SunQM-4s1: Is Born probability merely a special case of (the more generalized) non-Born probability (NBP)?

Yi Cao
A citizen scientist of QM
e-mail: yicaojob@yahoo.com

Abstract

In SunQM-4, to build a full-QM deduced Solar system's 3D probability density map with time-dependent orbital movement, we developed a non-Born probability (NBP) method. In the current paper, we showed that NBP is directly proportional to the wave function (rather than Born probability's conjugated-squared wave function). For a cos(x) wave function, its NBP is simply to lift-up wave function by one to make its min =0, and then divided by two to make its max =1, so its NBP = [1 + cos(x)]/2. The trigonometric formula [1 + cos(x)]/2 = [cos(x/2)}^2 revealed that for a planet doing orbital movement in Solar system, its NBP ground state is n=1, and its Born probability ground state is n = 1/2. We showed that NBP is valid not only for {N,n} QM's nLL QM state, but also for the 1D infinity deep square potential well QM, for the circular 1D QM, and for the plane wave QM. NBP demonstrates more direct and intuitive physical meaning than that of Born probability. In contrast to that Born probability is only for the standing wave function, NBP is for a uni-directional traveling wave function (which naturally includes the linearly combined two opposite-directional traveling waves, or a standing wave). Therefore, Born probability is possible merely a special case of (the more generalized) NBP. With NBP, we may explain the flute sound wave (an air mass density vibration) mechanics directly as the 1D quantum mechanics! This may significantly change our view on QM and its application on our daily-life-world's (Newtonian) physics. The planet formation through accretion is also discussed by using NBP.

Introduction

The SunQM series research articles [1] ~ [16] have demonstrated that the formation of Solar system (and planet-moon system) was governed by its {N,n} QM. In SunQM-4 [17], we showed that the full-QM deduction of Solar system's 3D probability density map with orbital movement needs to use non-Born probability (NBP) rather than the traditional Born probability. Since the NBP concept severely violates the traditional QM's rule, in the current paper, we try to find more evidence to support that NBP is a correct method. Note: for {N,n} QM nomenclature as well as the general notes for {N,n} QM model, please see SunQM-1 section VII. Note: Microsoft Excel's number format is often used in this paper, for example: x^2 = x^2, 3.4E+12 = $3.4*10^{12}$, 5.6E-9 = $5.6*10^{-9}$. Note: The reading sequence for SunQM series papers is: SunQM-1, 1s1, 1s2, 1s3, 2, 3, 3s1, 3s2, 3s6, 3s7, 3s8, 3s3, 3s9, 3s4, 3s10, 3s11, 4, and 4s1. Note: for all SunQM series papers, reader should check "SunQM-4s7: Updates and Q/A for SunQM series papers" for the most recent updates and corrections. Note: Because the topic of this paper is too controversial and I could be very wrong, "a citizen scientist of QM" is added under author's name.

I. Applying the non-Born probability (NBP) calculation to the 1D infinity deep square potential well QM (1D∞QM)

In SunQM-4, we were forced to define a non-Born probability (abbreviated as "NBP") as: 1) it is for a uni-directional traveling matter wave (in contrast to Born probability's standing matter wave), 2) its probability directly proportional to the wave function, or |ψ|^2 ∝ ψ (in contrast to Born probability's conjugated-squared |ψ|^2), and 3) it is calculated as to lift-up the wave function to make min =0 (rather than negative), and then times ½ to make the max =1 (see

SunQM-4's eq-52). If NBP is correct, then it must be able to explain all QM's probability, including 1D infinity deep square potential well QM (here we abbreviated it as "1D∞QM")'s probability. So in this section, we will explain how to use NBP for 1D∞QM.

I-a. 1D infinity deep square potential well can be used to represent a circular 1D orbit.

First let's review (from eq-1 through eq-4, and Figure 1) the known QM result of a particle in a 1D∞QM. As shown in Giancoli's text book (page 1031, eq-38-13, and eq-38-14), its QM state energy is

$$E_n = n^2 \frac{h^2}{8m\prime l\prime_n{}^2} \qquad \text{eq-1}$$

where *m*' is the particle's mass, and *l*' is the potential well's width. Or

$$E_n = n^2 E_1 \qquad \text{eq-2}$$

where E_1 is the ground state's state energy

$$E_1 = \frac{h^2}{8m\prime l\prime_n{}^2} \qquad \text{eq-3}$$

and its wave function is

$$\psi_n = \sqrt{\frac{2}{l\prime}} \sin\left(\frac{n\pi}{l\prime} x\right) \qquad \text{eq-4}$$

Figure 1 (a, b, c, from left to right) shows the plotted result of a particle in a 1D∞QM as QM state energy, wave function and probability density.

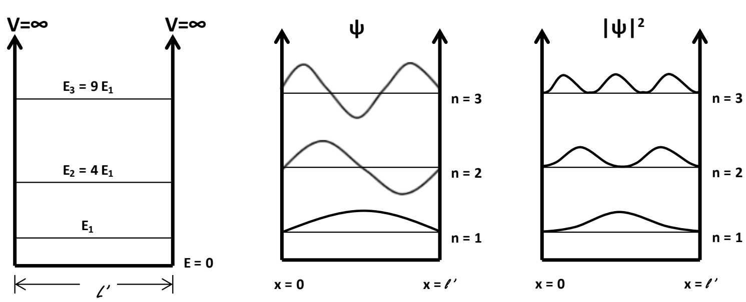


Figure 1 (a, b, c, from left to right). The QM result of a particle in an infinitely deep square well potential. 1a, QM state energy; 1b, wave function; 1c, probability density.

In 2015, after many tries, I realized that the circular 1D orbital standing waves (of de Broglie's matter wave) in Bohr's model can be perfectly represented by the standing waves in a 1D∞QM shown in Figure 1. Or, in other words, the QM of Bohr model and the QM of an infinitely deep square well potential are equivalent! Figure 2 shows the detailed explanation. First, we need to introduce two quantum numbers here: the quantum number n is used only for the r-dimension, and the quantum number j is used only for the φ-dimension. For Bohr model, n ≡ j, and each n can have only one j (see Figure 2a). However, for a 2j (j=1,2,3…) divisible planet model, n can be not equal to j, and each n can have countless j(s) (see Figure 2b, and also see sections after section I-a).

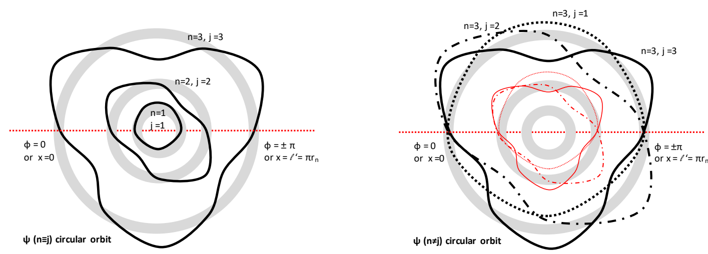


Figure 2a (left). Demonstration of circular 1D matter waves for Bohr model (n ≡ j, each n can have only one j).
Figure 2b (right). Demonstration of circular 1D matter waves for a 2j divisible planet model, n ≠ j, and each n can have countless j(s), only showed n=3 with j=1, 2, 3, and n=2 with j=1, 2, 3.

In Figure 3a, a circular 1D de Broglie matter wave in Bohr model with φ dimension quantum number j (=n) = 3 is displayed. Let us define that at point A (φ=0), the wave amplitude =0. Notice that for a de Broglie matter wave in Bohr model with any n number (n =j =1, 2, 3, …), the point B (φ=π) always has the wave amplitude =0. So both points A (φ=0) and B (φ=π) are always the wave nodes for any n (even or odd) in Bohr model. Imagine that we can pull point B away from point A, and make the AB circle line become an AB straight line, then a circular orbit traveling wave become a straight-line traveling wave which bounces back at points A and B. This is exactly the standing wave shown in Figure 3b (where j = 1, 2, and 3 modes are presented). From this, we can easily calculate the width of the (re-modeled) infinity deep potential well, *l'* = π * r_n. We can see that Figure 3b is a typical 1D infinity potential well's QM. The major difference between Figure 3b and Figure 1b is, in Figure 3b, the bouncing back standing wave opposites its amplitude at the wall (see the red arrows), while in Figure 1b, it is not clear the bouncing back standing wave opposites its amplitude at the wall or not.

