Objective Mathematics (OM),

Mathematics Built On A Circle And A Sphere

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Abstract

The author of this article asked himself: why is mathematics built on a mythical infinite straight line and plane, and what happens if it is built on a circle and a sphere - the most perfect objects of nature?

The author called the resulting theory Objective Mathematics (OM), bearing in mind that this theory operates on natural objects that exist in nature (a circle and a sphere) and does not use the axiomatic approach, in particular the infinite parallel straight lines and planes present in traditional mathematics (TM).

The constructs and proofs in this article are first made on a circle (one-dimensional OM), and then the resulting law is generalized to a sphere (two-dimensional OM) and 3-sphere (three-dimensional OM).

The results obtained 4 empirical laws and 21 laws.

The paper gives definitions of such concepts as:

• A harmonic four (quartet) on a circle,

• A logical three-dimensional count,

• Preliminary mathematics (Premathematics) in OM,

• Non-Euclidean geometry in OM,

• One-dimensional, two-dimensional, three-dimensional OM, etc.

As a result of the analysis, the author concludes that the universe is a three-dimensional sphere, where a ray of light is a large circle of this sphere.

What is a harmonic four (quartet) on a circle?

Take a circle ***I*** made from wire or plastic and mark 3 arbitrary points on it, which we will designate with letters ***A***, ***A'*** and ***X***. Place this circle on any sphere ***S*** of any size and use a spherical compass to draw a circle ***m*** that passes through points ***A*** and ***A'*** and belongs to ***S***. Then draw a single circle ***n*** on ***S*** that passes through the point ***X***, which is orthogonal (vertical) to both circles ***I*** and ***m***. The last circle ***n*** intersects ***I*** at another point ***X***', which is called the fourth harmonic point of the three given points ***A***, ***A'*** and ***X***. We can also say that the pair ***X X'*** harmoniously separates the pair ***A A'***.

It is very important to note here that OM is an experimental science. We take only two objects from nature, one of which is the most perfect object in nature, called the sphere, and the other - a spherical compass with which we draw circles on the sphere. Only a very small number of empirical laws are needed to create OM. Here, the angle between two circles is determined not by a number, but by the opening of a compass, which even a preschooler can understand (see below). It's interesting that the position of the fourth harmonic point does not depend on the size of the sphere ***S*** and the arbitrariness randomness of the circle ***m***. That is, the fourth harmonic of the given 3 points on the circle ***I*** is uniquely determined regardless of the arbitrariness of the sphere ***S*** passing through ***I*** and the circle ***m***. This ratio of points ***A***, ***A'***, ***X*** and ***X'*** is called the "Harmonic Quartet". Far-reaching conclusions can be drawn from this law.

The Law of the Harmonic Quartet is the cornerstone of continuous harmony in nature. This is the basis on which the superstructure of the exact laws of nature rises.

Now let us see how a one-dimensional OM is born from a harmonic quartet. If we leave points ***A*** and ***A'*** fixed on ***I*** and move only the point ***X***, then the point ***X'*** will also move, and we will get the symmetry (equivalent correspondence) ***X*** → ***X'***, which we will call harmony with respect to points ***A*** and ***A'***, and denote it by the letter ***a***. If we have another ***b*** -harmony with respect to the pair of points ***B*** and ***B'*** on the same circle ***I***, then we denote by ***a*** * ***b*** the symmetry obtained from the successive application of the harmonies ***a*** and ***b***.

It is immediately clear that ***a * a = 1***, where ***1*** is not a number, but is a symbol of symmetry, leaving all points on the circle stationary. It is easy to see that the symmetries ***a*** * ***b*** and ***b*** * ***a*** do not coincide. The following rule is very easy to prove: ***a*** * (***b*** * ***c***) = (***a*** * ***b***) * ***c*** for any 3 harmonies. The following law, which is very important in OM, is also easy to prove: ***a*** * ***b*** = ***b*** * ***a*** if and only if the pairs ***A A'*** and ***B B'*** form a harmonic quartet. Here begins the so-called logical count OM, which does not need a number. On the contrary, as we will see later, without this count there is no arithmetic. Only this reasonable count can answer the question "What is a number?".

The discovery of a logical three-dimensional Leibniz count, which dramatically enhances human thinking

The logical count found above can be easily generalized for two-dimensional, three-dimensional, and multidimensional OM, which I will discuss later. Leibniz dreamed of opening such a logical count that would simplify all mathematics in the same way that a system of 10 digits simplified all arithmetic. It is worth recalling that modern computer technology owes its success to the 2-digit system of calculus, which is the analogue of the

10-digit system. Leibniz believed that this logical count would enhance human thinking as much as no optical instrument could enhance human vision. He was not mistaken in this matter. Leibniz was dissatisfied with the coordinate geometry of R. Descartes and found in his mathematical analysis the myth of infinite small values and hoped that this logical count would put an end to difficulties in this area. He was not mistaken in this matter either.

Here I want to say that such a count can be used in different areas, for example, you can write a mathematical analysis using not a number, but a harmonic quartet. The slogan “Number is everything” can be called into question.

