Malus's Law and Bell's Theorem with local hidden variables

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Abstract

A local hidden variables solution of Malus's Law and a circumvention of Bell's Theorem. The solution for Bell assumes that antiparticles actually do travel backwards in time and therefore a Bell experiment begins not at the usual source of particle pairs but at the measurement of positrons and ends at the measurement of their paired partner electrons. The solution for Malus's Law assumes that the distribution of hidden variables in a polarised beam can be deduced by reverse-engineering Malus's Law intensity calculations. Malus's Law and Bell's Experiment can therefore each be explained using local hidden variables. A computer program is given to provide results of a Stern Gerlach detector using local hidden variables in a particle-at-a-time simulation.

Introduction

Several difficulties arose in my previous attempts (Refs. 1 and 2) to simulate Bell experiments (Ref. 3) using local hidden variables.

A. Generating pairs of simulated particles in Reference 1 (with local hidden variables being vectors representing particle spin) using a random-on-a-sphere method did **not** lead to a breaking of Bell's Inequalities. As the name suggests, random-on-a-sphere data produces an *unpolarised* beam with a random distribution of spin vectors pointing from the origin of a sphere to points on the surface of the sphere.

In the simulation in Reference 1, the correlation coefficient between Alice's and Bob's measurements was 0.4994 (ignoring arithmetical sign) for a total of one million pairs of particles. This simulation was carried out for a single pair of detector settings with a difference of 45^o separating the two detector setting angles. This correlation fails to break Bell's Inequality and would need to reach a value of 0.707 to achieve the quantum mechanics target correlation. By comparison, in a CHSH Bell experiment there are four pairs of detector setting per analysis and my result of 0.5 above is equivalent to CHSH S statistic of 2 whereas an S value of 2.8 is required as the quantum mechanical target.

B. The simulation for Bell's experiment in Reference 1 can also be used for a test of Malus's Law (Refs. 2 and 4). Malus's Law calculations, however, should be based on an incoming beam of polarised particles and the random-on-a-sphere method of generating particle hidden variables will not produce a polarised beam. It is shown in Reference 2 that a Malus intensity of 0.75 would be equivalent to a Bell quantum mechanics correlation of 0.5. To be equivalent to a Bell correlation of 0.707 the Malus intensity would need to be 0.8536. This can be seen because, for electrons, the Malus intensity at an angle of 45 degrees is the same as an intensity for an angle of 22.5 degrees for photons, which is $\cos^2 22.5^o = 0.8536$ (using the two-hundred-year-old Malus's Law empirical calculation). This means that the results for an actual, and successful, Bell experiment and for Malus's Law are equivalent when a polarised beam is used for Malus's Law.

C. So how can a successful result for a Bell experiment possibly be achieved without using an inputted polarised beam for each of Alice and Bob? And how does one generate a polarised beam in a computer simulation of either a Bell or a Malus experiment?

Summary

A huge assumption in this paper is that positrons physically do travel backwards in time. The effect of the time reversal is to dissociate 'entanglement' from the A and B measurements of Alice and Bob in a Bell experiment, so these measurements are no longer based on entangled pairs of particles.

The entanglement is still present in the experiment but is only important in creating a polarised antiparticle beam travelling from Alice to the Source which evolves into an equally (but anti-)polarised electron beam from the Source to Bob. The Bell experiment defaults to a Malus experiment with an added measurement of the unpolarised (time reversed and incoming) beam of positrons. Because it is, in this view, so directly connected to a Malus experiment, there is little surprise that the results break Bell's inequalities (see Tables 1 and 2). The Bell inequalities are based on experiments with entangled pairs whereas the new explanation of the experiment circumvents the inequalities by showing that the measurements are not made on entangled pairs and the results are caused by measurements on **polarised** beams. Entanglement is only important because it enforces polarised beams of electrons.

Probability is an essential feature of the use of local hidden variables in the Malus's Law calculation and this feature means that counterfactual definiteness should not be true for Bell experiments with time-reversal, and also why the Quantum Randi challenge cannot be met. But local hidden variables nevertheless explain Bell Experiment results, with some intrinsic lack of calculability during their times of flight in the experiment.

