# A New Solution of the Friedman Equations.

## ABSTRACT

A new cosmological model is suggested, a type of Big Bang cosmology but with a different redshift scale-factor relation and no cosmological constant. There is discussion of why observations often seem to support the possibly faulty Concordance Cosmology. In the new model there is an apparent matter density of about 0.3 but no coincidence problem as the real matter density is 1.0. There is a solution to the Hubble tension.

John Hunter[*]

**INTRODUCTION**

Since 1915 there has been apparently increasing evidence for a universe with dark matter, dark energy and a period of inflation, the LCDM model. The evidence has been so strong and from so many different types of investigation that the majority of cosmologists have understandably become supporters of the so called 'Concordance Cosmology'.

There are alternatives however, one of which is the subject of this paper. The layout is as follows.

In section 1 there is a brief review of some of the evidence in favour of the Concordance Cosmology (LCDM), some problems are also discussed. In section 2 there is a new cosmological model which has a new redshift scale-factor relation. The main features of the new cosmology are described. By mirroring the layout of the section 1 subsections, it is shown how the observed matter density and apparent cosmological constant might have been incorrectly inferred in LCDM. Section 3 describes a way that the tension between Hubble constant measurements can be resolved. Philosophical matters and detailed calculations are in the appendices.

### Section 1. Concordance Cosmology

Since the development of General Relativity by Einstein in 1915 cosmologists have developed cosmological models compatible with it.

[*]john@gravity.uk.com Member of the Institute of Physics.

In 1922 Friedman published his equations [Friedman, 1999]

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3} \qquad (1)$$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2} = \frac{8\pi G\rho + \Lambda c^2}{3} \qquad (2)$$

the symbols have their usual meanings and the Hubble parameter is

$$H(z) = \left(\frac{\dot{a}}{a}\right) \qquad (3)$$

where $z$ is the redshift of light from distant objects. The matter density is

$$\Omega_m = \frac{\rho}{\rho_{crit}} \qquad (4)$$

the critical density is

$$\rho_{crit} = \frac{3H(z)^2}{8\pi G} \qquad (5)$$

Concordance cosmology has a solution with a matter density $\Omega_m$ a dark energy density parameter $\Omega_\Lambda \approx 1 - \Omega_m$ and four other variable parameters, it has been shown to give a good match to data.

The universe is thought to originate in the Big Bang. A period of inflation followed. The universe then expanded with quantities, including the matter density, being related to the scale factor $a$.

The redshift is related to the scale factor as

$$a = \frac{1}{1+z} \qquad (6)$$

In LCDM the Hubble parameter is

$$H(z) = H_0\sqrt{\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}$$

$H_0$ is the local Hubble constant.　　　　　(7)

The co-moving distance is obtained from

$$D_M = \int_t^0 \frac{c}{a(t)} dt \qquad (8)$$

$$\frac{da}{dt} = \frac{da}{dz} \times \frac{dz}{dt} = -\frac{1}{(1+z)^2} \times \frac{dz}{dt} \qquad (9)$$

$$H(z) = \frac{\dot{a}}{a} = -\frac{1}{1+z} \times \frac{dz}{dt} \qquad (10)$$

So (8) can also be written as

$$D_M = \int_0^z \frac{c}{H(z)} dz \qquad (11)$$

Luminosity distance $D_L$ and angular diameter distance $D_A$ are from

$$D_L = (1+z)D_M \qquad (12)$$

and

$$D_A = \frac{D_M}{1+z} \qquad (13)$$

Alternatives to some of these equations are in section 2.

Below is a list of factors, in approximately historical order, that have led to the development and acceptance of the LCDM model, there is a review in Peebles [Peebles, 2002].

1.1 Redshift

The redshift of light from distant objects seems to indicate an expansion and support for a Big Bang type of model - although not all scientists believed that it was evidence of an expansion e.g. Zwicky [Zwicky,1929] who favoured a 'tired light' model.

1.2 Abundancies of the elements

Big Bang nuclear synthesis (BBN) successfully predicts the abundancies of the elements, adding further support to a Big Bang type of model. Typical values for the baryon density $\Omega_b h^2$ from BBN are 0.02166 [Cooke, 2018] where $H_0 = 100h$ kms$^{-1}$Mpc$^{-1}$.

1.3 Cosmic Microwave Background Radiation

The Cosmic Microwave Background Radiation (CMB) was discovered by Penzias and Wilson in 1965, a 2.73K Planck black body spectrum. It was immediately interpreted as the 'afterglow' from the Big Bang.

Remaining alternatives such as the Steady State model of Bondi, Gold and Hoyle [Bondi, 1948] and tired light models went out of fashion – although Hoyle, Burbidge and Narlikar [Hoyle, 1993] later published a "Quasi Steady State Cosmology".

1.4 Flatness and horizon problems

The Big Bang model still faced difficulties. The flatness problem is that the universe appears to be near critical density. The horizon problem is that the universe is homogeneous, even on scales so large that that the separate regions should not have had time to communicate with each other.

Inflation was developed in the 1970s and 80s and added to the Big Bang model [Guth, 1980], to help solve these problems.
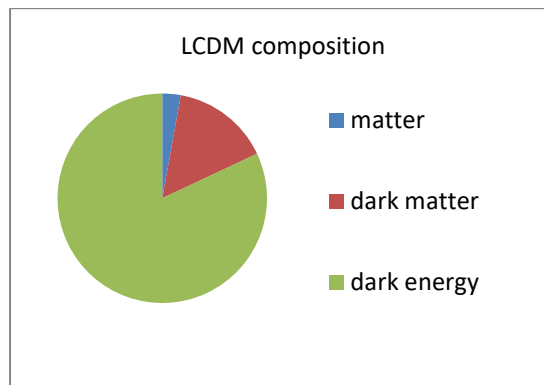
1.5 Matter density

Evidence for a low matter density came from the motion of stars around galaxies and from clusters of galaxies [Ferramacho, 2006] and [Allen, 2007]. It seems that as well as visible matter there must be dark matter. Even including dark matter, the total matter density seemed to be much less than one, a typical value for $\Omega_m$ from galaxy clusters being 0.2-0.3. So the next piece of evidence for LCDM was readily received.

