# On some Ramanujan equations: mathematical connections with various topics concerning Number Theory, $\phi$, $\zeta(2)$ and several parameters of Particle Physics. IV

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## Abstract

*In this paper we have described and analyzed some Ramanujan equations. We have obtained several mathematical connections between some topics concerning Number Theory, $\phi$, $\zeta(2)$ and various parameters of Particle Physics.*

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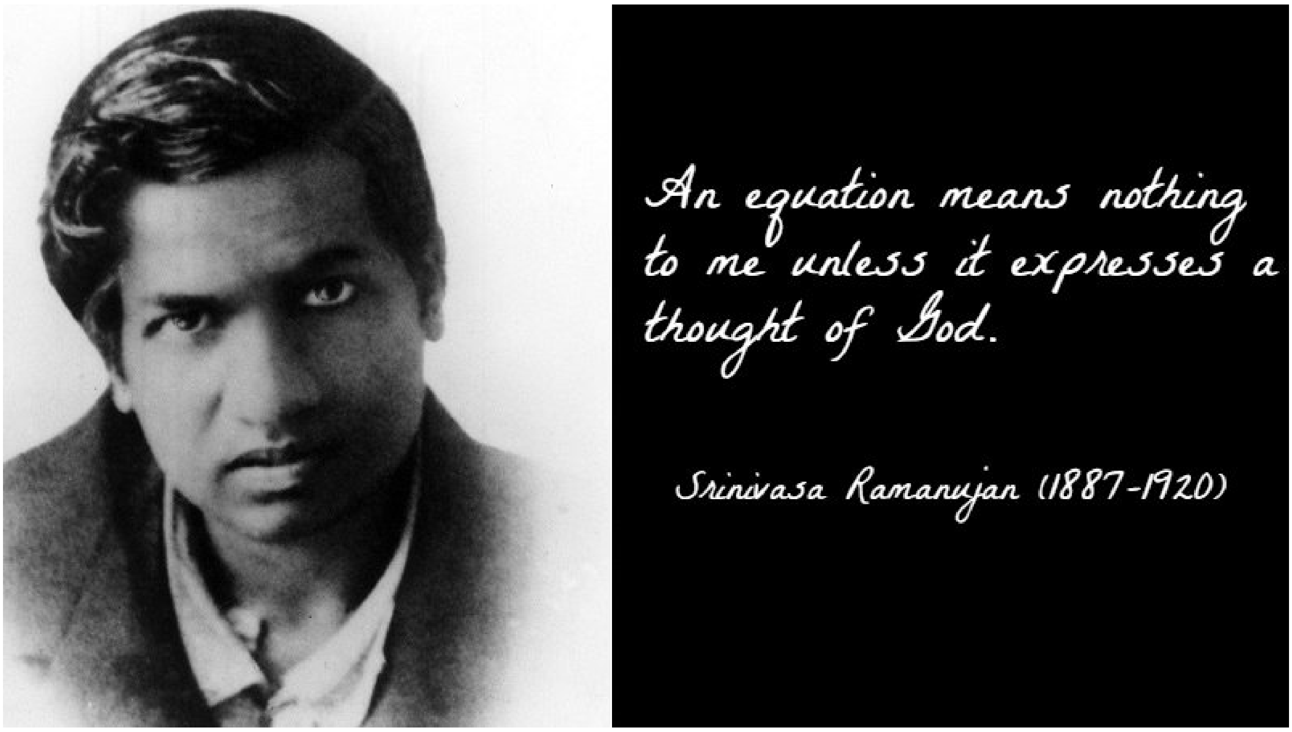
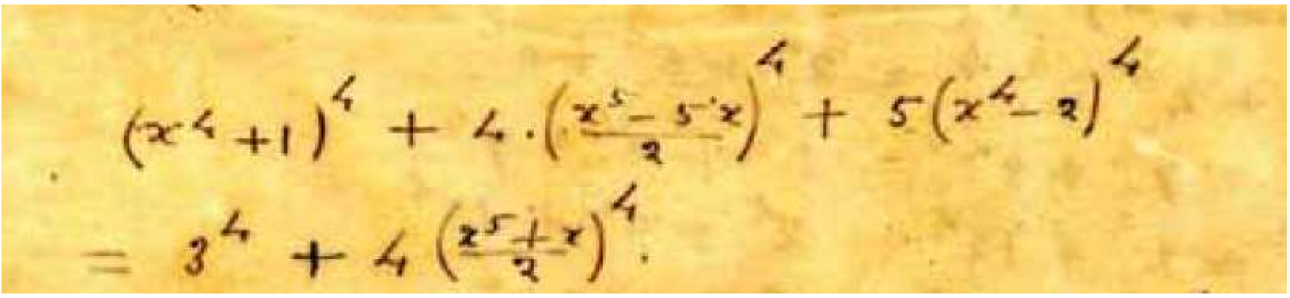



https://mobygeek.com/features/indian-mathematician-srinivasa-ramanujan-quotes-11012

**We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation**

From

Page 20 – **Manuscript Book 3 of Srinivasa Ramanujan**



$$(x^4+1)^4 + 4.\left(\frac{x^5-5.x}{2}\right)^4 + 5(x^4-2)^4$$
$$= 3^4 + 4\left(\frac{x^5+x}{2}\right)^4.$$

For x = 2, we obtain:

(2^4+1)^4+4((2^5-5*2)/2)^4+5(2^4-2)^4 = 3^4+4((2^5+2)/2)^4

**Input:**

$$(2^4+1)^4+4\left(\frac{1}{2}(2^5-5\times 2)\right)^4+5(2^4-2)^4 = 3^4+4\left(\frac{1}{2}(2^5+2)\right)^4$$

**Result:**

True

**Left hand side:**

$$(2^4+1)^4+4\left(\frac{1}{2}(2^5-5\times 2)\right)^4+5(2^4-2)^4 = 334165$$

**Right hand side:**

$$3^4+4\left(\frac{1}{2}(2^5+2)\right)^4 = 334165$$

334165

((((2^4+1)^4+4((2^5-5*2)/2)^4+5(2^4-2)^4)))^1/26

**Input:**

$$\sqrt[26]{(2^4+1)^4+4\left(\frac{1}{2}(2^5-5\times 2)\right)^4+5(2^4-2)^4}$$

**Result:**

$\sqrt[26]{334165}$

**Decimal approximation:**

1.631022818310082384146910024444202846598473656742990747650...

