

On some possible mathematical connections between various equations concerning the Mock Modularity closely related to $N = 4$ super Yang-Mills, ϕ , $\zeta(2)$ and some parameters of Particle Physics.

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Abstract

In this paper we have described some possible mathematical connections between various equations concerning the Mock Modularity closely related to $N = 4$ super Yang-Mills, ϕ , $\zeta(2)$ and some parameters of Particle Physics.

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<https://www.britannica.com/biography/Srinivasa-Ramanujan>

We want to highlight that the development of the various equations was carried out according to our possible logical and original interpretation.

For more information on the data entered for the development of the various equations, see the "Observations" section.

From:

Duality and Mock Modularity

Atish Dabholkar, Pavel Putrov, Edward Witten - arXiv:2004.14387v1 [hep-th] 29

Apr 2020

we have that:

$$\begin{aligned}
 \langle \bar{G} \rangle_y &= -i \left(\int_y h \right) \cdot e^{3\pi t/4} \overline{\eta(\tau)^3} \cdot \frac{1}{8\pi^2 \tau_2 \eta(\tau)^2 \overline{\eta(\tau)^2}} \cdot \frac{\widehat{\chi}_v^{u(1)_2}(\tau)}{2\pi\sqrt{2}} \\
 &= \frac{-3i \cdot e^{3\pi i/4}}{4\pi\sqrt{2}\tau_2\eta(\tau)^2} \cdot \sum_{n \in \mathbb{Z}} \bar{q}^{(n+v/2)^2}.
 \end{aligned} \tag{4.37}$$

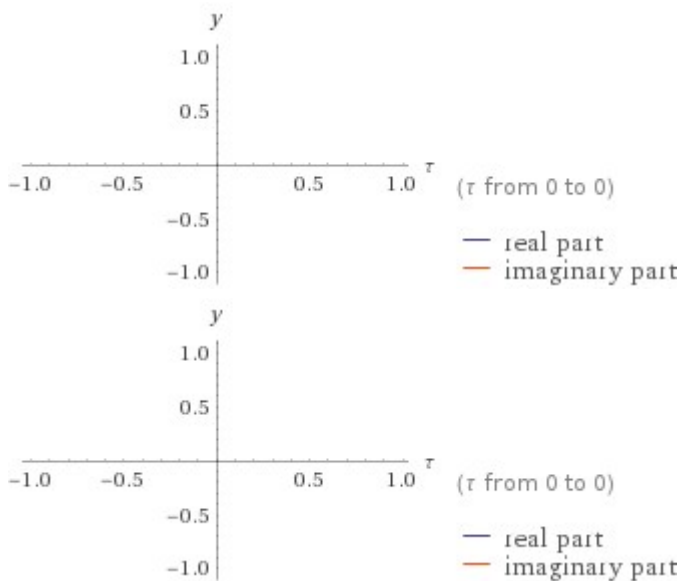
For:

Input:

$\eta(\tau)^2$

$\eta(\tau)$ is the Dedekind eta function

Plots:



Numerical roots:

$\tau \approx -2.74517 + 381.228 i \dots$

$$\tau \approx -0.499998 + 382.66 i \dots$$

$$\tau \approx 1.69666 \times 10^{-6} + 383.062 i \dots$$

$$\tau \approx 0.500001 + 382.66 i \dots$$

$$\tau \approx 2.74518 + 381.228 i \dots$$

From this last solution, we obtain:

$$2.74518 + 381.228i$$

Series expansion at $\tau = 0$:

$$\frac{i e^{-(25 i \pi)/(6 \tau)} (-1 + e^{(2 i \pi)/\tau})^2}{\tau}$$

Alternative representations:

$$\eta(\tau)^2 = \left(e^{1/12 \pi (i \tau)} \vartheta_3 \left(\frac{1}{2} (\tau + 1) \pi, e^{3 \pi i \tau} \right) \right)^2 \text{ for } \text{Im}(\tau) > 0$$

$$\eta(\tau)^2 = \left(\frac{\vartheta_2 \left(\frac{\pi}{6}, e^{(\pi i \tau)/3} \right)}{\sqrt{3}} \right)^2 \text{ for } (\text{Im}(\tau) > 0 \text{ and } |\text{Re}(\tau)| < 3)$$

Series representations:

$$\eta(\tau)^2 = \exp \left(\frac{i \pi \tau}{6} - 2 \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \frac{e^{2 i k n \pi \tau}}{k} \right)$$

$$\eta(\tau)^2 = \frac{1}{\left(\sum_{k=0}^{\infty} e^{2 i (-1/24+k) \pi \tau} p(k) \right)^2}$$

$$\eta(\tau)^2 = e^{(i \pi \tau)/6} \left(\sum_{k=-\infty}^{\infty} (-1)^k e^{i k (-1+3 k) \pi \tau} \right)^2$$

Ramanujan's τ_z function is defined by

$$\tau_z(t) = \frac{\Gamma(6 + i t) (2 \pi)^{-i t}}{f(6 + i t) \sqrt{\frac{\sinh(\pi t)}{\pi t \prod_{k=1}^{\infty} (k^2 + t^2)}}}$$

For $t = 1$, we obtain:

$$\frac{\Gamma(6+i) (2\pi)^{-i}}{[(6+i)\sqrt{\frac{\sinh(\pi)}{\pi \prod_{k=1}^5 (k^2+1)}}]}$$

Input interpretation:

$$\frac{\Gamma(6+i) (2\pi)^{-i}}{(6+i) \sqrt{\frac{\sinh(\pi)}{\pi \prod_{k=1}^5 (k^2+1)}}}$$

$\Gamma(x)$ is the gamma function
 $\sinh(x)$ is the hyperbolic sine function
 i is the imaginary unit

Result:

$$\left(\frac{30}{37} - \frac{5i}{37}\right) 2^{3/2-i} \pi^{1/2-i} \sqrt{221 \operatorname{csch}(\pi)} \Gamma(6+i) \approx 1893.3 - 568.069i$$

$\operatorname{csch}(x)$ is the hyperbolic cosecant function

$$(1893.3-568.069i)$$

Alternate forms:

$$\left(\frac{175}{1369} - \frac{60i}{1369}\right) 2^{3/2-i} \pi^{1/2-i} (6+i)! \sqrt{221 \operatorname{csch}(\pi)}$$

$$\left(\frac{30}{37} - \frac{5i}{37}\right) 2^{2-i} \sqrt{\frac{221}{e^\pi - e^{-\pi}}} \pi^{1/2-i} \Gamma(6+i)$$

Thence, from (4.37), we obtain:

$$\frac{((-3i \exp((3\pi i)/4)))}{((4\pi \sqrt{2} (2.74518 + 381.228i) (1893.3 - 568.069i)))} * \sum_{n=-\infty}^{+\infty} (\exp(2\pi i))^{((n+3/2)^2)}$$

$$\frac{((-3i \exp((3\pi i)/4)))}{((4\pi \sqrt{2} (2.74518 + 381.228i) (1893.3 - 568.069i)))} * \sum_{n=0}^2 (\exp(2\pi i))^{((n+3/2)^2)}$$

Input interpretation:

$$\frac{-3i \exp\left(\frac{1}{4} (3\pi i)\right)}{4\pi \sqrt{2} (2.74518 + 381.228i) (1893.3 + i \times (-568.069))} \sum_{n=0}^2 \exp\left(\frac{n+3}{2}\right)^2 (2\pi)$$

i is the imaginary unit

Result:

$$5.29513 \times 10^{26} - 2.80197 \times 10^{26} i$$

Input interpretation:

$$5.29513 \times 10^{26} - 2.80197 \times 10^{26} i$$

i is the imaginary unit

Result:

$$5.29513... \times 10^{26} - 2.80197... \times 10^{26} i$$

Polar coordinates:

$$r = 5.99078 \times 10^{26} \text{ (radius), } \theta = -27.886^\circ \text{ (angle)}$$

$$5.99078 * 10^{26}$$

From:

$$\frac{\partial Z_v}{\partial \bar{\tau}} = \frac{3}{16\pi i \tau_2^{3/2} \eta(\tau)^3} \sum_{n \in \mathbb{Z}} \bar{q}^{(n+v/2)^2}, \tag{4.38}$$

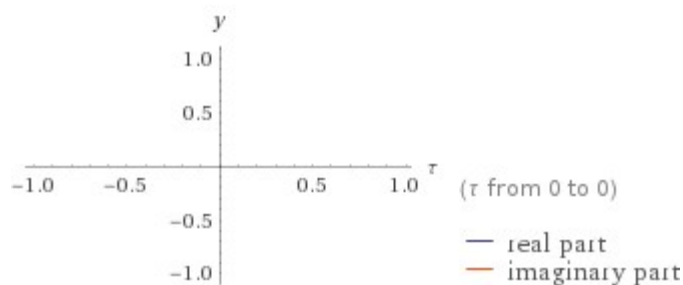
For:

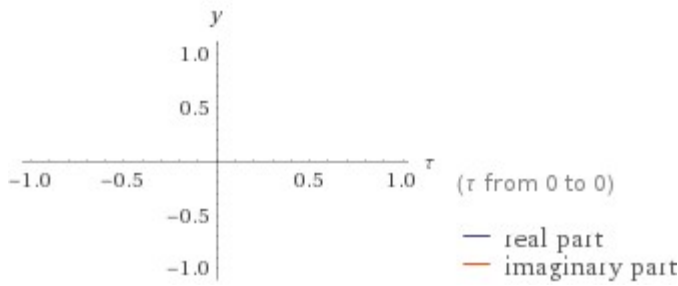
Input:

$$\eta(\tau)^3$$

$\eta(\tau)$ is the Dedekind eta function

Plots:





Numerical roots:

$$\tau \approx -2.04801 + 254.388 i...$$

$$\tau \approx -0.499999 + 255.395 i...$$

$$\tau \approx 1.43993 \times 10^{-6} + 255.708 i...$$

$$\tau \approx 0.500002 + 255.395 i...$$

$$\tau \approx 2.04801 + 254.388 i...$$

From this last solution, we obtain:

$$(2.04801+254.388i)$$

Series expansion at $\tau = 0$:

$$\frac{e^{-(i\pi)/(4\tau)}}{(-i\tau)^{3/2}}$$

Alternative representations:

$$\eta(\tau)^3 = \frac{1}{2} \vartheta_1'(0, e^{i\pi\tau}) \text{ for } (\text{Im}(\tau) > 0 \text{ and } |\text{Re}(\tau)| \leq 1)$$

$$\eta(\tau)^3 = \left(e^{1/12\pi(i\tau)} \vartheta_3\left(\frac{1}{2}(\tau+1)\pi, e^{3\pi i\tau}\right) \right)^3 \text{ for } \text{Im}(\tau) > 0$$

$$\eta(\tau)^3 = \left(\frac{\vartheta_2\left(\frac{\pi}{6}, e^{(\pi i\tau)/3}\right)}{\sqrt{3}} \right)^3 \text{ for } (\text{Im}(\tau) > 0 \text{ and } |\text{Re}(\tau)| < 3)$$

$$\eta(\tau)^3 = \frac{1}{2} \vartheta_2(0, e^{\pi i\tau}) \vartheta_3(0, e^{\pi i\tau}) \vartheta_4(0, e^{\pi i\tau}) \text{ for } (\text{Im}(\tau) > 0 \text{ and } |\text{Re}(\tau)| \leq 1)$$

Series representations:

$$\eta(\tau)^3 = \exp\left(\frac{i\pi\tau}{4} - 3 \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \frac{e^{2ikn\pi\tau}}{k}\right)$$

$$\eta(\tau)^3 = \frac{1}{\left(\sum_{k=0}^{\infty} e^{2i(-1/24+k)\pi\tau} p(k)\right)^3}$$

$$\eta(\tau)^3 = e^{(i\pi\tau)/4} \left(\sum_{k=-\infty}^{\infty} (-1)^k e^{ik(-1+3k)\pi\tau} \right)^3$$

$$3/(((16*\text{Pi}*i*((1893.3-568.069i)^{1.5})*(2.04801+254.388i)))) * \text{sum}((\exp(2\text{Pi}))^{(n+3/2)^2}), n=0 \text{ to } 2$$

Input interpretation:

$$\frac{3}{(16\pi i(1893.3 + i \times (-568.069))^{1.5})(2.04801 + 254.388i)} \sum_{n=0}^2 \exp\left(\left(n+\frac{3}{2}\right)^2\right) (2\pi)$$

i is the imaginary unit

Result:

$$-6.4431 \times 10^{24} - 3.07505 \times 10^{24} i$$

Input interpretation:

$$-6.4431 \times 10^{24} - 3.07505 \times 10^{24} i$$

i is the imaginary unit

Result:

$$-6.44310... \times 10^{24} - 3.07505... \times 10^{24} i$$

Polar coordinates:

$$r = 7.13929 \times 10^{24} \text{ (radius), } \theta = -154.487^\circ \text{ (angle)}$$

$$7.13929 * 10^{24}$$

Dividing the two results, we obtain:

$$(5.99078 * 10^{26} / 7.13929 * 10^{24})$$

Input interpretation:

$$\frac{5.99078 \times 10^{26}}{7.13929 \times 10^{24}}$$

Result:

83.91282606533702931243863185274726198263412748326514261222...

83.912826.....

From which:

$$1/\text{golden ratio} + (((((((sqrt(((113+5sqrt(505))/8)) + sqrt(((105+5sqrt(505))/8))))))^3)))^(1/14)*(5.99078*10^26 / 7.13929*10^24)$$

where, from Ramanujan expression

$$((((((((sqrt(((113+5sqrt(505))/8)) + sqrt(((105+5sqrt(505))/8))))))^3)))^(1/14)$$

we obtain:

$$\sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}} + \sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3} = 1,65578 \dots$$

Input interpretation:

$$\frac{1}{\phi} + \sqrt[14]{\left(\sqrt{\frac{1}{8}(113+5\sqrt{505})} + \sqrt{\frac{1}{8}(105+5\sqrt{505})}\right)^3} \times \frac{5.99078 \times 10^{26}}{7.13929 \times 10^{24}}$$

φ is the golden ratio

Result:

139.560...

139.560.... result practically equal to the rest mass of Pion meson 139.57 MeV

or:

$$\text{golden ratio} + \zeta(2)*(5.99078*10^26 / 7.13929*10^24)$$

Input interpretation:

$$\phi + \zeta(2) \times \frac{5.99078 \times 10^{26}}{7.13929 \times 10^{24}}$$

ζ(s) is the Riemann zeta function

ϕ is the golden ratio

Result:

139.649...

139.649....

and:

golden ratio + zeta(2)*(5.99078*10²⁶ / 7.13929×10²⁴)-11- π

Input interpretation:

$$\phi + \zeta(2) \times \frac{5.99078 \times 10^{26}}{7.13929 \times 10^{24}} - 11 - \pi$$

$\zeta(s)$ is the Riemann zeta function

ϕ is the golden ratio

Result:

125.508...

125.508... result very near to the Higgs boson mass 125.18 GeV

Performing the ln of the two results and dividing, we obtain:

$$(\ln(5.99078 \times 10^{26}) / \ln(7.13929 \times 10^{24}))$$

Input interpretation:

$$\frac{\log(5.99078 \times 10^{26})}{\log(7.13929 \times 10^{24})}$$

$\log(x)$ is the natural logarithm

Result:

1.077406...

1.077406...

From which:

$$1 + 1 / (\ln(5.99078 \times 10^{26}) / \ln(7.13929 \times 10^{24}))^6$$

Input interpretation:

$$1 + \frac{1}{\left(\frac{\log(5.99078 \times 10^{26})}{\log(7.13929 \times 10^{24})}\right)^6}$$

log(x) is the natural logarithm

Result:

1.639327...

$$1.639327\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

From the previous equation

$$\frac{-3 i \exp\left(\frac{1}{4} (3 \pi i)\right)}{4 \pi \sqrt{2} (2.74518 + 381.228 i) (1893.3 + i \times (-568.069))} \sum_{n=0}^2 \exp\left(\frac{n+3}{2}\right)^2 (2 \pi)$$

we obtain also:

$$\left(\frac{(-3 * i * \exp((3 * \text{Pi} * i) / 4))}{((4 * \text{Pi} * \text{sqrt}2 * (2.74518 + 381.228 i)) * ((1893.3 - 568.069 i)))} * \text{sum}((\exp(2 \text{Pi}))^{((n + 3 / 2)^2)}, n = 0 \text{ to } 2))\right)^{1 / 128}$$

Input interpretation:

$$\sqrt[128]{\frac{-3 i \exp\left(\frac{1}{4} (3 \pi i)\right)}{4 \pi \sqrt{2} (2.74518 + 381.228 i) (1893.3 + i \times (-568.069))} \sum_{n=0}^2 \exp\left(\frac{n+3}{2}\right)^2 (2 \pi)}$$

i is the imaginary unit

Result:

1.61881 - 0.00615533 i

Input interpretation:

1.61881 + i × (-0.00615533)

i is the imaginary unit

Result:

1.61881... -
0.00615533... i

Polar coordinates:

$r = 1.61882$ (radius), $\theta = -0.217859^\circ$ (angle)

1.61882 result that is a very good approximation to the value of the golden ratio 1.618033988749...

Possible closed forms:

$$\begin{aligned} & \pi \left[\text{root of } 5x^4 + 2x^3 + 7x^2 + x - 3 \text{ near } x = 0.515281 \right] + \\ & \quad i e^{1+1/e-2/\pi-\pi} \pi^{1-e} \sin(e\pi) \cos(e\pi) \approx 1.6188026288 - 0.0061553105 i \\ & \frac{1029 e}{550 \pi} + i e^{1+1/e-2/\pi-\pi} \pi^{1-e} \sin(e\pi) \cos(e\pi) \approx 1.6188152779 - 0.0061553105 i \\ & \frac{e + \log(2)}{\sqrt{2} + \log(2)} + i e^{1+1/e-2/\pi-\pi} \pi^{1-e} \sin(e\pi) \cos(e\pi) \approx 1.6188158674 - 0.0061553105 i \end{aligned}$$

We note that:

From:

RIEMANN'S HYPOTHESIS AND SOME INFINITE SET OF MICROSCOPIC UNIVERSES OF THE EINSTEIN'S TYPE IN THE EARLY PERIOD OF THE EVOLUTION OF THE UNIVERSE - JAN MOSER - arXiv:1307.1095v2 [physics.gen-ph] 28 Jul 2013

Corollary 2. On the Riemann hypothesis we have the following infinite set of a mathematical universes

$$\begin{aligned} (4.3) \quad R(t; t_0, \Lambda, \mu) &= \mu \frac{e}{\sqrt{\Lambda}} + \mathcal{O}\left(\frac{1}{t_0}\right), \\ \kappa e^2 \rho(t; t_0, \Lambda, \mu) &= \left(\frac{3}{\mu^2} - 1\right) \Lambda + \mathcal{O}\left\{\left(1 + \frac{1}{\mu^2}\right) \frac{1}{t_0}\right\}, \\ \kappa p(t; t_0, \Lambda, \mu) &= \left(1 - \frac{1}{\mu^2}\right) \Lambda + \mathcal{O}\left\{\left(1 + \frac{1}{\mu^2}\right) \frac{1}{t_0}\right\}, \\ t \in J(t_0), \quad \gamma' < t_0 < \gamma'', \quad \mu > 0, \quad t_0 \rightarrow \infty. \end{aligned}$$

Definition. Let

$$(4.5) \quad \begin{aligned} E_1(t; t_0, \Lambda, \mu) &= \kappa c^2 \rho - \kappa p, \\ E_2(t; t_0, \Lambda, \mu) &= \kappa c^2 \rho + \kappa p. \end{aligned}$$

Then we will call the set

$$(4.6) \quad F(t_0, \Lambda, \mu) = \{t \in (\gamma', \gamma'') : E_1(t) \geq 0, E_2(t) \geq 0, \rho(t) > 0\}, \quad t_0 \rightarrow \infty$$

the *physical domain of the universe* (4.3).

