On Geoengineering the Albedo Solution to Global Warming and Identifying Key Parameters

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Abstract

A solar geoengineering global warming model is developed with a re-radiation factor and results are shown to be consistent with the Planck parameter. The re-radiation factor is important in quantifying the relative global warming impact of the albedo effect compared to that of greenhouse gases (GHG). The potential reverse forcing due to a change in the Earth's global albedo compared to GHGs is illustrated. Results of modeling support solar geoengineering solutions with two key parameters from modeling: an albedo-GHG and a Planck-albedo parameter. Using these, it is concluded that a 1.5% solar geoengineering change in the global albedo could result in a significant resolution to global warming.

1 Introduction

Solar geoengineering is vital in global warming solutions as results can reverse trends and reduce the probability of a tipping point from occurring. In this paper, a geoengineering model that uses a re-radiation factor, which helps to quantify differences between changes in the global albedo versus greenhouse gas forcing, is developed. The re-radiation parameter is initially obtained in the absence of warming feedbacks with a unique value of 0.618 (or β=0.887). The re-radiation factor is a redefined variable taken from the effective emissivity constant of the planetary system. An application of the model is provided between two different time periods (1950 and 2019). In 2019, the re-radiation parameter takes GHG change and feedback effects into account. Then, the Planck feedback parameter is used to verify model consistency. The model illustrates a reasonable way to view the Earth's energy budget; simplifies estimates without the need for doubling theory, provides a number of useful insights in climatology estimates and allows for practical solar geoengineering calculations of global warming mitigation [1]. Specifically, a 1.6 albedo-GHG factor along and a Planck-Albedo parameter (having a value of $1\text{W/m}^2/^{\circ}\text{K}/\Delta\%$albedo) is obtained in modeling results. These values greatly simplify solar geoengineering [2, 3] calculations. Using these values, we exemplify a global warming albedo solution and provide a measure of feasibility [1].

2. Data and Method

To introduce the re-radiation model, we will often refer to the Planck parameter and its associated variables that play a key role in its development and verification. Therefore, an overview in Appendix A is provided which also includes a unique way to assess the parameter's value using an albedo approach (see Section A.1).

2.1 The Re-radiation Global Warming Model

In geoengineering, we are working with absorption and re-radiation, we define

$$P_{Total} = \sigma T_S^4 = \sigma\left(\frac{T_{TOA}}{\beta}\right)^4 \text{ and } P_\alpha = \sigma T_\alpha^4 = \sigma\left(\beta T_S\right)^4 \tag{1}$$

The definitions of T_{TOA}, T_S and β are provided in Appendix A (Eq. A-1, A-2, A-3). Consider a time when there is ***no feedback issues*** causing warming trends. Then by conservation of energy, the equivalent power re-radiated from GHGs in this model is dependent on P_α with

$$P_{GHG} = P_{Total} - P_\alpha = \sigma T_S^4 - \sigma T_\alpha^4 \tag{2}$$

To be consistent with Eq. A-1, $T_\alpha = T_{TOA}$, since typically $T_\alpha \approx 255^{\circ}$K and $T_s \approx 288^{\circ}$K, then in keeping with a common definition of the global beta (see Eq. A-4) for the moment $\beta = T_\alpha/T_s = T_{TOA}/T_S$.

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This allows us to write the dependence

$$P_{GHG} = \sigma T_S^4 - \sigma T_\alpha^4 = \frac{\sigma T_\alpha^4}{\beta^4} - \sigma T_\alpha^4 = \sigma T_\alpha^4 \left(\frac{1}{\beta^4} - 1 \right) \tag{3}$$

Note that when $\beta^4=1$, there are no GHG contributions. We now define a re-radiation parameter $f= \beta^4$. Consider the fraction of the blackbody re-radiated by GHGs given by

$$P_{GHG} = f\, P_\alpha = f\, \sigma\, T_\alpha^4 \tag{4}$$

It is important in geoengineering to view the re-radiation as part of the albedo effect. This is a key difference in how we view the total effect from short wavelength absorption by the inclusion of re-radiation effect [2]. Now in order to have consistency for f, we require from Equations 3 and 4

$$P_{GHG} = \sigma T_\alpha^4 \left(\frac{1}{f} - 1 \right) = f\, \sigma T_\alpha^4 \tag{5}$$

This dependence leads us to the solution of the quadratic expression

$$f^2 + f - 1 = 0 \text{ yielding } f_1 = 0.618034 = \beta^4,\ \beta = (0.618034)^{1/4} = 0.88664 \tag{6}$$

This is very close to the common value estimated for β (Appendix A) and this has been obtained through energy balance in the planetary system providing a self-determining assessment. In Section 2.3, we double check this model in another way by balancing energy. Then in Section 3 we will apply the modeling to demonstrate its capability and consistency with the Planck parameter.