1.63102281831008….≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

**All 26th roots of 334165:**

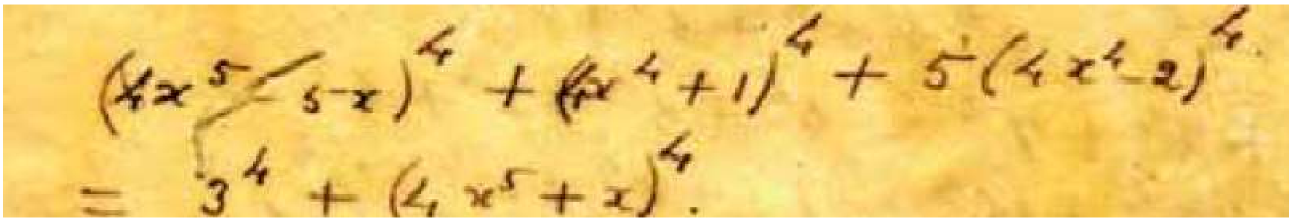
$\sqrt[26]{334165}\; e^0 \approx 1.631023$ (real, principal root)

$\sqrt[26]{334165}\; e^{(i\pi)/13} \approx 1.58363 + 0.39033\, i$

$\sqrt[26]{334165}\; e^{(2 i\pi)/13} \approx 1.44420 + 0.7580\, i$

$\sqrt[26]{334165}\; e^{(3 i\pi)/13} \approx 1.2208 + 1.0816\, i$

$\sqrt[26]{334165}\; e^{(4 i\pi)/13} \approx 0.9265 + 1.3423\, i$




For x = 2, we obtain:

(4*2^5-5*2)^4+(4*2^4+1)^4+5(4*2^4-2)^4 = 3^4+(4*2^5+2)^4

**Input:**

$(4\times 2^5 - 5\times 2)^4 + (4\times 2^4 + 1)^4 + 5\,(4\times 2^4 - 2)^4 = 3^4 + (4\times 2^5 + 2)^4$

**Result:**

True

**Left hand side:**

$(4\times 2^5 - 5\times 2)^4 + (4\times 2^4 + 1)^4 + 5\,(4\times 2^4 - 2)^4 = 285\,610\,081$

**Right hand side:**

$3^4+(4\times 2^5+2)^4 = 285\,610\,081$

285610081

From which, we obtain:

((((4*2^5-5*2)^4+(4*2^4+1)^4+5(4*2^4-2)^4)))^1/39

**Input:**

$\sqrt[39]{(4\times 2^5-5\times 2)^4+(4\times 2^4+1)^4+5\,(4\times 2^4-2)^4}$

**Result:**

$\sqrt[39]{285\,610\,081}$

**Decimal approximation:**

1.647459344300932062294224016118872869700218793101095369669...

$1.647459344300932\ldots\ldots\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

**All 39th roots of 285610081:**

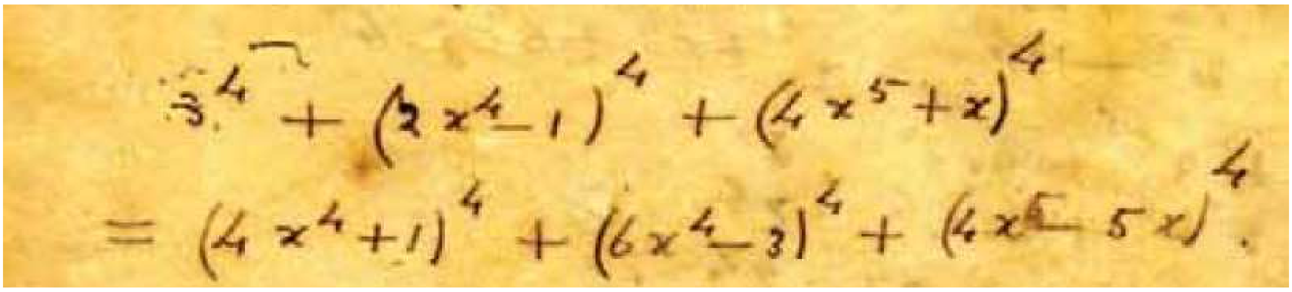
$\sqrt[39]{285\,610\,081}\; e^0 \approx 1.647459$ (real, principal root)

$\sqrt[39]{285\,610\,081}\; e^{(2\,i\,\pi)/39} \approx 1.62613+0.26427\,i$

$\sqrt[39]{285\,610\,081}\; e^{(4\,i\,\pi)/39} \approx 1.56268+0.5217\,i$

$\sqrt[39]{285\,610\,081}\; e^{(2\,i\,\pi)/13} \approx 1.45875+0.7656\,i$

$\sqrt[39]{285\,610\,081}\; e^{(8\,i\,\pi)/39} \approx 1.3170+0.9897\,i$



For x = 2, we obtain:

(3^4+(2*2^4-1)^4+(4*2^5+2)^4)=((4*2^4+1)^4+(6*2^4-3)^4+(4*2^5-5*2)^4)

**Input:**

$$3^4 + (2 \times 2^4 - 1)^4 + (4 \times 2^5 + 2)^4 = (4 \times 2^4 + 1)^4 + (6 \times 2^4 - 3)^4 + (4 \times 2^5 - 5 \times 2)^4$$

**Result:**

True

**Left hand side:**

$$3^4 + (2 \times 2^4 - 1)^4 + (4 \times 2^5 + 2)^4 = 286533602$$

**Right hand side:**

$$(4 \times 2^4 + 1)^4 + (6 \times 2^4 - 3)^4 + (4 \times 2^5 - 5 \times 2)^4 = 286533602$$

286533602

From which

((4*2^4+1)^4+(6*2^4-3)^4+(4*2^5-5*2)^4)^1/39

**Input:**

$$\sqrt[39]{(4 \times 2^4 + 1)^4 + (6 \times 2^4 - 3)^4 + (4 \times 2^5 - 5 \times 2)^4}$$

**Result:**

$$\sqrt[39]{286533602}$$

**Decimal approximation:**

1.6475957209812549626895903425407123031373645806296292 77091...

1.6475957209812549…..≈ $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

**All 39th roots of 286533602:**

$\sqrt[39]{286533602}\ e^0 \approx 1.647596$ (real, principal root)

$\sqrt[39]{286533602}\ e^{(2 i \pi)/39} \approx 1.62626 + 0.26429 i$

$\sqrt[39]{286533602}\ e^{(4 i \pi)/39} \approx 1.56280 + 0.5217 i$

$\sqrt[39]{286533602}\ e^{(2 i \pi)/13} \approx 1.45887 + 0.7657 i$

$\sqrt[39]{286533602}\ e^{(8 i \pi)/39} \approx 1.3172 + 0.9898 i$

We have the following results: 334165, 285610081, 286533602

((( 286533602 +285610081+ 334165)))^1/40

**Input:**

$\sqrt[40]{286533602 + 285610081 + 334165}$

**Result:**

$2^{3/40}\ \sqrt[40]{71559731}$

**Decimal approximation:**

1.655556349170576462553410230862877354076139063936090025415...