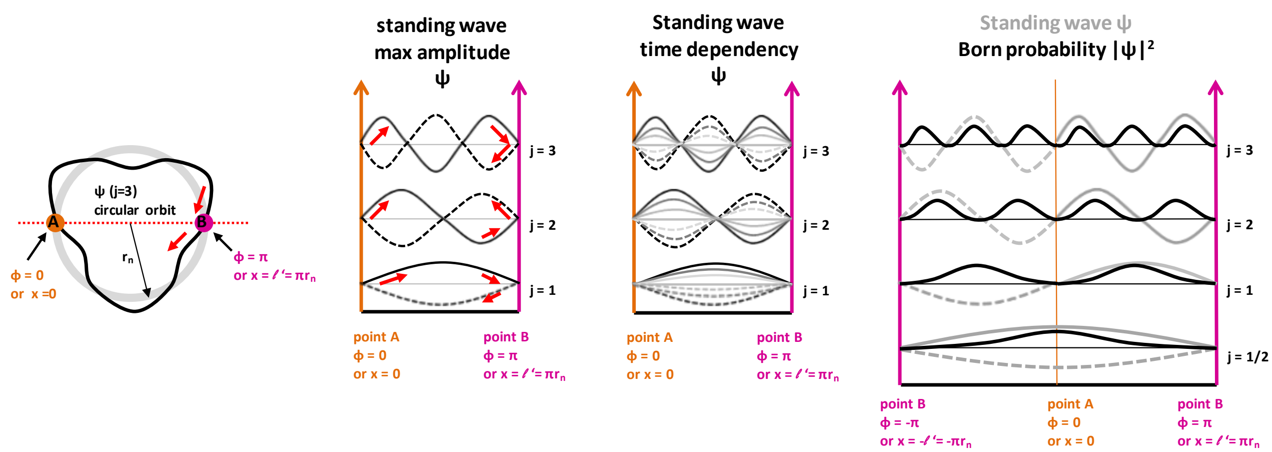


Figure 3 (a, b, c, d, from left to right). Bohr model's circular 1D orbit wave function ψ(n=3) (Figure 3a, left) is remodeled in a 1D∞QM (Figure 3b, middle-left, n=3, j=1, 2, 3). The real time changing of standing waves are shown in Figure 3c. The corresponding probability function |ψ|^2 is shown in Figure 3d (right) with 2j of peaks. The red arrows are the traveling directions of waves. The quantum number n is for r-dimension and j is for φ-dimension. For Bohr model, j ≡ n. But for a generalized model, j is independent of n.

To prove that Bohr model's circular orbit wave (Figure 3a) can be remodeled in a 1D∞QM (Figure 3b), let us re-calculate Bohr model's hydrogen atom energy transition by using the remodeled 1D∞QM. After applying the known conditions $r_n = a_0 * n^2$, and $l'_n = \pi * r_n = \pi * a_0 * n^2$, where a_0 is the Bohr radius, the QM state energy of 1D∞QM (eq-1) becomes

$$E_n = n^2 \frac{h^2}{8m\prime {l\prime_n}^2} = n^2 \frac{h^2}{8m\prime(\pi a_0 n^2)^2} = \frac{h^2}{8m\prime(\pi a_0 n)^2} \qquad \text{eq-5}$$

This E_n is a positive value and has $E_n \to \infty$ at $n \to 0$. In Bohr model's hydrogen, the electron's E_n (see the original Bohr model formula in Giancoli's book, eq-37-14a, and copied here as eq-6)

$$E_n = -\left(\frac{Z^2 e^4 m\prime}{8\varepsilon_0^2 h^2}\right)\left(\frac{1}{n^2}\right) \qquad \text{eq-6}$$

has a negative value and $E_n = 0$ at $n = \infty$. This is because 1D∞QM's energy eq-1 correlates to the {N,n} QM's φ-1D-diemensional QM (see SunQM-4's eq-20), while Bohr QM's energy correlates to the 3D-dimensional QM (see SunQM-4's eq-21). So in eq-5 we need to replace the E_n by $0 - E_n$, or the corrected E_n is

$$E_n = -\frac{h^2}{8m\prime(\pi a_0 n)^2} \qquad \text{eq-7}$$

In Table 1, we used 1D∞QM's eq-7 to re-calculate Bohr mode's orbit energy for n= 1, 2, and 3 (shown in column 3). When comparing to the classical calculation eq-8 (combined with $r_n = a_0 * n^2$, shown in column 4),

$$E_n = -\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Ze^2}{r_n} \qquad \text{eq-8}$$

or the calculation from the original Bohr model formula eq-6 (shown in column 5), they all showed the same result. Of cause, the calculated photon emission wave length is also the same between the 1D∞QM re-modeled result and the experimental result (see Table 1's bottom part). This clearly proved that Bohr model's 1D circular orbit wave (Figure 3a) can be remodeled in a 1D∞QM (Figure 3b).

Table 1. Demonstration of Bohr model result can be calculated by using a 1D∞QM.

	n=	E_n = - h^2 / [8m(π a_0 n)^2]	E_n = -1/2 * (1/4πε_0) *Ze^2/r_n	En = - [Z^2 * e^4 * m /(8 * ε0^2 *h^2)] / n^2
$E_{n=1}$	1	-2.18E-18	-2.18E-18	2.18E-18
$E_{n=2}$	2	-5.45E-19	-5.45E-19	5.45E-19
$E_{n=3}$	3	-2.42E-19	-2.42E-19	2.42E-19
$\lambda_{2\to1}$=c/f =hc / (E_2-E_1), (nm)		121.50		
observed H-atom $\lambda_{2\to1}$= (nm)		121.57		
$\lambda_{3\to1}$=c/f =hc / (E_3-E_1), (nm)		102.52		
observed H-atom $\lambda_{3\to1}$= (nm)		102.57		
$\lambda_{3\to2}$=c/f =hc / (E_3-E_2), (nm)		656.10		
observed H-atom $\lambda_{3\to2}$= (nm)		656.3		

Note: column 5 lost the negative sign due to Microsoft Excel (v2007 & v2019)'s software bug.

Furthermore, by using $2\pi * a_0 = \lambda_{n=1}$, and $h / \lambda_n = p_n$, and $p_n = m' * v_n$, we are able to deduce the particle version of Bohr's model from a 1D∞QM state energy formula eq-7:

$$E_n = -\frac{h^2}{8m\prime(\pi a_0 n)^2} = -\left(\frac{1}{2m'}\right)\left(\frac{h}{2\pi a_0}\right)^2\left(\frac{1}{n^2}\right) = -\left(\frac{1}{2m'}\right)(p_{n=1})^2\left(\frac{1}{n^2}\right) = -\left(\frac{1}{2}\right)m' v_1^2\left(\frac{1}{n^2}\right) = E_1\left(\frac{1}{n^2}\right) \qquad \text{eq-9}$$

where $E_1 = -(1/2) * m' * v_1^2$.

Note: Although I made above deduces independently in 2015 ~ 2016, I will not be surprised that if some other scientists had already published the similar result many years ago (because this is a 100 years old subject). If so, readers please inform me so that I can cite those publications (probably in SunQM-4s7). Sorry I am only a citizen scientist of QM, don't know much about the history that beyond what the general QM text books mentioned. Note: on 4/9/2020, I accidently found wiki "Particle in a ring", and it shows that circular 1D QM has been solved by other scientists (although no citation/source has been listed). The method used is very different than that of mine, so I decided to present my result as is.

Now let's discuss a little bit more on the probability curves in Figure 1c and Figure 3d. At j=3, Figure 1c shows there are 3 peaks in the potential well from x = 0 to x = +*l'*. In the QM theory of a particle in a 1D infinity deep potential well, this means that the probability of finding this particle in the well is spread more in the 0 to +*l'* range, rather than at the center position (which is *l'*/2). It does not mean that this single particle is divided into three pieces. Similarly, Figure 3d's j=3 shows that there are 2j=6 peaks in the φ-dimension from 0 to 2π (or from -π to +π). It also means that the probability of finding this particle in the circular is spread more evenly in the whole circle, rather than that the particle itself is divided into 6 pieces. However, if we are not bound with the traditional QM's explanation, Figure 3d's j=3 mode does can be explained as there are j = 3 (but not 2j =6) objects in the 1D circular orbit (see section II for details).

I-b. Using non-Born probability (NBP) to explain the 1D infinity deep potential well QM (1D∞QM)

Figure 3b showed that a standing wave is made of two equal but opposite traveling waves. The rightward (+x) traveling wave can be written as

$$\psi_{\mathrm{rightward}} \propto \sin\left[\frac{2\pi}{\lambda}(x - vt)\right] = \sin(kx - \omega t) = \sin(jx - \omega t) \qquad \text{eq-10}$$

where wave number k = 2*π/λ, angular frequency/velocity ω = 2*π*f, and v = λ*f (see Douglas C. Giancoli, Physics for Scientists & Engineers with Modern Physics, 4th ed. 2009, eq-15-10b, and eq-15-11). The last step in eq-10 is because in 1D infinity deep potential well, k = j (see Figure 3). The leftward (-x) traveling wave can be written as

$$\psi_{\mathrm{leftward}} \propto \sin(jx + \omega t) \qquad \text{eq-11}$$

So that the sum of eq-10 and eq-11 gives the time-dependent standing wave changes (as sown in Figure 3c):

$$\psi_{\mathrm{StandingWave}} = \psi_{\mathrm{rightward}} + \psi_{\mathrm{leftward}} \propto \sin(jx - \omega t) + \sin(jx + \omega t) = 2\sin(jx)\cos(\omega t) \qquad \text{eq-12}$$

The last step used the trigonometric sum-to-product identity

$$\sin(a) + \sin(b) = 2\sin\left[\frac{(a+b)}{2}\right]\cos\left[\frac{(a-b)}{2}\right] \qquad \text{eq-13}$$

Eq-12 can be treated as a spatial (of x) wave sin(j*x) with its amplitude is modulated by a time factor cos(ω * t). An illustrative plot of eq-3 is shown in Figure 3c (with relative amplitude value).

