The Riemann zeta function formula for the entire complex plane

How to calculate a center point of the zeta & eta function spirals on the complex plane

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Abstract

I am using the eta function spirals to find a formula for all the center points of any spiral on the complex plane
This is based on my previous ideas about the center points of the zeta function

(I am currently working on some really nice ideas that relate to this I will update this pdf when needed)

Base Spiral [Level 0]

$$Arm1 = f(x) = \sum_{k=1}^{x} \frac{(-1)^{k-1}}{k^s}$$

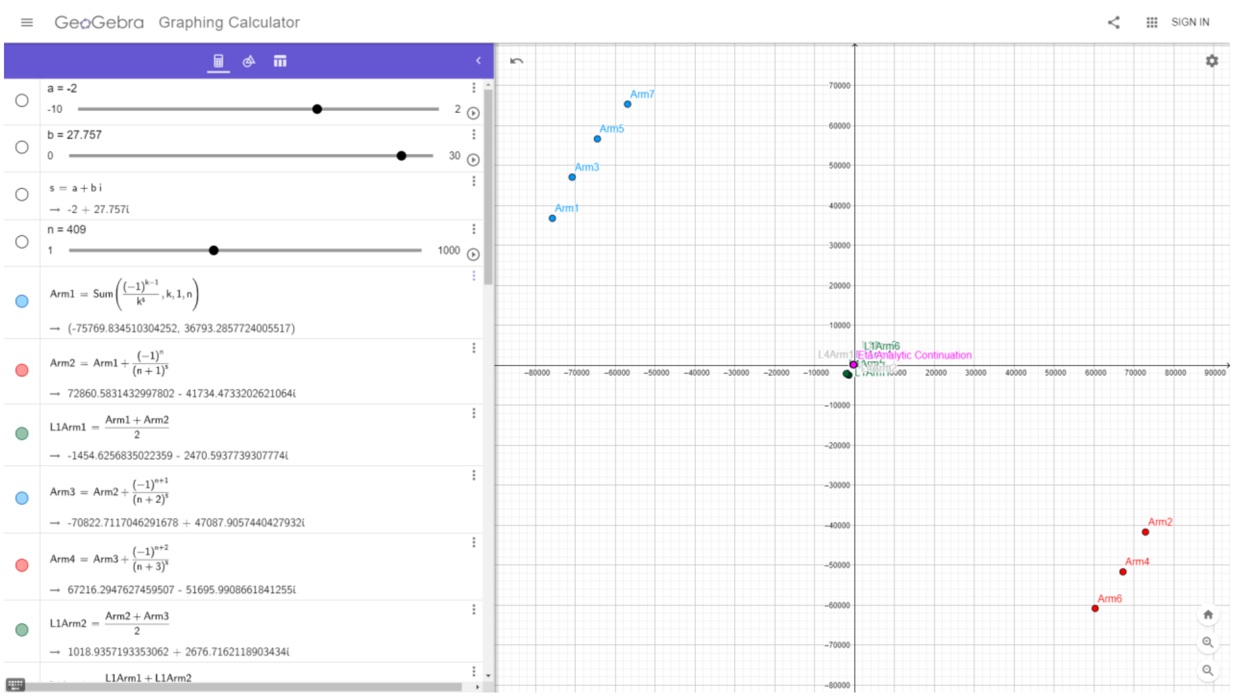
$$Arm2 = f(x+1) = \sum_{k=1}^{x+1} \frac{(-1)^{k-1}}{k^s} = \sum_{k=1}^{x} \frac{(-1)^{k-1}}{k^s} + \frac{(-1)^{x}}{(x+1)^s} = f(x) + \frac{(-1)^{x}}{(x+1)^s}$$

$$Arm3 = f(x+2) = \sum_{k=1}^{x+2} \frac{(-1)^{k-1}}{k^s} = \sum_{k=1}^{x+1} \frac{(-1)^{k-1}}{k^s} + \frac{(-1)^{x+1}}{(x+2)^s} = f(x+1) + \frac{(-1)^{x+1}}{(x+2)^s}$$