The theory of symmetry discovered by Evariste Galois effectively operates in OM

It is well known that modern science cannot exist without the theory of symmetry (or, as they say, the principle of symmetry, a group of symmetry, a group of transformations). It is also known that the founder of the theory of symmetry was the young Evariste Galois, who was persistently ignored by mathematicians in 1832. he died in a stupid duel at the age of 21. Before the battle, he wrote his theory in just 10 pages. After 120 years, German scientist German Weil writes that these 10 pages are the most ingenious document ever written by man. In the logical count of OM, the Galois theory is very effective. One of the results is that all the axioms in classical mathematics presented today can be proved in the so-called pre-mathematical section of OM and they become wonderful laws of nature.

Until now, TM has used the axiomatic approach as a basis. The biggest mistake Plato, Aristotle and Euclid made was that they did not understand the essence of Socrates’ method. Confronting Socrates, Plato said that "God is a geometer." The author of this article does not quite agree with this slogan, since he believes that geometry is built on a shaky basis of axioms. To justify the above, we need to go to a two-dimensional OM, which is the theory of all points and circles on a common sphere.

What is two-dimensional OM?

In OM, there are only empirical laws that are absolutely true, and no attempt can refute these laws. The first of these laws is the following:

I empirical law: Each circle belonging to a sphere has 2 centers, which are called opposite points of the sphere.

It follows that the same circle can be drawn around two centers, usually with different openings of the compass. A circle is called large if these opening are the same. The constructions are facilitated if you mark this opening once and for all on a compass, which is called the middle opening. For example, any parallel on the globe has two centers that are poles. If the openings are the same, the parallel is an equator, which is a large circle. If we have two points on two different sides of the equator that are on the same meridian and have the same latitude, then the centers of the circles passing through these points are on the equator. In general, the following law is true.

II empirical law: Each pair of points on the sphere uniquely defines a large circle on which the centers of all circles passing through these points are located. The large circle for these points is called the middle line.

Since the circle has 2 opposite centers, a new law follows from here, which is a logical consequence of the first two empirical laws:

Law 1: If a point belongs to a large circle, then its opposite point also belongs to the same circle.

Another law is very easy to prove:

Law 2: Two large circles on a sphere always intersect at opposite points.

It directly follows from here:

Law 3: Only one large circle passes through two non-opposite points of the sphere. Two opposite points intersect with an infinite number of circles, all of which are large (for example, the meridians of the globe).

With a compass, it is easy to build a large circle through 2 points. In fact, giving the compass a middle opening, let us draw 2 large circles around these points, which will intersect at 2 opposite points. Any of these new points will be the center of the large circle sought.

Law 4: Only one circle passes through each of three different points on the sphere.

Indeed, if we construct the middle line of the first and second points, do the same for the second and third points, then these two middle lines intersect only at opposite points, which will be a pair of centers of the desired circle. It is easy to see that the middle line of the third pair of points passes through the same opposite points.

Two circles on a sphere are called in contact with each other if they have only one common point, called the point of contact.

Law 5: If a circle ***a*** and a point ***M*** belonging to it are given, then through any point ***N*** of the sphere other than ***M*** there passes only one circle that is in contact with the circle ***a*** at the point ***M***.

Indeed, if ***O*** is one of the centers of the circle ***a***, then the centers should be on the middle line of the pair ***M N*** and the large circle passing through pair ***O M***. Therefore, the intersection points of these lines are the centers of the desired circle. We will soon see that Euclid's axiom of parallels is based on a peculiar representation of this law of nature.

Law 6: If a circle ***a*** is given and the point ***M*** belongs to it, then there is one large circle that touches ***a*** at the point ***M***.

Indeed, if we place the sharp end of the compass with the middle opening on point ***M*** and draw a large circle, then on this circle there should be a pair of centers of the desired circle. On the other hand, it should also be on the large circle passing through M O pair. Therefore, the intersection points of these two large circles will be the centers of the desired circle.

Now it is easy to understand how to measure the angle between two circles on a sphere by opening a compass. Remember that we do not have the right to use arithmetic and all mathematics.

First, let us take two meridians on the globe, which are large circles. These meridians cross the equator at two points ***A*** and ***B***. Place one end of the compass at point ***A*** and the other at point ***B***. The resulting opening of the compass is called the angle between these meridians. Now let us take 2 random circles ***a*** and ***b*** that intersect. At the intersection points, we construct the only large circles that touch these circles (***a*** and ***b***) (Law 6). The size of the angle between circles ***a*** and ***b***, will be considered the size of the angle between the last large circles. If two circles are orthogonal, the corresponding opening of the compass is the average opening.

Law 7: An arbitrary circle ***a*** and two arbitrary points ***X*** and Y are given on the sphere, of which Y belongs to ***a***. Only one circle passes through these two points, which is orthogonal to ***a***.

Indeed, the pair of centers of the desired circle must correspond to the intersection of two large circles, one of which is the middle line of the points ***X*** and Y, and the other - the large circle tangent to the point Y.