Born probability explanation: For a 1D∞QM's ψ = sin(j*x) matter wave function (see Figure 1b), its Born probability density function is |ψ|^2 = [sin(j*x)]^2 (see Figure 1c). Using the standard trigonometric formula

$$[\cos(\varphi/2)]^2 = \frac{1+\cos(\varphi)}{2} \qquad \text{eq-14}$$

or

$$[\sin(\varphi/2)]^2 = \frac{1-\cos(\varphi)}{2} \qquad \text{eq-15}$$

We then have

$$|\psi|^2 \propto [\sin(jx)]^2 = \frac{[1-\cos(2jx)]}{2} \qquad \text{eq-16}$$

non-Born probability (NBP) explanation: In NBP, the probability directly proportional to the wave function, or |ψ|^2 ∝ ψ = sin(j*x - ω * t). A normalized probability value has to be between 0 and 1, with minimum value = 0 and maximum value = 1. Therefore we need to lift up sin(j*x - ω * t) curve to 1 + sin(j*x - ω * t) to make its minimum =0, and then divide it by two to make its max =1. So it become

$$|\psi_{\mathrm{rightward}}|^2_{\mathrm{NBP}} = \frac{[1+\sin(jx-\omega t)]}{2} \qquad \text{eq-17}$$

and

$$|\psi_{\mathrm{leftward}}|^2_{\mathrm{NBP}} = \frac{[1+\sin(jx+\omega t)]}{2} \qquad \text{eq-18}$$

The total NBP for the two opposite traveling waves (equals to a standing wave) is the sum of eq-17 and eq-18,

$$|\psi|^2_{\mathrm{NBP}} \propto \frac{[1+\sin(jx-\omega t)]}{2} + \frac{[1+\sin(jx+\omega t)]}{2} = 1 + \sin(jx)\cos(\omega t) \qquad \text{eq-19}$$

Again, sin(j*x) *cos(ω * t) in Eq-19 can be treated as a spatial wave sin(j*x) with its amplitude is modulated by a time factor cos(ω*t). When integrating F(x,t) = sin(j*x) *cos(ω * t) with ∫ F(x,t) d(j*x) for the whole x range from 0 to 2π, it equals to zero, meaning at any time t, the averaged whole x range (from 0 to 2π)'s probability $|\psi(x)_{\text{x-averaged-NBP}}|^2 = 1$. When integrating F(x,t) = sin(j*x) *cos(ω * t) with ∫ F(x,t) d(ω * t) for the whole ω * t range from 0 to 2π, it also equals to zero, meaning at any position x, the averaged whole time period (ω * t from 0 to 2π)'s probability $|\psi(t)_{\text{t-averaged-NBP}}|^2 = 1$. An illustrative plot of eq-12 and eq-19 is shown in Figure 4.

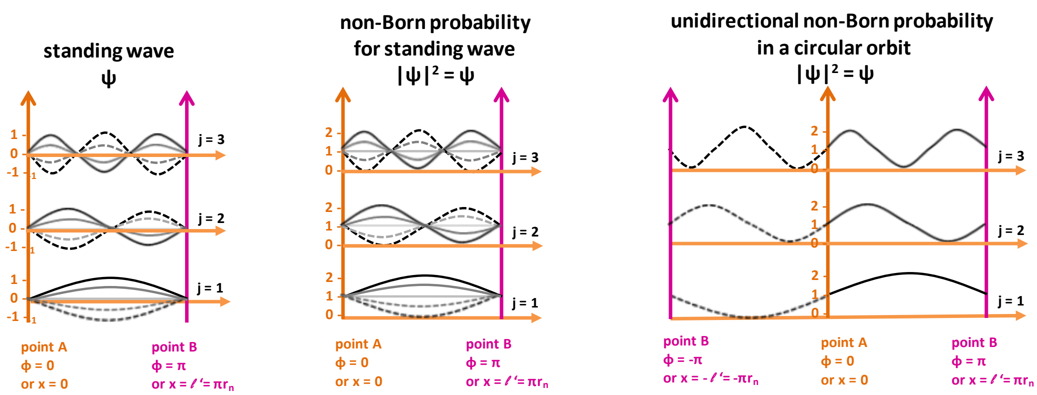


Figure 4a (left). Illustration of standing waves in 1D∞QM (with j=1, 2, 3) described by eq-12.
Figure 4b (middle). Illustration of NBP for the standing waves in 1D∞QM (with j=1, 2, 3) described by eq-19.
Figure 4c (right). Illustration of unidirectional NBP in a circular orbit (with j=1, 2, 3).

Discussion of 1D∞QM's or circular 1D QM's NBP in section I-b:

1) NBP can be used to explain a circular orbit based 1D∞QM. Notice that for j = odd number in Figure 4b, to make the averaged NBP=1, we need to count-in the backward NBP from φ=π to φ=2π besides from φ=0 to φ=π.

2) For 1D∞QM, the NBP function form (eq-19) is significantly different than the Born probability function form (eq-16): NBP curve's peak is fatter that Born probability curve's peak. The advantage of NBP is obvious: it can calculate a circular orbit's unidirectional traveling wave's probability density (see Figure 4c) while the Born probability can only be used for a standing wave (or a combined two opposite-directional traveling waves).

3) Comparing Figure 4c to Figure 3d, we see that for a planet that is doing circular orbital movement, because it must have a single mass (or probability) peak in a circular orbit, when using NBP, we need to use n=j=1 as the ground state. This is because at n=1, NBP equals to the lifted sin(x) wave and it has only one probability (or mass) peak. However, when using Born probability, we need to use n=1/2 as the ground state. This is because at n=1, Born probability [sin(x)]^2 produces two peaks in a circle, while only n=1/2 's Born probability [sin(x/2)]^2 will produce a single probability peak in a circle. The trigonometric relationship in eq-14 and eq-15 exactly reflect this relationship between NBP and Born probability! Now we can explain what is the meaning of eq-15 (which it is used in eq-16). Here we use eq-14 instead of eq-15 for the explanation (because it is more intuitive). [cos(x/2)]^2 in the left side of eq-14 means a wave function cos(x) 's Born probability (so it is |ψ|^2) calculated at the ground state (so it is n=1/2, or equivalent to cos(x/2)), and the right side is the NBP that equals to the wave function cos(x) itself (meaning n=1), although it is transformed as lift up the wave function cos(x) (by adding 1) to make the min =0, and then scale down the amplitude (by dividing 2) to make the max =1, so it ends as (1 + cos(x))/2. So the physical meaning of eq-14 is: it switches a ground state (n=1/2) Born probability into a ground state (n=1) NBP. The same explanation is applicable to eq-15 (although a little bit more complicated, see eq-43's explanation). This transformation is applicable not only to cos(x) or sin(x) wave, but also to cos(j*x) and sin(j*x) waves (like we did in eq-16, and also see section II). Notice that eq-14 relationship has also been used in SunQM-4. This is also the exact physical meaning of $\Phi(\varphi)_{equivalent}$ and $|\Phi(\varphi)_{equivalent}|$^2 mentioned in SunQM-4's eq-12 and eq-13.

4) Notice that in 1D∞QM's j ≥1 Born probability, we used to use sine wave because it makes the probability = 0 at the wall of the well. But for j=1/2, or for NBP, we better to use cosine wave because it is easier to interpret.

5) Repeat the key explanation of NBP calculation: use a cosine wave function cos(x) to replace sine wave function in 1D∞QM (because it is equivalent to a circular 1D QM in which sine wave and cosine wave are equivalent), lift-up wave by adding one to make min = 0, and dividing by two to normalized max = 1. So cos(x) 's NBP = [1+cos(x)]/2. Then using eq-14, cos(x) 's NBP = [1+cos(x)]/2 = [cos(x/2)]^2, which means a cos(x) wave's NBP (at n=1) equals to a cos(x) wave's Born probability at n=1/2.

6) The comparison of Born probabilities at the ground state n = 1/2 between the 1D∞QM and the harmonic oscillator potential well QM will be given in the future study.

II. Multiplier n 's matter wave cos(n*x), linear combination, and non-Born Prob

In this section we study how to apply NBP to cos(n*x) kind wave with either multi peaks or a single (wave packet) peak. Note: quantum number j is used only in 1D∞QM, quantum number n (and multiplier n', sometimes also written as n) is used in {N,n} QM, quantum number m is used in QM for φ-dimension, and in nLL QM state, m=n-1. However, in this section, for discussing the NBP of a cosine wave function, we treat cos(n*x), cos(m*x), or cos(j*x) the same and exchangeable. We also treat circular 1D QM and 1D∞QM exchangeable.

