Periodic Tables of Prime Numbers

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Abstract

If p is prime>2 and n is a integer with $0<n<p$, then the n/p fraction forms a repeating group (period) of decimal digits of length ℓ. If we do the same for each n from 1 to p-1 we will get p-1 such periods of digits of the same length. We will notice that many of these periods identify with each other as their difference is that they start with a different digit of the same period. From the groups that identify with each other, we leave only one, deleting the others. So we will finally have r periods of length ℓ digits, and will apply $r\times\ell+1=p$.

For example, 7 has only one period, 11 has five and 13 has two. In different numerical bases different numbers of periods for each prime appear. But if we have to do the same for 101, it will be very difficult. Things get worse when we work with bases that are different from 10. What we need is a simple method of locating these periods for each prime and for each base of the arithmetic system.

More specifically, these periods can be arranged in $\ell\times r$ dimension tables for each base, using only integers, without having to abandon the familiar base 10. This idea is applied with the following sentence.

Definitions

p : prime>2
B : numbering system basis, $1<B<p$, $\gcd(p,B)=1$
n : integer, $0<n<p$
ℓ : the length of the period forming the n/p fraction
r : number of periods of the p
b : basis of organization of periods of p in base B, $1<b<p$, as defined in the sentence

Sentence

There is at least one base b, for which the n/p fraction, with

$$n = b^{rx+y} \bmod p, \;\; -1<x<\ell, \; -1<y<r$$

will form the *y*-th period of *p* from the *x*-th digit.

Example with p=41, B=10, b=6

y\x	0	1	2	3	4
0	01	10→	18	16	37←
1	06↓	19↘	26	14	17↙
2	36	32	33	02	20
3	11	28	34	12	38
4	25	04	40	31	23
5	27	24	35	22	15
6	39	21	05	09	08
7	29↗	03	30	13↖	07↑

p=41

where (0,0)=1, (1,0)=B, (0,1)=b, and ℓ=5, r=8.

This table can be constructed through the relationship

$(x,y) = 6^{8x+y} \bmod 41 = n$

For example, $(3,4) = 6^{8\times3+4} \bmod 41 = 31$

Now we see that the period on the line y=4 starts with 25/41=0.<u>60975</u>, while the digit corresponding to position x=3 is 7. Indeed, fraction 31/41=0.<u>75609</u> forms this periodic number from digit 7.

In place of *b* we can even put the numbers 13, 28 and 35. It is noted that *p* always has a unique period of length *p*-1 digits at base *b* of the arithmetic system. There is another type of organization of the periods of a number *q*, compound or prime, in which this does not apply. For 41, this type of organization occurs when we put the numbers 3, 14, 27 and 38 in place of *b* and use the relationship $(x,y) = B^x b^y \bmod 41$.

The downside of this type of organization is that it does not apply to all prime and compisite numbers.

The methods for determining base b vary and are generally dependent on gcd(ℓ, r).

Numerical operations in periodic tables

Periodic tables provide the possibility of arithmetic operations in the arithmetic of mod p with vectors, such as multiplications, divisions, and inversions. In the table above there is an example of multiplication, 28×2 mod 41=15. In order for these operations to be possible using parallelograms we need to copy part of the table bottom and right.

The runners

Each element of the table corresponds to a vector. The elements corresponding to the unit vectors are noted in the arrow table. In all periodic tables these arrows are always in the same positions. For example, 29 in the lower left corner shows right and top. Thus, the operation 31×29 (mod 41) will give 38, which is located just above and to the right of 31.

Square periodic tables

For some bases B and for some prime numbers p, square tables are formed. That is, it will be ℓ=r. For B = 10 there are no square tables. In square tables each element of the line y=1 is a suitable base b, and no other elements of the table. In these tables there are never b bases that are suitable for the second type of organization of periods.

For example, at base B=4 o p=17 it has four periods of four elements with bases b = 6, 7, 10, 11.

y\x	0	1	2	3
0	01	04	16	13
1	06	07	11	10
2	02	08	15	09
3	12	14	05	03

$$(x,y) = 6^{4x+y} \bmod 17$$

In base B = 2, the smallest prime number that has a square table is 257, with the smallest base of b being 27. In one-dimensional tablets that form a period, base B plays the role of base b.

For more table constructions

B=10, p =/= 2, 5

The primes that have only one period are omitted.

p	ℓ	r	b
3	1	2	2
7	6	1	3
11	2	5	2, 6, 7, 8
13	6	2	6, 7
31	15	2	17
37	3	12	5, 13, 18, 19, 24, 32
41	5	8	6, 13, 28, 35
43	21	2	28
53	13	4	26, 27
67	33	2	12
71	35	2	62
73	8	9	5, 14, 20, 28, 39, 40
79	13	6	29, 35
83	41	2	59
89	44	2	30, 59

Interactive. https://www.geogebra.org/m/JrsAVqep

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