2.2 ***Re-radiation Model Applied to Two Different Time Periods***

Global warming can be exemplified by looking at two different time periods. The model applied for 1950 needs to be consistent with Eq. 2 and 4. Here we will

- assume no feedback issues causing a warming trend in 1950 so that from our model

$$P_{Total\_1950} = P_\alpha + P_{GHG} = P_\alpha + f_1 P_\alpha = P_\alpha (1 + f_1) = 1.618\, P_\alpha \tag{7}$$

where $P_\alpha = S_o \{0.25x(1 - Albedo)\}$ and S_o=1361W/m^2. Under the assumption of no changes in GHG and feedback issues, this provides a base number for our geoengineering estimates so that 1.618 becomes the 1950 albedo-GHG reference value. Since its value is related to the re-radiation parameter, it is subjected to changes due to variations in our aging climate system. As a reference value, it is constrained by the energy balance in Eq. 5 and as discussed in Section 2.3.

In 2019 due to global warming trends, this model is more complex and harder to separate out terms. However, we proceed similarly and results and verification will justify its continual use, then

$$P_{Total\,2019} = P_{\alpha'} + P_{GHG'+Feedback} \approx P_{\alpha'} + f_2 P_{\alpha'} \tag{8}$$

Here, $P_{GHG'+Feedback}$ includes the 1950 GHGs and 2019 increase with feedbacks such as water-vapor concentration, lapse rate effect and other changes such as increase in snow-ice albedo variations that are hard to separate out. That is, feedbacks are related to GHG increases and albedo change. $P_{\alpha'}$ represents the 2019 point in time with its albedo due to changes in UHI absorption, cloud absorption, ice and snow melting, and so forth that can be discerned. The model does not demand rigid accountability in its application (see Sec.3) but reasonable estimates are helpful. We note that f_2 is not a strict measure of the emissivity.

In 1950 f_1 defines the GHG re-radiation function (with no feedbacks) and is consistent with the estimates for beta. In 2019, it is more complex and according to Eq. 8, must include feedbacks. The value f_2 while close to the beta value in Eq. 6, is no longer identical as f_1 (see Equation 22). The value f_2 can also be assessed relative to f_1 as described in Section 2.3.2. However, in general, between the two time periods, we will find $P_{GHG} \approx P_{GHG'+Feedback}$ (see results in Section 3).

2.3 Energy Balance

Although f_1 has been uniquely defined in Eq. 6, this should also result from balancing the energy in and out of the global system.

2.3.1 Balancing P_{out} and P_{in} in 1950

To balance the energy in 1950, we start with Eq. 7. In equilibrium the radiation that leaves must balance P_α, from the energy absorbed, so that

$$\begin{aligned} Energy_{Out} &= (1-f_1)P_\alpha + (1-f_1)P_{Total} = (1-f_1)P_\alpha + (1-f_1)\{P_\alpha + f_1 P_\alpha\} \\ &= 2P_\alpha - f_1 P_\alpha - f_1^2 P_\alpha = Energy_{In} = P_\alpha \end{aligned} \tag{9}$$

This is consistent, so that in 1950 Eq. 9 requires the same quadratic solution as Eq. 6. It is also apparent that

$$P_\alpha = f_1 P_{Total\_1950} = \beta_1^4 P_{Total\_1950} \tag{10}$$

since

$$P_\alpha = f_1(P_\alpha + f_1 P_\alpha) \;\; or \;\; 1 = f_1(1+f_1) \tag{11}$$

The RHS of Eq. 11 is Eq. 6. This illustrates f_1 from another perspective as the fractional amount of total radiation in equilibrium. As a final check, an application in Section 3, Table 1 results, will illustrate that f_1 provides reasonable results.