From this rule it follows that from a given point ***X*** to a given circle ***a***, an infinite number of orthogonal circles can be drawn. To do this, you need to arbitrarily change the position of the point Y.

For example, from one pole of the globe to any parallel, an infinite number of orthogonal meridians can be drawn. In the case of poles, all these circles intersect at the other pole. But it turns out that this law remains true also for any point. However, we cannot prove this law without the law of a harmonious quartet. For a two-dimensional OM, it is advisable to adopt a new empirical law, which is equivalent to the law of a Harmonic Quartet and has the following wording:

III Empirical law: If all circles orthogonal to circle ***a*** are drawn from a given point on sphere ***X***, then the latter also intersect at point ***X'***, which is called the harmonic point ***X*** relative to circle ***a***.

Building point ***X'*** is very easy. It is necessary to draw a circle ***b*** passing through the point ***X*** and orthogonal to ***a***, which intersects ***a*** at two points. Then on ***b*** you need to find that unique point ***X'***, which together with point ***X*** will form a Harmonic Quartet with these two points. It is easier to do the following: from point ***X*** to ***a*** we draw two orthogonal circles, which again intersect at point ***X'***.

What is the harmony and logical count arising from it?

Now move the point ***X***, respectively, move the point ***X’***, and so we get a one-to-one correspondence over the entire sphere, which we call harmony with respect to the circle ***a*** and denote this harmony by the same letter ***a***.

Mathematicians know what inversion is relative to a circle on a plane. However, this correspondence is not one-to-one, which is one of the Achilles heels of our mathematics. This circumstance forces me to abandon the term called inversion, and replace it with the term harmony, known in projective geometry, and which is the cornerstone of the harmony of nature.

Law 8: If we use harmony ***a*** twice, all points on the sphere will return to their places. That is, ***a********a*** = 1

This statement obviously follows from Law III.

We, everywhere speaking symmetry, will understand one-to-one correspondence. If we have any α-symmetry acting on the sphere, then we denote by the symbol α^{-1} the symmetry, the use of which, after applying α, returns all points on the sphere back to their positions. That is, $\alpha*\alpha^{-1} = 1$. From ***Law*** 8 it follows that $\boldsymbol{a} = \boldsymbol{a}^{-1}$. That is, harmony is equal to its inverse.

If the harmonies ***a*** and ***b*** act on the sphere, it is easy to make sure that the symmetries ***a*** * ***b*** and ***b*** * ***a*** do not coincide in the general case.

Law 9: a * b = b * a only if the circles ***a*** and ***b*** are orthogonal to each other.

This law follows from the fact that harmony ***a*** leaves all orthogonal circles stationary.

IV Empirical Law: A closed curve belonging to a sphere is a circle if and only if an arbitrary circle passing through its arbitrary two points forms equal angles with it.

This is the last empirical law of two-dimensional OM. We have only 4 empirical laws. Let us see what theory can be developed on their basis. First, what follows directly from Law IV:

Law 10: If two arbitrary circles on a sphere intersect, then at both intersection points they form equal angles.

Again, it is easy to get two laws arising from Law IV, which are very important for the entirety of OM.

Law 11: Harmony does not change the size of the angle. We also say that the angle is invariant with respect to harmony.

Law 12: Harmony leads a circle to a circle.

As we have already seen, each circle on a sphere has its own harmony. We can apply these harmonies in sequence or (as they say in mathematics) multiply them. Each product consists of a finite number of harmonies, which again is a symmetry acting on the same sphere. We denote the set of such symmetries by the letter ***G*** and call it the group of symmetries. In this group, ***1*** is called a unit, which is a symbol of equal symmetry. If $\boldsymbol{\alpha} = \boldsymbol{a} * \boldsymbol{b}$, then $\boldsymbol{\alpha}^{-1} = \boldsymbol{b} * \boldsymbol{a}$. The following law is true: $(\boldsymbol{a} * \boldsymbol{b} *** \boldsymbol{c})^{-1} = \boldsymbol{c} * \boldsymbol{b} *** \boldsymbol{a}$. α-symmetry is called involutional if $\alpha \neq \boldsymbol{1}$ and $\alpha * \alpha = \boldsymbol{1}$ (also written as $\alpha^{-1} = \boldsymbol{1}$).

If $\boldsymbol{\alpha} = \boldsymbol{a} * \boldsymbol{b} *** \boldsymbol{c}$, then it is involutional only if $\boldsymbol{\alpha} = \boldsymbol{c} *** \boldsymbol{b} * \boldsymbol{a}$. The product of an odd number of harmonies can never be ***1***. Thus begins the logical count, which is associated with the group ***G***.

The German geometer Friedrich Bachmann in 1959 used this logical count. He built all geometry based on symmetry, which still inspires all geometers. Nevertheless, a similar count, discovered by Galois, is much stronger in OM than in traditional mathematics (TM).

