

Diagram of space-time. Quantum equation of the Doppler shift

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Abstract

Using the diagram of space-time, which represents intersections of the spheres depicting the front of an electromagnetic wave in the resting and moving frames with a plane going through the centres of the spheres, and the assumption that a quantum (photon) of spherical electromagnetic wave maintains its entity during the propagation of the wave, it is derived a quantum equation of the Doppler effect: $\sqrt{1-\beta^2} \tau^2 - 2 \cos \gamma \tau_0 \tau + \sqrt{1-\beta^2} \tau_0^2 = 0$, where τ_0 and τ are the periods of an electromagnetic wave in the frames of the source and the receiver, β is the relative speed of the frames divided by the speed of light in free space, and γ is the angle of aberration (the difference of the angles of slope of a ray in the frames of the source and the receiver). It is maintained that despite the differences between the obtained equation and Einstein's expression for the Doppler shift, the equation of time dilation doesn't need any alteration. Inapplicability of some fundamental principles of the special relativity to finite distances as well as inherent uncertainties in the measurements of space and time intervals is discussed.

1. Diagram of Space-time

Einstein's theory of special relativity (SR), the mathematics of which is based on the Lorentz transformation, has two major faults: it considers electromagnetic (EM) waves as plane waves (while they are in fact spherical) and it does not take into account the quantum nature of EM emission. This paper aims at making some corrections in consideration of those two factors. For that purpose, a space-time diagram is used.

This diagram of space-time represents intersections of two spheres with a plane going through their centres (Fig. 1). The spheres depict the same front of a spherical EM wave in two frames, from which one is at rest and the other is moving at a constant speed. It is implied that the centres of the spheres coincide in point O and the clocks in both centres show zero time when the emission starts from one of them. The radii of the spheres are proportional to elapsed times in corresponding frames. Because of that, those spheres may also be considered as time spheres without connection to any kind of waves. The main feature of this simplest type of space-time diagram (the one without acceleration and gravity) is that the radii of the time spheres and the distance between their centres satisfy the Pythagorean theorem; i.e., their connection gives the equation of time dilation.

The diagram may be considered as a kind of geometric substitute of the Lorentz transformation for EM waves. Here are some of its interesting features.

The intersections of a line of arbitrary slope passing through the centre of the moving sphere (it is shown below that the angle of slope is the same in both frames, as opposed to SR) with the spheres give the same point of the front of an EM wave in those frames.

In Fig. 1, the points A and B of the front of an EM wave in the resting frame are the same as the points A' and B' respectively in the moving frame. To show this, imagine that the line with an angle of slope α in the figure is an imaginary tube, in which a ray (or a point of the wave front) of an EM wave is traveling. Since the speed of light is independent of the speed of the emitter,

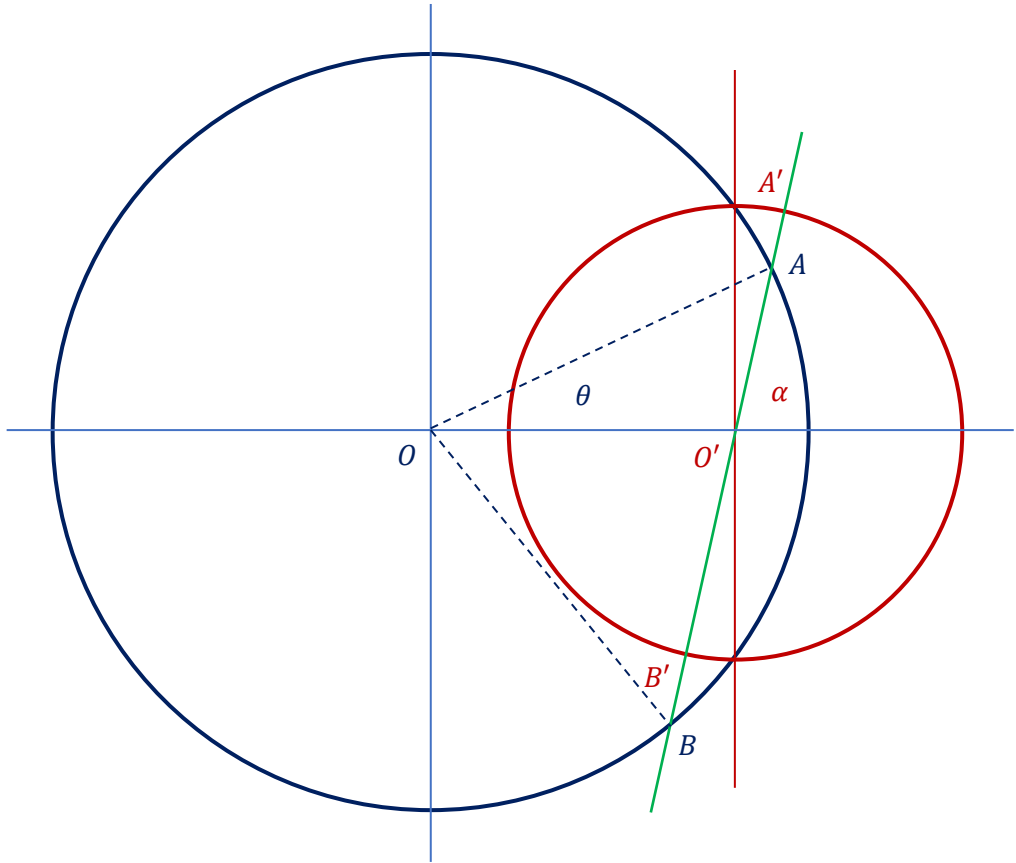


Fig. 1. The simplest type of space-time diagram.