II-a. Non-Born Probability explanation for cos(j*x) type multi-peak matter wave in circular 1D QM

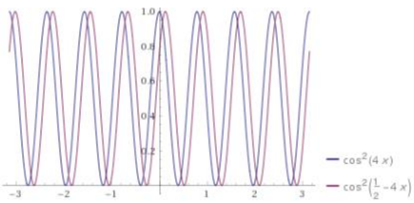
A Solar {N,n//q} QM structure's nLL orbit can also be treated as a circular 1D orbit in 1D∞QM. An eastward traveling wave (here we define that eastward = rightward = positive direction, and westward = leftward = negative direction) can be described by eq-10 in 1D∞QM. For easy explanation, let's use cosine wave instead of sine wave, so the circular 1D QM description of this wave is

$$\psi_{\mathrm{eastward}} \propto \cos(jx - \omega t) \qquad \text{eq-20}$$

And the circular 1D QM description of this wave's NBP is (lifting-up the wave function by one, and then dividing by two)

$$|\psi_{\mathrm{eastward}}|^2_{\mathrm{NBP}} = \frac{[1+\cos(jx-\omega t)]}{2} \qquad \text{eq-21}$$

Here we use $|\psi|^\wedge 2_{\mathrm{NBP}}$ to distinguish NBP from the traditional Born probability $|\psi|^\wedge 2$. Plot eq-21 with j=8 and with either ω * t = 0, or ω * t = 1 (in Figure 5a) shows that there are eight NBP peaks moving rightward. We can intuitively depict Figure 8a as that there are eight equal-size, equal-mass, and equal distanced objects doing circular orbit movement as shown in Figure 5b.



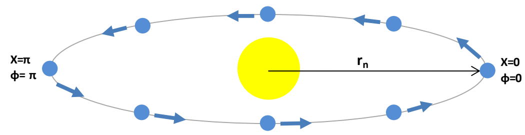


Figure 5a. Plot of (1 + cos(j *x - ω * t))/2, with j = 8 and with either ω * t = 0, or ω * t = 1 shows that there are eight NBP peaks moving rightward.
Figure 5b. The equivalent eight equal-size, equal-mass, and equal distanced objects doing eastward circular orbit movement.

The same wave and its Born probability in {N,n} QM's nLL orbit description has been given in SunQM-3s11's section III-c. However, the NBP in {N,n} QM's nLL orbit description was only briefly given in SunQM-4's section I-a. Now let's explain it in details. In{N,n} QM, any object in nLL QM state (doing circular orbital movement) can be described by Schrodinger equation's solution with the φ-dimensional wave (Eigen) function as either exp(imφ) or exp(-imφ). Then (same as shown in SunQM-3s11' eq-31), we can linearly combine these two wave functions to be

$$Y(l, \pm m) = \begin{cases} \dfrac{[Y(l,m) + Y(l,-m)]}{2} \propto \dfrac{[e^{im\varphi}(\sin\theta)^m + e^{-im\varphi}(\sin\theta)^m]}{2} = \cos(m\varphi)(\sin\theta)^m \ldots\ldots\ldots \text{when } m = \text{even} \\ \dfrac{[-Y(l,m) + Y(l,-m)]}{2} \propto \dfrac{[e^{im\varphi}(\sin\theta)^m + e^{-im\varphi}(\sin\theta)^m]}{2} = \cos(m\varphi)(\sin\theta)^m \ldots\ldots\ldots \text{when } m = \text{odd} \end{cases}$$

eq-22

Or simply,

$$Y(l, \pm m) = \cos(m\varphi)\,[\sin(\theta)]^m \qquad \text{eq-23}$$

where Y(*l*,m) is the spherical harmonic function, *l* = m = n-1, and n can either base n or multiplier n'. Because a linear combination of a partial differential equation (PDE)'s solution is still a valid solution of this PDE, eq-23 is still a solution of Schrodinger equation (although it represents a φ-dimensional standing wave mode). Then the corresponding φ-dimensional wave function is

$$\Phi(\varphi) \propto \cos(m\varphi) \qquad \text{eq-24}$$

and the corresponding NBP (which is wave function plus one divide two) is

$$|\Phi(\varphi)|^2_{NBP} = \frac{[1+\cos(m\varphi)]}{2} \qquad \text{eq-25}$$

Plot eq-25 with m=8 gives similar (but in standing still) curve as in Figure 5a. So now both circular 1D QM (or 1D∞QM) description (eq-20 and eq-21) and {N,n} QM nLL orbit description (eq-24 and eq-25) have the similar cosine wave function and NBP, and both descriptions are good for the situation in Figure 5b (although one is in standing mode, and one is in traveling mode).

II-b. Non-Born Probability explanation for a single-peak in circular 1D QM using a "citizen-scientist level" method

Then if all eight objects in Figure 5b are accreted to be one object at position φ=0 (or x=0), what is the corresponding wave function or NBP? In SunQM-4's eq-10 we used a "citizen-scientist level method". Now let's re-write it as

$$|\Phi(\varphi)|^2_{NBP} = \Phi(\varphi) \propto \cos(m\varphi) \to e^{im\varphi} = \left(e^{im\varphi/2}\right)^2 = \left(e^{i\varphi/2}\right)^{2m} = \{[\cos(\varphi/2) + i\ \sin(\varphi/2)]^2\}^m \to \{[\cos(\varphi/2)]^2\}^m = \left[\frac{1+\cos(\varphi)}{2}\right]^m = \left[\frac{1+\cos(\varphi)}{2}\right]^{(n-1)} \qquad \text{eq-26}$$

where m = n-1. With the knowledge we just learned, now we can explain the physical meaning of all steps in eq-26. exp(imφ) = [exp(imφ/2)]^2 means to switch from NBP (where ground state is n=1, and $|\psi|^2_{NBP}$ = ψ) to Born probability (where ground state is n=1/2, and the true |ψ|^2). exp(imφ) = [exp(iφ)]^m means to switch from m number of probability peaks (or m objects) to a single probability peak (or one accreted object). The switch of cos(mφ) to exp(imφ) is to make the above power index math operation possible, and the back switch is to make NBP a real value. The second last step is to switch NBP from a wave function form to a forever positive from (by lifting-up the wave function using eq-14). Note: although eq-26 can be written as {[1+cos(φ)]/2}^m =[cos(φ/2)]^(2m), but we don't want to do it because we want to present NBP as a function of its wave function cos(φ).

One way to show how m numbered probability peaks decreased into a single probability peak in eq-26 is to add the intermediate steps in that equation (using the citizen scientist level method):

$$|\Phi(\varphi)|^2_{NBP} \propto \cos(m\varphi) \to e^{im\varphi} = \left(e^{ib\varphi/2}\right)^{2m/b} \to \{[\cos(b\varphi/2)]^2\}^{m/b} = \left[\frac{1+\cos(b\varphi)}{2}\right]^{m/b} \qquad \text{eq-27a}$$

where m is {N,n} QM quantum number in φ-dimension, and b can be any integer between 1 and m that makes m/b equals to an integer. For example, let's set n=17, then m=n-1=16, so b can be 1, 2, 4, 8, 16 (to make m/b equals to an integer). If we choose b=1, 4, and 16, then the corresponding m/b=16, 4, and 1, or NBP function = {[1 + cos(φ)]/2}^16, {[1 + cos(4*φ)]/2}^4, and [1 + cos(16*φ)]/2. In Figure 6 we plot these three functions with each function's integration normalized to that of the single peak's {[1 + cos(φ)]/2}^16 integration. It clearly shows how 16 objects (or probability peaks) decreases step by step into one object (or peak) with the fixed amount of total mass.

Notice that eq-27a can also be deduced as eq-27b:

$$|\Phi(\varphi)|^2_{NBP} \propto \cos(m\varphi) \to e^{im\varphi} = \left(e^{ib\varphi}\right)^{m/b} \to [\cos(b\varphi)]^{m/b} \to \left[\frac{1+\cos(b\varphi)}{2}\right]^{m/b} \qquad \text{eq-27b}$$

The major difference is: in eq-27b, we directly use NBP's method cos(x) → [1+cos(x)]/2, while in eq-27a, we first calculate Born probability at n=1/2, then using eq-14 to switch Born probability into NBP. So eq-27a and eq-27b demonstrated that these two deductions end to the same NBP result.

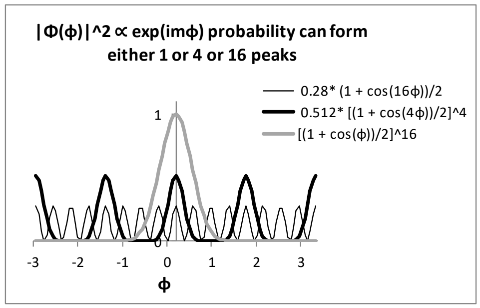


Figure 6. Plot of eq-27 to show how 16 objects (or NBP peaks) decreased step by step into one object (or NBP peak) with the fixed amount of total mass.