2.3.2 Warming Imbalance in 2019

The re-radiation parameters f_1 and f_2, are connected and from Eq. 6, 7 and 8 we have

$$f_2 = f_1 + \left(\frac{P_{2019}}{P_{\alpha'}} - \frac{P_{1950}}{P_\alpha}\right) = f_1 + \Delta f = \beta_1^4 + \Delta f \approx \beta_2^4 + \Delta f \tag{12}$$

In this way f_2 is a function of f_1=0.618 and the differences in the global warming residuals that is identified in Eq. 12 as Δf. The RHS of Eq. 12 (indicating that $\beta_1 \approx \beta_2$) will become apparent in application (Eq. 15 and 16) and verification.

3.0 Results and Discussion

Since the re-radiation parameter is fixed for f_1=0.618, to obtain T_{1950}=13.89°C (287.038°K), the only adjustable parameter left in our model is the global albedo. This requires an albedo value of 0.3008 (see Table 1) to obtain the correct value T_{1950}. This albedo number is reasonable and similar to values cited in the literature [4].

In 2019, the average temperature of the Earth is T_{2019}=14.84°C (287.99°K). Here we are not sure of the albedo value since it likely changed due to UHI increase, snow and ice melting and cloud coverage changes. The IPCC value in AR5 [6] is 0.294118 (100/340). However, this would represent a 3% change since 1950 which may be an overestimation. In this assessment, we will assume a low middle value of 1.2% change. Another reason for this choice will become apparent in the resulting analysis. Then, the f_2 parameter is adjusted to 0.6311 to obtain T_{2019}. Table 1 summarizes model results for the specified albedos and observed Earth's surface temperatures. The results yield $P_{Total\ 1950}$=384.935 W/m^2 and $P_{Total\ 2019}$=390.055 W/m^2.

Table 1 Model results

Year	**T(°K)**	**T_α(°K)**	***f_1, f_2***	**α, α'**	**P_α, $P_{\alpha'}$ (W/m^2)**	**$P_{GHG'+feedback}$ P_{GHG} (W/m^2)**	**P_{Total} (W/m^2)**
2019	287.991	254.83	0.63114	29.719	239.131	150.925	390.056
1950	287.041	254.51	0.6180	30.08	237.903	147.032	384.935
Δ2019-1950	**0.95**	0.328	**1.311%**	0.361 **(1.2%)**	**1.228**	3.893	**5.12**

From Table 1

$$\Delta P_{Total} = P_{2019} - P_{1950} = 5.121 W/m^2 \tag{13}$$

and

$$\Delta T_{Total} = T_{2019} - T_{1950} = 0.95°C \tag{14}$$

as modeled.

3.1 Showing Model Consistency with the Planck Parameter

To show model consistency, the forcing change, 5.121 W/m^2, resulting in a 0.95^{o}K rise, should agree with what is expected when using the Planck feedback parameter.

In order to show model consistency, we will need some exact values for beta using the temperatures in Table 1, these are from the two different time periods (see Eq. A-3)

$$\beta_{1950} = \frac{T_\alpha}{T_S} = \frac{T_{TOA}}{T_S} = \frac{254.51}{287.04} = 0.88667 \ and \ \beta^4_{1950} = 0.61809 \tag{15}$$

as required, and

$$\beta_{2019} = \frac{T_\alpha}{T_S} = \frac{T_{TOA}}{T_S} = \frac{254.83}{287.99} = 0.88485 \ and \ \beta^4_{2019} = 0.61304 \tag{16}$$

Although these two are very close, we use both values due to the need for high accuracy, self-consistency is required.

From Equation A-4 in the appendix, we note the Planck parameter from Table 1 can be estimated as

$$\lambda_o = -4\frac{\Delta R_{LWR}}{T_S} = -4\left(\frac{237.9W/m^2}{287.04°K}\right)_{1950} = -3.315W/m^2/°K \tag{17}$$

and

$$\lambda_o = -4\frac{\Delta R_{LWR}}{T_S} = -4\left(\frac{239.13W/m^2}{287.99°K}\right)_{2019} = -3.321W/m^2/°K \tag{18}$$

We note these are very close in value showing miner error and consistency with Planck parameter value, often taken as 3.3W/m^2/^{o}K. While there are only small differences between each beta and these two Planck parameters, final warming predictions using a Planck parameter method, requires values found from the model. This self-consistency helps in providing accuracy for estimating ΔT by reducing compounding error within the model. We then use the generalized form of Eq. 10 (with beta) for the long wavelength estimate in Equation A-4, yielding the warming change in terms of the total power and the Planck parameter method as

$$\Delta T = T_{1950} - T_{2019} = -4\left\{\left(\frac{\beta^4 P_{Total}}{\lambda_o}\right)_{1950} - \left(\frac{\beta^4 P_{Total}}{\lambda_o}\right)_{2019}\right\} \tag{19}$$