$$|O'A'| = \sqrt{1 - \beta^2} |OA|; \frac{|A'B'|}{|AB|} = \sqrt{\frac{1 - \beta^2}{1 - \beta^2 \sin^2 \alpha}}$$

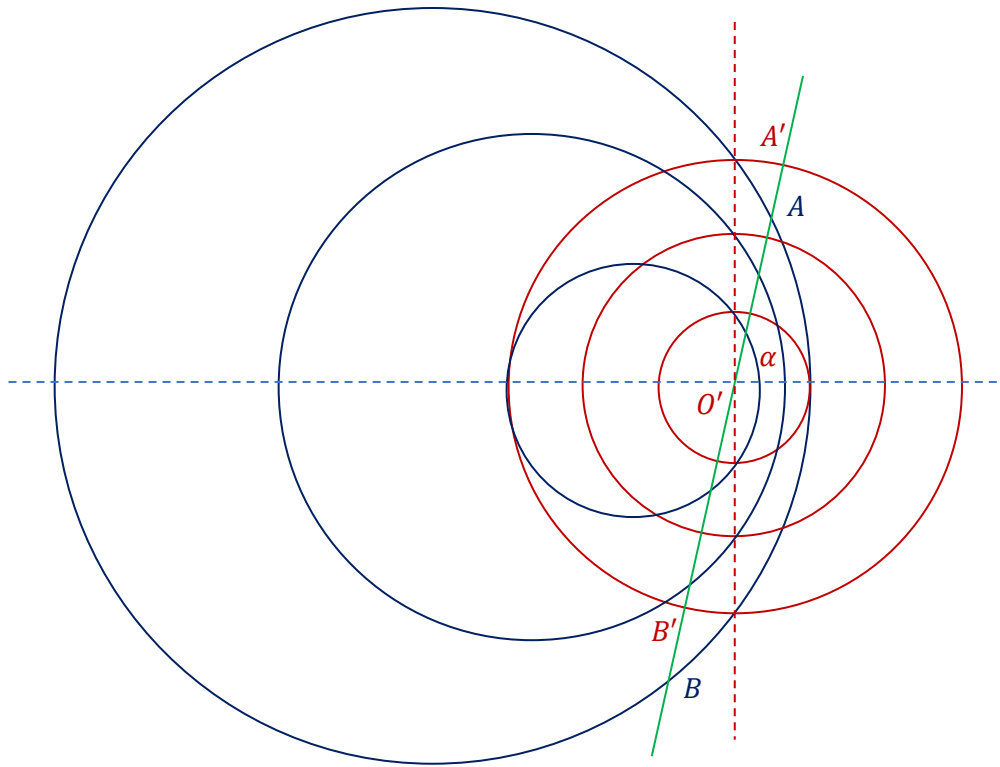


Fig. 2. Space-time diagram of several (three) wavelengths.

$$|O'A'| : |O'A| = \lambda_0 : \lambda_\alpha$$

$|OA| = ct$ and $|O'A'| = ct'$, where t and t' are the readings of the clocks in the resting O and moving O' points respectively, and c is the speed of light in free space; i.e., those segments are equal to the distances covered by the same point of the wave front in respective frames.

Fig. 2 shows several (in this case three) consecutive wave fronts with phase delays of 2π in the frames of the resting receiver and the moving source. The segment $|O'A|$ in the resting frame contains the same number of wave fronts (as well as the same points of the wave) as the segment $|O'A'|$ in the moving frame. It means that those segments represent the same distance in the resting and moving frames and that the lengths measured from moving points undergo changes. In particular, the units for measuring distances from the *moving* point O' to the wave front (“moving unites”) and the units for measuring the same distances from the *stationary* point O' (“resting unites”) must have the ratios of $|O'A|:|O'A'|$ and $|O'B|:|O'B'|$ in opposite directions, so that the speed of light remains the same in both frames. As it turns out in the text later, if the moving point represents a source of spherical EM waves, then those ratios are equal to the ratios of the wavelengths in the resting and moving frames, λ_0 and λ_α respectively (in Fig. 2, $|O'A'| = 3\lambda_0$ and $|O'A| = 3\lambda_\alpha$). So, the wavelengths of EM waves may be considered as “natural units” for measuring distances. In natural units, the wave fronts of EM waves represent the same concentric spheres with the source as the centre in any inertial frame. The smaller sphere in Fig. 1 represents the wave front in the moving frame in resting units as well as the wave front in the resting frame in moving units. (When a resting observer measures distances from the moving point of reference O' , that seems as if the observer is “teleported” into the moving frame. Because of this, the picture the observer gets should be the same as the one observed within the moving frame.)