$$Arm4 = f(x+3) = \sum_{k=1}^{x+3} \frac{(-1)^{k-1}}{k^s} = \sum_{k=1}^{x+2} \frac{(-1)^{k-1}}{k^s} + \frac{(-1)^{x+2}}{(x+3)^s} = f(x+2) + \frac{(-1)^{x+2}}{(x+3)^s}$$

$$Arm5 = f(x+4) = \sum_{k=1}^{x+4} \frac{(-1)^{k-1}}{k^s} = \sum_{k=1}^{x+3} \frac{(-1)^{k-1}}{k^s} + \frac{(-1)^{x+3}}{(x+4)^s} = f(x+3) + \frac{(-1)^{x+3}}{(x+4)^s}$$

$$Arm6 = f(x+5) = \sum_{k=1}^{x+5} \frac{(-1)^{k-1}}{k^s} = \sum_{k=1}^{x+4} \frac{(-1)^{k-1}}{k^s} + \frac{(-1)^{x+4}}{(x+5)^s} = f(x+4) + \frac{(-1)^{x+4}}{(x+5)^s}$$



we are going to average every two consecutive arms
this way we will create a new spiral that will rotate around the center point we are looking for

New spiral [Level 1]

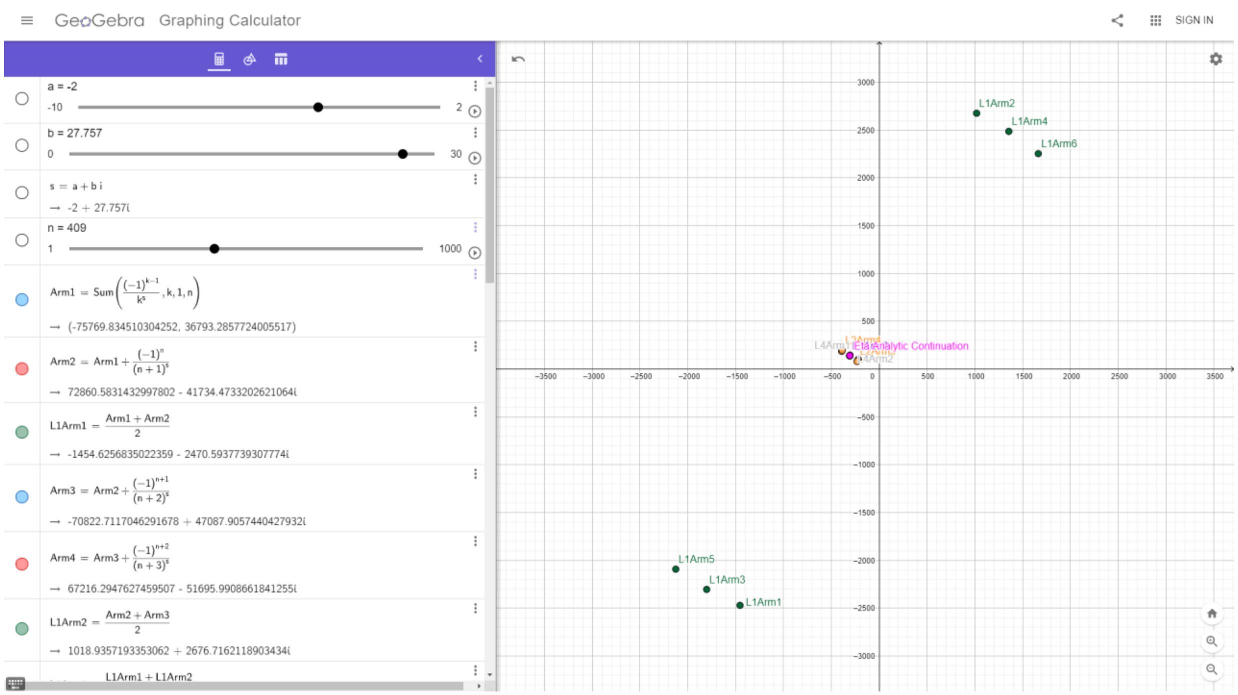
$$L1Arm1 = \frac{Arm1 + Arm2}{2}$$

$$L1Arm2 = \frac{Arm2 + Arm3}{2}$$

$$L1Arm3 = \frac{Arm3 + Arm4}{2}$$

$$L1Arm4 = \frac{Arm4 + Arm5}{2}$$

$$L1Arm5 = \frac{Arm5 + Arm6}{2}$$

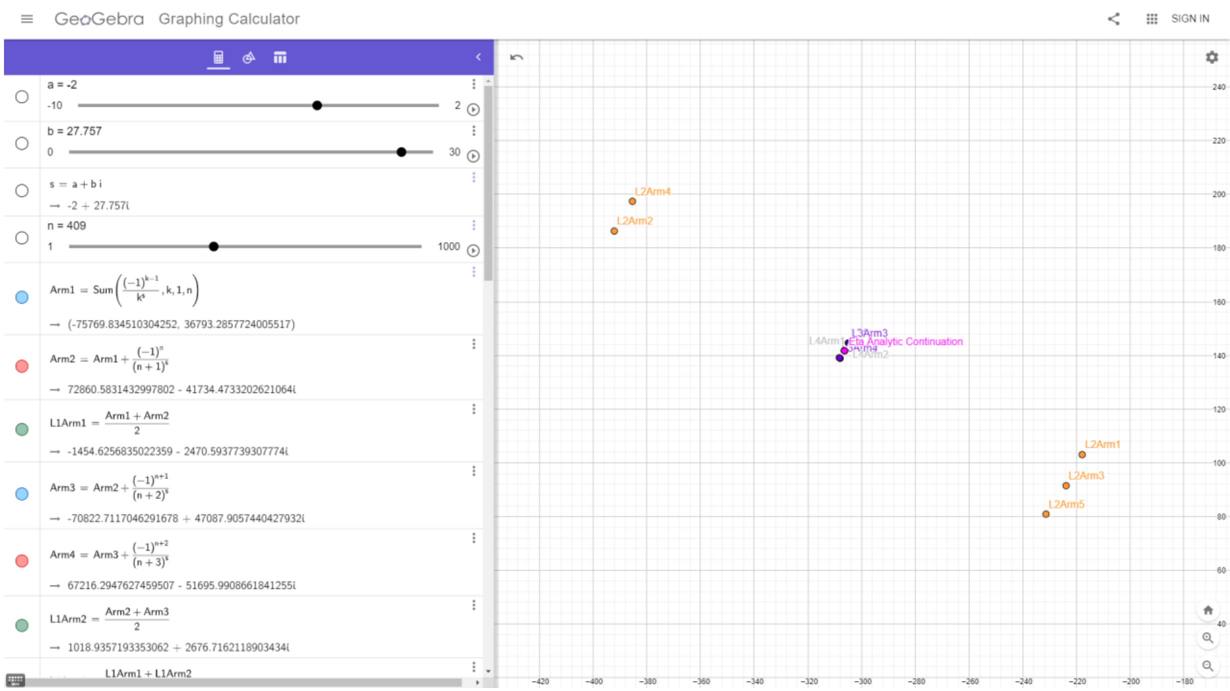


same idea …

$$L2Arm1 = \frac{L1Arm1 + L1Arm2}{2} = \frac{\frac{Arm1 + Arm2}{2} + \frac{Arm2 + Arm3}{2}}{2} = \frac{Arm1 + Arm2}{4} + \frac{Arm2 + Arm3}{4}$$