I said above that this logical count significantly enhances a person's thinking. I also said that TM is a consequence of coding a small part of OM - premathematics. It is time to give a rigorous proof of these allegations.

What is premathematics in OM?

Now let us take an arbitrary point ***P*** on the sphere and a group of those symmetries that leaves the point ***P*** fixed. Denote this group by $\boldsymbol{G_P}$. It is easy to see that G_P is a subgroup of ***G***. Mathematicians understand that each symmetry group has its own theory of invariants. The theory of invariants of the group $\boldsymbol{G_P}$ I will call premathematics. However, $\boldsymbol{G_P}$ is a subgroup of group ***G***, which allows me to say that TM is part of OM.

It is believed that TM works well in describing the laws of nature, however, as follows from the foregoing, TM is part of OM and, accordingly, it can work well for a limited area of nature. Therefore, OM is more suitable for describing nature on a global scale.

For example, crystallography is a local theory because the crystal is in a local region of the universe. That is why traditional geometry works here. Nevertheless, in this science the theory called pregeometry (see below) will work much better, which is part of premathematics. Despite this, premathematics has limited application in describing the global theory of nature.

The connection between TM and premathematics is visible from the well-known stereographic projection. If we place a sphere on a plane and project a plane from the top point ***P*** onto a sphere, then the plane is projected onto the sphere without point ***P***, and the line on the plane is projected as a circle passing through point P. Until now, this projection has not led to the appearance of premathematics within for the simple reason that mathematicians did not know the Harmonic Quartet on the circle.

In traditional geometry, the so-called "plane" concept appeared from a sphere taken from nature and a point P fixed on it, and the "straight line" concept appeared from a circle passing through a point P. If we use the

OM approach in geometry, then all abstract axioms and theorems become laws of nature. Abstract (fair to say, "mythical") geometry disappears.

There is no need to prove all geometry theorems for these new “lines” and the new “plane”. It is enough to prove the axioms. The Achilles heel of TM is that abstract axioms have not yet been proved, and this can be done with the help of OM after you have called a spade a spade. In particular, the plane and the straight line need a clear definition, which has not yet been done. Here we define a plane using a sphere, and the sphere does not need such a definition, because it is a simple and perfect object of nature.

We continue our incomplete proof. The first axiom of geometry states that only one line passes through two different points of the plane. It is important to note here that ***P*** is the end point of the sphere, but for a “plane” and a “straight line”, the same ***P*** is an infinitely distant point. This is the secret of infinity. After the discovery of this secret, the first axiom of geometry becomes the result of Law 4. Point ***P*** is the final (unchanged) point of all the “lines”. If two points on a sphere are given, then the circle passing through these points and the third point ***P*** is unique. This is the most convincing proof of the first axiom of geometry. If the circle belonging to the sphere does not pass through the point ***P***, then in geometry it simply remains a circle without quotes. By a “straight line” on a sphere, we must understand the arc of a circle passing through point ***P***. The terms “ray”, “angle” and “triangle” should be understood in the same way. Arcs of two circles passing through a point ***P*** on a sphere are called equal (or congruent) if there exists a symmetry in the group $\boldsymbol{G_P}$, which is the product of harmonies with respect to circles passing through the point ***P*** and which leads one of these arcs to the other. The equation of angles and triangles is defined similarly. If we assume that our sphere is a sphere of the earth, and the farmer takes measurements in a certain region around the point ***Q*** opposite to ***P***, then the arcs of all the circles passing through the point ***P*** and crossing this region will appear to the farmer as straight lines due to the huge size of the circles . All corners and triangles will look the same. It was here that the term geometry appeared about 7000 years ago. This is a theory that measures the surface of the earth that began with agriculture.

I will not give evidence of all axioms of geometry here, because they are obvious to any mathematician. But I want to prove the last axiom of geometry, which is the axiom of parallels, and which has been bothering all geometers for about 2000 years. Two “straight lines” on a sphere are called “parallel” to each other if they touch each other at point P. The axiom of parallels in geometry says that only one parallel line can be drawn through a given point relative to a given line. This axiom cannot be verified by experience, which is one of the Achilles heels in our mathematics. I already mentioned above that Law 5 is a proof of the axiom of parallels. Indeed, if in our traditional geometry we pass from the plane and the line to the sphere and circle passing through the point P, then the axiom of parallels becomes the law of nature formulated in Law 5. Only in this law the point ***M*** has to be replaced by the point ***P***.

The mathematician will say that in mathematics there is spherical geometry and its trigonometry, which are built on the sphere. In addition, there is a Riemannian geometry of space, the geodesic lines which were used by A. Einstein in his theory of relativity. Therefore, it cannot be said that mathematics is built only on a straight line and a plane. However, in addition to geodesic lines (large circles), there are also a large number of small circles on the sphere, the theory of which has not been studied. It is impossible to study this theory in nature, using our methods of intuitive mathematics. Here we need the method of Socrates, who believed that in science all concepts should be clearly defined, not abstract.

How was arithmetic born?

