the equations $T_{D,G,E,P,S,KG}\ \varphi = g$ (or 0) are the same and they can be mutually transformed and converted one to another. (A small „technical or technological" problem, but not abstract or mathematical problem is in the „imaginary" Schrödinger time and in the interpretation of some quantities. For example in quantum theory it has physical meaning $|\Psi|^2$, but this interpretation can be in the future changed or some alternative interpretation can be found). It is again assumed that the „definition area" and abstract spaces of fields and sources are suitably defined.

Specially $\varepsilon\mu = 1$ for the vacuum, $\varepsilon = 1/G$, $\mu = G$ (in the case of gravitation) and the „conductivity" of our space and vacuum for the gravitation is $\gamma = c/4\pi G$ (if it is assumed that the speed of the gravitation is c). It means that the value of the speed of light and gravitation is dependent on properties (on the conductivity for light and for gravitation) of the space and vacuum. This question and corresponding relations can be discussed in other article.

It is also evident that if these fields are transformed for example on the gravitational field, then this field in the terminology of general relativity curves space, which is not absolutely rigid (it can be determined „equivalent Young's modulus of elasticity" as well as for the Einstein equations, see some additional considerations) and it is in this field, which means that all fields can be mapped or described by the metric of curved spaces. If these fields are converted and transformed to quantum fields, then all fields can be described by the propagators, Feynmann diagrams, cross sections and reaction rates, see the considerations in the conclusion.

2.11. MATRIX FORM

These equations can be formally written in the matrix form:

$$T = \begin{pmatrix} T_D \\ T_{G,D} \\ T_{G,T} \\ T_E \\ T_{P,D} \\ T_{P,T} \\ T_S \\ T_{KG} \end{pmatrix} = \begin{pmatrix} 1, & -\Sigma_a D_d^{-1}, & a_1, & SD_d^{-1}, & a_2(D_d\langle v_d\rangle)^{-1}, & a_3\langle v_d\rangle^{-2} \\ 1, & -\Lambda_S, & c^2\Lambda, & -4\pi GM, & a_4(D_G\langle v_G\rangle)^{-1}, & -\varepsilon_G\mu_G c^{-2} \\ 1, & -\Lambda_S, & c^2\Lambda, & -4\pi GM, & -4\pi\mu_G\gamma_G c^{-2}, & -\varepsilon_G\mu_G c^{-2} \\ 1, & -a_5, & a_6, & 4\pi Q_E\varepsilon_E^{-1}, & -4\pi\mu_E\gamma_E c^{-2}, & -\varepsilon_E\mu_E c^{-2} \\ 1, & -k, & a_7, & 4\pi Q_P, & a_8(D_P\langle v_P\rangle)^{-1}, & -\varepsilon_P\mu_P c^{-2} \\ 1, & -k, & a_9, & 4\pi Q_P, & -4\pi\mu_P\gamma_P c^{-2}, & -\varepsilon_P\mu_P c^{-2} \\ 1, & -2mV\hbar^{-2}, & a_{10}, & Q_S, & -2im\hbar^{-1}, & -a_{11}\langle v\rangle^{-2} \\ 1, & -m_0^2c^2\hbar^{-2}, & a_{12}, & Q_{KG}, & -2ia_{13}m\hbar^{-1}, & -a_{14}c^{-2} \end{pmatrix} \begin{pmatrix} \Delta \\ 1 \\ \varphi^{-1} \\ \varphi^{-1} \\ \frac{\partial}{\partial t} \\ \frac{\partial^2}{\partial t^2} \end{pmatrix}$$

$$T\Phi = \begin{pmatrix} T_D \\ T_{G,D} \\ T_{G,T} \\ T_E \\ T_{P,D} \\ T_{P,T} \\ T_S \\ T_{KG} \end{pmatrix} (\varphi_D, \varphi_{GD}, \varphi_{GT}, \varphi_E, \varphi_{PD}, \varphi_{PT}, \varphi_S, \varphi_{KG}) = f$$

Now it is formally: $T\Phi = f$ or $T\Phi = 0$ and $\Phi = T^{-1} f$ or $\Phi = T^{-1}\, 0$.
If $TD = T_{GD} = T_{GT} = T_E = T_{PD} = T_{PT} = T_S = T_{KG}$, i.e. if $\Sigma_a D_d^{-1} = \Lambda_S = a_5 = k = 2\ mV\hbar^{-2} = (m_0\, c\hbar)^2$, $a_1 = c^2\Lambda =$
$= a_6 = a_7 = a_9 = a_{10} = a_{12}$, $S D_d^{-1} = -4\pi GM = 4\pi Q_E\varepsilon_E^{-1} = 4\pi Q_P = Q_S = Q_{KG}$, $a_2\,(D_d\langle v_d\rangle)^{-1} = a_4\,(D_G\langle v_G\rangle)^{-1} =$
$= -\,4\pi\mu_G\gamma_G c^{-2} = -\,4\pi\mu_E\gamma_E c^{-2} = a_8\,(D_P\langle v_P\rangle)^{-1} = -\,4\pi\mu_P\gamma_P c^{-2} = -2im\hbar^{-1} = -2ia_{13}m\hbar^{-1}$, $a_3\langle v_d\rangle^{-2} = -\varepsilon_G\mu_G c^{-2} = -\varepsilon_E\mu_E c^{-2} =$
$= -\varepsilon_P\mu_P c^{-2} = -a_{11}\,\langle v\rangle^{-2} = -a_{14}c^{-2}$ then $\varphi_D = \varphi_{GD} = \varphi_{GT} = \varphi_E = \varphi_{PD} = \varphi_{PT} = \varphi_S = \varphi_{KG} \equiv \varphi$ (under certain assumptions).

In this section, they are more detailed written some coefficients and some velocities because generally the average speed of the diffusion or diffusion waves can be different than the speed of particles in the given field.

2.12. OTHER CONSIDERATIONS

The typical standard form of the telegraph equation is (Φ is a field and f is a source)

$$(\Delta - a_0\, \partial^2/\partial t^2 - a_1\, \partial/\partial t - a_2)\, \Phi = f(\mathbf{r},t)$$

Formally, it can be written and defined: $\Delta = -\, \mathbf{p}^2\, \hbar^{-2}$, $\partial^2/\partial t^2 = -\, \mathbf{E}^2\, \hbar^{-2}$, $\partial/\partial t = -\, i\, \mathbf{E}\, \hbar^{-1}$
(as is usual in quantum mechanics)

Then:

$$(-\mathbf{p}^2 + a_0\, \mathbf{E}^2 + a_1\, i\, \hbar\, \mathbf{E} - a_2\, \hbar^2)\, \Phi = \hbar^2\, f(\mathbf{r}, t)$$

(A formal solution is: $\Phi = \hbar^2\, (-\mathbf{p}^2 + a_0\, \mathbf{E}^2 + a_1\, i\, \hbar\, \mathbf{E} - a_2\, \hbar^2)^{-1}\, f(\mathbf{r}, t)$)

And in the special case corresponding to „classical“ mechanics it gives the following equation:

$$(-\mathbf{p}^2 + a_0\, E^2 + a_1\, i\, \hbar\, E - a_2\, \hbar^2\,) = 0 \quad , \qquad \text{i.e.} \qquad \mathbf{p}^2 + a_2\, \hbar^2 = a_0\, E^2 + a_1\, i\, \hbar\, E$$

This dependence can describe a nonclassical „object“ or process for example with the dissipation or with some „internal“ source of energy or with a „programmed“ or „ simple intelligent“ behavior. Alternatively, it can also be considered as the „special“ dependence of $\hbar = \hbar\, (\mathbf{a, p, E})$ (for this model).

Similar considerations can be made for other equations of mathematical physics, see e.g. [6] (some equations are special cases of these equations).