Using Table 1, the temperature warming results is

$$\Delta T = -4\left(\frac{0.6181x384.935W/m^2/°K}{3.315W/m^2/°K} - \frac{0.61304x390.056W/m^2/°K}{3.3215W/m^2/°K}\right) = 0.947°K \tag{20}$$

This equation illustrates consistency of the re-radiation model with the Planck parameter showing surprising accuracy helping to verify the model from a different perspective.

3.1 Re-radiation Parameter Discussion

In Table 1, the measure of Δf=1.45% fractional increase is mainly due to re-radiation change and associated feedbacks. This is significant. From Eq. 7, 8 and 12

$$\Delta f = f_2 - f_1 = \left(\frac{P_{2019}}{P_{\alpha'}} - \frac{P_{1950}}{P_\alpha}\right) = \left(\frac{P_{GHG'+F}}{P_{\alpha'}} - \frac{P_{GHG}}{P_\alpha}\right) \tag{21}$$

Therefore, f is an estimate of climate re-radiation and Δf an estimate of its change and confounded with feedback effects. It is a measure of GHG forcing increase and the feedback relative to the initial 1950 radiation, and is generally helpful in looking at how our climate is working.

3.2 Comparisons Using the Albedo-GHG Factor

We can look at an important ratio, the power created by the albedo effect compared to GHGs in 1950. The initial radiation is P_α, and then according to Eq. 7 and Table 1, the energy is increased by P_{GHG} due to re-radiation fP_α that yields the ratio

$$\left\{\frac{P_\alpha + P_{GHG}}{P_{GHG}} = \frac{P_\alpha + f_1 P_\alpha}{f_1 P_\alpha} = \frac{1+f_1}{f_1} = \frac{1.62}{0.62} = 2.62\right\}_{1950} \qquad \text{also note that} \qquad \left\{\frac{1+f_2}{f_2} = 2.58\right\}_{2019} \tag{22}$$

We note the ratio is reduced in 2019 due to the addition ΔP_{GHG} and feedbacks. If f could eventually approach a catastrophic value of unity, this ratio reduces to a minimum of 2.

In this engineering view, a change in albedo forcing compared with a change in GHGs can be described. The variation in the energy due to an average albedo change and its re-radiation is

$$\Delta P_{\overline{\alpha}} = \Delta P_{\alpha'} + f_2 \Delta P_{\alpha'} = 1.631\ \Delta P_{\alpha'} \tag{23}$$

The average change in GHGs can be written in terms of *Δf*

$$\Delta P_{\overline{GHG'}} = \Delta f\ P_{GHG'} = 1.311\%\left(f_2\ P_{\alpha'}\right) = 0.827\% P_{\alpha'} \tag{24}$$

This resulting ratio from Table 1 is

$$\frac{\Delta P_{\overline{\alpha}}}{\Delta P_{\overline{GHG'}}} = \frac{\Delta P_{\alpha'}}{\Delta f}\frac{(1+f_2)}{P_{\alpha'}\ f_2} = \frac{1.228 W/m^2}{0.0131}\frac{1.631}{239.1 W/m^2\ 0.631} = 1.01 \tag{25}$$

Note that this ratio is of course dependent on the 2019 albedo 1.2% change, selected here to obtain unity for illustrative purposes. The ratio, $\Delta P_{\alpha'}/\Delta f$, is an interesting aspect of climate change. In 2019, if we have knowledge of values, we can compare the dominant aspect of the warming trend. It also provides us with a measure of solar reversibility

$$\Delta P_{\alpha'} \geq \Delta f\ \frac{P_{\alpha'}\ f_2}{(1+f_2)} 1.02 \approx 1.21 W/m^2 \tag{26}$$

This ratio is dependent on the change in the albedo compared with a GHG change. It may be helpful in assessing negative CO2 emissions vs an albedo reduction. Although, it is perhaps not the best way to assess geoengineering estimates. True values of Δα and Δf are not easily obtained in 2019. However, it avoids CO_2 doubling estimates, which are also difficult to evaluate. Furthermore, in some instances, a local change in ΔP_α can create excess increase in GHGs. This has been a concern with cool roofs in the winter which might require additional anthropogenic energy. This might be a good way to estimate by Eq. 26, whether such a change is beneficial by comparison.