As we know, TM is based on two pillars, one of which is geometry and the other is arithmetic. Now let us see how arithmetic was born. As we will see, arithmetic was born from a Harmonic Quartet of one-dimensional OM.

I already mentioned above that, on a circle taken from nature, there are a large number of harmonies that create one group. We denote this group by the letter ***H*** and recall that the logical count operates in this particular group. On each circle belonging to the sphere there is a symmetry group ***H*** that has much in common with OM. Let me remind the reader of mathematics that the law of a harmonic quartet, endowed with all the properties of the same law on a circle, is also applicable to the projection of a straight line. Let me also remind you that one-dimensional, two-dimensional, and multidimensional projective geometries are also built on a harmonic quartet. The same process occurs in one-dimensional, two-dimensional, and multidimensional OM. 1847 The German geometer Staudt discovered on a projective straight line a count that he called the Wurf algebra. Our ancestors discovered this count thousands of years ago for points of a circle and called it arithmetic. It should be recalled that Staudt arbitrarily fixes three points ***A***, ***B*** and ***P*** on the projection line. Then he takes the group that is born from harmonies on this line, which is an analog of our group ***H***, and the subgroup of this group that leaves the point ***P*** fixed. Then he selects 2 new subgroups from this subgroup, which relate to points ***A***, ***B*** and ***P*** and are commutative. Using the last subgroups, he defines the concepts of “sum” and “product” of points and called this theory the Wurf algebra. Then, taking ***A* = *0***, **B = 1, P = ∞**, you get arithmetic for the points of the projective line. That is, points on a projection of a straight line obey the same laws as numbers in arithmetic.

The same theory applies to one-dimensional OM for points of a circle. Thus, we get arithmetic not on the mythical line (as in our mathematics), but on the circle that exists in nature. We denote by $\boldsymbol{H_P}$ the subgroup of the group $\boldsymbol{H}$ on the circle that leaves the point $\boldsymbol{P}$ fixed. Just as the Wurf algebra is part of the vast theory that exists on the projection of a straight line, so arithmetic is part of the one-dimensional OM on the circle. All laws of one-dimensional OM extend to a circle in nature. Thus, the laws of one-dimensional OM are the laws of nature and now that arithmetic works on a circle, its dry axioms and many theorems become laws of nature. I would like to add for the mathematician that there is an important difference between this arithmetic and the arithmetic that we have known so far: in the first there is only one infinite remote point, which is the point $\boldsymbol{P}$, and in the second there are two infinitely remote points ***(+ ∞** and **- ∞).***

The problem of the consistency of arithmetic over the past 100 years ranks second in the list of D.Hilbert's problems. In fact, one-dimensional OM gives a solution to Hilbert's II problem. In OM there are no axioms and no contradictions. The laws of nature cannot contradict each other.

American mathematician Richard Bellman and many other scientists hypothesize that nature is much simpler than mathematical models with which a person tries to understand nature. Bellman is absolutely right about this. Nature is a very logical system that obeys only the laws of innate mathematics. The author of the article believes that a person who is well acquainted with OM has great opportunities in understanding nature and overcoming the problems facing humanity.

We continue the reasoning. As we presented arithmetic on a circle using a harmonic quartet, the theory of complex and hypercomplex numbers can be represented on a sphere and multidimensional spheres, again using the same harmonic quartet. However, this discovery is worthless for the simple reason that number is a weak tool. To illustrate, consider two examples.

Example 1: if $\boldsymbol{f} \in \boldsymbol{H}$ and the circle has a point $\boldsymbol{A}$, the iteration with the number $\boldsymbol{r}$ of which coincides with $\boldsymbol{A}$ (i.e. $\boldsymbol{f}^r(\boldsymbol{A}) = \boldsymbol{A}$) and $\mathbf{r \geq 2}$, then the iterations with the number r of all points on the circle coincide with this point ($\boldsymbol{f}^r$ is an identity).

Example 2: if there is a point on the circle at which all iterations are different, and this sequence has a boundary point, then this point is one of the fixed points $\boldsymbol{f}$. This point is also the boundary point for sequences obtained from all other points of this circle.

These two rules are true for all one-dimensional, two-dimensional, and multidimensional OM. I applied these laws about 30 years ago in the topology of asymptotic lines of closed striped surfaces in projective geometry. But, as far as I know, these laws have not yet been proved for fractional functions, which speaks in favor of the fact that the number is not a powerful tool.

What is non-Euclidean geometry in OM?

About 170 years ago, people became aware of the new geometry, the axioms of which and the axioms of our geometry differed only in the postulate (axiom) of parallel lines. Gauss, one of the pioneers of this geometry, called it non-Euclidean geometry. At first, this new geometry was accepted as a catastrophe of thought, and later, when it was applied to Einstein's theory, it began to be considered a great achievement of thought. Now we will call a spade a spade and will make sure that they are neither a catastrophe nor a great achievement.

