Einstein's field equations geometrized

Ilija Barukčić[1]

[1] Internist, Horandstrasse, DE-26441 Jever, Germany

Correspondence: Ilija Barukčić, Horandstrasse, DE-26441 Jever, Germany. Tel: 49-4466-333. E-mail: Barukcic@t-online.de

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ABASTRACT

Objective. The possibility of the geometrization of the stress-energy-tensor of energy/matter of Einstein's general theory of relativity and the problems associated with such an undertaking is reviewed again.

Methods.

The usual tensor calculus rules were used.

Results.

The stress-energy-tensor of matter of Einstein's general theory of relativity is geometrized. The stress-energy-tensor of the electromagnetic field is geometrized too.

Conclusions.

Einstein's field equations are completely geometrized.

Keywords: principium identitatis, principium contradictionis, theory of relativity, Unified field theory, causality

1. Introduction

Gravity[1] as the dominant interaction at large length scales is an essential part of cosmology and is equally deeply interrelated with energy[2], time[3] and space. Albert Einstein[4–6] introduced a new way of representing gravity by replacing the single gravitational potential and the associated field equation of Newton’s theory of gravitation. One of the basic features of Einstein’s theory of general relativity (GTR) and equally that what distinguishes GTR sharply from all other competing physical theories, is the geometrization of physical interaction which opened theoretically the possibility to understand the gravitational field as something like the manifestation of space - time curvature. Einstein’s point of view was too that the gravitational field can be described by mathematical tools like the metric tensor $g_{\mu\nu}$. However, this need not imply that gravity is and has to be reduced to pure geometry in its own right. In point of fact, Einstein’s stress-energy momentum tensor of GTR is equally a weak spot of his theory because this field is still **devoid[7] of any geometrical significance**. Various proposals for a unified field theory “a generalization of the theory of the gravitational field”[8] were influenced by the desire to **replace the stress-energy momentum tensor of Einstein’s general theory of relativity by geometrical structures**. In order to bring some order into the many different ways to include the electromagnetic field into a geometric setting, Einstein’s theory of general relativity[9] can serve as a point of departure for such an undertaking. However, it is not inevitably necessary to assume that ‘geometrization’ and ‘unification’ are incompatible. However, both need not to be conceptually identical either. Under optimal circumstances a complete geometrization of Einstein’s gravitational field equations has the potential to end up at a **unified field theory** in the sense of Weyl and Eddington’s classical field theory in which all fundamental interactions are described by objects of space-time geometry. Trying to answer these and similar questions[6,10] was the subject of many published manuscripts and is the main subject of this publication too.

2. Material and methods

The Royal Society of London and the Royal Astronomical Society announced at their joint meeting[11] on the sixth of November 1919 that astronomical observations made by a special British team during the solar eclipse on May 29 provided the first empirical test of the validity of Einstein's general theory of relativity. In order to obtain a kind of a deeper knowledge of the foundations of nature and of physics as such it appears to be reasonable to assure that new basic physical concepts are in accordance with Einstein's general of relativity[5] too. In point of fact, attempts to extend general relativity's geometrization of gravitational force to non-gravitational interactions, in particular, to electromagnetism[9], were not completely in vain.

2.1 Definitions

Some basic rules of tensor calculus can be found in secondary literature too.

Definition 2.1.1(Anti tensor).

Let $a_{\mu\nu}$ denote a certain co-variant tensor. Let $b_{\mu\nu}$, $c_{\mu\nu}$ ... denote other tensors. Let $S_{\mu\nu}$ denote another tensor. Let the relationship $a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + \ldots \equiv S_{\mu\nu}$ be given. The **anti tensor** of a tensor $a_{\mu\nu}$, denoted in general as $\underline{a}_{\mu\nu}$, is defined as

$$\begin{aligned} \underline{a}_{\mu\nu} &\equiv S_{\mu\nu} - a_{\mu\nu} \\ &\equiv b_{\mu\nu} + c_{\mu\nu} + \cdots \end{aligned} \tag{1}$$

There are circumstances were an anti-tensor is identical with an anti-symmetrical tensor, but both are not identical as such.

Definition 2.1.2(Einstein's field equations).

Let $R_{\mu\nu}$ denote the **Ricci tensor**[12] of 'Einstein's general theory of relativity'[5], a geometric object developed by Gregorio Ricci-Curbastro (1853 – 1925) which is able to encapsulate the degree to which a certain geometry of a given metric differs from the geometry of ordinary Euclidean space. Let R denote the **Ricci scalar**, the trace of the Ricci curvature tensor with respect to the metric and equally the simplest curvature invariant of a

Riemannian manifold. Ricci scalar curvature is the contraction of the Ricci tensor and is written as R without subscripts or arguments. Let Λ denote the Einstein's cosmological constant. Let $\underline{\Lambda}$ denote the "anti cosmological constant"[13]. Let $g_{\mu\nu}$ denote the metric tensor of Einstein's general theory of relativity. Let $G_{\mu\nu}$ denote Einstein's[14] curvature tensor, the trace-reversed Ricci tensor. Let $\underline{G}_{\mu\nu}$ denote the "anti tensor"[15] of Einstein's curvature tensor. Let $E_{\mu\nu}$ denote stress-energy tensor of energy. Let $\underline{E}_{\mu\nu}$ denote tensor of non-energy, the anti-tensor of the stress-energy tensor of energy. Let $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$ and $d_{\mu\nu}$ denote the four basic fields of nature were $a_{\mu\nu}$ is the stress-energy tensor of ordinary matter, $b_{\mu\nu}$ is the stress-energy tensor of the electromagnetic field. Let c denote the speed of the light in vacuum, let γ denote Newton's gravitational "constant"[16–19]. Let π denote the number pi. Einstein's field equation, published by Albert Einstein[4] for the first time in 1915, and finally 1916[5] but later with the "cosmological constant"[6] term relate the curvature of spacetime to the energy and momentum as

$$\begin{aligned}
(R_{\mu\nu}) - \left(\left(\frac{R}{2}\right) \times (g_{\mu\nu})\right) + (\Lambda \times g_{\mu\nu}) &\equiv \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\nu} \\
&\equiv \left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times 4 \times T_{\mu\nu} \\
&\equiv G_{\mu\nu} + (\Lambda \times g_{\mu\nu}) \\
&\equiv R_{\mu\nu} - \underline{E}_{\mu\nu} \\
&\equiv E_{\mu\nu} \\
&\equiv a_{\mu\nu} + b_{\mu\nu} \\
LEFT\ SIDE &\equiv RIGHT\ SIDE
\end{aligned} \tag{2}$$

There are circumstances were an anti-tensor is identical with an anti-symmetrical tensor. However, an anti-tensor and an anti-symmetrical tensor are not identical as such. In point of fact, the left-hand side of the Einstein's field equations represents only one part (Ricci curvature) of the geometric structure (i.e. Weyl curvature).