II-c. Non-Born Probability explanation for a single-object in circular 1D QM using a linear combination of NBP

In SunQM-3s11 section III-c from eq-35 to eq-45, we have deduced a linear combination of cos[(m+ δ)φ)] wave for the φ-dimensional wave function and then used it for Born probability calculation. Here we use the same method for NBP calculation (note: it had been mentioned briefly in SunQM-4 from eq-12 through eq-16 without much explanation). First, we construct a (normalized) linear combination of a group of cos[(m+ δ)φ)] with the integer number m (= n-1) that is deviated by a small integer δ (with |δ| << m):

$$\Phi(\varphi) \propto \frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos[(\mathrm{m}+\delta)\varphi]$$

eq-28

where 1/(1+2* δ) is the normalization factor. When we plot eq-28 at m = 1024 and δ = 36 (shown in Figure 7a), it shows that a wave packet is formed beyond many single cos[(m+ δ)φ)] waves, and the envelop of this wave packet (almost) perfectly fits to [cos(φ)]^m curve's one peak at φ=0, or we can write it as

$$\Phi(\varphi) \propto \frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos[(\mathrm{m}+\delta)\varphi] \rightarrow [\cos(\varphi)]^{m}$$

eq-29

There are two ways to calculate NBP: a) for eq-28, we should calculate NBP as lift-up the wave of eq-28 to make min=0 by subtracting its baseline -[cos(φ)]^m (which equivalent to add [cos(φ)]^m), and then dividing by 2 to make max =1:

$$|\Phi(\varphi)|^2_{\mathrm{NBP}} = \left\{\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos[(\mathrm{m}+\delta)\varphi] + [\cos(\varphi)]^{m}\right\}\Big/2$$

eq-30

b) for eq-29, we should calculate NBP as lift-up the wave of eq-29 to make min=0 by adding one to cos(φ) and then dividing by 2 to make max =1:

$$|\Phi(\varphi)|^2_{\mathrm{NBP}} = \Phi(\varphi) \propto \frac{1}{1+2\delta}\sum_{-\delta}^{+\delta}\cos[(\mathrm{m}+\delta)\varphi] \to [\cos(\varphi)]^m = \left[\frac{1+\cos(\varphi)}{2}\right]^m \quad \text{eq-31}$$

So eq-31 (in which NBP is calculated by using a more sophisticated math with the linear combination of wave functions) has the same form as eq-26 (where NBP is calculated by using a citizen scientist level method). Comparing the plot of eq-30 and eq-31 (see Figure 7b), we see that eq-31 kind of NBP curve's peak is always a little bit fatter than that of eq-30 (eq-30 has the same peak width as that of wave packet). This is caused by the wave's amplitude is compressed to ½ in NBP.

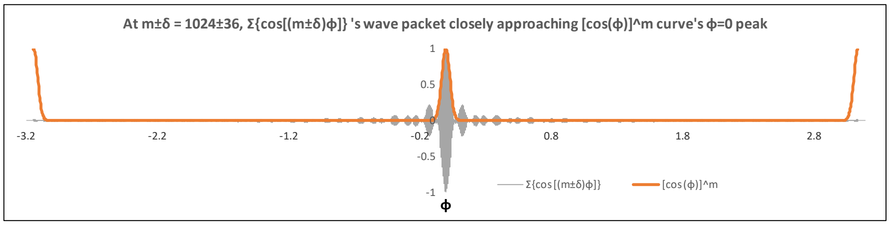


Figure 7a. Plot eq-28 at m = 1024 and δ = 36 shows that a wave packet, and its envelop (almost) perfectly fits to [cos(φ)]^m curve's one peak at φ=0.

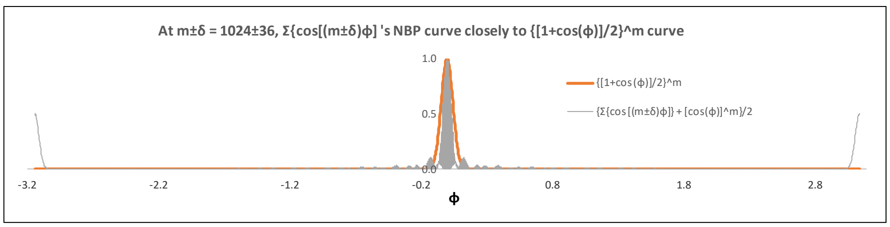


Figure 7b. Comparing the NBP density curves of eq-31 and eq-30 (ignoring the fake peaks at φ = ±π because those come from the [cos(φ)]^m).

III. An alternative (and relative independent) way to prove NBP of |Φ(φ)|^2 * |T(t)|^2 ∝ exp(imφ) * [exp(-i*$\omega_{n,ph}$*t)]^2 in SunQM-4's eq-46 (or eq-47) is correct.

What we want to do in this section is to use an alternative (and relative independent) deduction to support the non-Born probability calculation method in SunQM-4's eq-47 (or eq-46). According to one explanation in wiki "Uncertainty principle" section "Wave mechanics interpretation", and combined with SunQM-4 section-I 's result, a 1D wave packet can be created by using a collection of different n's plane waves:

$$\Psi(\mathrm{x},\mathrm{t}) = \sum_{\mathrm{n}} \mathrm{A_n} e^{i(\mathrm{p_n x}/\hbar - \omega_{\mathrm{n,ph}}\mathrm{t})} \quad \text{eq-32}$$

Because eq-32 is the wave function, not the probability function, so that $\omega_{n,ph}$ has to be the wave packet's phase angular frequency/velocity, not the group angular frequency/velocity. For the φ-dimensional only circular 1D wave packet, x should be replaced by r_n * φ. From Bohr model we have $r_n = r_1$ * n^2, $v_n = v_1$ /n, and p_n * r_n = n * ħ. So in eq-32, the p_n * x/ ħ = p_n * r_n *φ / ħ = n * ħ*φ / ħ = n * φ, then we have

$$\Psi(\varphi,\mathrm{t}) = \Psi(\mathrm{x} = \mathrm{r_n}\varphi,\mathrm{t}) = \sum_{\mathrm{n}} \mathrm{A_n} e^{i\left(\frac{\mathrm{p_n x}}{\hbar} - \omega_{\mathrm{n,ph}}\mathrm{t}\right)} = \sum_{\mathrm{n}} \mathrm{A_n} e^{i(\mathrm{n}\varphi - \omega_{\mathrm{n,ph}}\mathrm{t})} \quad \text{eq-33}$$

From SunQM-4's eq-40 we have $\omega_{n,ph} = \omega_n$ * n/2, where $\omega_{n,ph}$ is the wave packet's phase angular frequency/velocity, and ω_n is the wave packet's group angular frequency/velocity. Then, put into eq-33, we have

$$\Psi(\varphi, t) = \sum_n A_n e^{i(n\varphi - n\omega_n t/2)} = \sum_n A_n e^{in\varphi} e^{-in\omega_n t/2} \qquad \text{eq-34}$$

This is the circular 1D's plane wave function. To obtain the probability, again we have to use SunQM-4 section I 's result, that is, its spatial portion's probability function equals to its wave function |Φ(φ)|^2 = Φ(φ) = exp(i * n * φ), and its time portion is squared |T(t)|^2 = [T(t)]^2 = [exp(-i * n *ω_n *t / 2)]^2. So for each n,

$$|\Psi_n(\varphi, t)|^2_{NBP} \propto e^{in\varphi}\left(e^{-in\omega_n t/2}\right)^2 = e^{i(n\varphi - n\omega_n t)} = e^{in(\varphi - \omega_n t)} \qquad \text{eq-35}$$

Plot the real portion of eq-35 cos[n*(φ - ω_n* t)] with n=128 shows a (probability) wave with n=128 cycles evenly distributed in φ-dimension's whole range from -π to -π (see Figure 8a). Then, after sum a spectrum of n(s),

$$\sum_n |\Psi_n(\varphi, t)|^2_{NBP} = \sum_n e^{in(\varphi - \omega_n t)} \qquad \text{eq-36}$$

we will obtain a (traveling) wave packet with the peak at φ - ω_n* t = 0, and with the group angular frequency/velocity = ω_n. Plot the real portion of eq-36 (as shown in eq-37) with n =128, δ =1 (or δ =6, or δ =36), is shown in Figure 8b (or 8c, 8d),

$$\frac{1}{1+2\delta}\sum_{-\delta}^{+\delta} \cos[(n+\delta)(\varphi - \omega_n t)] \qquad \text{eq-37}$$

It shows that as the |δ| value increases, a single (probability) wave packet is gradually formed in φ-dimension from -π to +π with its peak at φ - ω_n* t = 0, and the envelop of this wave packet is closely resembled by formula [cos(φ)]^n. Then we can use NBP's eq-27b kind transformation to obtain eq-38:

$$\sum_n |\Psi_n(\varphi, t)|^2_{NBP} = \sum_n e^{in(\varphi - \omega_n t)} \to [\cos(\varphi - \omega_n t)]^n = \left[\frac{1+\cos(\varphi - \omega_n t)}{2}\right]^n \qquad \text{eq-38}$$

Comparing eq-38 with SunQM-4's eq-48, there are the same. So (hopefully) we have proved that SunQM-4's eq-48 is correct (by using a relatively independent method). Honestly to say, eq-38's deduction may not be a mathematically strict deduction. But at least this deduction favors (rather than disfavors) SunQM-4's eq-48.