Indeed, let us put an arbitrary circle on a sphere and take all the orthogonal circles to this circle. Each of these circles creates harmony on the sphere. All these harmonies give rise to a symmetry group denoted by the letter ***Ga***. An experienced mathematician realizes that I am building a Poincare model not on a plane, but on a sphere. It is clear that ***Ga*** is one of the subgroups of ***G***. The circle divides the sphere into 2 domains, denoted by the letters ***S*** and ***S'***. It is also clear that each symmetry belonging to ***Ga*** leaves not only the circle ***a*** stationary, but also each of these domains. On circle ***a***, each orthogonal circle is divided into two arcs, one of which belongs to ***S*** and the other to ***S'***. From now on, by an arc we mean only arcs of orthogonal circles ***a***. Two arcs belonging to the same domain are called intersecting if they have a common point inside the domain. The same arcs are called parallel to each other if the common point belongs to the circle ***a***. Finally, the same arcs are called differential if there is no common point. Using these clearly defined terms, we will now have a theory in nature, which in the language of abstract and indefinite terms is called non-Euclidean geometry.

Let us continue our proof. First, we prove the following law, which is a generalization of the above Law 7.

Law 13: If the circle ***a*** is given on the sphere and 2 arbitrary points are located on one side of ***a***, then only one circle passes through these points, which is orthogonal to ***a***.

Indeed, let us take these 2 points and their harmonic points on the circle. According to Law III, these 4 points are on the desired circle. If we apply Law 13 for arcs, we get:

Law 14: Only one arc passes through two different points belonging to the same domain.

This law is a proof of the first axiom of non-Euclidean geometry. To understand this, you need to get rid of the language of abstract and vague terms in mathematics. Now, under the indefinite non-Euclidean term plane, let us understand one of the two domains ***S*** and ***S'*** separated by the circle ***a***. Therefore, instead of the points of the plane, we must take all the internal points of this domain. By the indefinite term of a straight line we mean the orthogonal arc to ***a***. After this renaming, it will become clear that Law 14 is indeed proof of the 1st axiom of non-Euclidean geometry. This axiom claims that only one straight line passes through two points, now, after exposition in a non-abstract language, it has become a law of nature.

I will not give evidence of all the axioms here; this is a pleasant lesson for every mathematician. But I want to give a proof of the axiom of parallels where the Euclidean and non-Euclidean geometries are different. Using abstract terms, this axiom sounds like this: from a point ***M*** that is not on a straight line, you can draw two disjoint lines in the direction of this line. If we now turn to the language of clear terms, this axiom will become a logical consequence of the following law:

Law 15: If a point ***M*** and an arc ***m*** not passing through this point are arbitrarily given on the domain, then through ***M*** one can draw two arcs parallel to the arc ***m***.

Indeed, let the arc ***m*** intersect the circle ***m*** at points ***A*** and ***B***. According to Law 7, only one circle passes through points ***A*** and ***M***, which is orthogonal to ***a***. Denote the arc of this circle by ***1***st. By the same law, one circle also passes through points ***B*** and ***M***, orthogonal to ***a***. Denote the arc of this circle by the letter ***n***. According to the above definition, arcs ***1*** and ***n*** are parallel to ***m***. The law is proven.

Arcs ***1*** and ***n*** intersecting at point ***M*** form 2 pairs of opposite angles. All arcs passing through one pair intersect ***m***, but arcs passing through another do not. Thus, through ***M*** there passes an infinite number of arcs (in the abstract language, lines) that do not intersect ***m***. The non-Euclidean parallel axiom is completely proven.

I would like to mention the mathematics that in non-Euclidean geometry there are two types of curves, one of which is called the eclidistant, and the other is the horocycle. So far, these curves have also been myths. On a sphere, these curves turn into circles. The eclidistant is the circle that intersects the circle ***a*** non-orthogonally, and the horocycle is the circle that touches ***a*** from the inside of the domain.

Now the question of which of the Euclidean and non-Euclidean geometries is more accurate becomes meaningless. Both are absolutely correct theories on the sphere, which are widely used in the process of discovering the secrets of nature.

It is clearly seen here that Euclidean geometry is not a special case of non-Euclidean geometry. Indeed, a point on ***a*** sphere is a special case of a circle. If we make the circle shrink to a point ***P*** decreasing, then this statement will be obvious. It is interesting to know how harmony ***a*** changes in this case. It becomes a transforming correspondence, which brings all points of the sphere to point ***P***. Denote this correspondence by the same letter ***P***.

In the general case, if we have points ***A*** and ***B*** on the sphere, it makes sense to compose the product of their transforming correspondences, which is written in the form ***A*** * ***B***. It is easy to see that ***A*** * ***B*** and ***B*** * ***A*** are not the same thing.

Law 16: Points ***A*** and ***B*** on a sphere coincide with it only if ***A*** * ***B*** = ***B*** * ***A***.

The proof of this law is obvious. If we have α symmetry and a point ***A*** on the sphere, then the product α * ***A*** is also not without meaning. It is easy to see that:

Law 17: The point ***A*** on a sphere is a fixed point of symmetry ***α*** only if ***α*** * ***A*** = ***A*** * α.

