Wave Property in Non-inertial Reference Frame

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The velocity of a wave depends on the choice of reference frame. The relative motion between the rest frames of the wave source, the observer, and the wave determines the apparent wavelength and apparent period in each rest frame. The apparent period is different from the original period unless the wave source and the observer occupy the same rest frame. The apparent wavelength is identical to the original wavelength unless the relative motion between the wave source and the wave is non-inertial. The observed wavelength is identical to all observers. A time varying wavelength is an indication that a remote star is in non-inertial motion during star birth. The inertial force corresponding to the non-inertial relative motion between the rest frames can not be identified as any fundamental force. A neutral object in the non-inertial motion is not attracted by electric force. The massless microwave in the non-inertial reference frame is not attracted by gravitational force.

I. INTRODUCTION

The wave property depends on the choice of reference frame. The velocity of wave is zero in the rest frame of the wave. Correspondingly, the frequency of the wave is also zero in the rest frame of the wave. In another reference frame, there will be frequency and velocity for the same wave. The apparent period depends on the relative motion between the new reference frame and the rest frame of the wave. Furthermore, the non-inertial relative motion between the rest frame of the wave source and the rest frame of the wave changes not only the wave period but also the wavelength. The observed wavelength will vary with time.

The effect of non-inertial motion on the wave property are investigated with constant acceleration applied to the observer, the emitter and eventually to the wave. The constant acceleration is chosen over arbitrary acceleration to keep the verification concise.

II. PROOF

Consider one dimensional motion.

F_1 is the rest frame of the initial wave crest.

F_2 is the rest frame of the wave emitter.

F_3 is the rest frame of the observer.

A wave emitter emits periodic wave. The oscillation period is T. The wavelength is λ. The initial distance between the wave emitter and the observer is D. The elapsed time is conserved in all reference frames[1,2]. The oscillation period T is conserved in F_1, F_2, and F_2.

A. Accelerating Observer

F_3 is subject to a constant acceleration relative to F_1.

1. Rest Frame Of Emitter

Let F_2 move at the velocty of -V relative to F_1. Let F_3 be subject to a constant acceleration A with an initial velocity of -V relative to F_1. In F_2, the distance between the observer and the emitter is D initially. Under constant acceleration, the distance is

$$d = D + \frac{A}{2}t^2 \tag{1}$$

The nth wave crest is emitted at $t = nT$, $n \geq 0$. The nth wave crest reaches the observer at $t = t_n$. The distance between the observer and the emitter at t_n is

$$d_n = D + \frac{A}{2}t_n^2 \tag{2}$$

This is also the distance travelled by the nth wave crest from the emitter to the observer.

$$d_n = V(t_n - nT) \tag{3}$$

From equations (2,3),

$$V(t_n - nT) = D + \frac{A}{2}t_n^2 \tag{4}$$

Define K as the time for the observer to accelerate to the speed of wave in F_2.

$$K = \frac{V}{A} \tag{5}$$

From equations (4,5),

$$t_n = K \pm \sqrt{K^2 - 2\frac{D}{A} - 2KnT} \tag{6}$$

The initial wave crest is emitted at n=0.

$$t_0 = K \pm \sqrt{K^2 - 2\frac{D}{A}} \tag{7}$$

If $D = 0$, the initial crest takes no time to reach the observer. $t_0 = 0$. Therefore,

$$t_0 = K - \sqrt{K^2 - 2\frac{D}{A}} \tag{8}$$

From equations (6,8),

$$t_n = K - \sqrt{K^2 - 2\frac{D}{A} - 2KnT} \tag{9}$$

The apparent period to the observer is

$$T_n = t_n - t_{n-1} \tag{10}$$

From equation (9),

$$K^2 - 2\frac{D}{A} - 2KnT \geq 0 \tag{11}$$

The last wave crest to reach the observer is

$$n \leq \frac{K}{2T} - \frac{D}{VT} \tag{12}$$

The apparent wavelength to the observer is the distance between the nth wave crest and the (n-1)th wave crest at $t = t_{n-1}$. From equations (2,3), the wavelength is

$$\lambda_n = d_{n-1} - V(t_{n-1} - nT) = VT \tag{13}$$

The apparent wavelength is conserved in both F_1 and F_2.

The apparent speed of the wave from equations (10,13) is

$$\frac{\lambda_n}{T_n} = \frac{VT}{t_n - t_{n-1}} \tag{14}$$

The apparent wavelength remains constant over time while the apperent period inscreases with time until the wave is too slow to reach the observer.