Since for

$$t \in J(t_0), \quad t_0 > K > 0,$$

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where K is sufficiently big, we have (see (4.3), (4.6))

$$\begin{aligned} E_1 &= 2 \left(\frac{2}{\mu^2} - 1 \right) \Lambda + \mathcal{O} \left\{ \left(1 + \frac{1}{\mu^2} \right) \frac{1}{t_0} \right\}, \\ E_2 &= \frac{2}{\mu^2} \Lambda + \mathcal{O} \left\{ \left(1 + \frac{1}{\mu^2} \right) \frac{1}{t_0} \right\}, \\ \kappa c^2 \rho &= \left(\frac{3}{\mu^2} - 1 \right) \Lambda + \mathcal{O} \left\{ \left(1 + \frac{1}{\mu^2} \right) \frac{1}{t_0} \right\}, \end{aligned}$$

5.1. In the case

$$\mu = \epsilon$$

with ϵ being an arbitrarily small fixed value, we obtain from (4.3) the following infinite subset of the universes

$$(5.1) \quad \begin{aligned} R(t; t_0, \Lambda, \epsilon) &= \epsilon \frac{c}{\sqrt{\Lambda}} + \mathcal{O} \left(\frac{1}{t_0} \right), \\ \kappa c^2 \rho(t; t_0, \Lambda, \epsilon) &= \left(\frac{3}{\epsilon^2} - 1 \right) \Lambda + \mathcal{O} \left\{ \left(1 + \frac{1}{\epsilon^2} \right) \frac{1}{t_0} \right\}, \\ \kappa p(t; t_0, \Lambda, \epsilon) &= \left(1 - \frac{1}{\epsilon^2} \right) \Lambda + \mathcal{O} \left\{ \left(1 + \frac{1}{\epsilon^2} \right) \frac{1}{t_0} \right\}, \\ t &\in J(t_0), \quad \gamma' < t_0 < \gamma'', \quad t_0 \rightarrow \infty. \end{aligned}$$

Corollary 5. On the Riemann hypothesis there is an infinite set of the microscopic (see (5.2), (5.3)) universes (5.1) (a subset of the set (4.3)) such that the state equation (see (5.4))

$$(5.5) \quad \frac{p(t; t_0, \Lambda, \epsilon)}{c^2 \rho(t; t_0, \Lambda, \epsilon)} = -\frac{1}{3} + \frac{2\epsilon^2}{9 - 3\epsilon^2} + \mathcal{O} \left(\frac{1}{\Lambda t_0} \right), \quad t \in J(t_0), \quad t_0 \rightarrow \infty.$$

From:

$$E_2 = \frac{2}{\mu^2} \Lambda + \mathcal{O} \left\{ \left(1 + \frac{1}{\mu^2} \right) \frac{1}{t_0} \right\} \quad (\text{a})$$

we obtain:

$$2/2 * 1.1056e-52 + ((1+1/2)*1/\infty) =$$

$1.1056 * 10^{-52}$, that is the value of Cosmological Constant

From:

$$\kappa c^2 \rho = \left(\frac{3}{\mu^2} - 1 \right) \Lambda + \mathcal{O} \left\{ \left(1 + \frac{1}{\mu^2} \right) \frac{1}{t_0} \right\} \quad (\text{b})$$

we obtain:

$$(3/2-1)*1.1056e-52 + ((1+1/2)*1/\infty)$$

Input interpretation:

$$\left(\frac{3}{2} - 1 \right) \times 1.1056 \times 10^{-52} + \left(1 + \frac{1}{2} \right) \times \frac{1}{\infty}$$

Result:

$$5.528 \times 10^{-53}$$

$5.528 * 10^{-53}$

From the ratio between (a) and (b):

Input interpretation:

$$\frac{\frac{2}{2} \times 1.1056 \times 10^{-52} + \left(1 + \frac{1}{2} \right) \times \frac{1}{\infty}}{\left(\frac{3}{2} - 1 \right) \times 1.1056 \times 10^{-52} + \left(1 + \frac{1}{2} \right) \times \frac{1}{\infty}}$$

Result:

2

2

and from the inverse:

$$\frac{((3/2-1)*1.1056e-52+((1+1/2)*1/infinity))}{((2/2 * 1.1056e-52+((1+1/2)*1/infinity)))}$$

Input interpretation:

$$\frac{\left(\frac{3}{2} - 1\right) \times 1.1056 \times 10^{-52} + \left(1 + \frac{1}{2}\right) \times \frac{1}{\infty}}{\frac{2}{2} \times 1.1056 \times 10^{-52} + \left(1 + \frac{1}{2}\right) \times \frac{1}{\infty}}$$

Result:

0.5

0.5

Rational form:

$$\frac{1}{2}$$

From:

$$\frac{p(t; t_0, \Lambda, \epsilon)}{c^2 \rho(t; t_0, \Lambda, \epsilon)} = -\frac{1}{3} + \frac{2\epsilon^2}{9 - 3\epsilon^2} + \mathcal{O}\left(\frac{1}{\Lambda t_0}\right) \quad (5.5)$$

we obtain:

$$-1/3 + (2 * (1.2183e-60)^2) / (9 - 3 * (1.2183e-60)^2) + O(1/infinity)$$

Input interpretation:

$$-\frac{1}{3} + \frac{2(1.2183 \times 10^{-60})^2}{9 - 3(1.2183 \times 10^{-60})^2} + \mathcal{O}\left(\frac{1}{\infty}\right)$$

Result:

$O(0) - 0.333333$

Alternate forms:

$O(0) - 0.333333$

$0.333333 (3 O(0) - 1)$

Decimal approximation:

1.618812083207842836501100130228768765693091859684179712493...

1.618812083.... result that is a very good approximation to the value of the golden ratio 1.618033988749...

Property:

$\frac{2197}{432\pi}$ is a transcendental number

Alternative representations:

$$\frac{\left(\frac{1}{\frac{2(-1)}{3}} + -\frac{2}{3}\right)^3 (-1)}{2\pi} = -\frac{\left(-\frac{2}{3} + \frac{1}{\frac{2(-1)}{3}}\right)^3}{360^\circ}$$

$$\frac{\left(\frac{1}{\frac{2(-1)}{3}} + -\frac{2}{3}\right)^3 (-1)}{2\pi} = \frac{-\left(-\frac{2}{3} + \frac{1}{\frac{2(-1)}{3}}\right)^3}{-2i \log(-1)}$$

$$\frac{\left(\frac{1}{\frac{2(-1)}{3}} + -\frac{2}{3}\right)^3 (-1)}{2\pi} = -\frac{\left(-\frac{2}{3} + \frac{1}{\frac{2(-1)}{3}}\right)^3}{2 \cos^{-1}(-1)}$$

Series representations:

$$\frac{\left(\frac{1}{\frac{2(-1)}{3}} + -\frac{2}{3}\right)^3 (-1)}{2\pi} = \frac{2197}{1728 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

$$\frac{\left(\frac{1}{\frac{2(-1)}{3}} + -\frac{2}{3}\right)^3 (-1)}{2\pi} = \frac{2197}{1728 \sum_{k=0}^{\infty} \frac{(-1)^{1+k} 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}}$$

$$\frac{\left(\frac{1}{\frac{2(-1)}{3}} + -\frac{2}{3}\right)^3 (-1)}{2\pi} = \frac{2197}{432 \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)}$$

Integral representations:

$$\frac{\left(\frac{1}{\frac{2(-1)}{3}} + -\frac{2}{3}\right)^3 (-1)}{2\pi} = \frac{2197}{1728 \int_0^1 \sqrt{1-t^2} dt}$$

$$\frac{\left(\frac{1}{\frac{2(-1)}{3}} + -\frac{2}{3}\right)^3 (-1)}{2\pi} = \frac{2197}{864 \int_0^\infty \frac{1}{1+t^2} dt}$$

$$\frac{\left(\frac{1}{\frac{2(-1)}{3}} + -\frac{2}{3}\right)^3 (-1)}{2\pi} = \frac{2197}{864 \int_0^1 \frac{1}{\sqrt{1-t^2}} dt}$$

We observe that:

$$\sqrt[128]{\frac{-3i \exp\left(\frac{1}{4}(3\pi i)\right)}{4\pi\sqrt{2}(2.74518 + 381.228i)(1893.3 + i \times (-568.069))} \sum_{n=0}^2 \exp\left(\frac{n+\frac{3}{2}}{2}\right)^2 (2\pi)}$$

$r = 1.61882$ (radius), $\theta = -0.217859^\circ$ (angle)

1.61882

and that:

$$\frac{\left(-\frac{2}{\frac{1}{3}} + -\frac{2}{3}\right)^3}{2\pi}$$

$$\frac{2197}{432\pi}$$

1.618812083207842836501100130228768765693091859684179712493...

1.618812083....

The two results are practically equal. Thence, we obtain the following possible interesting mathematical connection:

$$\left(\sqrt[128]{\frac{-3i \exp\left(\frac{1}{4}(3\pi i)\right)}{4\pi\sqrt{2}(2.74518 + 381.228i)(1893.3 + i \times (-568.069))} \sum_{n=0}^2 \exp\left(\frac{n+\frac{3}{2}}{2}\right)^2 (2\pi)} \right) = 1.61882 \Rightarrow$$

$$\Rightarrow \left(\frac{\left(-\frac{\frac{1}{2}}{1} + -\frac{2}{3}\right)^3}{2\pi} \right) = 1.618812083\dots$$

From:

New expressions for Riemann's functions $\xi(s)$ and $\Xi(t)$ – *Srinivasa Ramanujan*
 Quarterly Journal of Mathematics, XLVI, 1915, 253 – 260

We have that:

$$\begin{aligned} & n^{-\frac{3}{2}} \int_0^{\infty} v^{-\frac{1}{2}s} dv \int_0^{\infty} x e^{-\pi v x^2/n} \left(\frac{1}{e^{2\pi x} - 1} - \frac{1}{2\pi x} \right) dx \\ &= n^{-\frac{3}{2}} \int_0^{\infty} x \left(\frac{1}{e^{2\pi x} - 1} - \frac{1}{2\pi x} \right) dx \int_0^{\infty} v^{-\frac{1}{2}s} e^{-\pi v x^2/n} dv \\ &= \pi^{\frac{1}{2}(s-2)} n^{-\frac{1}{2}(s+1)} \Gamma\left(1 - \frac{1}{2}s\right) \int_0^{\infty} x^{s-1} \left(\frac{1}{e^{2\pi x} - 1} - \frac{1}{2\pi x} \right) dx \\ &= -\frac{n^{-\frac{1}{2}(s+1)}}{4\pi\sqrt{\pi}} \Gamma\left(-\frac{s}{2}\right) \Gamma\left(\frac{s-1}{2}\right) \xi(s), \end{aligned} \tag{6}$$

For

$$\xi(s) = (s - 1)\Gamma\left(1 + \frac{1}{2}s\right)\pi^{-\frac{1}{2}s}\zeta(s).$$

$$s = \sigma + it, \text{ where } 0 < \sigma < 1.$$

$$\alpha\beta = \pi^2, \text{ and } t \text{ is real,}$$

$$t = 1/3 \text{ and } s = 1/2 + 1/3 i$$

$$(1/2+1/3i-1) \text{ gamma } (1+1/2*(1/2+1/3i)) * \text{Pi}^{(-1/2*(1/2+1/3i))} * \text{zeta } (1/2+1/3i)$$

Input:

$$\left(\frac{1}{2} + \frac{1}{3}i - 1\right)\Gamma\left(1 + \frac{1}{2}\left(\frac{1}{2} + \frac{1}{3}i\right)\right)\pi^{-1/2(1/2+1/3i)}\zeta\left(\frac{1}{2} + \frac{1}{3}i\right)$$

$\Gamma(x)$ is the gamma function
 $\zeta(s)$ is the Riemann zeta function
 i is the imaginary unit

Exact result:

$$\left(-\frac{1}{2} + \frac{i}{3}\right)\pi^{-1/4-i/6}\zeta\left(\frac{1}{2} + \frac{i}{3}\right)\Gamma\left(\frac{5}{4} + \frac{i}{6}\right)$$

Decimal approximation:

0.495846082017514605932172824641474404651251707881683234276...

(using the principal branch of the logarithm for complex exponentiation)

0.4958460820.....

Alternate forms:

$$-\frac{13}{72}\pi^{-1/4-i/6}\zeta\left(\frac{1}{2} + \frac{i}{3}\right)\Gamma\left(\frac{1}{4} + \frac{i}{6}\right)$$

$$\left(-\frac{82}{229} + \frac{72i}{229}\right)\pi^{-1/4-i/6}\left(\frac{5}{4} + \frac{i}{6}\right)!\zeta\left(\frac{1}{2} + \frac{i}{3}\right)$$

$n!$ is the factorial function

Alternative representations:

$$\left(\frac{1}{2} + \frac{i}{3} - 1\right)\Gamma\left(1 + \frac{1}{2}\left(\frac{1}{2} + \frac{i}{3}\right)\right)\pi^{1/2(1/2+i/3)(-1)}\zeta\left(\frac{1}{2} + \frac{i}{3}\right) =$$

$$\left(-\frac{1}{2} + \frac{i}{3}\right)\exp\left(-\log\Gamma\left(1 + \frac{1}{2}\left(\frac{1}{2} + \frac{i}{3}\right)\right) + \log\Gamma\left(2 + \frac{1}{2}\left(\frac{1}{2} + \frac{i}{3}\right)\right)\right)\pi^{1/2(-1/2-i/3)}\zeta\left(\frac{1}{2} + \frac{i}{3}, 1\right)$$

$$\left(\frac{1}{2} + \frac{i}{3} - 1\right) \Gamma\left(1 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{3}\right)\right) \pi^{1/2(1/2+i/3)(-1)} \zeta\left(\frac{1}{2} + \frac{i}{3}\right) =$$

$$\left(-\frac{1}{2} + \frac{i}{3}\right) (1)_{\frac{1}{2}\left(\frac{1}{2} + \frac{i}{3}\right)} \pi^{1/2(-1/2-i/3)} \zeta\left(\frac{1}{2} + \frac{i}{3}, 1\right)$$

$$\left(\frac{1}{2} + \frac{i}{3} - 1\right) \Gamma\left(1 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{3}\right)\right) \pi^{1/2(1/2+i/3)(-1)} \zeta\left(\frac{1}{2} + \frac{i}{3}\right) =$$

$$\frac{\left(-\frac{1}{2} + \frac{i}{3}\right) G\left(2 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{3}\right)\right) \pi^{1/2(-1/2-i/3)} \zeta\left(\frac{1}{2} + \frac{i}{3}, 1\right)}{G\left(1 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{3}\right)\right)}$$

Series representations:

$$\left(\frac{1}{2} + \frac{i}{3} - 1\right) \Gamma\left(1 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{3}\right)\right) \pi^{1/2(1/2+i/3)(-1)} \zeta\left(\frac{1}{2} + \frac{i}{3}\right) =$$

$$\pi^{-1/4-i/6} \Gamma\left(\frac{5}{4} + \frac{i}{6}\right) \sum_{n=0}^{\infty} \frac{\sum_{k=0}^n (-1)^k (1+k)^{1/2-i/3} \binom{n}{k}}{1+n}$$

$$\left(\frac{1}{2} + \frac{i}{3} - 1\right) \Gamma\left(1 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{3}\right)\right) \pi^{1/2(1/2+i/3)(-1)} \zeta\left(\frac{1}{2} + \frac{i}{3}\right) =$$

$$\left(-\frac{1}{2} + \frac{i}{3}\right) \pi^{-1/4-i/6} \Gamma\left(\frac{5}{4} + \frac{i}{6}\right) \sum_{k=0}^{\infty} \frac{\left(\left(\frac{1}{2} + \frac{i}{3}\right) - s_0\right)^k \zeta^{(k)}(s_0)}{k!} \quad \text{for } s_0 \neq 1$$

$$\left(\frac{1}{2} + \frac{i}{3} - 1\right) \Gamma\left(1 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{3}\right)\right) \pi^{1/2(1/2+i/3)(-1)} \zeta\left(\frac{1}{2} + \frac{i}{3}\right) =$$

$$\frac{\left(1 - \frac{2i}{3}\right) 2^{-1+i/3} \pi^{-1/4-i/6} \Gamma\left(\frac{5}{4} + \frac{i}{6}\right) \sum_{n=0}^{\infty} 2^{-1-n} \sum_{k=0}^n (-1)^k (1+k)^{-1/2-i/3} \binom{n}{k}}{-2^{i/3} + \sqrt{2}}$$

Integral representations:

$$\left(\frac{1}{2} + \frac{i}{3} - 1\right) \Gamma\left(1 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{3}\right)\right) \pi^{1/2(1/2+i/3)(-1)} \zeta\left(\frac{1}{2} + \frac{i}{3}\right) =$$

$$\frac{\left(\frac{1}{2} - \frac{i}{3}\right) \pi^{-1/4-i/6} \Gamma\left(\frac{5}{4} + \frac{i}{6}\right)}{(1 - 2^{1/2-i/3}) \Gamma\left(\frac{1}{2} + \frac{i}{3}\right)} \int_0^{\infty} \frac{t^{-1/2+i/3}}{1+e^t} dt$$

$$\left(\frac{1}{2} + \frac{i}{3} - 1\right) \Gamma\left(1 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{3}\right)\right) \pi^{1/2(1/2+i/3)(-1)} \zeta\left(\frac{1}{2} + \frac{i}{3}\right) =$$

$$\frac{\left(1 - \frac{2i}{3}\right) 2^{-1+i/3} \pi^{-1/4-i/6} \left(\int_0^{\infty} \frac{t^{-1/2+i/3}}{1+e^t} dt\right) \int_0^1 \log^{1/4+i/6}\left(\frac{1}{t}\right) dt}{(-2^{i/3} + \sqrt{2}) \Gamma\left(\frac{1}{2} + \frac{i}{3}\right)}$$

$$\left(\frac{1}{2} + \frac{i}{3} - 1\right) \Gamma\left(1 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{3}\right)\right) \pi^{1/2(1/2+i/3)(-1)} \zeta\left(\frac{1}{2} + \frac{i}{3}\right) =$$

$$\frac{\left(1 - \frac{2i}{3}\right) 2^{-3/2+(2i)/3} \pi^{-1/4-i/6} \left(\int_0^1 \log^{1/4+i/6}\left(\frac{1}{t}\right) dt\right) \int_0^{\infty} t^{1/2+i/3} \operatorname{sech}^2(t) dt}{(-2^{i/3} + \sqrt{2}) \Gamma\left(\frac{3}{2} + \frac{i}{3}\right)}$$

From:

$$-\frac{n^{-\frac{1}{2}(s+1)}}{4\pi\sqrt{\pi}}\Gamma\left(-\frac{s}{2}\right)\Gamma\left(\frac{s-1}{2}\right)\xi(s),$$

n is real,

we obtain:

$$\left(\frac{(-0.25^{-(1/2((3/2+i/3))))}{(4*\text{Pi}*\text{sqrt}(\text{Pi}))} * \text{gamma}(-(1/2+1/3i)*1/2) * \text{gamma}(1/2(-1/2+i/3)) * (0.4958460820)\right)$$

Input interpretation:

$$-\frac{0.25^{-1/2(3/2+i/3)}}{4\pi\sqrt{\pi}}\Gamma\left(-\left(\frac{1}{2} + \frac{1}{3}i\right) \times \frac{1}{2}\right)\Gamma\left(\frac{1}{2}\left(-\frac{1}{2} + \frac{i}{3}\right)\right) \times 0.4958460820$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

$$-0.951190... - 0.223768... i$$

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$$r = 0.977157 \text{ (radius), } \theta = -166.762^\circ \text{ (angle)}$$