A computer program is shown in the Appendix which generates local hidden variable data made particle-at-a-time for a polarised beam of electrons which is inputted into a Stern Gerlach detector. The program counts the electrons passing through the detector and shows that the count gives an intensity of beam which agrees with Malus's Law. Although the calculation is for individual electrons, and that gives the correct results, it is probable that the static distribution of hidden variables (Figure A) masks an underlying dynamic occupation of that distribution by the individual electrons so that it is ultimately a probabilistic result which leads to uncertainty as to which hidden variables are passing through the detector/filter.

Polarised beams and Malus's Law calculations

Distribution of particles within a polarised beam

A polarised beam is a collection of particles after they have passed through a filter or a detector, such as a Stern Gerlach detector. The degree of uniformity of alignment for each particle depends on how much effort is put into controlling/measuring the beam. For the purposes of this paper it is assumed that in a polarised beam the particles have local hidden variables which are vectors with a range of spin axes centred on a vector axis of polarisation. The Malus's Law formula can be used to reverse engineer the distribution of spin axes in an incoming polarised beam.

In 1808 Malus showed empirically that the intensity of plane polarised light after being passed through a polarising filter at an angle of θ to the plane polarised light had an intensity equal to $\cos^2 \theta$ times the original intensity (Ref. 4, Wikipedia). It is assumed here that Malus's Law is experimentally correct and that the intensity of the filtered light is dependent on the number of photons passing through the filter.

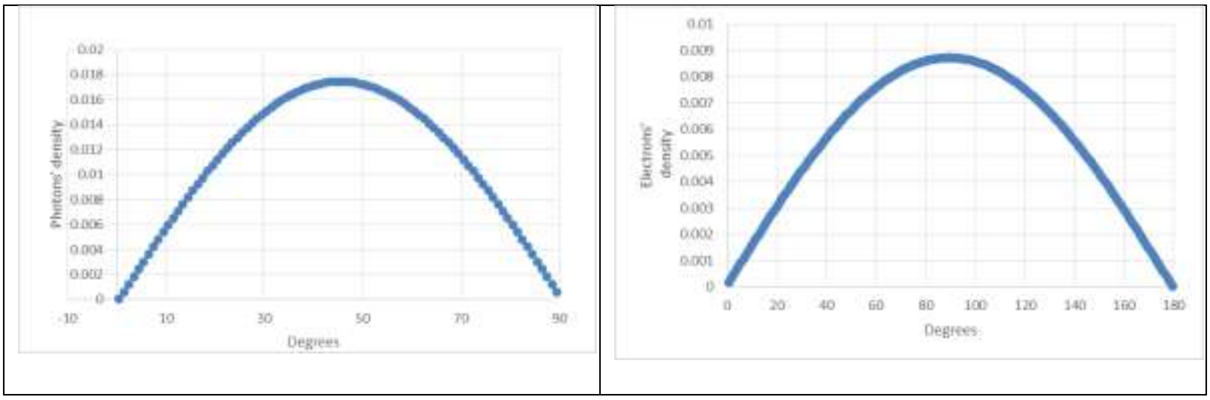
For Malus's Law, if vector **a** is the initial polarisation direction of a beam of photons, and if vector **b** is the polarisation direction of the subsequent filter, then if angle **a** – **b** is 45^o there will be a proportion equal to $\cos^2 45^o$ [that is, 0.5, or 50 per cent] of the beam passing through the filter. When angle **a – b** is 90^o no photons will pass through the filter as $\cos^2 90^o$ is zero. The simplest way to visualise this is if the initial polarised beam of photons has a spread of spin axes pointing not over an entire hemisphere but over a cone taking in the polar region down to the 45^o parallel, or circle of latitude, by analogy with the earth's globe. This cone subtends (in a two-dimensional projection) an angle of 90^o at the central point of the hemisphere.

For electron beams, the corresponding angle in Malus's Law is halved, so that when angle **a – b** is 90^o then $\cos^2(90^o/2) = 0.5$ which allows half of the electrons through the filter. For electrons, the modified Malus's Law becomes Intensity = $I_o \cos^2 (\theta/2)$, where I_o is the initial intensity. The visualisation for the electrons is a beam occupying a full hemisphere subtending a (two-dimensional) angle of 180^{o} at the centre of the hemisphere.

It has already been noted above that a random distribution of local hidden variables does not explain Malus's Law using my simulations so now the actual distribution of hidden variables of a polarised beam needs to be investigated. Fortunately, the Malus's Law calculations can indicate the distributions of local hidden variables in polarised beams. The resulting distributions are shown in Figure A.

