An unintentional repetition of the Ramanujan formula for π, and some independent mathematical mnemonic tools for calculating with exponentiation and with dates

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Abstract

The paper is consisted of contentually unrelated sections, where each section could be one short paper. Section 1 deals with the process of an unintentional repetition of one of the Ramanujan formulas, and the author did not know it before. The process of the unintentional repetition of the Ramanujan formula is interesting for estimating, for instance, the physical background for guessing of dimensionless physical constants. Section 2 shows an approximation which helps at memorizing the square roots of integers up to 10. Section 3 shows the specialities of the squares of integers at the last digits. Section 4 shows how the last two digits are repeated for 2 to the sequential integer powers, and how to find out this. Section 5 contains some mathematical peculiarities at calculating dates. All sections contain mnemonic methods to help us memorize and calculate. These sections belong to pedagogical and recreational mathematics, maybe even something more is here.

I. REPETITION OF THE RAMANUJAN APPROXIMATION FOR π

The author wanted to calculate the volume of a sphere with radius π. (It was a specific purpose, which he may will describe in the next paper. The purpose was not to obtain an approximation for π.) Its volume (V) equals

$$V = \frac{4\pi^4}{3} = 129.878788045 \tag{1}$$

Behind the decimal point the author recognized a repeating pattern, which is a sign for a fairly simple fraction. He inserted this to the right-hand part of the Eq. (1), and obtained the following very small deviation after the fraction

$$V = \frac{4\pi^4}{3} = 130 - \frac{4}{33} + 1.66549 \times 10^{-7} \tag{2}$$

$$\frac{4}{33} = 0.1212121212.. \tag{3}$$

If we use this formula to calculate π in the opposite direction, we obtain

$$\pi_{r=\pi} = \left(97.5 - \frac{1}{11}\right)^{1/4} = 14159265\mathbf{2582646} \tag{4}$$

Where $\pi_{r=\pi}$ is this approximation for π. But, the true value for π is

$$\pi = \pi_{r=\pi} + 1.007147254483698 \times 10^{-9} = 3.14159265\mathbf{3589793} \tag{5}$$

If we look for a short enough and memorable procedure for π that is also accurate enough, it probably wins. Of course, this is not a formula for all decimal places.

Then the author recalculated this $\pi_{r=\pi}$ with Google. He did not expect it, but beside the calculation, Google also gave him a link to this calculated $\pi_{r=\pi}$ and to a formula that is a derivative of his formula:

$$\left(9^2 + \frac{19^2}{22}\right)^{1/4} = 3.14159265\mathbf{2582646} \tag{6}$$

Then he put only the result of his calculation of $\pi_{r=\pi} = 3.14159265258$ in Google and found that this formula had already been derived by Srinivasa Ramanujan, a well-known former Indian mathematician.[1] His original formula is:

$$\sqrt[4]{3^4 + 2^4 + \frac{1}{2 + (2/3)^2}} = 3.14159265\mathbf{2582646} \tag{7}$$

But it is also fascinating that Google was able to find this.

We wonder what is the probability that the author came across such a formula for π when exploring various relations. Due to this has happened again after Ramanujan, there is probably not a large number of short and precise approximate formulas for π.

The advantage of derivation of Eqs. (1) to (4) according to Eqs. (7) is that it can be better remembered because it is enough to remember only four points:
1. We calculate the **volume** of a sphere with the radius π.
2. 3. **Decimals** that deviate from the nearest **integer** should be changed to a **fraction**.
4. And we calculate π in the **opposite direction**.

This can also help at some statistical estimation of what the probability is that the searchers for a numerological formula for the fine structure constant[2] can find a correct formula, this means a formula with the physical background. The author, too, was such a searcher.[3,4] Otherwise, In Ref.[3] he has presented only formulas that use the fine structure constant, not formulas that calculate it directly. But he also obtained one direct formula for the fine structure constant.[4]

After calculating the result in Eq. (4) we can also look at the digits of $\pi^4 = 94.409091034$. True, even here the attentive observer may notice that we have repetitive decimals, which therefore means that we can convert them into some simple fraction. (It is, however, somewhat less obvious than at $4\pi^4/3 = 129.878788045$.) This obvious pattern otherwise diminishes possibility of an extra accident where the author found this formula, maybe a lot of people found it after Srinivas Ramanujan.