0.977157

Alternative representations:

$$\frac{\left(\Gamma\left(-\frac{1}{2}\left(\frac{1}{2} + \frac{i}{3}\right)\right)\Gamma\left(\frac{1}{2}\left(-\frac{1}{2} + \frac{i}{3}\right)\right)0.495846\right)(-1)0.25^{-1/2(3/2+i/3)}}{4\pi\sqrt{\pi}} = \frac{0.495846\left(-1 + \frac{1}{2}\left(-\frac{1}{2} - \frac{i}{3}\right)\right)!\left(-1 + \frac{1}{2}\left(-\frac{1}{2} + \frac{i}{3}\right)\right)!0.25^{1/2(-3/2-i/3)}}{4\pi\sqrt{\pi}}$$

$$\frac{\left(\Gamma\left(-\frac{1}{2}\left(\frac{1}{2} + \frac{i}{3}\right)\right)\Gamma\left(\frac{1}{2}\left(-\frac{1}{2} + \frac{i}{3}\right)\right)0.495846\right)(-1)0.25^{-1/2(3/2+i/3)}}{4\pi\sqrt{\pi}} = \frac{1}{4\pi\sqrt{\pi}}0.495846 \times 0.25^{1/2(-3/2-i/3)} \exp\left(\log G\left(1 + \frac{1}{2}\left(-\frac{1}{2} - \frac{i}{3}\right)\right) - \log G\left(\frac{1}{2}\left(-\frac{1}{2} - \frac{i}{3}\right)\right)\right) \exp\left(\log G\left(1 + \frac{1}{2}\left(-\frac{1}{2} + \frac{i}{3}\right)\right) - \log G\left(\frac{1}{2}\left(-\frac{1}{2} + \frac{i}{3}\right)\right)\right)$$

$$\frac{\left(\Gamma\left(-\frac{1}{2}\left(\frac{1}{2} + \frac{i}{3}\right)\right)\Gamma\left(\frac{1}{2}\left(-\frac{1}{2} + \frac{i}{3}\right)\right)0.495846\right)(-1)0.25^{-1/2(3/2+i/3)}}{4\pi\sqrt{\pi}} =$$

$$\frac{0.495846 G\left(1 + \frac{1}{2}\left(-\frac{1}{2} - \frac{i}{3}\right)\right)G\left(1 + \frac{1}{2}\left(-\frac{1}{2} + \frac{i}{3}\right)\right)0.25^{1/2(-3/2-i/3)}}{G\left(\frac{1}{2}\left(-\frac{1}{2} - \frac{i}{3}\right)\right)G\left(\frac{1}{2}\left(-\frac{1}{2} + \frac{i}{3}\right)\right)(4\pi\sqrt{\pi})}$$

Series representations:

$$\frac{\left(\Gamma\left(-\frac{1}{2}\left(\frac{1}{2} + \frac{i}{3}\right)\right)\Gamma\left(\frac{1}{2}\left(-\frac{1}{2} + \frac{i}{3}\right)\right)0.495846\right)(-1)0.25^{-1/2(3/2+i/3)}}{4\pi\sqrt{\pi}} =$$

$$\frac{0.350616 e^{0.231049i} \Gamma\left(-\frac{1}{4} - \frac{i}{6}\right)\Gamma\left(-\frac{1}{4} + \frac{i}{6}\right)}{\pi \exp\left(\pi \mathcal{A}\left[\frac{\operatorname{sig}(\pi-x)}{2\pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (\pi-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}} \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{\left(\Gamma\left(-\frac{1}{2}\left(\frac{1}{2} + \frac{i}{3}\right)\right)\Gamma\left(\frac{1}{2}\left(-\frac{1}{2} + \frac{i}{3}\right)\right)0.495846\right)(-1)0.25^{-1/2(3/2+i/3)}}{4\pi\sqrt{\pi}} =$$

$$\frac{12.6222 e^{0.231049i} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(-\frac{1}{4} - \frac{i}{6}\right)^{k_1} \left(-\frac{1}{4} + \frac{i}{6}\right)^{k_2} \Gamma^{(k_1)}(1)\Gamma^{(k_2)}(1)}{k_1!k_2!}}{(-1.5+i)(1.5+i)\pi\sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k}}$$

$$\frac{\left(\Gamma\left(-\frac{1}{2}\left(\frac{1}{2} + \frac{i}{3}\right)\right)\Gamma\left(\frac{1}{2}\left(-\frac{1}{2} + \frac{i}{3}\right)\right)0.495846\right)(-1)0.25^{-1/2(3/2+i/3)}}{4\pi\sqrt{\pi}} =$$

$$\frac{0.350616 e^{0.231049i} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(-\frac{1}{4} - \frac{i}{6} - z_0\right)^{k_1} \left(-\frac{1}{4} + \frac{i}{6} - z_0\right)^{k_2} \Gamma^{(k_1)}(z_0)\Gamma^{(k_2)}(z_0)}{k_1!k_2!}}{\pi\sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k}}$$

for $(z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$

Integral representations:

$$\frac{\left(\Gamma\left(-\frac{1}{2}\left(\frac{1}{2} + \frac{i}{3}\right)\right)\Gamma\left(\frac{1}{2}\left(-\frac{1}{2} + \frac{i}{3}\right)\right)0.495846\right)(-1)0.25^{-1/2(3/2+i/3)}}{4\pi\sqrt{\pi}} =$$

$$-\frac{1}{\pi\sqrt{\pi}} 0.350616 e^{0.231049i} \operatorname{csc}\left(\frac{1}{24}(-3+2i)\pi\right)$$

$$\operatorname{csc}\left(-\frac{1}{24}(3+2i)\pi\right) \left(\int_0^{\infty} t^{-5/4-i/6} \sin(t) dt\right) \int_0^{\infty} t^{-5/4+i/6} \sin(t) dt$$

$$\frac{\left(\Gamma\left(-\frac{1}{2}\left(\frac{1}{2} + \frac{i}{3}\right)\right)\Gamma\left(\frac{1}{2}\left(-\frac{1}{2} + \frac{i}{3}\right)\right)0.495846\right)(-1)0.25^{-1/2(3/2+i/3)}}{4\pi\sqrt{\pi}} =$$

$$-\frac{1}{\pi\sqrt{\pi}}0.350616e^{0.231049i}\left(\int_0^\infty e^{-t}t^{-5/4-i/6}\left(1-e^{-t}\sum_{k=0}^n\frac{(-t)^k}{k!}\right)dt\right)$$

$$\int_0^\infty e^{-t}t^{-5/4+i/6}\left(1-e^{-t}\sum_{k=0}^n\frac{(-t)^k}{k!}\right)dt \text{ for } \left(n \in \mathbb{Z} \text{ and } 0 \leq n < \frac{1}{4}\right)$$

$$\frac{\left(\Gamma\left(-\frac{1}{2}\left(\frac{1}{2} + \frac{i}{3}\right)\right)\Gamma\left(\frac{1}{2}\left(-\frac{1}{2} + \frac{i}{3}\right)\right)0.495846\right)(-1)0.25^{-1/2(3/2+i/3)}}{4\pi\sqrt{\pi}} =$$

$$\frac{1.40246e^{0.231049i}\pi\mathcal{A}^2}{\sqrt{\pi}\int_L e^t t^{1/4+i/6} dt \int_L e^t t^{1/4-i/6} dt}$$

From which, we obtain:

$$\left(\frac{\left(\left(\left(\left(\left(-0.25^{-1/2\left(\left(\frac{3}{2}+i/3\right)\right)}\right)\right)\right)\right)\right)}{\left(\left(4*\text{Pi}*\text{sqrt}(\text{Pi})\right)\right)} * \text{gamma}\left(-\left(x+1/3i\right)*1/2\right) * \text{gamma}\left(1/2\left(-1/2+i/3\right)\right) * \left(0.4958460820\right)\right)\right)\right)\right) = (-0.951190-0.223768i)$$

Input interpretation:

$$-\frac{0.25^{-1/2(3/2+i/3)}}{4\pi\sqrt{\pi}}\Gamma\left(-\left(x+\frac{1}{3}i\right)\times\frac{1}{2}\right)\Gamma\left(\frac{1}{2}\left(-\frac{1}{2}+\frac{i}{3}\right)\right)\times 0.4958460820 =$$

$$-0.951190 + i \times (-0.223768)$$

$\Gamma(x)$ is the gamma function
 i is the imaginary unit

Result:

$$(0.198653 + 0.148543 i)\Gamma\left(\frac{1}{2}\left(-x - \frac{i}{3}\right)\right) = -0.95119 - 0.223768 i$$

Alternate forms:

$$(0.198653 + 0.148543 i)\Gamma\left(-\frac{x}{2} - \frac{i}{6}\right) = -0.95119 - 0.223768 i$$

$$(3.6113 - 1.57393 i) + \Gamma\left(-\frac{x}{2} - \frac{i}{6}\right) = 0$$

Alternate form assuming x is positive:

$$(3.6113 - 1.57393 i) + \Gamma\left(\frac{1}{6}(-3x - i)\right) = 0$$

Numerical solutions:

$$x \approx -10.7611 - 10.7396 i \dots$$

$$x \approx -8.20557 - 4.42894 i \dots$$

$$x \approx 0.5 + 4.53581 \times 10^{-8} i \dots$$

$$x \approx 1.51076 - 0.657755 i \dots$$

$$x \approx 4.21602 - 0.245925 i \dots$$

From this solution, we obtain:

$$0.5 + 4.53581 e^{-8} i$$

Input interpretation:

$$0.5 + 4.53581 \times 10^{-8} i$$

i is the imaginary unit

Result:

$$0.5 + 4.53581 \dots \times 10^{-8} i$$

Polar coordinates:

$$r = 0.5 \text{ (radius), } \theta = 5.19766 \times 10^{-6} \text{ (angle)}$$

$$0.5 = 1/2$$

and:

$$\left(\frac{(-0.25)^{-1/2} \left(\frac{3}{2} + i/3 \right)}{4 \pi \sqrt{\pi}} \right) / \left((4 \pi \sqrt{\pi}) \right) * \text{gamma} \left(-\left(\frac{1}{2} + x i \right) * \frac{1}{2} \right) * \text{gamma} \left(\frac{1}{2} \left(-\frac{1}{2} + i/3 \right) \right) * (0.4958460820) = (-0.951190 - 0.223768 i)$$

Input interpretation:

$$-\frac{0.25^{-1/2} \left(\frac{3}{2} + i/3 \right)}{4 \pi \sqrt{\pi}} \Gamma \left(-\left(\frac{1}{2} + x i \right) \times \frac{1}{2} \right) \Gamma \left(\frac{1}{2} \left(-\frac{1}{2} + \frac{i}{3} \right) \right) \times 0.4958460820 = -0.951190 + i \times (-0.223768)$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

$$(0.198653 + 0.148543 i) \Gamma \left(\frac{1}{2} \left(-i x - \frac{1}{2} \right) \right) = -0.95119 - 0.223768 i$$

Alternate form:

$$(0.198653 + 0.148543 i) \Gamma \left(-\frac{i x}{2} - \frac{1}{4} \right) = -0.95119 - 0.223768 i$$

Alternate form assuming x is positive:

$$(3.6113 - 1.57393 i) + \Gamma \left(-\frac{i x}{2} - \frac{1}{4} \right) = 0$$

Numerical solutions:

$$x \approx -0.324422 - 1.01076 i \dots$$

$$x \approx 0.333333 - 3.48953 \times 10^{-7} i \dots$$

$$x \approx 5.0751 + 9.07377 i \dots$$

From this solution, we obtain:

$$(0.333333 - 3.48953 \times 10^{-7} i)$$

Input interpretation:

$$0.333333 - 3.48953 \times 10^{-7} i$$

i is the imaginary unit

Result:

$$0.333333 \dots - 3.48953 \dots \times 10^{-7} i$$

Polar coordinates:

$$r = 0.333333 \text{ (radius)}, \quad \theta = -0.0000599807^\circ \text{ (angle)}$$

$$0.333333 = 1/3$$

From which:

$$-1/(2\pi) \left(\frac{0.5 + 4.53581 \times 10^{-8} i}{-0.333333 - 3.48953 \times 10^{-7} i} \right) + \left(\frac{1}{0.5 + 4.53581 \times 10^{-8} i} \right) \left(\frac{-1}{-0.333333 - 3.48953 \times 10^{-7} i} \right) \right)^3$$

Input interpretation:

$$\frac{\left(-\frac{0.5 + 4.53581 \times 10^{-8} i}{0.333333 - 3.48953 \times 10^{-7} i} + \frac{1}{\frac{0.5 + 4.53581 \times 10^{-8} i}{0.333333 - 3.48953 \times 10^{-7} i}} \right)^3}{2\pi}$$

i is the imaginary unit

Result:

$$1.61881 \dots + 2.12484 \dots \times 10^{-6} i$$

Polar coordinates:

$r = 1.61881$ (radius), $\theta = 0.0000752059^\circ$ (angle)

1.61881 result that is a very good approximation to the value of the golden ratio 1.618033988749... and almost equal to the result of the below expression:

$$\left(\sqrt[128]{\frac{-3i \exp\left(\frac{1}{4}(3\pi i)\right)}{4\pi\sqrt{2}(2.74518 + 381.228i)(1893.3 + i \times (-568.069))} \sum_{n=0}^2 \exp\left(\frac{(n+\frac{3}{2})^2}{2\pi}\right)} \right) = 1.61882$$

We have also:

1. The principal object of this paper is to prove that if the real parts of α and β are positive, and $\alpha\beta = \pi^2$, and t is real, then

$$\begin{aligned} & \alpha^{-\frac{1}{4}} \left\{ \frac{1}{1+t^2} + 4\alpha \int_0^\infty \left(\frac{1}{3^2+t^2} - \frac{\alpha}{1!7^2+t^2} + \frac{\alpha^2}{2!11^2+t^2} - \dots \right) \frac{x dx}{e^{2\pi x} - 1} \right\} \\ & - \beta^{-\frac{1}{4}} \left\{ \frac{1}{1+t^2} + 4\beta \int_0^\infty \left(\frac{1}{3^2+t^2} - \frac{\beta}{1!7^2+t^2} + \frac{\beta^2}{2!11^2+t^2} - \dots \right) \frac{x dx}{e^{2\pi x} - 1} \right\} \\ & = \frac{\pi^{-\frac{3}{4}}}{4t} \Gamma\left(\frac{-1+it}{4}\right) \Gamma\left(\frac{-1-it}{4}\right) \Xi\left(\frac{t}{2}\right) \sin\left(\frac{t}{8} \log \frac{\beta}{\alpha}\right). \end{aligned} \tag{1}$$

$$\xi\left(\frac{1}{2} + \frac{1}{2}it\right) = \Xi\left(\frac{1}{2}t\right),$$

$(1/2+1/2*i*1/3)0.4958460820$

Input interpretation:

$$\left(\frac{1}{2} + \frac{1}{2}i \times \frac{1}{3}\right) \times 0.4958460820$$

i is the imaginary unit

Result:

0.247923041... +
0.08264101367... i

Polar coordinates:

$r = 0.261334$ (radius), $\theta = 18.4349^\circ$ (angle)

0.261334

$$(8/5 * \pi * 5/8 * \pi) = \pi^2$$

Input:

$$\frac{8}{5} \pi \times \frac{5}{8} \pi = \pi^2$$

Result:

True

Thence, from:

$$\begin{aligned} & \alpha^{-\frac{1}{4}} \left\{ \frac{1}{1+t^2} + 4\alpha \int_0^\infty \left(\frac{1}{3^2+t^2} - \frac{\alpha}{1! 7^2+t^2} + \frac{\alpha^2}{2! 11^2+t^2} - \dots \right) \frac{x dx}{e^{2\pi x} - 1} \right\} \\ & - \beta^{-\frac{1}{4}} \left\{ \frac{1}{1+t^2} + 4\beta \int_0^\infty \left(\frac{1}{3^2+t^2} - \frac{\beta}{1! 7^2+t^2} + \frac{\beta^2}{2! 11^2+t^2} - \dots \right) \frac{x dx}{e^{2\pi x} - 1} \right\} \\ & = \frac{\pi^{-\frac{3}{4}}}{4t} \Gamma\left(\frac{-1+it}{4}\right) \Gamma\left(\frac{-1-it}{4}\right) \Xi\left(\frac{t}{2}\right) \sin\left(\frac{t}{8} \log \frac{\beta}{\alpha}\right). \end{aligned}$$

we obtain:

$$\begin{aligned} & (\pi^{-(3/4)}) / (4 * 1/3) * \text{gamma}(1/4 * (-1 + 1/3 * i)) * \text{gamma}(1/4 * (-1 - 1/3 * i)) * \\ & (0.261334) * \sin(\left(\left(\left(1/3 * 1/8 \ln\left(\left(\left(8/5\right)\pi\right)/\left(\left(5/8\right)\pi\right)\right)\right)\right)\right)) \end{aligned}$$

Input interpretation:

$$\frac{\pi^{-3/4}}{4 \times \frac{1}{3}} \Gamma\left(\frac{1}{4} \times \left(-1 + \frac{1}{3} i\right)\right) \Gamma\left(\frac{1}{4} \times \left(-1 - \frac{1}{3} i\right)\right) \times 0.261334 \sin\left(\frac{1}{3} \times \frac{1}{8} \log\left(\frac{8}{5} \frac{\pi}{\pi}\right)\right)$$

$\Gamma(x)$ is the gamma function
 $\log(x)$ is the natural logarithm
 i is the imaginary unit

Result:

0.0691038...

0.0691038...**Alternate form:**

0.0691038

Alternative representations:

$$\frac{\left(\Gamma\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)\right)0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3\times 8}\right)}{\frac{4}{3}} =$$

$$\frac{0.261334G\left(1+\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)G\left(1+\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\cos\left(\frac{\pi}{2}-\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3\times 8}\right)\pi^{-3/4}}{\frac{4}{3}G\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)G\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)}$$

$$\frac{\left(\Gamma\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)\right)0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3\times 8}\right)}{\frac{4}{3}} =$$

$$\frac{0.261334G\left(1+\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)G\left(1+\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\cos\left(\frac{\pi}{2}+\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3\times 8}\right)\pi^{-3/4}}{\frac{4}{3}G\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)G\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)}$$

$$\frac{\left(\Gamma\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)\right)0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3\times 8}\right)}{\frac{4}{3}} =$$

$$0.261334\cos\left(\frac{\pi}{2}-\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3\times 8}\right)\left(1\right)_{-1+\frac{1}{4}\left(-1-\frac{i}{3}\right)}\left(1\right)_{-1+\frac{1}{4}\left(-1+\frac{i}{3}\right)}\pi^{-3/4}$$

Series representations:

$$\frac{\left(\Gamma\left(\frac{1}{4}\left(-1 + \frac{i}{3}\right)\right)\pi^{-3/4} \Gamma\left(\frac{1}{4}\left(-1 - \frac{i}{3}\right)\right)\right) 0.261334 \sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)}{\frac{4}{3}} = -\frac{1}{(-9 + i^2)\pi^{3/4}} 56.4481$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 12^{-k_2-k_3} (-3-i)^{k_2} (-3+i)^{k_3} J_{1+2k_1}\left(\frac{1}{24} \log\left(\frac{64}{25}\right)\right) \Gamma^{(k_2)}(1) \Gamma^{(k_3)}(1)}{k_2! k_3!}$$

$$\frac{\left(\Gamma\left(\frac{1}{4}\left(-1 + \frac{i}{3}\right)\right)\pi^{-3/4} \Gamma\left(\frac{1}{4}\left(-1 - \frac{i}{3}\right)\right)\right) 0.261334 \sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)}{\frac{4}{3}} =$$

$$-\frac{1}{(-9 + i^2)\pi^{3/4}} 28.2241 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{(1+2k_1)! k_2! k_3!} (-1)^{k_1} 2^{-3-6k_1-2k_2-2k_3} \times$$

$$3^{-1-2k_1-k_2-k_3} (-3-i)^{k_2} (-3+i)^{k_3} \log^{1+2k_1}\left(\frac{64}{25}\right) \Gamma^{(k_2)}(1) \Gamma^{(k_3)}(1)$$

$$\frac{\left(\Gamma\left(\frac{1}{4}\left(-1 + \frac{i}{3}\right)\right)\pi^{-3/4} \Gamma\left(\frac{1}{4}\left(-1 - \frac{i}{3}\right)\right)\right) 0.261334 \sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)}{\frac{4}{3}} =$$

$$\frac{1}{\pi^{3/4}} 0.392001 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_2! k_3!} (-1)^{k_1} J_{1+2k_1}\left(\frac{1}{24} \log\left(\frac{64}{25}\right)\right) \left(-\frac{1}{4} - \frac{i}{12} - z_0\right)^{k_2}$$

$$\left(-\frac{1}{4} + \frac{i}{12} - z_0\right)^{k_3} \Gamma^{(k_2)}(z_0) \Gamma^{(k_3)}(z_0) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$\frac{\left(\Gamma\left(\frac{1}{4}\left(-1 + \frac{i}{3}\right)\right)\pi^{-3/4} \Gamma\left(\frac{1}{4}\left(-1 - \frac{i}{3}\right)\right)\right) 0.261334 \sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)}{\frac{4}{3}} = \frac{1}{\pi^{3/4}} 0.196001$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{(1+2k_1)! k_2! k_3!} (-1)^{k_1} 24^{-1-2k_1} \log^{1+2k_1}\left(\frac{64}{25}\right) \left(-\frac{1}{4} - \frac{i}{12} - z_0\right)^{k_2}$$

$$\left(-\frac{1}{4} + \frac{i}{12} - z_0\right)^{k_3} \Gamma^{(k_2)}(z_0) \Gamma^{(k_3)}(z_0) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