Notice that in eq-36, we cannot make the un-equal equation in eq-39 to become equal. This is because the left side of eq-39 is to use linear combination to transform wave into a wave packet (and it is correct), while the right side exp[i* n *(φ - ω_n* t)] = {exp[i *(φ - ω_n* t)]}^n itself is also to transform wave into a wave packet, then the sum makes it double do the wave packet transformation, ant it is not correct.

$$\sum_n A_n e^{in(\varphi - \omega_n t)} \neq \sum_n A_n \left[e^{i(\varphi - \omega_n t)}\right]^n \qquad \text{eq-39}$$

We can apply this calculation to the Solar {N,n} QM structure. For example, in SunQM-4's eq-80, we can use eq-36 to represent any planet's φ-dimensional probability density. For all eight planets, their multiplier n' > 1E+9, so the sum-range from n' - δ = n' - 1E+6 to n' + δ = n' + 1E+6 will give fairly good wave packet for the size of this planet. Notice that at n >> 1, eq-35 is equivalent to SunQM-4's eq-46 (copied and then approximated as eq-40 here)

$$|\Phi(\varphi)|^2\, |T(t)|^2 \propto e^{im\varphi}\left[e^{-i\left(\frac{n}{2}\right)\omega_n t}\right]^2 = e^{i[m\varphi - n\omega_n t]} \approx e^{i[n\varphi - n\omega_n t]} \qquad \text{eq-40}$$

where m = n-1. Also notice that one of the major differences between eq-38 (or eq-36) and SunQM-4's eq-46 is that there is no positive precession n/(n-1) *ω_n in eq-38 (or eq-36).

At first we may think $|\Phi(\varphi)|^2 * |T(t)|^2 \propto \exp(im\varphi) * [\exp(-i*\omega_{n,ph}*t)]^2$ in SunQM-4's eq-46 is very awkward: how can a non-squared $\Phi(\varphi)$ (meaning $|\Phi(\varphi)|^2 = \Phi(\varphi)$) times a squared $T(t)$ ended as a physical meaningful NBP? With the above knowledge learned, now we can explain it: after using SunQM-4's eq-13 (copied here as eq-41)

$$|\Phi(\varphi)|^2 \propto e^{im\varphi} = \left(e^{\frac{im\varphi}{2}}\right)^2 \propto \left|\Phi(\varphi)_{\text{equivalent}}\right|^2 \qquad \text{eq-41}$$

the non-squared $|\Phi(\varphi)|^2 = \Phi(\varphi)$ function (a ground state of n=1 under NBP) become a true squared function $|\Phi(\varphi)_{equivalent}|^2$ (a ground state of n=1/2 under Born probability). Then $|\Phi(\varphi)_{equivalent}|^2 * |T(t)|^2$ become the true squared (Born probability) function.

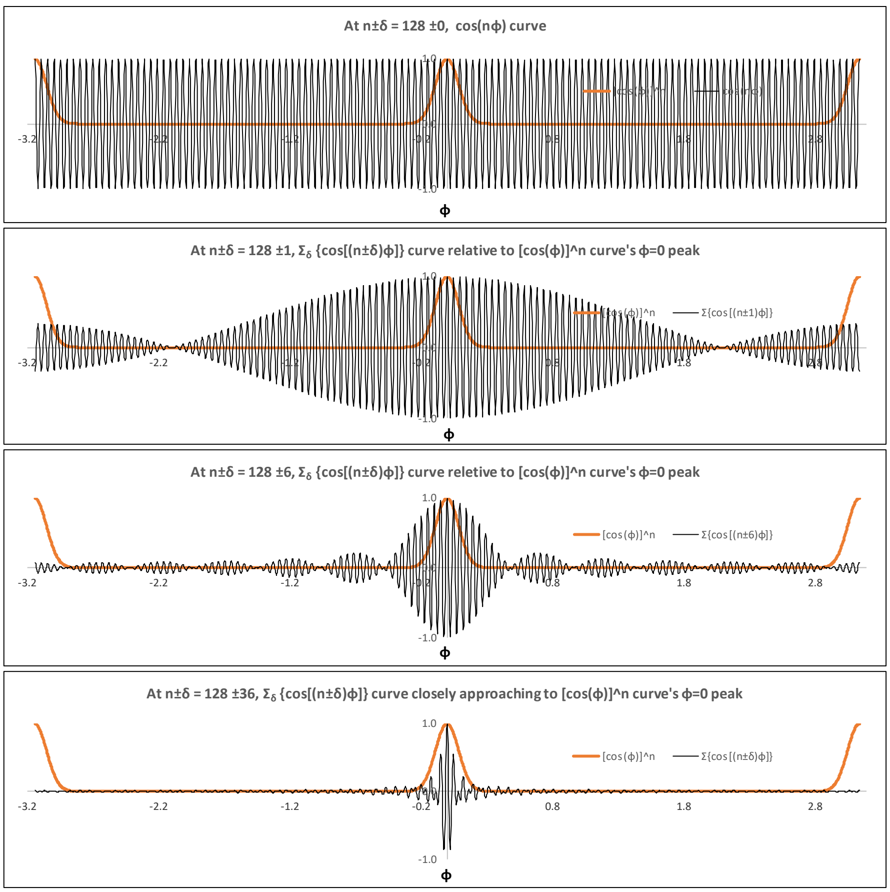


Figure 8a. Plot the real part of eq-35 with t = 0, n=128 shows a (NBP) wave with n=128 cycles evenly distributed in φ-dimension (from -π to -π).
Figure 8b, c, d. Plot the real part of eq-36 with t = 0, n = 128, and δ = 1, 6, and 36 shows a (NBP) wave packet is gradually formed in φ-dimension (from -π to +π), and its peak is narrowed as δ value increases.

IV. Explanation of the planet accretion using wave function and NBP.

IV-a. The process of wave function's linear combination can be used to demonstrate how mass in a belt accretes into a planet

Figure 8 can be directly used to demonstrate how a planet (represented by Figure 8d) is gradually formed by accreting the mass inside a belt (represented by Figure 8a, b, c). We can easily guess out that the reason why the accretion causes a belt to form a planet (and not the other way around) is that in the φ-1D dimension, a planet has the lowest QM state energy, while a belt has the highest QM state energy. However, as a citizen QM level scientist, I am no able to calculate out the state energy for each of four QM states represented by Figure 8.

In the case of planet Jupiter, although almost all mass in {2,2}o orbit space has accreted into a single planet Jupiter, there is tiny amount of mass (known as the Trojan asteroids, see wiki "Trojan (celestial body)") still unaccreted. Currently there are three major unaccreted mass groups in orbit {2,2}. They are "Greeks" (π/3 ahead of Jupiter in the {2,2} orbit at L4 position), "Trojans" (π/3 after of Jupiter in the {2,2} orbit at L5 position), and "Hildas" (opposite of Jupiter in the {2,2} orbit at L3 position). Then (for the simplest way), we can use the superposition of two QM states to describe Jupiter (at the first QM state) and the three unaccreted mass groups (at the second QM state). For the first QM state (the major probability one), we still use Figure 8d's φ=0 peak to represent Jupiter (which equivalents to use eq-38 to represent Jupiter's wave function or NBP). For the second QM state (the minor probability one), we can use Figure 9's φ= π (minor) peak to represent Jupiter, and then use the three major wave packets to represent "Greeks", "Hildas", and "Trojans".

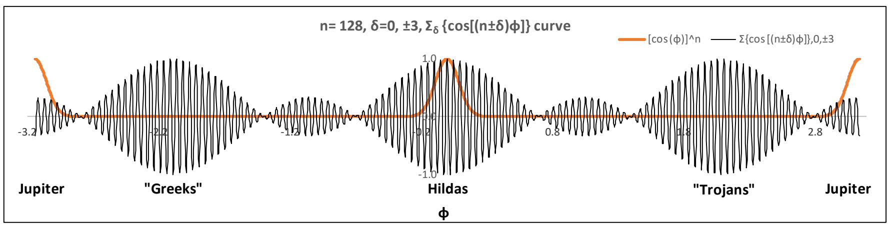


Figure 9. Plotting of the real part of eq-35 (as shown in eq-37) with t = 0, n = 128, and δ = -3, 0, and 3 shows three major wave packets (or NBP peaks) evenly distributed in φ-dimension (from -π to +π). If assign Jupiter at φ= π, then these three major wave packets can be used to describe "Greeks", "Trojans", and "Hildas" as shown in the figure.

