

# Discrete Mass Neutron Stars

B. F. Riley

Following on from an analysis of black hole and neutron star mass in gravitational wave events we show that the mass of a neutron star in general is related through an inverse 5/2 power law to a sub-Planckian mass scale – in the range 7-10 TeV – within one or more of three geometric sequences of mass scales that descend from Planck scale and are occupied by the particles. The three common ratios are  $1/\pi$ ,  $2/\pi$  and  $1/e$ . The results provide more evidence for a stringy theory of quantum gravity.

The (super-Planckian) mass  $M$  of a neutron star is inserted into equation (1): the resulting value of  $m$  is a sub-Planckian mass scale in the range 7-10 TeV. Planck units are used, which explains the apparently unbalanced dimensions.

$$2m^{-5} = M^2 \quad (1)$$

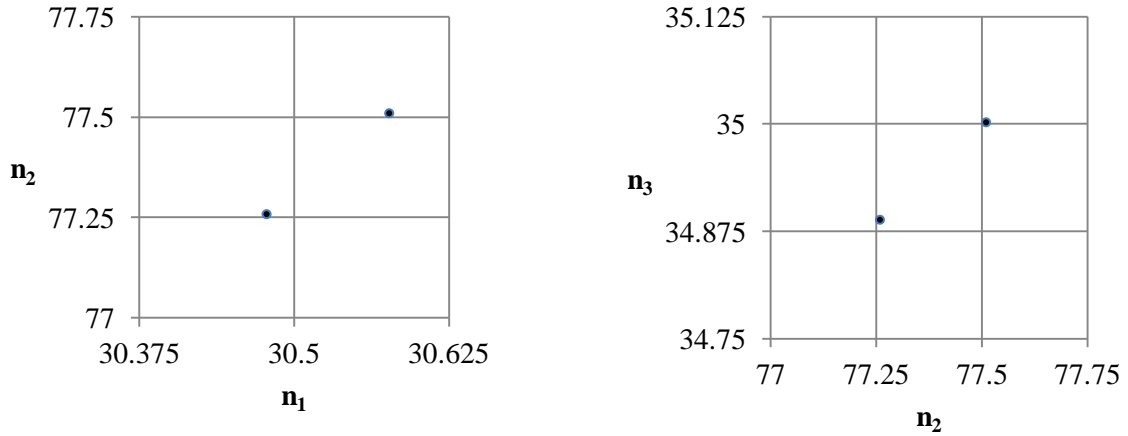
The sub-Planckian scales resulting from (1) coincide with mass levels and sub-levels of fractional level-number in Sequences 1, 2 and 3, which descend geometrically from Planck scale with common ratio  $\pi^{-1}$ ,  $(\pi/2)^{-1}$  and  $e^{-1}$ , respectively. Each mass level is related to the Planck Mass through an exponential factor, e.g.  $\pi^{-n_1}$  where  $n_1$  is an integer or a half-integer, quarter-integer, eighth-integer etc. The sequences may derive from the geometry of a warped spacetime [1]; the exponents  $n_1$ ,  $n_2$  and  $n_3$  may be the winding numbers (integer and fractional) of intersecting D-branes that wrap cycles of Planckian compact spaces and whose intersections are the domains of the particles. Equation (1) was most recently applied to the black holes and neutron stars of gravitational wave events [2]. The origin, and implications for neutron stars, of equation (1) will be discussed later.

Precise data are taken from a review of neutron star masses and radii by Özel and Freire [3] and are shown in Table 1. The ten values of neutron star mass used here are those for which the stated uncertainty is  $\leq 0.005 M_\odot$ .