Integral representations:

$$\frac{\left(\Gamma\left(\frac{1}{4}\left(-1 + \frac{i}{3}\right)\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}\left(-1 - \frac{i}{3}\right)\right)\right)0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)}{\pi^{3/4}} =$$

$$\frac{0.00816669\Gamma\left(-\frac{1}{4} - \frac{i}{12}\right)\Gamma\left(\frac{3}{12}(-3 + i)\right)\log\left(\frac{64}{25}\right)}{\pi^{3/4}} \int_0^1 \cos\left(\frac{1}{24}t \log\left(\frac{64}{25}\right)\right) dt$$

$$\frac{\left(\Gamma\left(\frac{1}{4}\left(-1 + \frac{i}{3}\right)\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}\left(-1 - \frac{i}{3}\right)\right)\right)0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)}{\pi^{3/4}} =$$

$$\frac{0.0326668\pi^{5/4}\mathcal{A}^2\log\left(\frac{64}{25}\right)}{\int_L \oint e^t t^{1/4+i/12} dt \int_L \oint e^t t^{1/4-i/12} dt} \int_0^1 \cos\left(\frac{1}{24}t \log\left(\frac{64}{25}\right)\right) dt$$

$$\frac{\left(\Gamma\left(\frac{1}{4}\left(-1 + \frac{i}{3}\right)\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}\left(-1 - \frac{i}{3}\right)\right)\right)0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)}{\pi^{3/4}} =$$

$$\frac{0.00816669\sqrt[4]{\pi}\mathcal{A}\log\left(\frac{64}{25}\right)\sqrt{\pi}}{\int_L \oint e^t t^{1/4+i/12} dt \int_L \oint e^t t^{1/4-i/12} dt} \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{e^{s-\log^2\left(\frac{64}{25}\right)/(2304s)}}{s^{3/2}} ds \text{ for } \gamma > 0$$

Multiple-argument formulas:

$$\frac{\left(\Gamma\left(\frac{1}{4}\left(-1 + \frac{i}{3}\right)\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}\left(-1 - \frac{i}{3}\right)\right)\right)0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)}{\pi^{3/4}} =$$

$$\frac{0.0692966\cos\left(\frac{1}{48}\log\left(\frac{64}{25}\right)\right)\Gamma\left(-\frac{1}{8} - \frac{i}{24}\right)\Gamma\left(\frac{3}{8} - \frac{i}{24}\right)\Gamma\left(\frac{1}{24}(-3 + i)\right)\Gamma\left(\frac{9+i}{24}\right)\sin\left(\frac{1}{48}\log\left(\frac{64}{25}\right)\right)}{\pi^{3/4}\sqrt{\pi}^2}$$

Alternate form:

1.65661

Alternative representations:

$$1 + \frac{1}{\sqrt[8]{\frac{2}{\left(\Gamma\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)\right)0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3\times 8}\right)}\frac{4}{3}}} =$$

$$1 + \frac{1}{\sqrt[8]{\frac{2}{0.261334\cos\left(\frac{\pi}{2}-\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3\times 8}\right)\left(1\right)^{-1+\frac{1}{4}\left(-1-\frac{i}{3}\right)}\left(1\right)^{-1+\frac{1}{4}\left(-1+\frac{i}{3}\right)}\pi^{-3/4}}\frac{4}{3}}} =$$

$$1 + \frac{1}{\sqrt[8]{\frac{2}{\left(\Gamma\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)\right)0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3\times 8}\right)}\frac{4}{3}}} = 1 +$$

$$\frac{1}{\sqrt[8]{\frac{2}{0.261334\left(1\right)^{-1+\frac{1}{4}\left(-1-\frac{i}{3}\right)}\left(1\right)^{-1+\frac{1}{4}\left(-1+\frac{i}{3}\right)}\left(-e^{-i\log\left(\frac{8\pi}{5(5\pi)}\right)/(3\times 8)}+e^{i\log\left(\frac{8\pi}{5(5\pi)}\right)/(3\times 8)}\right)\pi^{-3/4}}\frac{4(2i)}{3}}} =$$

$$\begin{aligned}
& 1 + \frac{1}{\sqrt[8]{\frac{2}{\left(\Gamma\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)\right)0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3\times 8}\right)}\right)} = \\
& \left(1 + 1 / \left(\left(2 / \frac{1}{\frac{4}{3}} 0.261334 \cos\left(\frac{\pi}{2} - \frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3\times 8}\right)\right) \right. \right. \\
& \quad \left. \left. \exp\left(-\log G\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right) + \log G\left(1+\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)\right) \right. \right. \\
& \quad \left. \left. \exp\left(-\log G\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right) + \log G\left(1+\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\right)\pi^{-3/4} \right)^{\wedge (1/8)} = \right. \\
& \left. 1 + \frac{0.748003}{\sqrt[8]{\frac{\exp\left(-\log G\left(1+\frac{1}{4}\left(-1-\frac{i}{3}\right)\right) - \log G\left(1+\frac{1}{4}\left(-1+\frac{i}{3}\right)\right) + \log G\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right) + \log G\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\right)\pi^{3/4}}{\cos\left(\frac{\pi}{2} - \frac{1}{24}\log\left(\frac{64}{25}\right)\right)}} \right)
\end{aligned}$$

Series representations:

$$\begin{aligned}
& 1 + \frac{1}{\sqrt[8]{\frac{2}{\left(\Gamma\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)\right)0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)}\frac{4}{3}}} = \\
& -\frac{1}{(-9+i^2)\pi^{3/4}} 1.51819 \left(5.92809 \pi^{3/4} - 0.658677 i^2 \pi^{3/4} + \right. \\
& \quad \left(-\left((-9+i^2)\pi^{3/4} \right) / \left(\sum_{k=0}^{\infty} (-1)^k J_{1+2k} \left(\frac{1}{24} \log\left(\frac{64}{25}\right) \right) \right) \right. \\
& \quad \quad \left. \left(\sum_{k=0}^{\infty} \frac{12^{-k} (-3-i)^k \Gamma^{(k)}(1)}{k!} \right) \sum_{k=0}^{\infty} \frac{12^{-k} (-3+i)^k \Gamma^{(k)}(1)}{k!} \right) \right)^{7/8} \\
& \quad \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_2! k_3!} (-1)^{k_1} 12^{-k_2-k_3} (-3-i)^{k_2} (-3+i)^{k_3} \\
& \quad \left. J_{1+2k_1} \left(\frac{1}{24} \log\left(\frac{64}{25}\right) \right) \Gamma^{(k_2)}(1) \Gamma^{(k_3)}(1) \right)
\end{aligned}$$

$$\begin{aligned}
& 1 + \frac{1}{\sqrt[8]{\frac{2}{\left(\Gamma\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)\right)0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)}\frac{4}{3}}} = \\
& -\frac{1}{(-3+i)(3+i)\pi^{3/4}} 1.15358 \left(7.80181 \pi^{3/4} - 0.866868 i^2 \pi^{3/4} + \right. \\
& \quad 1.31607 \left(-\left((-9+i^2)\pi^{3/4} \right) / \left(\sum_{k=0}^{\infty} (-1)^k J_{1+2k} \left(\frac{1}{24} \log\left(\frac{64}{25}\right) \right) \right) \right. \\
& \quad \quad \left. \left(\sum_{k=0}^{\infty} \frac{12^{-k} (-3-i)^k \Gamma^{(k)}(1)}{k!} \right) \sum_{k=0}^{\infty} \frac{12^{-k} (-3+i)^k \Gamma^{(k)}(1)}{k!} \right) \right)^{7/8} \\
& \quad \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_2! k_3!} (-1)^{k_1} 4^{-k_2-k_3} \left(-1-\frac{i}{3}\right)^{k_2} \left(-1+\frac{i}{3}\right)^{k_3} \\
& \quad \left. J_{1+2k_1} \left(\frac{1}{24} \log\left(\frac{64}{25}\right) \right) \Gamma^{(k_2)}(1) \Gamma^{(k_3)}(1) \right)
\end{aligned}$$

$$\begin{aligned}
& 1 + \frac{1}{\sqrt[8]{\frac{2}{\left(\Gamma\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)\right)0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3\times 8}\right)}\frac{4}{3}}} = \\
& -\frac{1}{(-9+i^2)\pi^{3/4}}1.39219\left(6.46463\pi^{3/4}-0.718292i^2\pi^{3/4}+\right. \\
& \left(-\left((-9+i^2)\pi^{3/4}\right)/\left(\sum_{k=0}^{\infty}\frac{(-1)^k24^{-1-2k}\log^{1+2k}\left(\frac{64}{25}\right)}{(1+2k)!}\right)\right. \\
& \left.\left.\left(\sum_{k=0}^{\infty}\frac{12^{-k}(-3-i)^k\Gamma^{(k)}(1)}{k!}\right)\sum_{k=0}^{\infty}\frac{12^{-k}(-3+i)^k\Gamma^{(k)}(1)}{k!}\right)\right)^{7/8} \\
& \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\left((-1)^{k_1}2^{-3-6k_1-2k_2-2k_3}\times 3^{-1-2k_1-k_2-k_3}(-3-i)^{k_2}\right. \\
& \left.(-3+i)^{k_3}\log^{1+2k_1}\left(\frac{64}{25}\right)\Gamma^{(k_2)}(1)\Gamma^{(k_3)}(1)\right)/((1+2k_1)!k_2!k_3!)
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& 1 + \frac{1}{\sqrt[8]{\frac{2}{\left(\Gamma\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)\right)0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3\times 8}\right)}\frac{4}{3}}} = \\
& 1 + \frac{0.597908}{\sqrt[8]{\frac{\csc\left(\frac{1}{12}\log\left(\frac{8}{5}\right)\right)\log\left(\frac{8}{5}\right)}{12\pi^{5/4}\mathcal{A}^2\log\left(\frac{64}{25}\right)}\int_L\phi e^t t^{1/4+i/12} dt \int_L\phi e^t t^{1/4-i/12} dt}}
\end{aligned}$$

$$1 + \frac{1}{\sqrt[8]{\frac{2}{\left(\Gamma\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)\right)0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)}}} =$$

$$\frac{4}{3}$$

$$0.597908$$

$$1 + \frac{1}{\sqrt[8]{\frac{\csc\left(\frac{1}{12}\log\left(\frac{8}{5}\right)\right)\log\left(\frac{8}{5}\right)}{12\pi^{5/4}\mathcal{A}^2\log\left(\frac{64}{25}\right)}\int_L \phi e^t t^{1/4+i/12} dt \int_L \phi e^t t^{1/4(1-i/3)} dt}}$$

$$1 + \frac{1}{\sqrt[8]{\frac{2}{\left(\Gamma\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)\right)0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)}}} =$$

$$\frac{4}{3}$$

$$0.597908$$

$$1 + \frac{1}{\sqrt[8]{\frac{\mathcal{A}^2 \csc\left(\frac{1}{12}\log\left(\frac{8}{5}\right)\right)\log\left(\frac{8}{5}\right)}{12\pi^{5/4}\log\left(\frac{64}{25}\right)}\int_L \phi e^{-t} (-t)^{1/4+i/12} dt \int_L \phi e^{-t} (-t)^{1/4-i/12} dt}}$$

Multiple-argument formulas:

$$1 + \frac{1}{\sqrt[8]{\frac{2}{\left(\Gamma\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)\right)0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)}}} =$$

$$\frac{4}{3}$$

$$0.656841$$

$$1 + \frac{1}{\sqrt[8]{\frac{\pi^{3/4}\sqrt{\pi}^2}{\cos\left(\frac{1}{48}\log\left(\frac{64}{25}\right)\right)\Gamma\left(-\frac{1}{8}-\frac{i}{24}\right)\Gamma\left(\frac{3}{8}-\frac{i}{24}\right)\Gamma\left(\frac{1}{24}(-3+i)\right)\Gamma\left(\frac{9+i}{24}\right)\sin\left(\frac{1}{48}\log\left(\frac{64}{25}\right)\right)}}}}$$

$$1 + \frac{1}{\sqrt[8]{\frac{2}{\left(\Gamma\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)\right)0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3\times 8}\right)}}{\frac{4}{3}}} = 0.558232$$

$$1 + \frac{1}{\sqrt[8]{\frac{\pi^{11/4}}{\cos\left(\frac{1}{48}\log\left(\frac{64}{25}\right)\right)\Gamma\left(\frac{1}{36}\left(-3-i\right)\right)\Gamma\left(\frac{1}{4}-\frac{i}{36}\right)\Gamma\left(\frac{7}{12}-\frac{i}{36}\right)\Gamma\left(\frac{1}{36}\left(-3+i\right)\right)\Gamma\left(\frac{9+i}{36}\right)\Gamma\left(\frac{21+i}{36}\right)\sin\left(\frac{1}{48}\log\left(\frac{64}{25}\right)\right)}}{\frac{4}{3}}}} = 0.748003$$

And, we have also:

$$47 \times \frac{1}{\left(\frac{2}{\left(\frac{\pi^{-3/4}}{4 \times \frac{1}{3}}\right) \Gamma\left(\frac{1}{4}\left(-1+\frac{1}{3}i\right)\right) \Gamma\left(\frac{1}{4}\left(-1-\frac{1}{3}i\right)\right)\right) 0.261334 \sin\left(\frac{1}{3} \times \frac{1}{8} \ln\left(\frac{(8/5)\pi}{(5/8)\pi}\right)\right)}\right)}$$

Input interpretation:

$$47 \times \frac{1}{\frac{2}{\frac{\pi^{-3/4}}{4 \times \frac{1}{3}} \Gamma\left(\frac{1}{4}\left(-1+\frac{1}{3}i\right)\right) \Gamma\left(\frac{1}{4}\left(-1-\frac{1}{3}i\right)\right) 0.261334 \sin\left(\frac{1}{3} \times \frac{1}{8} \log\left(\frac{8\pi}{5(5\pi)}\right)\right)}}$$

$\Gamma(x)$ is the gamma function
 $\log(x)$ is the natural logarithm
 i is the imaginary unit

Result:

1.62394...

1.62394....

Alternate form:

1.62394

Alternative representations:

$$\begin{aligned}
 & \frac{\frac{47}{2}}{\frac{4}{3}} = \\
 & \frac{\Gamma\left(\frac{1}{4}(-1+\frac{i}{3})\right)\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1-\frac{i}{3})\right) \left(0.261334 \sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)\right)}{\frac{4}{3}} \\
 & \frac{47}{2} \\
 & \frac{0.261334 \cos\left(\frac{\pi}{2} - \frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right) \Gamma\left(-1+\frac{1}{4}(-1-\frac{i}{3})\right) \Gamma\left(-1+\frac{1}{4}(-1+\frac{i}{3})\right)\pi^{-3/4}}{\frac{4}{3}} \\
 & \frac{47}{2} = \\
 & \frac{\Gamma\left(\frac{1}{4}(-1+\frac{i}{3})\right)\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1-\frac{i}{3})\right) \left(0.261334 \sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)\right)}{\frac{4}{3}} \\
 & \frac{47}{2} \\
 & \frac{0.261334 \Gamma\left(-1+\frac{1}{4}(-1-\frac{i}{3})\right) \Gamma\left(-1+\frac{1}{4}(-1+\frac{i}{3})\right) \left(-e^{-\left(\frac{i \log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)} + e^{\left(\frac{i \log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)}\right) \pi^{-3/4}}{\frac{4(2i)}{3}}
 \end{aligned}$$

$$\frac{\frac{47}{2}}{\frac{4}{3}} = \frac{\Gamma\left(\frac{1}{4}(-1+i)\right)\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1-i)\right) \left(0.261334 \sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)\right)}{\frac{4}{3}}$$

$$47/2 / \frac{1}{\frac{4(2i)}{3}} 0.261334 \exp\left(-\log G\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right) + \log G\left(1+\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)\right)$$

$$\exp\left(-\log G\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right) + \log G\left(1+\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\right)$$

$$\left(-e^{\left(i \log\left(\frac{8\pi}{5(5\pi)}\right)\right)/(3 \times 8)} + e^{\left(i \log\left(\frac{8\pi}{5(5\pi)}\right)\right)/(3 \times 8)} \right) \pi^{-3/4}$$

Series representations:

$$\frac{\frac{47}{2}}{\frac{4}{3}} = -\frac{1}{(-9+i^2)\pi^{3/4}} 1326.53$$

$$\frac{\Gamma\left(\frac{1}{4}(-1+i)\right)\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1-i)\right) \left(0.261334 \sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)\right)}{\frac{4}{3}}$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 12^{-k_2-k_3} (-3-i)^{k_2} (-3+i)^{k_3} J_{1+2k_1}\left(\frac{1}{24} \log\left(\frac{64}{25}\right)\right) \Gamma^{(k_2)}(1) \Gamma^{(k_3)}(1)}{k_2! k_3!}$$

$$\frac{\frac{47}{2}}{\frac{4}{3}} = -\frac{1}{(-9+i^2)\pi^{3/4}} 663.266 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{(1+2k_1)! k_2! k_3!} (-1)^{k_1} 2^{-3-6k_1-2k_2-2k_3} \times$$

$$3^{-1-2k_1-k_2-k_3} (-3-i)^{k_2} (-3+i)^{k_3} \log^{1+2k_1}\left(\frac{64}{25}\right) \Gamma^{(k_2)}(1) \Gamma^{(k_3)}(1)$$

$$\frac{\frac{47}{2}}{\frac{\Gamma\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)\left(0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3\times 8}\right)\right)}{\frac{1}{\pi^{3/4}}9.21202\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\frac{1}{k_2!k_3!}(-1)^{k_1}J_{1+2k_1}\left(\frac{1}{24}\log\left(\frac{64}{25}\right)\right)\left(-\frac{1}{4}-\frac{i}{12}-z_0\right)^{k_2}\left(-\frac{1}{4}+\frac{i}{12}-z_0\right)^{k_3}\Gamma^{(k_2)}(z_0)\Gamma^{(k_3)}(z_0)\text{ for } (z_0 \notin Z \text{ or } z_0 > 0)}}$$

$$\frac{\frac{47}{2}}{\frac{\Gamma\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)\left(0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3\times 8}\right)\right)}{\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\sum_{k_3=0}^{\infty}\frac{1}{(1+2k_1)!k_2!k_3!}(-1)^{k_1}24^{-1-2k_1}\log^{1+2k_1}\left(\frac{64}{25}\right)\left(-\frac{1}{4}-\frac{i}{12}-z_0\right)^{k_2}\left(-\frac{1}{4}+\frac{i}{12}-z_0\right)^{k_3}\Gamma^{(k_2)}(z_0)\Gamma^{(k_3)}(z_0)\text{ for } (z_0 \notin Z \text{ or } z_0 > 0)}}$$

Integral representations:

$$\frac{\frac{47}{2}}{\frac{\Gamma\left(\frac{1}{4}\left(-1+\frac{i}{3}\right)\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}\left(-1-\frac{i}{3}\right)\right)\left(0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3\times 8}\right)\right)}{\frac{0.191917\Gamma\left(-\frac{1}{4}-\frac{i}{12}\right)\Gamma\left(\frac{1}{12}(-3+i)\right)\log\left(\frac{64}{25}\right)}{\pi^{3/4}}\int_0^1\cos\left(\frac{1}{24}t\log\left(\frac{64}{25}\right)\right)dt}}$$

$$\begin{aligned}
& \frac{47}{2} = \\
& \frac{\Gamma\left(\frac{1}{4}(-1+\frac{i}{3})\right)\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1-\frac{i}{3})\right) \left(0.261334 \sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)\right)}{\frac{4}{3} \frac{0.767669 \pi^{5/4} \mathcal{A}^2 \log\left(\frac{64}{25}\right)}{\oint_L \phi e^t t^{1/4+i/12} dt \oint_L \phi e^t t^{1/4-i/12} dt} \int_0^1 \cos\left(\frac{1}{24} t \log\left(\frac{64}{25}\right)\right) dt} \\
& \frac{47}{2} = \\
& \frac{\Gamma\left(\frac{1}{4}(-1+\frac{i}{3})\right)\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1-\frac{i}{3})\right) \left(0.261334 \sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)\right)}{\frac{4}{3} \frac{0.191917 \sqrt[4]{\pi} \mathcal{A} \log\left(\frac{64}{25}\right) \sqrt{\pi}}{\oint_L \phi e^t t^{1/4+i/12} dt \oint_L \phi e^t t^{1/4-i/12} dt} \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{e^{s-\log^2\left(\frac{64}{25}\right)/(2304s)}}{s^{3/2}} ds \text{ for } \gamma > 0}
\end{aligned}$$