For example, ***a*** * ***A*** = ***A*** * ***a*** only and only if point ***A*** belongs to circle ***a***. In the logical count of OM there are many such laws that I am not talking about here. These laws are the cornerstones of logical human thinking. It should be noted that the solution of the II Hilbert problem is given using the last two laws.

I mentioned above that each element (symmetry) of the ***Ga*** group moves the spherical points so that the domains ***S*** and ***S'*** and the circle ***A*** remain motionless. Now let us see what happens when harmony with respect to the circle ***a*** acts on the sphere. Law III directly implies:

Law 18: Any harmony on the sphere with respect to the circle ***a*** leaves all points of this circle stationary, but swaps the domains ***S*** and ***S***'. Each orthogonal ***a*** circle also remains stationary.

If we now imagine that non-Euclidean geometries were constructed in both ***S*** and ***S'***, then the harmony ***a*** creates a one-to-one correspondence between these two geometries, which in mathematics is called isomorphism.

Law 19: Harmony with respect to any circle ***a*** on a sphere is an isomorphism of non-Euclidean geometries constructed in different domains of this circle.

As we will see, this law has its counterpart in three-dimensional OM and is one of the most fundamental laws of nature.

What is three-dimensional OM?

After deciphering the terms, a straight line and a plane ad infinitum revealed that the first is a circle with a fixed point ***P***, and the second is a sphere with the same fixed point ***P***, which are objects of nature and have nothing to do with infinity. It also became clear that our traditional mathematics was born from premathematics, after renaming changes to its objects. It has been said that TM is a premathematic version in nature. Premathematics was born from 4 empirical laws of OM. Consequently, the axioms of our one-dimensional and two-dimensional mathematics are rational consequences of only four laws of OM. Let us recall that the number of these axioms reaches thousands. This fact allows me to argue that the abstract axiomatic method is ineffective and that OM significantly improves human thinking.

As we have seen, a one-dimensional OM acts on a circle, which is a very rich theory. All theorems that exist on a projection straight line after decryption become the laws of one-dimensional OM. All theorems of arithmetic and number theory also become laws of OM and nature. Here the final solution was received by

Hilbert's II problem. Much richer two-dimensional OM. All our two-dimensional mathematics is a variant of premathematics, and premathematics is part of OM.

As we have already seen, one-dimensional and two-dimensional TMs are a figment of the human imagination, where intuitive concepts of lines and planes leading to infinity are allowed. In one-dimensional and two-dimensional OM, this is not. The circle and sphere cannot have an intuitive origin for the simple reason that they are the simplest objects of nature. The motto "The most ingenious - the simplest" is not just good words. Now, after getting acquainted with one-dimensional and two-dimensional OM, the following postulate is inevitable for us:

Postulate. The Universe is a three-dimensional sphere, where a ray of light is a large circle of this sphere.

This postulate is not new for physicists, but physicists here do not know the most important thing, which is three-dimensional OM. Until now, physicists have identified a ray of light with a straight line. This dogma became destructive for optics for the simple reason that optics also became incomprehensible. Indeed, as we have seen, the term "straight line" is a "different" name for a circle passing through a fixed point ***P*** on a sphere. (Soon we will see that the straight line of our space in three-dimensional OM is a circle belonging to a three-dimensional sphere, which again passes through a fixed point ***P*** in space. Therefore, this dogma states that all rays of light in the universe (that is, in nature) pass through a fixed point ***P***, which cannot be true. All the rays of light emanating from the Sun, from other stars, galaxies and quasars cannot pass through one point in the universe.

Gauss, the king of mathematicians, was also deceived by this dogma. It is known that in Euclidean geometry the sum of the internal angles of a triangle is 180°, while in non-Euclidean geometry the sum is less than 180° and is a variable. In other words, the farther the corners of the triangle are from each other, the greater the difference between 180° and this value. Given this fact, Gauss measured the sum of the internal angles of a triangle formed by three rays of light passing through the peaks of three famous hills in Prussia. He hoped that this amount would be significantly less than 180°. The experiment showed an unexpected result. This value exceeded 180° by 15 arc seconds. Later, Gauss was convinced that 15 seconds did not exceed the error of the experiment, and this experiment was forgotten.

Now these 15 seconds speak in favor of my postulate. It is known that the sum of the internal angles of a triangle, consisting of 3 large circles on a sphere, is also a variable, but always greater than 180°. In my opinion, if you experiment and measure the angles of a triangle formed by three distant satellites in space (the farther the distance between them, the better), then the sum of these three angles will be more than 180°. If we measure the area of this triangle, we can, using a well-known formula, calculate the approximate size of the entire universe (that is, a three-dimensional sphere). The information obtained can lead to great discoveries in the field of cosmology.

OM is an absolutely empirical theory that can be used to describe nature. As we have already seen, in OM, using only 4 empirical laws, one can imagine all two-dimensional mathematics, where the existing axioms were proved and became laws of nature.

What is one of the fundamental laws of inanimate nature?

