Vibration of Yukawa Potential dependent Time and Extended Klein-Gordon Equation in Rindler Space-Time

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ABSTRACT

Atom's nucleus force understand by Yukawa potential independent time. We study Yukawa potential dependent about time. We make Klein-Gordon equation is satisfied by Yukawa potential dependent about time. Yukawa potential satisfy Proca equation or Klein-Gordon equation. If we represent Yukawa potential dependent time in Rindler space-time, this Yukawa potential satisfy the extended Klein-Gordon equation in Rindler space-time. We understand Yukawa force in Rindler space-time.

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Key words: Nucleus vibration;

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1. Introduction

Atom's nucleus force understand by Yukawa potential. We study Yukawa potential dependent about time. We make Klein-Gordon equation is satisfied by Yukawa potential dependent about time.

At first, Yukawa potential V describes nucleus's combine force in semi-classical method.[7]

$$V = -\frac{g^2}{r}\exp(-\frac{m_\pi rc}{\hbar})$$

g is real number, m_π is the meson's mass (1)

Klein-Gordon equation is satisfied by Yukawa potential V.

$$-\partial_i\partial^i V + \frac{m^2c^2}{\hbar^2}V = -\nabla^2 V + \frac{m_\pi{}^2c^2}{\hbar^2}V = 0$$

$$V = -\frac{g^2}{r}\exp(-\frac{m_\pi rc}{\hbar}) \quad (2)$$

If we focus Klein-Gordon equation make 4-dimential partial differential equation about Yukawa potential ϕ dependent time,

$$\frac{m_\pi{}^2c^2}{\hbar^2}\phi + \partial_\mu\partial^\mu\phi = \frac{m_\pi{}^2c^2}{\hbar^2}\phi + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi - \nabla^2\phi = 0 \quad (3)$$

In this time, Yukawa potential ϕ dependent time is.

$$\phi = -\frac{g^2}{r}\exp(-\frac{m_\pi rc}{\hbar}) + A_0\sin\omega t, \quad \text{Frequency } \omega = \frac{m_\pi c^2}{\hbar} \quad (4)$$

Absolutely, if we calculate, Eq(3) is satisfied by Eq(4). Yukawa potential ϕ is vibrated about the amplitude A_0, but we know the nuclear strong force doesn't vibrate about time in inertial frame..

2. Yukawa potential dependent time from Extended Klein-Gordon Equation in Rindler-Space-Time

Rindler coordinates are

$$ct = (\frac{c^2}{a_0}+\xi^1)\sinh(\frac{a_0}{c}\xi^0) \ , \ x = (\frac{c^2}{a_0}+\xi^1)\cosh(\frac{a_0}{c}\xi^0) - \frac{c^2}{a_0}$$

$$y = \xi^2, z = \xi^3 \quad (5)$$

If we write Yukawa potential ϕ in inertial frame,

$$\phi = -\frac{g^2}{r}\exp(-\frac{m_\pi rc}{\hbar}) + A_0\sin\omega t, \quad \text{Frequency } \omega = \frac{m_\pi c^2}{\hbar} \quad (6)$$

If we rewrite Yukawa potential ϕ_ξ in Rindler space-time,

$$\phi = \phi^1 + \phi^2 = \phi_\xi = \phi_\xi{}^1 + \phi_\xi{}^2 \quad (7)$$

$$\phi^1 = \phi_\xi{}^1 = -\frac{g^2}{\sqrt{x^2+y^2+z^2}} \exp(-\frac{m_\pi c}{\hbar}\sqrt{x^2+y^2+z^2})$$

$$= -\frac{g^2}{\sqrt{\{(\frac{c^2}{a_0}+\xi^1)\cosh(\frac{a_0}{c}\xi^0)-\frac{c^2}{a_0}\}^2+(\xi^2)^2+(\xi^3)^2}} \exp[-\frac{m_\pi c}{\hbar}\sqrt{\{(\frac{c^2}{a_0}+\xi^1)\cosh(\frac{a_0}{c}\xi^0)-\frac{c^2}{a_0}\}^2+(\xi^2)^2+(\xi^3)^2}\,] \tag{8}$$

And,

$$\phi^2 = \phi_\xi{}^2 = A_0 \sin\omega t = A_0 \sin[\omega\{(\frac{c}{a_0}+\frac{\xi^1}{c})\sinh(\frac{a_0\xi^0}{c})\}] \tag{9}$$

This Yukawa potential satisfy the extended Klein-Gordon equation. At first, energy and momentum are in Rindler space-time[1],

$$E_\xi = i\hbar\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^0},\ \vec{p}_\xi = -i\hbar\vec{\nabla}_\xi \tag{10}$$