The segments $|AB|$ and $|A'B'|$ in Fig. 2 are the same segment measured in resting and moving unites respectively. This segment is equal to the distance covered by light along the tube from O' to A (or B) and back, or from O' to A' (or B') and back. The ratio: $\frac{|A'B'|}{|AB|} = \sqrt{\frac{1-\beta^2}{1-\beta^2 \sin^2 \alpha}}$, where $\beta = \frac{v}{c}$ and v is the speed of the moving frame, represents the equation of Lorentz-Fitzgerald contraction [1].

When describing propagation of EM waves in different frames, especially that of the spherical ones or of those emitted from a point source, it is convenient to make use of the so-called directions (or angles) of reception and emission.

The direction of reception is the direction of a ray (or a normal to the wave front) in the frame of the receiver. The observer has to direct a spyglass along that direction in order to see the point from which the emission has started. The direction of the same ray in the frame of the source is the direction of emission. Let's call the angles those directions make with the line connecting the origins of the frames of the source and the receiver the angles of reception and emission accordingly.

In Fig. 1, if the source is in the moving point O' and the receiver in the resting point A , then the direction \overrightarrow{OA} is the direction of reception (with θ as the angle of reception), and the direction $\overrightarrow{O'A}$ is the direction of emission (with α as the angle of emission), and vice versa if the source is in the resting point O and the receiver in the moving point A .

The absolute value of the difference between the angles of emission and reception of a ray is sometimes called the angle of aberration. The use of this angle seems to be preferable in some cases.

2. Doppler shift: the distance between the source and the receiver is equal to one wavelength

In Fig. 3, a source of EM waves, S' , is moving to the right from point S with velocity $v = \beta c$. At the moment when points S and S' coincide, the clocks in those points have the same readings $t_0 = t'_0 = 0$. Let τ_0 be the period of the EM wave in the frame of the source (which is equal to the period of internal oscillations of the source). Due to time dilation, in the observer's frame the period of internal oscillations of the source is $\tau'_0 = \frac{\tau_0}{\sqrt{1-\beta^2}}$. The period of the wave in the frame of the receiver depends on the angle of propagation of a ray.

Let's consider the moment of time when the clock in point S shows $t = \tau'_0$, and, accordingly, the clock in point S' reads $t' = \tau_0$. The wave front has just arrived from point S at the resting receiver in point O . The next wave front with a phase delay of 2π starts emitting from point S' , and will arrive at the receiver after the time interval $\tau = \frac{|S'O|}{c}$. Thus, τ is equal of the wave period measured by the resting observer in point O .

Sometimes it is convenient to consider not the wave itself, but a sequence of short pulses emitted by the source with the same frequency. In Fig. 3, out of two consecutive pulses one is emitted from point S and the other - from point S' . Both pulses are received by the receiver in point O . The period of time between the emission of those pulses in the frame of the source is τ_0 , and the time period between the received pulses measured by the observer shall be $\tau = \frac{|S'O|}{c}$.

In the figure, the angle θ is the angle of reception of the first pulse, $|SO| = c\tau'_0$, $|SS'| = \beta c\tau'_0$, and $|S'O| = c\tau$. It is important to note that the angle α is not only the angle of slope of the moving tube in which the first pulse is traveling, but also that of the direction of the second pulse in the frame of the receiver. Thus, the angle α is the angle of reception of the second pulse as well as the angle of emission of the first pulse measured from the frame of the receiver.

From $\Delta(SOS')$:

$$\sqrt{1-\beta^2} \tau^2 + 2\beta \cos \alpha \tau_0 \tau - \sqrt{1-\beta^2} \tau_0^2 = 0. \quad (1)$$

The solutions are:

$$\tau = \tau_0 \frac{(-\beta \cos \alpha \pm \sqrt{1-\beta^2 \sin^2 \alpha})}{\sqrt{1-\beta^2}}; \quad (2)$$

or for the first solution:

$$\tau = \tau_0 \frac{\sqrt{1-2\beta \cos \theta + \beta^2}}{\sqrt{1-\beta^2}}. \quad (3)$$

Now let's see how all this looks in the frame of the source.