1.65555634917057646… result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

**Alternate form:**

root of $x^{40} - 572477848$ near $x = 1.65556$

**All 40th roots of 572477848:**

$2^{3/40}\ \sqrt[40]{71559731}\ e^{0} \approx 1.65556$ (real, principal root)

$2^{3/40}\ \sqrt[40]{71559731}\ e^{(i \pi)/20} \approx 1.63517 + 0.25899 i$

$2^{3/40}\ \sqrt[40]{71559731}\ e^{(i \pi)/10} \approx 1.57453 + 0.5116 i$

$2^{3/40}\ \sqrt[40]{71559731}\ e^{(3 i \pi)/20} \approx 1.47511 + 0.7516 i$

$2^{3/40}\ \sqrt[40]{71559731}\ e^{(i \pi)/5} \approx 1.3394 + 0.9731 i$

We have also:

((( 286533602 +285610081+ 334165)))/2909+89

where 2909 and 89 are Eisenstein numbers

**Input:**

$$\frac{286533602 + 285610081 + 334165}{2909} + 89$$

**Exact result:**

$$\frac{572736749}{2909}$$

**Decimal approximation:**

196884.4101065658301821931935372980405637676177380543141973...

196884.41010656… 196884 is a fundamental number of the following *j*-invariant

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots$$

(In mathematics, Felix Klein's *j*-invariant or *j* function, regarded as a function of a complex variable *τ*, is a modular function of weight zero for SL(2, Z) defined on the upper half plane of complex numbers. Several remarkable properties of *j* have to do with its *q* expansion (Fourier series expansion), written as a Laurent series in terms of *q* = *e*2*πiτ* (the square of the nome), which begins:

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots$$

Note that *j* has a simple pole at the cusp, so its *q*-expansion has no terms below *q*−1.

All the Fourier coefficients are integers, which results in several almost integers, notably Ramanujan's constant:

$$e^{\pi\sqrt{163}} \approx 640320^3 + 744.$$

The asymptotic formula for the coefficient of *qn* is given by

$$\frac{e^{4\pi\sqrt{n}}}{\sqrt{2}\,n^{3/4}},$$

as can be proved by the Hardy–Littlewood circle method)

From:

https://oeis.org/search?q=104164&sort=&language=&go=Search

A041020 Numerators of continued fraction convergents to sqrt(14).

3, 4, 11, 15, 101, 116, 333, 449, 3027, 3476, 9979, 13455, 90709, **104164**, 299037, 403201, 2718243, 3121444, 8961131, 12082575, 81456581, 93539156, 268534893, 362074049, 2440979187, 2803053236, 8047085659 (list; graph; refs; listen; history; text; internal format)

The formula is:

(3+4*x+11*x^2+15*x^3+11*x^4-4*x^5+3*x^6-x^7)/(1-30*x^4+x^8)

Multiplying the previous results, we obtain:

(286533602 * 285610081 * 334165)

**Input:**

$286533602 \times 285610081 \times 334165$

**Result:**

27347022768402161398730

27347022768402161398730

From this result, dividing from the above formula, we have:

27347022768402161398730/(3+4*x+11*x^2+15*x^3+11*x^4-4*x^5+3*x^6-x^7)/(1-30*x^4+x^8)

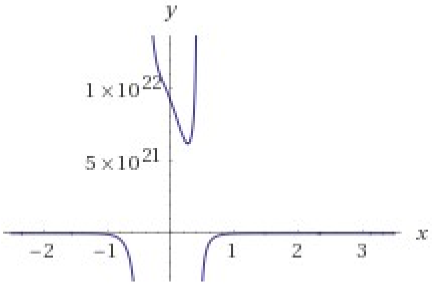
**Input:**

$$\frac{\frac{27347022768402161398730}{3+4x+11x^2+15x^3+11x^4-4x^5+3x^6-x^7}}{1-30x^4+x^8}$$

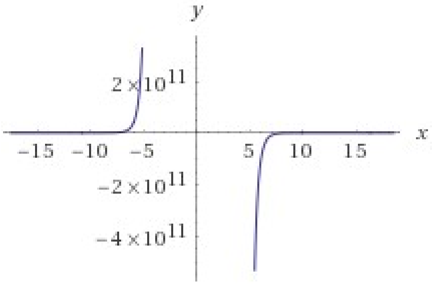
**Result:**

$$\frac{27347022768402161398730}{(-x^7+3x^6-4x^5+11x^4+15x^3+11x^2+4x+3)(x^8-30x^4+1)}$$

## Plots:




(x from −2.5 to 3.5)




(x from −17.5 to 18.5)

## Alternate forms:

$$-\frac{27\,347\,022\,768\,402\,161\,398\,730}{(x^7 - 3x^6 + 4x^5 - 11x^4 - 15x^3 - 11x^2 - 4x - 3)(x^8 - 30x^4 + 1)}$$

$$-\frac{27\,347\,022\,768\,402\,161\,398\,730}{(x^8 - 30x^4 + 1)(x(x(x(x(x((x-3)x+4)-11)-15)-11)-4)-3)}$$

$$-(27\,347\,022\,768\,402\,161\,398\,730 / (x(x(x(x(x(x(x(x(x(x(x(x(x((x-3)x+4)-11)-45)+79)-124)+327)+451)+327)+124)+79)-15)-11)-4)-3))$$

## Expanded form:

$$-\left(27\,347\,022\,768\,402\,161\,398\,730 \middle/ \left(x^{15} - 3x^{14} + 4x^{13} - 11x^{12} - 45x^{11} + 79x^{10} - 124x^9 + 327x^8 + 451x^7 + 327x^6 + 124x^5 + 79x^4 - 15x^3 - 11x^2 - 4x - 3\right)\right)$$

## Properties as a real function:

### Domain

$\{x \in \mathbb{R} : x \neq -2.3397$ and $x \neq -0.427406$ and $x \neq 0.427406$ and $x \neq 2.3397$ and $x \neq 3.31181\}$

### Range

$\{y \in \mathbb{R} : y \neq 0\}$

$\mathbb{R}$ is the set of real numbers

**Series expansion at x = 0:**

$$\frac{27347022768402161398730}{3} - \frac{109388091073608645594920\,x}{9} - \frac{464899387062836743778410\,x^2}{27} + \frac{1777556479946140490917450\,x^3}{81} + \frac{81330045713228027999823020\,x^4}{243} + O(x^5)$$

(Taylor series)