Definition 2.1.3 (Weyl's tensor).

Let $R_{kl\mu\nu}$ denote the Riemann curvature tensor. Let $W_{kl\mu\nu}$ denote Weyl's[20] curvature tensor, the traceless component of the Riemann tensor $R_{kl\mu\nu}$. Let $\underline{W}_{kl\mu\nu}$ denote anti Weyl's curvature tensor. In general, the Riemann tensor can be split[21] into a symmetric (**massless**) **traceless** Weyl's curvature tensor $W_{kl\mu\nu}$ and a **trace-full massive** part $\underline{W}_{kl\mu\nu}$ as

$$R_{kl\mu\nu} \equiv \underbrace{W_{kl\mu\nu}}_{Weyl's\ curvature\ tensor} + \underbrace{\underline{W}_{kl\mu\nu}}_{Anti\ Weyl's\ curvature\ tensor} \tag{3}$$

The Weyl tensor $W_{kl\mu\nu}$ differs from the Riemann curvature tensor $R_{kl\mu\nu}$. The Weyl tensor $W_{kl\mu\nu}$ does not convey information on how the volume of a certain body changes, but rather how the shape of this body is distorted by tidal forces. A further manipulation (decomposition) of this equation before yields the relationship

$$\begin{aligned} \underbrace{g^{kl} R_{kl\mu\nu}} &\equiv \underbrace{g^{kl} W_{kl\mu\nu}} + \underbrace{g^{kl} \underline{W}_{kl\mu\nu}} \\ R_{\mu\nu} &\equiv W_{\mu\nu} + \underline{W}_{\mu\nu} \end{aligned} \tag{4}$$

while the Ricci tensor $R_{\mu\nu}$ comes from the contraction of the Riemann tensor $R_{kl\mu\nu}$ and *Weyl's tensor*, denoted as $W_{\mu\nu}$, comes from the contraction of the *Weyl's[20] curvature tensor* $W_{kl\mu\nu}$, while anti Weyl's tensor, denoted as $\underline{W}_{\mu\nu}$, comes from the contraction of anti Weyl's curvature tensor $\underline{W}_{kl\mu\nu}$. In general relativity, the Weyl curvature exists in free space too and is related to the propagation of **gravitational waves**[21,22] through regions of space devoid of matter (vacuum solution of Einstein's field equation). Einstein's field equations simplify under these circumstances as

$$\begin{aligned} (\underline{\boldsymbol{W}}_{\mu\nu}) - \left(\left(\frac{\boldsymbol{R}}{2}\right) \times (\boldsymbol{g}_{\mu\nu})\right) + (\boldsymbol{\Lambda} \times \boldsymbol{g}_{\mu\nu}) + (\boldsymbol{W}_{\mu\nu}) &\equiv \left(\frac{2 \times \boldsymbol{\pi} \times \boldsymbol{\gamma} \times \boldsymbol{T}}{\boldsymbol{c}^4}\right) \times \boldsymbol{g}_{\mu\nu} \\ &\equiv a_{\mu\nu} + b_{\mu\nu} \\ LEFT\ SIDE &\equiv RIGHT\ \ SIDE \end{aligned} \tag{5}$$

This equation draws out key highlights of Einstein's field equations in much greater detail than the original one field equations due to the presence of Weyl's tensor. In point of fact, one of the many justified questions is whether there may exist circumstances where the three basic fields of nature $a_{\mu\nu}$, $b_{\mu\nu}$, $c_{\mu\nu}$, (except gravitational waves $d_{\mu\nu}$) are determined as

$$\begin{aligned}(a_{\mu\nu}) + (b_{\mu\nu}) + (c_{\mu\nu}) &\equiv \underline{W}_{\mu\nu} \\ &\equiv (R_{\mu\nu}) - (\underbrace{d_{\mu\nu}}) \\ &\equiv (R_{\mu\nu}) - (W_{\mu\nu})\end{aligned} \tag{6}$$

Especially, there is some evidence[23] that Einstein's tensor $G_{\mu\nu}$ is not completely identical with $\underline{W}_{\mu\nu}$. Therefore, the probability is mounting with some slight extent that

$$\begin{aligned}(d_{\mu\nu}) &\equiv (R_{\mu\nu}) - \left(\underbrace{(a_{\mu\nu}) + (b_{\mu\nu}) + (c_{\mu\nu})}\right) \\ &\equiv (R_{\mu\nu}) - (\underline{\boldsymbol{W}}_{\mu\nu}) \\ &\equiv g^{kl} W_{kl\mu\nu} \\ &\equiv W_{\mu\nu}\end{aligned} \tag{7}$$

Definition 2.1.4 (The Einstein's curvature tensor).

The physics of gravitation as described by general relativity demands that the curvature of spacetime is determined by the presence of energy-momentum to manifest itself as gravity and vice versa. In turn, matter or energy and momentum acts on spacetime to create curvature. Under conditions of Einstein's general theory of relativity, Einstein's tensor of curvature denoted as $G_{\mu\nu}$ is defined/derived/determined as

$$\begin{aligned}G_{\mu\nu} &\equiv (R_{\mu\nu}) - \left(\left(\frac{R}{2}\right) \times (g_{\mu\nu})\right) \\ &\equiv a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} - b_{\mu\nu} \\ &\equiv a_{\mu\nu} + d_{\mu\nu} + c_{\mu\nu} - d_{\mu\nu} \\ &\equiv a_{\mu\nu} + c_{\mu\nu} \\ &\equiv G \times g_{\mu\nu}\end{aligned} \tag{8}$$

where G is equal to

$$\begin{aligned}\boldsymbol{G} &\equiv \boldsymbol{S} - \left(\frac{\boldsymbol{R}}{\boldsymbol{2}}\right) \\ &\equiv \boldsymbol{E} + \boldsymbol{t} - \left(\frac{\boldsymbol{R}}{\boldsymbol{2}}\right) \\ &\equiv \left(\frac{\boldsymbol{2} \times \boldsymbol{\pi} \times \boldsymbol{\gamma} \times \boldsymbol{T}}{\boldsymbol{c^4}}\right) + \left(\frac{\boldsymbol{R}}{\boldsymbol{2}} - \boldsymbol{\Lambda}\right) - \left(\frac{\boldsymbol{R}}{\boldsymbol{2}}\right) \\ &\equiv \left(\frac{\boldsymbol{2} \times \boldsymbol{\pi} \times \boldsymbol{\gamma} \times \boldsymbol{T}}{\boldsymbol{c^4}}\right) - \boldsymbol{\Lambda}\end{aligned} \tag{9}$$

Definition 2.1.5 (The tensor of non-energy).