Multiple-argument formulas:

$$\begin{aligned}
& \frac{47}{2} = \\
& \frac{\Gamma\left(\frac{1}{4}(-1+\frac{i}{3})\right)\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1-\frac{i}{3})\right) \left(0.261334 \sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)\right)}{\frac{4}{3} \frac{1.62847 \cos\left(\frac{1}{48} \log\left(\frac{64}{25}\right)\right) \Gamma\left(-\frac{1}{8} - \frac{i}{24}\right) \Gamma\left(\frac{3}{8} - \frac{i}{24}\right) \Gamma\left(\frac{1}{24}(-3+i)\right) \Gamma\left(\frac{9+i}{24}\right) \sin\left(\frac{1}{48} \log\left(\frac{64}{25}\right)\right)}{\pi^{3/4} \sqrt{\pi}^2} \\
& \frac{47}{2} = \\
& \frac{\Gamma\left(\frac{1}{4}(-1+\frac{i}{3})\right)\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1-\frac{i}{3})\right) \left(0.261334 \sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)\right)}{\frac{4}{3} \frac{1}{\pi^{3/4} \sqrt{\pi}^2} 3.25694 \Gamma\left(-\frac{1}{8} - \frac{i}{24}\right) \Gamma\left(\frac{3}{8} - \frac{i}{24}\right) \Gamma\left(\frac{1}{24}(-3+i)\right) \Gamma\left(\frac{9+i}{24}\right) \left(-0.75 \sin\left(\frac{1}{72} \log\left(\frac{64}{25}\right)\right) + \sin^3\left(\frac{1}{72} \log\left(\frac{64}{25}\right)\right)\right)}
\end{aligned}$$

$$\frac{\frac{47}{2}}{\frac{\Gamma\left(\frac{1}{4}(-1+\frac{i}{3})\right)\pi^{-3/4}\Gamma\left(\frac{1}{4}(-1-\frac{i}{3})\right)\left(0.261334\sin\left(\frac{\log\left(\frac{8\pi}{5(5\pi)}\right)}{3 \times 8}\right)\right)}{\frac{1}{\pi^{3/4}\sqrt{\pi^2}}2.44271\Gamma\left(-\frac{1}{8}-\frac{i}{24}\right)\Gamma\left(\frac{3}{8}-\frac{i}{24}\right)\Gamma\left(\frac{1}{24}(-3+i)\right)\Gamma\left(\frac{9+i}{24}\right)\left(\cos^2\left(\frac{1}{72}\log\left(\frac{64}{25}\right)\right)\sin\left(\frac{1}{72}\log\left(\frac{64}{25}\right)\right)-0.333333\sin^3\left(\frac{1}{72}\log\left(\frac{64}{25}\right)\right)\right)}$$

We have that:

where $R^2/2$ is the ratio of the areas of two \mathbb{CP}^1 's. In this case the effective two-dimensional theory is closely related to a $(0,1)$ sigma model with $S^1 \times \mathbb{R}^3$ target, where R is the radius of S^1 . The difference comes from rescaling of the lattice of winding numbers and momenta along S^1 by the overall $\sqrt{2}$ factor. The formula (5.3) then reads

$$\frac{\partial Z_v}{\partial \tau} = \frac{(R + 2/R)}{16\pi i \tau_2^{3/2} \eta(\tau)^4} \sum_{\substack{n \in \mathbb{Z}^2 \\ n = v \pmod{2}}} \bar{q}^{(n_1/R + R n_2/2)^2/4} q^{(n_1/R - R n_2/2)^2/4}, \quad (5.10)$$

where $v \in \mathbb{Z}_2^2$.

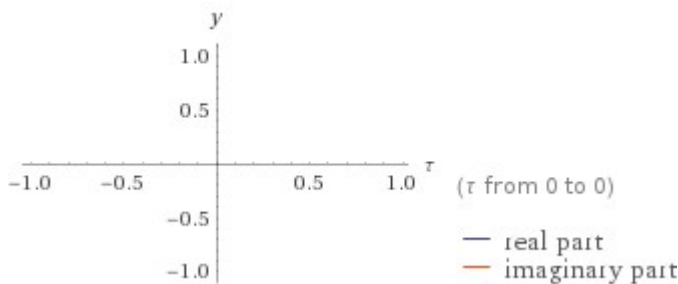
We obtain, from:

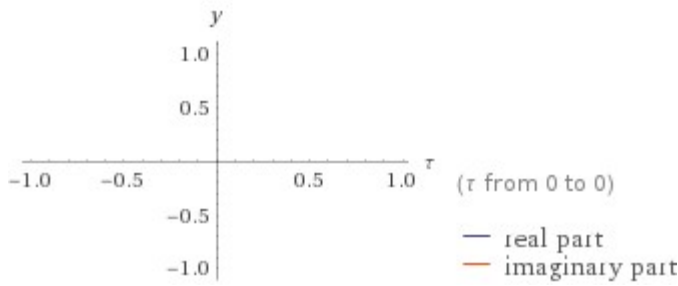
Input:

$$\eta(\tau)^4$$

$\eta(\tau)$ is the Dedekind eta function

Plots:





Numerical roots:

$$\tau \approx -1.91383 + 190.882 i \dots$$

$$\tau \approx -0.499998 + 191.763 i \dots$$

$$\tau \approx 1.98552 \times 10^{-6} + 192.031 i \dots$$

$$\tau \approx 0.500002 + 191.763 i \dots$$

$$\tau \approx 1.91383 + 190.882 i \dots$$

From this last solution, we obtain:

$$(1.91383 + 190.882i)$$

Series expansion at $\tau = 0$:

$$\frac{e^{-(i\pi)/(3\tau)}}{\tau^2}$$

Alternative representations:

$$\eta(\tau)^4 = \left(e^{1/12 \pi (i\tau)} \vartheta_3 \left(\frac{1}{2} (\tau + 1) \pi, e^{3\pi i \tau} \right) \right)^4 \text{ for } \text{Im}(\tau) > 0$$

$$\eta(\tau)^4 = \left(\frac{\vartheta_2 \left(\frac{\pi}{6}, e^{(\pi i \tau)/3} \right)}{\sqrt{3}} \right)^4 \text{ for } (\text{Im}(\tau) > 0 \text{ and } |\text{Re}(\tau)| < 3)$$

Series representations:

$$\eta(\tau)^4 = \exp \left(\frac{i\pi\tau}{3} - 4 \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \frac{e^{2ikn\pi\tau}}{k} \right)$$

$$\eta(\tau)^4 = \frac{1}{\left(\sum_{k=0}^{\infty} e^{2i(-1/24+k)\pi\tau} p(k) \right)^4}$$

$$\eta(\tau)^4 = e^{(i\pi\tau)/3} \left(\sum_{k=-\infty}^{\infty} (-1)^k e^{ik(-1+3k)\pi\tau} \right)^4$$

From

$$\frac{\partial Z_v}{\partial \bar{\tau}} = \frac{(R + 2/R)}{16\pi i \tau_2^{3/2} \eta(\tau)^4} \sum_{\substack{n \in \mathbb{Z}^2 \\ n \equiv v \pmod{2}}} \bar{q}^{(n_1/R + R n_2/2)^2/4} q^{(n_1/R - R n_2/2)^2/4}, \quad (5.10)$$

for $R = 8$; $n_1 = 3$; $n_2 = 5$

we obtain:

$$(8+2/8) / (((16*\text{Pi}*i*((1893.3-568.069i)^{1.5})*(1.91383+190.882i))))*(e^{(2\text{Pi})^{((3/8+20)^2)/4}}*(e^{(2\text{Pi})^{((3/8-20)^2)/4}})$$

Input interpretation:

$$\frac{8 + \frac{2}{8}}{16 \pi i (1893.3 + i \times (-568.069))^{1.5} (1.91383 + 190.882 i)} (e^{2\pi})^{1/4 (3/8+20)^2} (e^{2\pi})^{1/4 (3/8-20)^2}$$

i is the imaginary unit

Result:

$$-7.72551... \times 10^{537} - 3.70585... \times 10^{537} i$$

Polar coordinates:

$$r = 8.568360740421015 \times 10^{537} \text{ (radius), } \theta = -154.37340132794072^\circ \text{ (angle)}$$

$$8.568360740421015 * 10^{537}$$

Alternative representations:

$$\frac{\left((e^{2\pi})^{1/4 (3/8+20)^2} (e^{2\pi})^{1/4 (3/8-20)^2} \right) \left(8 + \frac{2}{8} \right)}{16 \pi i (1893.3 - i 568.069)^{1.5} (1.91383 + 190.882 i)} = \frac{\left(8 + \frac{2}{8} \right) (e^{360^\circ})^{1/4 (-20+3/8)^2} (e^{360^\circ})^{1/4 (20+3/8)^2}}{2880^\circ i (1.91383 + 190.882 i) (1893.3 - 568.069 i)^{1.5}}$$

$$\frac{\left((e^{2\pi})^{1/4 (3/8+20)^2} (e^{2\pi})^{1/4 (3/8-20)^2} \right) \left(8 + \frac{2}{8} \right)}{16 \pi i (1893.3 - i 568.069)^{1.5} (1.91383 + 190.882 i)} = \frac{\left(\exp^{2\pi(z)} \right)^{1/4 (3/8+20)^2} \exp^{2\pi(z)} \left(8 + \frac{2}{8} \right)}{16 \pi i (1893.3 - i 568.069)^{1.5} (1.91383 + 190.882 i)} \text{ for } z = 1$$

$$\frac{\left((e^{2\pi})^{1/4 (3/8+20)^2} (e^{2\pi})^{1/4 (3/8-20)^2} \right) \left(8 + \frac{2}{8} \right)}{16 \pi i (1893.3 - i 568.069)^{1.5} (1.91383 + 190.882 i)} =$$

$$\frac{\left(8 + \frac{2}{8} \right) (e^{-2i \log(-1)})^{1/4 (-20+3/8)^2} (e^{-2i \log(-1)})^{1/4 (20+3/8)^2}}{16 i^2 (1.91383 + 190.882 i) \log(-1) (1893.3 - 568.069 i)^{1.5}}$$

Integral representations:

$$\frac{\left((e^{2\pi})^{1/4 (3/8+20)^2} (e^{2\pi})^{1/4 (3/8-20)^2} \right) \left(8 + \frac{2}{8} \right)}{16 \pi i (1893.3 - i 568.069)^{1.5} (1.91383 + 190.882 i)} =$$

$$\frac{0.00135064 e^{800 \int_0^\infty 1/(1+t^2) dt} \left(e^{4 \int_0^\infty 1/(1+t^2) dt} \right)^{9/128}}{(1893.3 - 568.069 i)^{1.5} i (0.0100262 + i) \int_0^\infty \frac{1}{1+t^2} dt}$$

$$\frac{\left((e^{2\pi})^{1/4 (3/8+20)^2} (e^{2\pi})^{1/4 (3/8-20)^2} \right) \left(8 + \frac{2}{8} \right)}{16 \pi i (1893.3 - i 568.069)^{1.5} (1.91383 + 190.882 i)} =$$

$$\frac{0.00135064 e^{800 \int_0^\infty \sin(t)/t dt} \left(e^{4 \int_0^\infty \sin(t)/t dt} \right)^{9/128}}{(1893.3 - 568.069 i)^{1.5} i (0.0100262 + i) \int_0^\infty \frac{\sin(t)}{t} dt}$$

$$\frac{\left((e^{2\pi})^{1/4 (3/8+20)^2} (e^{2\pi})^{1/4 (3/8-20)^2} \right) \left(8 + \frac{2}{8} \right)}{16 \pi i (1893.3 - i 568.069)^{1.5} (1.91383 + 190.882 i)} =$$

$$\frac{0.000675319 e^{1600 \int_0^1 \sqrt{1-t^2} dt} \left(e^{8 \int_0^1 \sqrt{1-t^2} dt} \right)^{9/128}}{(1893.3 - 568.069 i)^{1.5} i (0.0100262 + i) \int_0^1 \sqrt{1-t^2} dt}$$

From which:

$$\ln\left(\frac{\left(\left(\left(\left(8+\frac{2}{8}\right) / \left(\left(16 \cdot \pi \cdot i \cdot \left(\left(1893.3 - 568.069 i\right)^{1.5}\right) \cdot \left(1.91383 + 190.882 i\right)\right)\right)\right) \cdot \left(e^{2\pi}\right)^{\left(\left(\frac{3}{8}+20\right)^2 / 4\right)} \cdot \left(e^{2\pi}\right)^{\left(\left(\frac{3}{8}-20\right)^2 / 4\right)}\right)\right)\right) - 2\pi$$

Input interpretation:

$$\log\left(\frac{8 + \frac{2}{8}}{16 \pi i (1893.3 + i \times (-568.069))^{1.5} (1.91383 + 190.882 i)} \left(e^{2\pi} \right)^{1/4 (3/8+20)^2} \left(e^{2\pi} \right)^{1/4 (3/8-20)^2} \right) - 2\pi$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Result:

1232.353... -
2.694324... i

Polar coordinates:

$r = 1232.3560313964369745$ (radius), $\theta = -0.12526698484791731^\circ$ (angle)

[1232.3560313964369745](#) result practically equal to the rest mass of Delta baryon
1232

Alternative representations:

$$\log \left(\frac{\left((e^{2\pi})^{1/4 (3/8+20)^2} (e^{2\pi})^{1/4 (3/8-20)^2} \right) \left(8 + \frac{2}{8} \right)}{16 \pi i (1893.3 - i 568.069)^{1.5} (1.91383 + 190.882 i)} \right) - 2 \pi =$$

$$-2 \pi + \log_e \left(\frac{\left(8 + \frac{2}{8} \right) (e^{2\pi})^{1/4 (-20+3/8)^2} (e^{2\pi})^{1/4 (20+3/8)^2}}{16 i (1.91383 + 190.882 i) \pi (1893.3 - 568.069 i)^{1.5}} \right)$$

$$\log \left(\frac{\left((e^{2\pi})^{1/4 (3/8+20)^2} (e^{2\pi})^{1/4 (3/8-20)^2} \right) \left(8 + \frac{2}{8} \right)}{16 \pi i (1893.3 - i 568.069)^{1.5} (1.91383 + 190.882 i)} \right) - 2 \pi =$$

$$-2 \pi + \log(a) \log_a \left(\frac{\left(8 + \frac{2}{8} \right) (e^{2\pi})^{1/4 (-20+3/8)^2} (e^{2\pi})^{1/4 (20+3/8)^2}}{16 i (1.91383 + 190.882 i) \pi (1893.3 - 568.069 i)^{1.5}} \right)$$

$$\log \left(\frac{\left((e^{2\pi})^{1/4 (3/8+20)^2} (e^{2\pi})^{1/4 (3/8-20)^2} \right) \left(8 + \frac{2}{8} \right)}{16 \pi i (1893.3 - i 568.069)^{1.5} (1.91383 + 190.882 i)} \right) - 2 \pi =$$

$$-2 \pi - \text{Li}_1 \left(1 - \frac{\left(8 + \frac{2}{8} \right) (e^{2\pi})^{1/4 (-20+3/8)^2} (e^{2\pi})^{1/4 (20+3/8)^2}}{16 i (1.91383 + 190.882 i) \pi (1893.3 - 568.069 i)^{1.5}} \right)$$

Series representations:

$$\log \left(\frac{\left((e^{2\pi})^{1/4 (3/8+20)^2} (e^{2\pi})^{1/4 (3/8-20)^2} \right) \left(8 + \frac{2}{8} \right)}{16 \pi i (1893.3 - i 568.069)^{1.5} (1.91383 + 190.882 i)} \right) - 2 \pi =$$

$$-2 \pi + \log \left(-1 + \frac{33 (e^{2\pi})^{25 609/128}}{64 (1893.3 - 568.069 i)^{1.5} i (1.91383 + 190.882 i) \pi} \right) -$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{0.00270128 e^{400 \pi} (e^{2\pi})^{9/128}}{(1893.3 - 568.069 i)^{1.5} (0.0100262 i \pi + i^2 \pi)} \right)^{-k}}{k}$$

$$\log \left(\frac{\left((e^{2\pi})^{1/4 (3/8+20)^2} (e^{2\pi})^{1/4 (3/8-20)^2} \right) \left(8 + \frac{2}{8} \right)}{16 \pi i (1893.3 - i 568.069)^{1.5} (1.91383 + 190.882 i)} \right) - 2 \pi =$$

$$-2 \pi + 2 \pi \mathcal{A} \left[\frac{\arg \left(\frac{33 (e^{2\pi})^{25609/128}}{64 (1893.3 - 568.069 i)^{1.5} i (1.91383 + 190.882 i) \pi} - x \right)}{2 \pi} \right] + \log(x) -$$

$$\sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{0.00270128 e^{400 \pi} (e^{2\pi})^{9/128}}{(1893.3 - 568.069 i)^{1.5} (0.0100262 i \pi + i^2 \pi)} - x \right)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log \left(\frac{\left((e^{2\pi})^{1/4 (3/8+20)^2} (e^{2\pi})^{1/4 (3/8-20)^2} \right) \left(8 + \frac{2}{8} \right)}{16 \pi i (1893.3 - i 568.069)^{1.5} (1.91383 + 190.882 i)} \right) - 2 \pi =$$

$$-2 \pi + 2 \pi \mathcal{A} \left[\frac{\pi - \arg \left(\frac{33 (e^{2\pi})^{25609/128}}{64 (1893.3 - 568.069 i)^{1.5} i (1.91383 + 190.882 i) \pi z_0} \right) - \arg(z_0)}{2 \pi} \right] +$$

$$\log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{0.00270128 e^{400 \pi} (e^{2\pi})^{9/128}}{(1893.3 - 568.069 i)^{1.5} (0.0100262 i \pi + i^2 \pi)} - z_0 \right)^k z_0^{-k}}{k}$$

Integral representations:

$$\log \left(\frac{\left((e^{2\pi})^{1/4 (3/8+20)^2} (e^{2\pi})^{1/4 (3/8-20)^2} \right) \left(8 + \frac{2}{8} \right)}{16 \pi i (1893.3 - i 568.069)^{1.5} (1.91383 + 190.882 i)} \right) - 2 \pi =$$

$$-2 \pi + \int_1^{\infty} \frac{0.00270128 e^{400 \pi} (e^{2\pi})^{9/128}}{(1893.3 - 568.069 i)^{1.5} (0.0100262 i \pi + i^2 \pi)} \frac{1}{t} dt$$

$$\log \left(\frac{\left((e^{2\pi})^{1/4 (3/8+20)^2} (e^{2\pi})^{1/4 (3/8-20)^2} \right) \left(8 + \frac{2}{8} \right)}{16 \pi i (1893.3 - i 568.069)^{1.5} (1.91383 + 190.882 i)} \right) - 2 \pi = -2 \pi +$$

$$\frac{1}{2 \pi \mathcal{A}} \int_{-\mathcal{A} \infty + \gamma}^{\mathcal{A} \infty + \gamma} \frac{\left(-1 + \frac{33 (e^{2\pi})^{25609/128}}{64 (1893.3 - 568.069 i)^{1.5} i (1.91383 + 190.882 i) \pi} \right)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds$$

for $-1 < \gamma < 0$

$\Gamma(x)$ is the gamma function

From

$$\widehat{\chi_{0,0}^{-u(2)}}_1(\tau; z) = \frac{1}{\eta(\tau)^2} \sum_{n \in \mathbb{Z}^2} q^{\frac{n_1^2 + n_2^2}{2} + (n_1 + n_2)z} = \left[\frac{\vartheta_3(\tau; z)}{\eta(\tau)} \right]^2$$

we obtain:

$$1/(2.74518+381.228i) * (\exp(2\pi i))^40$$

Input interpretation:

$$\frac{1}{2.74518 + 381.228 i} \exp^{40}(2 \pi)$$

i is the imaginary unit

Result:

$$2.66862... \times 10^{104} - \\ 3.70596... \times 10^{106} i$$

Polar coordinates:

$$r = 3.70606 \times 10^{106} \text{ (radius), } \theta = -89.5874^\circ \text{ (angle)}$$

$$3.70606 * 10^{106}$$

$$\ln(((1/(2.74518+381.228i) * (\exp(2\pi i))^40)))$$

Input interpretation:

$$\log\left(\frac{1}{2.74518 + 381.228 i} \exp^{40}(2 \pi)\right)$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Result:

$$245.38399... - \\ 1.5635956... i$$

Polar coordinates:

$$r = 245.389 \text{ (radius), } \theta = -0.365086^\circ \text{ (angle)}$$

$$245.389$$

Alternative representations:

$$\log\left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i}\right) = \log_e\left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i}\right)$$

$$\log\left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i}\right) = \log(a) \log_a\left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i}\right)$$