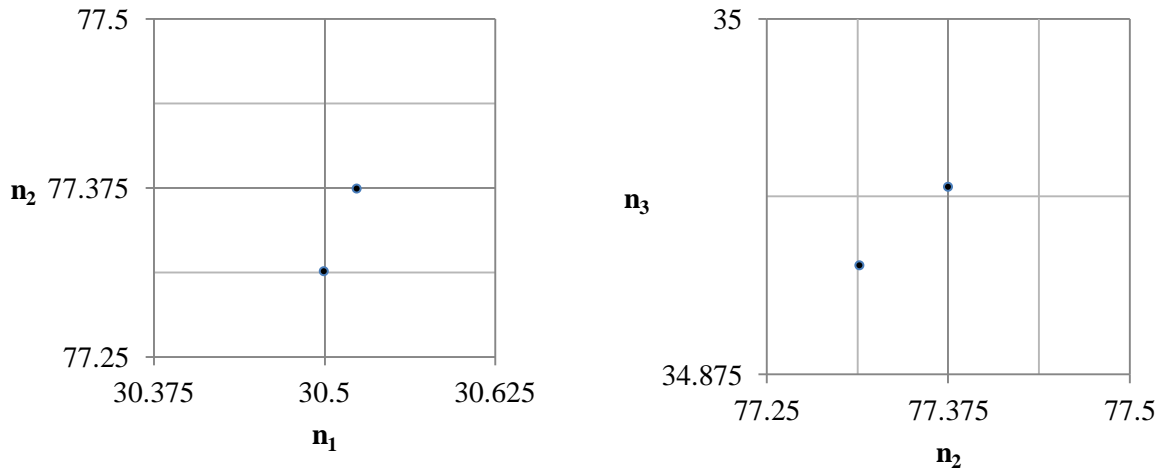
System	Neutron star masses ( $M_\odot$ )	Comments
J0453+1559	1.559(5), 1.174(4)	Double NS system
J0737–3039	1.3381(7), 1.2489(7)	Double NS system
B1534+12	1.3330(4), 1.3455(4)	Double NS system
B1913+16	1.4398(2), 1.3886(2)	Double NS system
J1807–2500B	1.3655(21)	Millisecond pulsar; nature of companion uncertain
J0337+1715	1.4378(13)	Millisecond pulsar; white dwarf companion

**Table 1:** The neutron star masses of the analysis.

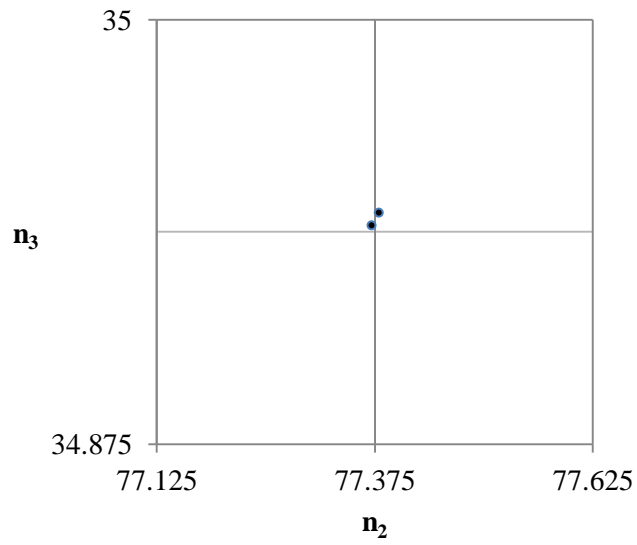
The occupation of sub-levels in Sequences 1, 2 and 3 is shown in Figures 1-5. As usual in the Planck Model [4], mass levels of ‘low order’ (integer, half-integer, quarter-integer level-number) and especially near-coincident such levels are preferred locations for the scales. We see that neutron stars assume discrete masses that are codified through (1).



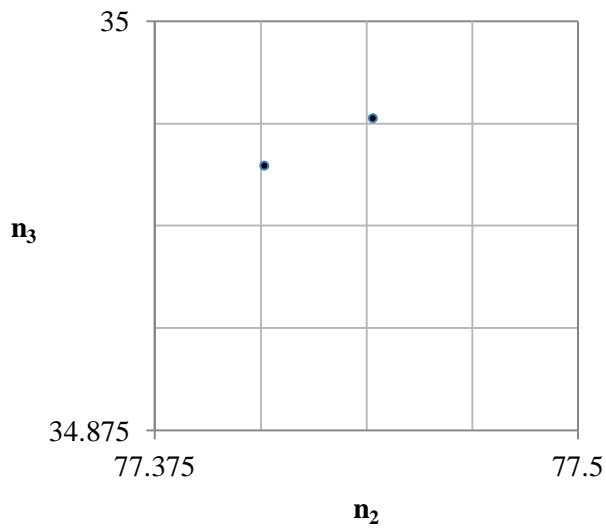
**Figure 1:** Sub-Planckian mass scales corresponding through (1) to neutron star masses from system J0453+1559 on the mass levels of Sequences 1, 2 and 3. The mass scale corresponding to the more massive NS is of greater level-number than the mass scale corresponding to the less massive NS. Greater level-number means smaller sub-Planckian mass scale. Marker size is an indication of uncertainty in location.



