Conditions for stable equilibrium of the upright human body in the absence and presence of external influence

Professor Karlo Moistsrapishvili
Georgian State University Of Physical Culture and Sports
Tbilisi, Georgia
e-mail: karlo.moistsrapishvili@gmail.com

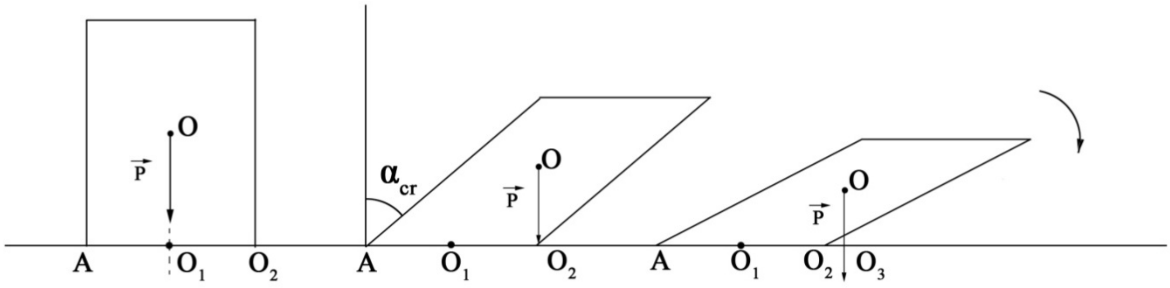
Introduction: This paper combines two previous scientific works [1, 2]. It investigates conditions of the equilibrium stability of a cylinder (shaped homogeneous body) in order to study the equilibrium stability of a human body of different proportions. A new method to determine a human body's common centre of gravity is proposed. An optimal bending forward angle at which the degree of the equilibrium stability of an upright standing person reaches its maximum value is defined and relevant conclusions are drawn.

In addition, external forces are also introduced, which may disturb the stable equilibrium and cause the body to tumble. The paper also introduces the concept of the coefficient of the stable equilibrium and proposes a formula for its calculation. Finally, a formula is set up to obtain the maximum value of external forces under which the body still maintains the state of equilibrium.

Key words: equilibrium stability; common center of gravity; optimal bending; critical angles, stability coefficient, force of gravity, external force, overturning.

PREAMBLE

The equilibrium of a body and the ability to maintain stability is a significant factor in all areas of human physical activities including sports. We'll examine conditions of the equilibrium stability based on the example of a cylindrical homogeneous body (Fig.1). Its center of gravity (O) matches the mechanical center and its equilibrium stability is at a maximum when in strictly vertical position (Fig.1a) [1].



a b c

Fig.1

Process of gradual bending of a cylindrical body with a fixed base

In this position the vertical projection of the cylindrical body's center of gravity onto the base area matches with the mechanical center of the base (O_1) and the moment of gravity of the cylinder $M = |\vec{P}| \cdot \frac{d}{2}$. As for the center of rotation (O_2), it has maximum value and serves as a resisting function (i.e. is a resisting moment), thus ensuring maximum degree of the equilibrium stability (|P|- gravity value, d=|AO_2|- base diameter, $\frac{d}{2}$ -arm of gravity).

With the cylinder's gradual deviation (conditional upon the fixed base) both, the arm of gravity and the value of the resisting moment diminish resulting in the decrease of the cylinder's equilibrium stability.

At the position given in Fig.1b, the cylinder's vertical projection of gravity onto the base area coincides with the center of rotation and the resisting moment (a), therefore, the degree of equilibrium stability becomes practically zero (critical angle of deviation - α_{cr}). At the subsequent deviation of the cylinder, the gravity moment changes its sign, becomes rotational moment and the body begins to topple over (Fig.1c).

Considering the above, one may conclude: the degree of the body's equilibrium stability becomes maximum when the vertical projection of its center of gravity onto the base area coincides with the mechanical center of the base; i.e. while the vertical projection of the body's gravity is within the base area, the body maintains its equilibrium position but when the projection leaves the area, the body's equilibrium fails and toppling begins.