Sometimes it is used the equation: $(\Delta\Delta + c^{-2}\, \partial^2/\partial t^2)\, \Phi = f(\mathbf{r}, t)$

Then: $(c^2\, \mathbf{p}^4 - \hbar^2\, \mathbf{E}^2)\, \Phi = c^2\, \hbar^4\, f(\mathbf{r}, t)$

A formal solution: $\Phi = (\Delta\Delta + c^{-2}\, \partial^2/\partial t^2)^{-1}\, f(\mathbf{r},t) = (c^2\, \mathbf{p}^4 - \hbar^2\, \mathbf{E}^2\,)^{-1}\, c^2\, \hbar^4\, f(\mathbf{r},t)$

A special „nonclassical dependence and behavior“: $c^2\, \mathbf{p}^4 = \hbar^2\, E^2 \quad (E = \pm\, cp^2/\, \hbar)$

Once more example (an oscillating fiber in the gravitational field at one end fixed and with the zero source):

$$(x\, \partial^2/\partial x^2 + \partial/\partial x - g^{-1}\, \partial^2/\partial t^2)\, \Phi = 0 \qquad \text{(the „source“ of Bessel functions)}$$

Then: $(-\, x\, g\, \mathbf{p}_x^{\,2} + i\, g\, \hbar\, \mathbf{p}_x + \mathbf{E}^2)\, \Phi = 0$

A „nonclassical“ dependence: $E^2 = x\, g\, p_x^{\,2} - i\, g\, \hbar\, p_x$

Alternatively, it can again be seen as the dependence of $\hbar = (x\, g\, p_x^{\,2} - E^2)\, (i\, g\, p_x)^{-1}$ (in this model)

For these operators can be investigated for example equations with characteristic numbers and functions, e.g. equations in the form: $\mathbf{E}\Psi_n = E_n\Psi_n$ and $\mathbf{E}^2\Phi_n = E_n^{\,2}\Phi_n$, $(\Psi_n = \Phi_n)$ and similarly for the telegraph operator $\mathbf{T}\Psi_n = t_n\Psi_n$ and for other operators.

If $\mathbf{A}\, \Phi = f_1$ and $\mathbf{B}\, \Phi = f_2$ where **A** and **B** are some operators, Φ is a field and f_1, f_2 are sources, then $(\mathbf{A} + \mathbf{B})\, \Phi = f_1 + f_2$.

This means that as well e.g. $(\mathbf{T_S} + \mathbf{T_{KG}})\, \Phi = \mathbf{T_T}\, \Phi = f_S + f_{KG}$ where $\mathbf{T_S}$ and $\mathbf{T_{KG}}$ are the Schrödinger and Klein-Gordon operators, Φ is a field, $\mathbf{T_T}$ is corresponding telegraph operator and f_S , f_{KG} are sources.
Generally $(\mathbf{T_D} + \mathbf{T_G} + \mathbf{T_E} + \mathbf{T_P} + \mathbf{T_S} + \mathbf{T_{KG}})\, \Phi = \mathbf{T_T}\, \Phi = f_D + f_G + f_E + f_P + f_S + f_{KG} = f$

$\mathbf{T_i}$ are the operators used in this paper (diffusion, gravitational, electromagnetic, Proca, Schrödinger, Klein-Gordon), Φ is a field and f_i are corresponding sources. The formal solution is $\Phi = \mathbf{T_T}^{-1}$ (f).
It is known that the Lorentz transformation **L** is given by the relation: $\mathbf{x_2} = \mathbf{L}\, \mathbf{x_1} \Leftrightarrow \Box_2 = \Box_1$ where $\Box$ is the d`Alembert operator ($\Box \equiv \Delta$ - $c^{-2}\, \partial^2/\partial t^2$ and **x** = (**r,** ict) for example) or arbitrary other invariant operator (with **L** transformation). If $\Box$ is invariant (at this transformation **L**) then also c^2 and c is invariant. This means that if it is considered the relation: $\mathbf{x_2} = \mathbf{F}\, \mathbf{x_1} \Leftrightarrow \mathbf{T}_2 = \mathbf{T}_1$ where **T** is the telegraph operator, then **L** is the special case of the transformation **F** (for **T** = $\Box$). **x** can be for example some space-time coordinates in arbitrary dimension. Analogous considerations can be made also for some other more general operators (for example with higher order) and it can be studied invariance or noninvariance of the operators with higher time and space derivatives. This means that usual relativistic physics is only the special case of "some other physics". "Global and local" transformation properties of the eq. **T**f = g are given by the transformation properties of **T** and f or g, the transformational properties one of the quantities are given by the transformational properties of another two quantities. Again, it is assumed that the abstract or "real" spaces of fields and sources are suitable defined.

2.13. „GENERALIZED" QUANTUM EQUATIONS

As already was said, the quantum formalism can be generalized to "arbitrary" operator equations. For example, for the operator equation $\mathbf{A}\, \Psi_n = \lambda_n\, \Psi_n$ can be written $\mathbf{A}$ [kg m^2 s^{-2}] $\Psi_n = \lambda_n$ [kg m^2 s^{-2}] Ψ_n (both sides of the equation were multiplied by the dimension of energy). Then **A** [kg m^2 s^{-2}] is formally some "generalized Hamiltonian" and has the dimension of energy and λ_n [kg m^2 s^{-2}] is a characteristic number, which has also the meaning and dimension of energy. It can be made direct "normalization" on the energy, i.e. $\lambda_n = k\, E_n$ where E_n is some "energy" and then: $\mathbf{A}\, \Psi_n = \lambda_n\, \Psi_n = k\, E_n\, \Psi_n \Leftrightarrow (k^{-1}\, \mathbf{A})\, \Psi_n \equiv \mathbf{H}\Psi_n = E_n\, \Psi_n$ (under certain assumptions).
k^{-1} **A** can be again seen as an operator of energy or the Hamiltonian. Analogously for the equation $\partial\Psi/\partial t = \mathbf{A}\, \Psi \Leftrightarrow i\, \hbar\, \partial\Psi/\partial t = \hbar\, \mathbf{A}\, i\, \Psi \equiv \mathbf{H}\Psi$ and it is visible and clear, that usual quantum physics is only the special case of these equations, so as it can be expected and so as it should be. For imaginary operator **A** will be **H** real so as it usual and for real operator **A** will be **H** imaginary, which is new generalization of the notion "Hamiltonian". For complex operator **A** will be also **H** complex operator with the meaning of some Hamiltonian.

(Formally $\partial\Psi/\partial t$ or $\lambda_n\, \Psi_n$ is a source and Ψ or Ψ_n is a field). Similar reasoning can be done for the Klein-Gordon, Dirac and other equations.

Analogously can be analyzed further mathematical equations and operators. Usual „physical behavior and laws" are only special cases of these equations and operators. These operators can be considered as special observables and quantities of quantum algebra, see [48], or generalized observables and quantities of quantum physics, see e.g. [17,18]. The quantity ħ and physical dimensions are not necessary in quantum algebra and they are only auxiliary concepts and notions. Some similar considerations can be made also in quantum (differential) geometry, but these things will be probably described in another paper. It seems to be evident that it is possible to convert "arbitrary" mathematical equation on an abstract or real or generalized physical equation by means of the multiplication by a physical dimension. This creates certain large class of equivalency.

The important property is the invariance or noninvariance of given operator at the time reversion. It is to see that all terms with $\partial/\partial t$, $\partial^3/\partial t^3$ and so on, are not invariant when $t \to -t$ and all even time derivatives are time invariant. The size of the noninvariance will be dependent on the coefficients at these derivatives and also on another time-noninvariant functions or sources $f(t) \neq f(-t)$ i.e. $0 \neq \delta = f(t) - f(-t)$ or on fields φ which will be in these operators. Analogously for the integral operators $I = D^{-1}$. The typical time noninvariant function is for example the exponential decay law. If CPT = 1, then, as is known, $CP = T^{-1}$ where C, P and T is the transformation of charge, parity and time. Similarly, for C and P. As was mentioned, if **T**f = g, then the „symmetry" or transformation properties are given by two terms of **T**, f or g, the third term is determined by the remaining terms.

