Some thoughts on the unification of quantum and classical mechanics

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Abstract

The paper discusses some of the issues related to the unification of quantum theory with classical theory with the aim of providing pointers as to how to go about achieving this.

Anselm of Aosta was a Benedictine monk in the 11th Century who rose through the ranks of the church to become Archbishop of Canterbury and eventually a Saint. Anselm thought that belief in God was more than just an article of faith but was also rational. He sought to show this by proving the existence of God using logic. The essence of his argument was that if we postulate that God exists then we can, through a series of logical steps, prove that god exists. Such a circular argument is of course invalid, although it is often used to reinforce an argument. It is simply not the case that by assuming something to be true you can prove that therefore it must be true.

This has important implications for quantum physics because quantum theory is itself based on an assumption. It was originally proposed by John W Nicholson (1881-1955). He observed that the units of Planck's constant were the same as those for angular momentum. So he reasoned that the orbital angular momentum of electron in the hydrogen atom was related to Planck's constant. He went one step further and argued that the orbital angular momentum could only take on values which were an integer multiple of Planck's constant. He thus "quantised" angular momentum. Niels Bohr then used this assumption to develop his mathematical model for the hydrogen atom.

This idea that angular momentum is quantised is a cornerstone of quantum theory. Not only did Bohr use it to create a mechanical model for the atom, but so did Louis de Broglie and Erwin Schrödinger. De Broglie argued that the electron could be viewed as a wave, and showed that his waves formed a harmonic series when viewed in the context of the atom. However his waves were the result of assuming that the wavelength is related to Planck's constant, which in turn he assumed to be quantised in exactly the same way as Bohr. Schrödinger developed a set of equations to describe de Broglie's waves and this too incorporates Nicholson's postulate. Indeed the simplest way to derive Schrödinger's wave equation is to substituted quantised angular momentum into the canonical form of an undamped second order differential equation.

Given that quantum theory rests on the assumption that angular momentum is quantised means that it is not possible validate quantum theory and prove that angular momentum is quantised by consideration of quantum theory itself. Since we cannot prove the validity of quantum theory from the perspective of quantum theory, we must do so using classical mechanics. A problem arises that if we try to do so using classical Newtonian mechanics, while we obtain a stable atom, it only has a single energy level. In order to prove quantum theory or indeed develop a valid quantum theory it is necessary that we somehow modify Newtonian mechanics in such a way as to present us with an infinite number of energy levels in and in such a way that the differences between energy levels matches those of the empirically derived Rydberg formula.

This is exactly what Niels Bohr did when he derived his eponymous model for the hydrogen atom. He argued that the model presented by Newtonian mechanics was incorrect because it did not take account of the idea that angular momentum was quantised into units of Planck's constant. By modifying classical Newtonian mechanics in this way Bohr was able to derive a model which matched the energy levels of the Rydberg formula.

While it is not possible to prove that a postulate is true based on the truth of the assumption, the obverse is not the case. It is possible to disprove postulate by first assuming that it is true and then showing that this leads to a contradiction, a paradox or an absurdity. Such proofs are referred to as *Reductio ad Absurdum* and are commonplace throughout mathematics and date back to ancient times. A good example is Euclid's proof that the square root of two is irrational. Euclid first postulates that the square root of two is rational and then shows that this leads to a contradiction and hence that it cannot be rational. Indeed the so called 'scientific method' is itself based on the underlying logic of *reductio ad absurdum*. This requires that we first put forward an assumption or postulate and based on this develop a model. The model is then tested against experimental or empirical data and if it fails the postulate underpinning the model is deemed to be incorrect. In this case the absurdity is the failure of the experiment used to test the model, but otherwise the logic is essentially the same. It should be noted that the scientific method can never prove a postulate to be true, it can only ever be used to show that a postulate is false.

The assumption that angular momentum is quantised is just such a case where we can apply the logic of reductio ad absurdum. Using this assumption we can derive Bohr's model for the hydrogen atom, however the model requires that changes in the energy level of the atom occur when the electron moves from one orbit to another without ever occupying anywhere in between the two orbits. This was quickly dubbed the "quantum leap" and is clearly a physically impossibility. It was recognised that this was sufficient to render the Bohr model invalid, but it also means that the assumption that lies behind it is also invalid. That is angular momentum cannot be quantised, at least not in the way that Nicholson and Bohr describe.