The dependence of the critical angle values (α_{cr}) on the cylindrical body ($\frac{H}{d}$), where (H) is the height of the cylinder and (d) is the base diameter.

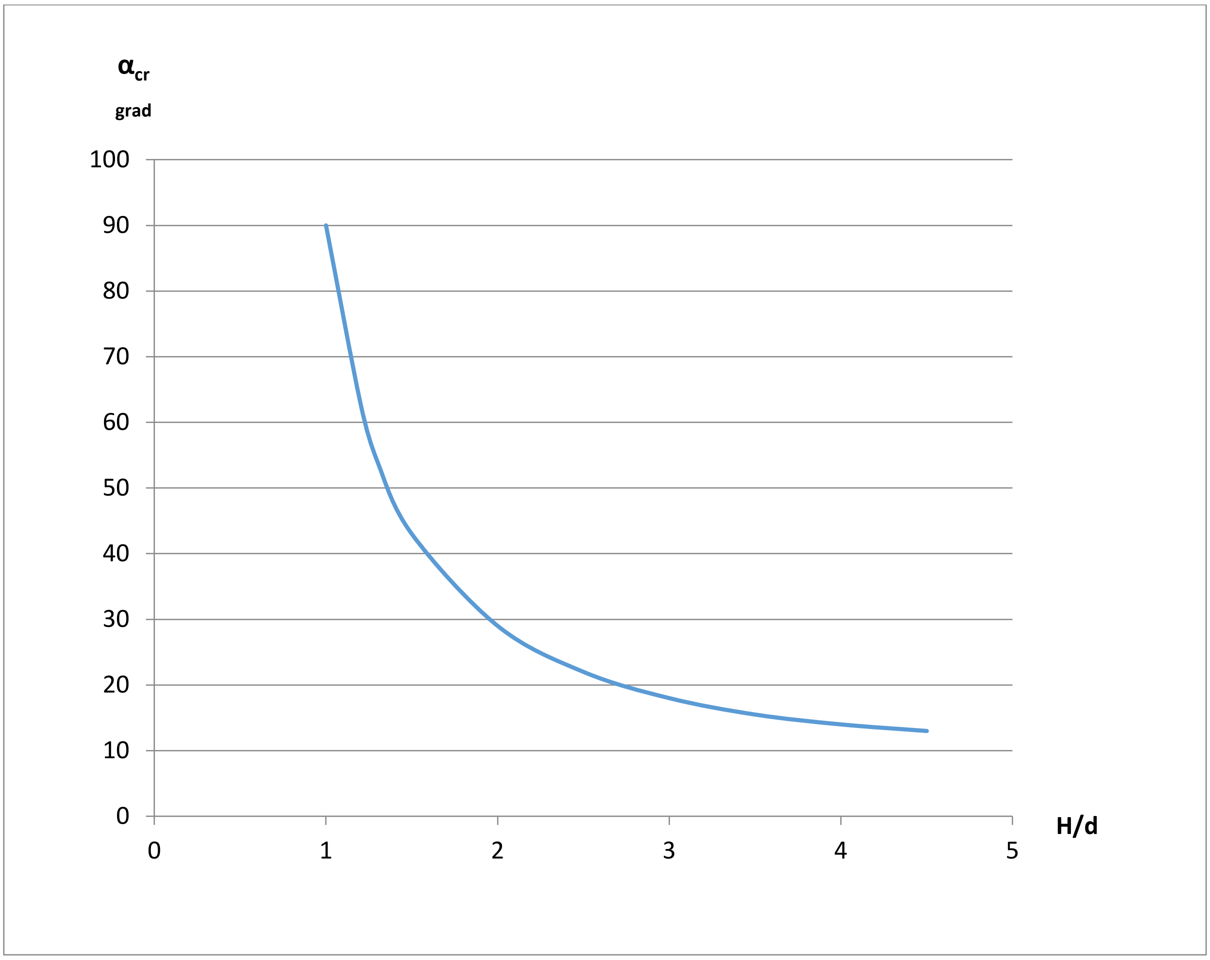
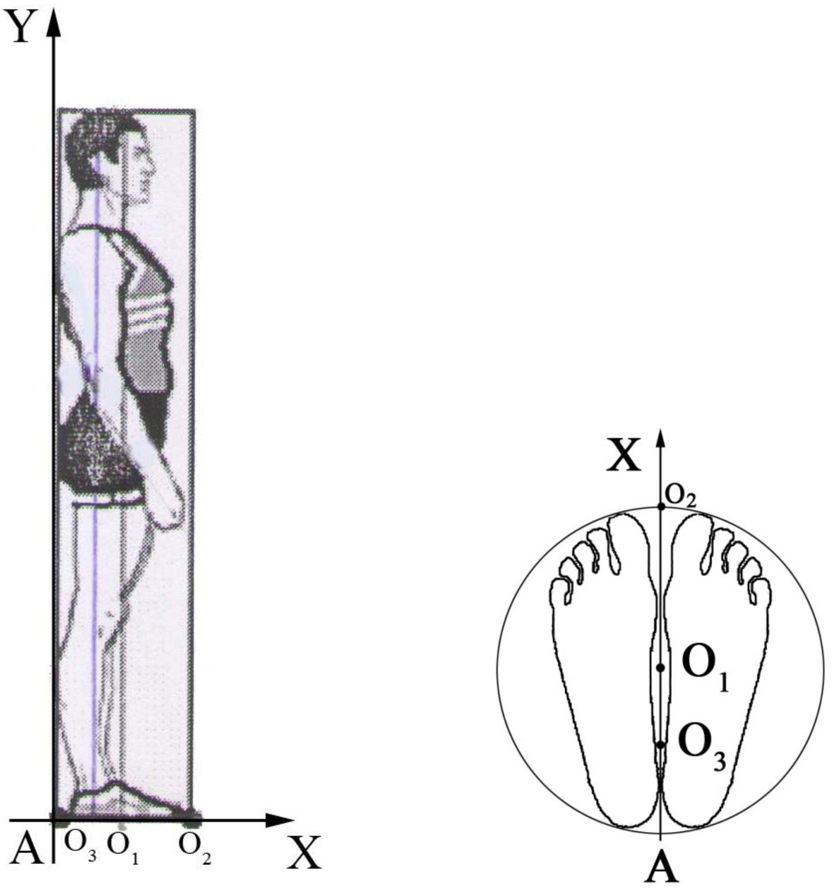