2.14. GRAVITATION
(Einstein and Friedman's equations)

It is possible to say „gravitation curves space or space-time", (which is not or as far as is not absolutely rigid or empty), which means that it is possible to write the equation **A**g = 0 or **A**g = f, i.e. formally $g = \mathbf{A}^{-1}$ (0) or $g = \mathbf{A}^{-1}$ (f), where g is a metric, **A** is a suitable operator and f is a source. The space in which there is our Universe is not absolutely rigid or empty. (Under certain assumptions it can be, in special cases, also $\mathbf{A} = fg^{-1}$, for example $(d/dx)(x^2) = 2x \equiv g$ and $(d/dx) = 2xx^{-2} = gx^{-2} = 2/x$ in this case and so on). (Euclid's or flat space is a special case with special metric g). **A** and g or f depend on the intensity of sources, on the „rigidity" or stiffness of considered (real or abstract) space, on the gravitational and cosmological constant G, Λ (if it is considered the real or abstract space with these constants) and on some further parameters. (The concept and notion of „rigidity“ (or stiffness) usually is not considered in this context, but generally it can be considered). The metric of given space (or space-time) is then g = g(G, Λ, f, M, P, **r**, t). (P are other parameters). The metric is generally in all equations and operators that describe physical phenomena and processes, which means that G and Λ are also in all physical equations including equations of quantum physics, but in most cases their influence is small (the space-time is usually considered as flat). However, it is possible to study abstract mathematical models with variable parameters G, Λ, ħ and some others, then their influence can be significant, which will have an influence for example on the cross sections and reaction rates of gravitational and other interactions (see also considerations in the chapter about the „pocket“ universe). In limit, the metric is Euclidean. The relation **A**g = 0 or f is usually written in the form of „simple“ Einstein’s equations (1915) and derived Friedman’s equations (1922) for the Friedman-Robertson-Walker metric (and with further state equations). In these equations the notion and concept „rigidity“ or stiffness is not used explicitly, but implicitly it is contained in them and also explicitly it can be inserted into these equations. „Equivalent Young’s coefficient of elasticity“ can be also defined. (In simple Newton’s model of gravitation or for Coulomb’s forces, the rigidity or stiffness may be formally defined as $K = -F/r = -ma/r = G\,mM/r^3$ (or qQ/r^3), where M and m are masses and q and Q are charges, which corresponds to $K = m\,\omega^2$ and $F = -Kr$. Usually this relation is used for vibration motions, but a generalization to rotating motions is possible. Analogously it can be formally considered and generalized (for gravitational, electromagnetic and other fields) the equation $F/S = E\,\delta l/l$, where F/S is the force per unit area (the surface density of forces), δl/l is the length change (in corresponding space) or the change of a testing length and E is the modulus of elasticity. (It is obvious that similar considerations can be formally defined and implemented under certain assumptions, into the special theory of relativity also). Einstein’s eq. is (as is known): $\mathbf{G} = 8\pi\,\mathbf{T}$ (in geometrodynamics units, **G** is the Einstein tensor and **T** is the tensor of energy-momentum). In components, the Einstein equation is

$$R_{ik} - (1/2)\,R\,g_{ik} - \Lambda_E\,g_{ik} = (8\pi G/c^4)\,T_{ik}$$

R_{ik} is the Ricci tensor, g_{ik} -metric, R – the scalar curvature (invariant), G the gravitational constant, Λ_E -Einstein’s cosmological constant (Einstein, 1917) and c is the speed of light. Λ_E may be included in T_{ik} and interpreted as the density of mass and energy of the vacuum. Sometimes it is written $\varepsilon_v = (\Lambda c^2)(8\pi G)^{-1} + Q_F$, where ε_v is the density of energy of the vacuum and Q_F is the contribution of classical and quantum fields, see e.g. [30].

The tensor T_{ik} can be derived by means of the Lagrangian of mass and matter and the left side of the equation can be derived from the Lagrangian of geometry, see e.g. [9, 4]. Indices i, k = 1 to 4, often they are used indices μ, ν.

„ The rigidity“ (stiffness) of the space, which corresponds to this equation probably, is not yet determined and defined, but it can be determined. As the example of a curved space it is possible to consider the sheet of the paper, which is in the gravitational field and the stiffness of this paper will have an influence on the metric and curvature of this paper. From the theoretical point of view, the Seeliger constant can be also inserted into the Einstein and Friedman equations, for example as $\Lambda_S\,\mathbf{b}\,g$, where **b** is a suitable operator and g is the metric. Not only gravitation but also every force curves the space in which or on which affects and which is not absolutely rigid or which is not empty. The curvature of these spaces depends on the cross sections of these interactions (including gravitational interaction) with these spaces or with the mass and matter in these spaces.

The mass and energy of the physical vacuum and preexisting matter, which is in our physical space and which is not known, “visible” and observable till the present time, can be much larger than the mass and energy of the observed Universe, similarly as the mass of the Earth is much larger than the mass of all things that are and were created on its surface. For $\Lambda_E > 0$ the vacuum gives the energy into the Universe and for $\Lambda_E < 0$ vacuum takes in the energy from the Universe (takes back). Details of corresponding mechanisms will be explained in the future. Generally the vacuum can be considered as a reservoir and positive (or negative) source of energy (and the mass) of the Universe. The total cosmological constant $\Lambda = \Lambda_1 + \Lambda_2 + \Lambda_3$, where Λ_1 is a proper density of the mass and energy of the vacuum, Λ_2 is the positive density which is given into the Universe (accelerated expansion) and Λ_3 is the negative density which is taken back from the Universe and all these constant may have a dependence on (**r**,t). Laws of conservation must be generally considered for the Universe and vacuum as the whole. Gravitational charges are negative sources, it is not accidental and it must have some reasons and consequences. As was said nothing is accidental. Accident is “God” of sillies (pošetilých), as is known. Accidentallity usually expresses the unknowlidge of detailed mechanism, which can be for some people suitable, but not for all. Accidentallity can be considered in abstract, theoretical and approximate models (probability, statistics, quantum physics). Real world is not accidental (random) (náhodný). It can be consequently considered, that a part of energy is taken back from the Universe into the physical vacuum and space via these negative sources. It can be assumed that $\Lambda_1 \gg \Lambda_2 > |\Lambda_3|$ in the mean at present and also $\rho(\Lambda_1) \gg \rho_{universe}$ where ρ are corresponding densities. Negative Λ_3 probably corresponds to Λ which was derived in the previous part of this article. “The strength” of interactions of the vacuum with our universe or Metagalaxy is dependent on the values of interaction constants and on the values of the cross sections of these interactions.

Einstein’s equations can be taken across on Friedman’s (1922) (or Friedman-Lemaitre) equations; see for example [29, 30, 36, 44] and some considerations are also in [45]. (The Friedman eq. is only the first eq.).

$$H^2 \equiv (dR/dt\ R^{-1})^2 = a\ 8\pi\rho G/3 - kc^2/R^2 + \Lambda_E\ c^2/3 = H_1{}^2 + H_\Lambda{}^2 \qquad (1)$$

$$(d^2R/dt^2) / R = -4\pi G(\rho + 3p/c^2)/3 + \Lambda_E\ c^2/3 \qquad (2)$$

Into the eq. (1) it was added the parameter a, usually a = 1, R (t) is the scale factor for the distance in comoving coordinates, p is the isotropic pressure, ρ is the total mass-energy density, k is the curvature (after rescaling k = 0 for spatially flat geometry, and 1 for a closed spherical universe, and –1 for a hyperbolic universe with the shape of the saddle in two-dimensional analogy, as is known). Here it is necessary to say, that an object or its form is seen as the whole only in the space with the dimension at least of 1 greater than is the dimension of the object, as it follows from visual imagination. A simple colored and finite N dimensional object must be in N dimensional space observed from various sides and directions to determine it. H is the Hubble parameter and $q = -R^{-1}H^{-2}d^2R/dt^2$ is the deceleration parameter.

The cosmological constant may be considered as an independent source term, $\Lambda_E = K_1\ \pi\rho_\Lambda G/c^2 = C\Lambda$ where K_1 and C are parameters that can be time and space dependent and Λ can be the “total” cosmological constant. „Effective“ Λ_E in Einstein’s equations, which has influence on the Universe at present, can be much smaller than Λ, as is known. If Λ_E is positive, then Λ_E and the vacuum give the energy into the Universe, and if it is negative, then it draws and takes back the energy from the Universe, as was said. If $\Lambda_E = -8\pi\rho G/c^2$, then for k = 0 it is H = 0, as it is to see from the eq. (1), (for $\Lambda_E < -8\pi\rho G/c^2$ and k = 0, this first equation gives the imaginary solution for H and it is the question of how to interpret it, in fact it means the immediate change of the velocity to the negative value and the decrease of the volume).

For $\Lambda_E \geq 4\pi G\rho c^{-2}$ (for p = 0) the acceleration of the expansion is positive, for $\Lambda_E < 4\pi G\rho c^{-2}$ the acceleration is negative and the expansion will cross after certain time into decreasing of the volume (as it is to see from the second equation). Seeliger`s constant is not explicitly included in the Einstein and Friedman equations, but it can be put into these equations and so it is possible to obtain and describe more general models, as was already said.

For a = 1/2, the eq. (1) is (in the main) equivalent to the Huyghens force (A. D. 1659), see [47], $F = mv^2/R$ for k = 0, $\Lambda_E = 0$ and $\rho = M/V = M\ ((4/3)\pi\ R^3)^{-1}$, V is a volume and R is considered as the distance, because $v = (GMR^{-1})^{1/2}$. For a = 1, the eq. (1) gives „2nd cosmic velocity“ $v = (2GMR^{-1})^{1/2}$. Similarly 3rd cosmic velocity. It is considered v = dR/dt. The eq. (2) gives directly (and it is „equivalent“) the Newton force (A. D. 1680), $m\ (d^2R/dt^2) = -GmMR^{-2}$ for p = 0, $\Lambda_E = 0$ and the mass m. It is clear that the Friedman eqs. can be used not only for the description of the dynamics of the observable universe, but also for the description of two Newton masses with the gravitational interaction for example. The Coriolis force (A. D. 1832) should be also derivable from the Einstein eqs. It is evident that both generalized Friedman-Lemaitre eqs. can formally give whole Newton gravitation under certain assumptions.