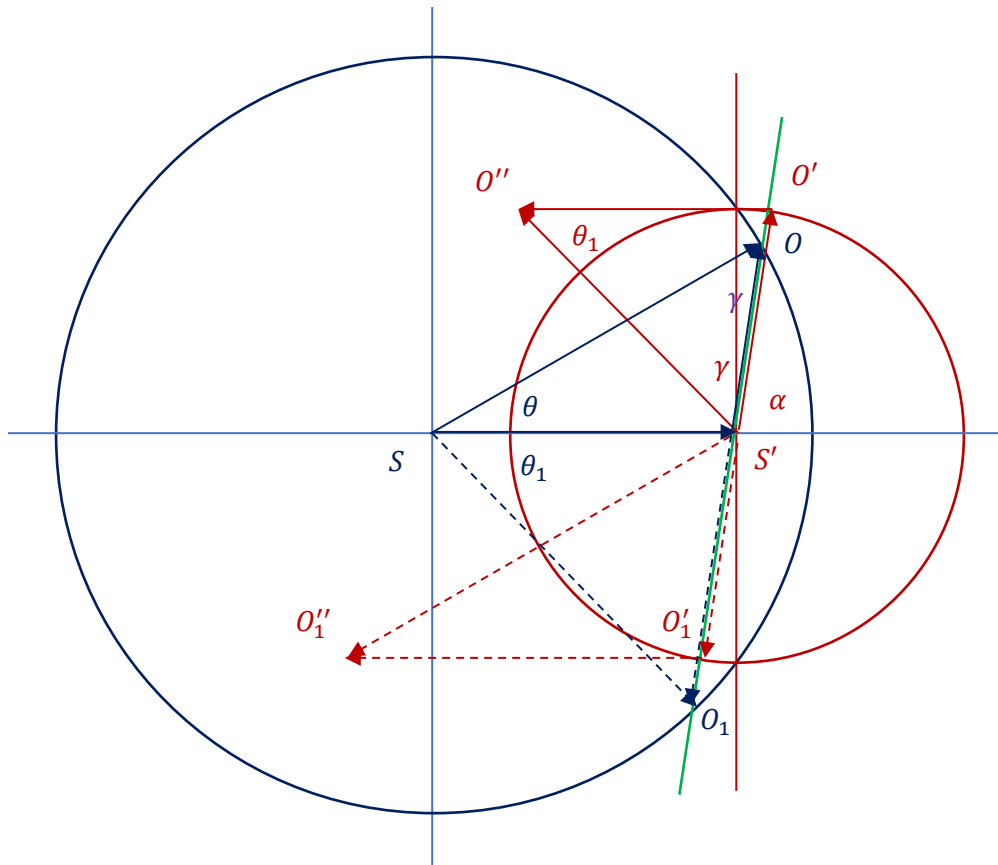


Fig. 3. The distance between the source and the receiver is equal to one wavelength.

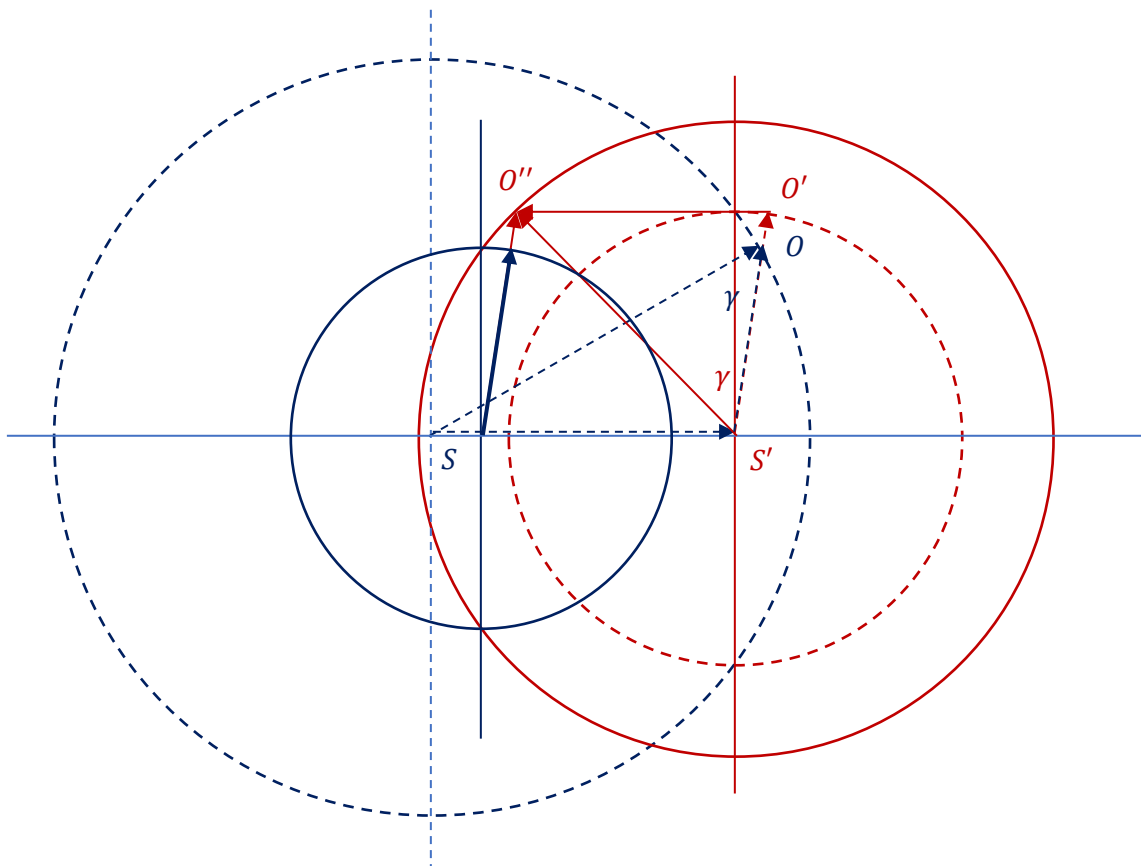


Fig. 3a. The distance between the source and the receiver is equal to one wavelength: the final diagram