2. Inertial Wave In Rest Frame of Observer

In F_3, the rest frame of the observer, the emitter accelerates away with a constant acceleration of A. The motion of the wave crest may be inertial in F_3. Assume the speed of the wave crest to be constant.

The nth wave crest is emitted at t=nT. The distance between the emitter and the observer in F_3 is

$$d_n = D + \frac{A}{2}(nT)^2 \tag{15}$$

This is also the distance travelled by the nth wave crest to the observer at $t = t_n$.

$$d_n = V(t_n - nT) \tag{16}$$

From equations (15,16),

$$t_n = nT + \frac{1}{V}D + \frac{A}{2V}(nT)^2 \tag{17}$$

Equation (17) is not covariant to equation (9). The speed of the wave crest can not be constant in F_3.

3. Non-inertial Wave In Rest frame of Observer

In F_3, the wave crest decelerates to reach the observer. The nth wave crest is emitted at t=nT. The distance between the emitter and the observer is

$$d_n = D + \frac{A}{2}(nT)^2 \tag{18}$$

This is also the distance travelled by the nth wave crest to the observer at $t = t_n$.

$$d_n = (V - AnT)(t_n - nT) - \frac{A}{2}(t_n - nT)^2 \tag{19}$$

From equations (18,19),

$$t_n = K \pm \sqrt{K^2 - 2KnT - \frac{2}{A}D} \tag{20}$$

Equation (20) is covariant to equation (6). The speed of the wave crest is indeed not constant in F_3.

From equations (6,20),

$$t_n = K - \sqrt{K^2 - 2KnT - \frac{2}{A}D} \tag{21}$$

The observed wavelength is the distance between the nth wave crest and the (n-1)th wave crest at $t = t_{n-1}$.

From equations (15,16), the wavelength is

$$\lambda_n = d_n - ((V - AnT)(t_{n-1} - nT) - \frac{A}{2}(t_{n-1} - nT)^2) \tag{22}$$

$$= VT \tag{23}$$

The observed wave period is

$$T_n = t_n - t_{n-1} \tag{24}$$

The observed wave velocity is

$$\frac{\lambda_n}{T_n} = \frac{VT}{t_n - t_{n-1}} \tag{25}$$

The observed wavelength in F_3 is identical to the original wavelength in F_1. The observed period increases over time.

4. Temporary Acceleration In Rest Frame Of Observer

Let the acceleration of the observer stop at $t = mT$. The observer moves at the velocity of -V relative to F_1 initially.

For $n > m$, the nth wave crest is emitted at $t = nT$ in F_3. The distance between the emitter and the observer at $t = nT$ is

$$d_n = D + \frac{A}{2}(mT)^2 + AmT(n - m)T \tag{26}$$

This is also the distance travelled by the nth wave crest to the observer at $t = t_n$.

$$d_n = (V - AmT)(t_n - nT) \tag{27}$$

From equations (26,27),

$$t_n = \frac{nTV + D - \frac{A}{2}(mT)^2}{V - AmT} \tag{28}$$

The observed period is

$$T_n = t_n - t_{n-1} = \frac{TV}{V - AmT} \tag{29}$$

The observed wavelength from equations (26,27) is

$$\lambda_n = d_n - (V - AmT)(t_{n-1} - nT) \tag{30}$$

$$= (V - AmT)T_n = TV \tag{31}$$

The observed wave velocity from equations (29,31) is

$$\frac{\lambda_n}{T_n} = V - AmT \tag{32}$$

The observed wavelength is identical to the original wavelength in the rest frame of the wave. The observed period is different from the original period in the rest frame of the emitter.

5. Temporary Acceleration In Rest Frame Of Emitter

Let the acceleration of the observer stop at $t = mT$. The observer moves at the velocity of -V relative to F_1 initially.

In F_2, the rest frame of the emitter, the motion of the observer becomes inertial after mT. The nth wave crest is emitted at nT. For $n > m$, the distance between the observer and the emitter is

$$d_n = D + \frac{A}{2}(mT)^2 + AmT(t_n - mT) \tag{33}$$

This is also the distance travelled by the nth wave crest to the observer at $t = t_n$.

$$d_n = V(t_n - nT) \tag{34}$$

From equations (33,34),

$$t_n = \frac{VnT + D - \frac{A}{2}(mT)^2}{V - AmT} \tag{35}$$

The apparent period is

$$T_n = t_n - t_{n-1} = \frac{TV}{V - AmT} \tag{36}$$

The apparent wavelength from equations (33,34) is

$$\lambda_n = d_{n-1} - V(t_{n-1} - nT) = VT \tag{37}$$

The apparent wave velocity from equations (36,37) is

$$\frac{\lambda_n}{T_n} = V - AmT \tag{38}$$

The apparent wavelength in the rest frame of the emitter is identical to the original wavelength in the rest frame of the wave. The apparent period is different from the original period.