Energy-Momentum equation is in Rindler space-time[1],

$$E_\xi{}^2 = \vec{p}_\xi c\cdot\vec{p}_\xi c + m^2c^4 \tag{11}$$

Hence, normal Klein-Gordon equation is in Rindler-spacetime,

$$\frac{m_\pi{}^2c^2}{\hbar^2}\phi_\xi + \frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}\frac{\partial^2}{(\partial\xi^0)^2}\phi_\xi - \nabla_\xi{}^2\phi_\xi = 0 \tag{12}$$

In this time, we focus the gauge Λ equation in Rindler space-time[1],

$$\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}\frac{\partial^2}{(\partial\xi^0)^2}\Lambda - \nabla_\xi{}^2\Lambda - \frac{\partial\Lambda}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)} = 0 \tag{13}$$

Hence, Eq(12) change extended Klein-Gordon equation in Rindler space-time.

Extended Klein-Gordon Equation is in Rindler space-time,

$$\frac{m_\pi{}^2c^2}{\hbar^2}\phi_\xi{}^1 + \frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}\frac{\partial^2\phi_\xi{}^1}{(\partial\xi^0)^2} - \nabla_\xi{}^2\phi_\xi{}^1 - \frac{\partial\phi_\xi{}^1}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)} = 0 \tag{14}$$

And

$$\frac{m_\pi{}^2c^2}{\hbar^2}\phi_\xi{}^2 + \frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}\frac{\partial^2\phi_\xi{}^2}{(\partial\xi^0)^2} - \nabla_\xi{}^2\phi_\xi{}^2 - \frac{\partial\phi_\xi{}^2}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)} = 0 \tag{15}$$

Hence,

$$\frac{m_\pi{}^2c^2}{\hbar^2}\phi_\xi+\frac{1}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2}\frac{\partial^2\phi_\xi}{(\partial\xi^0)^2}-\nabla_\xi{}^2\phi_\xi-\frac{\partial\phi_\xi}{\partial\xi^1}\frac{a_0}{c^2}\frac{1}{(1+\frac{a_0}{c^2}\xi^1)}=0 \qquad (16)$$

Eq(8) ,Eq(9), Yukawa potentials $\phi_\xi{}^1,\phi_\xi{}^2$ satisfy Eq(14),Eq(15), extended Klein-Gordon equations in Rindler space-time. Therefore, Eq(7), Yukawa potential ϕ_ξ satisfy Eq(16), extended Klein-Gordon equation in Rindler space-time

Yukawa force $\vec{f}$ is

$$\vec{f}=-\vec{\nabla}\phi=-\frac{g^2}{r^3}[\exp(-\frac{m_\pi rc}{\hbar})](1+\frac{m_\pi rc}{\hbar})\vec{r} \qquad (17)$$

In this time, Yukawa force $\vec{f}_\xi$ is Rindler space-time,

$$\vec{f}_\xi=-\vec{\nabla}_\xi\phi_\xi=-\vec{\nabla}_\xi\phi_\xi{}^1-\vec{\nabla}_\xi\phi_\xi{}^2=-\frac{g^2}{r^3}[\exp(-\frac{m_\pi rc}{\hbar})](1+\frac{m_\pi rc}{\hbar})(x\cosh(\frac{a_0\xi^0}{c}),\xi^2,\xi^3)$$
$$-\frac{\omega}{c}A[\cos(\omega t)](\sinh(\frac{a_0\xi^0}{c}),0,0) \qquad (18)$$

Hence, according to Yukawa force $\vec{f}_\xi$ in Rindler space-time, the nuclear force strongly acts and vibrates in accelerated frame rather than inertial frame in x-axis.

3. Conclusion

We found Yukawa potential dependent time. Hence, the nuclear strong force vibrates about time in Rindler spacetime. We found Yukawa potential mechanism in Rindler Space-time. We understand nuclear force in Rindler space-time.

References

[1]S.Yi, “Electromegnetic Wave Function and Equation, Lorentz Force in Rindler Space-time”, International Journal of Advanced Research in Physical Science,**5**,9(2018)

[2]J.M. Normand, A Lie group: Rotations in Quantum Mechanics(North-Holland Pub. Co., 1980)

[3]J.D. Bjorken & S. D. Drell, Relativistic Quantum Field(McGraw- Hill Co., 1965)

[4]P.Bergman,Introduction to the Theory of Relativity(Dover Pub. Co.,Inc., New York,1976),Chapter V

[5]R.L.Liboff, Quantum Mechanics(Addison-Wesley Publishing Co., Inc.,1990)

[6]A.Beiser, Concept of Modern Physics(McGraw-Hill,Inc.,1991)

[7]H. Yukawa, “On the interaction of elementary particle. I”, Proceedings of the Physico-Mathematical

Society of Japan. 3rd Series 17(1935):48-573