It is important to simplify further to provide a more productive approach. In reverse solar geoengineering a global warming solution, it is helpful to have simple reliable values. In this view, the 1.6 albedo-GHG factor (which is reasonably accurate) is an important engineering number. Another important engineering value is described by a Planck-albedo parameter.

3.3 The Planck-Albedo Parameter

The albedo changes and ΔP_α in Table 1, are: $\%\Delta\alpha = 1.2\%$ and 1.228W/m^2, respectively. We note that we can define a unique Planck-albedo parameter $\Lambda_{\%\Delta\alpha} = \Delta P_\alpha / \%\Delta albedo$. To illustrate from Table 1

$$\Lambda_{\%\Delta\alpha} = 1.023\ \text{W/m}^2/\Delta\%\text{albedo} \tag{27}$$

This parameter can also be expressed per degree (noting the 0.95°K change in Table 1)

$$\Lambda_{\%\Delta\alpha\Delta T} \approx 1W/m^2/\Delta\%albedo/^{\circ}K \tag{28}$$

The helpful parameter [3] is featured here as a modeling tool. We term it the Planck-albedo parameter, since it relates to blackbody (P_α) absorption. A simple numeric example is given in the conclusion to illustrate how it provides helpful estimates along with the albedo-GHG factor. This interesting parameter simplifies from the basic assessments of the two different time periods (see also Eq. A-8) as

$$\Lambda_{\%\Delta\alpha} = \frac{\left(\Delta E_o\right)_\alpha}{\frac{\alpha_1-\alpha_2}{\alpha_1}100} = \frac{E_o\left(\alpha_1-\alpha_2\right)}{\frac{\alpha_1-\alpha_2}{\alpha_1}100} = E_o\alpha_1/100 \approx 1W/m^2/\%\Delta albedo \tag{29}$$

where E_o=340 W/m^2 and when α_1 is 0.294118, the value 1.000W/m^2/Δ%albedo is obtained. We note the value 29.4118% (100/340) is given in AR5 [6]. The parameter's relationship to λ_α is

$$\lambda_\alpha = \Lambda_{\%\Delta\alpha\Delta T}\ x\ \%\Delta\alpha \tag{30}$$

and appropriate feedback parameters could include the re-radiation albedo-GHG factor in 2019 [2], for example

$$\lambda_\alpha^{\dagger} = \Lambda_{\%\Delta\alpha\Delta T}\ x\ \%\Delta\alpha\ \ (1+f_2) \tag{31}$$

3.4 A Simplified Reverse Forcing Solution

The albedo-GHG and the Planck-Albedo parameter may now be combined in order to provide a simple solar geoengineering solution estimate for reverse forcing

$$\Delta P_{\mathrm{Re}v\_S} = -\Lambda_{\%\Delta\alpha\Delta T}\ \ \%\Delta\alpha\ (1+f_2)\ A = \Delta P_T\,(1+f_2)\ A \tag{32}$$

with effective results

$$Effect = \frac{\Delta P_{\mathrm{Re}v\_S}}{\Delta P_{Total}} \tag{33}$$

and from A-14 $\Delta P_{\mathrm{Re}v\_LWR} = \beta^4 \Delta P_{\mathrm{Re}v\_S}$ the temperature reduction is

$$\Delta T_{\mathrm{Re}v} = -\frac{\beta^4 \Delta P_{\mathrm{Re}v\_S}}{\lambda_o} \tag{34}$$

Here ΔP_{Rev} is the reverse forcing, A is an estimate of the anticipated GW amplification (feedback) reduction, and ΔP_T is the reverse forcing from the target area. The equation provides a fairly simple and practical way to estimate ΔP_{Rev}. An example is provided in the conclusion**.** In solar geoengineering, anticipating an allowance for the climate system to equilibrate [13] may be unnecessary, since the lagged transient climate response is anticipated to be similar. That is, a positive or negative albedo change is likely not to have a strong hysteresis effect.

4.0 Conclusion

In this paper, we provided a re-radiation global warming model. The model shows consistency with the Planck parameter. We noted that the re-radiation parameter increased by about 1.31% due to global warming from 1950 to 2019, illustrating the warming from a different perspective. From the model, a helpful albedo-GHG parameter was quantified having a value of 1.6.