Figure A **Plots of the intensity of particles within polarised beams for photons (polarised along polar axis represented by 45^{o}) and electrons (polarised along polar axis represented by 90^{o})**

Photons Electrons



In Figure A, the intensity of electrons at angles 0^{o} or 180^{o} is zero which means that there are very few electrons with hidden variables pointing at or near the equator of the hemisphere. An angle of 90^{o} represents hidden variables pointing at the (say) north pole, and this is the region for maximum intensity and is along the vector of polarisation. For photons, the intensity is minimum for hidden variables pointing at 0^{o} or 90^{o} which are photons pointing at the 45^{th} parallel or circle of latitude. The maximum intensity is reached for photons pointing direct along the axis of polarisation, here represented by the direction of the north pole which, for photons, is represented by the angle of 45^{o}. These curves are proportional to the derivatives of $\sin^2 \theta$ and $\sin^2 (\theta/2)$ respectively: they are $\sin 2\theta$ and $0.5 * \sin \theta$ respectively. (Further explanation of these formulae are given at the end of the Appendix.) These curves can be used in a simulation of a Malus Experiment to represent the distribution of local hidden variables in a polarised beam of photons or electrons.

Polarised particles passing through a filter at angle θ

In the ‘electrons’ diagram in Figure A, the distribution of electrons is at a maximum when the hidden variables point at the north pole (at 90^o). Say a filter is set at an angle of 10^o to the pole and we wish to see how many of these electrons pass through this filter. To see this, rotate the electron figure in Figure A by 10^o clockwise and see where the polarised and filtered areas overlap. The electron spins originally pointing between 0^o and 10^o for the initial polarised beam are no longer pointing in the 0^o to 180^o range of the filtered beam and so do not pass through the filter. All the other electrons in the polarised beam are pointing within the hemisphere of the filter beam and so all polarised electrons pointing between 10^o and 180^o pass through the filter. This amount can be found by integrating between 10^o and 180^o on the intensity curve in Figure A or simply by calculating $\cos^2 (10^o/2)$ using Malus’s Law for electrons, which gives a cumulative total.

The same procedure works for photons. In general, if the filter is rotated θ degrees clockwise away from the polarisation pole vector then the amount of photons passing through the filter is found by integrating the curve between θ and 90^o or by calculating $\cos^2 \theta$.

Distribution of polarised particle spin vectors after passing through a filter at angle θ

After passing through the filter, the particles will redistribute to be as in Figure A but with the maximum intensity now pointing along the filter vector angle instead of along the initial polarisation angle. It is not known how the particles rearrange themselves into this pattern. This is the point at which knowledge about the evolution of individual local hidden variables breaks down. After the filtering, or a measurement, there is only statistical information about the distribution of hidden variables for the filtered beam as a whole. It is also assumed here that even in the initial polarised beam there may be an underlying and unknown dynamic for individual particles. The overall patterns in Figure A may be static but the picture for individual particles may be dynamic. This dynamism means that there is uncertainty as to which individual particles would pass through a second filter. Moreover, even if we were to know that an individual particle passed through a filter, we could not later re-identify that particular particle’s hidden variable in the filtered beam and so cannot track an individual simulated particle’s local hidden variable throughout the measurement.

Computer program to simulate a particle-at-a-time Malus's Law experiment using local hidden variables

Code for a computer program is given in the Appendix to simulate Malus's Law results for passing a beam of particles through a filter or a Stern Gerlach detector. More details of the program are also given in the Appendix. Rather than just use Malus's Law formula direct, the formulae underlying Figure A have been used to generate distributions of local hidden variables in a polarised beam in a particle-at-a-time simulation. The outcomes are shown in Table 1.

Table 1 Simulation of Malus's Law results for electrons, with filter angle of θ

Polarisation angle * θ^{o}	Numbers of electrons passing through filter, out of an incident 100000 electrons in a simulation	Malus intensity $= \cos^2(\theta /2)$ (or $0.5 + 0.5*\cos\theta$)	Equivalent Bell correlation = -1 + 2 * Malus intensity (or simply $\cos\theta$)
30	93377	0.933	0.866
45	85627	0.854	0.707
60	75007	0.75	0.5
90	50098	0.5	0

* To get the equivalent complete table for photons, divide the polarisation angle, in the first column only, by 2.

Although local hidden variables are generated for individual simulated particles in the computer program, it is likely that the individual particles have variables which are not static. This introduces probability into the results and implies non-calculability of maintaining track of spins of individual particles.