**Series expansion at x = ∞:**

$$-\frac{27347022768402161398730}{x^{15}} - \frac{82041068305206484196190}{x^{16}} - \frac{136735113842010806993650}{x^{17}} - \frac{382858318757630259582220}{x^{18}} + O\left(\left(\frac{1}{x}\right)^{19}\right)$$

(Laurent series)

**Derivative:**

$$\frac{d}{dx}\left(\frac{27347022768402161398730}{(3+4x+11x^2+15x^3+11x^4-4x^5+3x^6-x^7)(1-30x^4+x^8)}\right) = \left(27347022768402161398730\left(15x^{14}-42x^{13}+52x^{12}-132x^{11}-495x^{10}+790x^9-1116x^8+2616x^7+3157x^6+1962x^5+620x^4+316x^3-45x^2-22x-4\right)\right) / \left(\left(-x^7+3x^6-4x^5+11x^4+15x^3+11x^2+4x+3\right)^2\left(x^8-30x^4+1\right)^2\right)$$

**Indefinite integral:**

$$\int \frac{27347022768402161398730}{(3+4x+11x^2+15x^3+11x^4-4x^5+3x^6-x^7)(1-30x^4+x^8)}\,dx = \frac{1}{20608}\,13673511384201080699365\left(8\sum_{\{\omega:\,\omega^7-3\omega^6+4\omega^5-11\omega^4-15\omega^3-11\omega^2-4\omega-3=0\}}\left(116\omega^6\log(x-\omega)-539\omega^5\log(x-\omega)+962\omega^4\log(x-\omega)-1736\omega^3\log(x-\omega)-172\omega^2\log(x-\omega)+2709\omega\log(x-\omega)+1931\log(x-\omega)\right)/\left(7\omega^6-18\omega^5+20\omega^4-44\omega^3-45\omega^2-22\omega-4\right)-\sum_{\{\omega:\,\omega^8-30\omega^4+1=0\}}\frac{1}{\omega^7-15\omega^3}\left(116\omega^7\log(x-\omega)-191\omega^6\log(x-\omega)-75\omega^5\log(x-\omega)+79\omega^4\log(x-\omega)-3476\omega^3\log(x-\omega)+5721\omega^2\log(x-\omega)+2245\omega\log(x-\omega)-2361\log(x-\omega)\right)\right) + \text{constant}$$

(assuming a complex-valued logarithm)

log(x) is the natural logarithm

**Limit:**

$$\lim_{x\to\pm\infty} \frac{27\,347\,022\,768\,402\,161\,398\,730}{(3+4x+11x^2+15x^3+11x^4-4x^5+3x^6-x^7)(1-30x^4+x^8)} = 0$$

For the above formula equal to 104164, we obtain:

(3+4*x+11*x^2+15*x^3+11*x^4-4*x^5+3*x^6-x^7)/(1-30*x^4+x^8) = 104164

**Input:**

$$\frac{3+4x+11x^2+15x^3+11x^4-4x^5+3x^6-x^7}{1-30x^4+x^8} = 104\,164$$

**Alternate forms:**

$$\frac{x(x(x(15-x(x((x-3)x+4)-11))+11)+4)+3}{x^8-30x^4+1} = 104\,164$$

$$-\frac{x^7-3x^6+4x^5-11x^4-15x^3-11x^2-4x-3}{x^8-30x^4+1} = 104\,164$$

**Expanded form:**

$$\frac{11x^4}{x^8-30x^4+1} + \frac{4x}{x^8-30x^4+1} + \frac{3}{x^8-30x^4+1} - \frac{x^7}{x^8-30x^4+1} + \frac{3x^6}{x^8-30x^4+1} - \frac{4x^5}{x^8-30x^4+1} + \frac{15x^3}{x^8-30x^4+1} + \frac{11x^2}{x^8-30x^4+1} = 104\,164$$

**Real solutions:**

$x \approx -2.3397046645462967841$

$x \approx -0.42740326903658959021$

$x \approx 0.42739747475995044542$

$x \approx 2.3396988702645115458$

we take the solutions 0.427397474759

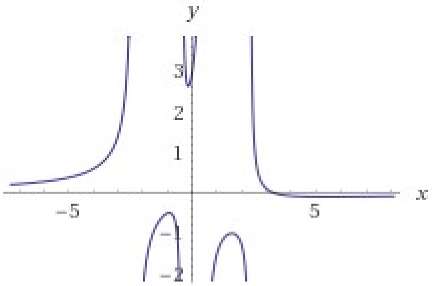
With regard the function, we have the following mathematical description:

(3 + 4 x + 11 x^2 + 15 x^3 + 11 x^4 - 4 x^5 + 3 x^6 - x^7)/(1 - 30 x^4 + x^8)

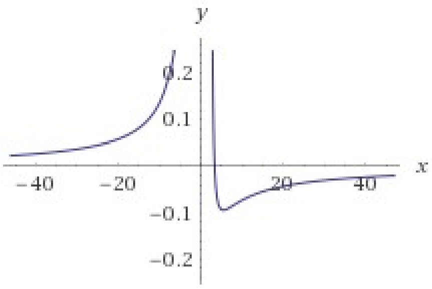
**Input:**

$$\frac{3+4x+11x^2+15x^3+11x^4-4x^5+3x^6-x^7}{1-30x^4+x^8}$$

**Plots:**




(x from −7.4 to 8.1)




(x from −46 to 46.8)

**Alternate forms:**

$$-\frac{x^7-3x^6+4x^5-11x^4-15x^3-11x^2-4x-3}{x^8-30x^4+1}$$

$$\frac{x(x(x(15-x(x((x-3)x+4)-11))+11)+4)+3}{x^8-30x^4+1}$$

$$\frac{x(x(x(x(x((3-x)x-4)+11)+15)+11)+4)+3}{(x^4-30)x^4+1}$$

**Expanded form:**

$$\frac{11x^4}{x^8-30x^4+1}+\frac{4x}{x^8-30x^4+1}+\frac{3}{x^8-30x^4+1}-\frac{x^7}{x^8-30x^4+1}+\frac{3x^6}{x^8-30x^4+1}-\frac{4x^5}{x^8-30x^4+1}+\frac{15x^3}{x^8-30x^4+1}+\frac{11x^2}{x^8-30x^4+1}$$

**Real root:**

$x \approx 3.31181$

3.31181

## Complex roots:

$x \approx -0.625763 - 0.430402\,i$

$x \approx -0.625763 + 0.430402\,i$

$x \approx 0.0564406 - 0.54459\,i$

$x \approx 0.0564406 + 0.54459\,i$

$x \approx 0.413417 - 2.2512\,i$

## Properties as a real function:

### Domain

$\{x \in \mathbb{R} : x \neq -2.3397 \text{ and } x \neq -0.427406 \text{ and } x \neq 0.427406 \text{ and } x \neq 2.3397\}$