Fig.2

Dependence of the critical angle value of the cylindriform body on its geometry $\alpha_{cr} = f\left(\frac{H}{d}\right)$

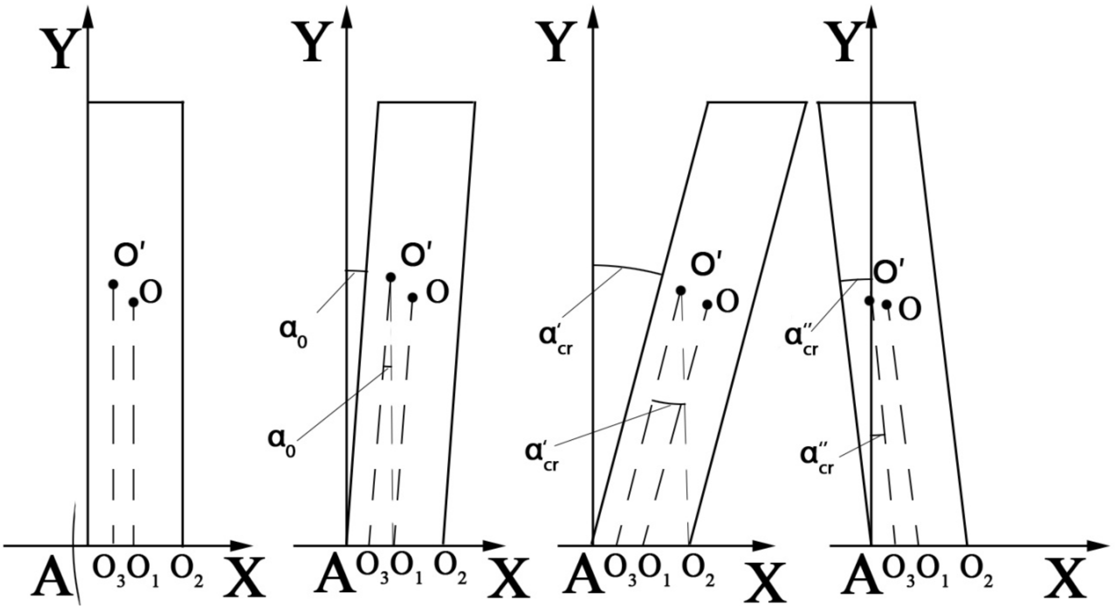
MAIN PART

Fig.4a clearly demonstrates that If an upright standing human is placed into an imaginary (virtual) cylinder (Fig.3a), the body weight will be distributed unevenly inside the cylinder and the general center of gravity (GCG) of the person will not coincide with the cylinder's mechanical center. The GCG (O') of the human and the mechanical center of the cylinder (O) are displaced considerably against each other due to the specificity of the human anatomy.



a b

Fig.3
a-An upright standing human body placed into an imaginary (virtual) cylinder.
b-Feet position on the base area



a b c d

Fig.4

a – Position of the general gravity center (O') of an upright standing human

b – Optimal position of a human body when the degree of equilibrium stability is maximal and the optimal bending forward angle (α_o) correlates with it.

c - Critical condition occurs when a human bends forward and the degree of equilibrium stability reaches zero and falling over of the body takes place; critical angle of bending forward $\alpha'_{cr.}$.

d - Critical situation occurs when a human bends backward; critical angle $\alpha'_{cr.}$. Body topples over backward.

Feet position of a human on cylinder's base is given in Fig.3b, where point O_1 is a projection of the cylinder's mechanical center (O) (Fig.4a) coinciding with the mechanical center of the base area (O_1), whereas point O_3 is a projection of the person's GCG onto the same area; since the GCG projections of the man (O') and the cylinder's mechanical center on the base area are reciprocally displaced, the equilibrium stability degree cannot have its maximum value; to succeed, the upright standing human should bend forward (without separating the feet away from the foothold) at an optimal angle (α_o) so that the vertical projection of the person's GCG (of O' point) onto the base area matches its mechanical center (O_1). The given situation is presented in Fig.4b and enables to determine the optimal angle of bending [1]:

$$\alpha_0 = \arcsin\left(\frac{|O_3O_1|}{|O_3O'|}\right). \qquad (1)$$

In much the same way, critical angles of bending forward α'_{cr} and backward α''_{cr} can be determined by the following formulas (Fig.4c and 4d):

$$\alpha'_{cr} = \arcsin\left(\frac{|O_3O_2|}{|O_3O'|}\right), \qquad (2)$$

$$\alpha''_{cr} = \arcsin\left(\frac{|AO_3|}{|O_3O'|}\right). \qquad (3)$$

It is well known {1,2} that: $|AO_3| = |O_3O_1| = \frac{d}{4}$, $|O_3O_2| = \frac{3}{4}d$, $|O_2O_1| = \frac{d}{2}$, and $|O_3O'| = h$

Taking the above into consideration, formulas (1), (2) and (3) will become:

$$\alpha_0 = \arcsin\left(\frac{d}{4h}\right), \qquad (1')$$

$$\alpha'_{cr} = \arcsin\left(\frac{3d}{4h}\right), \qquad (2')$$

$$\alpha''_{cr} = \arcsin\left(\frac{d}{4h}\right), \qquad (3')$$

Where d=IAO_2I is the diameter of the base area (length of the person's foot), h – the height of GCG position from the same area.