If it is written the mass and matter term on the left side of the equations (sources) and geometric terms are on the right side (geometry), (which means from the mathematical point of view that mass and matter is equivalent to geometry. This consideration can be also made with the Einstein eqs, see some considerations in „Conclusion“), then will be:

$$GM = (R\ (dR/dt)^2 + k\ c^2\ R - (1/3)\ \Lambda_E\ c^2\ R^3)/(2a), \text{ (a = 1 in the standard form of FE.)} \qquad (1a)$$

$$GM_1 = -R^2\ d^2R/dt^2 + (1/3)\ \Lambda_E\ c^2\ R^3,\ M_1 = M + 4\ \pi\ R^3\ p\ c^{-2} \qquad (2a)$$

The „stiffness“ of the space will be

$$K_1 = m((1/3)\ \Lambda\ c^2 - d^2R/dt^2\ /R) = GmM_1/R^3 \qquad (2b)$$

$$K = m(R^{-2}\ (dR/dt)^2 + k\ c^2\ R^{-2} - (1/3)\ \Lambda\ c^2)/(2a) = GmM/R^3 \qquad (1b)$$

$K = K_1$ for p = 0. (m is some „test“ mass, it is possible to consider m =1 kg, $M = (4/3)\ \pi R^3\ \rho$)

(The volume is considered for flat space.)

The Friedman equations with the Seeliger constant can be written in the form:

$$(dR/dt)^2 = 2a\ G\ M\ R^{-1} \exp(-\Lambda_S^{1/2} R) - k\ c^2 + (1/3)\ \Lambda\ c^2 R^2 \qquad (1c)$$

$$(d^2R/dt^2) = -\ G\ M\ (R^{-2} + \Lambda_S^{1/2}/R) \exp(-\Lambda_S^{1/2} R) - 4\ \pi\ G\ pR\ c^{-2} + (1/3)\ \Lambda\ c^2 R \qquad (2c)$$

or $$(d^2R/dt^2) = -\ G\ M_1\ (R^{-2} + \Lambda_{S,1}^{1/2}/R) \exp(-\Lambda_{S,1}^{1/2} R) + (1/3)\ \Lambda\ c^2 R \qquad (2c')$$

It was modified the „potential“ GM/R in the first and corresponding „force“ GM/R^2 in the second eq., and the „force“ for the modified potential GM_1/R in the second eq. For p = 0 it is $\Lambda_{S,1} = \Lambda_S$. Theoretically, some other alternatives can be also considered. It is also considered $\Lambda_S \geq 0$, for $\Lambda_S < 0$ as it was considered in the first part of this paper, it gives imaginary values and the interpretation must be other. If it is written, FE = A (EE), where A is an algorithm for the derivation of the Friedman eqs. from the Einstein eq. then the stiffness K and the Seeliger constant Λ_S can be inserted into the Einstein eq. formally by EE = A^{-1} (FE) where FE are the modified Friedmann eqs. and EE is the modified Einstein eq. If E (K, Λ, Λ_S) g (K, Λ, Λ_S) = **T** (K, Λ, Λ_S) are modified Einstein eqs. then usual Einstein equations correspond to $\Lambda_S = 0$, i.e. **E** (Λ, 0) g (Λ, 0) = **T** (Λ, 0).

Usually it is argumented that $\Lambda_S = 0$, because the absorption of the gravitation is zero, however the absorption of electromagnetic fields and particle fields (neutrons and others) can be significant, which means that also the absorption of gravitational equivalents can be significant, i.e. $\Lambda_S \neq 0$. The absorption of gravitation is given by the cross sections for the absorption of the gravitons. It is also possible to modify corresponding sources, i.e. sources in T_{ik} or in ρ.

If these Friedman equations are used for example for simple quantum systems with discrete values $R_n = n^2\ \hbar^2 /(GM^2\ m)$ where R_n are real or „probabilistic“ discrete values, then it is possible for these systems to consider corresponding „quantum“ Friedman eqs. with $R = R_n$ and analogously the Einstein eqs. EE = A^{-1} (FE). Connection with quantum differential geometry and quantum physics will be probably described in other paper. Another way, which is usually used in quantum gravitation, is presented for example in [30].

2.15. Λ ESTIMATION

The value of Λ (or Λ_E), which has influence on the expansion at „present“, can be estimated. The force F between two objects M and m is (as usually under certain assumptions), see [30]

$$F = F_G + F_\Lambda = -\ G\ M\ m\ r^{-2} + (1/3)\ \Lambda_E\ c^2\ m\ r\ , \text{ i.e. } d^2r/dt^2 = -\ G\ M\ r^{-2} + (1/3)\ c^2\ \Lambda_E\ r,$$

which is in the main the second Friedman eq. (p = 0).

The potential that corresponds to this force is the combination of Newton’s attractive potential and the potential of a linear harmonic oscillator with $k = -\Lambda_E\ c^2 /3$. The distance is r.
(The potential $\varphi = -\ (G\ M/r + \Lambda_E\ c^2\ r^2 /6)$)

Then the component of the force and acceleration, which corresponds to Λ_E is

$$F_\Lambda = m\ a_\Lambda = (1/3)\ c^2\ \Lambda_E\ m\ r \qquad \Rightarrow \qquad a_\Lambda = (1/3)\ c^2\ \Lambda_E\ r$$

The change of the velocity, as the result of this acceleration in the time Δt is (again under certain assumptions):

$$\Delta v = \int a(t)dt = (1/3)\ c^2\ \Lambda_E \int r(t)\ dt = (1/3)\ c^2\ \Lambda_E\ \langle r\rangle\ \Delta t \qquad \Rightarrow \qquad \Lambda_E = 3\ \Delta v\ (c^2 \langle r\rangle\ \Delta t)^{-1}$$

$\langle r\rangle$ - is the mean distance r in this time interval

These relations follow also from the Taylor series and the Friedman equations, see below.
If it is assumed that some part of the change of the distance between the Moon and the Earth (and all another distances) is the consequence of this force and it is for example 1 cm/year or 1 mm/year, see [41], then the corresponding „present“ value is Λ_E = 8.72 E-47 cm^{-2} or Λ_E = 8.72 E-48 cm^{-2}. Similarly, for the Sun and the Earth and other planets, see the following table. This is a „mean“ or „background“ cosmological constant (in our solar system), the cosmological constant in some „strong“ fields and in other parts of the universe need not be the same and it can vary in time. For example, if this value is combined with the cosmological constant of the hydrogen atom Λ_H , as it was determined above, then will be $\Lambda = \Lambda_E + \Lambda_H$.

In the literature, see e.g. [4, 30, 36] it is stated (already quite long) the approximate value $\Lambda_E \leq 1$ E-53 cm^{-2} or $\leq$ E-55 cm^{-2}, but it is possible that this value may be higher. The value in [36, (2010)] is approximately $\Lambda = \Omega_\Lambda 3H_0^2/c^2 = 0.74/5.5E51m^2 = 1.345454E\text{-}56\ cm^{-2}$.

For example, the numerical value of „the gravitational and cosmological" force between the Sun and the Earth is then: $F_G = 3.54E22\ kgm/s^2$, $F_\Lambda = 2.35E10\ kgm/s^2$, $(F_G >> F_\Lambda)$

The value of the acceleration is: $a_G = 5.93E\text{-}3\ m/s^2$, $a_\Lambda = 3.91E\text{-}15\ m/s^2$,

For the Sun and the „center" of the Galaxy (under the approximation of a „central" mass) it is:

$F_G = 3.90E20\ kgm/\ s^2$, $F_\Lambda = 1.28E25\ kgm/\ s^2$, $(F_G << F_\Lambda)$ $(\Lambda = 8.72E\text{-}47cm^{-2})$

$a_G = 1.96E\text{-}10\ m/s^2$, $a_\Lambda = 6.45E\text{-}6\ m/s^2$

For $\Lambda = 1.345E\text{-}56\ cm^{-2}$ is $F_\Lambda = 2.128E15\ kgms^{-2}$.