1.6 Supernovae and the cosmological constant.

The Luminosity distances of supernovae seem to show an accelerating expansion [Riess, 1998] and [Perlmutter, 1998]. LCDM could accommodate this data with a cosmological constant. The cosmological constant was also necessary to account for the low matter density and have a flat universe where the density components add up to one. The pie chart in Figure 1 shows the composition of the universe for the LCDM model.

Figure 1  Composition of the universe, LCDM



## LCDM composition

- matter
- dark matter
- dark energy

### 1.7 Anisotropies in the CMBR

Measurements of the size of the acoustic peaks of the power spectrum of the anisotropies in the Cosmic Microwave Background Radiation have enabled parameters in the LCDM model to be measured, WMAP [Hinshaw, 2012] finding $\Omega_m$= 0.281, $\Omega_\Lambda$= 0.719, $H_0$ = 69.7kms⁻¹Mpc⁻¹ and PLANCK [Aghanim, 2018]  $\Omega_m$= 0.315, $\Omega_\Lambda$ = 0.68, $H_0 = 67.4$ kms⁻¹Mpc⁻¹

### 1.8 Baryon Acoustic Oscillations

A relatively new method, BAO observations involve measurements of the sound horizon, which are thought to provide a 'standard ruler' [Aubourg, 2014], for a review see Bassett [Bassett, 2009]. BAO on the whole, support LCDM.  This method is often combined with CMB and supernovae data. Using an 'inverse distance ladder' approach, the results give a low Hubble constant typically of 67.3 kms⁻¹Mpc⁻¹

### 1.9 Hubble parameter data

By measuring the differential ageing of galaxies at different redshifts [Jimenez, 2001], the Hubble parameter can be found from equation (10)

$$H(z) = \frac{\dot{a}}{a} = -\frac{1}{1+z} \times \frac{dz}{dt}$$

The results are consistent with LCDM but also compatible with $H(z) = (1 + z)H_0$.  Data from different groups correlated with CMB and BAO data [Farooq, 2016] show a transition from deceleration to acceleration at about $z$ = 0.78
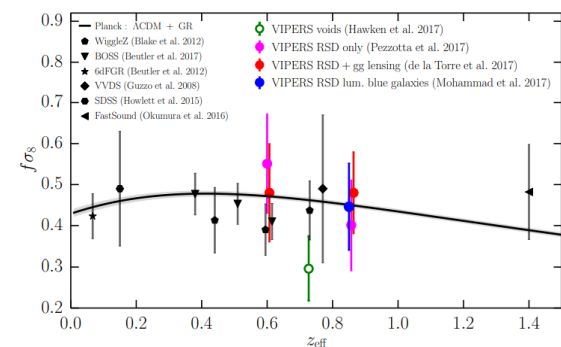
### 1.10 Measurements of the Hubble constant

Recent local methods of determining the Hubble constant give 74.03 kms⁻¹Mpc⁻¹ [Riess, 2019], in tension, 4.4$\sigma$, with values from CMB and BAO. Local measurements do not appear to agree with the value for $H_0$ deduced by LCDM.  This is the so called 'Hubble tension'.

### 1.11 Large Scale Structure

Measurements of the growth of large scale structure measure the quantity $f\sigma_8(z)$ where $f(z)$ is the growth factor and $\sigma_8$ is the amplitude of the matter fluctuations on a scale of 8 Mpc.

Figure 2  Growth parameter against redshift



The graph of Figure 2 [Guzzo, 2018] shows the LCDM match to the growth data.  The new model (section 2.11) gives a good match with a constant $f\sigma_8(z)$ of about 0.47.

The above is quite a long list!  The fact that LCDM successfully accounts for nearly all these observations is an astounding success.  There is also a degree of consistency, with apparent evidence for dark energy and dark matter coming from different types of investigation. The fact that scientists have designed and carried out such experiments, with such precision, is a credit to the world's physicists, astronomers, cosmologists and engineers.

We have now entered the era of precision cosmology and we are in a position, with further upcoming experiments, to subject LCDM to even more detailed scrutiny.

However there are some problems with LCDM. No matter how successfully it matches the observations, it cannot be denied that, philosophically speaking, its foundations are not so sound. It relies on three phenomena which have no theoretical understanding and have not been subject to local tests - dark energy, dark matter and inflation.

Whilst observationally, on the whole, LCDM has been successful, there is the Hubble tension that has recently risen to over 4 standard deviations. There is also the Lyman Alpha Forest anomaly, LCDM not giving a good match to BAO data at $z=2.34$. The beginning of the universe is also a problem - there would have been a state of infinite density and pressure. More matter than antimatter would have to be created.

Perhaps just as serious is the coincidence problem. LCDM does not account for why the matter density parameter and the dark energy density parameter should be the same order of magnitude at the present time.

There has also been a kind of inter-dependency of methods, whereby conclusions from one e.g. supernovae data is used to bolster another, e.g. BAO, or BAO data is used to calibrate CMB data.

In the next section a model is suggested which might help with some of the issues above. After introducing the main features of the model, the subsections in section 2 mirror those of section 1, to enable any changes necessary to be discussed. Section 2 illustrates how it's possible for concordance cosmology to be incorrect, yet still appear to match observations.

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## Section 2. An Alternative Solution

There are other solutions of the Friedman equations. If we are guided by the principle of simplicity we can look for a flat solution, $k = 0$, with a constant expansion parameter $H$

$$a = e^{-Ht} \qquad (14)$$

an expanding solution, with the convention that $t$ is positive into the past.