At such a calculation of π, the calculation time is also important. It took maybe two weeks since the time the author decided to calculate the volume with the radius π to the time when he found out Ramanujan's formula. During this time, the author filled less than 564 cells in an Excel worksheet. Thus, the calculation itself took a few hours. As a comparison, he spent less than an hour for the raw formula of the electron (Ref.[3] (Eq. 1)). Before that, he used about a month to find a formula for the fine structure constant[4] that is independent of the gravitational coupling constants of the elementary particles.

II. PRESENTATION OF SOME APPROXIMATIONS AND MNEMONIC PROCEDURES FOR SQUARE ROOTS OF INTEGERS UP TO 10

Between $\sqrt{1}$ and $\sqrt{4}$ there are two non-integer square roots of integers, between $\sqrt{4}$ and $\sqrt{9}$ there are four non-integer square roots of integers, between $\sqrt{9}$ and $\sqrt{16}$ there are six non-integer roots of integers and so on.

In school, we learned by heart the root of the numbers $\sqrt{2} = 1.41$ and $\sqrt{3} = 1.73$, and now the author is going to show the readers how to remember the next few non-integer roots of integers more easily.

If the non-integer roots between $\sqrt{9}$ and $\sqrt{16}$ were located linearly, then it will be valid

$$\sqrt{4} = 2 \tag{8}$$
$$\sqrt{5} = 2.2 \tag{9}$$
$$\sqrt{6} = 2.4 \tag{10}$$
$$\sqrt{7} = 2.6 \tag{11}$$
$$\sqrt{8} = 2.8 \tag{12}$$
$$\sqrt{9} = 3 \tag{13}$$

But in fact, it is true as follows

$$\sqrt{4} = 2 \tag{14}$$
$$\sqrt{5} = 2.23607 = 2.2 + 0.03607 \tag{15}$$
$$\sqrt{6} = 2.44949 = 2.4 + 0.04949 \tag{16}$$
$$\sqrt{7} = 2.64575 = 2.6 + 0.04575 \tag{17}$$
$$\sqrt{8} = 2.82834 = 2.8 + 0.02843 \tag{18}$$
$$\sqrt{9} = 3 \tag{19}$$

Therefore, the deviation from the linearity is quite small. Due to the divisibility by 5, the linearity is quickly evident and verifiable here. In addition, the next decimals or roundings are also fairly constant in these four calculations.

This above fact is related to the fact that $11^2 = 121$ is close to 125, $12^2 = 144$ is close to 150, $13^2 = 169$ is close to 175, and $14^2 = 196$ is close to 200.

The calculations of the $\sqrt{2}$ and $\sqrt{3}$ could also be helped by this principle:

$$\sqrt{1} = 1 \tag{20}$$

$$\sqrt{2} = 1.41421 = 1 + 1/3 + 0.08088 \tag{21}$$

$$\sqrt{3} = 1.73205 = 1 + 2/3 + 0.06538 \tag{22}$$

$$\sqrt{4} = 2 \tag{23}$$

Now the deviation from the linearity is larger, still ever it is not very large, and both deviations are similar. Thus, this principle is worth remembering.

It is also worth using such an analysis for $\sqrt{10}$:

$$\sqrt{10} = 3.16228 = 3 + (1/7 = 0.14286) + 0.01942 \tag{24}$$

We can see here that the linear approximation is coincidentally equal to 22/7, which is also an approximation for π. Otherwise, it is said many times that the value of $\sqrt{10}$ can also be an approximation for π.

Then it is also important that there is no big difference between 1/7 and 1/5, so consequently the value of 3.2 can be an approximation for $\sqrt{10}$, which can be seen in the following formula:

$$\sqrt{10} = 3 + 0.2 - 0.03772 \tag{25}$$

III. THE LAST DIGITS AT THE SQUARES OF INTEGERS

Let us look at the formula $x^2 = y$, where x are integers, we will call the last digits in these numbers as x_0 and y_0. Then we present the relations between the last digits in Table I.