Corresponding „spirals" of the motion, see [41], for a circular trajectory will be approximately

$(x, y) = r\ (\cos \omega t, \sin \omega t)$, $r = r_0\ (1 + Ht) + 1/2\ a_\Lambda\ t^2 + O_1$,

or alternatively $r = r_0 + r\ Ht + 1/2\ a_\Lambda\ t^2 + O_2$ $(a_\Lambda = (1/3)\ c^2\ \Lambda_E\ r)$

For an elliptical trajectory will be valid this approximate equation:

$(x, y) = (a \cos \omega t, b \sin \omega t)$, $(a, b) = (a_0\ , b_0)\ (1 + Ht) + 1/2\ a_\Lambda\ t^2 + O_1$,

or alternatively $(a, b) = (a_0\ , b_0) + (a, b)\ Ht + 1/2\ a_\Lambda\ t^2 + O_2$ $(a_\Lambda = (a,b)\ (1/3)\ c^2\ \Lambda_E\ )$

(O_1 and O_2 are remains and further terms, for example from another forces).

It is used the parabolic approximation, which is sufficient for the purposes of this article and for shorter time intervals (hundreds and thousands of years). In the cubic approximation will be also the term $a_3\ t^3$. These eqs. can be used for planetary systems and similar relations will be hold also in quantum physics, for example, for Bohr`s radius and other values $r_N = r_1\ N^2$, cross-sections, etc. These questions and corresponding relations can be discussed in other article. It is to see, that the values of the acceleration are very small for these macro and mega systems, which means that these objects are „practically free" even though they move in circular orbits. (The forces are great because the masses are great). A more detailed alternative explanation of why this is so, can be done in another article. Sometimes it is written, see e.g. [9], that these bound systems do not expand, but this consideration is not right probably.

If it is used the Taylor or Mac Laurin series [4,6] then it is (**V** is the shift operator and $D = d/dt$, $D^n = d^n/dt^n$)

$$r(t + \alpha) = \mathbf{V}\ r(t) = e^{\alpha D}\ r(t) = (1 + \alpha D + 2!^{-1}\ \alpha^2\ D^2 + ...)\ r(t)$$

If it is also used physical "terminology" and the Friedman-Lemaitre equations [29,30,36] then it is (for G, c and π const, generally π can be considered as variable, this can be described in other paper, π = const is only special case)

$$R(t_0 + t) \equiv R(t_0) + vt + \tfrac{1}{2}\ at^2 + ... + O(t) \equiv R(t_0)\{1 + (1/3)\ t\ (8\pi\ G\rho + \Lambda c^2 - 3\ k\ c^2\ R^{-2}(t_0))^{1/2} +$$
$$+ (1/6)\ t^2\ (\Lambda c^2 - 4\pi\ G(\rho + 3pc^{-2}) + (1/18)\ t^3\ [R^{-1}(t_0)(\Lambda c^2 - 4\pi\ G(\rho + 3pc^{-2})dR(t_0)/dt +$$
$$+ (c^2 d\Lambda/dt - 4\pi G(d\rho/dt + 3c^{-2}\ dp/dt)]\} + O(t)$$

Similarly can be written further terms in this series.

O (t) is remain, $(O_n\ (t) = (t^{n+1}\ /(n+1)!)\ d^{n+1}\ R\ (\theta t)/dt^{n+1})$, where $0 < \theta < 1$
$a = d^2R/dt^2 = a_\Lambda + a_\rho + a_p = 3^{-1}\ \Lambda(t_0)\ c^2\ R(t_0) - (4/3)\pi G\rho(t_0)R(t_0) - 4\pi Gp(t_0)R(t_0)$

$v = dR/dt = HR$
$H = +- (8\pi G\rho/3 + \Lambda c^2/3 – kc^2R^{-2})^{1/2}$
The sign + is realized in our Metagalaxy or observable Universe, the sign – can be realized in another Metagalaxy or in our Metagalaxy when it will be in some time $8\pi G\rho/3 + \Lambda c^2/3 – kc^2R^{-2} = 0$. The next epoch can have negative H.
If they are used the numerical values from [36 (2010)], i.e. H_0 = 23.331E-19 s^{-1}, Λ = 1.345E-56 cm^{-2} and
$\rho_m + \rho_b + \rho_\gamma + \rho_\nu \cong (0.26+0.044)\ \rho_c$, $\rho_c = 3H_0^2/(8\pi G)$, then it is

$R(t_0 + t)/R(t_0)$ = 1 + 23.33E-19[s^{-1}] t + (1/2) t^2 (4.035E-34 - 8.273E-37)[s^{-2}] + O(t)
$R(t_0$ + 1 year)/$R(t_0) \cong$ 1 + 83.988E-12 + 20.11E-20 + O(t)
$R(t_0$ + 1 year) - $R(t_0) \cong$ 8.4E-12 $R(t_0)$
For R = 3.844E10 cm (the distance between the Earth and Moon) it gives R (t_0 + 1 year) – R (t_0) ≅ 32.29E-2 cm = 0.32 cm and analogously for the distance between the Sun and Earth, the Sun and the center of the Galaxy and other distances. As can be easily verified, the corresponding distances for “table” values of a_Λ, a_ρ, a_p are very small.

2.16. G AND Λ IN „STANDARD“ QUANTUM EQUATIONS **

For every object of the micro world and macro world it is possible to write the equations of quantum physics (as was said and as is known) because the operators Δ and ∇ and also the potentials are valid for arbitrary distances, (the potential can be zero on certain distance).

For macroscopic objects, these solutions give classical solutions in the limit. It is therefore possible to write the Schrödinger and other quantum equations for planetary and galactic systems, atoms, pendulums and so on and corresponding expansion of the space and objects that are in this space, will have an influence on all these systems and it will be to change some quantities which are in these equations or which are derived from them. These equations and quantities will be dependent on G and Λ, and also on the Seeliger constant.

The Seeliger (or Laplace, 1805) constant is important constant in theoretical considerations and it expresses the „absorption“ of gravitation, in the limit is zero. Usually it is argumented that the absorption of gravitation does not exist, but from the theoretical point of view it may be given into Einstein and Friedman’s equations, as already was said. Its value will be given by the cross sections for interactions (inelastic scattering) of assumed gravitons (Blochincev, 1934) (which create the gravitation field) with the physical vacuum or with the „real“ space and with another „mass and matter“ (see also considerations in the first part of this paper). In theoretical models this constant will have an influence also on other carriers of interactions, gluons, photons or gamma and on „weak“ bosons and its value can be different for these gauge bosons. (In some equations Λ is written without index E because in fields and in the vicinity of sources can be and will be $\Lambda \neq \Lambda_E$).

The Schrödinger equation with the gravitational and cosmological constant is

$$i \hbar\ \partial/\partial t\ \Psi = (- \hbar^2 (2m)^{-1} \Delta + V(\mathbf{r},t) + V_G(\mathbf{r},t) + V_\Lambda(\mathbf{r},t))\ \Psi$$

where V is usual potential energy (electromagnetic, oscillator and other forms of the potential energy), V_G is gravitational potential energy and $V_\Lambda = - (1/6)\ c^2\ \Lambda\ m\ r^2$. The time-independent equation is

$$(- \hbar^2 (2m)^{-1} \Delta + V(\mathbf{r}) + V_G(\mathbf{r}) + V_\Lambda(\mathbf{r}))\ \Phi = E\ \Phi$$

or

$$(- \hbar^2 (2m)^{-1} \Delta + V(\mathbf{r}) + V_G(\mathbf{r}) + V_\Lambda(\mathbf{r}))\ \Phi_n = E_n\ \Phi_n$$

for characteristic numbers and functions.

If it is considered the central oscillator potential, $V(r) = (1/2)\ k\ r^2$, where $\omega_0 = (k/m)^{1/2}$ and $V_G(\mathbf{r}) = 0$ then the energy levels will be given with $V_\Lambda = - \Lambda\ r^2\ c^2\ m/6$ by

$$E_n = \hbar\ \omega\ (n + 3/2), \qquad \omega = [(k/m - (1/3)\ c^2\ \Lambda)]^{1/2} = (\omega_0^2 - (1/3)\ c^2\ \Lambda)^{1/2}$$

The matrix element is $\langle f\ |V + V_G + V_\Lambda|\ i\rangle$ and similarly invariant amplitudes and various cross sections.

In the theoretical models in which all potential terms (gravitational, electromagnetic, strong and other) and charges will be comparable, the cross sections for various processes (including production of new particles in gravitational interactions and for the annihilation and creation of particles) will be also comparable.