Equations (1) and (2) reduce to

$$3H^2 = -4\pi G \left(\rho + \frac{3p}{c^2}\right) + \Lambda c^2 \qquad (15)$$

$$3H^2 = 8\pi G\rho + \Lambda c^2 \qquad (16)$$

with zero cosmological constant, $\Lambda = 0$,

$$\rho = \frac{3H^2}{8\pi G} \qquad (17)$$

the universe is at the critical density

$$G = \frac{3H^2}{8\pi\rho} \qquad (18)$$

and

$$p = -\rho c^2 \qquad (19)$$

With similarities to the Einstein - De Sitter [Einstein 1932] model. Is such a situation possible? It seems as though, that when the universe expands, the matter density should reduce and the above solution should be ruled out. However observations show that (17) is true, at least approximately and at the present time.

This is really the flatness problem as noted by Dicke [Dicke, 1970]. It's similar to the 'coincidence problem', that the matter density has a value $\sim$ 0.5 so that the dark energy density has a similar value, in a flat universe. This important 'coincidence' has not been properly accounted for.

If we look for a way that (17) is always true, we can find the following interpretation.

In LCDM the scale factor determines the distance between galaxies - there is now an important distinction with the new model.

The changing scale factor $a$ now applies not only to space, but to matter too, and all distances. This includes atoms, people, all distances, the sizes of stars, galaxies and the distance between all objects now depend on the scale factor. All physical constants which contain length dimensions are changed by the changing scale factor too.

Every quantity $Q$ with $n$ length dimensions changes according to $Qa^n$ so, for example

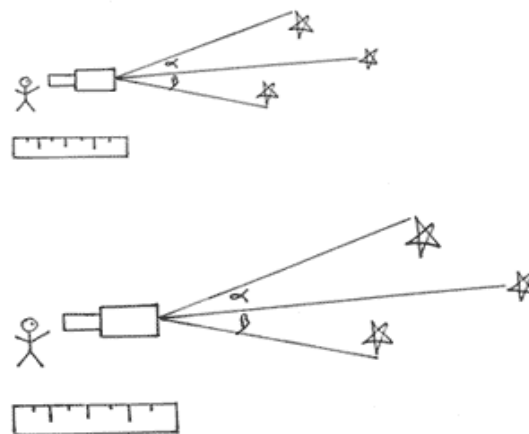Table 1 Changes of physical quantities

| Physical Quantity | Change with time |
|---|---|
| Planck's constant $h$ | $he^{-2Ht}$ |
| Masses $m$ | $m$ constant |
| Fine structure constant $\alpha$ | $\alpha$ constant |
| Gravitational constant $G$ | $Ge^{-3Ht}$ |
| Pressure $p$ | $pe^{Ht}$ |
| Speed of light $c$ | $ce^{-Ht}$ |
| Density $\rho$ | $\rho e^{3Ht}$ |

With $t$ being applied in the cosmologists sense, positive into the past. Equations (1), (2) and (17) (and in fact all physics equations true today), remain valid with this system, it's a kind of global conformal transformation, but a continuous and ongoing transformation.

This model universe is expanding yet static. Expanding in the sense that there is a continuous expansion (of all length scales) – but static in the sense that it would be impossible for any observer to measure the expansion locally. For them the universe could be regarded as static - for example $G$ appears constant in time if measured locally.

Measurements at a distance would also yield a null result. If we tried to measure any change in the fine structure constant $\alpha$ [Murphy, 2016], for distant stars, since Plancks constant $h$, the relative permittivity of free space $\varepsilon_0$ and the speed of light $c$ all change, the exponential factors cancel and $\alpha$ is left unchanged.

Figure 3 Cartoon to show the expanding universe



Attempts to measure the changes of other physical quantities at a distance would result either in an unchanged time measurement, (as it has no length dimension) an unchanged angle (also no length dimension) or a ratio of lengths – also unchanged.

The universe would always appear to be at critical density from (17), if the correct value of the expansion constant $H$ is used (see section 2.1).

In appendix A it's suggested that gravity is *caused* by the expansion, with $G$ having the value to conserve energy as the expansion occurs – hence equation (17) . It's also suggested that the strength of gravity reduces when the mass to radius ratio of a region of matter approaches $c^2/G$.

The pressure term arises as there is a balance between gravity and the explosions that occur for any region of matter which becomes so dense that the mass to radius ratio approaches $c^2/G$, such as the centres of galaxies.

So although static on the largest scales, there is a great deal of motion on a smaller scale. Peculiar velocities occur, also the interaction and flowing of matter. The Big Bang (or Bangs) is included in this model and occur when large quantities of matter collapse under gravity and then 'bounce'. So although apparently static in terms of scale factor, the universe is in a dynamic equilibrium.

The following subsections have the same structure as the subsections of section 1.

## 2.1 Redshift

The Big Bang (or Bangs) is thought to have occurred long ago and motions of the galaxies through space can now be ignored. The continuing expansion of all length scales causes a redshift as follows.