To get around this slight inconvenience, physicists will often say that the Bohr model is obsolete and that our view of the world has moved on, that reality is not what it seems, that particles do not exist until they are observed etc. However it is a false premise to proceed along these lines when the underlying assumption has already been shown to be invalid. What all of these circumlocutions amount to is simply another way of trying to describe the quantum leap but without using the words "quantum" or "leap". The electron for example is described as a wave function which "collapses" when it is observed to reveal the position or the velocity of the electron itself, which is now viewed as being in its particle form rather that its wave form. The process of collapsing is just another way of describing the instantaneous transformation of the electron into a particle which then exists at some point in space by denying that it existed as a particle prior to this transformation. In reality all such descriptions are simply euphemisms for the quantum leap.

The fact that the Bohr model leads to the absurdity of the quantum leap clearly demonstrates that the assumption that angular momentum is quantised is false. To then argue that it is correct to assume that angular momentum is quantised if we change to viewing the electron as a wave rather than as a particle is equally invalid. It is akin to telling Euclid that the square root of two is a rational number if we change the context in which we view it. It is logically inconsistent to accept that the quantum leap is a physical impossibility and to still assert that angular momentum is quantised. Once a postulate is shown to be invalid it remains irredeemably invalid whatever the context.

We do however know that something is quantised and that this is connected with the discrete energy levels of the atom. This is because there is a relationship between the various energy levels which is linked to a harmonic series. In the case of de Broglie these harmonics are thought to be related to the orbit of the electron as standing waves, existing as multiples of the base orbital frequency.

Wherever we see a harmonic series in nature there must be an accompanying sampling process or quantisation taking place. This comes about when we consider the Fourier representation of a harmonic series.   The Fourier representation of a harmonic series is unity at the base or fundamental frequency and at every integer multiple of the base frequency and has a value of zero everywhere else on the jω axis. For real entities this extends along both positive and negative jω axes to infinity. Such a function is commonly referred to as Dirac Comb.   The Fourier transform of a Dirac Comb is itself another Dirac Comb, only this time in either the time domain or the space domain. Such a Dirac comb is a sampling function or, if you prefer, a quantisation function.

All of which begs the question that if angular momentum is not quantised but we know that something must be quantised, what exactly is the variable of quantisation inherent in the structure of the atom?

A possible clue lies in the timeline of events leading up to Bohr's model. For the better part of 300 years it was assumed that Newton's version of classical mechanics was complete. Then in 1905 Einstein showed that it was not, he showed that time and distance and mass varied according to the relative velocity of the observer and the observed. Bohr's model was first published in 1912 and even at the time was acknowledged to have problems. Bohr chose to ignore relativity, which at the time was not well understood and even rejected by some physicists. So just at the time when we needed to investigate the idea that Newtonian mechanics were incorrect, the idea that one aspect of them is incorrect emerged. Given the timing of the discovery of relativity and the attempts to describe the dynamics of the atom, it seems highly likely that relativity lies at the heart of any misconceptions we might have about classical mechanics.

In summary: quantum theory can only be validated from the perspective of classical theory and not from within quantum theory itself. This means that there has to be something wrong in our current understanding of classical mechanics, since as it is currently understood the mechanics of the atom simply do not work. The presence of a harmonic series in the mechanics of the atom means that something is sampled or quantised. Niels Bohr sought to suggest it is angular momentum that is quantised into units of Planck's constant. However this assumption leads to an absurdity, the quantum leap, and so cannot itself be valid. Despite this the assumption has been carried forward without question into subsequent models of the atom. This means that we must seek an alternative explanation based on quantisation of some other variable. It is highly likely that whatever is quantised it is connected in some way with the effects of relativity.

We will ultimately gauge the success of any new postulates and models for the atom based on the scientific method, which requires that whatever we postulate as the deficiency in classical mechanics is tested by experiment. In the meantime we can suggest a few pointers to a successful model. First, of course, it must have an infinite number of stable states whose differences return the correct values for the energy levels of the atom by matching those predicted by the Rydberg formula. The dynamics must be such that the orbital radius of the electron remains the same in all the various energy levels, since anything other than this would require the existence of the quantum leap or its latter day equivalent. It should also address the issues of which Bohr was unaware or chose to ignore; these include an explanation for the existence and the value of the Fine Structure Constant, an explanation for the existence of and value of Zero Point energy, an explanation as to why the orbiting electron does not emit synchrotron radiation and it must be seen to fully take into account the effects of relativity. Finally, since all of the variables involved are continuous, it should provide an explanation as to how exactly such continuous variables can interact so as to only be able to take on the discrete values. In effect this is the mechanism which underlies quantisation and is the causal link that de Broglie was unable to find.

The challenge that physics needs to address is first to recognise that unification of quantum and classical mechanics is essential to our understanding of the universe and that the major obstacle to such a unification is the belief that current models based on the quantisation of angular momentum are correct but incomplete. It is to recognise that the problems with quantum theory lie in its very origins and not in some peripheral oversight.