### Range

$\{y \in \mathbb{R} : y \leq -0.492398 \text{ or } y \geq -0.0943637\}$

$\mathbb{R}$ is the set of real numbers

## Series expansion at x = 0:

$3 + 4x + 11x^2 + 15x^3 + 101x^4 + O(x^5)$

(Taylor series)

## Series expansion at x = ∞:

$-\frac{1}{x} + \frac{3}{x^2} - \frac{4}{x^3} + \frac{11}{x^4} + O\left(\left(\frac{1}{x}\right)^5\right)$

(Laurent series)

## Derivative:

$$\frac{d}{dx}\left(\frac{3 + 4x + 11x^2 + 15x^3 + 11x^4 - 4x^5 + 3x^6 - x^7}{1 - 30x^4 + x^8}\right) = \frac{1}{(x^8 - 30x^4 + 1)^2}\left(x^{14} - 6x^{13} + 12x^{12} - 44x^{11} + 15x^{10} - 246x^9 + 92x^8 - 24x^7 + 443x^6 + 678x^5 + 340x^4 + 404x^3 + 45x^2 + 22x + 4\right)$$

## Indefinite integral:

$$\int \frac{3+4x+11x^2+15x^3+11x^4-4x^5+3x^6-x^7}{1-30x^4+x^8}\,dx = \sum_{\{\omega:\,4096\omega^4+2048\omega^3-1152\omega^2+160\omega-23=0\}} \omega \log(16x^2 - 1536x\omega^3 - 832x\omega^2 + 376x\omega - 39x - 16) + \text{constant}$$

(assuming a complex-valued logarithm)

log(x) is the natural logarithm

**Local maxima:**

$$\max\left\{\frac{3+4x+11x^2+15x^3+11x^4-4x^5+3x^6-x^7}{1-30x^4+x^8}\right\}\approx -0.49240 \text{ at } x\approx -0.93272$$

$$\max\left\{\frac{3+4x+11x^2+15x^3+11x^4-4x^5+3x^6-x^7}{1-30x^4+x^8}\right\}\approx -0.99981 \text{ at } x\approx 1.6053$$

**Local minima:**

$$\min\left\{\frac{3+4x+11x^2+15x^3+11x^4-4x^5+3x^6-x^7}{1-30x^4+x^8}\right\}\approx 2.6395 \text{ at } x\approx -0.16385$$

$$\min\left\{\frac{3+4x+11x^2+15x^3+11x^4-4x^5+3x^6-x^7}{1-30x^4+x^8}\right\}\approx -0.094364 \text{ at } x\approx 5.4448$$

**Limit:**

$$\lim_{x\to\pm\infty}\frac{3+4x+11x^2+15x^3+11x^4-4x^5+3x^6-x^7}{1-30x^4+x^8}=0$$

From

$$\frac{3+4x+11x^2+15x^3+11x^4-4x^5+3x^6-x^7}{1-30x^4+x^8}$$

we obtain, for x = 3.31181:

(3 + 4 *3.31181 + 11 *3.31181^2 + 15 *3.31181^3 + 11 *3.31181^4 - 4 *3.31181^5 + 3 *3.31181^6 -3.31181^7)/(1 - 30 *3.31181^4 + 3.31181^8)

**Input interpretation:**

$$(3+4\times 3.31181+11\times 3.31181^2+15\times 3.31181^3+11\times 3.31181^4-4\times 3.31181^5+3\times 3.31181^6-3.31181^7)/(1-30\times 3.31181^4+3.31181^8)$$

**Result:**

$1.509291969490175659022373138050798412032928017044615 0\ldots \times 10^{-7}$

1.50929196949…*10^-7

From which:

0.1/(1.5092919694901756590223731380507984120329280170446150 × 10^-7)^(((35 ζ(3))/(3 log(2) log^2(3) log^3(2 π))))

**Input interpretation:**

$$0.1/(1.5092919694901756590223731380507984120329280170446150 \times 10^{-7})^{(35\,\zeta(3))/(3\log(2)\log^2(3)\log^3(2\pi))}$$

$\zeta(s)$ is the Riemann zeta function

$\log(x)$ is the natural logarithm

**Result:**

$2.62538... \times 10^{17}$

2.62538…*$10^{17}$

Furthermore, adding 1/(27347022768402161398730), we obtain:

1/(27347022768402161398730)+(((0.1/(1.5092919694901756590223731380507984120329280170446150 × 10^-7)^(((35 ζ(3))/(3 log(2) log^2(3) log^3(2 π)))))))

**Input interpretation:**

$$\frac{1}{27\,347\,022\,768\,402\,161\,398\,730} + 0.1/(1.5092919694901756590223731380507984120329280170446150 \times 10^{-7})^{(35\,\zeta(3))/(3\log(2)\log^2(3)\log^3(2\pi))}$$

$\zeta(s)$ is the Riemann zeta function

$\log(x)$ is the natural logarithm

**Result:**

$2.62538... \times 10^{17}$

2.62538…*$10^{17}$

From which:

sqrt[1/10^17[1/(27347022768402161398730)+(((0.1/(1.5092919694901756590223731380507984120329280170446150 × 10^-7)^(((35 ζ(3))/(3 log(2) log^2(3) log^3(2 π)))))))]

**Input interpretation:**

$$\sqrt{\left(\frac{1}{10^{17}}\left(\frac{1}{27\,347\,022\,768\,402\,161\,398\,730}+0.1\Big/\left(1.50929196949017565902237313805079841203292801704461 50\times 10^{-7}\right)^{(35\,\zeta(3))/(3\log(2)\log^2(3)\log^3(2\pi))}\right)\right)}$$

$\zeta(s)$ is the Riemann zeta function

$\log(x)$ is the natural logarithm

**Result:**

1.6203009737098780021402990729522403154675926240031462637 75...

1.620300973709…. result that is a good approximation to the value of the golden ratio 1.618033988749...