IV-b. 1D∞QM can also be used to show how mass in a belt accretes into a planet

We can also use 1D∞QM to show how mass in a belt accretes into a planet. For example, using Figure 6 as a template, divide an object into 2 pieces, 4 pieces, 8 pieces, …, and calculate out the energy E_j for each quantum state at j=1, 2, 4, 8, ... (similar as that of eq-1 and Figure 1a, although the formula is completely different). Because $E_1 < E_2 < E_4 < E_8$, it can be concluded that the mass accreting process decreases the 1D∞QM state energy while the mass dividing process increases the 1D∞QM state energy. Therefore, j=1 is the ground state in which all mass accreted as one object. It sounds easy and straightforward, but unfortunately after over three year of trying, I am still not able to deduce out a convincingly correct E_j formula (again due to my citizen scientist level math capability).

V. Explanation of an orbital moving planet's θ-dimensional standing wave's NBP.

For a planet in Solar {N,n} QM structure doing orbital movement in φ-dimension, we have explained it as the wave function exp(+imφ) correlates to planet's eastward orbital rotation's $\omega_{n,ph}$, while exp(-imφ) correlates to the westward orbital rotation's $\omega_{n,ph}$ (see SunQM-4's eq-7). Because for an eastward rotational planet, it can be thought as the westward orbital rotation's $\omega_{n,ph}$ =0, so exp(-imφ) ∝ exp[i*$\omega_{n,ph,west}$*t) = exp[i*0*t) = 1 (this is SunQM-4's eq-7). So we have a good physical meaning for the φ-dimension's NBP calculation |Φ(φ)|^2 ∝ Φ(φ) ∝ exp(imφ) in SunQM-4's eq-9. Then how to calculate the same planet's θ-dimensional probability in Solar {N,n} QM structure? According to Solar {N,n} QM theory, after > 99% of mass quantum collapsed (r-dimensionally) into N-1 super shell, the leftover < 1% mass in N super shell further collapsed in θ-dimension to the equator θ = π/2 under nLL QM effect, and then standing still there (not doing the unidirectional movement in θ-dimension) since then. It is obvious that a planet (or a belt)'s matter wave in Solar {N,n} QM structure's θ-dimension is a standing wave Θ(θ) ∝ [sin(θ)]^m, and it seems we should use Born probability |Θ(θ)|^2 ∝ [sin(θ)]^(2m) for calculation (as shown in SunQM-3s11's eq-33). However, we have showed that the correct calculation is the NBP form |Θ(θ)|^2 ∝ [sin(θ)]^m (shown in SunQM-4's eq-50). After learned the knowledge from section-I, now we know that we can use the ground state is n=1/2 (rather than n=1) to explain. First, we still need to change the θ-dimensional wave function from sine wave to cosine wave Θ(θ) ∝ [sin(θ)]^m = [cos(θ')]^m, where θ = π/2 – θ'. Then we can have

$$|\Theta(\theta)|_{NBP}^2 \propto \cos(\theta')^{(n-1)} \propto \left\{\frac{1+\cos(\theta\prime)}{2}\right\}^{(n-1)} = \left[\cos\left(\frac{\theta\prime}{2}\right)^2\right]^{n-1} \qquad \text{eq-42}$$

The explanation of eq-42 is: after the first ∝ is the wave function, after the second ∝ is the NBP calculation (the lifted-up wave and divided by 2), and last item [cos(θ'/2)]^2 is a standing wave's Born probability at n=1/2. We can explain [cos(θ'/2)]^2 as a standing wave between θ' = –π/2 and θ' = +π/2, and its Born probability peak at θ' = 0 (or θ = π/2 – θ'= π/2). Thus, an orbital moving planet's θ-dimensional standing wave's NBP (at n=1) can be directly explained as Born probability at n=1/2.

We also can directly use SunQM-4's eq-51 (a citizen-scientist level method, copied as eq-43 here) to explain:

$$|\Theta(\theta)|_{NBP}^2 \propto \sin(\theta)^{(n-1)} = \left[\cos\left(\frac{\pi}{2}-\theta\right)\right]^{(n-1)} \to \left[e^{i\left(\frac{\pi}{2}-\theta\right)}\right]^{(n-1)} = \left[e^{\frac{i\left(\frac{\pi}{2}-\theta\right)}{2}}\right]^{2(n-1)} \to \left\{\cos\left[\left(\frac{\pi}{2}-\theta\right)/2\right]\right\}^{2(n-1)} = \left\{\left[1+\cos\left(\frac{\pi}{2}-\theta\right)\right]/2\right\}^{(n-1)} = \left\{\frac{[1+\sin(\theta)]}{2}\right\}^{(n-1)} \qquad \text{eq-43}$$

The last item is the NBP calculation (the lifted-up sine wave and divided by 2), then after the second "→" item {cos[(π/2 – θ)/2]}^2 is a n=1/2 standing wave's Born probability (though it is not easy to figure out). So from SunQM-3s11 to SunQM-4, |Θ(θ)|^2 changed from =[sin(θ)]^(2m) to =[sin(θ)]^m, the physical meaning is that the planet's matter wave still in standing (or opposite bi-directional traveling), but the ground state changed from n=1 to n=1/2! Or, the ground state at n=1/2 is the true reason that causes |Θ(θ)|^2 = Θ(θ) = $[\Theta(\theta)_{\text{equivalent}}]$^2.

Here is a summary for an orbital moving planet's NBP: Its φ-dimensional NBP (at the circular 1D QM's ground state n=j=1) describes a uni-directional traveling wave's probability. Its θ-dimensional NBP (at n=1) describes a bi-directional traveling wave (or standing wave)'s probability (which equivalents to a Born probability at the 1D∞QM's ground state with n=j=1/2). Its r-dimensional NBP may also be explained as that of θ-dimensional NBP.

VI. An alternative (and independent) way to prove that the NBP's positive precession n/(n-1) *ω_n is correct

Moved to SunQM-4s4.

VII. NBP shows more direct and intuitive physical meaning than that of Born probability

In NBP, the wave function itself is the probability density function. In Figure 10, we use the 1D air mass density wave (e.g., a sound wave in a flute [18]) as the example. Normally the air density = 1.225 kg/m^3. When making a sound wave, supposing the air density vibrated at 1.225 ±0.125 kg/m^3 as shown in Figure 10a (Note: sorry that the value of ±0.125 kg/m^3 is from my pure guess. As a citizen scientist, I don't know how to calculate the right one. If readers know, please teach me). In the classical wave mechanics, we extract the density vibration part (±0.125 kg/m^3) out of the constant background density (1.225 kg/m^3) and then represent it as a wave shown in Figure 10b (with the wave amplitude ±1). With the new concept of NBP, we can describe this physical phenomenon (an air mass density vibration, or a sound wave) by a 1D infinity deep square potential well QM. Then the NBP density of this sound wave is the sound wave itself (see Figure 10c) with minimum probability =0 and maximum probability =1. So now we can understand NBP's true physical meaning: NBP is directly proportional to the wave, so it directly reflects the mass density change (just like a sound wave directly reflects the air mass density change)! In this way, NBP shows more direct and intuitive physical meaning than that of Born probability. We believe that this should be true not only for mass density, but also for other physical variables (e.g., pressure, temperature, even electromagnetic wave's intensity, etc.).

A flute sound wave (or an air mass vibration) is a physics phenomenon that exists in our daily-life-world, and in our daily-life-world's length scale. It has been always explained by the Newtonian physics, and it can never be explained by the traditional QM. So the discovery of NBP (that directly linked a sound wave mechanics to the quantum mechanics) itself is a tremendous leap-forward in QM. If this is correct, then {N,n} QM explains not only the central-force field produced (Newtonian) physics (like the Solar system), but also many (if not all) our daily-life-world's (Newtonian) physics (like a flute sound wave, etc.).

The establish of NBP concept has enormous impact on the QM physics: it allows us to think about the 3D QM wave form in the real-world space directly from the 3D space mass distribution. For example, according to NBP, Earth ball's 3D QM wave has ball-like shape, which is |R(1,0)|*Y(0,0)|; The spiral galaxy tells us that the galaxy's (linearly combined) 3D QM wave is spiral shape; Then the Virgo supercluster at the size of {10,1//6}, the largest cosmic structure that is still dominated by its gravity force, should also have the (linearly combined) QM 3D wave shape like its mass distribution in 3D. (Note: NBP map is mass density map, but may or may not be the wave function map, because NBP may be the linear combination of wave functions, not a single one wave function).

Furthermore, if we believe that NBP should be valid not only for |nLL> QM state, but also for all |n,*l*,m> QM states, then Earth's global weather change (which is caused by the air mass density, or pressure, or moisture differences between one place and another) may also be directly mapped by the (linearly combined) θφ-2D Y(*l*,m) wave functions. Therefore, in our next study (SunQM-4s2), we will use NBP method to study Earth's atmosphere pattern and its effect on the extreme weather.