If it considered the usual equation: $(\Delta - \Lambda_S)\varphi + c^2 \Lambda - 4\pi\rho G = 0$
and the potential in the form : $\varphi = - GMr^{-1} \exp(- \Lambda_S^{1/2} r) – c^2 \Lambda r^2 /6$

then the „standard“ force will be

$$F = - m \, \mathrm{grad}\, \varphi = - GMm (r^{-2} + \Lambda_S^{1/2} /r) \exp (- \Lambda_S^{1/2} r) + mc^2 \Lambda r/3$$

and the potential energy will be $V = m\varphi$.

The Schroedinger eq. will be

$$i \hbar \, \partial\Psi/\partial t = (- \hbar^2 /2m \, \Delta - GMmr^{-1} \exp(- \Lambda_S^{1/2} r) – c^2 \Lambda mr^2 /6 – (4\pi\varepsilon_0)^{-1} Ze^2 /r + 1/2 \, m \, \omega^2 r^2) \Psi$$

and similarly for the Dirac or Klein-Gordon equation. (The Coulomb and harmonic oscillator potential were added). These equations can be used for example for atomic phenomena and also for „point“ planetary and Sun masses, but as is to see, the simple Schr. eq. can give theoretical possibilities and values that are beyond usual physical values and corresponding numbers belong rather into mathematics than into physics. For example for the point Sun - Earth system (which is only abstract) and other similar systems with the gravitational potential is R_1 = 2.348E-137 m and E_1 = -16.89E180 J. In fact, it is necessary to modify simple equations, potentials or to limit the space distances or use other models. For high densities of gravitational sources and for the high density of the gravitational energy can also play a role the gravitational Lamb shift and the gravitational polarization of the vacuum. This question can be described in another paper.

If it is considered the „Seeliger absorption“ of the electromagnetic field, then the Hamiltonian will be

$$\mathbf{H} = - \hbar^2 /2m \, \Delta - GMmr^{-1} \exp(- \Lambda_S^{1/2} r) – c^2 \Lambda mr^2 /6 – (4\pi\varepsilon_0)^{-1} Ze^2 r^{-1} \exp(- \Lambda_1^{1/2} r) + 1/2 \, m \, \omega^2 r^2$$

where Λ_1 is the „Seeliger“ constant for the electromagnetic field.
The special case $\Lambda_1 = \Lambda_S = \Lambda = \omega = G = 0$, $Z = 1$ gives usual atom H. (Λ_S may be different from zero, it is enough $G = 0$). From the mathematical point of view it is the „standard“ mathematical eq., which can be physically interpreted.

The Dirac eq. gives

$$[\mathbf{E} - e\varphi - \boldsymbol{\alpha} \, (c \, \mathbf{p} – e \, \mathbf{A}) - \beta \, mc^2] \, \Psi = 0, \quad \mathbf{E} = i \hbar \, \partial/\partial t, \quad \mathbf{p} = - i \hbar \, \mathrm{grad}$$

For $\mathbf{A} = 0$, and generalized $e\varphi = V$, the Hamiltonian will be

$$\mathbf{H} = V + c \, \boldsymbol{\alpha} \, \mathbf{p} + \beta \, mc^2 = c \, \boldsymbol{\alpha} \, \mathbf{p} + \beta \, mc^2 - GMmr^{-1} \exp (- \Lambda_S^{1/2} r) – c^2 \Lambda mr^2 /6 – (4\pi\varepsilon_0)^{-1} Ze^2 r^{-1} \exp (- \Lambda_1^{1/2} r) + 1/2 \, m \, \omega^2 r^2$$

where

$$V = - GMmr^{-1} \exp (- \Lambda_S^{1/2} r) – c^2 \Lambda mr^2 /6 – (4\pi\varepsilon_0)^{-1} Ze^2 r^{-1} \exp (- \Lambda_1^{1/2} r) + 1/2 \, m \, \omega^2 r^2.$$

e**A** may be generalized for so called vector potentials of the gravitational field, so as it is sometimes considered. In some cases, these vector potentials should be derived from the Einstein eq. (analogously as the gravitational potential) and the theory will have „higher“ symmetry as it is sometimes written.

The K. G. eq. will be $(\mathbf{E}^2 – H^2) \, \Psi = 0$.

In all these equations, V can be further generalized an written in the form

$$V = \Sigma_i \, (-G_i \, Q_{1i} \, Q_{2i} \, r^{-1} \exp(- \Lambda_{Si}^{1/2} r) – c^2 \, \Lambda_i \, Q_{1i} \, r^2 /6 + + 1/2 \, Q_{1i} \, \omega_i^2 \, r^2$$

where G_i are interaction constants, Q_{ji} are generalized „charges“, sources or masses and Λ_{Si} , Λ_i is „a set of the generalized Seeliger and cosmological constants“, because from the theoretical point of view for each charge or source may be different the Seeliger and also „cosmological“ constant. This means that the physical vacuum or „real space“ can have different effects on various kinds of charges, similarly as it is in the case of inertia:

$$(F_1, F_2) = (k_1\, m\, a\, ,\ k_2\, q\, a)\ ,\quad (E_1, E_2) = ½\,(k_1\, m\, v^2,\ k_2\, q\, v^2)$$

$k_1 = 1$ for the mass m and $k_2 = 0$ for the electric charge q;
F is the force and a is the acceleration, which means that the inertia for the charge q is zero, it is nonzero for the quantity m (gravitational mass) in the standard Newton law, see [50]. Analogously for the kinetic energy E (nonrelativistic approximation) and other similar quantities.

In quantum physics it is considered the fine-structure constant $\alpha_E = e^2/4\pi\varepsilon_0\, \hbar c = 1/137.036$. Similarly for quantum gravitational systems can be used, (as is known), „the gravitational fine-structure constant“ $\alpha_G = Gm^2/\hbar c = (m/M_P)^2$, which gives for example for the „unit“ mass m = 1kg the value α_G = 2.111E15 and for m_e = 9.109E-31 kg it is α_G = 1.752E-45. For m_g = 1.8593 E-9 kg it is $\alpha_G = \alpha_E$. For this value, the electrostatic force between the electron and positron is the same as the gravitational force between two m_g masses as is known.

The Planck mass is $M_P = (\hbar c/G)^{1/2}$ = 2.1767E-8 kg.

2.17. TABLE OF SOME APPROXIMATE PARAMETERS OF THE SUN SYSTEM

The following table contains some approximate parameters of the solar system. In this table can be inserted some further values (see another paper). Quantum numbers are not stated usually, they are stated as the matter of interest. Analogously, the quantum numbers of the orbital and spin moment can be stated. Usual physical relations and data were used, see [42, 43] and also the approximation $l\,(l+1) = l^2$ for $l >> 1$. The meaning of the quantities is clear. It is also possible to state the total values of additive physical quantities, but they are determined mainly of the values for the Sun, as seen. The spin-orbit energy is calculated by the relation, see [45, 3], which has the form $V_{LS} = (2\, m^2 c^2\, r)^{-1}\, (dV/dr)\, \mathbf{LS}$. **L** and **S** is the orbital and spin momentum and V(r) is the potential energy of the central gravitational field. For some objects are not known some real values (for example spin moment) and so only model values are used. Generally, every quantity $Q_i = k_i\, .q_i$, where $k_i = k_i\,(\mathbf{r},t)$ are correction coefficients, $k_i = 1$ is special case.

Quantity:	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
Object:												
Mercury	5,79E+07	3,30E+23	4,79E+04	1,58E+28	3,78E+32	1,51E+02	1,31E+22	4,99E+08	3,96E-02	1,51E-15	8,68E+72	8,72E-47
Venus	1,08E+08	4,87E+24	3,50E+04	1,71E+29	2,99E+33	2,82E+02	5,52E+22	1,38E+10	1,13E-02	2,83E-15	1,75E+74	8,72E-47
Earth	1,50E+08	5,97E+24	2,98E+04	1,78E+29	2,65E+33	3,89E+02	3,54E+22	2,33E+10	5,93E-03	3,91E-15	2,52E+74	8,72E-47
Mars	2,28E+08	6,42E+23	2,41E+04	1,55E+28	1,87E+32	5,93E+02	1,64E+21	3,82E+09	2,56E-03	5,95E-15	3,35E+73	8,72E-47
Jupiter	7,78E+08	1,90E+27	1,31E+04	2,49E+31	1,63E+35	2,02E+03	4,16E+23	3,86E+13	2,19E-04	2,03E-14	1,83E+77	8,72E-47
Saturn	1,43E+09	5,69E+26	9,67E+03	5,50E+30	2,66E+34	3,71E+03	3,71E+22	2,12E+13	6,52E-05	3,73E-14	7,43E+76	8,72E-47
Uran	2,87E+09	8,68E+25	6,84E+03	5,94E+29	2,03E+33	7,47E+03	1,40E+21	6,51E+12	1,61E-05	7,50E-14	1,61E+76	8,72E-47
Neptune	4,50E+09	1,02E+26	5,48E+03	5,59E+29	1,53E+33	1,17E+04	6,69E+20	1,20E+13	6,56E-06	1,18E-13	2,36E+76	8,72E-47
Pluto	5,91E+09	1,30E+22	4,75E+03	6,18E+25	1,47E+29	1,54E+04	4,95E+16	2,01E+09	3,81E-06	1,54E-13	3,45E+72	8,72E-47
Moon	3,84E+05	7,35E+22	1,02E+03	7,52E+25	3,85E+28	1,00E+00	1,98E+20	7,38E+05	2,70E-03	1,00E-17	2,73E+68	8,72E-47
Sun	2,47E+17	1,99E+30	2,20E+05	4,38E+35	4,81E+40	6,42E+11	3,90E+20	1,28E+25	1,96E-10	6,45E-06	1,02E+90	8,72E-47