Under conditions of Einstein's general theory of relativity, the tensor of non-energy, denoted as $\underline{E}_{\mu\nu}$, or the anti tensor of the stress energy tensor of energy is defined/derived/determined as follows:

$$
\begin{aligned}
\underline{E}_{\mu\nu} &\equiv (R_{\mu\nu}) - \left(\frac{4 \times 2 \times \pi \times \gamma}{c^4}\right) \\
&\equiv \left(\left(\frac{R}{2}\right) \times (g_{\mu\nu})\right) - (\Lambda \times g_{\mu\nu}) \\
&\equiv \left(\frac{R}{2} - \Lambda\right) \times g_{\mu\nu} \\
&\equiv c_{\mu\nu} + d_{\mu\nu} \\
&\equiv \underline{E} \times g_{\mu\nu} \\
&\equiv t \times g_{\mu\nu}
\end{aligned}
\tag{10}
$$

where

$$
t \equiv \underline{E} \equiv \left(\frac{R}{2} - \Lambda\right) \tag{11}
$$

Definition 2.1.6 (The anti Einstein's curvature tensor $\underline{G}_{\mu\nu}$ or the tensor or non-curvature).

Under conditions of Einstein's general theory of relativity, the anti Einstein tensor, the tensor of non-curvature, denoted as $\underline{G}_{\mu\nu}$, is defined/derived/determined

$$
\begin{aligned}
\underline{G}_{\mu\nu} &\equiv (R_{\mu\nu}) - (G_{\mu\nu}) \\
&\equiv (R_{\mu\nu}) - \overbrace{\left(R_{\mu\nu} - \left(\frac{R}{2}\right) \times (g_{\mu\nu})\right)} \\
&\equiv \left(\frac{R}{2}\right) \times (g_{\mu\nu}) \\
&\equiv a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu} - a_{\mu\nu} - c_{\mu\nu} \\
&\equiv b_{\mu\nu} + d_{\mu\nu} \\
&\equiv \underline{G} \times g_{\mu\nu}
\end{aligned}
\tag{12}
$$

where $\underline{G}$ = (R/2).

Definition 2.1.7 (The tensor $d_{\mu\nu}$ (neither curvature nor momentum)).

Under conditions of Einstein's general theory of relativity, the tensor of neither curvature nor momentum, denoted as $d_{\mu\nu}$, is defined/derived/determined as follows:

$$\begin{aligned} d_{\mu\nu} &\equiv \left(\frac{R}{2}\right) \times (g_{\mu\nu}) - (b_{\mu\nu}) \\ &\equiv \left(\frac{R}{2} \times (g_{\mu\nu})\right) - (\Lambda \times (g_{\mu\nu})) - (c_{\mu\nu}) \\ &\equiv d \times g_{\mu\nu} \end{aligned} \tag{13}$$

where

$$\begin{aligned} d &\equiv \left(\frac{R}{2}\right) - b \\ &\equiv \left(\frac{R}{2}\right) - \left(\left(\frac{1}{4 \times 4 \times \pi}\right) \times \left((F_{\mu c} \times F^{\mu c}) - (F_{de} \times F^{de})\right)\right) \end{aligned} \tag{14}$$

while F is called the (traceless) Faraday/electromagnetic/field strength tensor. Until further proof/disproof, we assume the following

$$\begin{aligned} W_{\mu\nu} &\equiv \left(\frac{R}{2}\right) \times (g_{\mu\nu}) - (b_{\mu\nu}) \\ &\equiv \left(\frac{R}{2} \times (g_{\mu\nu})\right) - (\Lambda \times (g_{\mu\nu})) - (c_{\mu\nu}) \\ &\equiv d \times g_{\mu\nu} \end{aligned} \tag{15}$$

Definition 2.1.8 (The tensor $c_{\mu\nu}$).

Under conditions of Einstein's general theory of relativity, the tensor of non-momentum and curvature, denoted as $c_{\mu\nu}$, is defined/derived/determined as follows:

$$
\begin{aligned}
c_{\mu\nu} &\equiv (G_{\mu\nu}) - (a_{\mu\nu}) \\
&\equiv \overbrace{(a_{\mu\nu}) + (c_{\mu\nu})} - (a_{\mu\nu}) \\
&\equiv \underbrace{\left(\frac{R}{2} \times (g_{\mu\nu})\right) - \left(\Lambda \times (g_{\mu\nu})\right)} - (d_{\mu\nu}) \\
&\equiv (c_{\mu\nu}) + (d_{\mu\nu}) - (d_{\mu\nu}) \\
&\equiv (b_{\mu\nu}) - \left(\Lambda \times (g_{\mu\nu})\right) \\
&\equiv c \times g_{\mu\nu}
\end{aligned}
\tag{16}
$$

where c is equal to

$$
\boldsymbol{c \equiv \left(\left(\frac{1}{4 \times 4 \times \pi}\right) \times \left((F_{\mu c} \times F^{\mu c}) - (F_{de} \times F^{de})\right)\right) - \Lambda} \tag{17}
$$

Definition 2.1.9 (The tensor $b_{\mu\nu}$).

The co-variant stress-energy tensor of the electromagnetic field, in this context denoted by $b_{\mu\nu}$, is of order two while its components can be displayed by a 4×4 matrix too. Under conditions of Einstein's general theory of relativity, the tensor $b_{\mu\nu}$ denotes the stress-energy tensor of the electromagnetic field expressed more compactly and, in a coordinate independent form as

$$
\begin{aligned}
b_{\mu\nu} &\equiv (E_{\mu\nu}) - (a_{\mu\nu}) \\
&\equiv \overbrace{(a_{\mu\nu}) + (b_{\mu\nu})} - (a_{\mu\nu}) \\
&\equiv \left(\frac{R}{2} \times (g_{\mu\nu})\right) - (d_{\mu\nu}) \\
&\equiv \overbrace{(b_{\mu\nu}) + (d_{\mu\nu})} - (d_{\mu\nu}) \\
&\equiv \left(\frac{1}{4 \times 4 \times \pi}\right) \\
&\quad \times \\
&\quad \left((F_{\mu c} \times F^{\mu c}) - (F_{de} \times F^{de})\right) \times g_{\mu\nu} \\
&\equiv b \times g_{\mu\nu}
\end{aligned}
\tag{18}
$$

where (as can be seen in the following of this paper) it is

$$b \equiv \left(\left(\frac{1}{4 \times 4 \times \pi}\right) \times \left(\left(F_{\mu c} \times F^{\mu c}\right) - \left(F_{de} \times F^{de}\right)\right)\right) \tag{19}$$

while F is called the (traceless) Faraday/electromagnetic/field strength tensor.