In the next section, Bell's Experiment results will be shown to depend on Malus's Law. That is why it was thought to be important to derive Malus's Law here based on local hidden variables in a particle-at-a-time simulation so that local hidden variables could be the basis for an explanation of Bell's Experiment as well as Malus's Law.

A re-casting of the time order of events in a Bell's Theorem experiment

The re-casting of the time direction for antiparticles

In a Bell Experiment two researchers, Alice and Bob, make sets of measurements A and B independently on the spins of entangled pairs of particles (Ref. 3). The particle pairs are created at a source and are entangled as they travel to the detectors of Alice and Bob. The measurement results of Alice and Bob are put into contingency tables after the experiment and correlation coefficients are calculated. For a single pair of detector settings, a typical difference in detector angles used is 45^o. Bell's Theorem shows that, when using local hidden variables to represent particle spin axes, this correlation cannot exceed 0.5 in absolute magnitude. But actual experiments result in a correlation coefficient of up to 0.707.

One way of having polarised beams in a Bell experiment is to assume that antiparticles travel backwards in time. So that when Alice measures positrons, they have travelled from outside the normal scope of the experiment and then became polarised on measurement by Alice, after which (in reverse time) they travelled back to the Source of the emission of the pairs. The positrons give rise to electrons at the Source and the electrons are also polarised (due to entanglement) along Alice's spin vector setting. So Bob measures electrons polarised in the direction of Alice's setting. In the converse situation, Alice similarly measures electrons polarised in the direction of Bob's setting. Although entanglement is necessary to enforce the polarisations, the cause of the breaking of Bell's Inequalities is **polarisation** of the incoming electron beams. Entanglement, in this setting, plays an important but secondary role. As the beams of electrons are polarised there is no difficulty in explaining results which break Bell's inequalities as Malus's Law provides such results as indicated in Table 1. In Table 1 the Malus results for a detector setting of 45^o for electrons is shown to be equivalent to a Bell correlation of 0.707, which exceeds the Bell Inequality limit of a correlation of 0.5. But Bell's Inequalities are more circumvented than broken as the measurements are not made on a pair of entangled particles. The electrons are entangled and polarised on measurement but the partner positrons are not polarised and not entangled on their measurement.

A traditional view of a Bell experiment or simulation starts with the creation of a particle-antiparticle entangled pair. The two particles undergo a time of flight towards researchers Alice and Bob who each measure one of the entangled pair with measurement apparatuses set at angles a and b respectively to a given reference axis. Alice makes measurement A and Bob makes measurement B where A and B take the values (+1, +1) or (+1, -1) or (-1, +1) or (-1, -1). This is repeated for a population of particle pairs. The correlation coefficient between the set of A and B experimental values is given by –cos (a-b) verified using experiments. This correlation is in general larger (in absolute magnitude) than what might be expected in classical physics, which corresponds to the value given by $2*\theta/\pi - 1$ on a zig-zag curve, where θ is in radians. This exceeding of the classical expectation involves breaking Bell's Inequalities. It is commonly required that the breaking of the inequalities requires one or more of the following conditions to fail: locality, reality, counterfactual determinism, and no-conspiracy. (Ref. 3, Wikipedia)

The purpose of this paper is not to deny Bell's Theorem as the truth of that Theorem is here accepted. For simulation calculations, though, it is simpler to use a quarter of the normal Bell experiment in which Alice makes measurements only at a single angle a and Bob makes measurements only at a single angle b. Where the optimum value of a-b is 45 degrees. In a CHSH experiment, with a-b= 45 degrees, using a quarter experiment revises the target experimental S statistic to 0.5 * sqrt 2 (= 0.707) which is larger than the classical result of 0.5. This result is equal to the absolute magnitude of the correlation between Alice's and Bob's measurements. In simulations one can alternatively use cloned pairs of particles and a target correlation of $\cos\theta$ rather than $-\cos\theta$ so the overall sign of the target correlation is dependent on context.

Let us assume that a positron is travelling backwards in time and say a photon with, say, +1 spin is also travelling backwards in time. The electron and the photon with spin -1 both travel forwards in time. (See Discussion section.)