From

$$\frac{27\,347\,022\,768\,402\,161\,398\,730}{(-x^7+3x^6-4x^5+11x^4+15x^3+11x^2+4x+3)(x^8-30x^4+1)}$$

putting x = 0.427397474759 ≈ 0.42739747476, we obtain:

27347022768402161398730/(((3+4*0.42739747476+11*0.42739747476^2+15*0.42739747476^3+11*0.42739747476^4-4*0.42739747476^5+3*0.42739747476^6-0.42739747476^7))/(1-30*0.42739747476^4+0.42739747476^8)

**Input interpretation:**

$$27\,347\,022\,768\,402\,161\,398\,730\Big/\left(\left(3+4\times0.42739747476+11\times0.42739747476^2+15\times0.42739747476^3+11\times0.42739747476^4-4\times0.42739747476^5+3\times0.42739747476^6-0.42739747476^7\right)\Big/\left(1-30\times0.42739747476^4+0.42739747476^8\right)\right)$$

**Result:**

$2.62538138011085192825775217743717924946745285499499 39\ldots\times10^{17}$

$2.6253813801\ldots*10^{17}$

From

(3+4*x+11*x^2+15*x^3+11*x^4-4*x^5+3*x^6-x^7)/(1-30*x^4+x^8)

for x = 0.427397474759, we obtain:

(3+4*0.427397474759+11*0.427397474759^2+15*0.427397474759^3+11*0.427397474759^4-4*0.427397474759^5+3*0.427397474759^6-0.427397474759^7)/(1-30*0.427397474759^4+0.427397474759^8)

**Input interpretation:**

$$\left(3+4\times 0.427397474759+11\times 0.427397474759^2+15\times 0.427397474759^3+11\times 0.427397474759^4-4\times 0.427397474759^5+3\times 0.427397474759^6-0.427397474759^7\right)/\left(1-30\times 0.427397474759^4+0.427397474759^8\right)$$

**Result:**

104163.9882662083417117788099682997017887854658184992382824...

104163.988266….. ≈ 104164

We remember that:

((( 286533602 * 285610081* 334165)))

$286\,533\,602\times 285\,610\,081\times 334\,165$

27 347 022 768 402 161 398 730

$2.7347022768402161398730\times 10^{22}$

27347022768402161398730

Dividing the two obtained results, we have:

27347022768402161398730/104163.9882662083417117788099682997017887854658184992382824

**Input interpretation:**

$$\frac{27\,347\,022\,768\,402\,161\,398\,730}{104\,163.9882662083417117788099682997017887854658184992382824}$$

**Result:**

$2.625381691272449242181434646163001525933525152664 7785... \times 10^{17}$

$2.625381691272...*10^{17}$

**Decimal form:**

262538169127244924.21814346461630015259335251526 6

262538169127244924.21814…. result near to the value of the Ramanujan's constant

From Ramanujan's constant

$$e^{\pi\sqrt{163}} \approx 640320^3 + 744.$$

we obtain:

640320^3+744

**Input:**

$640\,320^3 + 744$

**Result:**

262 537 412 640 768 744

262537412640768744

**Scientific notation:**

$2.62537412640768744 \times 10^{17}$

$2.625374126407...*10^{17}$

Now, from:

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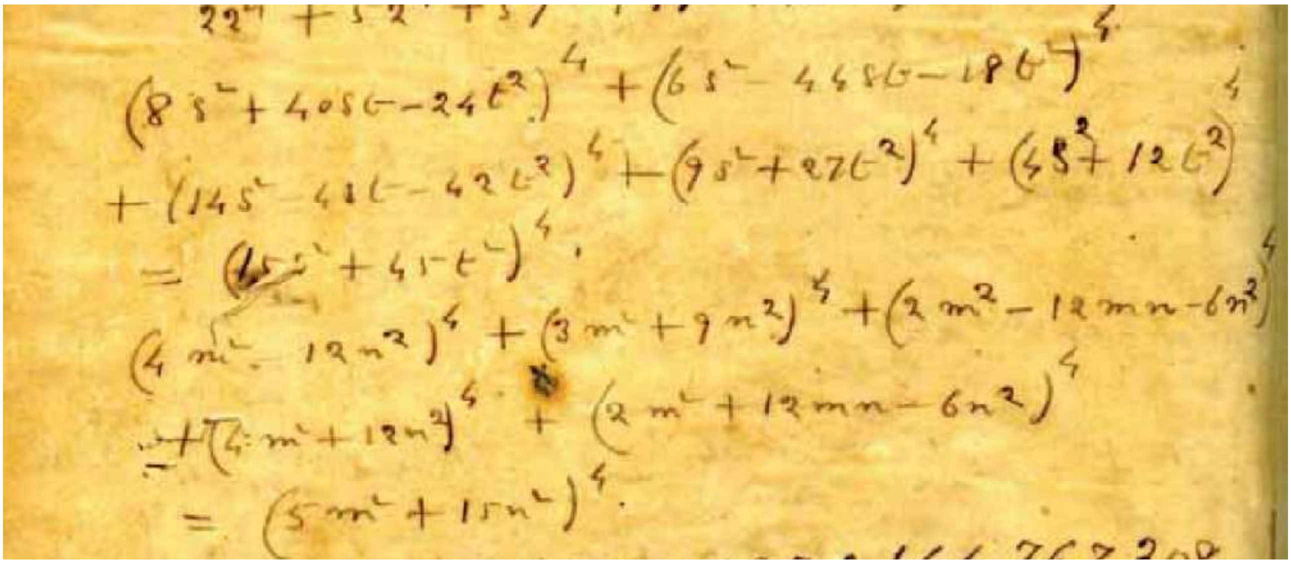


$$(8s^2+40st-24t^2)^4+(6s^2-44st-18t^2)^4+(14s^2-4st-42t^2)^4+(9s^2+27t^2)^4+(4s^2+12t^2)^4=(15s^2+45t^2)^4.$$

$$(4m^2-12n^2)^4+(3m^2+9n^2)^4+(2m^2-12mn-6n^2)^4+(4m^2+12n^2)^4+(2m^2+12mn-6n^2)^4=(5m^2+15n^2)^4.$$

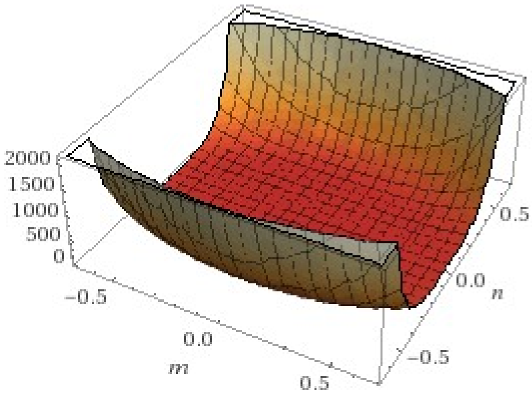
(5m^2+15n^2)^4

**Input:**
$(5\,m^2+15\,n^2)^4$

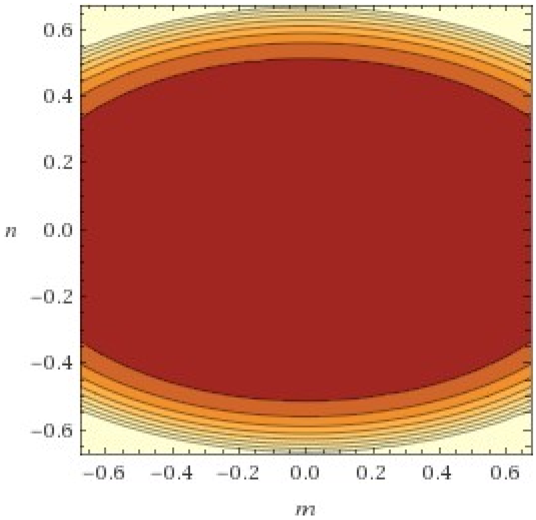
**Values:**

| $m$ | |
|---|---|
| 0 | $50\,625\,n^8$ |
| 1 | $(15\,n^2+5)^4$ |
| 2 | $(15\,n^2+20)^4$ |
| 3 | $(15\,n^2+45)^4$ |