New spiral [Level 2]

$$L2Arm1 = \frac{L1Arm1 + L1Arm2}{2} = \frac{Arm1 + Arm2}{4} + \frac{Arm2 + Arm3}{4}$$
$$L2Arm2 = \frac{L1Arm2 + L1Arm3}{2} = \frac{Arm2 + Arm3}{4} + \frac{Arm3 + Arm4}{4}$$
$$L2Arm3 = \frac{L1Arm3 + L1Arm4}{2} = \frac{Arm3 + Arm4}{4} + \frac{Arm4 + Arm5}{4}$$
$$L2Arm4 = \frac{L1Arm4 + L1Arm5}{2} = \frac{Arm4 + Arm5}{4} + \frac{Arm5 + Arm6}{4}$$



same idea …

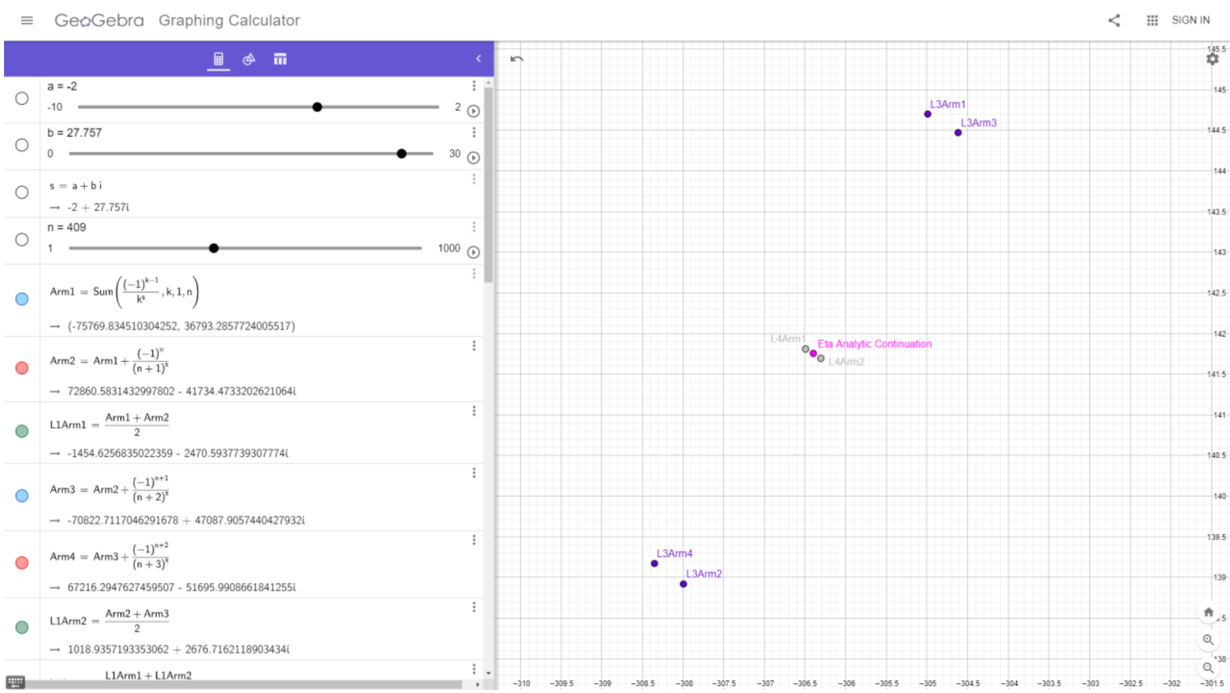
$$L3Arm1 = \frac{L2Arm1 + L2Arm2}{2} = \frac{\frac{Arm1 + Arm2}{4} + \frac{Arm2 + Arm3}{4} + \frac{Arm2 + Arm3}{4} + \frac{Arm3 + Arm4}{4}}{2} = \frac{Arm1}{2^3} + 3\frac{Arm2}{2^3} + 3\frac{Arm3}{2^3} + \frac{Arm4}{2^3}$$