If in Fig. 3, the direction of time flow is reversed, the triangle stays the same, but the source and the receiver swap places: the source (in point O) is stationary now and the receiver (in point S') is moving in opposite direction; angle α is the angle of emission of the first pulse, as well as the angle of reception of the second pulse measured from the frame of the source, and angle θ - the angle of emission of the second pulse. The period between the emitted pulses in the frame of the source is τ and the period between the received pulses measured by the observer is τ_0 . Thus, the form of the equation is the same in the frame of the source, but τ and τ_0 swap places:

$$\sqrt{1 - \beta^2} \tau_0^2 + 2\beta \cos \alpha \tau \tau_0 - \sqrt{1 - \beta^2} \tau^2 = 0.$$

To get the aforementioned case of direct flow of time in the frame of the source (where τ_0 is the period in the frame of the source and τ - in the observer's frame), in the last equation the sign before the cosine needs to be changed, or instead of α it has to be used $\alpha_1 = (\pi - \alpha)$:

$$\sqrt{1 - \beta^2} \tau^2 - 2\beta \cos \alpha_1 \tau_0 \tau - \sqrt{1 - \beta^2} \tau_0^2 = 0. \quad (4)$$

This equation is obviously algebraically identical to equation (1), but it refers to $\Delta(S'O'O'')$, where $|S'O'| = c\tau_0$, $|S'O''| = c\tau'$, $|O'O''| = \beta c\tau'$, and $\tau' = \frac{\tau}{\sqrt{1 - \beta^2}}$. Angle α_1 is the angle of emission of the first pulse and the angle of reception of the second pulse from the point of view of the frame of the source, and θ_1 is the angle of emission of the second pulse.

It is obvious that $\Delta(S'O'O'') \sim \Delta(O_1S'S)$. The angle α in Fig. 3, as well as in Fig. 1, between the line representing a tube, and the line connecting the centres of the spheres as well as the angle between the pulses, γ , is the same in both frames. The angle between the pulses, $\gamma = |\alpha - \theta| = |\alpha_1 - \theta_1|$ is also the angles of aberration for both pulses. Using this angle, Eq. (1), or Eq. (4), takes the following form:

$$\sqrt{1 - \beta^2} \tau^2 - 2 \cos \gamma \tau_0 \tau + \sqrt{1 - \beta^2} \tau_0^2 = 0. \quad (5)$$

The solutions of this equation are:

$$\tau = \tau_0 \frac{\cos \gamma \mp \sqrt{\beta^2 - \sin^2 \gamma}}{\sqrt{1 - \beta^2}}. \quad (6)$$

One solution is for the observer in point O , and the other - for the observer in point O_1 .

The square root from the product of those solutions gives τ_0 , and

$$\frac{1}{\tau} = \frac{1}{\tau_0} \frac{\cos \gamma \pm \sqrt{\beta^2 - \sin^2 \gamma}}{\sqrt{1 - \beta^2}}.$$

Eq. (5) represents the answer of spherical EM waves to the principle of relativity.

It has to be noted that the form of equations (1) and (4) was predicted in [2] using an entirely different approach.

A final version of the space-time diagram, when the distance between the source and the receiver is equal to one wavelength, is given in Fig. 3a: $|SO| = c\tau_0'$ and $|S'O| = c\tau$ are the ways from the source to the receiver made by two points of the wave front a period-of-the-wave apart in the observer's frame, and $|S'O'| = c\tau_0$ and $|S'O''| = c\tau'$ are the ways of the same points in the frame of the source.

3. Time Dilation

In the above discussion it was implied that Einstein's equation of time dilation was absolutely correct. Let's ascertain that in fact it does not require alteration.

Consider two very short pulses emitted by a resting source from point S' (Fig. 3). The receiver is moving with velocity $v = \beta c$. It receives the first pulse in point O' and the other – in point O'' . The time period between the pulses emitted is τ_0 in the frame of the source and τ'_0 in the frame of the receiver; the time period between the received pulses measured by the observer is τ , and the same interval of time measured in the frame of the source is τ' . Let's see whether or not the time periods designated by letters without primes and the ones by the same letters with primes (or the same time periods measured in resting and moving frames) have the same values.

In Fig. 3, it is assumed that the second pulse is issued exactly at the moment when the first pulse arrives at the receiver. So: $|S'O'| = c\tau_0$, $|S'O''| = c\tau'$, $|O'O''| = \beta c\tau'$, and γ is the angle between the directions of the pulses. From $\Delta(S'O'O'')$:

$$(1 - \beta^2)\tau'^2 - 2\tau_0\tau' \cos \gamma + \tau_0^2 = 0. \quad (7)$$

In order to get the form of an appropriate equation in the frame of the receiver, let's use the method of reversing the flow of time. In that case, all velocities reverse their directions and the source and the receiver swap places; i.e., the moving source emits the first pulse from point O'' and the second one from O' . Those pulses are received by the receiver in point S' . Now: $|O''S'| = c\tau'_0$, $|O''O'| = \beta c\tau'_0$, $|O'S'| = c\tau$, and