**3D plot:**




**Contour plot:**




**Geometric figure:**

line

**Alternate forms:**

$$625\,m^8 + n^2\left(7500\,m^6 + n^2\left(33\,750\,m^4 + n^2\left(67\,500\,m^2 + 50\,625\,n^2\right)\right)\right)$$

$$m^2\left(m^2\left(m^2\left(625\,m^2 + 7500\,n^2\right) + 33\,750\,n^4\right) + 67\,500\,n^6\right) + 50\,625\,n^8$$

$$625\left(m^4 + 6\,m^2\,n^2\right)^2 + 11\,250\,n^4\left(m^4 + 6\,m^2\,n^2\right) + 50\,625\,n^8$$

**Expanded form:**

$$625\,m^8 + 7500\,m^6\,n^2 + 33\,750\,m^4\,n^4 + 67\,500\,m^2\,n^6 + 50\,625\,n^8$$

**Roots:**

$$n = -\frac{i\, m}{\sqrt{3}}$$

$$n = \frac{i\, m}{\sqrt{3}}$$

**Integer root:**

$m = 0\,,\quad n = 0$

**Polynomial discriminant:**

$\Delta = 0$

**Property as a function:**

**Parity**

even

**Derivative:**

$$\frac{\partial}{\partial m}\left(\left(5\, m^2 + 15\, n^2\right)^4\right) = 5000\, m\, \left(m^2 + 3\, n^2\right)^3$$

**Indefinite integral:**

$$\int \left(5\, m^2 + 15\, n^2\right)^4 dm = \frac{625\, m^9}{9} + \frac{7500\, m^7\, n^2}{7} + 6750\, m^5\, n^4 + 22\,500\, m^3\, n^6 + 50\,625\, m\, n^8 + \text{constant}$$

**Global minimum:**

$\min\{(5\, m^2 + 15\, n^2)^4\} = 0$ at $(m, n) = (0, 0)$

**Definite integral over a disk of radius R:**

$$\iint_{m^2+n^2<R^2} \left(5\, m^2 + 15\, n^2\right)^4 dm\, dn = \frac{28\,375\, \pi\, R^{10}}{8}$$

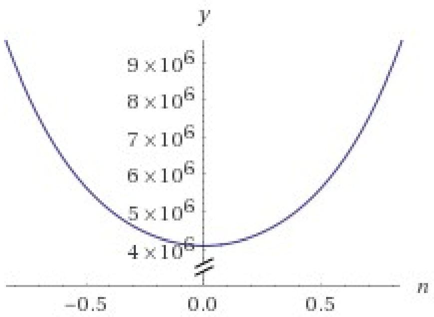
For m = 3, we obtain:

(15n^2+45)^4

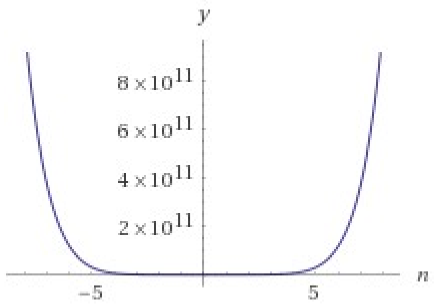
**Input:**

$(15\, n^2 + 45)^4$

## Plots:




(n from −0.8 to 0.8)




(n from −8.5 to 8.5)

## Values:

| $n$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $(15 n^2 + 45)^4$ | 12960000 | 1215506⋱ 25 | 104976⋱ 0000 | 659750⋱ 0625 | 3111696⋱ 0000 |

## Alternate forms:

$50625 (n^2 + 3)^4$

$(((50625 n^2 + 607500) n^2 + 2733750) n^2 + 5467500) n^2 + 4100625$

$50625 (n^4 + 6 n^2)^2 + 911250 (n^4 + 6 n^2) + 4100625$

## Expanded form:

$50625 n^8 + 607500 n^6 + 2733750 n^4 + 5467500 n^2 + 4100625$

## Complex roots:

$n = -i \sqrt{3}$

$n = i \sqrt{3}$

$i\sqrt{3} = \sqrt{3}$

that is the ratio between the gravitating mass $M_0$ and the Wheelerian mass q

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$
$$q = \frac{(3\sqrt{3})\, M_s}{2}.$$

(see: Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019)

**Polynomial discriminant:**

$\Delta = 0$

**Property as a function:**

**Parity**

even

**Derivative:**

$$\frac{d}{dn}((15\, n^2 + 45)^4) = 405\,000\, n\, (n^2 + 3)^3$$

**Indefinite integral:**

$$\int (45 + 15\, n^2)^4\, dn = 5625\, n^9 + \frac{607500\, n^7}{7} + 546\,750\, n^5 + 1\,822\,500\, n^3 + 4\,100\,625\, n + \text{constant}$$

**Global minimum:**

$$\min\{(15\, n^2 + 45)^4\} = 4\,100\,625 \text{ at } n = 0$$

For n = 3, we have:

(15*3^2+45)^4

**Input:**

$(15 \times 3^2 + 45)^4$

**Result:**

1 049 760 000

$1.04976 \times 10^9$

1049760000

Performing the 41th root, we obtain:

((((15*3^2+45)^4)))^1/41

where 41 is an Eisenstein number

**Input:**

$$\sqrt[41]{(15 \times 3^2 + 45)^4}$$

**Result:**

$$5^{4/41} \times 6^{8/41}$$

**Decimal approximation:**

1.659688504173702634917200078477822811228605794868646013841...

1.6596885041…. result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

and we obtain also:

1/(31*43*4)(15*3^2+45)^4 + 5

where 31 and 43 are prime numbers, while 4 is a Lucas number

**Input:**

$$\frac{1}{31 \times 43 \times 4} (15 \times 3^2 + 45)^4 + 5$$

**Exact result:**

$$\frac{262\,446\,665}{1333}$$

**Decimal approximation:**

196884.219804951237809452363090772693173293323330832708177 0...

196884.2198….