Series representations:

$$\log\left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i}\right) =$$

$$\log\left(-1 + \frac{\exp^{40}(2\pi)}{2.74518 + 381.228i}\right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{\exp^{40}(2\pi)}{2.74518 + 381.228i}\right)^k}{k}$$

$$\log\left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i}\right) = 2\pi \mathcal{A} \left[\frac{\arg\left(-x + \frac{\exp^{40}(2\pi)}{2.74518 + 381.228i}\right)}{2\pi} \right] +$$

$$\log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} \left(-x + \frac{\exp^{40}(2\pi)}{2.74518 + 381.228i}\right)^k}{k} \quad \text{for } x < 0$$

$$\log\left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i}\right) = \left[\frac{\arg\left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i} - z_0\right)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) +$$

$$\left[\frac{\arg\left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i} - z_0\right)}{2\pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i} - z_0\right)^k z_0^{-k}}{k}$$

Integral representations:

$$\log\left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i}\right) = \int_1^{\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i}} \frac{1}{t} dt$$

$$\log\left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i}\right) = \frac{1}{2\pi \mathcal{A}} \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{\left(-1 + \frac{\exp^{40}(2\pi)}{2.74518 + 381.228i}\right)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds$$

for $-1 < \gamma < 0$

$$1/4 \left(\ln\left(\frac{1}{2.74518 + 381.228i} \cdot (\exp(2\pi i))^{40}\right) + 11 \right)$$

Input interpretation:

$$\frac{1}{4} \left(\log\left(\frac{1}{2.74518 + 381.228i} \exp^{40}(2\pi)\right) + 11 \right)$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Result:

64.095997... -
0.39089889... i

Polar coordinates:

$r = 64.0972$ (radius), $\theta = -0.349422^\circ$ (angle)

$64.0972 \approx 64$

Alternative representations:

$$\frac{1}{4} \left(\log \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228 i} \right) + 11 \right) = \frac{1}{4} \left(11 + \log_e \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228 i} \right) \right)$$

$$\frac{1}{4} \left(\log \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228 i} \right) + 11 \right) = \frac{1}{4} \left(11 + \log(a) \log_a \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228 i} \right) \right)$$

Series representations:

$$\begin{aligned} \frac{1}{4} \left(\log \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228 i} \right) + 11 \right) = \\ \frac{11}{4} + \frac{1}{4} \log \left(-1 + \frac{\exp^{40}(2\pi)}{2.74518 + 381.228 i} \right) - \frac{1}{4} \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{\exp^{40}(2\pi)}{2.74518 + 381.228 i} \right)^k}{k} \end{aligned}$$

$$\begin{aligned} \frac{1}{4} \left(\log \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228 i} \right) + 11 \right) = \frac{11}{4} + \frac{1}{2} \pi \mathcal{A} \left[\frac{\arg \left(-x + \frac{\exp^{40}(2\pi)}{2.74518 + 381.228 i} \right)}{2\pi} \right] + \\ \frac{\log(x)}{4} - \frac{1}{4} \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} \left(-x + \frac{\exp^{40}(2\pi)}{2.74518 + 381.228 i} \right)^k}{k} \quad \text{for } x < 0 \end{aligned}$$

$$\begin{aligned} \frac{1}{4} \left(\log \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228 i} \right) + 11 \right) = \\ \frac{11}{4} + \frac{1}{4} \left[\frac{\arg \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228 i} - z_0 \right)}{2\pi} \right] \log \left(\frac{1}{z_0} \right) + \frac{\log(z_0)}{4} + \\ \frac{1}{4} \left[\frac{\arg \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228 i} - z_0 \right)}{2\pi} \right] \log(z_0) - \frac{1}{4} \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228 i} - z_0 \right)^k z_0^{-k}}{k} \end{aligned}$$

Integral representations:

$$\frac{1}{4} \left(\log \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i} \right) + 11 \right) = \frac{11}{4} + \frac{1}{4} \int_1^{\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i}} \frac{1}{t} dt$$

$$\frac{1}{4} \left(\log \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i} \right) + 11 \right) = \frac{11}{4} + \frac{1}{8\pi} \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{\left(-1 + \frac{\exp^{40}(2\pi)}{2.74518 + 381.228i}\right)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

27*1/4 (((ln(((1/(2.74518+381.228i) * (exp(2Pi))^40))) + 11))))-(8/5)

Input interpretation:

$$27 \times \frac{1}{4} \left(\log \left(\frac{1}{2.74518 + 381.228i} \exp^{40}(2\pi) \right) + 11 \right) - \frac{8}{5}$$

log(x) is the natural logarithm

i is the imaginary unit

Result:

1728.9919... -
10.554270... i

Polar coordinates:

r = 1729.02 (radius), θ = -0.349746° (angle)

1729.02

This result is very near to the mass of candidate glueball **f₀(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

With regard 27 (From Wikipedia):

*“The fundamental group of the complex form, compact real form, or any algebraic version of E₆ is the cyclic group **Z/3Z**, and its outer automorphism group is the cyclic group **Z/2Z**. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E₆ plays a role in some grand unified theories”.*

Alternative representations:

$$\frac{27}{4} \left(\log \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i} \right) + 11 \right) - \frac{8}{5} = \frac{27}{4} \left(11 + \log_e \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i} \right) \right) - \frac{8}{5}$$

$$\begin{aligned} \frac{27}{4} \left(\log \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i} \right) + 11 \right) - \frac{8}{5} = \\ \frac{27}{4} \left(11 + \log(a) \log_a \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i} \right) \right) - \frac{8}{5} \end{aligned}$$

Series representations:

$$\begin{aligned} \frac{27}{4} \left(\log \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i} \right) + 11 \right) - \frac{8}{5} = \\ \frac{1453}{20} + \frac{27}{4} \log \left(-1 + \frac{\exp^{40}(2\pi)}{2.74518 + 381.228i} \right) - \frac{27}{4} \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{\exp^{40}(2\pi)}{2.74518 + 381.228i} \right)^k}{k} \end{aligned}$$

$$\begin{aligned} \frac{27}{4} \left(\log \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i} \right) + 11 \right) - \frac{8}{5} = \\ \frac{1453}{20} + \frac{27}{2} \pi \mathcal{A} \left[\frac{\arg \left(-x + \frac{\exp^{40}(2\pi)}{2.74518 + 381.228i} \right)}{2\pi} \right] + \frac{27 \log(x)}{4} - \\ \frac{27}{4} \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} \left(-x + \frac{\exp^{40}(2\pi)}{2.74518 + 381.228i} \right)^k}{k} \quad \text{for } x < 0 \end{aligned}$$

$$\begin{aligned} \frac{27}{4} \left(\log \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i} \right) + 11 \right) - \frac{8}{5} = \\ \frac{1453}{20} + \frac{27}{4} \left[\frac{\arg \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i} - z_0 \right)}{2\pi} \right] \log \left(\frac{1}{z_0} \right) + \frac{27 \log(z_0)}{4} + \\ \frac{27}{4} \left[\frac{\arg \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i} - z_0 \right)}{2\pi} \right] \log(z_0) - \frac{27}{4} \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i} - z_0 \right)^k z_0^{-k}}{k} \end{aligned}$$

Integral representations:

$$\frac{27}{4} \left(\log \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i} \right) + 11 \right) - \frac{8}{5} = \frac{1453}{20} + \frac{27}{4} \int_1^{\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i}} \frac{1}{t} dt$$

$$\frac{27}{4} \left(\log \left(\frac{\exp^{40}(2\pi)}{2.74518 + 381.228i} \right) + 11 \right) - \frac{8}{5} =$$

$$\frac{1453}{20} + \frac{27}{8\pi} \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{\left(-1 + \frac{\exp^{40}(2\pi)}{2.74518+381.228i}\right)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

From which:

$$[27 \cdot \frac{1}{4} \left(\left(\ln \left(\frac{1}{2.74518 + 381.228i} \right) \cdot (\exp(2\pi))^{40} \right) + 11 \right) - \frac{8}{5}]^{1/15}$$

Input interpretation:

$$\sqrt[15]{27 \times \frac{1}{4} \left(\log \left(\frac{1}{2.74518 + 381.228i} \exp^{40}(2\pi) \right) + 11 \right) - \frac{8}{5}}$$

log(x) is the natural logarithm
i is the imaginary unit

Result:

$$1.64381662... - 0.000668947409... i$$

Polar coordinates:

$$r = 1.64382 \text{ (radius), } \theta = -0.0233164^\circ \text{ (angle)}$$

$$1.64382 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

From the following ratio, performing the 13th root, we obtain:

$$\left(\left(\left(\frac{1}{2.74518+381.228i} \cdot (\exp(2\pi))^{40} \right) / \left[\frac{8+2/8}{16\pi i (1893.3-568.069i)^{1.5}} \cdot (1.91383+190.882i) \right] \right) \cdot (e^{2\pi})^{((3/8+20)^2/4)} \cdot (e^{2\pi})^{((3/8-20)^2/4)} \right)^{1/13}$$

Input interpretation:

$$\sqrt[13]{\frac{\frac{1}{2.74518+381.228i} \exp^{40}(2\pi)}{\frac{8+2/8}{16\pi i (1893.3+i \times (-568.069))^{1.5} (1.91383+190.882i)} (e^{2\pi})^{1/4 (3/8+20)^2} (e^{2\pi})^{1/4 (3/8-20)^2}}}$$

i is the imaginary unit

Result:

$$6.55406... \times 10^{-34} + 5.71508... \times 10^{-35} i$$

Polar coordinates:

$r = 6.5789254122036805 \times 10^{-34}$ (radius), $\theta = 4.983536512965587^\circ$ (angle)

$6.5789254122036805 \times 10^{-34}$ result very near to the value of Planck constant

$6.62607015 \times 10^{-34}$

or:

$$\frac{1}{(2\pi)^{13}} \left(\frac{1}{(2.74518+381.228i) \exp^{40}(2\pi)} \right) / \left[\frac{8+\frac{2}{8}}{16\pi i (1893.3+i(-568.069))^{1.5} (1.91383+190.882i)} \right] \left(e^{2\pi} \right)^{1/4 (3/8+20)^2} \left(e^{2\pi} \right)^{1/4 (3/8-20)^2} \right)^{1/13}$$

Input interpretation:

$$\frac{1}{2\pi^{13}} \sqrt[13]{ \frac{\frac{1}{2.74518+381.228i} \exp^{40}(2\pi)}{\frac{8+\frac{2}{8}}{16\pi i (1893.3+i(-568.069))^{1.5} (1.91383+190.882i)} (e^{2\pi})^{1/4 (3/8+20)^2} (e^{2\pi})^{1/4 (3/8-20)^2}} } }$$

i is the imaginary unit

Result:

$1.04311... \times 10^{-34} + 9.09583... \times 10^{-36} i$

Polar coordinates:

$r = 1.04706849958510083 \times 10^{-34}$ (radius), $\theta = 4.983536512965587^\circ$ (angle)

$1.04706849958510083 \times 10^{-34}$ result very near to the value of reduced Planck constant

$1.054571817 \times 10^{-34}$

Alternative representations:

$$\frac{1}{2\pi^{13}} \sqrt[13]{ \frac{\frac{\exp^{40}(2\pi)}{(2.74518+381.228i) \left(\left(8+\frac{2}{8} \right) (e^{2\pi})^{1/4 (3/8+20)^2} (e^{2\pi})^{1/4 (3/8-20)^2} \right)}}{16\pi i (1893.3-i 568.069)^{1.5} (1.91383+190.882i)} } } = \frac{2\pi}{2\pi} \sqrt[13]{ \frac{\frac{\exp^{40}(2\pi)}{(2.74518+381.228i) \left(\left(8+\frac{2}{8} \right) (w^a)^{1/4 (3/8+20)^2} (w^a)^{1/4 (3/8-20)^2} \right)}}{16\pi i (1893.3-i 568.069)^{1.5} (1.91383+190.882i)} } } \text{ for } a = \frac{2\pi}{\log(w)}$$

$$\begin{aligned}
& \frac{\sqrt[13]{\frac{\exp^{40}(2\pi)}{(2.74518+381.228i)\left(\left(8+\frac{2}{8}\right)(e^{2\pi})^{1/4}(3/8+20)^2(e^{2\pi})^{1/4}(3/8-20)^2\right)}}{16\pi i(1893.3-i568.069)^{1.5}(1.91383+190.882i)}}{2\pi} = \\
& \frac{\sqrt[13]{\frac{\exp^{40}(2\pi)}{(2.74518+381.228i)\left(\left(8+\frac{2}{8}\right)(x^{2\pi})^{1/4}(3/8+20)^2(x^{2\pi})^{1/4}(3/8-20)^2\right)}}{16\pi i(1893.3-i568.069)^{1.5}(1.91383+190.882i)}}{2\pi} \quad \text{for } z = e \\
& \frac{\sqrt[13]{\frac{\exp^{40}(2\pi)}{(2.74518+381.228i)\left(\left(8+\frac{2}{8}\right)(e^{2\pi})^{1/4}(3/8+20)^2(e^{2\pi})^{1/4}(3/8-20)^2\right)}}{16\pi i(1893.3-i568.069)^{1.5}(1.91383+190.882i)}}{2\pi} = \\
& \left(\frac{\sqrt[13]{\frac{\exp^{40}(2\pi)}{(2.74518+381.228i)\left(\left(8+\frac{2}{8}\right)\left(1+\frac{2}{-1+\coth(\pi)}\right)^{1/4}(-20+3/8)^2\left(1+\frac{2}{-1+\coth(\pi)}\right)^{1/4}(20+3/8)^2\right)}}{16i(1.91383+190.882i)\pi(1893.3-568.069i)^{1.5}}}{2\pi} \right) = \\
& \left(\frac{\sqrt[13]{\frac{(1893.3-568.069i)^{1.5}i(1.91383+190.882i)\pi\exp^{40}(2\pi)}{(2.74518+381.228i)\left(1+\frac{2}{-1+\coth(\pi)}\right)^{25609/128}}}}{2^{7/13}\sqrt[13]{33}\pi} \right)
\end{aligned}$$

Series representations:

$$\begin{aligned}
& \frac{\sqrt[13]{\frac{\exp^{40}(2\pi)}{(2.74518+381.228i)\left(\left(8+\frac{2}{8}\right)(e^{2\pi})^{1/4}(3/8+20)^2(e^{2\pi})^{1/4}(3/8-20)^2\right)}}{16\pi i(1893.3-i568.069)^{1.5}(1.91383+190.882i)}}{2\pi} = \\
& \frac{0.498872\sqrt[13]{\frac{(1893.3-568.069i)^{1.5}i(0.0100262+i)\pi\exp^{40}(2\pi)}{(0.00720089+i)\left(\sum_{k=0}^{\infty}\frac{2^k\pi^k}{k!}\right)^{25609/128}}}}{\pi}
\end{aligned}$$

$$\frac{\sqrt[13]{\frac{\exp^{40}(2\pi)}{(2.74518+381.228i)\left(\left(8+\frac{2}{8}\right)(e^{2\pi})^{1/4}(3/8+20)^2(e^{2\pi})^{1/4}(3/8-20)^2\right)}}{16\pi i(1893.3-i568.069)^{1.5}(1.91383+190.882i)}}}{\frac{2\pi}{\pi}} =$$

$$0.498872 \sqrt[13]{\frac{(1893.3-568.069i)^{1.5} i(0.0100262+i)\pi \exp^{40}(2\pi)}{(0.00720089+i)\left(\sum_{k=-\infty}^{\infty} I_k(2\pi)\right)^{25609/128}}}$$

$$\frac{\sqrt[13]{\frac{\exp^{40}(2\pi)}{(2.74518+381.228i)\left(\left(8+\frac{2}{8}\right)(e^{2\pi})^{1/4}(3/8+20)^2(e^{2\pi})^{1/4}(3/8-20)^2\right)}}{16\pi i(1893.3-i568.069)^{1.5}(1.91383+190.882i)}}}{\frac{2\pi}{\pi}} =$$

$$0.498872 \sqrt[13]{\frac{(1893.3-568.069i)^{1.5} i(0.0100262+i)\pi \exp^{40}(2\pi)}{(0.00720089+i)\left(\sum_{k=-\infty}^{\infty} (-1)^k I_k(-2\pi)\right)^{25609/128}}}$$

and:

$$\left(\frac{\left(\frac{8+2/8}{(16\pi i(1893.3-568.069i)^{1.5})(1.91383+190.882i)}\right) \cdot (e^{2\pi})^{1/4} (3/8+20)^2 \cdot (e^{2\pi})^{1/4} (3/8-20)^2}{[1/(2.74518+381.228i) \cdot (\exp(2\pi))^{40}]}\right)^{1/2064}$$

Input interpretation:

$$\sqrt[2064]{\frac{\frac{8+\frac{2}{8}}{16\pi i(1893.3+i(-568.069))^{1.5}(1.91383+190.882i)} \cdot (e^{2\pi})^{1/4} (3/8+20)^2 \cdot (e^{2\pi})^{1/4} (3/8-20)^2}{\frac{1}{2.74518+381.228i} \exp^{40}(2\pi)}}$$

i is the imaginary unit

Result:

$$1.618058... - 0.0008864267... i$$

Polar coordinates:

$$r = 1.6180583501519062213 \text{ (radius), } \theta = -0.03138855361848480^\circ \text{ (angle)}$$

1.6180583501519062213 result that is a very good approximation to the value of the golden ratio 1.618033988749...

Now, we have that:

The bosonic part of the effective action, after coupling to background gauge fields of $SO(6)_R$, contains the following Wess-Zumino term [18, 32, 33]:

$$S_{4d \text{ WZ}} = 2\pi i \frac{n_W}{2} \int_{\Xi^5} \eta_5 \quad (3.16)$$

with

$$\begin{aligned} \eta_5 := & \frac{1}{120\pi^3} \epsilon_{I_1 I_2 I_3 I_4 I_5 I_6} [(D_{i_1} \hat{\Phi})^{I_1} (D_{i_2} \hat{\Phi})^{I_2} (D_{i_3} \hat{\Phi})^{I_3} (D_{i_4} \hat{\Phi})^{I_4} (D_{i_5} \hat{\Phi})^{I_5} \\ & + \frac{5}{2} F_{i_1 i_2}^{I_1 I_2} (D_{i_3} \hat{\Phi})^{I_3} (D_{i_4} \hat{\Phi})^{I_4} (D_{i_5} \hat{\Phi})^{I_5} + \frac{15}{4} F_{i_1 i_2}^{I_1 I_2} F_{i_3 i_4}^{I_3 I_4} (D_{i_5} \hat{\Phi})^{I_5}] \hat{\Phi}^{I_6} dx^{i_1} \wedge dx^{i_2} \wedge dx^{i_3} \wedge dx^{i_4} \wedge dx^{i_5}, \end{aligned} \quad (3.17)$$

where Φ^I , $I = 0, \dots, 5$ are the six scalar fields of the unbroken $U(1)$, $\hat{\Phi}^I := \Phi^I / \|\Phi\|$, $\|\Phi\|^2 := \Phi^I \Phi^I$, $(D_i \Phi)^I := \partial_i \Phi^I - A_i^I \Phi^J$, A is the background $SO(6)_R$ connection and F is its curvature.