TABLE I. The relations between the last digits, x_0 and y_0, in Eq. $x^2 = y$ are shown. The negative digits are calculated from the relations $x_0 - 10 \to x_0$ and $y_0 - 10 \to y_0$.

$\boldsymbol{x_0}$	$\boldsymbol{y_0}$	$\boldsymbol{y_0 - x_0}$	$\boldsymbol{x_0}$	$\boldsymbol{y_0}$	$\boldsymbol{y_0 - x_0}$
1	1	0	1	1	0
2	4	2	2	4	2
3	9	-4	3	-1	-4
4	6	2	4	-4	2
5	5	0	5	5	0
6	6	0	-4	-4	0
7	9	2	-3	-1	2
8	4	-4	-2	4	-4
9	1	2	-1	1	2
0	0	0	0	0	0

In the third column, we can see how $y_0 - x_0$ behaves the same in the $x_0 = 1...5$ block as in the $x_0 = 6...0$ block. The symmetry is also inside each block, i.e. for $x_0 = 1$ and 5, for $x_0 = 2$ and 4, and so on.

In the fourth column we can see one simplification that we can choose some negative numbers for x_0, i.e. $x_0 \to x_0 - 10$, and x_0 gives the same result for y_0 as $-x_0$ gives it. In the fifth column, y_0's are presented also as negative. This does not affect the calculation of $y_0 - x_0$ in the sixth column, which is therefore equal to the third column. This is also a tool for remembering, one example is remembering of a multiplication table. Due to these minus signs, it is enough to be able to know calculation of the multiplication table from $x_0 = 1$ to 5, from some aspects.

The control of the last numbers can thus be used as a control at the calculation of the squares, and for many other calculations. Such analysis can be continued for the higher integer exponents, which also give similar symmetries.

IV. THE REPETITION OF THE LAST DIGITS AT THE 2 TO THE SEQUENTIAL INTEGER POWERS

For the number 2 to the sequential integer powers, $2^n = y$, we can see that the sequence of the last digit $y_0 = 2, 4, 8, 6$ is repeated.

Let us look at how the sequence of the last two digits, i.e. y_1 and y_0, is repeated. Table II presents 2 to sequential integer powers, and the last two digits are in bold.

TABLE II. Presentation how the last two digits (in bold) change in 2 to sequential integer powers.

2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}	2^{11}	2^{12}	2^{13}
02	**04**	**08**	**16**	**32**	**64**	1**28**	2**56**	5**12**	10**24**	20**48**	40**96**	81**92**

We wonder when this sequence repeats itself again. Let us look at $2^{12} = 40\mathbf{96}$. To this we can say that the last digits of -04 have appeared. Further exponentiation will result in -96, which is equivalent to 04. This happens at $2^{22} = 41943\mathbf{04}$. From there, the sequence is repeated again, so at $2^{32} = 42949672\mathbf{96}$, for example, -04 is repeated. But 02 never happens again.

Thus, at such a logical reasoning, it is not necessary to calculate all the calculations to see this rule.

The number 2 to the integer powers are often used, so if we know some rules, we can easier remember these numbers, or we can easier control these calculations.

V. SOME SIMPLIFICATIONS IN THE CALCULATION OF DATES

A. Use of the dates of the first Sundays of the months

Some calculations with dates by heart are useful because if it is easier to remember which day of the week was on a particular date, we will remember events better. To calculate which day of the week is a particular day in a particular month, it is important in which day the first Sunday of that month is. Let us call the number of this day of the month as S. If two months have identical S's, all days of the month and weeks will match between those two months.

Among the first things we need to master to calculate dates is the sequence S throughout the year. For example, in 2018, this was the sequence of all S's, as evident in Table III:

TABLE III. A sequence of all S's in year 2018. It was a non-leap year.

Jan	Feb	Mar	**Apr**	May	Jun	**Jul**	Aug	Sep	**Oct**	Nov	Dec
7	4	4	**1**	6	3	**1**	5	2	**7**	4	2

That year, 2018, was a non-leap year. In the year 2024, which will be a leap year, this sequence of all S's is as evident in Table IV.