In order to determine the height of the GCG position of the upright standing human (h=IO_3O'I) by means of the height (H), we make use of Phidias Number (or "Fibonacci Number" and "Golden Ratio" equivalently). Phidias Number (φ=1.6118) determines the ratio between the human height H and the height of the human umbilicus (h'):

$$\varphi = \frac{H}{h'} \tag{4}$$

Also, it is well known that the GCG of an upright standing person is below the umbilicus by 0.05 for a male and by 0.1·H for a female. Taking into consideration the above and formula (4) we derive for a male body:

$$h = H \cdot \left(\frac{1}{\varphi} - 0{,}05\right), \tag{5}$$

$$h = 0{,}5680 \cdot H;$$

and for a female body:

$$h = H \cdot \left(\frac{1}{\varphi} - 0{,}1\right), \tag{5'}$$

$$h = 0{,}5180 \cdot H.$$

If formula (5) is taken into consideration, formulas (1'), (2') and (3') will become:

$$\alpha_0 = \arcsin\left(\frac{1}{4\left(\frac{1}{\varphi}-0{,}05\right)} \cdot \frac{d}{H}\right), \tag{6}$$

$$\alpha'_{cr} = \arcsin\left(\frac{3}{4\left(\frac{1}{\varphi}-0{,}05\right)} \cdot \frac{d}{H}\right), \tag{7}$$

$$\alpha''_{cr} = \arcsin\left(\frac{1}{4\left(\frac{1}{\varphi}-0{,}05\right)} \cdot \frac{d}{H}\right). \tag{8}$$

Taking into consideration that $\varphi = 1{,}618$, we receive:

$$\alpha_0 = \arcsin\left(0{,}44 \cdot \frac{d}{H}\right), \tag{6'}$$

$$\alpha'_{cr} = \arcsin\left(1{,}32 \cdot \frac{d}{H}\right), \tag{7'}$$

$$\alpha''_{cr} = \arcsin\left(0{,}44 \cdot \frac{d}{H}\right). \tag{8'}$$

In case of female and taking into consideration (5'), formulas (6'), (7') and (8')will become:

$$\alpha_0 = \arcsin\left(0{,}48 \cdot \frac{d}{H}\right), \qquad (6'')$$

$$\alpha'_{cr} = \arcsin\left(1{,}44 \cdot \frac{d}{H}\right), \qquad (7'')$$

$$\alpha''_{cr} = \arcsin\left(0{,}48 \cdot \frac{d}{H}\right). \qquad (8'')$$

It is generally known that the Phidias Number is the most harmonic ratio in the nature of things; sculptors, painters and architects of the ancient Egypt, of the antique epoch and Renaissance used it. Leonardo da Vinci used the Number in his famous work on “the human figure proportion”.

For a man of the Da Vinci proportion, both optimal α_0 and critical α_{cr}' , α_{cr}'' have the following value:

$$\alpha_0 = 3{,}7^0, \ \alpha'_{cr} = 11{,}15^0, \ \alpha''_{cr} = 3{,}7^0.$$

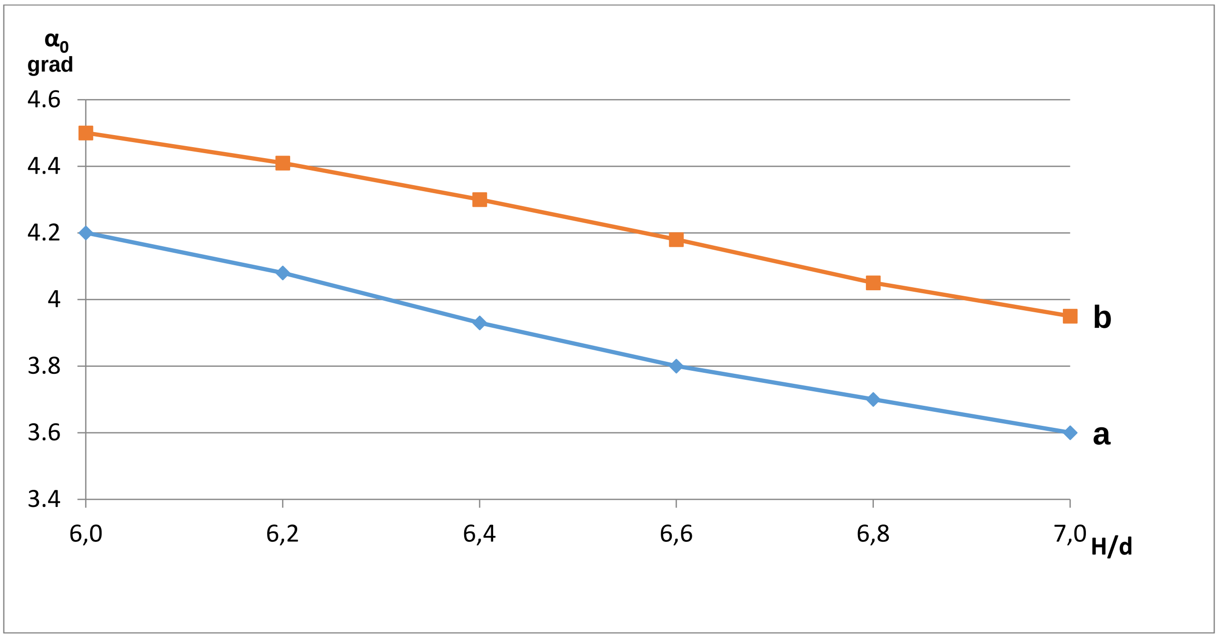


Fig.5

Dependence of the optimal angle value (α_0) on the man’s proportion $\left(\frac{H}{d}\right)$.

a-for men; b-for women (The average value $\left(\frac{H}{d}\right)$ for a proportional man varies within 6÷7)

Comparing the given diagrams (Fig. 5) one can conclude that the degree of the equilibrium stability of women is higher than that of men of similar proportion caused by a lower position of their (women) GCG. Similarly, a formula determining skier’s optimal angle of bending forward can be drawn:

$$\alpha_0 = \arcsin\left[\frac{1}{H}\left(1{,}32 \cdot d + \frac{n \cdot l}{0{,}568}\right)\right]$$

Where d-length of foot, l-length of ski, H-height of skier, n-coefficient determining the part of the ski length made up by the distance from the tip of foot to the geometric center of the part of the ski's bearing area.

The above analysis addresses the situation with no external force. Let us now introduce the effects of external force in our investigation. For the sake of simplicity, we introduce the coefficient of stability as applied to a stationary ankle joint (special footwear in sports where skis or skates are used provide such relative immobility of ankle joint):

$$K_{st.} = \frac{M_{res}}{M_{tur}} \qquad (9),$$

where M_{res} – resisting moment of gravity

M_{tur} – moment (torque) of external force

If $K_{st.} \geq 1$, body maintains state of equilibrium, while if $K_{st.} < 1$ – it loses equilibrium and overturns.

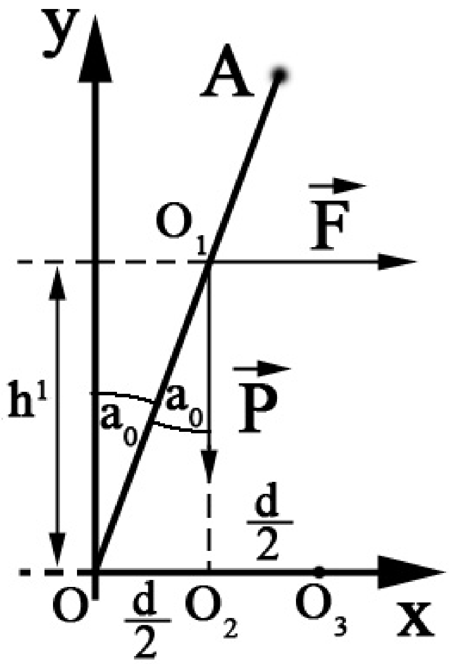


Fig. 6. Schematic representation of human body in a position with the maximum degree of stability [1].