Quantity:	13.	14.	15.	16.	17.	18.	19.	20.	21.
Object:									
Mercury	9,15E+38	8,68E+72	4,88E+06	7,86E+35	9,75E+29	9,24E+63	58,650	1,03E+16	1,24E-06
Venus	1,85E+40	1,75E+74	1,21E+07	7,13E+37	-2,13E+31	-2,02E+65	-243,020	4,71E+16	-2,99E-07
Earth	2,66E+40	2,52E+74	1,28E+07	9,72E+37	7,08E+33	6,71E+67	0,997	6,96E+18	7,29E-05
Mars	3,53E+39	3,35E+73	6,79E+06	2,96E+36	2,10E+32	1,99E+66	1,026	7,20E+16	7,08E-05
Jupiter	1,94E+43	1,84E+77	1,43E+08	3,88E+42	6,82E+38	6,47E+72	0,414	1,09E+22	1,76E-04

Saturn	7,85E+42	7,44E+76	1,21E+08	8,27E+41	1,35E+38	1,28E+72	0,444	4,75E+20	1,64E-04
Uranus	1,70E+42	1,62E+76	5,11E+07	2,27E+40	-2,30E+36	-2,18E+70	-0,718	1,41E+18	-1,01E-04
Neptune	2,51E+42	2,38E+76	4,95E+07	2,50E+40	2,71E+36	2,57E+70	0,671	5,42E+17	1,08E-04
Pluto	3,65E+38	3,46E+72	2,30E+06	6,85E+33	-7,80E+28	-7,39E+62	-6,387	7,84E+09	-1,14E-05
Moon	2,89E+34	2,74E+68	3,48E+06	8,88E+34	2,36E+29	2,24E+63	27,320	3,63E+12	2,66E-06
Sun	1,08E+56	1,02E+90	1,39E+09	3,85E+47	1,12E+42	1,06E+76	25,000	2,69E+20	2,91E-06

Quantity:	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**
Object:								
Mercury	0,24	8,25717E-07	7,00	0,01	1,03E+16	1,0000	6,04E+23	4,86E+01
Venus	0,62	3,23574E-07	3,40	177,30	-4,71E+16	-0,9989	3,19E+24	9,09E+01
Earth	1,00	1,98994E-07	0,00	23,45	6,39E+18	0,9174	2,58E+29	1,26E+02
Mars	1,88	1,05794E-07	1,85	25,19	6,52E+16	0,9049	7,44E+27	1,91E+02
Jupiter	11,86	1,67789E-08	1,31	3,12	1,09E+22	0,9985	5,99E+34	6,54E+02
Saturn	29,45	6,75714E-09	2,48	26,73	4,24E+20	0,8931	1,11E+34	1,20E+03
Uranus	84,02	2,36846E-09	0,77	97,86	-1,92E+17	-0,1368	1,16E+32	2,41E+03
Neptune	164,79	1,20758E-09	1,77	28,30	4,77E+17	0,8805	1,47E+32	3,78E+03
Pluto	247,92	8,02669E-10	17,14	122,50	-4,21E+09	-0,5373	4,44E+23	4,96E+03
Moon	27,32d	2,66792E-06	5,10	6,70	3,60E+12	0,9932	3,14E+23	3,23E-01
Sun	2,1494E+08	9,2583E-16		7,30	2,67E+20	0,9919	1,63E+36	2,07E+11

Quantity:

1. d - distance [km]
2. m – mass [kg]
3. v - orbital speed [m/s]
4. p - momentum [kgm/s]
5. E - kinetic energy [kg $(m/s)^2$]
6. v_Λ- speed of expansion as the consequence of Λ [cm/year]
7. F_{gr}- gravitational force [$kgms^{-2}$]
8. F_Λ- Λ force (cosmological force) [$kgms^{-2}$]
9. a_{gr}- gravitational acceleration [ms^{-2}]
10. a_Λ- Λ acceleration [ms^{-2}]
11. N - quantum number
12. Λ - cosmological constant [cm^{-2}]
13. L – orbital momentum (moment of momentum, [r,p] [$kgm^2 s^{-1}$]
14. l – orbital quantum number
15. D – diameter [m]
16. J – moment of inertia [kgm^2]
17. S – spin momentum [$kgm^2 s^{-1}$]
18. s – spin quantum number
19. T – rotational period [d]
20. E_{LS} - L-S energy (spin-orbital energy) [kg $(m/s)^2$]
21. Ω - rotational velocity [s^{-1}]
22. Orbital period, T_{orb} [sider. year]
23. Angular velocity, ω [1/s]
24. Orbital inclination, i [degree]
25. Axial tilt, j [degree]
26. LS energy with cosj [kg $(m/s)^2$]
27. cosj

28. Spin energy, $1/2\ J\omega^2$ [kg (m/s)^2]
29. v_H – speed of expansion as the consequence of H_0 [cm/year]

This table describes, in the main and approximately, the quantum and also classical state of the solar system and it can be in the future enlarged and specified. In principle, analogous tables and quantum states can be in the future approximately written for other known planetary systems and galaxies or part of the Metagalaxy. It will be difficult to compute these quantum states in whole, but it can be considered a main quantum system for example the galaxy and planetary systems as quantum subsystems (and so on). The center of the galaxy causes a small shift of the energy levels in these subsystems, i.e. also in solar system, approximately it will be $m_i\ \varphi(r_i,t)$, where m_i is the mass of the corresponding planet and φ is the gravitational potential given mainly by the mass of the center of the galaxy. In principle every macro object (and micro object) can have some quantum number or numbers, discrete for bound systems and continuous for free systems. The quantum description can be made from below or from above. Some further parameters and interesting consequences will be probably presented in another article. It is also evident that the number of circulations of the Sun around the center of our Galaxy is approximately 20.94 (under certain assumptions). It can serve as „a big and slow clock“. It is also quite clearly visible (with the help of other information), how much time is maximally left.

If $r \le [(2/5c^2)(GM)^{1/2}R_{pl}^2\omega_{rot.pl.}\cos j]^{2/3}$ then $E_{ls} \ge E_{kin}$. For "point" Sun and Mercury model it gives r = 0.4 m. It can be also compared the ratio E_{ls}/E_{kin} for the solar system and for atoms (the hydrogen atom). For a simple model of the central field with $E = (1/2)\ mv^2 + M^2/(2mr^2) + U(r) + E_{ls} + E_s$, where $M = |[\mathbf{r}, \mathbf{p}]| = mr^2 d\varphi/dt$ and (r,φ) are polar coordinates, it is $t = \int\{(2/m)[E-U(r)-E_{ls}-E_s]-M^2/(mr)^2\}^{-1/2}dr + t_0$,
$\varphi = \int Mr^{-2}\{2m[E-U(r)-E_{ls}-E_s]-(M/r)^2\}^{-1/2}dr + \varphi_0$ and $\Delta\varphi = 2\int_{r,min}^{r,max} Mr^{-2}\{2m[E-U(r)-E_{ls}-E_s]-(M/r)^2\}^{-1/2}$, see e.g. [9]. It is possible that this spin-orbital energy was not up to now considered.

3. <u>CONCLUSION</u>

Physics and mathematics are abstract sciences, which means that they search and investigate properties and characteristics that are common for more things or phenomena and which are independent on the area of the use. Consider the following algebraic scheme:

$$1 \leftrightarrow 2 \leftrightarrow 3 \leftrightarrow 4 \leftrightarrow \ldots\ldots\ldots\ldots \quad \ldots\ldots\ldots\ldots \leftrightarrow N \leftrightarrow N+1$$

1, 2, 3, 4, ..., N, N+1 are for example sets of fields

A_i operators between these sets, i.e. $(i+1) = A_i\ i$

A_i^{-1} inversion operators (if they exist)

It is evident that under certain assumptions, phenomena and processes (mathematical, physical, chemical, economic, biological, etc.) that are described by the fields 1 can be „equivalently“ described by any other set of the fields i (i = 1, 2, ... , N+1), i.e., from the mathematical point of view, there are classes of equivalency (or similarity) of these fields, phenomena and processes.