The time-reversed Bell experiment starts with incoming antiparticles measured as +1 by, say, Alice. Alice will in addition measure some antiparticles as -1 and also Bob will measure some antiparticles as +1 and others as -1. But staying with Alice's measurements of +1 antiparticles, these antiparticles travel backwards in time to the 'source' of the pairs. Next, electrons travel forwards in time from the Source to Bob and these electrons are entangled with their antiparticle positron partners such that their local hidden variables will be the same for the particles as for the antiparticles except for a sign change in their spin vector directions. This beam of particles will be polarised in the vector **a** direction, as the partner antiparticles will be polarised in the –**a** direction.

As noted in the section on Malus's Law, local hidden variables cannot be identified for an individual particle after a measurement, even in a computer simulation. Alice has made a measurement on a positron and Bob has made a subsequent measurement on its paired electron and so individualised local hidden variables cannot be used here in a particle-at-a-time simulation. However, this experiment is treatable as a local hidden variable situation where the hidden variables do exist but are not calculable for individual particles. The particles are treated probabilistically in their distributions of hidden variables before and after measurement and this is the same probabilistic treatment as used in the Malus's Law section.

Reversed-time Bell experiment results expressed as intensities and as correlation coefficients

Alice measured A = +1 for her half of her antiparticles. These antiparticles were precursors of a beam of particles travelling towards Bob as a beam polarised along vector -**a**. Malus's Law shows that the intensity of the beam measured by Bob is $\cos^2(\theta/2)$ for electrons where θ = angle a – b. This value is needed for the cell of Table 1 where A = +1 and B = -1. As noted earlier, Alice's measurements of A = +1 on her antiparticles form one quarter of the total measurements so she contributes $0.25*\cos^2\theta/2$ to this cell of Table 2. Similarly Bob's measurement of B = -1 on his antiparticles contributes another $0.25*\cos^2\theta/2$ to this cell making a total of $0.5*\cos^2\theta/2$ for the final cell value. Table 2 only has one degree of freedom as the marginal values are fixed, so the other three cells of results are filled in automatically once one cell value is known.

Table 2 Bell results for electrons with a – b = θ (using a negative correlation target)

Results as proportions	B = 1	B = -1	Total
A = 1	$P_{++} = 0.5*\sin^2\theta/2$	$0.5*\cos^2\theta/2$	0.5
A = -1	$0.5*\cos^2\theta/2$	$0.5*\sin^2\theta/2$	0.5
Total	0.5	0.5	1.0

It is easy to show that p_{++} = (1 + correlation coefficient)/4 and, conversely,

the correlation coefficient = $4 * p_{++} - 1$

So, the correlation coefficient between A and B measurements = $4 * 0.5* \sin^2\theta/2 - 1$

$= 2* \sin^2\theta/2 - 1 = (\sin^2\theta/2 - 1) + \sin^2\theta/2 = \sin^2\theta/2 - \cos^2\theta/2 = -\cos\theta.$

This agrees with the Quantum Mechanical result for the correlation in a Bell test and exceeds the attenuated correlation found for the classical calculation where the Bell's Inequalities are not exceeded.

Note that the intensity in a cell of a 2x2 Bell table is only half of the directly equivalent intensity in the cell in a 2x1 Malus table.

Discussion

My paper (Fearnley, 2017) gives a computer program to simulate a particle-at-a-time Bell experiment for electrons where $\theta = 45^o$. Generating local hidden vectors to simulate individual particles failed to produce a correlation of $-\cos 45^o$ in a particle-at-a-time simulation of a Bell experiment for electrons. That paper showed that Bell's Inequalities were not broken in that random-on-a-sphere simulation of a Bell Experiment.

My paper (Fearnley, 2019) links results of simulations of Bell's Experiments to results from Malus's Law and shows that, despite interesting interconnections of the tables of results for the two experiments, the Bell's inequalities were still not broken in particle-at-a-time local hidden variable simulations and also that Malus's Law was not explained by such a local hidden variable simulation using random-on-a-sphere unpolarised data. The latter failure is not surprising as Malus's Law requires the inputted beam to be polarised which is not the case for random-on-a-sphere simulated data.

The Feynman and Stueckelberg interpretation proposed a positron to be a positive energy electron travelling backwards in time. A positive energy electron travelling backwards in time can also be interpreted as a positive energy 'electron' (that is, a positron) travelling forward in time but with a reversed electric charge. Feynman also used advanced and retarded (in time) waves in his PhD thesis (Refs. 7 and 8) although this is not generally regarded as a physical reversal of time.