Definition 2.1.10 (The stress-energy tensor of ordinary matter $a_{\mu\nu}$).

Under conditions of Einstein's general theory of relativity, the stress-energy tensor of ordinary matter, denoted as $a_{\mu\nu}$, is defined/derived/determined as follows:

$$\begin{aligned} a_{\mu\nu} &\equiv a_{\mu\nu} + b_{\mu\nu} - b_{\mu\nu} \\ &\equiv a_{\mu\nu} + c_{\mu\nu} - c_{\mu\nu} \\ &\equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \\ &\quad - \left(\left(\frac{1}{4 \times 4 \times \pi}\right) \times \left(\left(F_{\mu c} \times F^{\mu c}\right) - \left(F_{de} \times F^{de}\right)\right) \right) \times g_{\mu\nu} \\ &\equiv R_{\mu\nu} - b_{\mu\nu} - c_{\mu\nu} - d_{\mu\nu} \\ &\equiv G_{\mu\nu} + \left(\Lambda \times \left(g_{\mu\nu}\right)\right) - b_{\mu\nu} \\ &\equiv a \times g_{\mu\nu} \end{aligned} \tag{20}$$

while

$$a \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) - \left(\left(\frac{1}{4 \times 4 \times \pi}\right) \times \left(\left(F_{\mu c} \times F^{\mu c}\right) - \left(F_{de} \times F^{de}\right)\right) \right) \tag{21}$$

Definition 2.1.11 (The stress-energy momentum tensor $E_{\mu\nu}$).

Under conditions of Einstein's general theory of relativity, the stress-energy momentum tensor, denoted as $E_{\mu\nu}$, is defined/derived/determined as follows:

$$
\begin{aligned}
\left(E_{\mu\nu}\right) &\equiv \left(\frac{4\times 2\times \pi\times \gamma}{c^4}\right) \times T_{\mu\nu}\\
&\equiv \left(\frac{2\times \pi\times \gamma}{c^4}\right) \times 4 \times T_{\mu\nu}\\
&\equiv \left(\frac{2\times \pi\times \gamma\times T}{c^4}\right) \times g_{\mu\nu}\\
&\equiv a_{\mu\nu} + b_{\mu\nu}\\
&\equiv \underbrace{\left(R_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right)\times\left(g_{\mu\nu}\right)\right)} + \Lambda\times g_{\mu\nu}\\
&\equiv G_{\mu\nu} + \Lambda\times g_{\mu\nu}\\
&\equiv \overbrace{a_{\mu\nu} + c_{\mu\nu}} + \left(\Lambda\times g_{\mu\nu}\right)\\
&\equiv R_{\mu\nu} - \underline{E}_{\mu\nu}\\
&\equiv E \times g_{\mu\nu}
\end{aligned}
\tag{22}
$$

while E is determined a

$$
E \equiv \left(\frac{2\times \pi\times \gamma\times T}{c^4}\right) \tag{23}
$$

Definition 2.1.12 (The Ricci tensor $R_{\mu\nu}$).

Under conditions of Einstein's general theory of relativity, the Ricci tensor, denoted a $R_{\mu\nu}$, is defined/derived/determined as follows:

$$
\begin{aligned}
R_{\mu\nu} &\equiv a_{\mu\nu} + b_{\mu\nu} + c_{\mu\nu} + d_{\mu\nu}\\
&\equiv \overbrace{\underbrace{\left(\frac{4\times 2\times \pi\times \gamma}{c^4}\right)\times T_{\mu\nu}}} + \overbrace{\underbrace{\left(\left(\frac{R}{2}\right)\times\left(g_{\mu\nu}\right)\right) - \left(\Lambda\times g_{\mu\nu}\right)}}\\
&\equiv E_{\mu\nu} + \underline{E}_{\mu\nu}\\
&\equiv G_{\mu\nu} + \underline{G}_{\mu\nu}\\
&\equiv (E+t) \times g_{\mu\nu}\\
&\equiv S \times g_{\mu\nu}
\end{aligned}
\tag{24}
$$

while S is defined/derived/determined as follows:

$$
\begin{aligned}
S &\equiv E + t\\
&\equiv \left(\frac{2\times \pi\times \gamma\times T}{c^4}\right) + \left(\frac{R}{2} - \Lambda\right)
\end{aligned}
\tag{25}
$$

Definition 2.1.13 (The Ricci scalar R).

Under conditions of Einstein's general theory of relativity, the Ricci scalar curvature R as the trace of the Ricci curvature tensor $R_{\mu\nu}$ with respect to the metric tensor $g_{\mu\nu}$ is determined at each point in space-time by lamda Λ and *anti-lamda* $\underline{\Lambda}$ [13] as

$$\begin{aligned} R &\equiv g^{\mu\upsilon} \times R_{\mu\upsilon} \\ & \quad g^{\mu\upsilon} \times \overbrace{g_{\mu\upsilon} \times S} \\ &\equiv \underbrace{g^{\mu\upsilon} \times g_{\mu\upsilon}} \times S \\ &\equiv 4 \times S \\ &\equiv \Lambda + \underline{\Lambda} \end{aligned} \tag{26}$$

From this definition follows the relationship S = (R/4) or that

$$\frac{R}{S} \equiv +4 \equiv g^{\mu\upsilon} \times g_{\mu\upsilon} \tag{27}$$

A Ricci scalar curvature *R* which is *positive* at a certain point indicates that the volume of a small ball about the point has smaller volume than a ball of the same radius in Euclidean space. In contrast to this, a *Ricci* scalar curvature R which is *negative* at a certain point indicates that the volume of a small ball is larger than it would be in Euclidean space. In general, we obtain

$$R \times g_{\mu\upsilon} \equiv (\Lambda \times g_{\mu\upsilon}) + (\underline{\Lambda} \times g_{\mu\upsilon}) \tag{28}$$

or

$$(4 \times S) \times g_{\mu\upsilon} \equiv (\Lambda \times g_{\mu\upsilon}) + (\underline{\Lambda} \times g_{\mu\upsilon}) \tag{29}$$

or

$$S \times g_{\mu\upsilon} \equiv \left(\frac{\Lambda}{4} \times g_{\mu\upsilon}\right) + \left(\frac{\underline{\Lambda}}{4} \times g_{\mu\upsilon}\right) \tag{30}$$

or

$$R_{\mu\upsilon} \equiv \mathrm{S} \times g_{\mu\upsilon} \equiv \left(\frac{\Lambda}{4} \times g_{\mu\upsilon}\right) + \left(\frac{\underline{\Lambda}}{4} \times g_{\mu\upsilon}\right) \tag{31}$$