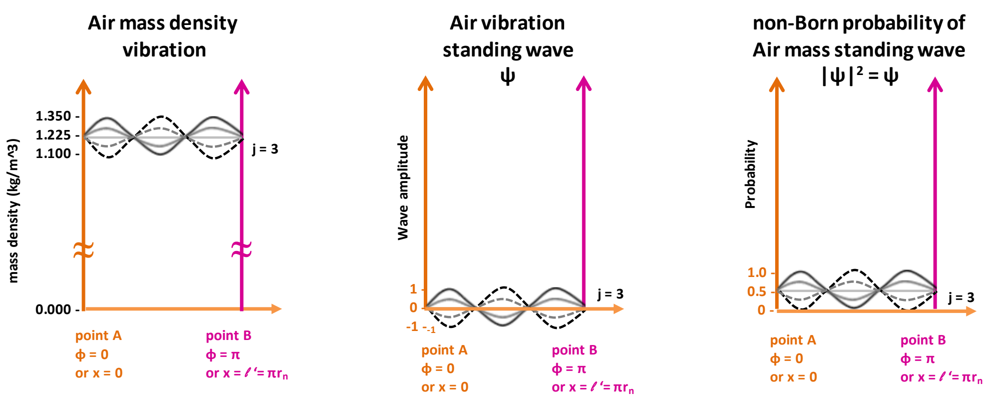


Figure 10. Illustration of how 1D air mass density vibration (Figure 10a, left) is explained as a wave (Figure 10b, middle), or as a NBP (Figure 10c, right) in a 1D infinity deep square potential well QM.

VIII. Is the Born probability merely a special case of (the more generalized) non-Born probability?

In wave mechanics, a standing wave is a special case of (the more generalized) traveling wave. For two opposite directional waves that forms standing wave, we can use either Born probability or two NBP to describe. But for a uni-directional traveling wave, we can only use NBP (but not Born probability) to describe. Therefore, is the Born probability merely a special case of (the more generalized) non-Born probability? If this is correct, then there are many QM probability explanations need to be re-written. For example, the Double-slit experiment need to use NBP (not Born probability) to explain, simply because its wave function is a uni-directional traveling wave, not a (bi-directional) standing wave!

Conclusion

Non-Born probability calculates probability density for a uni-directional traveling (matter) wave, which naturally includes the linearly combined two opposite-directional traveling waves (or a standing wave), and its probability is directly proportional to the wave function. Born probability only calculates probability density for the standing wave. Therefore, Born probability is possibly to be a special case of (the more generalized) non-Born probability. [1 + cos(x)]/2 = [cos(x/2)}^2 bridges NBP's ground state n=1 to Born probability's ground state n=1/2. NBP bridges the sound wave (an air mass vibration) mechanics to the quantum mechanics.

References

[1] Yi Cao, SunQM-1: Quantum mechanics of the Solar system in a {N,n//6} QM structure. http://vixra.org/pdf/1805.0102v2.pdf （original submitted on 2018-05-03）

[2] Yi Cao, SunQM-1s1: The dynamics of the quantum collapse (and quantum expansion) of Solar QM {N,n} structure. http://vixra.org/pdf/1805.0117v1.pdf （submitted on 2018-05-04）

[3] Yi Cao, SunQM-1s2: Comparing to other star-planet systems, our Solar system has a nearly perfect {N,n//6} QM structure. http://vixra.org/pdf/1805.0118v1.pdf （submitted on 2018-05-04）

[4] Yi Cao, SunQM-1s3: Applying {N,n} QM structure analysis to planets using exterior and interior {N,n} QM. http://vixra.org/pdf/1805.0123v1.pdf （submitted on 2018-05-06）

[5] Yi Cao, SunQM-2: Expanding QM from micro-world to macro-world: general Planck constant, H-C unit, H-quasi-constant, and the meaning of QM. http://vixra.org/pdf/1805.0141v1.pdf （submitted on 2018-05-07）

[6] Yi Cao, SunQM-3: Solving Schrodinger equation for Solar quantum mechanics {N,n} structure. http://vixra.org/pdf/1805.0160v1.pdf （submitted on 2018-05-06）

[7] Yi Cao, SunQM-3s1: Using 1st order spin-perturbation to solve Schrodinger equation for nLL effect and pre-Sun ball's disk-lyzation. http://vixra.org/pdf/1805.0078v1.pdf （submitted on 2018-05-02）

[8] Yi Cao, SunQM-3s2: Using {N,n} QM model to calculate out the snapshot pictures of a gradually disk-lyzing pre-Sun ball. http://vixra.org/pdf/1804.0491v1.pdf （submitted on 2018-04-30）

[9] Yi Cao, SunQM-3s3: Using QM calculation to explain the atmosphere band pattern on Jupiter (and Earth, Saturn, Sun)'s surface. http://vixra.org/pdf/1805.0040v1.pdf （submitted on 2018-05-01）

[10] Yi Cao, SunQM-3s6: Predict mass density r-distribution for Earth and other rocky planets based on {N,n} QM probability distribution. http://vixra.org/pdf/1808.0639v1.pdf (submitted on 2018-08-29)

[11] Yi Cao, SunQM-3s7: Predict mass density r-distribution for gas/ice planets, and the superposition of {N,n//q} or |qnlm> QM states for planet/star. http://vixra.org/pdf/1812.0302v2.pdf (submitted on 2019-03-08)

[12] Yi Cao, SunQM-3s8: Using {N,n} QM to study Sun's internal structure, convective zone formation, planetary differentiation and temperature r-distribution. http://vixra.org/pdf/1808.0637v1.pdf (submitted on 2018-08-29)

[13] Yi Cao, SunQM-3s9: Using {N,n} QM to explain the sunspot drift, the continental drift, and Sun's and Earth's magnetic dynamo. http://vixra.org/pdf/1812.0318v2.pdf (submitted on 2019-01-10)

[14] Yi Cao, SunQM-3s4: Using {N,n} QM structure and multiplier n' to analyze Saturn's (and other planets') ring structure. . http://vixra.org/pdf/1903.0211v1.pdf (submitted on 2019-03-11)

[15] Yi Cao, SunQM-3s10: Using {N,n} QM's Eigen n to constitute Asteroid/Kuiper belts, and Solar {N=1..4,n} region's mass density r-distribution and evolution. http://vixra.org/pdf/1909.0267v1.pdf (submitted on 2019-09-12)

[16] Yi Cao, SunQM-3s11: Using {N,n} QM's probability density 3D map to build a complete Solar system with time-dependent orbital movement. http://vixra.org/pdf/1912.0212v1.pdf (original submitted on 2019-12-11)

[17] Yi Cao, SunQM-4: Using full-QM deduction and {N,n} QM's non-Born probability density 3D map to build a complete Solar system with orbital movement. http://vixra.org/pdf/2003.0556v1.pdf (submitted on 2020-03-25)

[18] Joe Wolfe, Flute acoustics: an introduction to how a flute works.
https://newt.phys.unsw.edu.au/jw/fluteacoustics.html#overview.

[19] A series of my papers that to be published (together with current paper):
SunQM-4s1: Is Born probability merely a special case of (the more generalized) non-Born probability (NBP)?
SunQM-4s2: Using {N,n} QM and NBP to analyze Earth's atmosphere pattern and the extreme weather.
SunQM-4s3: Schrodinger equation and {N,n} QM.
SunQM-4s4: More explanations on non-Born probability (NBP)'s positive precession in {N,n}QM.
SunQM-4s7: Addendums, Updates and Q/A for SunQM series papers
SunQM-5: A new version of QM based on interior {N,n}, multiplier n', |R(n,l)|^2 * |Y(l,m)|^2 guided mass occupancy, and RF, and its application from string to universe (drafted in April 2018).
SunQM-5s1: White dwarf, neutron star, and black hole re-analyzed by using the internal {N,n} QM (drafted in April 2018).

[20] Major QM books, data sources, software I used for this study:
Douglas C. Giancoli, Physics for Scientists & Engineers with Modern Physics, 4th ed. 2009.
David J. Griffiths, Introduction to Quantum Mechanics, 2nd ed., 2015.
John S. Townsed, A Modern Approach to Quantum Mechanics, 2nd ed., 2012.
Stephen T. Thornton & Andrew Rex, Modern Physics for scientists and engineers, 3rd ed. 2006.
James Binney & David Skinner, The Physics of Quantum Mechanics, 1st ed. 2014.
Wikipedia at: https://en.wikipedia.org/wiki/
(Free) online math calculation software: WolframAlpha (https://www.wolframalpha.com/)
(Free) online spherical 3D plot software: MathStudio (http://mathstud.io/)
(Free) offline math calculation software: R
Microsoft Excel, Power Point, Word.
Public TV's space science related programs: PBS-NOVA, BBC-documentary, National Geographic-documentary, etc.
Journal: Scientific American.