From:

$$\begin{aligned} \eta_5 := & \frac{1}{120\pi^3} \epsilon_{I_1 I_2 I_3 I_4 I_5 I_6} [(D_{i_1} \hat{\Phi})^{I_1} (D_{i_2} \hat{\Phi})^{I_2} (D_{i_3} \hat{\Phi})^{I_3} (D_{i_4} \hat{\Phi})^{I_4} (D_{i_5} \hat{\Phi})^{I_5} \\ & + \frac{5}{2} F_{i_1 i_2}^{I_1 I_2} (D_{i_3} \hat{\Phi})^{I_3} (D_{i_4} \hat{\Phi})^{I_4} (D_{i_5} \hat{\Phi})^{I_5} + \frac{15}{4} F_{i_1 i_2}^{I_1 I_2} F_{i_3 i_4}^{I_3 I_4} (D_{i_5} \hat{\Phi})^{I_5}] \hat{\Phi}^{I_6} dx^{i_1} \wedge dx^{i_2} \wedge dx^{i_3} \wedge dx^{i_4} \wedge dx^{i_5}, \end{aligned}$$

$$(1*2*3*4*5*6)/(120*\text{Pi}^3) *$$

$$\begin{aligned} & (((((1*(2(\sqrt{3}))*(3(\sqrt{3})^2)*(4(\sqrt{3})^3)*(5(\sqrt{3})^4)+5/2*(3(\sqrt{3})^2)*(4(\sqrt{3})^3) \\ &)*(5(\sqrt{3})^4)+15/4*(5(\sqrt{3})^4)))))) * (\sqrt{3})^6 \end{aligned}$$

Input:

$$\frac{2 \times 3 \times 4 \times 5 \times 6}{120 \pi^3}$$

$$\frac{\left((2\sqrt{3})(3\sqrt{3}^2)(4\sqrt{3}^3)(5\sqrt{3}^4) + \frac{5}{2}(3\sqrt{3}^2)(4\sqrt{3}^3)(5\sqrt{3}^4) + \frac{15}{4}(5\sqrt{3}^4) \right)}{\sqrt{3}^6}$$

Result:

$$\frac{162 \left(\frac{117315}{4} + 12150\sqrt{3} \right)}{\pi^3}$$

Decimal approximation:

263187.1342915112103703153578499265891880830999143791042780...

263187.1342915112...

Property:

$\frac{162 \left(\frac{117315}{4} + 12150 \sqrt{3} \right)}{\pi^3}$ is a transcendental number

Alternate forms:

$$\frac{10935 (869 + 360 \sqrt{3})}{2 \pi^3}$$

$$\frac{9502515}{2} + 1968300 \sqrt{3}$$
$$\frac{\quad}{\pi^3}$$

$$\frac{9502515 + 3936600 \sqrt{3}}{2 \pi^3}$$

Series representations:

$$\frac{1}{120 \pi^3} \left((2 \sqrt{3} (3 \sqrt{3}^2) ((4 \sqrt{3}^3) (5 \sqrt{3}^4)) + \frac{1}{2} (3 \sqrt{3}^2) 5 (4 \sqrt{3}^3) (5 \sqrt{3}^4) + \frac{15}{4} (5 \sqrt{3}^4) \sqrt{3}^6) (2 \times 3 \times 4 \times 5 \times 6) = \right.$$
$$\left. \frac{45 \sqrt{2}^{10} \left(\sum_{k=0}^{\infty} 2^{-k} \binom{1}{k} \right)^{10} \left(5 + 40 \sqrt{2}^5 \left(\sum_{k=0}^{\infty} 2^{-k} \binom{1}{k} \right)^5 + 32 \sqrt{2}^6 \left(\sum_{k=0}^{\infty} 2^{-k} \binom{1}{k} \right)^6 \right)}{2 \pi^3} \right)$$

$$\frac{1}{120 \pi^3} \left((2 \sqrt{3} (3 \sqrt{3}^2) ((4 \sqrt{3}^3) (5 \sqrt{3}^4)) + \frac{1}{2} (3 \sqrt{3}^2) 5 (4 \sqrt{3}^3) (5 \sqrt{3}^4) + \frac{15}{4} (5 \sqrt{3}^4) \sqrt{3}^6) \right.$$
$$\left. (2 \times 3 \times 4 \times 5 \times 6) = \frac{1}{2 \pi^3} 45 \sqrt{2}^{10} \left(\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^{10} \right.$$
$$\left. \left(5 + 40 \sqrt{2}^5 \left(\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^5 + 32 \sqrt{2}^6 \left(\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^6 \right) \right)$$

Alternate form:

$$\frac{9 \sqrt[3]{\frac{15}{2} (869 + 360 \sqrt{3})}}{\pi}$$

All 3rd roots of $(162 (117315/4 + 12150 \sqrt{3}))/\pi^3$:

$$\frac{3 \sqrt[3]{6 \left(\frac{117315}{4} + 12150 \sqrt{3} \right)} e^{i0}}{\pi} \approx 64.08 \text{ (real, principal root)}$$

$$\frac{3 \sqrt[3]{6 \left(\frac{117315}{4} + 12150 \sqrt{3} \right)} e^{(2i\pi)/3}}{\pi} \approx -32.04 + 55.50 i$$

$$\frac{3 \sqrt[3]{6 \left(\frac{117315}{4} + 12150 \sqrt{3} \right)} e^{-(2i\pi)/3}}{\pi} \approx -32.04 - 55.50 i$$

Series representations:

$$\left(\frac{1}{120 \pi^3} \left((2 \sqrt{3} (3 \sqrt{3}^2) ((4 \sqrt{3}^3) (5 \sqrt{3}^4))) + \frac{1}{2} (3 \sqrt{3}^2) 5 (4 \sqrt{3}^3) (5 \sqrt{3}^4) + \frac{15}{4} (5 \sqrt{3}^4) \sqrt{3}^6 \right) (2 \times 3 \times 4 \times 5 \times 6) \right)^{(1/3)} = \sqrt[3]{\frac{5}{2}} 3^{2/3} \left(\frac{1}{\pi^3} \sqrt{2}^{-10} \left(\sum_{k=0}^{\infty} 2^{-k} \binom{1}{k} \right)^{10} \left(5 + 40 \sqrt{2}^5 \left(\sum_{k=0}^{\infty} 2^{-k} \binom{1}{k} \right)^5 + 32 \sqrt{2}^6 \left(\sum_{k=0}^{\infty} 2^{-k} \binom{1}{k} \right)^6 \right) \right)^{(1/3)}$$

$$\left(\frac{1}{120 \pi^3} \left((2 \sqrt{3} (3 \sqrt{3}^2) ((4 \sqrt{3}^3) (5 \sqrt{3}^4))) + \frac{1}{2} (3 \sqrt{3}^2) 5 (4 \sqrt{3}^3) (5 \sqrt{3}^4) + \frac{15}{4} (5 \sqrt{3}^4) \sqrt{3}^6 \right) (2 \times 3 \times 4 \times 5 \times 6) \right)^{(1/3)} = \sqrt[3]{\frac{5}{2}} 3^{2/3} \left(\frac{1}{\pi^3} \sqrt{2}^{-10} \left(\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^{10} \left(5 + 40 \sqrt{2}^5 \left(\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^5 + 32 \sqrt{2}^6 \left(\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)^6 \right) \right)^{(1/3)}$$

Property:

$$\frac{3^{2/13} 26 \sqrt[26]{2 \left(\frac{117315}{4} + 12150 \sqrt{3} \right)}}{\pi^{3/26}} \text{ is a transcendental number}$$

Alternate forms:

$$\frac{3^{7/26} 26 \sqrt[26]{\frac{4345}{2} + 900 \sqrt{3}}}{\pi^{3/26}}$$

$$\frac{3^{7/26} 26 \sqrt[26]{\frac{5}{2} (869 + 360 \sqrt{3})}}{\pi^{3/26}}$$

All 26th roots of $(162 (117315/4 + 12150 \text{sqrt}(3)))/\pi^3$:

$$\frac{3^{2/13} 26 \sqrt[26]{2 \left(\frac{117315}{4} + 12150 \sqrt{3} \right)} e^0}{\pi^{3/26}} \approx 1.61611 \text{ (real, principal root)}$$

$$\frac{3^{2/13} 26 \sqrt[26]{2 \left(\frac{117315}{4} + 12150 \sqrt{3} \right)} e^{(i \pi)/13}}{\pi^{3/26}} \approx 1.56915 + 0.38676 i$$

$$\frac{3^{2/13} 26 \sqrt[26]{2 \left(\frac{117315}{4} + 12150 \sqrt{3} \right)} e^{(2 i \pi)/13}}{\pi^{3/26}} \approx 1.4310 + 0.7510 i$$

$$\frac{3^{2/13} 26 \sqrt[26]{2 \left(\frac{117315}{4} + 12150 \sqrt{3} \right)} e^{(3 i \pi)/13}}{\pi^{3/26}} \approx 1.2097 + 1.0717 i$$

$$\frac{3^{2/13} 26 \sqrt[26]{2 \left(\frac{117315}{4} + 12150 \sqrt{3} \right)} e^{(4 i \pi)/13}}{\pi^{3/26}} \approx 0.9181 + 1.3300 i$$

Series representations:

$$\left(\frac{1}{120\pi^3} \left((2\sqrt{3} (3\sqrt{3}^2) ((4\sqrt{3}^3) (5\sqrt{3}^4))) + \frac{1}{2} (3\sqrt{3}^2) 5 (4\sqrt{3}^3) (5\sqrt{3}^4) + \frac{15}{4} (5\sqrt{3}^4) \right) \sqrt{3}^6 (2 \times 3 \times 4 \times 5 \times 6) \right)^{(1/26)} = \sqrt[26]{\frac{5}{2} \sqrt[13]{3}}$$

$$\left(\frac{1}{\pi^3} \sqrt{2}^{10} \left(\sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^{10} \left(5 + 40 \sqrt{2}^5 \left(\sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^5 + 32 \sqrt{2}^6 \left(\sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k} \right)^6 \right) \right)^{(1/26)}$$

$$\left(\frac{1}{120\pi^3} \left((2\sqrt{3} (3\sqrt{3}^2) ((4\sqrt{3}^3) (5\sqrt{3}^4))) + \frac{1}{2} (3\sqrt{3}^2) 5 (4\sqrt{3}^3) (5\sqrt{3}^4) + \frac{15}{4} (5\sqrt{3}^4) \right) \sqrt{3}^6 (2 \times 3 \times 4 \times 5 \times 6) \right)^{(1/26)} = \sqrt[26]{\frac{5}{2} \sqrt[13]{3}} \left(\frac{1}{\pi^3} \sqrt{2}^{10} \left(\sum_{k=0}^{\infty} \frac{(-\frac{1}{2})^k (-\frac{1}{2})_k}{k!} \right)^{10} \left(5 + 40 \sqrt{2}^5 \left(\sum_{k=0}^{\infty} \frac{(-\frac{1}{2})^k (-\frac{1}{2})_k}{k!} \right)^5 + 32 \sqrt{2}^6 \left(\sum_{k=0}^{\infty} \frac{(-\frac{1}{2})^k (-\frac{1}{2})_k}{k!} \right)^6 \right) \right)^{(1/26)}$$

$$\left(\frac{1}{120\pi^3} \left((2\sqrt{3} (3\sqrt{3}^2) ((4\sqrt{3}^3) (5\sqrt{3}^4))) + \frac{1}{2} (3\sqrt{3}^2) 5 (4\sqrt{3}^3) (5\sqrt{3}^4) + \frac{15}{4} (5\sqrt{3}^4) \right) \sqrt{3}^6 (2 \times 3 \times 4 \times 5 \times 6) \right)^{(1/26)} = \frac{1}{\sqrt{2}} \sqrt[13]{3} \sqrt[26]{5} \left(\frac{1}{\pi^3 \sqrt{\pi}^{16}} \left(\sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right)^{10} \left(20 \sqrt{\pi}^6 + 5 \sqrt{\pi} \left(\sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right)^5 + 2 \left(\sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s) \right)^6 \right) \right)^{(1/26)}$$

Integral representation:

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-a-s)}{z^s} ds}{(2\pi i)\Gamma(-a)} \text{ for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

From the previous expression:

$$\frac{162 \left(\frac{117315}{4} + 12150 \sqrt{3} \right)}{\pi^3}$$

we obtain:

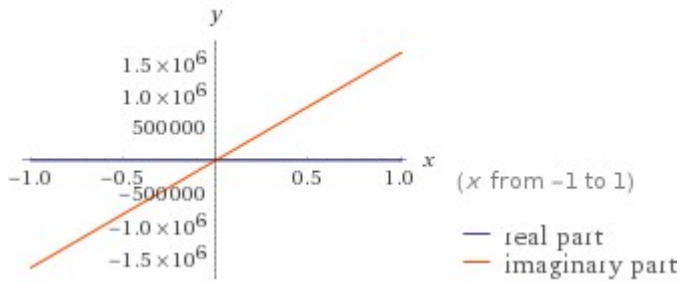
$$2\pi i \int \frac{162 \left(\frac{117315}{4} + 12150 \sqrt{3} \right)}{\pi^3} dx$$

Indefinite integral:

$$2\pi i \int \frac{162 \left(\frac{117315}{4} + 12150 \sqrt{3} \right)}{\pi^3} dx \approx \text{constant} + (1.65365 \times 10^6 i)x$$

i is the imaginary unit

Plot:



Alternate forms:

$$\frac{2i \left(\frac{9502515}{2} + 1968300 \sqrt{3} \right) x}{\pi^2}$$

$$\frac{10935i(869 + 360 \sqrt{3})x}{\pi^2}$$

$$\left(\frac{9502515i}{\pi^2} + \frac{3936600i\sqrt{3}}{\pi^2} \right) x$$

Expanded form:

$$\frac{3936600i\sqrt{3}x}{\pi^2} + \frac{9502515ix}{\pi^2}$$

Alternate form assuming x is real:

$$i \left(\frac{3936600\sqrt{3}x}{\pi^2} + \frac{9502515x}{\pi^2} \right)$$

From which:

$$(1653650 i)^{1/28}$$

Input:

$$\sqrt[28]{1653650 i}$$

i is the imaginary unit

Exact result:

$$\sqrt[56]{-1} \sqrt[14]{5} \sqrt[28]{66146}$$

Decimal approximation:

1.664958863306002091680376630399897976112261181858873161... +
0.09350208417171580014397855156967156367913740400496943868... i

Polar coordinates:

$r \approx 1.66758$ (radius), $\theta \approx 3.21429^\circ$ (angle)

1.66758 result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

We note also that, from the formula of coefficients of the '5th order' mock theta function $\psi_1(q)$: (A053261 OEIS Sequence)

$$\sqrt{\phi} \times \exp(\pi \sqrt{n/15}) / (2 \cdot 5^{1/4} \sqrt{n}) \quad \text{for } n = 406$$

we obtain:

$$\sqrt{\phi} \times \exp(\pi \sqrt{406/15}) / (2 \cdot 5^{1/4} \sqrt{406}) - 1364 - 123$$

where 123 and 1364 are Lucas numbers

Input:

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{406}{15}}\right)}{2 \sqrt[4]{5} \sqrt{406}} - 1364 - 123$$

ϕ is the golden ratio

Exact result:

$$\frac{e^{\sqrt{406/15} \pi} \sqrt{\frac{\phi}{406}}}{2 \sqrt[4]{5}} - 1487$$

Decimal approximation:

263187.9778297700583780435022250914375775558628106800762274...

263187.97782977.... result practically equal to the previous value
263187.1342915112....

Property:

$$-1487 + \frac{e^{\sqrt{406/15} \pi} \sqrt{\frac{\phi}{406}}}{2 \sqrt[4]{5}} \text{ is a transcendental number}$$

Alternate forms:

$$\frac{1}{4} \sqrt{\frac{5 + \sqrt{5}}{1015}} e^{\sqrt{406/15} \pi} - 1487$$

$$\frac{\sqrt{\frac{1}{203} (1 + \sqrt{5})} e^{\sqrt{406/15} \pi}}{4 \sqrt[4]{5}} - 1487$$

$$\frac{5^{3/4} \sqrt{203 (1 + \sqrt{5})} e^{\sqrt{406/15} \pi} - 6\,037\,220}{4060}$$

Series representations:

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{406}{15}}\right)}{2 \sqrt[4]{5} \sqrt{406}} - 1364 - 123 = \left(-14\,870 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (406 - z_0)^k z_0^{-k}}{k!} + 5^{3/4} \right. \\ \left. \exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{406}{15} - z_0\right)^k z_0^{-k}}{k!}\right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi - z_0)^k z_0^{-k}}{k!} \right) / \\ \left(10 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (406 - z_0)^k z_0^{-k}}{k!} \right) \text{ for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\begin{aligned}
& \frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{406}{15}}\right)}{2 \sqrt[4]{5} \sqrt{406}} - 1364 - 123 = \\
& \left(-14870 \exp\left(i \pi \left[\frac{\arg(406-x)}{2 \pi} \right] \right) \sum_{k=0}^{\infty} \frac{(-1)^k (406-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \right. \\
& \quad 5^{3/4} \exp\left(i \pi \left[\frac{\arg(\phi-x)}{2 \pi} \right] \right) \exp\left(\pi \exp\left(i \pi \left[\frac{\arg\left(\frac{406}{15}-x\right)}{2 \pi} \right] \right) \sqrt{x} \right. \\
& \quad \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{406}{15}-x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \sum_{k=0}^{\infty} \frac{(-1)^k (\phi-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) / \\
& \left(10 \exp\left(i \pi \left[\frac{\arg(406-x)}{2 \pi} \right] \right) \sum_{k=0}^{\infty} \frac{(-1)^k (406-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
\end{aligned}$$

$$\begin{aligned}
& \frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{406}{15}}\right)}{2 \sqrt[4]{5} \sqrt{406}} - 1364 - 123 = \\
& \left(\left(\frac{1}{z_0} \right)^{-1/2 [\arg(406-z_0)/(2 \pi)]} z_0^{-1/2 [\arg(406-z_0)/(2 \pi)]} \left(-14870 \left(\frac{1}{z_0} \right)^{1/2 [\arg(406-z_0)/(2 \pi)]} \right. \right. \\
& \quad \left. \left. z_0^{1/2 [\arg(406-z_0)/(2 \pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (406-z_0)^k z_0^{-k}}{k!} + \right. \right. \\
& \quad \left. \left. 5^{3/4} \exp\left(\pi \left(\frac{1}{z_0} \right)^{1/2 [\arg\left(\frac{406}{15}-z_0\right)/(2 \pi)]} z_0^{1/2 (1+[\arg\left(\frac{406}{15}-z_0\right)/(2 \pi)])} \right. \right. \right. \\
& \quad \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{406}{15}-z_0\right)^k z_0^{-k}}{k!} \right) \left(\frac{1}{z_0} \right)^{1/2 [\arg(\phi-z_0)/(2 \pi)]} \right. \\
& \quad \left. \left. z_0^{1/2 [\arg(\phi-z_0)/(2 \pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi-z_0)^k z_0^{-k}}{k!} \right) \right) / \\
& \left(10 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (406-z_0)^k z_0^{-k}}{k!} \right)
\end{aligned}$$

Let

In this paper, we will consider one particular twisting, originally studied in the present context in [1]. With this twisting, a formal argument shows that the partition function on a compact four-manifold X is holomorphic in τ or equivalently in $q = \exp(2\pi i\tau)$. Furthermore, if a certain curvature condition (eqn. (2.58) in [1]) is satisfied, the evaluation of the path integral can formally be argued to localize on the contribution of ordinary Yang-Mills instantons. (Without this curvature condition, one localizes on the solutions of a more complicated system of equations.) The contribution to the path integral from the component of field space with instanton number¹ n is then $a_n q^n$, where a_n is the Euler characteristic of the instanton number n moduli space \mathcal{M}_n . Thus the partition function after summing over bundles of all values of the instanton number is expected to be

$$Z = \sum_n a_n q^n. \quad (1.1)$$

$$\omega_4 := \frac{1}{64\pi^2} \epsilon_{a_1 a_2 a_3 a_4 a_5} Z^{a_1} dZ^{a_2} dZ^{a_3} dZ^{a_4} dZ^{a_5}$$

The resulting twisted theory thus has unbroken $Spin(4)_R \times U(1)_R$ global symmetry in addition to the $U(1)_{\ell'} \times SU(2)_r$ holonomy group. Note that since $U(1)_R$ is abelian, it remains unbroken even after turning on a non-trivial background. There are then four scalar supercharges which we denote as Q_A ($A = 1, 2$) and $Q_{\dot{A}}$ ($\dot{A} = 1, 2$) and which transform as $(1, 2, 1)_{+1}^0 \oplus (1, 1, 2)_{-1}^0$ respectively under $SU(2)_r \times Spin(4)_R \times U(1)_{\ell'} \times U(1)_R$ where the superscript denotes the $U(1)_{\ell'}$ charge and the subscript denotes the $U(1)_R$ charge.