Einstein's field equations are determined as

$$R_{\mu\upsilon} - \left(\left(\frac{R}{2}\right) \times (g_{\mu\upsilon})\right) + \Lambda \times g_{\mu\upsilon} \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\upsilon} \tag{32}$$

Rearranging we obtain

$$R_{\mu\nu} - \left(\left(\frac{R}{2}\right) \times (g_{\mu\nu})\right) - \left(\left(\frac{R}{2}\right) \times (g_{\mu\nu})\right) + \Lambda \times g_{\mu\nu} \equiv \left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times (g_{\mu\nu})\right) \tag{33}$$

or

$$R_{\mu\nu} - (R \times (g_{\mu\nu}) - \Lambda \times g_{\mu\nu}) \equiv \left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) - \left(\left(\frac{R}{2}\right) \times (g_{\mu\nu})\right) \tag{34}$$

or

$$R_{\mu\nu} + \left(\left(\frac{R}{2}\right) \times (g_{\mu\nu})\right) - \left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu}\right) \equiv (R \times (g_{\mu\nu}) - \Lambda \times g_{\mu\nu}) \tag{35}$$

The *anti lamda field* can be derived from Einstein's field equation as

$$\underline{\Lambda} \times g_{\mu\nu} \equiv R_{\mu\nu} + \left(\left(\frac{R}{2}\right) \times (g_{\mu\nu})\right) - \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\nu} \tag{36}$$

In other words, the anti lamda field can be simplified as

$$\underline{\Lambda} \times g_{\mu\nu} \equiv \left(S + \left(\frac{R}{2}\right) - \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right)\right) \times g_{\mu\nu} \tag{37}$$

However, doubts are more than justified whether there can be identified any kind of a particle which can be ascribed to such a more or less purely artificial and unrealistic field without any practical significance. Mathematics' as one product of human mind and imagination can reach us far while reality need not to follow at all. Nonetheless, as found before, it is S = (R/4) and *anti lambda* $\underline{\Lambda}$ can be calculated as

$$\begin{aligned} \underline{\Lambda} &\equiv \left(S + \left(\frac{R}{2}\right) - \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right)\right) \\ &\equiv \left(\left(\frac{R}{4}\right) + \left(\frac{R}{2}\right) - \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right)\right) \end{aligned} \tag{38}$$

or as

$$\underline{\Lambda} \equiv \left(\left(\frac{3 \times R}{4}\right) - \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right)\right) \tag{39}$$

The cosmological constant Λ can also be written algebraically as part of the stress–energy tensor of energy/matter, a second order tensor as the source of gravity (energy density). In this context, the cosmological constant Λ can be determined as

$$\Lambda \equiv \left(\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) + \left(\frac{R}{4}\right)\right) \tag{40}$$

which forces us to accept that

$$\left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \equiv \left(\Lambda - \left(\frac{R}{4}\right)\right) \tag{41}$$

Table 1 may provide a preliminary overview of the definitions of the four basic fields of nature.

		Curvature		
		Yes	No	
Momentum	Yes	$\mathbf{a}_{\mu\nu}$	$\mathbf{b}_{\mu\nu}$	$E \times g_{\mu\nu}$
	No	$\mathbf{c}_{\mu\nu}$	$\mathbf{d}_{\mu\nu}$	$\underline{t} \times g_{\mu\nu}$
		$G_{\mu\nu}$	$\underline{G}_{\mu\nu}$	$S \times g_{\mu\nu}$

Table 1. Four basic fields of nature

Einstein's field equations simplify as

$$\left(S \times g_{\mu\nu}\right) - \left(t \times g_{\mu\nu}\right) \equiv \left(E \times g_{\mu\nu}\right) \tag{42}$$

Under conditions where the Hamiltonian operator H is equal to **H=E**, the Hamiltonian operator and the wave function ψ is equal[24] to **ψ =t**, Einstein's field equation changed to

$$\left(S \times g_{\mu\nu}\right) - \left(\psi \times g_{\mu\nu}\right) \equiv \left(H \times g_{\mu\nu}\right) \tag{43}$$

Definition 2.1.14 (The inverse metric tensor $g^{\mu\nu}$ and the metric tensor $g_{\mu\nu}$).

Under conditions of Einstein's general[5] theory of relativity, it is

$$g^{\mu\nu} \times g_{\mu\nu} \equiv +4 \tag{44}$$

or

$$\frac{1}{g^{\mu\nu}} \times \frac{1}{g_{\mu\nu}} \equiv \frac{1}{+4} \tag{45}$$

where $g^{\mu\nu}$ is the matrix inverse of the metric tensor $g_{\mu\nu}$. The inverse metric tensor or the metric tensor, which is always symmetric, allow tensors to be transformed into each other. Einstein replaced the scalar Newtonian gravitational potential from Poisson's equation for gravity in general relativity by the metric tensor $g_{\mu\nu}$. Einstein's emphasizes this issue using the following statement *"... in the general theory of relativity ... must be ... the tensor $g_{\mu\nu}$ of the gravitational potential"*[25 (p. 88)]

Definition 2.1.15 (Laue's scalar T).

Max von Laue (1879-1960) proposed a scalar[26] as the contraction or as the trace of the stress–energy momentum tensor $T_{\mu\nu}$, denoted as T and written without subscripts or arguments. However, Einstein himself criticized Laue's scalar[27] T. Under conditions of Einstein's general[5] theory of relativity, it is

$$T \equiv g^{\mu\upsilon} \times T_{\mu\upsilon} \tag{46}$$

where $T_{\mu\nu}$ "*denotes the co-variant energy tensor of matter*" [25 (p. 88)]. Einstein explained further in the text: "*Considered phenomenologically, this energy tensor is composed of that of the electromagnetic field and of matter in the narrower sense.*" [25 (p. 93)]

Definition 2.1.16 (Index raising).