New spiral [Level 3]

$$L3Arm1 = \frac{L2Arm1 + L2Arm2}{2} = \frac{Arm1}{2^3} + 3\frac{Arm2}{2^3} + 3\frac{Arm3}{2^3} + \frac{Arm4}{2^3}$$

$$L3Arm2 = \frac{L2Arm2 + L2Arm3}{2} = \frac{Arm2}{2^3} + 3\frac{Arm3}{2^3} + 3\frac{Arm4}{2^3} + \frac{Arm5}{2^3}$$

$$L3Arm3 = \frac{L2Arm3 + L2Arm4}{2} == \frac{Arm3}{2^3} + 3\frac{Arm4}{2^3} + 3\frac{Arm5}{2^3} + \frac{Arm6}{2^3}$$



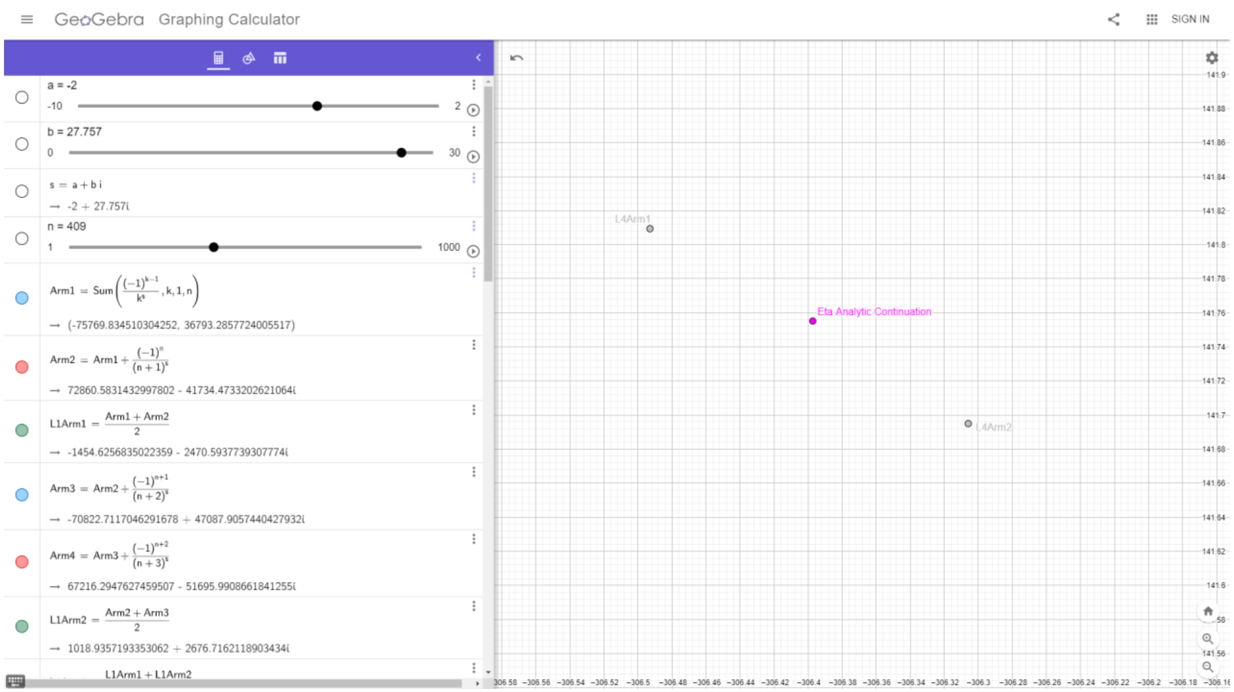
same idea …

$$L4Arm1 = \frac{L3Arm1 + L3Arm2}{2} = \frac{\frac{Arm1}{2^3} + 3\frac{Arm2}{2^3} + 3\frac{Arm3}{2^3} + \frac{Arm4}{2^3} + \frac{Arm2}{2^3} + 3\frac{Arm3}{2^3} + 3\frac{Arm4}{2^3} + \frac{Arm5}{2^3}}{2} = \frac{Arm1}{2^4} + 4\frac{Arm2}{2^4} + 6\frac{Arm3}{2^4} + 4\frac{Arm4}{2^4} + \frac{Arm5}{2^4}$$