An assumption in my paper is that positrons physically do travel backwards in time. The effect of the time reversal is to dissociate 'entanglement' of pairs from the A and B measurements of Alice and Bob in a Bell Experiment. In the time-reversed experiment, Alice's measurement does not depend on the path between the Source and her detector apparatus, as that path is reached by the antiparticles after her measurement A. So the measurements A and B are not measured on entangled pairs in the time-reversed experiment.

The entanglement is important so that the polarised nature of the antiparticle beam travelling from Alice to the Source after measurement A evolves into an equally (but anti-)polarised particle beam from the Source to Bob. So Bob, who measures at spin angle vector **b**, receives a beam of electrons polarised in direction vector **a**. The distribution of local hidden variables for positrons before they arrive at Alice is completely unknown. They can be assumed to be unpolarised and described by 'random-on-a-sphere' and that half of the antiparticles will be measured as +1 by Alice and the remainder will be measured to be -1.

I have not been able to devise an experiment to demonstrate which way the positrons are travelling in time. To determine the direction of travel through time requires proving that an incoming beam is unpolarised, and unpolarised beams always pass 50 per cent of the beam through a filter at any polarisation angle. All outgoing beams are polarised. Of course incoming and outgoing are time-direction dependent terms themselves. The demonstration would need to be carried out ideally on a beam of antiparticles. Is a beam of antiparticles, coming (in our macroscopic time direction) from a source of their creation, polarised or not?

In the time-reversed explanation of the Bell Experiment, entanglement is only used to provide a beam of electrons polarised along vector **a** to be subsequently measured by Bob along vector **b**. So the Bell experiment defaults to a Malus experiment with an added measurement of an inputted unpolarised beam of positrons. Because it is, in this view, so directly connected to a Malus experiment, there is little surprise that the results break Bell's inequalities. The Bell inequalities are based on experiments with entangled pairs whereas the new explanation of the experiment circumvents the inequalities by showing that the measurements are not made on entangled pairs. (See Tables 1 and 2.)

Measurements, A and B, are correlated to obtain a Bell Experiment result but hidden variables cannot be followed/calculated through a measurement and so an individualised and trackable local hidden variable simulation cannot be made for a Bell experiment. Probability is introduced into the Malus's Law calculation and this feature means that counterfactual definiteness should not be true for Bell experiments with time-reversal, and also why the Quantum Randi challenge cannot be met.

With respect to a second assumption, the distributions of local hidden variables or spin axis densities in a polarised beam (Figure A) have fixed density patterns but a fixed pattern could be based on a static underlying cloud of particles or a dynamic aggregate of underlying particles. Whichever is the case, an individual particle's local hidden variable cannot be tracked through a measurement because probability is required in deciding the outcome of the measurement. Outcomes of measurements are calculable for aggregates but not for individuals.

An unusual assumption in the paper is that a photon with spin +1 is the antiparticle of a photon with spin -1. Reference 6 (Fearnley, 2019), however, details his preon model in which these two entities are exact antiparticles of one another.

Conclusion

1. In conclusion, the results of a time-reversed Bell experiment is explainable using a probabilistic treatment of local hidden variables.

2. The empirical formula for Malus's Law has been used to reverse-engineer a distribution of local hidden variables of particles in polarised beams. A Visual Basic (MS Excel) computer program has been created using a particle-at-a-time simulation to replicate Malus's Law results, using local hidden variables. Although described as particle-at-a-time, the calculation uses an element of probability such that the destination ('filtered in' or 'filtered out') of an individual particle after encountering the filter is not calculable despite the existence of the local hidden variables. This software can be used to calculate intensities of polarised beams passing through Stern Gerlach detectors. (The Appendix gives the software coding and an explanation of the formulae used.)

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References

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APPENDIX Computer program to produce results for Malus's Law intensities using local hidden variables in a particle-at-a-time simulation