If the energy of a photon emitted (subscript 1) from a distant star towards an observer is conserved.

$$h_0 f_0 = h_1 f_1 \tag{20}$$

Since Planck's constant was lower in the past, there is a redshift of received light according to

$$f_0 = f_1 \frac{h_1}{h_0} \tag{21}$$

$$\lambda_0 = \lambda_1 e^{2Ht} \tag{22}$$

This leads to a new redshift – scale factor relation. The redshift of received light is given by

$$z = \frac{\lambda_1 e^{2Ht} - \lambda_1}{\lambda_1} \tag{23}$$

$$1 + z = e^{2Ht} = \frac{1}{a^2} \tag{24}$$

so

$$a = \frac{1}{\sqrt{1+z}} \tag{25}$$

An object, a distance $d$ away, would have an apparent velocity $v$, depending on the redshift.

$$\frac{v}{c} = z = e^{2Hd/c} - 1 \approx \frac{2Hd}{c} \tag{26}$$

$$v = 2Hd \tag{27}$$

comparing with Hubble's law, the expansion constant is half of the Hubble constant approximately 37.5 kms$^{-1}$Mpc$^{-1}$

$$H = \frac{H_0}{2} \tag{28}$$

Bassett [Bassett, 2013] mentions models where there is a 'redshift remapping' and discusses possible observational constraints on models that violate distance duality. Distance duality is preserved in this model (section 2.6). Wojtak [Wojtak, 2016] demonstrate that redshift remappings are degenerate with dark energy and they mention that a metric based interpretation of redshift guarantees agreement with the observed black body spectrum of the CMB.

The alternative equations (25) and (28) lead to changes in other formulae commonly used in cosmology and as Basset cautions, this can lead to changes in data points. Although similar in some ways to the Big Bang model with an expanding universe, the model has different formulae for luminosity and angular diameter distance. These are derived in section 2.6

## 2.2 Abundancies of the elements

The Big Bang(s) occurred in this model so the success of the predictions of the proportions of the elements is still retained.

Measurements of deuterium abundance from quasars and Big Bang nuclear synthesis (BBN) gave the baryon density $\Omega_b h^2 = 0.02166$ (section 1.2). With the new value for $h$ of about 0.375, half of the traditional value, the new value for $\Omega_b$ is four times larger, about 15 - 16% of the universe.

## 2.3 Cosmic Microwave Background Radiation

This still occurs in a similar way to the Big Bang model, due to the explosions that occur when collapsing matter reaches a high enough mass to radius ratio, $c^2/G$.

## 2.4 Flatness and horizon problems

The flatness problem does not occur in this model. The universe is naturally always at critical density, equations (17) and (18)

$$\rho = \frac{3H^2}{8\pi G}$$

and

$$G = \frac{3H^2}{8\pi \rho}$$

The interpretation of this, discussed more in Appendix A, is that gravity is caused by the expansion. It has the strength necessary to conserve energy as the universe expands.

The matter density is always 1.0 although could be measured to be 0.25 (depending on the method used) as the $H_0$ would be used in the denominator of (4) instead of the real expansion constant $H$.

The horizon problem is a problem for the Big Bang model, with a definite start for time. In the new model there is no beginning of time, and no particular single Big Bang event, although, over the last few billion years, there would have been many enormous explosive events, and perhaps one larger than the others.

2.5 Matter density

The low matter density (0.2-0.3) inferred from the X-ray gas mass fraction, was mentioned in section 1.5. The matter density is from
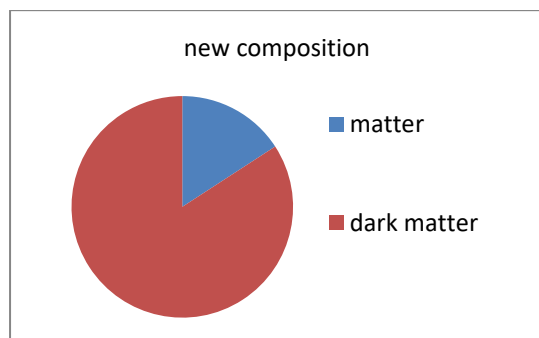
$$\Omega_m = \frac{\Omega_b}{f_{gas}(1+0.19h^{0.5})} \tag{29}$$

[Allen 2002] The denominator contains $h$ and $f_{gas}$. $f_{gas}$ is determined by measuring the mass of gas in a cluster from X-ray luminosity and the total mass of the cluster. The situation is complicated as both $d_A$ and $\rho_{crit}$ are used in the estimate of $f_{gas}$.

The main change needed, however, would be to the numerator. $\Omega_b$ is calculated from an $\Omega_b h^2$ value of 0.0205 (in 2001), obtained from the Deuterium to Hydrogen ratio in quasars. Hence $\Omega_b$ should be four times larger, this increases $\Omega_M$ by a similar factor to approximately 1.0.

The same factor four change applies to matter density from the CMB as discussed in section 2.7 Some dark matter still seems necessary. Figure 4 shows the composition of the universe in the new model.

Figure 4 Composition of the universe, new model



2.6 Supernovae and the cosmological constant

A new formula for luminosity distance is derived.

The co-moving distance is still (8)

$$D_M = \int_t^0 \frac{c}{a(t)} dt$$

Using (25)

$$a = \frac{1}{\sqrt{1+z}}$$

$$\frac{da}{dt} = \frac{da}{dz} \times \frac{dz}{dt} = -\frac{1}{2(1+z)^{3/2}} \times \frac{dz}{dt} \tag{30}$$

$$H(z) = H = \frac{\dot{a}}{a} = \frac{-1}{2(1+z)} \times \frac{dz}{dt} \tag{31}$$

$$dt = \frac{-1}{2H(1+z)} dz \tag{32}$$

$$D_M = \int_0^z \frac{c}{2H\sqrt{1+z}} dz \tag{33}$$

(33) is different to equation (11), so

$$D_M = \frac{2c}{H_0}\left(\sqrt{1+z} - 1\right) \tag{34}$$

Flux depends on the Luminosity distance as

$$F = \frac{L}{4\pi D_L^2} \tag{35}$$

There are two factors diminishing the flux arriving, which both depend on $(1+z)$, (not $\sqrt{1+z}$), the energy of the photon and the increased time of arrival due to the redshift.

Luminosity distance is $D_M$ multiplied by the factor $(1+z)$.