We also found an engineering factor that we termed the Planck-albedo parameter, which is about $\Lambda_{\%\Delta\alpha\Delta T} \approx 1W/m^2/\Delta\%albedo/^{\circ}K$. These findings can be helpful in quickly estimating the effect of an albedo change on global warming and in assessing λ_α. These results along with our model support solar geoengineering solutions [3, 7-9].

For example, Feinberg 2020 [1] suggested a goal of 1.5% geoengineering albedo change. Using Equation 32, with a decrease in water-vapor feedback anticipated, we might use a value of A≈2 [10], then

$$\Delta P_{\mathrm{Re}v\_S} = -1\mathrm{W/m^2/\%\ x\ 1.5\%\ x\ (1+f_2)\ x\ 2} = -4.9\ \mathrm{Watt/m^2} \tag{35}$$

This estimate can be compared with the re-radiation model results in Table 1 showing a forcing of 5.21 W/m^2 to obtain the relative effect of 94% from Eq. 33 for this particular geoengineering solution. Equation 35 expressed in terms of reverse temperature warming results is then from Eq. 34

$$\Delta T_{\mathrm{Rev}} = \frac{0.61 x 4.9 W/m^2}{\lambda_o} = -0.9°K \tag{36}$$

This would indicate a significant resolution to the current warming trend. As one might suspect, a 1.5% albedo change requires a lot of modified area. Feasibility is discussed in more detail in Feinberg's 2020 paper [1]. Results indicate the required percent of area change with proper hotspot targets, and such area would be roughly 12 times smaller than a non-hotspot area. Cooling estimates are also provided relative to UHI area target sizes. Other solar geoengineering solutions have been proposed [7-9].

Appendix A

Overview of Planck Feedback Parameter

Estimates on the Planck feedback parameter are varied, typically between -3.8W/m^2/^{o}K and -3.21W/m^2/^{o}K with some values as large as -7.1W/m^2/^{o}K [11]. The IPCC AR4 [12] lists a value of -3.21W/m^2/^{o}K. Numerous authors have developed different expressions [11]. A typical estimate starts with

$$F_{TOA} = (1-\alpha)\, S_o/4 - \sigma(\beta T_S)^4 = (1-\alpha)\, S_o/4 - R_{LWR} \tag{A-1}$$

where S$_o$=1361W/m^2, F$_{TOA}$ is the radiation budget at the top of the atmosphere, R$_{OLW}$ is the outgoing long wave radiation (a function of surface temperature and albedo), σ is the Stefan-Boltzmann constant and β is described in this section below and is redefined in terms of a re-radiation parameter in this paper. Then the Planck parameter λ_o can be calculated as

$$\lambda_o = \partial F_{TOA}/\partial T_S = -\partial R_{OLW}/\partial T_S \tag{A-2}$$

This result is

$$\lambda_o = -4\beta^4 \sigma T_S^3 = -4\beta\sigma T_{TOA}^3 = -\frac{4R_{OLW}}{T_S} \tag{A-3}$$

where β varies in the literature from 0.876 to 0.887 (averaging=0.8815) and Ts=288^{o}K [12]. This yields -3.37W/m^2/^{o}K<λ_o<-3.21W/m^2/^{o}K. However, from Eq. A-3, β is often taken as the ratio

$$\beta = T_{TOA}/T_S = 255°K/288°K = 0.8854 \text{ and } \beta^4 = 0.615 \tag{A-4}$$

A common assessment uses T$_{TOA}$=255^{o}K, so that λ_o =-3.33W/m^2/^{o}K. Another expression developed by Schlesinger [6] is dependent on the albedo and surface temperature as

$$\lambda_o = S_o(1-\alpha)/T_S \tag{A-5}$$

When S$_o$=1361, 0.294118<α<0.3, and Ts=288 ^{o}K then -3.308W/m^2/^{o}K >λ_o>-3.3358W/m^2/^{o}K , respectively.