Our postulate states that the entire universe is a three-dimensional sphere. The three-dimensional sphere is infinite but does not extend to infinity (sometimes it is called infinite, but not unlimited, which causes misunderstanding). Just as a pair of points on a circle divides it into 2 arcs, a circle divides 2 spherical segments into a sphere, and a sphere divides a three-dimensional sphere (universe) into 2 three-dimensional domains. Now let us denote this sphere with the letter ***a***, and domains with the letters ***S*** and ***S'***.

In one of these domains, we take an arbitrary point ***X*** and take all the spheres that pass through this point and are orthogonal to ***a***. An arbitrary pair of such two spheres intersects each other along a circle ***l*** passing through the point ***X***. ***l*** intersects the sphere ***a*** at points ***A*** and ***B***. Let ***X'*** be the point that harmoniously separates the pair AB from point ***X***. It follows from the law of the Harmonic Quartet that the position of the point ***X'*** depends only on the choice of the point ***X***. ***X*** → ***X'*** is a symmetry - an involutionary one-to-one correspondence of the entire universe. Let us again denote this symmetry by the letter ***a*** and call it harmony with respect to the sphere ***a***. All harmonies with respect to the universe give rise to a huge group, which we denote by the letter ***F***. ***F*** does not differ in its properties from the previous ***G*** and ***H***. Here also ***a*** * ***a*** = ***1***, ***a*** * ***b*** = ***b*** * ***a***, only if the spheres ***a*** and ***b*** are orthogonal to each other, etc. Here, also, any symmetry ***α*** belonging to ***F***, a sphere leads to a sphere, angles are invariant, a circle leads to a circle, a Harmonic Quartet to a Harmonic Quartet and so on. Here you can prove many laws that describe the structure of the whole universe, but now this is not my goal. The physicist says my postulate is not new. However, he does not know anything about group ***F*** and the logical count, with the help of which the structure of the whole universe (that is, a three-dimensional sphere) is revealed. Here, as an example, I will prove one of the laws of inanimate nature. The following law is obvious:

Law 20: The harmony belonging to the three-dimensional sphere leaves all points of the sphere motionless, but changes the position of the ***S*** and ***S'*** domains. Each sphere orthogonal to ***a*** also remains stationary.

That is, if ***X*** belongs to ***S***, then ***X'*** belongs to ***S'*** and vice versa. Now let us build a non-Euclidean dimension of space in area ***S***, just as we built a dimension of a plane. Consider the set of spheres that are orthogonal to the ***a*** sphere. All harmonies with respect to these spheres give rise to the symmetry group ***Fa***, which is a subgroup of

F. It is easy to see that each symmetry belonging to ***Fa*** leaves both the sphere ***a*** and the regions ***S*** and ***S'*** immobile. Each sphere orthogonal to ***a*** is divided into two sectors, one of which belongs to ***S*** and the other to ***S'***. These sectors are called non-Euclidean planes: one in ***S*** and the other in ***S'***. Each circle orthogonal to the sphere ***a*** is also divided into two arcs called non-Euclidean straight lines, one of which belongs to ***S*** again and the other to ***S'***. Thus, non-Euclidean straight lines and planes are no longer myths. These are objects belonging to spheres ***S*** and ***S'***. It is also easy to prove here that all non-Euclidean axioms are true for these new types of lines and planes. Therefore, all theorems will be correct. That is, these axioms and theorems are renamed, rethought, and become laws of nature.

Thus, I managed to build 2 non-Euclidean spaces inside ***S*** and ***S'*** domains (one in each). It is immediately clear that harmony with respect to the sphere ***a*** is an isomorphism between the two theories. This isomorphism is one of the fundamental laws of inanimate nature and is formulated as follows:

Law 21*:* any sphere ***a*** existing in nature divides the entire universe into two ***S*** and ***S***' domains (balls), in each of which non-Euclidean geometry acts. Harmony ***a*** with respect to sphere ***a*** is an isomorphism between the two theories.

Conclusion

This article attempts to create the foundations of mathematics that does not use the axiomatic construction method, in particular, concepts such as the infinite straight line and plane and the answer is found to the question: what happens if the mathematics is built on a circle and a sphere - the most perfect objects of nature?

The author called the resulting theory Objective Mathematics (OM), bearing in mind that this theory operates on natural objects that exist in nature.

The constructions and proofs in the article are first made on a circle (one-dimensional OM), and then the resulting law is generalized to a sphere (two-dimensional OM) and then to a three-dimensional sphere (three-dimensional OM).

The results obtained 4 empirical laws and 21 laws.

The paper gives definitions of such concepts as:

• Harmonic Four (Quartet) on a circle,

• Logical three-dimensional count,

• Preliminary mathematics (Premathematics) in OM,

• Non-Euclidean geometry in OM,

• Three-dimensional OM, etc.

The inspiration of the article is the method of Socrates, who believed that it is necessary to operate only with clear and understandable terms and definitions for humans. Also, the theory of symmetry of Evarista Galois played an important role in the creation of OM.

The article claims that the created theory can be effective in studying the laws of nature.