B. Accelerating Emitter

F_1 is the rest frame of the initial wave crest.
F_2 is the rest frame of the wave emitter.
F_3 is the rest frame of the obserer.

Let F_2 be accelerated relative to F_1. The non-inertial motion of F_2 can not be passed on to any object detached from F_2. Every subsequent wave crest has its own rest frame which is different from both F_1 and F_2. The wave property is drastically altered by the non-inertial motion of F_2.

1. Rest Frame Of Observer

Let the emitter be subject to a constant acceleration of A relative to F_1. The initial velocity of the emitter is -V relative to F_1. An observer is moving at the velocity of -V relative to F_1.

In the rest frame of the observer, the nth wave crest is emitted at $t = nT$. The distance to the emitter is

$$d_n = D + \frac{A}{2}(nT)^2 \tag{39}$$

This is also the distance travelled by the nth wave crest to the observer at $t = t_n$.

$$d_n = (V - AnT)(t_n - nT) \tag{40}$$

From equations (39,40),

$$t_n = \frac{nTV + D - \frac{A}{2}(nT)^2}{V - AnT} \tag{41}$$

The observed period is

$$T_n = t_n - t_{n-1} \tag{42}$$

The observed wavelength is

$$\lambda_n = d_n - (V - AnT)(t_{n-1} - nT) \tag{43}$$

$$= (V - AnT)T_n \tag{44}$$

The observed wave speed from equations (42,44) is

$$\frac{\lambda_n}{T_n} = V - AnT \tag{45}$$

The relative motion between the emitter and the wave is non-inertial. As a result, the speed of wave crest is not constant in the rest frame of the emitter.

For a similar example, consider a marble ball resting on the floor of a bus. An observer standing on the street observes the marble staying in the same position in the bus and concludes that the bus moves at a constant speed. If the marble moves to other position, the bus must be accelerating.

2. *Rest Frame Of Emitter*

Let the emitter be subject to a constant acceleration of A relative to F_1. The initial velocity of the emitter is -V relative to F_1. An observer is moving at the velocity of -V relative to F_1. In the rest frame of the emitter, both the observer and the wave crest accelerate away.

The nth wave crest is emitted at $t = nT$. The distance to the observer at $t = t_n$ is

$$d_n = D + \frac{A}{2} t_n^2 \tag{46}$$

This is also the distance travelled by the nth wave crest to the observer at t_n.

$$d_n = V(t_n - nT) + \frac{A}{2}(t_n - nT)^2 \tag{47}$$

From equations (46,47),

$$t_n = \frac{D + VnT - \frac{A}{2}(nT)^2}{V - AnT} \tag{48}$$

The apparent period is

$$T_n = t_n - t_{n-1} \tag{49}$$

The apparent wavelength is

$$\lambda_n = d_{n-1} - (V(t_{n-1} - nT) + \frac{A}{2}(t_{n-1} - nT)^2) \tag{50}$$

$$= (V - AnT)T_n \tag{51}$$

The apparent wave speed from equations (49,51) is

$$\frac{\lambda_n}{T_n} = V - AnT \tag{52}$$

The relative motion between the emitter and the wave is non-inertial. All properties of the wave depend on the reference frame.

For a similar example, consider a marble ball resting on the floor of a bus. An observer sits in the bus. If the ball remains stationary then the observer concludes that the bus is in an inertial motion. If the marbles moves, the bus must be accelerating.

For another example, replace the bus with a spaceship. The movement of the marble ball indicates the non-inertial motion of the spaceship.

3. *Temporary Acceleration In Rest Frame of Observer*

Let the emitter be subject to constant acceleration of A relative to F_1. The acceleration stops at $t = mT$. The initial velocity of the emitter is -V relative to F_1. An observer is moving at the velocity of -V relative to F_1.