It is clear from Fig. 6, that $M_{res} = |\vec{P}| \cdot \frac{d}{2}$ and $M_{tur} = |\vec{F}| \cdot h'$, when point O_3 is the center of rotation. External influences are represented by $|\vec{F}|$, which is the horizontal effect of all external forces. It is directed towards the gradient of a human body as applied to point O_1 - common center of mass (CCM) of a human body.

Explanation of fig.1:

$|OA| = H$ – human height;

$|OO_1| = h$ – level of body's common center of mass;

$|OO_3| = d$ – foot length;

$\vec{P}$ – force of gravity on human body, including vertical effect of external forces (body weight);

$\vec{F}$ – horizontal component of resultant external forces;

O_1 – body's common center of mass

O_2 – geometrical center of the area of a foot

O_3 – body hinge point;

α_0 – Optimum slope angle of a human's body bending forward, at which the maximum degree of stability of equilibrium is achieved [2].

With respect to the center of rotation O_3 (Fig. 6) the following is true:

$$M_{res} = |\vec{P}| \cdot \frac{d}{2} \quad \text{and} \quad M_{tur} = |\vec{F}| \cdot h' \qquad (10)$$

As $d = 2 \cdot h \cdot \sin\alpha_0$ and $h' = h \cdot \cos\alpha_0$, we receive:

$$M_{res} = |\vec{P}| \cdot h \cdot \sin\alpha_0 \quad \text{and} \quad M_{tur} = |\vec{F}| \cdot h \cdot \cos\alpha_0 \qquad (11)$$

By simple transformations we receive:

$$K_{st} = \frac{|\vec{P}|}{|\vec{F}|} \cdot tg\, \alpha_0 \qquad (12)$$

Paper [1] defined the optimum angle α_0 for men:

$$\alpha_0 = arc\sin\left(0{,}44\frac{d}{H}\right) \qquad (13)$$

And for women:

$$\alpha_0 = arc\sin\left(0{,}48\frac{d}{H}\right) \qquad (14)$$

Considering (5) and (6) the formula (4) will result in the following:

$$K_{st} = \frac{|\vec{P}|}{|\vec{F}|} \cdot \mathrm{tg}\left[arc\sin\left(0{,}44\frac{d}{H}\right)\right] \qquad (15) \text{ for men}$$

$$K_{st} = \frac{|\vec{P}|}{|\vec{F}|} \cdot \mathrm{tg}\left[arc\sin\left(0{,}48\frac{d}{H}\right)\right] \qquad (16) \text{ for women}$$

When $K_{st} = 1$ $\quad |\vec{F}| = |\vec{F}|_{max}$ (17)

$|\vec{F}|_{max}$ – the maximum value of the external force at which body maintains in a state of equilibrium;

When $\left|\vec{F}\right| > \left|\vec{F}\right|_{max}$ (т.е. $K_{st} < 1$) body overturns.

Considering condition (9), from (7) and (8) we deduce:

$$\left|\vec{F}\right|_{max} = \left|\vec{P}\right| \cdot \mathrm{tg}\left[arc\sin\left(0{,}44\frac{d}{H}\right)\right] \qquad (18) \quad \text{for men}$$

$$\left|\vec{F}\right|_{max} = \left|\vec{P}\right| \cdot \mathrm{tg}\left[arc\sin\left(0{,}48\frac{d}{H}\right)\right] \qquad (19) \quad \text{for women}$$

From these formulae we can obtain numerical values of $\left|\vec{F}\right|_{max}$ for human bodies of different parameters.

Based on the above, we can conclude:

1. When $\left|\vec{F}\right| \leq \left|\vec{F}\right|_{max}$ body maintains its balance;
2. When $\left|\vec{F}\right| > \left|\vec{F}\right|_{max}$ body overturns;
3. Given the same external effects and identical anthropometry, stability in women is greater than in men.

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