$$\tau^2 - 2\tau'_0\tau \cos \gamma + (1 - \beta^2)\tau_0'^2 = 0, \quad (8)$$

In general, Eqs. (7) and (8) are different, which makes it possible to determine whether the source or the receiver is resting or moving. If there is no difference between the values with and without primes (which, for example, is almost the case if the pulses are of sound waves at low velocities of the source and the receiver) and Eq. (7) holds true, then the source is resting and the receiver is moving relative to the medium in which the pulses (or wave) propagate(s) with some constant velocity c , and if Eq. (8) holds - the opposite is true. When considering EM waves in free space, it is obvious that Eqs. (7) and (8) must have the same form. That happens when the values with and without primes have the following relations: $\tau'_0 = \frac{\tau_0}{\sqrt{1-\beta^2}}$ and $\tau' = \frac{\tau}{\sqrt{1-\beta^2}}$, which, in both cases, give Eq. (5). (It can be proved that for the events having physical sense, those relations are unique.)

The symmetry of Eq. (5) in regard to τ_0 and τ implies that the angle γ between the directions of the first and second pulses cannot be different in the frames of the source and the receiver. So, Eq. (5) is identical in both frames.

Now, if between those two pulses some other pulses are also emitted by the source and received by the receiver, then the sum of the time intervals between the pulses emitted equals to $|S'O'|/c$ and the sum of the time intervals between the pulses received equals to $|S'O''|/c$. Since those sums do not depend on the number and the regularity of the pulses, each particular time interval between the pulses must undergo the same effect of time dilation as their sums. Using the triangles constructed in the similar way as $\Delta(S'O'O'')$, it can be shown that the same is true for any other possible time intervals prior as well as after emittance of those two pulses.

It may be noted that the above reasoning about time dilation is not restricted to one-dimensional cases and does not involve any spatial effects of relativity.

Thus, there is no reason to doubt absolute correctness of Einstein's equation for time dilation, which cannot be said about his equations for the Doppler shift and aberration.

Finally, let's mention that Eq. (5) includes all four major triangles in Fig. 3.

4. The distance between the source and the receiver is greater than one wavelength: quantum equation of the Doppler shift

Until now the discussion didn't contain anything that would contradict the current rules of the classical physics. The contradiction starts when the distance between the source and the receiver becomes greater than one wavelength.

According to Plank's hypothesis, the emission of EM waves occurs in the form of quanta of discrete energies. According to Einstein's hypothesis, the absorption of EM waves occurs in the form of photons of discrete energies. Logically, one can assume that during the propagation of EM waves, a quantum (photon) maintains its discrete nature, its entity. It means that if certain points of the wave are associated with some quantum (photon) in a given frame, this association shall be maintained in any other frame. To make a photon (quantum), two consecutive wave fronts with phase delay of 2π must arrive at the receiver. For instance, in Fig. 4, if points O'_1 and O'_2 are the initial and final points of a quantum in the frame of the source, then O_1 and O_2 shall be the initial and final points of the same quantum in the frame of the receiver. When a photon is absorbed by a receiver in any frame, then all of its points shall be absorbed independent of whether the receiver is resting or moving. The simplest classical model of a quantum is an entity of two very short pulses, the time interval between which is equal to the period of the wave.

For instance, if in the frame of the source a *resting* receiver in point O'_1 absorbs the photon $[O'_1, O'_2]$ with a period of τ_0 , in the observer's frame this absorption is taking place in the process of displacement of this receiver from point O_1 to point O_3 . Thus, in the observer's frame the photon becomes $[O_3, O_2]$, where $|O_3 O_2| = \frac{|O'_1 O'_2|}{\sqrt{1-\beta^2}} = \frac{c\tau_0}{\sqrt{1-\beta^2}}$.

If in the frame of the source the absorption of a photon is taking place during the receiver's displacement from point O'_1 to point O'_3 , then the absorbed photon in the frame of the source shall be $[O'_3, O'_2]$, which in the observer's frame corresponds to the photon $[O_1, O_2]$. Obviously, $|O_1 O_2| = |O'_1 O'_2| \frac{(-\beta \cos \alpha + \sqrt{1-\beta^2} \sin^2 \alpha)}{\sqrt{1-\beta^2}} = |O'_3 O'_2| \sqrt{1-\beta^2} = c\tau$.

According to Huygens' principle, each point of space in which the front of a wave arrives, becomes itself a source of a secondary spherical wave (so-called wavelet). In Fig. 4, points O'_1 and O'_3 are the points of the consecutive wave fronts with a phase delay of 2π issued from the same point O'_2 . In the observer's frame, this point has moved from O to O_2 , and the points of the wave fronts with phase delay of 2π arrive at the receiver in point O_1 .

From the classical point of view, points O'_1 and O'_3 belong to the wavelets issued from different points of space, because the points of each wave front arriving at the receiver have to be the closest ones to it (according to Fermat's principle). That seems impossible from the quantum viewpoint: a quantum cannot be issued from different points of space, since it must remain a quantum even when flow of time is reversed. Thus, in an EM wave, all points associated with a

quantum (photon) must originate from a single point of space in the frame of the source and arrive at a single point of space in the frame of the receiver, with a phase difference of 2π between the first and the last points of the association.

