The Huygens Formula is Exact for a Very Large Angle The Moon-Earth System as a Gigantic Pendulum Clock

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Abstract

In this paper, we look at the Earth and Moon as a gigantic pendulum clock. To find its periodicity, we used Huygens pendulum periodic calculations. The Huygens formula should supposedly only hold for a very small angle, but remarkably, we find it to be accurate for a very large angle clock, namely 360 degrees. The Huygens formula predicts the orbital period of the Moon to be approximately 27.42 days, which is only about 0.3% off from the official number of the Moon's orbital period. Actually, the Huygens formula is exact for a very large angle, which is equal to 360 degrees.

Key words: Huygens pendulum formula, Huygens clock, Small angle approximation, Earth, Moon.

1 Introduction

Huygens [1] was likely the first to derive the formula for the period of an ideal mathematical pendulum

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{1}$$

where L is the length of the pendulum and g is the gravitational acceleration. The gravitational acceleration at the surface of the Earth is known experimentally to be about $g \approx .9.81\ m/s^2$. From modern Newtonian gravity, including the gravitational constant G, we also know that the gravitational acceleration is given by

$$g = \frac{GM}{R^2} \tag{2}$$

The Huygens pendulum clock derivation is assumed to be a good approximation only for a small angle. However, it also seems to be accurate for very large angles, as we will see.

Here we will consider the Earth and Moon system as a pendulum clock. The Moon will be the end of the pendulum. The length of the pendulum is then the distance from the Earth to the Moon, which is about 384,400 km. Next, we need the gravitational acceleration from the Earth at this distance; for this we also need the mass of the Earth, which is approximately 5.972×10^{24} kg. It is given by

$$g = \frac{GM_E}{R^2} = \frac{G \times 5.972 \times 10^{24}}{384400000^2} \tag{3}$$

So, we have that the pendulum period of the Earth-Moon clock is

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{384400000}{\frac{G\times 5.972\times 10^{24}}{384400000^2}}} \approx 2369422\ seconds$$

There are $24 \times 60 \times 60 = 86400$ seconds in a 24-hour period, so the pendulum time is $2369422/86400 = 27.42$ days in this pendulum clock, which is very close to the official numbers for the number of days the Moon uses to travel around the Earth. Interestingly, this also shows that the small angle approximation is also very accurate for a 360 degree angle and that it is also very accurate when we are close to 360 degrees.

The Moon moves around the Earth in 360 degrees. The Huygens formula is assumed to be an approximation because for small angles, $\sin[\theta] \approx \theta$. However, I have talked with several physicists who do not seem to understand that even if the Huygens formula is not accurate for a large angle, it is amazingly exact for a very large angle, e.g., one equal to 360 degrees. This is because we have

$$\sin[\theta = 0] = \sin[\theta = 360] = \theta$$

Thus, for a 360 angle (2π), or a full circle, the Huygens formula is not only a good approximation, it is exact. If the Moon had moved in a perfect circle around the Earth and was not influenced by other gravity objects, such as the Sun, then the Huygens formula would be a perfect description of the Earth-Moon pendulum clock. The formula is a good approximation for small angles. It is a bad approximation for large angles, but it is again a good approximation for very large angles, and it is exact for a full circle.

References

[1] C. Huygens. *Horologium oscillatorium sive de motu pendularium.* 1673.