See end of program for an explanation of the formulae and method used. Any text on a line after an apostrophe is only a commentary and not executed by the program. This simulation is for electrons, not for photons, but it is easy to calculate results for photons direct from those for electrons.

```
Sub AJFModel()
' ------------------------------------------------------------------------------------------------------------------
' Microsoft Excel visual basic program to produce results for Malus intensities using
' local hidden variables in a particle-at-a-time simulation
'
' Simulates a Stern-Gerlach detector
' ------------------------------------------------------------------------------------------------------------------
' PRELIMINARY WORK:

' Set the dimensions of variables, their sizes and types
  Dim nb As Long  ' B measurement
  Dim np, nm As Long ' counts of filtered particles
  Dim ithparticle As Long   ' ithparticle is the index of the ith particle
  Dim TotalNoOfParticles As Long   ' a long integer used to store the total number of particles to be generated in one run
  Dim x, ya, za, yb, zb, length, meanB, total As Double  '  double length real numbers
  Dim theta, cumprobability, pi, p, pp As Double   '  double length real numbers

  Randomize ' this randomizes the random-number generator, to avoid using the same set of random numbers in every run

' Define constants to be used in the program
  TotalNoOfParticles = 100000 ' this sets the total number of particles which are to be generated   <<<*******INPUT
  pi = 3.1415926535        ' pi radians is equivalent to an 180 degree angle

' Set angle theta (angle in 2D between incoming polarisation angle and the polarisation angle of the filter)
  theta = 45  ' This inputs the polarisation angle to be used in this run, in degrees           <<<*******INPUT
  theta = theta * pi / 180  ' this converts the polarisation angle from degrees to radians

' Set counters to zero before use
  np = 0   ' count of number of particles passing through the filter
  nm = 0   ' count of number of particles failing the filter

' Pick a polarisation vector for detector b (Bob).  This is the filter vector.
' Bob's unit vector has components yb and zb calculated to give theta.  Theta was inputted earlier.
' Alice's vector a is represented by a constant unit vector along the Z axis so za = 1 and ya = 0, in all runs.
' The 2D plane used here is the (y,z) plane.
        ya = 0
        za = 1
        yb = Sin(theta)
        zb = Cos(theta)

  x = ya * yb + za * zb   'a step in the calculation of angle between spin vectors a and b, intended to be theta.
  theta = Atn(-x / Sqr(-x * x + 1)) + 2 * Atn(1)   '  excel's formula to give the arccos function
' Need now to turn theta (decimal in radians) into IntegerTheta (an integer angle in degrees)
  theta = theta * 180 / pi

' END OF PRELIMINARY WORK
' -----------------

'  -----------------
```

```
' MAIN PART OF PROGRAM

' Generate particles in a for/next loop
  For ithparticle = 1 To TotalNoOfParticles

' generate cumulative probability value at random between 0.001 and 0.999
  cumprobability = Rnd  ' cumulative probability is a random number on a uniform distribution between 0 and 1
  p = cumprobability
  pp = 1 - 2 * p  ' see end of program for an explanation of formulae

  Angle = Atn(-pp / Sqr(-pp * pp + 1)) + 2 * Atn(1)   ' excel's formula to give the arccos function in radians
  nb = -1     ' alternatively value could be set at 0 instead of -1.  For a particle which fails the filter

  If Angle > theta * pi / 180 Then nb = 1  ' for a particle which passes through the filter

' INCREMENT THE PLUS AND MINUS ARRAY COUNTERS FOR THE iTH PARTICLE
  If nb < 0 Then nm = nm + 1  ' accumulator for particles which  fails to pass the filter
  If nb > 0 Then np = np + 1  ' accumulator for particles which pass through the filter

10 Next ithparticle  ' jump to top of FOR/NEXT loop for generation of the next particle
'END OF MAIN PART OF PROGRAM  --------------------------------


' WRITE RESULTS TO SPREADSHEET ' set which row and column will be used as starting point to write results to spreadsheet
   Range("a1").Select

   ActiveCell.Offset(0, 1).Formula = "theta = "
   ActiveCell.Offset(6, 2).Formula = "frequencies"
'  A =1 always in a malus experiment.  Indicating particles have initially been polarised by Alice.
'  So the measurement, B, at the filter is made on an already polarised beam.
'  Calculation of B.  B = 1 if the particle passes the filter and zero if not.
   ActiveCell.Offset(6, 3).Formula = "B = 1"
   ActiveCell.Offset(6, 4).Formula = "B = 0"    'using zero rather than -1
   ActiveCell.Offset(6, 5).Formula = "Total"
   ActiveCell.Offset(12, 2).Formula = "mean of B"    'using zero rather than -1
   ActiveCell.Offset(15, 2).Formula = "Total N of particles"