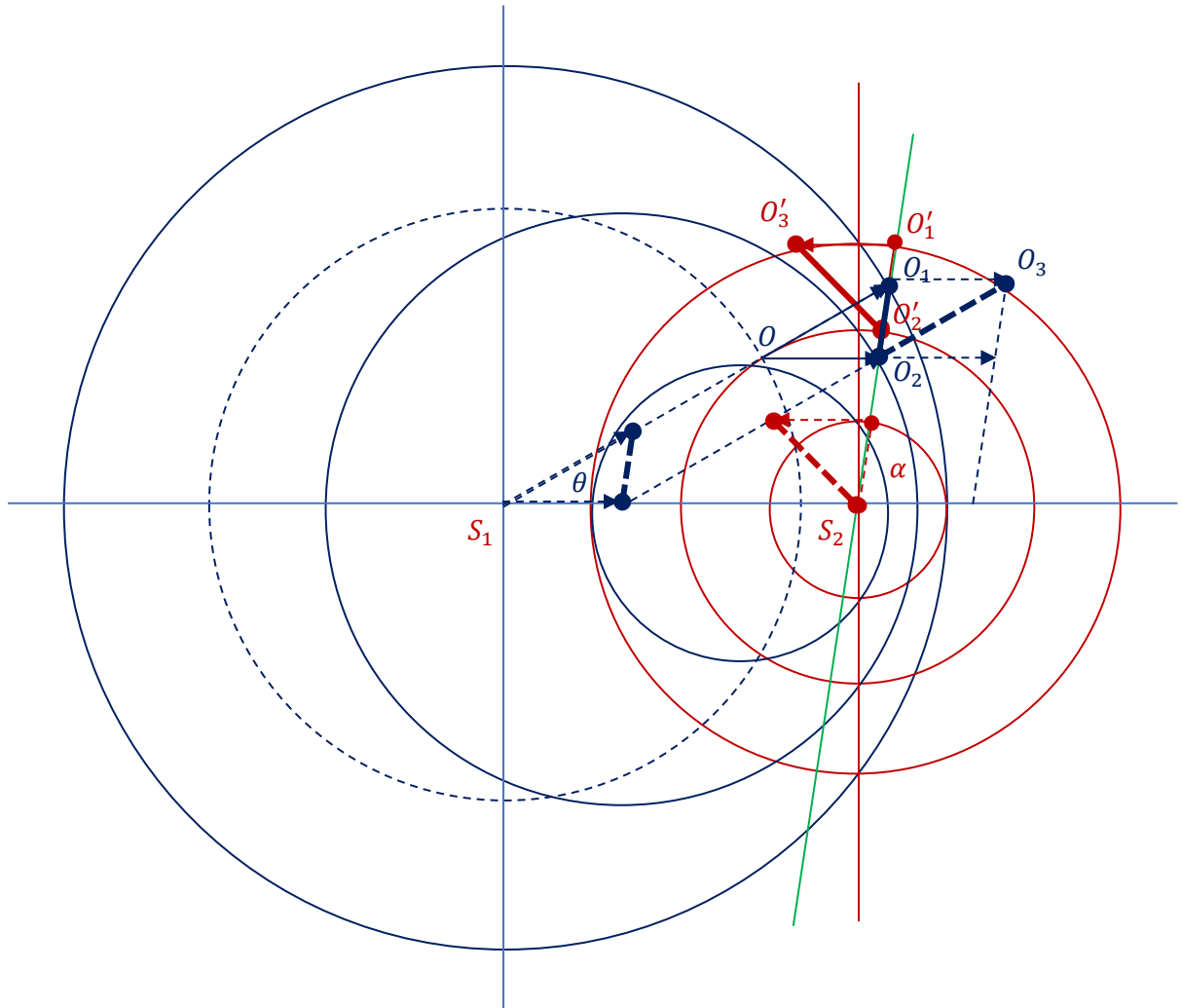


Fig. 4. Space-time diagram: quantum mechanism.
A quantum is the same at any distance from a uniformly moving source.

In Fig. 2, along the sloping line, i.e., in any direction from the source (point O'), the distances between consecutive wave fronts with phase delay of 2π are the same. In the classical physics, a resting receiver if located on this line at different distances from the source shall get different periods (or wavelengths). Namely, the period of the wave (and a photon) measured at different distances from the source varies from $\tau_\alpha = \tau_0 \frac{(-\beta \cos \alpha + \sqrt{1 - \beta^2 \sin^2 \alpha})}{\sqrt{1 - \beta^2}}$ (when the distance between the source and the receiver is at minimum, i.e. equals to one wavelength) to $\tau_\alpha = \tau_0 \frac{\sqrt{1 - \beta^2}}{1 - \beta \cos \alpha}$ (when the distance between the source and the receiver is infinite). That seems to be in conflict with the conservation laws and/or the quantization principle as well as some fundamental principles of SR.