New spiral [Level 4]

$$L4Arm1=\frac{L3Arm1+L3Arm2}{2}=\frac{Arm1}{2^4}+4\frac{Arm2}{2^4}+6\frac{Arm3}{2^4}+4\frac{Arm4}{2^4}+\frac{Arm5}{2^4}$$

$$L4Arm2=\frac{L3Arm2+L3Arm3}{2}=\frac{Arm2}{2^4}+4\frac{Arm3}{2^4}+6\frac{Arm4}{2^4}+4\frac{Arm5}{2^4}+\frac{Arm6}{2^4}$$



same idea …

$$L5Arm1=\frac{Arm1}{2^5}+5\frac{Arm2}{2^5}+10\frac{Arm3}{2^5}+10\frac{Arm4}{2^5}+5\frac{Arm5}{2^5}+\frac{Arm6}{2^5}$$

I think we got the idea no?

General Summation Formula

$$L1Arm1 = \frac{Arm1}{2^1} + \frac{Arm2}{2^1}$$

$$L2Arm1 = \frac{Arm1}{2^2} + 2\frac{Arm2}{2^2} + \frac{Arm3}{2^2}$$

$$L3Arm1 = \frac{Arm1}{2^3} + 3\frac{Arm2}{2^3} + 3\frac{Arm3}{2^3} + \frac{Arm4}{2^3}$$

$$L4Arm1 = \frac{Arm1}{2^4} + 4\frac{Arm2}{2^4} + 6\frac{Arm3}{2^4} + 4\frac{Arm4}{2^4} + \frac{Arm5}{2^4}$$

$$L5Arm1 = \frac{Arm1}{2^5} + 5\frac{Arm2}{2^5} + 10\frac{Arm3}{2^5} + 10\frac{Arm4}{2^5} + 5\frac{Arm5}{2^5} + \frac{Arm6}{2^5}$$

New spiral [Level n-1]

$$Level(n-1)Arm1 = \binom{n-1}{0}\frac{Arm1}{2^{n-1}} + \binom{n-1}{1}\frac{Arm2}{2^{n-1}} + \binom{n-1}{2}\frac{Arm3}{2^{n-1}} + \binom{n-1}{3}\frac{Arm4}{2^{n-1}} + \binom{n-1}{4}\frac{Arm5}{2^{n-1}} + \ldots + \binom{n-1}{n-1}\frac{Arm(n)}{2^{n-1}}$$

(center point need at least two arms at level n-1 so there will be center point at level n so …)

$$Level(n-1)Arm2 = \binom{n-1}{0}\frac{Arm2}{2^{n-1}} + \binom{n-1}{1}\frac{Arm3}{2^{n-1}} + \binom{n-1}{2}\frac{Arm4}{2^{n-1}} + \binom{n-1}{3}\frac{Arm5}{2^{n-1}} + \binom{n-1}{4}\frac{Arm6}{2^{n-1}} + \ldots + \binom{n-1}{n-1}\frac{Arm(n+1)}{2^{n-1}}$$

New spiral [Level n]

$$Level(n) = \binom{n}{0}\frac{Arm1}{2^n} + \binom{n}{1}\frac{Arm2}{2^n} + \binom{n}{2}\frac{Arm3}{2^n} + \binom{n}{3}\frac{Arm4}{2^n} + \binom{n}{4}\frac{Arm5}{2^n} + \ldots + \binom{n}{n}\frac{Arm(n+1)}{2^n}$$

we need only to add the limit lol

Center Point

this is the center point:

$$f(x)=\sum_{k=1}^{x}\frac{(-1)^{k-1}}{k^s}$$

$$\lim_{n\to\infty}\left[\binom{n}{0}\frac{f(x+0)}{2^n}+\binom{n}{1}\frac{f(x+1)}{2^n}+\binom{n}{2}\frac{f(x+2)}{2^n}+\binom{n}{3}\frac{f(x+3)}{2^n}+\binom{n}{4}\frac{f(x+4)}{2^n}+\ldots+\binom{n}{n}\frac{f(x+n)}{2^n}\right]$$

$$\lim_{n\to\infty}\left[\binom{n}{0}\frac{\sum_{k=1}^{x}\frac{(-1)^{k-1}}{k^s}}{2^n}+\binom{n}{1}\frac{\sum_{k=1}^{x+1}\frac{(-1)^{k-1}}{k^s}}{2^n}+\binom{n}{2}\frac{\sum_{k=1}^{x+2}\frac{(-1)^{k-1}}{k^s}}{2^n}+\binom{n}{3}\frac{\sum_{k=1}^{x+3}\frac{(-1)^{k-1}}{k^s}}{2^n}+\binom{n}{4}\frac{\sum_{k=1}^{x+4}\frac{(-1)^{k-1}}{k^s}}{2^n}+\ldots+\binom{n}{n}\frac{\sum_{k=1}^{x+n}\frac{(-1)^{k-1}}{k^s}}{2^n}\right]$$

let rearrange

this is the formula for eta function for the entire complex plane ☺

$$\eta(s)=\lim_{n\to\infty}\left[\sum_{k=1}^{x}\binom{n}{0}\frac{(-1)^{k-1}}{2^n k^s}+\sum_{k=1}^{x+1}\binom{n}{1}\frac{(-1)^{k-1}}{2^n k^s}+\sum_{k=1}^{x+2}\binom{n}{2}\frac{(-1)^{k-1}}{2^n k^s}+\sum_{k=1}^{x+3}\binom{n}{3}\frac{(-1)^{k-1}}{2^n k^s}+\sum_{k=1}^{x+4}\binom{n}{4}\frac{(-1)^{k-1}}{2^n k^s}+\ldots+\sum_{k=1}^{x+n}\binom{n}{n}\frac{(-1)^{k-1}}{2^n k^s}\right]$$

lets check for s=0 just for the fun of it ^_^

$$\text{if }\sum_{k=1}^{x}\frac{(-1)^{k-1}}{k^s}=1\text{ then }\frac{1}{2^n}\left[\binom{n}{0}+\binom{n}{2}+\binom{n}{4}+\ldots+\binom{n}{n}\right]=\frac{2^{n-1}}{2^n}=\frac{1}{2}$$

$$\text{if }\sum_{k=1}^{x}\frac{(-1)^{k-1}}{k^s}=0\text{ then }\frac{1}{2^n}\left[\binom{n}{1}+\binom{n}{3}+\binom{n}{5}+\ldots+\binom{n}{n}\right]=\frac{2^{n-1}}{2^n}=\frac{1}{2}$$

either way the value will be 0.5 just it should be on the analytic continuation value ^_^

this is the formula for The Riemann zeta function for the entire complex plane ☺

$$\zeta(s)=\left(\frac{1}{1-2^{1-s}}\right)\cdot\lim_{n\to\infty}\left[\sum_{k=1}^{x}\binom{n}{0}\frac{(-1)^{k-1}}{2^n k^s}+\sum_{k=1}^{x+1}\binom{n}{1}\frac{(-1)^{k-1}}{2^n k^s}+\sum_{k=1}^{x+2}\binom{n}{2}\frac{(-1)^{k-1}}{2^n k^s}+\sum_{k=1}^{x+3}\binom{n}{3}\frac{(-1)^{k-1}}{2^n k^s}+\sum_{k=1}^{x+4}\binom{n}{4}\frac{(-1)^{k-1}}{2^n k^s}+\ldots+\sum_{k=1}^{x+n}\binom{n}{n}\frac{(-1)^{k-1}}{2^n k^s}\right]$$