TABLE IV. A sequence of all S's as will be in year 2024. It will be a leap year.

Jan	Feb	Mar	**Apr**	May	Jun	**Jul**	Aug	Sep	**Oct**	Nov	Dec
7	4	3	**7**	5	2	**7**	4	1	**6**	3	1

The differences between these two sequences are the same for all non-leap or leap years. At this, we need to respect that if we calculate the absolute value of the difference greater than 3, we must add or subtract 7 so as to obtain a difference for which the absolute value is less than 4.

The first mnemonic tool here is how to remember these two sequences in Tables III and IV. The rule is that the differences of S's within consecutive months with step 3 are on average the smallest. For example, for year 2018, let us look at these three sub-series in Table V:

TABLE V. The sub-series of all S's in year 2018, as shown in steps by 3.

Jan	**Apr**	**Jul**	**Oct**
7	**1**	**1**	**7**
Feb	May	Aug	Nov
4	6	5	4
Mar	Jun	Sep	Dec
4	3	2	2

The differences within these three sub-series are small and this is easier to remember. (Of course, the previous rule for differences must be respected, i.e. if the absolute value of the difference is greater than 3, then we add or subtract 7 from this difference so that the absolute value of this difference becomes less than 4.)

Each doubled S is inside one sub-series. Only one tripled $S$is inside two sub-series.

Also in these three series, it is essential to remember the differences between successive S 's, not so much the values themselves, because these differences are the same for each non-leap year. (If we remember shape of such a sub-series curve, we remember these differences.)

We can make similar series for the leap year 2024.

B. Match of the last digits of the dates and of days of the week

When remembering days of the week in the dates with a certain S, the specifics of these combinations are also useful. The first peculiarity is when in some Monday of the month the last digit of the day in the month is equal to 1, for instance, Monday 21th. This is in this case of a month with $S = 6$. In general, however, this happens in months with $S = 7$, 3, 6, and conditionally in $S = 2$ if it has 31 days.

(In Tables III, IV and V we can write 0 instead of 7 and it becomes more transparent and understandable, because in this case the last digit of the date is divisible by 3, except that we have to add 7 to 2 to be divisible by 3. In addition, we obtain smaller numbers on Tables III, IV, and V, so it is more transparent. Thus we change the rule a bit instead of calculate with only the first Sundays, but this rule is not necessary and is not the only possible for these calculations.)

Another mnemonic specialty is when we also have months where in a given week the last digit of the day number of the month is larger for 5 than the day number of the week, for example in month with $S = 4$ Monday is the 26th of the month. This happens in the months with $S = 2$, 5, 1 and 4.

Thus, the next mnemonic tool and mathematical peculiarity in memorizing dates is that these first and second specialties cover all S's, and they overlap only at $S = 2$ and only if the month has 31 days.

A further special feature and mnemonic tool is Friday the 13th, when for the month it is valid $S = 1$. Then the next special feature is the month with $S = 0$, where the first day of the month starts at Monday.

Then there are other peculiarities and mnemonic aids at calculating dates, which I will not mention here.

This calculation of dates is not as mathematically elegant or outstanding as the above methods, but it is practically more useful, this benefit is the aid at remembering various events.

VI. CONCLUSION

Unrelated sections shows some ideas on calculations with exponents and mnemonic procedures. The first section also describes the unintentional discovery of the formula for π, which was already found by Ramanujan,[1] and was hitherto unknown to the author. Then the author processed some mnemonic tools at various calculations of exponentiations. In the last section, however, some mnemonic help in calculating dates is presented.

In all of them, we can help us at memorizing; and here is an example of repeating the calculation from the Ramanujan's formula, the procedure is important because it is a new step towards assessing how such guessing can help find a numerological formula for fine structure constant that also has a physical background.

It is known that chess players can improve their memory for positions and moves if they practise chess a lot. Similarly someone who deeply understand mathematical principles of calculations remembers them better, and better calculates.

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