In the rest frame of the observer, the nth wave crest is emitted at $t = nT$. The distance to the emitter is

$$d_n = D + \frac{A}{2}(mT)^2 + AmT(nT - mT) \tag{53}$$

This is also the distance travelled by the nth wave crest to the observer at $t = t_n$.

$$d_n = (V - AmT)(t_n - nT) \tag{54}$$

From equations (53,54),

$$t_n = \frac{nTV + D - \frac{A}{2}(mT)^2}{V - AmT} \tag{55}$$

The observed period is

$$T_n = t_n - t_{n-1} = \frac{TV}{V - AmT} \tag{56}$$

The observed wavelength is

$$\lambda_n = d_n - (V - AmT)(t_{n-1} - nT) = VT \tag{57}$$

From equations (56,57), the observed wave velocity is

$$\frac{\lambda_n}{T_n} = V - AmT \tag{58}$$

The wavelength is conserved in F_1, F_2 and F_3. It is independent of the motion of the observer. The wave crest travels at a lower speed due to the deceleration.

4. *Temporary Acceleration In Rest Frame Of Emitter*

Let the emitter be subject to a constant acceleration of A relative to F_1. The acceleration stops at $t = mT$. The initial velocity of the emitter is -V relative to F_1. An observer is moving at the velocity of -V relative to F_1.

In the rest frame of the emitter, both the observer and the wave crest accelerate away until $t = mT$. The nth wave crest is emitted at $t = nT$. The distance to the observer at $t = t_n$ is

$$d_n = D + \frac{A}{2}(mT)^2 + (t_n - mT)mTA \tag{59}$$

This is also the distance travelled by the wave crest to the observer at $t = t_n$.

$$d_n = V(t_n - nT) \tag{60}$$

From equations (59,60),

$$t_n = \frac{nTV + D - \frac{A}{2}(mT)^2}{V - mTA} \tag{61}$$

The apparent period to the observer is

$$T_n = t_n - t_{n-1} = \frac{TV}{V - AmT} \tag{62}$$

The apparent wavelength is

$$\lambda_n = d_{n-1} - V(t_{n-1} - nT) = VT \tag{63}$$

The apparent velocity to the observer is

$$\frac{\lambda_n}{T_n} = V - AmT \tag{64}$$

The wavelength is conserved in F_1, F_2 and F_3. It is independent of the motion of the observer. The wave crest travels at a lower speed as a result of the acceleration.

C. Accelerating Wave

Let both emitter and observer be stationary relative to F_2. The distance between them is D. Let F_2 be subject to the acceleration of A with an initial velocity of -V relative to F_1, the rest frame of the initial wave crest.

In F_2, the nth wave crest is emitted at $t = nT$. The distance to the observer at $t = t_n$ is

$$d_n = D \tag{65}$$

This is also the distance travelled by the nth wave crest to the observer.

$$d_n = V(t_n - nT) - \frac{A}{2}(t_n - nT)^2 \tag{66}$$

From equations (65,66),

$$t_n = nT + K - \sqrt{K^2 - 2\frac{D}{A}} \tag{67}$$

The apparent period to the observer is

$$T_n = t_n - t_{n-1} = T \tag{68}$$

The apparent wavelength is

$$\lambda_n = D - V(t_{n-1} - nT) + \frac{A}{2}(t_{n-1} - nT)^2 \tag{69}$$

$$= \frac{AT^2}{2} + AT\sqrt{K^2 - 2\frac{D}{A}} \tag{70}$$

The apparent velocity to the observer is

$$\frac{\lambda_n}{T_n} = \frac{AT}{2} + A\sqrt{K^2 - 2\frac{D}{A}} \tag{71}$$

The relative motion between the wave and the observer is non-inertial. If the arbitrary A is chosen to match the Coulomb force, the microwave will appear to be subject to the influence of Coulomb force. If A is chosen to match the gravity, the microwave will appear to be subject to the influence of gravitational force.

However, the wave is not charged and can not be subject to electric force. The wave is massless and is not subject to gravity. The inertial force can not be identified with any fundamental force.

D. Apparent Wave Period

The observed period is different from the original wave period if there is relative motion between the emitter and the observer, This is commonly known as the Doppler effect[3].

TABLE I. Apparent Period

	Observer Rest frame	**Emitter Rest frame**
Constant Acceleration of Observer	$t_n - t_{n-1}$	$t_n - t_{n-1}$
Temporary Acceleration of Observer	$\frac{TV}{V-AmT}$	$\frac{TV}{V-AmT}$
Constant Acceleration of Emitter	$t_n - t_{n-1}$	$t_n - t_{n-1}$
Temporary Acceleration of Emitter	$\frac{TV}{V-AmT}$	$\frac{TV}{V-AmT}$
Constant Acceleration of Wave	T	T

Table I shows how the observed period depends on the choice of reference frame and the acceleration. The observed period varies over time for non-inertial motion. The observed period remains constant for inertial motion.