Angular diameter distance is from $D_M$ divided by $\sqrt{1+z}$. This would be true for a cosmology with redshift scale-factor relation (25), with the objects retaining their usual sizes. As the object being viewed was smaller (in the past) relative to the observer, the angular diameter distance is reduced by another factor $\sqrt{1+z}$

$$D_A = \frac{2c}{H_0(1+z)}\left(\sqrt{1+z} - 1\right) \tag{36}$$

$$D_L = \frac{2c}{H_0}(1+z)(\sqrt{1+z} - 1) \tag{37}$$

Distance duality still applies.

Plotting the distance moduli for supernovae binned data [Betoule, 2014] gives Figure 5. It shows the new model, top curve. LCDM with a matter density of 0.3 is the middle curve, and matter density of 1.0 is the bottom curve.

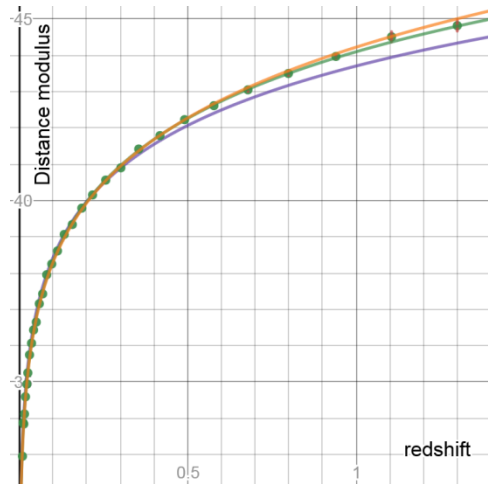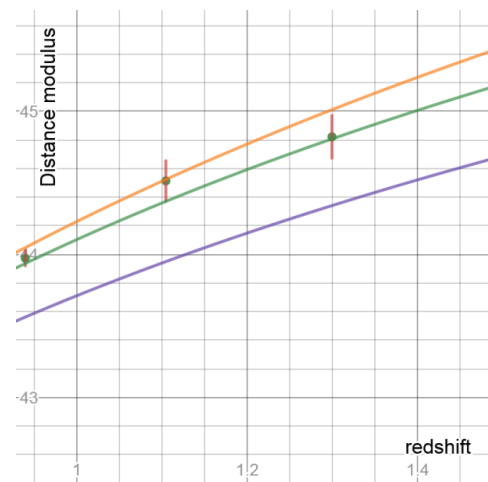Figure 5  New model, LCDM $\Omega_m$ = 0.3 and 1.0



Figure 6  An enlargement of Figure 5



LCDM has two variable parameters, $\Omega_m$ and $H_0$. The new model has only one, $H_0$.

By looking at the binomial expansion of LCDM and the new model, we would expect LCDM to give a best fit $\Omega_m$ of slightly under 1/3.

For the new model

$$D_M = \frac{c}{2H}\left(z - \frac{z^2}{4} + \cdots\right) \qquad (38)$$

For LCDM

$$D_M = \frac{c}{H_0}\left(z - \frac{3mz^2}{4} + \cdots\right) \qquad (39)$$

Details are in Appendix B. By comparing (38) and (39) there is a match if $3m = 1$, where $m$ is short for $\Omega_m$. Most of the data points are at low values of $z$, so if LCDM is not correct but is varying the matter density to match data from the new model, we would expect it to predict $\Omega_m$ = 1/3.

The matter density inferred from supernovae [Abbot, 2018] is 0.331.

2.7 Anisotropies in the CMBR

As mentioned in section 1.7, WMAP9 [Hinshaw, 2012] finds $\Omega_m$= 0.2815 from $\Omega_m h^2$ = 0.1368 $\pm$ 0.005 and a $h$ value of 0.697 $\pm$ 0.02. If $h$ is halved, $\Omega_m$ becomes four times as large, giving an $\Omega_m$ value of 1.126 $\pm$ 0.10.

There is a similar situation for PLANCK data.

So we can see why LCDM often gives values for the matter density varying between 0.25 and 0.333. Data from supernovae tend to give matter density values towards 0.333, values from $\Omega_m h^2$ give approximately 0.25

Often studies combine data from different methods and decide on a value between 0.25 and 0.333. In a flat universe the dark energy density parameter $\Omega_\Lambda$ is deduced to be between 0.667 and 0.75, but is really 0.

2.8 Baryon Acoustic Oscillations

Does the new model lead to any changes for BAO?

The BAO observations measure the standard ruler both parallel and perpendicular to the line of sight. Parallel measurements constrain $c/H(z)$ and perpendicular measurements constrain $d_A$

In section 2.6 eqn (11) was changed to (33)

$$D_M = \int_0^z \frac{c}{2H\sqrt{1+z}}\,dz$$

This might affect the interpretation of the parallel measurements. The parallel component has been identified as a probable source of the Hubble tension [Wu, 2020]. This tension is the subject of section 3.

Reports [Alam, 2016] on the values of $d_M$ from BAO data are in Table 2, with the predictions from the new model using (34)

$$D_M = \frac{2c}{H_0}(\sqrt{1+z} - 1)$$

With $H_0 = 70$ kms$^{-1}$Mpc$^{-1}$

Table 2  BAO measurements

| Redshift | Average $D_M$ measurement | Prediction of the new model |
|---|---|---|
| 0.38 | $1509 \pm 24$ | 1497 |
| 0.51 | $1974 \pm 28$ | 1961 |
| 0.61 | $2298 \pm 36$ | 2304 |

2.9 Hubble parameter data

From the reworking of equations in section 2.6 equation (10) became (31)

$$2H(z) = \frac{\dot{a}}{a} = -\frac{1}{(1+z)} \times \frac{dz}{dt}$$

i.e. the formulae for finding the Hubble parameter and $H_0$ are still valid, but the method measures twice the real expansion parameter, $2H$.