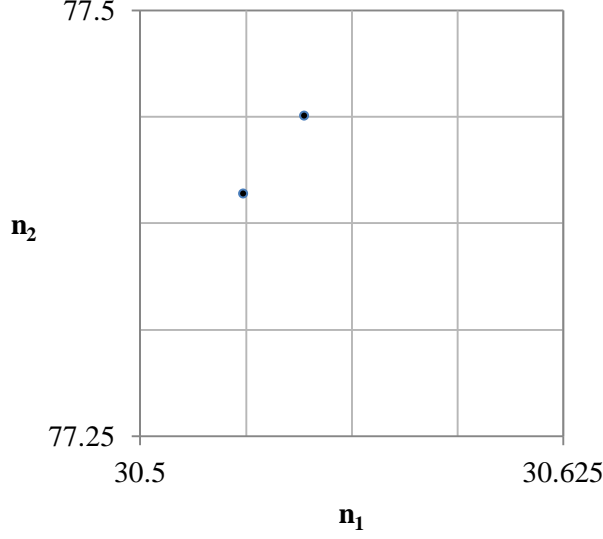
**Figure 2:** Sub-Planckian mass scales corresponding through (1) to neutron star masses from system J0737-3039 on the mass levels of Sequences 1, 2 and 3.



**Figure 3:** Sub-Planckian mass scales corresponding through (1) to neutron star masses from system B1534+12 on the mass levels of Sequences 2 and 3. Symmetrical arrangements such as this are common in the Planck Model [4].



**Figure 4:** Sub-Planckian mass scales corresponding through (1) to neutron star masses from system B1913+16 on the mass levels of Sequences 2 and 3.



**Figure 5:** Sub-Planckian mass scales corresponding through (1) to neutron star masses from systems J1807–2500B and J0337+1715 on the mass levels of Sequences 1 and 2.

The Quantum/Classical connection was found in an analysis of the cosmological constant [5]. By equating the vacuum energy in the 5-sphere of  $AdS_5 \times S^5$  spacetime at the length scale of the Bohr radius  $a_0$  and on the boundary of the observable universe, radius  $R_{OU} = 14.3$  Gpc [6], we found the precise equation

$$2a_0^5 = R_{OU}^2 \quad (2)$$

The implications of this equation alone are profound but it was the associated equations

$$2m^{-5} = R^2 \quad (3)$$

and

$$2r^{-5} = M^2 \quad (4)$$

where  $m$  and  $r$  are corresponding sub-Planckian mass and length scales and  $R$  and  $M$  are cosmological and astrophysical length (radius) and mass scales that have been investigated extensively. Two examples: (3) has been used to relate the radii of Main Sequence stars to the masses of atoms [7]; (4) has been used to relate the masses of planetary bodies to sub-Planckian length scales on the levels of the Planck Sequences (Sequences 1, 2 and 3) [8].

Equation (4) is of particular interest to us here. Since  $r$  in this equation is the radius of an unknown object we have written the equation as in (1) in order to relate neutron star masses to sub-Planckian mass scales with which we are familiar rather than to sub-Planckian length scales with which we are unfamiliar. Note, though, that when we applied (3) and (4) to the canonical  $1.4 M_{\odot}$  neutron star with radius 11.7 km we found that  $r = 2m$ , precisely [9]. Compare this equation with that of the Schwarzschild black hole:  $R_S = 2M_{\bullet}$ . For the canonical neutron star (3) and (4) tell us that

$$R_{NS} = 2^{5/2} M_{NS} \quad (5)$$

It seems that physics on cosmological and astrophysical scales derives from physics on length scales shorter than the Bohr radius through the Quantum/Classical connection. Geometry is key in the

quantum world. The ‘quantum’ scales relating to the ‘classical’ scales of astrophysical bodies tend to be found on coincident levels and low-order sub-levels of mass scale and length scale. At the corresponding locations in the higher-dimensional spacetime the local geometry is most similar to that at Planck scale.

## References

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