From the quantum viewpoint, the period (or wavelength) of an EM wave, and, correspondingly the energy and the momentum of the absorbed photon, for a given direction are the same at any distance between the source and the receiver, as it can be seen in Fig. 2.

Thus, the equations which are obtained for a single (the first) wavelength of the EM wave remain the same for any number of wavelengths (for any distance between the source and the receiver), which is a logical consequence of Huygens' and Plank's hypotheses.

Eq. (5), as well as Eq. (1), may be considered as a quantum equation of the Doppler shift, and Fig. 3a - as a space-time diagram of a quantum.

As it can be seen in Fig. 1, the equation of aberration now is:

$$\sin \alpha = \frac{\sin \theta}{\sqrt{1 - 2\beta \cos \theta + \beta^2}}. \quad (9)$$

In contrast to Einstein's equations for the Doppler shift and aberration for two inertial frames with relative velocity β , which may be written this way:

$$\tau = \tau_0 \frac{1 - \beta \cos \theta}{\sqrt{1 - \beta^2}}; \tau_0 = \tau \frac{1 + \beta \cos \alpha}{\sqrt{1 - \beta^2}}; \cos \alpha = \frac{\cos \theta - \beta}{1 - \beta \cos \theta}, \quad (10)$$

Eqs. (2), (3) and (9) are not symmetrical in regard to the angles of reception and emission. This asymmetry (different geometry) is not caused by the movement of the source or the receiver; it implies only that the frame of the source is unique and can be singled out among other inertial frames (in contrast to SR). In particular, among all inertial frames, the frame of the source is the only one with no Doppler shift.

The symmetry of equations (10) indicates that there is no particular "frame of source" for the plane EM waves. Each one of the angles θ and α may be chosen arbitrarily to be the angle of emission the other being the angle of reception and vice versa. Also, since β , and accordingly the speed of light, is the same in those equations, there is no need in introducing the aether as medium for propagation of EM waves in order to meet the principle of relativity of uniform motion.

The trouble is that those equations are approximations at infinity of the classical equations (involving time dilation) that for finite distances are not symmetrical in regard to the angles of reception and emission, due to the fact that they contain the distance (or time necessary for light for covering this distance) between the source and receiver. If those classical equations were true, it would require of the EM waves to have a (frame of) source for emitting and a medium (the aether) for propagation, the necessity of both of which would disappear at infinity.

Thus, for the distances between the source and the receiver up to infinite, some fundamental principles of SR are not generally applicable. The necessity those distances to be infinite, was first noted by Einstein himself in his very first paper on SR [3]. Strangely, this most important fact has been universally ignored or underestimated since then.

The truth seems to be in the fact that the use of the principles of the classical physics (including SR), without taking into account the quantum nature of EM waves, is unjustified and leads to incorrect results. There are two exemptions: i) the distance between the source and the receiver is equal to one wavelength, where the classical and quantum expressions for the Doppler shift are the same, and ii) the case is one-dimensional (the velocities of the source and the receiver are always directed along their connecting line), where the classical equation does not depend on distance and coincides exactly with Einstein's equation, which, for its part, coincides with the

quantum equation. Those circumstances must be taken into account while using in practice as well as experimentally proving the principles of SR.

It is interesting to consider the angles when the geometric factor of the Doppler shift disappears and only the relativistic one remains. By a simple logic, that must happen when two points belonging to two consecutive wave fronts with a 2π -phase delay take the same time for traveling from the source to the receiver in a given frame. For the quantum equation, $\tau = \tau_0\sqrt{1 - \beta^2}$ when $\cos \alpha = \frac{\beta}{2}$, and $\tau = \frac{\tau_0}{\sqrt{1 - \beta^2}}$ when $\cos \theta = \frac{\beta}{2}$, and those relations do not depend on distance between the source and the receiver. By the rules of classical physics, the geometric factor shall disappear when $\cos \alpha = \frac{\beta\tau_0}{2t_0}$ and $\cos \theta = \frac{\beta\tau}{2t}$, where t_0 and t are the times of light's travel from the source to the receiver, and τ_0 and τ the periods of the source's internal oscillations in the source's and the receiver's frames respectively. Those relations too are valid for any distances between the source and the receiver. According to Einstein's equations, which are approximations of the classical ones, the geometric factor disappears at the angles $\alpha = \frac{\pi}{2}$ and $\theta = \frac{\pi}{2}$, and that, of course, would happen only at infinity.

When $\alpha = \frac{\pi}{2}$ (or $\cos \theta = \beta$), the quantum equation gives: $\tau = \tau_0$. This case is discussed in [4].

Finally, let's note that all above discussion concerns the distances from one wavelength of an EM wave up to infinity. What about the shorter distances?

EM waves may be considered "natural tools" (in the absence of a ruler [2]) for measuring distances, with their wavelengths as the smallest units of measurement, since there is no "fraction of the quantum". Using such a tool, speaking of the distances shorter than its wavelength would be mere speculation. Any measurement of distance implies some inherent inaccuracy, "uncertainty", the size of which equals to the length of the EM wave used as a measuring tool. The same is true, of course, for the time period necessary for light for covering a distance in free space and the period of the wave as the unit of its measurement.

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