A.1 Estimating the Planck Parameter with an Albedo Method

Consider a global albedo change corresponding to 1^{o}K rise from solar absorption letting

$$F_{TOA} = 0 = (1-\alpha)E_o - \sigma(T_S)^4 \tag{A-6}$$

where E$_o$=S$_o$/4. Then a 1^{o}K change is

$$\Delta T_S = T_2 - T_1 = \left(\frac{E_o}{\sigma}(1-\alpha_2)\right)^{1/4} - \left(\frac{E_o}{\sigma}(1-\alpha_1)\right)^{1/4} = 1°K \tag{A-7}$$

Here we will use the AR5 albedo starting value of 0.294118 [6]. We find that the corresponding albedo change is 0.28299 when E$_o$=340W/m^2. This corresponds to

$$\Delta E_o = E_o\left\{\left(1-\alpha_2\right)-\left(1-\alpha_1\right)\right\} = E_o\left(\alpha_1-\alpha_2\right) = 3.784W/m^2 \qquad \text{(A-8)}$$

Since this is for a 1^oK rise, then it can also be written as

$$\lambda_{1K}=3.784\text{W/m}^2/^o\text{K} \qquad \text{(A-9)}$$

We note this is related to the surface value, then

$$\lambda_{1K} = -4\sigma T_s^3 \qquad \text{(A-10)}$$

By comparison to above we have

$$\lambda_o=\lambda_{1K}\,\beta = -3.784\text{W/m}^2/^o\text{K}=-3.349\text{W/m}^2/^o\text{K} \qquad \text{(A-11)}$$

This is very close to the -3.33 $\text{W/m}^2/^o\text{K}$ value obtained in the traditional manner.

References

1. Feinberg, A. (May 2020) On Implementing an Albedo Solution with UHI Global Warming and Cooling Estimates, *Vixra 2005.0184,* DOI: 10.13140/RG.2.2.33637.22244
2. Winton, M. (2005) Surface Albedo Feedback Estimates for the AR4 Climate Models, *AMS*, https://journals.ametsoc.org/doi/10.1175/JCLI3624.1
3. Feinberg, A. (2020) Urban Heat Island Amplification Estimates on Global Warming Using an Albedo Model, preprint: *Vixra 2003.0088 DOI: 10.13140/RG.2.2.32758.14402/15 (submitted).*
4. Stephens, G., O'Brien, D., Webster, P., Pilewski, P., Kato, S, Li, J, (2015) The albedo of Earth, *Rev. of Geophysics*, https://doi.org/10.1002/2014RG000449
5. Schlesinger, M.E. (1986) Equilibrium and transient climatic warming induced by increased atmospheric CO2. *ClimateDynamics,Vol.1,35–51*
6. Hartmann, D.L., A.M.G. Klein Tank, M. Rusticucci, L.V. Alexander, S. Brönnimann, Y. Charabi, F.J. Dentener, E.J. Dlugokencky, D.R. Easterling, A. Kaplan, B.J. Soden, P.W. Thorne, M. Wild and P.M. Zhai, (2013) Observations: Atmosphere and Surface. In: Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the *Fifth Assessment Report of the Intergovernmental Panel on Climate Change* [Stocker, T.F., D. Qin, G.-K. Plattner, M. Tignor, S.K. Allen, J. Boschung, A. Nauels, Y. Xia, V. Bex and P.M. Midgley (eds.)]. *Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.*
7. D. Dunne, (2018), Six ideas to limit global warming with solar geoengineering, CarbonBrief, https://www.carbonbrief.org/explainer-six-ideas-to-limit-global-warming-with-solar-geoengineering
8. A. Cho (2016), To fight global warming, Senate calls for study of making Earth reflect more light, Science, https://www.sciencemag.org/news/2016/04/fight-global-warming-senate-calls-study-making-earth-reflect-more-light
9. Levinson, R., Akbari, H. (2010) Potential benefits of cool roofs on commercial buildings: conserving energy, saving money, and reducing emission of greenhouse gases and air pollutants. *Energy Efficiency* **3,** 53–109 https://doi.org/10.1007/s12053-008-9038-2
10. Dessler A. E. ,Zhang Z., Yang P., (2008), Water-vapor climate feedback inferred from climate fluctuations, 2003–2008, *Geophysical Research Letters*, https://doi.org/10.1029/2008GL035333
11. Kimoto, K. (2006) On the Confusion of Planck Feedback Parameters, Energy & Environment (2009)
12. Soden, B.J. and Held I.M.,: An Assessment of Climate Feedbacks in Coupled Ocean Atmosphere Models. *J. Climate,Vol.19,3354–3360*
13. Armour, K. (2017) Energy budget constraints on climate sensitivity in light of inconstant climate feedbacks, *Nature Climate Change*