For an order-2 tensor, twice multiplying by the contra-variant metric tensor and contracting in different indices[28] raises each index. In simple words, it is

$$F^{\substack{1\ 3\\ \mu\ c}} \equiv g^{\substack{1\ 2\\ \mu\ \upsilon}} \times g^{\substack{3\ 4\\ c\ d}} \times F_{\substack{\upsilon\ d\\ 2\ 4}} \tag{47}$$

or more professionally

$$F^{\mu c} \equiv g^{\mu\upsilon} \times g^{cd} \times F_{\upsilon d} \tag{48}$$

2.2 Axioms

2.2.1 Axioms in general

Making errors is indeed only human. However, it is important to be able to recognize erroneous thinking as such. Axioms[29] and rules chosen carefully can be of use to recognize and to avoid logical inconsistency and equally may be of help to prevent science from supporting particular ideologies. Rightly or wrongly, long lasting advances in our knowledge of nature are enabled by suitable axioms[30] too. In this context, Einstein himself reiterates again the importance of implementing axioms in science. "*Die wahrhaft großen Fortschritte der Naturerkenntnis sind auf einem der Induktion fast diametral entgegengesetzten Wege entstanden*."[31 (p. 17)] Einstein's previous position now been translated into English: *The truly great advances in our understanding of nature originated in a manner almost diametrically opposed to induction*. It is worth mentioning in this matter, Einstein himself advocated especially basic laws (axioms) and conclusions derived from such axioms as a main logical foundation of any 'theory'. "*Grundgesetz (Axiome) und Folgerungen zusammen bilden das was man eine 'Theorie' nennt*." [31 (p. 17)] Albert Einstein's (1879-1955) message translated into English as: "*Basic law (axioms) and conclusions together form what is called a 'theory'* " has still to get round. However, it is currently difficult to ignore completely these historical and far reaching words of wisdom. The same taken more seriously and put into practice, will yield an approach to fundamental scientific problems which is more creative and sustainably logically consistent. Historically, Aristotle himself already cited *the law of excluded middle* and *the law of contradiction* as examples of axioms. However, *lex identitatis* is an axiom too, which possess the potential to serve as the most basic and equally as the simplest axiom of science. Something which is really just itself is equally different from everything else. In point of fact, is such an equivalence which everything has to itself inherent or must the same be constructed by human mind and consciousness. Following Gottfried Wilhelm Leibniz (1646-1716): "*Chaque chose est ce qu'elle est. Et dans autant d'exemples qu'on voudra A est A, B est B.*" [32 (p. 327)] or A = A, B = B or +1 = +1. In this context, lex contradictionis, the negative of lex identitatis, or +0 = +1 is of no minor importance too.

2.2.2 Axiom I. Lex identitatis

To say that +1 is identical to +1 is to say that both are the same.

AXIOM1. LEX IDENTITATIS.

$$g^{\mu v} \times g_{\mu v} - 3 \equiv +1 \tag{49}$$

or

$$+1 \equiv +1 \tag{50}$$

However, even such a numerical identity which seems in itself wholly unproblematic, for it indicates just to a relation which something has to itself and nothing else, is still subject to controversy. Another increasingly popular view is that the same numerical identity implies the controversial view that we are talking about two different numbers +1. The one +1 is on the left side on the equation, the other +1 is on the right side of an equation. The basicness of the relation of identity implies the contradiction too while circularity is avoided. In other words, how can the same +1 be identical with itself and be equally different from itself? We may usefully state that identity is an utterly problematic notion and might be the most troublesome of all.

2.2.3 Axiom II. Lex contradictionis

AXIOM2. LEX CONTRADICTIONIS.

$$\left((g^{\mu v} \times g_{\mu v}) - 3\right) - \left((g^{\mu v} \times g_{\mu v}) - 3\right) \equiv +1 \tag{51}$$

or

$$+0 \equiv +1 \tag{52}$$

A considerable obstacle to understanding contemporary usage of the term contradiction, however, is that contradiction does not seem to be a unitary one. How can something be both, itself (a path is a straight line from the standpoint of a co-moving observer at a certain point in space-time) and the other of itself, its own opposition (the same path is not a straight line, the same path is curved, from the standpoint of a stationary observer at a certain point in space-time)[33]. We may simply deny the existence of objective or of any other contradictions. However, even if it remains justified especially according to Einstein's special theory of relativity that it is not guaranteed that the notion of an absolute contradiction is not secured, Einstein's special theory of relativity insist that contradictions are objective and real[33]. This fact highlights and implies the question too, why should and how can an identical be a contradictory too?

2.2.4 Axiom III. Lex negationis

AXIOM3. LEX NEGATIONIS.

$$\neg \times 0 \quad \equiv \quad +1 \tag{53}$$

where ¬ denotes the (natural "*determinatio negatio est*"[34]/logical[35]) process of negation[36].

Theorem 3.1 (THE GEOMETRICAL FORM OF THE STRESS-ENERGY TENSOR OF THE ELECTROMAGNETIC FIELD $b_{\mu\nu}$).

All mater considered as some deformation of space-time would enable a combined theory of electromagnetism and gravitation in terms of geometry too. However, it is of course quite natural to ask whether nature's all (long range) fields (of force) have a common origin at all? In particular, is the electromagnetic field a property of spacetime or merely something being embedded in spacetime? Einstein's mathematics of the theory of general relativity related the gravitational field to the curvature of spacetime, while the electromagnetic field remains completely unrelated to the spacetime geometry. Unfortunately, the geometrization of the electromagnetic fields within the frame of Einstein's theory of general relativity has been left behind by Einstein as an unsolved problem. In recent years, the work on the geometrization of electromagnetism has begun to increase. Many different trials proposed its own way to extend the geometry of general relativity that would, so it seemed, serve as a geometrization of the electromagnetic field as well. However, the most of these trials[9,37,38] were thus far not widely accepted or are totally ignored by scientific community. While wrong papers don't make it through history on the long run, we will have to face the fact that the obvious public ignorance of such publications appears to provide a kind of an indirect proof that these and similar papers are simply wrong because ignoring the same suffices to date. With regard to the conceptual differences between the geometrized gravitational field and the classical Maxwellian theory of the electromagnetic filed the obviously more than insurmountable old borders remain at least in scientist's consciousness. Nonetheless and needless to say that only a minority of inveterate optimists try once and again to dispute accepted wisdom in this context.

CLAIM.

In general, the completely geometrical form of the stress-energy momentum tensor of the electromagnetic field $b_{\mu\nu}$ is given by

$$b_{\mu v} \equiv \left(\left(\frac{1}{4 \times 4 \times \pi}\right) \times \left(\left(F_{\mu c} \times F^{\mu c}\right) - \left(F_{de} \times F^{de}\right)\right)\right) \times g_{\mu v} \tag{54}$$

PROOF BY MODUS PONENS.