There is a problem however.  Time dilation has not been taken into account when applying the method.  The measured $dt$ would be lower than the real $dt$ by a factor $(1+z)$.  For example if it appears that 0.1Gyr has passed in the ageing of galaxies then really 0.2Gyr might have passed and the real $dt$ should be bigger.  This reduces $H(z)$ by a factor $(1+z)$, so instead of measurements showing an apparent $H(z) = H_0(1+z)$ relation, the data is compatible with a constant $H(z)$.

Table 3  Hubble parameter data

| $z$ | $H(z)$ | $H(z)/1+z$ |
|---|---|---|
| 0.199 | 75.0 | 62.6 |
| 0.24 | 79.69 | 64.3 |
| 0.35 | 82.1 | 60.8 |
| 0.57 | 92.4 | 57.8 |
| 0.6 | 87.9 | 54.9 |
| 0.73 | 92.3 | 53.3 |
| 2.34 | 222 | 66.5 |

Table 3 shows data from different groups, compiled by Meng [Meng, 2015].  Some of the data points (with lowest standard deviation compared to redshift) are chosen to illustrate the approximate $H(z) = (1+z)H_0$ relation, the errors in the middle column are up to $\pm 7$ kms$^{-1}$Mpc$^{-1}$

The method uses spectroscopic dating - often a spectral feature D(4000) [Moresco, 2020] to determine ages.  However galaxies and stars will evolve slower with time dilation and all features depending on their evolution will be affected, similarly with any age differences.  So the last column is showing us the real $H(z)$, or $2H$.
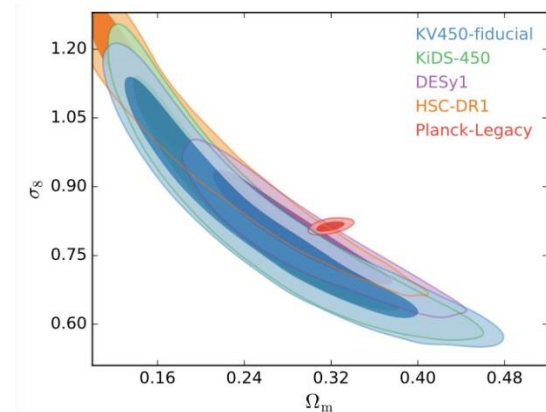
2.10 Measurements of the Hubble constant

There are some changes to the methods used to find the value of $H_0$. They are discussed in the next section.

2.11 Large Scale Structure

Figure 2 of section 1.11 shows an apparently constant value for $f\sigma_8(z)$ with little or no variation with redshift.  In the new model, $f(\Omega_m)$ is predicted to be constant at $-1 + \sqrt{5/2} \approx 0.5811$ (Appendix D).

Figure 7  $\sigma_8$ against $\Omega_m$ from cosmic shear



Data from KV450 and Figure 7 [Hildebrandt, 2018], show that $\sigma_8$ is about 0.81, so $f\sigma_8(z)$ is constant $\approx$ 0.47 a good match to the data of Figure 2.

The data for an apparently low matter density universe comes from different sources, but since they usually depend on values of $\Omega_b h^2$ or $\Omega_m h^2$, these matter densities could be four times as large, if the expansion parameter is halved. Supernovae data can be matched by the alternative model with constant $H$ and fewer variable parameters. These factors indicate that it might be possible to have a solution to the Friedman equations with a cosmological constant of zero. The alternative model also resolves the Hubble tension.

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### Section 3. The Hubble Tension

Over recent years there has been tension in the values of the Hubble constant measured locally or via an inverse distance ladder approach. For a review [Riess, 2020]. The tension has now reached $4.4\sigma$ with local measurements being higher, around 74 kms⁻¹Mpc⁻¹, whereas the inverse distance ladder method, with BAO, being lower around 67.3 kms⁻¹Mpc⁻¹

The value for $H_0$ is derived in three different ways, from CMB data, BAO and the local distance method. Some changes are needed but compatibility is found at 75-76 kms⁻¹Mpc⁻¹.

3.1 The CMB data of PLANCK and WMAP

From Planck data [Aghanim, 2018], $\Omega_m h^2$ = 0.1430 if we assume that $\Omega_m = 1$ then $h = 0.37815$ and $H_0 = 75.63$ kms⁻¹Mpc⁻¹

From WMAP [Hinshaw, 2012], the values are $\Omega_m h^2 = 0.1367$, if $\Omega_m = 1$ then $h = 0.3697$ and $H_0 = 73.95$ kms⁻¹Mpc⁻¹

3.2 BAO measurements

BAO measurements parallel to the line of sight constrain $H(z)$. Due to the changes made in section 2.6, equation (11) became equation (33) and we can determine the real expansion parameter by putting

$$\frac{c}{2H_{true}\sqrt{1+z}} = \frac{c}{H_{LCDM}(z)} \qquad (40)$$

This is because the LCDM modelling for $H(z)$ would have had to include the $\sqrt{1+z}$ factor, but the true value wouldn't.

The data point with lowest standard deviation [Aubourg 2015] is at $z = 0.57$, the modelling would need to pass near that point, so

$$2H_{true}\sqrt{1.57} = H_{LCDM}(0.57) \qquad (41)$$

from (7) with $\Omega_m = 0.321$, $\Omega_k = 0$

$$H_{LCDM}(0.57) = 67.3\sqrt{0.321(1.57)^3 + 1 - 0.321}$$

This in an approximation and depends on the parameters, the above are from Planck data [Aghanim, 2018], but there is some choice here.