   ActiveCell.Offset(20, 2).Formula = "cos squared (theta/2) ="
   ActiveCell.Offset(21, 2).Formula = "notional Bell correlation ="

   ActiveCell.Offset(0, 2).Formula = theta
   ActiveCell.Offset(7, 3).Formula = np        '  A=1 and B=1
   ActiveCell.Offset(7, 4).Formula = nm        '  A=1 and B=0
   ActiveCell.Offset(7, 5).Formula = np + nm    'total number of particles
    total = np + nm
    meanB = np / total
   ActiveCell.Offset(12, 3).Formula = meanB
   ActiveCell.Offset(15, 3).Formula = total
    theta = theta * pi / 180
   ActiveCell.Offset(20, 3).Formula = Cos(theta / 2) * Cos(theta / 2)    '  Malus formula applied direct
   ActiveCell.Offset(21, 3).Formula = 2 * Cos(theta / 2) * Cos(theta / 2) - 1  '  the equivalent Bell correlation

' NOW USER CAN GO TO RESULTS SPREADSHEET TO INSPECT 2X1 MALUS TABLE AND EQUIVALENT BELL CORRELATION


' EXLANATION OF FORMULAE

' See Figure A of the report: "Malus's Law and Bell's Theorem with local hidden variables".

' Malus's law uses a formula for the intensity of a polarised beam of electrons passing through a filter at an angle
' phi as being proportional to cos squared (phi/2) where phi is the angle between the polarisation vector of the
' incoming beam and the polarisation vector of the filter.
'
' A perhaps simpler way of writing cos squared (phi/2) is 0.5 + 0.5 * cos phi.
```

```
'
'  This function is high valued near to zero degrees because a small angle of phi (between beam and filter) passes many
'  particles.  But this means that there are few particles incoming with local hidden variables (or spin vectors)
'  pointing away from the incoming polarisation vector.  This means that electrons with spin vectors pointing at zero or
'  180 degrees in Figure A are scarce while the bulk of them have a central tendency pointing at around 90 degrees,
'  which is the direction of the polarising vector.
'
'  Another way of seeing this is that the intensity for angle theta can be derived by integrating the distribution of
'  individual spin vectors between theta and 180 degrees in Figure A.
'
'  To find the distribution of individual spin vectors (at angles of  phi) is is necessary to differentiate not
'  cos squared (phi/2)
'  but 1 - cos squared (phi/2),
'  and 1 - cos squared (phi/2) = 0.5 - 0.5 * cos phi
'  which differentiates to  0.5 sin phi.
'  This is the curve which is plotted for electrons in Figure A of the report: "Malus's Law and Bell's Theorem with
'  local hidden variables".
'
'  Next we need to know how this curve was implemented in this simulation.
'  Spin vectors for particles have been generated at random.
'
'  First, by generating a random number between 0 and 1 to represent a random cumulative probability of a particle
'  lying on the spin vector distribution curve.  Next, we need to find out what the spin vector angle (psi) is, for the
'  generated particle, corresponding to that random cumulative probability (p).
'
'  If we integrate the electron spin vector density 0.5 * sin x  between x = 0 and x = psi, we get 0.5(-cos psi - -cos(0)).
'  = -0.5 * cos psi +0.5.
'
'  Set this integral to be p, the random cumulative probability for the generated electron.
'
'  So  p = -0.5 * cos psi +0.5
'  and hence cos psi = 1-2p.
'
'  and psi is the angle whose cosine is 1-2p.
'
' That is commonly called arccos(1-2p)
'
' But in MS visual basic the arccos function does not exist and so the function used is a convoluted, but standard,
' combination of arctan functions.
'
'  So now we have a particle with a known spin vector angle psi.  If this angle is greater than theta then the particle
'  passes the filter, where theta is the angle between the incoming polarisation vector and the filter spin vector axis.
'
'  This software can be used to find intensities of polarised beams passing through combinations of Stern Gerlach
'  detectors.  This software does not deal with unpolarised beams but any unpolarised beam is known to pass 50 per cent
'  of its particles through a filter set at any spin angle.
'
'  These formula and the distribution of individual particle's spin vectors in Figure A are two dimensional.  The
'  polarising filters are set in a real experiment using three dimensions.  A two dimensional treatment is satisfactory
'  however because the three dimensional distribution of spin vectors in a polarised beam is symmetrical about its
'  polarisation axis.  Take any 2D plane where the polarisation spin vector lies completely on that plane and the
'  distribution of particle spin vectors has 2D form 0.5 * sin theta.  So in a sequence of 3D polariation vectors in
'  a Stern Gerlach experiment, each succeeding filtered intensity can be treated in the 2D plane defined by containing
'  both incoming and outgoing polarisation vectors.

End
End Sub
```