If the premise of modus ponens

$$(+1) \equiv (+1) \tag{55}$$

is true, **then** the following conclusion

$$b_{\mu v} \equiv \left(\left(\frac{1}{4 \times 4 \times \pi}\right) \times \left(\left(F_{\mu c} \times F^{\mu c}\right) - \left(F_{de} \times F^{de}\right)\right)\right) \times g_{\mu v} \tag{56}$$

is also true, the absence of any technical errors presupposed. The premise

$$(+1) \equiv (+1) \tag{57}$$

is true. Multiplying this premise by the stress-energy momentum tensor of the electromagnetic field $b_{\mu\nu}$, we obtain

$$(+1) \times b_{\mu v} \equiv (+1) \times b_{\mu v} \tag{58}$$

or

$$b_{\mu v} \equiv b_{\mu v} \tag{59}$$

Rearranging equation according to the definition above, it is

$$b_{\mu v} \equiv \left(\frac{1}{4 \times \pi}\right) \times \left(\left(\left(F_{\mu c} \times F_{v d} \times g^{cd}\right)\right) - \left(\left(\frac{1}{4}\right) \times \left(F_{de} \times F^{de}\right) \times g_{\mu v}\right)\right) \tag{60}$$

Rearranging equation before again it is

$$b_{\mu\nu} \equiv \left(\frac{1}{4\times\pi}\right)\times\left(\left(1\times\left(F_{\mu c}\times F_{\nu d}\times g^{cd}\right)\right)-\left(\frac{\left(F_{de}\times F^{de}\right)}{4}\right)\times g_{\mu\nu}\right) \tag{61}$$

and equally

$$b_{\mu\nu} \equiv \left(\frac{1}{4\times\pi}\right)\times\left(\left(\frac{4\times\left(F_{\mu c}\times F_{\nu d}\times g^{cd}\right)}{4}\right)-\left(\frac{\left(F_{de}\times F^{de}\right)}{4}\right)\times g_{\mu\nu}\right) \tag{62}$$

According to definition above, this equation simplifies as

$$b_{\mu\nu} \equiv \left(\frac{1}{4\times\pi}\right)\times\left(\left(\frac{\left(g^{\mu\nu}\times g_{\mu\nu}\right)\times\left(F_{\mu c}\times F_{\nu d}\times g^{cd}\right)}{4}\right)-\left(\frac{\left(F_{de}\times F^{de}\right)}{4}\right)\times g_{\mu\nu}\right) \tag{63}$$

and equally as

$$b_{\mu\nu} \equiv \left(\frac{1}{4\times\pi}\right)\times\left(\left(\left(\frac{\left(g^{\mu\nu}\times F_{\mu c}\times F_{\nu d}\times g^{cd}\right)}{4}\right)\times g_{\mu\nu}\right)-\left(\left(\frac{\left(F_{de}\times F^{de}\right)}{4}\right)\times g_{\mu\nu}\right)\right) \tag{64}$$

A further simplification of the relationship before yields the stress-energy momentum tensor of the electromagnetic field $b_{\mu\nu}$ determined by the metric tensor of general relativity $g_{\mu\nu}$ as

$$b_{\mu\nu} \equiv \left(\left(\frac{1}{4\times\pi}\right)\times\left(\left(\frac{1}{4}\right)\times\left(\left(F_{\mu c}\times g^{\mu\nu}\times g^{cd}\times F_{\nu d}\right)-\left(F_{de}\times F^{de}\right)\right)\right)\right)\times g_{\mu\nu} \tag{65}$$

However, the term

$$\left(\left(\frac{1}{4}\right)\times\left(\left(F_{\mu c}\times g^{\mu\nu}\times g^{cd}\times F_{\nu d}\right)-\left(F_{de}\times F^{de}\right)\right)\right) \tag{66}$$

of the equation before can be simplified further. For an order-2tensor, twice multiplying by the contra-variant metric tensor and contracting in different indices[28] raises each index. In other words, it is according to definition view lines before

$$F^{\mu c} \equiv g^{\mu\nu} \times g^{cd} \times F_{\nu d} \tag{67}$$

which simplifies the term before as

$$\left(\left(\frac{1}{4}\right)\times\left(\left(F_{\mu c}\times F^{\mu c}\right)-\left(F_{de}\times F^{de}\right)\right)\right) \tag{68}$$

or as

$$\left(\frac{1}{4}\right) \times \left(\left(F_{\mu c} \times F^{\mu c}\right) - \left(F_{de} \times F^{de}\right)\right) \tag{69}$$

This relationship simplifies the geometrical form of the stress-energy momentum tensor of the electromagnetic field $b_{\mu\nu}$ further as

$$b_{\mu\upsilon} \equiv \left(\left(\frac{1}{4 \times 4 \times \pi}\right) \times \left(\left(F_{\mu c} \times F^{\mu c}\right) - \left(F_{de} \times F^{de}\right)\right)\right) \times g_{\mu\upsilon} \tag{70}$$

while F is called the (traceless) Faraday/electromagnetic/field strength tensor. The stress-energy momentum tensor of the electromagnetic field is geometrized completely, our conclusion is true.

QUOD ERAT DEMONSTRANDUM.

Theorem 3.2 (THE GEOMETRICAL FORM OF THE STRESS-ENERGY TENSOR OF EENERGY/MATTER).

The starting point of Einstein's theory of general relativity was that gravity as such is at least a property of space-time geometry. Consequently, Einstein published[5] a geometric theory of gravitation while Einstein's initial hope to construct a purely geometric theory of gravitation in which even the sources of gravitation themselves would be of geometric origin has still not been fulfilled. Einstein's field equations have a source term, the stress-energy tensor of matter, radiation, vacuum et cetera, which is of order two and is still devoid of any geometry and free of any geometrical significance.

CLAIM.

In general, the completely geometrical form of the stress-energy momentum tensor of Einstein's theory of general relativity is given by

$$\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\upsilon} \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\upsilon} \tag{71}$$

PROOF BY MODUS PONENS.

If the premise of modus ponens

$$(+1) \equiv (+1) \tag{72}$$

is true, **then** the following conclusion

$$\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu v} \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu v} \tag{73}$$

is also true, the absence of any technical errors presupposed. The premise

$$(+1) \equiv (+1) \tag{74}$$

is true. Multiplying this premise by the stress-energy momentum tensor of matter, we obtain

$$(+1) \times \left(\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu v}\right) \equiv (+1) \times \left(\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu v}\right) \tag{75}$$

or

$$\left(\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu v}\right) \equiv \left(\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu v}\right) \tag{76}$$

This equation simplifies as

$$\left(\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu v}\right) \equiv \left(\left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times 4 \times T_{\mu v}\right) \tag{77}$$

According to definition above this equation is rearranged as

$$\left(\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu v}\right) \equiv \left(\left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times \left(g_{\mu v} \times g^{\mu v}\right) \times T_{\mu v}\right) \tag{78}$$

or as

$$\left(\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu v}\right) \equiv \left(\left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times \left(g^{\mu v} \times T_{\mu v}\right) \times \left(g_{\mu v}\right)\right) \tag{79}$$

Referring to the definition of Laue's scalar T, this equation can be rearranged as

$$\left(\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu v}\right) \equiv \left(\left(\frac{2 \times \pi \times \gamma}{c^4}\right) \times T \times \left(g_{\mu v}\right)\right) \tag{80}$$

The desired geometrical representation of the stress-energy momentum tensor energy/matter of Einstein's general theory of relativity follows as

$$\left(\frac{8 \times \pi \times \gamma}{c^4}\right) \times T_{\mu\upsilon} \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\upsilon} \quad (81)$$

In other words, our conclusion is true.