BAO is not used alone to find $H_0$ the method is often used with supernovae or CMB data, but typically it's found that (41) makes $2H_{true}$ about 9 to 11% larger than previous BAO results, about $74 \pm 1.2$ kms⁻¹Mpc⁻¹

3.3 Direct local distance method

Riess [Riess, 2019] recently measured the value of $H_0$ directly and found 74.03 kms⁻¹Mpc⁻¹

Details of the calculations used are in [Riess, 2016]. The derived value for $H_0$ is proportional to

$$X = 1 + \frac{1}{2}(1 - q_0)z - \frac{1}{6}[1 - q_0 - 3q_0^2 + j_0]z^2$$

Equation (42), see also equations (4) and (5) of [Riess, 2016], $q_0$ is the deceleration parameter

$$q_0 = -\frac{a\ddot{a}}{\dot{a}^2} \qquad (43)$$

A value of $q_0$ of - 0.55 is used to determine the local value of $H_0$ and a jerk parameter $j_0 = 1$

In the new model from (14)

$$a = e^{-Ht}$$

so $q_0 = -1$ and the jerk parameter is still $j_0 = 1$

For the new model $X$ simplifies to

$$X_{new} = 1 + z \qquad (44)$$

For the local method it simplifies to

$$X_{local} = 1 + 0.775z - 0.27375z^2 \qquad (45)$$

X is then determined from 600 supernovae with redshifts between 0.023 and 0.15 (Figure 8 of [Riess, 2016], page 47). The average redshift is approximately $z = 0.06$ and from (44) and (45) $X_{new}/X_{local}$ at $z = 0.06$ is 1.01385

So the new estimate of $H_0$ is $74.03 \times 1.01385 = 75.06$ kms$^{-1}$Mpc$^{-1}$

3.4 A combined estimate for the Hubble constant

The amended data from the methods above and lensing data from H0LiCOW [Wong 2019] are summarised in Table 4.

Table 4  Values of the Hubble constant

| Method | The Hubble Constant (kms$^{-1}$Mpc$^{-1}$) |
|---|---|
| CMB (WMAP) | $73.95 \pm 1.4$ |
| CMB (Planck) | $75.63 \pm 0.29$ |
| BAO | $74.0 \pm 2.0$ |
| Local | $75.06 \pm 1.4$ |
| Lensing | $73.3 \pm 1.8$ |

The errors are subjective, depending on the complexity of the method. The errors on the CMB measurements have been lowered as now the only uncertainty is from the measurement of $\Omega_m h^2$, for Planck it's as low as 0.0011. For BAO the process seems to involve many uncertainties. The local method and lensing errors are unchanged.

It's clear that the amended local method and the amended CMB method, especially Planck, seem most accurate and agreement can be found at a value for $H_0$ of 75-76 kms$^{-1}$Mpc$^{-1}$.

**Summary**

It is possible that there has been a serious and longstanding misunderstanding of the redshift scale-factor relation, starting in about 1920. Since then a cosmology has been designed, with the best of intentions, based on General Relativity. It has needed modification, culminating nowadays with the LCDM model which has been successful in matching most observations. There are philosophical difficulties however, described earlier and also singularity problems, the coincidence problem and the Hubble tension.

The alternative model proposed has no cosmological constant hence no coincidence problem. Singularities are avoided by the strength of gravity , $G$, reducing for dense regions of matter. Then universe would be in dynamic equilibrium – collapsing matter 'bouncing' back out. The Big Bang would be one of the 'bounces'.

The strength of gravity is determined by the rate of 'expansion', in the new model the expansion applies equally to all lengths and quantities with length dimensions - space, physical constants and material objects too. The redshift is thus due to the energy of an emitted photon being lower in the past.

The advantages of the new model are as follows. It is simple philosophically, with time symmetry and scale symmetry. There is no need for a cosmological constant or inflation, no coincidence problem and no unsettled questions of why and when inflation or dark energy began or ended. Singularity problems are avoided and the cause of gravity and the value of $G$ is explained. There is a solution to the Hubble tension with $H_0$ being found, from all methods to be 75-76 kms$^{-1}$Mpc$^{-1}$.

The value of the expansion constant $H$, half of the Hubble constant, has no explanation in the new model, it remains constant and is a parameter of the model. The number of parameters is reduced.

We would have to come to terms with the notion that all length scales expand (not just space) and at first this might seem difficult to get used to. However by adopting that point of view a difficult question is also removed i.e. why in LCDM does space expand but matter doesn't ?

Changing our cosmological model, if that proves necessary, is not a quick or easy thing for us to do.  A reanalysis of data by experts in various fields of physics and astronomy would be needed.  A century of work would not be in vain however.   Many of the observations and experiments made during that time would be vital in checking and refining the model.  It is also an achievement for us to have reached the stage where it can be seen by experiment that our cosmology needs changing.  After a period of readjustment we may end up with a clearer understanding of our universe and the physical laws that govern it.

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## Appendix A.  The strength of gravity

The type of expansion proposed in this model ensures conservation of energy as the universe expands - (without any slowing of the expansion due to gravity).

Imagine a mass $m$,  it's rest energy varies during the expansion (in the absence of gravity) as

$$mc^2 e^{2Ht} \qquad\qquad (A1)$$

Energy would not be conserved.

With gravity included, the total energy of the mass varies as

$$\left(mc^2 - \frac{GMm}{R}\right) e^{2Ht} \qquad\qquad (A2)$$

where the second term represents contributions from the rest of the universe of mass $M$ and radius $R$.  Small numerical constants are omitted for simplicity.  Energy can be conserved if the quantity in the bracket of (A2) $= 0$.

$$G = \frac{Rc^2}{M} \qquad\qquad (A3)$$

Formula (18), from the Friedman equations can be regarded as a necessary condition.

$$G = \frac{3H^2}{8\pi\rho}$$

It is to conserve energy as the universe expands. Gravity and the value of $G$ is *caused* by the expansion, but does not change the rate of expansion, which is constant.