The $\mathcal{N} = 4$ vector multiplet contains the gauge field and six scalar fields in the untwisted theory. The gauge field is not affected by twisting, and splits into the following two irreducible representations¹⁷:

$$A^{\pm} (2, 1, 1)_0^{\pm 1} \quad (A.1)$$

corresponding to the Hodge decomposition of a 1-form on a Kähler manifold into (1,0) and (0,1) forms. Similarly, the exterior derivative splits as $d = d^+ + d^-$:

$$d^{\pm} (2, 1, 1)_0^{\pm} \quad (A.2)$$

χ is the Euler characteristic, that in the case of the K3 surface is equal to $\chi = 24$

$$Z = \sum_n a_n q^n.$$

$$q = \exp(2\pi i\tau)$$

$$\begin{aligned}
\eta_4 &= \omega_4 + \frac{4!}{64\pi^2} \left\{ -A(Z^4 dZ^4 + Z^5 dZ^5)(Z^1 dZ^2 dZ^3 + Z^2 dZ^3 dZ^1 + Z^3 dZ^1 dZ^2) \right. \\
&\quad \left. + A((Z^4)^2 + (Z^5)^2) dZ^1 dZ^2 dZ^3 \right. \\
&\quad \left. - \frac{1}{3} dA(Z^1 dZ^2 dZ^3 Z^2 dZ^3 dZ^1 + Z^3 dZ^2 dZ^1) \right\} \\
&= \omega_4 + \frac{4!}{64\pi^2} \left\{ A(Z^1 dZ^1 + Z^2 dZ^2 + Z^3 dZ^3)(Z^1 dZ^2 dZ^3 + Z^2 dZ^3 dZ^1 + Z^3 dZ^1 dZ^2) \right. \\
&\quad \left. + A(1 - (Z^1)^2 - (Z^2)^2 - (Z^3)^2) dZ^1 dZ^2 dZ^3 \right. \\
&\quad \left. - \frac{1}{3} dA(Z^1 dZ^2 dZ^3 + Z^2 dZ^3 dZ^1 + Z^3 dZ^2 dZ^1) \right\} \\
&= \omega_4 + \frac{4!}{64\pi^2} \left\{ AdZ^1 dZ^2 dZ^3 - \frac{1}{3} dA(Z^1 dZ^2 dZ^3 + Z^2 dZ^3 dZ^1 + Z^3 dZ^2 dZ^1) \right\} \\
&= \omega_4 - \frac{1}{8\pi^2} d \left\{ A(Z^1 dZ^2 dZ^3 + Z^2 dZ^3 dZ^1 + Z^3 dZ^2 dZ^1) \right\} \quad (C.11)
\end{aligned}$$

From

$$\omega_4 := \frac{1}{64\pi^2} \epsilon_{a_1 a_2 a_3 a_4 a_5} Z^{a_1} dZ^{a_2} dZ^{a_3} dZ^{a_4} dZ^{a_5}$$

and:

$$\omega_4 - \frac{1}{8\pi^2} d \left\{ A(Z^1 dZ^2 dZ^3 + Z^2 dZ^3 dZ^1 + Z^3 dZ^2 dZ^1) \right\}$$

we obtain:

$$\begin{aligned}
&1/(64*\text{Pi}^2)*(24*48*72*96*120)*(24*\exp(2\text{Pi}))-1/(8*\text{Pi}^2) \\
&2*(((24*\text{Exp}(2\text{Pi}))+24*2*\exp(2\text{Pi})^2+24*3*\exp(2\text{Pi})^3)))
\end{aligned}$$

Input:

$$\begin{aligned}
&\frac{1}{64\pi^2} (24 \times 48 \times 72 \times 96 \times 120) (24 \exp(2\pi)) - \\
&\frac{1}{8\pi^2} (2 (24 \exp(2\pi) + 24 \times 2 \exp^2(2\pi) + 24 \times 3 \exp^3(2\pi)))
\end{aligned}$$

Exact result:

$$\frac{358318080 e^{2\pi}}{\pi^2} - \frac{24 e^{2\pi} + 48 e^{4\pi} + 72 e^{6\pi}}{4\pi^2}$$

Decimal approximation:

$$1.9160742124269596842234904494988765573112091531837892... \times 10^{10}$$

$$1.91607421242695.... * 10^{10}$$

Alternate forms:

$$\frac{6 e^{2\pi} (-59\,719\,679 + 2 e^{2\pi} + 3 e^{4\pi})}{\pi^2}$$

$$\frac{358\,318\,074 e^{2\pi}}{\pi^2} - \frac{12 e^{4\pi}}{\pi^2} - \frac{18 e^{6\pi}}{\pi^2}$$

Series representations:

$$\frac{(24 \times 48 \times 72 \times 96 \times 120) (24 \exp(2\pi))}{64 \pi^2} - \frac{2 (24 \exp(2\pi) + 24 \times 2 \exp^2(2\pi) + 24 \times 3 \exp^3(2\pi))}{8 \pi^2} = \frac{6 \left(\sum_{k=0}^{\infty} \frac{1}{k!} \right)^{2\pi} (-59\,719\,679 + 2 \left(\sum_{k=0}^{\infty} \frac{1}{k!} \right)^{2\pi} + 3 \left(\sum_{k=0}^{\infty} \frac{1}{k!} \right)^{4\pi})}{\pi^2}$$

$$\frac{(24 \times 48 \times 72 \times 96 \times 120) (24 \exp(2\pi))}{64 \pi^2} - \frac{2 (24 \exp(2\pi) + 24 \times 2 \exp^2(2\pi) + 24 \times 3 \exp^3(2\pi))}{8 \pi^2} = \frac{3 e^{8 \sum_{k=0}^{\infty} \frac{(-1)^k}{(1+2k)}} \left(-59\,719\,679 + 2 e^{8 \sum_{k=0}^{\infty} \frac{(-1)^k}{(1+2k)}} + 3 e^{16 \sum_{k=0}^{\infty} \frac{(-1)^k}{(1+2k)}} \right)}{8 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^2}$$

$$\frac{(24 \times 48 \times 72 \times 96 \times 120) (24 \exp(2\pi))}{64 \pi^2} - \frac{2 (24 \exp(2\pi) + 24 \times 2 \exp^2(2\pi) + 24 \times 3 \exp^3(2\pi))}{8 \pi^2} = \frac{6 \left(-59\,719\,679 + 2 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{2\pi} + 3 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{4\pi} \right) \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{2\pi}}{\pi^2}$$

Integral representations:

$$\frac{(24 \times 48 \times 72 \times 96 \times 120) (24 \exp(2 \pi))}{64 \pi^2} - \frac{2 (24 \exp(2 \pi) + 24 \times 2 \exp^2(2 \pi) + 24 \times 3 \exp^3(2 \pi))}{8 \pi^2} = \frac{3 e^4 \int_0^\infty \frac{1}{(1+t^2)} dt \left(-59719679 + 2 e^4 \int_0^\infty \frac{1}{(1+t^2)} dt + 3 e^8 \int_0^\infty \frac{1}{(1+t^2)} dt \right)}{2 \left(\int_0^\infty \frac{1}{1+t^2} dt \right)^2}$$

$$\frac{(24 \times 48 \times 72 \times 96 \times 120) (24 \exp(2 \pi))}{64 \pi^2} - \frac{2 (24 \exp(2 \pi) + 24 \times 2 \exp^2(2 \pi) + 24 \times 3 \exp^3(2 \pi))}{8 \pi^2} = \frac{3 e^4 \int_0^\infty \frac{\sin(t)}{t} dt \left(-59719679 + 2 e^4 \int_0^\infty \frac{\sin(t)}{t} dt + 3 e^8 \int_0^\infty \frac{\sin(t)}{t} dt \right)}{2 \left(\int_0^\infty \frac{\sin(t)}{t} dt \right)^2}$$

$$\frac{(24 \times 48 \times 72 \times 96 \times 120) (24 \exp(2 \pi))}{64 \pi^2} - \frac{2 (24 \exp(2 \pi) + 24 \times 2 \exp^2(2 \pi) + 24 \times 3 \exp^3(2 \pi))}{8 \pi^2} = \frac{3 e^8 \int_0^1 \sqrt{1-t^2} dt \left(-59719679 + 2 e^8 \int_0^1 \sqrt{1-t^2} dt + 3 e^{16} \int_0^1 \sqrt{1-t^2} dt \right)}{8 \left(\int_0^1 \sqrt{1-t^2} dt \right)^2}$$

and:

$$\ln\left(\frac{(1/(64 \cdot \pi^2) \cdot (24 \cdot 48 \cdot 72 \cdot 96 \cdot 120) \cdot (24 \cdot \exp(2\pi)) - 1/(8 \cdot \pi^2) \cdot 2 \cdot ((24 \cdot \exp(2\pi) + 24 \cdot 2 \cdot \exp(2\pi)^2 + 24 \cdot 3 \cdot \exp(2\pi)^3))))))^{1/(2\pi)}\right)$$

Input:

$$\log\left(\frac{1}{64 \pi^2} (24 \times 48 \times 72 \times 96 \times 120) (24 \exp(2 \pi)) - \frac{1}{8 \pi^2} (2 (24 \exp(2 \pi) + 24 \times 2 \exp^2(2 \pi) + 24 \times 3 \exp^3(2 \pi)))\right)^{\left(\frac{1}{2 \pi}\right)}$$

log(x) is the natural logarithm

Exact result:

$$2\pi \sqrt{\log\left(\frac{358318080 e^{2\pi}}{\pi^2} - \frac{24 e^{2\pi} + 48 e^{4\pi} + 72 e^{6\pi}}{4 \pi^2}\right)}$$

Decimal approximation:

1.654734576752129681878834699067592000937900857824734366260...

1.654734576.... result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

Alternate forms:

$$2\pi \sqrt{2\pi + \log\left(-\frac{6(-59\,719\,679 + 2e^{2\pi} + 3e^{4\pi})}{\pi^2}\right)}$$

$$2\pi \sqrt{2\pi + \log(6) + \log(59\,719\,679 - 2e^{2\pi} - 3e^{4\pi}) - 2\log(\pi)}$$

$$2\pi \sqrt{\log\left(\frac{358\,318\,080e^{2\pi}}{\pi^2} + \frac{-24e^{2\pi} - 48e^{4\pi} - 72e^{6\pi}}{4\pi^2}\right)}$$

Alternative representations:

$$\log\left(\frac{(24 \times 48 \times 72 \times 96 \times 120)(24 \exp(2\pi))}{64\pi^2} - \frac{2(24 \exp(2\pi) + 24 \times 2 \exp^2(2\pi) + 24 \times 3 \exp^3(2\pi))}{8\pi^2}\right)^{\wedge\left(\frac{1}{2\pi}\right)} = 2\pi \sqrt{\log_e\left(-\frac{2(24 \exp(2\pi) + 48 \exp^2(2\pi) + 72 \exp^3(2\pi))}{8\pi^2} + \frac{22\,932\,357\,120 \exp(2\pi)}{64\pi^2}\right)}$$

$$\log\left(\frac{(24 \times 48 \times 72 \times 96 \times 120)(24 \exp(2\pi))}{64\pi^2} - \frac{2(24 \exp(2\pi) + 24 \times 2 \exp^2(2\pi) + 24 \times 3 \exp^3(2\pi))}{8\pi^2}\right)^{\wedge\left(\frac{1}{2\pi}\right)} = \left(\log(a) \log_a\left(-\frac{2(24 \exp(2\pi) + 48 \exp^2(2\pi) + 72 \exp^3(2\pi))}{8\pi^2} + \frac{22\,932\,357\,120 \exp(2\pi)}{64\pi^2}\right)\right)^{\wedge\left(\frac{1}{2\pi}\right)}$$

Series representations:

$$\log\left(\frac{(24 \times 48 \times 72 \times 96 \times 120) (24 \exp(2 \pi))}{64 \pi^2} - \frac{2 (24 \exp(2 \pi) + 24 \times 2 \exp^2(2 \pi) + 24 \times 3 \exp^3(2 \pi))}{8 \pi^2}\right) \wedge \left(\frac{1}{2 \pi}\right) =$$

$$\left(2 i \pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi}\right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \pi^{-2k} z_0^{-k} (358 318 074 e^{2 \pi} - 12 e^{4 \pi} - 18 e^{6 \pi} - \pi^2 z_0)^k}{k}\right) \wedge \left(\frac{1}{2 \pi}\right)$$

$$\log\left(\frac{(24 \times 48 \times 72 \times 96 \times 120) (24 \exp(2 \pi))}{64 \pi^2} - \frac{2 (24 \exp(2 \pi) + 24 \times 2 \exp^2(2 \pi) + 24 \times 3 \exp^3(2 \pi))}{8 \pi^2}\right) \wedge \left(\frac{1}{2 \pi}\right) =$$

$$\left(\log\left(-1 + \frac{358 318 080 e^{2 \pi}}{\pi^2} - \frac{24 e^{2 \pi} + 48 e^{4 \pi} + 72 e^{6 \pi}}{4 \pi^2}\right) - \sum_{k=1}^{\infty} \frac{\pi^{2k} \left(\frac{1}{-358 318 074 e^{2 \pi} + 12 e^{4 \pi} + 18 e^{6 \pi} + \pi^2}\right)^k}{k}\right) \wedge \left(\frac{1}{2 \pi}\right)$$

$$\log\left(\frac{(24 \times 48 \times 72 \times 96 \times 120) (24 \exp(2 \pi))}{64 \pi^2} - \frac{2 (24 \exp(2 \pi) + 24 \times 2 \exp^2(2 \pi) + 24 \times 3 \exp^3(2 \pi))}{8 \pi^2}\right) \wedge \left(\frac{1}{2 \pi}\right) =$$

$$\left(2 i \pi \left[\frac{\arg\left(\frac{358 318 080 e^{2 \pi}}{\pi^2} - \frac{24 e^{2 \pi} + 48 e^{4 \pi} + 72 e^{6 \pi}}{4 \pi^2} - x\right)}{2 \pi}\right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \pi^{-2k} x^{-k} (358 318 074 e^{2 \pi} - 12 e^{4 \pi} - 18 e^{6 \pi} - \pi^2 x)^k}{k}\right) \wedge \left(\frac{1}{2 \pi}\right)$$

$\left(\frac{1}{2 \pi}\right)$ for $x < 0$

Integral representations:

$$\log\left(\frac{(24 \times 48 \times 72 \times 96 \times 120) (24 \exp(2 \pi))}{64 \pi^2} - \frac{2 (24 \exp(2 \pi) + 24 \times 2 \exp^2(2 \pi) + 24 \times 3 \exp^3(2 \pi))}{8 \pi^2}\right) \wedge \left(\frac{1}{2 \pi}\right) =$$

$$2 \pi \sqrt{\int_1^{-\frac{6 e^{2 \pi} (-59 719679 + 2 e^{2 \pi} + 3 e^{4 \pi})}{\pi^2}} \frac{1}{t} dt}$$

$$\log\left(\frac{(24 \times 48 \times 72 \times 96 \times 120) (24 \exp(2\pi))}{64 \pi^2} \frac{2 (24 \exp(2\pi) + 24 \times 2 \exp^2(2\pi) + 24 \times 3 \exp^3(2\pi))}{8 \pi^2}\right)^{\wedge}\left(\frac{1}{2\pi}\right) = (2\pi)^{-1/(2\pi)}$$

$$2\pi \sqrt{-i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\pi^{2s} (358\,318\,074 e^{2\pi} - 12 e^{4\pi} - 18 e^{6\pi} - \pi^2)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}$$

for $-1 < \gamma < 0$

From the formula of coefficients of the '5th order' mock theta function $\psi_1(q)$:
(A053261 OEIS Sequence)

$\sqrt{\phi} \times \exp(\pi \sqrt{n/15}) / (2 \cdot 5^{1/4} \sqrt{n})$ for $n = 1197.947$
where 1197.947 is very near to the rest mass of Sigma baryon 1197.449

$$\sqrt{\phi} \times \exp(\pi \sqrt{1197.947/15}) / (2 \cdot 5^{1/4} \sqrt{1197.947})$$

Input interpretation:

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{1197.947}{15}}\right)}{2 \sqrt[4]{5} \sqrt{1197.947}}$$

ϕ is the golden ratio

Result:

$$1.91607... \times 10^{10}$$

1.91607... * 10¹⁰ result practically equal to the previous value
1.91607421242695.... * 10¹⁰

Series representations:

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{1197.95}{15}}\right)}{2 \sqrt[4]{5} \sqrt{1197.95}} = \frac{\exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (79.8631 - z_0)^k z_0^{-k}}{k!}\right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi - z_0)^k z_0^{-k}}{k!}}{2 \sqrt[4]{5} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1197.95 - z_0)^k z_0^{-k}}{k!}}$$

for (not $(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)$)

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{1197.95}{15}}\right)}{2 \sqrt[4]{5} \sqrt{1197.95}} = \left(\exp\left(i \pi \left\lfloor \frac{\arg(\phi - x)}{2 \pi} \right\rfloor\right) \right. \\ \left. \exp\left(\pi \exp\left(i \pi \left\lfloor \frac{\arg(79.8631 - x)}{2 \pi} \right\rfloor\right)\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (79.8631 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \\ \left. \sum_{k=0}^{\infty} \frac{(-1)^k (\phi - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) / \\ \left(2 \sqrt[4]{5} \exp\left(i \pi \left\lfloor \frac{\arg(1197.95 - x)}{2 \pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k (1197.95 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right)$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{1197.95}{15}}\right)}{2 \sqrt[4]{5} \sqrt{1197.95}} = \left(\exp\left(\pi \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(79.8631 - z_0)/(2 \pi) \rfloor}\right) \right. \\ \left. z_0^{1/2 (1 + \lfloor \arg(79.8631 - z_0)/(2 \pi) \rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (79.8631 - z_0)^k z_0^{-k}}{k!} \right) \\ \left(\frac{1}{z_0}\right)^{-1/2 \lfloor \arg(1197.95 - z_0)/(2 \pi) \rfloor + 1/2 \lfloor \arg(\phi - z_0)/(2 \pi) \rfloor} \\ z_0^{-1/2 \lfloor \arg(1197.95 - z_0)/(2 \pi) \rfloor + 1/2 \lfloor \arg(\phi - z_0)/(2 \pi) \rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi - z_0)^k z_0^{-k}}{k!} \Bigg) / \\ \left(2 \sqrt[4]{5} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (1197.95 - z_0)^k z_0^{-k}}{k!} \right)$$

Observations

From:

https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJIQxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRsIBDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that $p(9) = 30$, $p(9 + 5) = 135$, $p(9 + 10) = 490$, $p(9 + 15) = 1,575$ and so on are all divisible by 5. Note that here the n 's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of $p(n)$ that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n 's separated by $5^3 = 125$ units, saying that the corresponding $p(n)$'s should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ . The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson:

125 GeV for $T = 0$ and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \dots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and $4096 = 64^2$

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π , ϕ , $1/\phi$, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n , form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the n th Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.

Fibonacci numbers are also closely related to Lucas numbers, in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.^[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803.....

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ , the golden ratio.^[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies^[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **$f_0(1710)$ scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson π) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

References

Duality and Mock Modularity

Atish Dabholkar, I Pavel Putrov, I Edward Witten - arXiv:2004.14387v1 [hep-th] 29 Apr 2020

RIEMANN'S HYPOTHESIS AND SOME INFINITE SET OF MICROSCOPIC UNIVERSES OF THE EINSTEIN'S TYPE IN THE EARLY PERIOD OF THE EVOLUTION OF THE UNIVERSE - *JAN MOSER* - arXiv:1307.1095v2 [physics.gen-ph] 28 Jul 2013

New expressions for Riemann's functions $\xi(s)$ and $\Xi(t)$ – *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLVI, 1915, 253 – 260