In physics, it is often used the following approximate scheme "Sch":

$$\begin{array}{lllll} & & H(\varphi) \leftrightarrow \Psi \leftrightarrow |\Psi|^2 \leftrightarrow (\rho,\mathbf{j}) \leftrightarrow \langle f|S_1|i\rangle \leftrightarrow M \leftrightarrow \sigma \leftrightarrow R \\ & & \updownarrow \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \updownarrow \\ Sch \equiv \{\varphi \leftrightarrow & (I + aID + bDI + D)\varphi \leftrightarrow & L(\varphi) \leftrightarrow S(\varphi) \leftrightarrow P(\varphi) \leftrightarrow \Sigma F(\varphi) \leftrightarrow M \leftrightarrow \sigma \leftrightarrow R\ \} \\ & & \updownarrow \\ & & \mathbf{T}(\varphi) \leftrightarrow \mathbf{G}(g) \leftrightarrow g(\varphi) \end{array}$$

φ - is a set of fields (classical or quantum fields)
I, D – are integral and differential operators, a and b are coefficients or functions or further operators
L –Lagrangians
H - Hamiltonians

S - actions
P - propagators
Σ (F) - sums of the Feynman diagrams
T - tensors of energy-momentum
G - Einstein's tensor
g - metrics of space-times (determined for example by „the Einstein operator" (from the Einstein equations))
σ - cross sections
R - reaction rates (for example in the form of a vector)
M - invariant amplitudes
ψ - wave function
S_1 - S matrix (or an "evolution" operator)
$j^\mu = (\rho,\mathbf{j})$ **-** density of probability and flux or current density
|i> and |f> - initial and final state
Etc.

If exist inversion mappings $(I + aID + bDI + D)^{-1}$, L^{-1}, and so on, then it is possible to go from the right to the left and vice versa. This means that sets of fields and phenomena which these fields describe, can be transformed and „equivalently" described for example by the metric of some spaces or space-times and also by corresponding propagators, the Feynman diagrams, amplitudes, reaction rates and cross sections, etc. and vice versa the metric of spaces and space-times can be transformed on the propagators, the Feynman diagrams, cross sections, amplitudes, reaction rates, fields and on field equations. It is also evident, from this scheme, that it is possible "equivalently" to use enough wide set of operators (more or less arbitrary), i.e. not only currently used "basic" operators (including the Einstein operator) and propagators and thus create new theories that can be "calibrated" by the old theories or fields, analogously as it is made the calibration of instruments or devices (by means of standards) that are used for measurements. This basic scheme can be changed in individual parts and also as the whole. If some scheme "Sch" will describe physics or other set of things, phenomena and processes then also scheme A_i (Sch) ≡ Sch (i) can be under certain assumptions an equivalent description of these things because Sch = A_i^{-1} (Sch (i)), if there is inversion mapping A_i^{-1}. From the mathematical point of view, classes of equivalency or "similarity" will be created, but this "basic" scheme "Sch" is and always will be very important and probably will be always and for ever and for all ages and at all times developed and improved. Houses, villages, towns, cities and capitals can be built otherwise, only some basic principles, functions and properties must be valid and preserved, the present appearance, placement and another things are given and determined by historical and other circumstances. Similarly and analogously, physics, mathematics and another sciences can be built up and created otherwise, (again only some basic principles, functions and properties must be valid and preserved); the present state is mainly given and determined by historical and other circumstances. All physical quantities, observables, variables and constants (classical, quantum, relativistic and nonrelativistic) and many of mathematical and other quantities belong into this scheme, which means from the algebraic point of view, that under certain assumptions, for arbitrary two quantities **a** and **b** (from this scheme) including constants can be written the relation **a** = f (**b**).

For example, the prices of shares on the exchange stocks (Dow Jones index and others) can be equivalently described by propagators and Feynmann diagrams, see [20], cross sections, reaction rates etc. or by the metric of "curved" (or flat) economic space-times (e.g. by using the "Einstein" operator ("ad absurdum") or any other "arbitrary" operator or by using some special and suitable economic operators). Similarly, biological fields can be transformed and described by the metric of "biological" spaces and space-times or by means of the corresponding propagators, Feynmann diagrams, cross sections, etc. and vice versa. These quantities must be suitable defined. Particle and nuclear fields (i.e. not only gravitational fields) can be also transformed and converted under certain assumptions, on the metric of corresponding space-times and vice versa. The metric of gravitational and Einstein's spaces can be transformed on the propagators and sums of Feynmann diagrams, amplitudes, reaction rates and cross sections. For example, (formally) if $g = GTLD\varphi$ and $R = r\sigma M\Sigma FPSLD\varphi$ then $\varphi = (GTLD)^{-1}g = (r\sigma M\Sigma FPSLD)^{-1}R$, $g = GTLD(r\sigma M\Sigma FPLSD)^{-1}R$, $R = r\sigma M\Sigma FPSLD(GTLD)^{-1}g$ and so on. Similarly can be considered for example the relation g = g (ψ) and ψ = ψ(g). For quantum fields (or for creation operators) φ^+ can be suitable defined and considered some „quantum" (or „creation") metrics $g^+ = gTLD\varphi^+$, which give (after the application on the „vacuum or basic state (or space) |0> ") the corresponding metrics and spaces. Analogously for the annihilation operators. „Arbitrary" abstract, mathematical or geometric model can be materialized and quantized and vice versa the arbitrary "material" or quantum model can be geometrized and abstracted. Also the "constants " (for example Planck's, gravitational, etc.) can be "arbitrary", the models with currently used constants are only special cases of general models.

In mathematics (see e.g. Wikipedia and other references or web sources), it is sometimes used the equation $\partial g/\partial t = k R (g)$, (Hamilton-Ricci flow). R (g) is the Ricci curvature of the metric g; k is a coefficient of proportionality (usually k = -2). The "telegraph" generalization is obvious: $\partial g/\partial t + b\, \partial^2 g/\partial t^2 = k R (g)$. This equation will describe not only flow (or diffusion) but also wave properties of the object, which is described by this equation. Further terms $\Sigma_i\, b_i\, \partial^i g/\partial t^i$ in these equations give further „fractal – wave" structures (on these objects) with various amplitudes and frequencies, i.e. these terms give „dynamic fractal – wave and flowing or temperature or diffuse" objects and fields (as usually, under certain assumptions) as it seems to be evident.

Detail analysis of all these things is out of possibilities of this article, but the situation in the "global" is clear. By this way can be obtained new connections and more "light" in the understanding and description of these phenomena and processes. If it will be a time, then will be written some another unexpected and interesting things and consequences. If the gravitational field is considered as a system of gravitons, then it is possible to use absolute and straight or flat Euclid's space and time. Location of this field is in individual gravitons. Local space-time coordinates (flat or curved) can be expressed as functions of these Euclid's space-time coordinates in the same or in higher dimension. (It can be also used for suitable mapping more Euclid's space times with the same dimension and naturally with higher dimension as well). For this system of gravitons can be defined a positive density of energy, but it is necessary to make some changes, as was said above. In this case, the theory of gravitation will be in fact, more or less and in the main, by the part of the mathematical theory of sources. Many of these ideas and concepts can be also used in further fields (electromagnetic, strong etc.).

It is good, if can be used and if exist absolute, direct and straight things, notions, concepts and ideas, not only in theology or mathematics, but also in physics and sometimes also in politics, economics, etc. (and not only relative and curved things, notions, concepts and ideas). By words of Aesop (Aisopos, 300 B.C., Phaedrus, 1st Century A.D., Romulus, 10th Century A.D., Anonymous, 12th Century A.D., [49]): "Eagles see other things (and otherwise) than fishes. Fishes need not believe to eagles that it is true that what they see and eagles can doubt about that what fishes see".

But it is possible and probable that some things, which are in the order of all things, will be forever hidden and unrevealed. Only some "basic and necessary principles and knowledge" are gradually and step-by-step unveiled, everything the others remain hidden and unobserved. It is also possible another consideration, that in future (in limit) "all" will be known and unveiled, in reality (from various reasons) not for all but in principle for all or for many or for some. If some basic and important principles are not good and true or are not kept then also results are not good. This is valid generally and it is necessary to have it in memory.

1/3/2010 (in the main)

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