For a large stationary mass

$$\left(mc^2 - \frac{GMm}{R} - \frac{Gm^2}{r}\right)e^{2Ht} \qquad (A4)$$

and using (A3)

$$G_{reduced} = \frac{c^2}{\frac{M+m}{R}+\frac{m}{r}} = \frac{G}{1+\frac{Gm}{rc^2}} \qquad (A5)$$

a reduction in the strength of gravity for regions of matter where $m/r$ approaches $c^2/G$. These arguments give an indication that a future gravitational theory, based on General Relativity, should include the feature that the strength of gravity reduces for dense regions of matter.

The above formula is for a large stationery mass, and work is still ongoing to decide whether it applies to masses in motion in the same way. It's interesting to wonder however, that, as the earth gets nearer or further away from the sun over a year, whether a cyclic change in $G$ would be measured.

We would expect, from (A5), an annual cyclic variation in $G$ of amplitude $1.69 \times 10^{-10}G$ . In [Matsuo, 2013] such a variation of the Earth's gravity field has been noted, (Fig 1a) of Matsuo. It's from Satellite Laser Ranging (SLR) data - and has the same amplitude and period. These variations are being interpreted as being due to mass redistributions of ice and water.

**Appendix B. Binomial Expansions**

Binomial expansion for the new model, from (34)

$$D_M = \frac{2c}{H_0}(\sqrt{1+z} - 1)$$

omitting the $c/H_0$ and for small $z$

$$= 2\left(1 + \frac{1}{2}z - \frac{1}{4}z^2 - 1\right) \qquad (B1)$$

$$= z - \frac{1}{4}z^2 \qquad (B2)$$

For LCDM from (11)

$$D_M = \int_0^z \frac{c}{H(z)}dz$$

$$= \int_0^z \frac{c}{H_0\sqrt{\Omega_m(1+z)^3+\Omega_k(1+z)^2+\Omega_\Lambda}}dz \qquad (B3)$$

$$= \int_0^z (m(1+z)^3 + 1 - m)^{-1/2}dz \qquad (B4)$$

a flat universe approximation, again omitting the $c/H_0$ and using $m$ for $\Omega_m$ gives (B5)

$$= \int_0^z (m(1 + 3z + 3z^2 + \cdots) + 1 - m)^{-1/2}dz$$

$$= \int_0^z (1 + 3mz + 3mz^2)^{-1/2}dz \qquad (B6)$$

$$= \int_0^z \left(1 - \frac{3}{2}mz + \cdots\right)dz \qquad (B7)$$

$$= z - \frac{3m}{4}z^2 \qquad (B8)$$

Comparing (B2) and (B8) there is a match for low $z$ if $\Omega_m$ is 1/3. Most of the supernovae are at low $z$.

Higher values of $z$ have a match, (using equal Hubble's constants) at values under 1/3.

Using graphical software we can see that at $z = 0.5$ it's $\Omega_m = 0.25$, $z = 1.0$ it's $\Omega_m = 0.19$ and at $z = 1.5$ it's $\Omega_m = 0.155$ so we would expect supernovae data to give values for $\Omega_m$ of slightly under 1/3.

**Appendix C. Time symmetry and antimatter**

The universe in the new model is infinitely old. There are explosive events due to collapsing matter reaching the critical mass/radius ratio, as described in Appendix A and section 2. However there is no definite beginning of time. If we were to be able to film such a universe and then run the film backwards, it would look the same - on the largest scales. This type of universe obeys the 'perfect cosmological principle'. The universe is isotropic, homogeneous and will always look the same.

Other areas of physics have laws with time symmetry. If we go forward in time far enough - will the region of the universe near us collapse and then 'bounce' in a type of Big Bang?

In the cosmologically reversed time universe, matter collapses, and bounces back out in a colossal explosion (just as our Big Bang would have originated from collapsing matter). Stars galaxies and planets would form and life would evolve (as the fundamental laws of electromagnetism and atomic physics are time symmetric).

If we do a time reversal

$$t \to -t \qquad\qquad (C1)$$

$$H \to -H \qquad\qquad (C2)$$

the equations of the new model look the same, e.g. (14)

$$a = e^{-Ht}$$

equation (18) shows that gravity would still be attractive.

$$G = \frac{3H^2}{8\pi\rho}$$

The model is unchanged by a change of time direction.

To conserve the measured CPT symmetry, a time reversed universe would have a charge conjugation and a parity reversal. It would be an antimatter universe.

One of the problems of Big Bang cosmology is that we are left with the question of why there is more matter than antimatter.

In the new cosmology, there is no cosmological distinction between past and future. So there is no distinction between a $+t$ matter universe and a $-t$ antimatter universe. It is valid to claim that we are living in the $-t$ antimatter universe.

So perhaps the fact that we normally only observe matter isn't a mystery after all. Even in a time reversed universe, we would still observe time flowing forward and observe matter to be normal matter (instead of antimatter), there would be a redshift and gravity would still be attractive.

**Appendix D.   LSS - Growth factor**

From perturbation theory, in LCDM the growth factor $D$ is a solution of

$$\ddot{D} + 2H(z)\dot{D} - \frac{3}{2}\Omega_m H_0^2 (1+z)^3 D = 0 \qquad (D1)$$

In the new model $\Omega_m$ remains constant at 1.0

and $H(z) = H$

$$\ddot{D} + 2H\dot{D} - \frac{3}{2}H^2 D = 0 \qquad\qquad (D2)$$

for a solution of the form $D = e^{kHt}$

$$k^2 + 2k - \frac{3}{2} = 0 \qquad\qquad (D3)$$

$$k = \sqrt{(5/2)} - 1 \approx 0.5811 \qquad\qquad (D4)$$

$$f(\Omega_m) = \frac{1}{H}\frac{\dot{D}}{D} = k \qquad\qquad (D5)$$

so in the new model $f(z)$ is constant at about 0.5811