TABLE II. Arrival Time (t_n)

	Observer Rest frame	**Emitter Rest frame**
Constant Acceleration of Observer	$K - Q_n$	$K - Q_n$
Temporary Acceleration of Observer	V_m	V_m
Constant Acceleration of Emitter	V_n	V_n
Temporary Acceleration of Emitter	V_m	V_m
Constant Acceleration of Wave	$nT + K - Q$	$nT + K - Q$

Q_n and Q are defined as

$$Q_n = \sqrt{K^2 - 2KnT - 2\frac{D}{A}} \tag{72}$$

$$Q = \sqrt{K^2 - 2\frac{D}{A}} \tag{73}$$

V_n and V_m are defined as

$$V_n = \frac{VnT + D - \frac{A}{2}(nT)^2}{V - AnT} \tag{74}$$

$$V_m = \frac{VnT + D - \frac{A}{2}(mT)^2}{V - AmT} \tag{75}$$

E. Apparent Wavelength

The observed wavelength is identical to the original wavelength unless the relative motion between the wave and the emitter is non-inertial.

Hence, the observed wavelength is a good indication of the non-inertial motion of the wave source. During star birth, the non-inertial motion of the stars will cause the observed wavelength to change with time.

TABLE III. Apparent Wavelength

	Observer Rest frame	**Emitter Rest frame**
Constant Acceleration of Observer	VT	VT
Temporary Acceleration of Observer	VT	VT
Constant Acceleration of Emitter	$(V - AnT)T_n$	$(V - AnT)T_n$
Temporary Acceleration of Emitter	VT	VT
Constant Acceleration of Wave	$\frac{1}{2}AT^2 + ATQ$	$\frac{1}{2}AT^2 + ATQ$

F. Apparent Wave Velocity

The apparent wave velocity depends on the relative motion between F_1 and either F_2 or F_3. A marble ball resting on the floor of a bus will move if the bus acclerates. A static wave in a reference frame will move if that reference frame is accelerated.

The non-inertial motion of the observer can be detected by the observed wave velocity in the rest frame of the observer. The radial acceleration of a remote star can be determined from the observed radial velocity.

A similar example is a marble ball resting on the floor of a bus. The observer inside the bus can detect the non-inertial motion of the observer from the movement of the ball. The observer outside the bus can detect the acceleration of the bus from the velocity of the marble ball relative to the ground.

TABLE IV. Apparent Wave Velocity

	Observer Rest frame	**Emitter Rest frame**
Constant Acceleration of Observer	$\frac{VT}{t_n - t_{n-1}}$	$\frac{VT}{t_n - t_{n-1}}$
Temporary Acceleration of Observer	$V - AmT$	$V - AmT$
Constant Acceleration of Emitter	$V - AnT$	$V - AnT$
Temporary Acceleration of Emitter	$V - AmT$	$V - AmT$
Constant Acceleration of Wave	$\frac{1}{2}AT + AQ$	$\frac{1}{2}AT + AQ$

III. CONCLUSION

The observed wavelength is identical to the original wavelength unless the relative motion between the wave source and the wave is non-inertial. The wavelength does not depend on the rest frame of the observer. During star birth, the stars in the non-inertial motion can emit radiation whose wavelength will vary with time.

The apparent period of the wave depends on the rest frame of the observer. For an approaching galaxy, the observed period will decrease while the observed wavelength remains identical to the original wavelength. This is known as blueshift in astronomy.

The apparent period is constant over time only if the relative motion between the wave source and the observer is inertial. Consequently, the apparent wave velocity also depends on the choice of reference frame. The blueshift of the spectrum indicates the light from the remote galaxy speeds up. The redshift of the spectrum indicates the light slows down as the result of the receding motion of the remote galaxy.

The inertial force from non-inertial relative motion should not be mistaken as any fundamental force. Two marble balls resting on the floor of a bus will move in the same motion if the bus accelerates. One ball is positively charged. The other ball is neutral. Such inertial force can not be identified as the electric force acting on the neutral ball.

Similarly, a massive marble ball and the massless light inside the bus move in a linear motion in the transverse direction. The non-inertial motion of the bus in the longitudinal direction converts the linear motion of the ball and the light into projectile motion. Such relative motion can not be identified as the electric force acting on the neutral ball nor the gravitational force acting on the massless light wave.

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