QUOD ERAT DEMONSTRANDUM.

Remark.

From the geometrical point of view the stress–energy momentum tensor of energy/matter $E_{\mu\nu}$ is more or less identical with the metric tensor $g_{\mu\nu}$, enriched only by view constants and a scalar as

$$E_{\mu\upsilon} \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu\upsilon} \quad (82)$$

Using geometrized units where $\gamma = c = 1$, the geometrical form of the stress–energy momentum tensor of energy/matter $E_{\mu\nu}$ can be rewritten as

$$E_{\mu\upsilon} \equiv (2 \times \pi \times T) \times g_{\mu\upsilon} \quad (83)$$

However, describing the fundamental stress–energy momentum tensor energy/matter, the source term of the gravitational field in Einstein's general theory of relativity, as an inherent geometrical structure, as being determined or dependent on the metric field $g_{\mu\nu}$ is associated with several and far reaching consequences. Theoretically, the properties of energy, momentum, mass, stress et cetera need no longer to be seen as intrinsic properties of matter with the consequence that the properties which material systems possess could be determined in virtue of their relation to space-time structures too. The question thus far arises whether the energy tensor $E_{\mu\nu}$ at the end could be in different aspects less fundamental than the metric field $g_{\mu\nu}$ itself. Briefly, is and why is matter more fundamental[39] than space-time? In contrast to such a position, is the assumption justified that *without* the space-time structure encoded in the metric *no* energy tensor? To bring it to the point, can space-time (and its geometric structure) exist without matter and if yes, what kind of existence could this be? Einstein's starting point was to derive space-time structure from the properties of material systems. However, the theorem before does not exclude the possibility that the energy tensor depends on the metric field too or is completely determined by the metric field. Consequently, the matter fields themselves are derivable from the structure of space-time or the very definition of an energy tensor is determined by space-time structures too. Thus far, the question is not answered definitely, which came first, *either* space-time structure *or* energy tensor. So it is reasonable to ask, is the energy-momentum tensor of matter only dependent on the structure of space-time or even determined by the structure of space-time or both or none? In other words, granddaddies *either* chicken *or* the egg dilemma is revitalized and asking for an innovative and a comprehensive solution and may end up in an Anti-Machian[40] theory. However, this leads us at this point too far afield.

Theorem 3.3 (EINSTEIN'S FIELD EQUATION'S COMPLETELY GEOMETRIZED).

Now, we can derive a completely geometrical form of Einstein's field equations.

CLAIM.

In general, the completely geometrical form of Einstein's field equations is given by

$$\left(S \times g_{\mu v}\right) - \left(\left(\frac{R}{2}\right) \times \left(g_{\mu v}\right)\right) + \left(\Lambda \times g_{\mu v}\right) \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu v} \tag{84}$$

PROOF BY MODUS PONENS.

If the premise of modus ponens

$$(+1) \equiv (+1) \tag{85}$$

is true, **then** the following conclusion

$$\left(S \times g_{\mu v}\right) - \left(\left(\frac{R}{2}\right) \times \left(g_{\mu v}\right)\right) + \left(\Lambda \times g_{\mu v}\right) \equiv \left(\frac{2 \times \pi \times \gamma \times T}{c^4}\right) \times g_{\mu v} \tag{86}$$

is also true, the absence of any technical errors presupposed. The premise

$$(+1) \equiv (+1) \tag{87}$$

is true. Multiplying this premise by the stress-energy momentum tensor of matter, we obtain

$$(+1)\times\left(\left(\frac{8\times\pi\times\gamma}{c^4}\right)\times T_{\mu\upsilon}\right) \equiv (+1)\times\left(\left(\frac{8\times\pi\times\gamma}{c^4}\right)\times T_{\mu\upsilon}\right) \quad (88)$$

or

$$\left(\left(\frac{8\times\pi\times\gamma}{c^4}\right)\times T_{\mu\upsilon}\right) \equiv \left(\left(\frac{8\times\pi\times\gamma}{c^4}\right)\times T_{\mu\upsilon}\right) \quad (89)$$

This equation simplifies according to the theorem before as

$$\left(\left(\frac{8\times\pi\times\gamma}{c^4}\right)\times T_{\mu\upsilon}\right) \equiv \left(\frac{2\times\pi\times\gamma\times T}{c^4}\right)\times g_{\mu\upsilon} \quad (90)$$

Einstein's field equations are given as

$$(R_{\mu\upsilon}) - \left(\left(\frac{R}{2}\right)\times(g_{\mu\upsilon})\right) + (\Lambda\times g_{\mu\upsilon}) \equiv \left(\frac{2\times\pi\times\gamma\times T}{c^4}\right)\times g_{\mu\upsilon} \quad (91)$$

Since $R_{\mu\nu} = S\times g_{\mu\nu}$, we obtain Einstein's field equations completely geometrized as

$$(S\times g_{\mu\upsilon}) - \left(\left(\frac{R}{2}\right)\times(g_{\mu\upsilon})\right) + (\Lambda\times g_{\mu\upsilon}) \equiv \left(\frac{2\times\pi\times\gamma\times T}{c^4}\right)\times g_{\mu\upsilon} \quad (92)$$

In other words, our conclusion is true.

QUOD ERAT DEMONSTRANDUM.

5. Discussion

Einstein was one of the first to use explicitly the term "*unified field theory*" in the title[41] of his publication in 1925. In the following, Einstein himself published more than thirty technical papers on this topic. However, Einstein's unified field theory program, besides of his justified insistence on the possibility and desirability of a unified field theory, independent of any geometry preferred, required a substantial amount of new mathematical preliminaries and methods[9,15] too and was on the level of the mathematical possibilities at Einstein's time technically in vain. At the heart of any enterprise to geometrize all fundamental interactions of nature and to provide *a completely geometrized[8] theory of relativity* is the assumption too that geometrization could eventually lead to a unification of all known physical interactions which would simplify at least the work of view scientists. This publication has been able to geometrize Einstein’s field equation is completely. This is of importance and of use not only in astronomy and cosmology but in the *(computer-aided) design and development of (medical) drugs* too.

5. Conclusion

Einstein’s field equations are is geometrized completely.

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