196884 is a fundamental number of the following $j$-invariant

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots$$

(In mathematics, Felix Klein's $j$-invariant or $j$ function, regarded as a function of a complex variable $\tau$, is a modular function of weight zero for SL(2, Z) defined on the upper half plane of complex numbers. Several remarkable properties of $j$ have to do with its $q$ expansion (Fourier series expansion), written as a Laurent series in terms of $q = e2\pi i\tau$ (the square of the nome), which begins:

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots$$

Note that $j$ has a simple pole at the cusp, so its $q$-expansion has no terms below $q-1$.

All the Fourier coefficients are integers, which results in several almost integers, notably Ramanujan's constant:

$$e^{\pi\sqrt{163}} \approx 640320^3 + 744.$$

The asymptotic formula for the coefficient of $qn$ is given by

$$\frac{e^{4\pi\sqrt{n}}}{\sqrt{2}\,n^{3/4}},$$

as can be proved by the Hardy–Littlewood circle method)

and again:

1/19(((5*3^2+15*3^2)^4))^1/2 +24

**Input:**

$$\frac{1}{19}\sqrt{(5\times3^2+15\times3^2)^4}+24$$

**Exact result:**

$$\frac{32856}{19}$$

**Decimal approximation:**

1729.2631578947368421052631578947368421052631578947368421052...

1729.263157…..

This result is very near to the mass of candidate glueball **$f_0$(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Performing the 15th root:

(((1/19(((5*3^2+15*3^2)^4))^1/2 +24)))^1/15

**Input:**

$$\sqrt[15]{\frac{1}{19}\sqrt{(5\times 3^2+15\times 3^2)^4}+24}$$

**Result:**

$$\sqrt[15]{\frac{3}{19}}\sqrt[5]{2}\ 37^{2/15}$$

**Decimal approximation:**

1.6438319070688570620417622076048536607185456956493923579 95...

$1.6438319\ldots. \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

**Alternate form:**

root of $19x^{15} - 32856$ near $x = 1.64383$

**All 15th roots of 32856/19:**

$$\sqrt[15]{\frac{3}{19}}\sqrt[5]{2}\ 37^{2/15}\ e^0 \approx 1.64383 \text{ (real, principal root)}$$

$$\sqrt[15]{\frac{3}{19}}\ \sqrt[5]{2}\ 37^{2/15}\ e^{(2 i \pi)/15} \approx 1.5017 + 0.6686\, i$$

$$\sqrt[15]{\frac{3}{19}}\ \sqrt[5]{2}\ 37^{2/15}\ e^{(4 i \pi)/15} \approx 1.0999 + 1.2216\, i$$

$$\sqrt[15]{\frac{3}{19}}\ \sqrt[5]{2}\ 37^{2/15}\ e^{(2 i \pi)/5} \approx 0.5080 + 1.5634\, i$$

$$\sqrt[15]{\frac{3}{19}}\ \sqrt[5]{2}\ 37^{2/15}\ e^{(8 i \pi)/15} \approx -0.17183 + 1.63483\, i$$

From which:

(((1/19(((5*3^2+15*3^2)^4))^1/2 +24)))^1/15 - (21+5)1/10^3

**Input:**

$$\sqrt[15]{\frac{1}{19}\sqrt{(5\times 3^2 + 15\times 3^2)^4} + 24} - (21+5)\times\frac{1}{10^3}$$

**Result:**

$$\sqrt[15]{\frac{3}{19}}\ \sqrt[5]{2}\ 37^{2/15} - \frac{13}{500}$$

**Decimal approximation:**

1.617831907068857062041762207604853660718545695649392357995...

1.617831907068…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

**Alternate forms:**

$$\frac{500\ \sqrt[5]{2}\ \sqrt[15]{3}\ 19^{14/15}\times 37^{2/15} - 247}{9500}$$

$$(\text{root of } 19 x^{15} - 32856 \text{ near } x = 1.64383) - \frac{13}{500}$$

$$\frac{1}{500}\left(500\sqrt[15]{\frac{3}{19}}\ \sqrt[5]{2}\ 37^{2/15} - 13\right)$$

Now, we have that:

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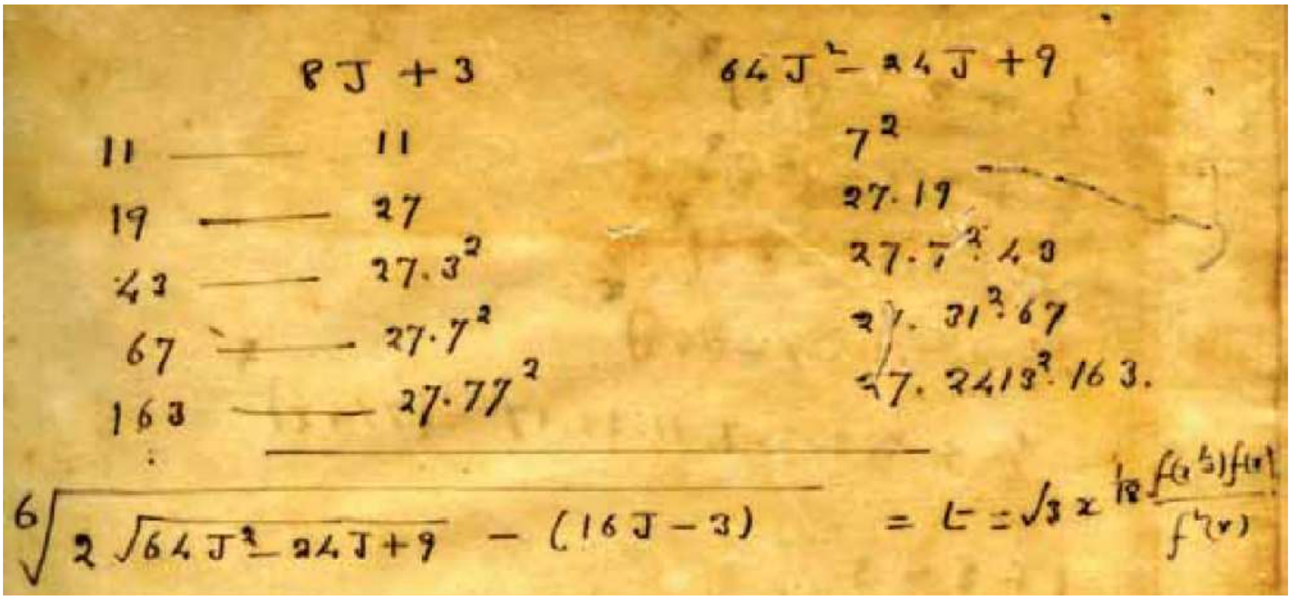
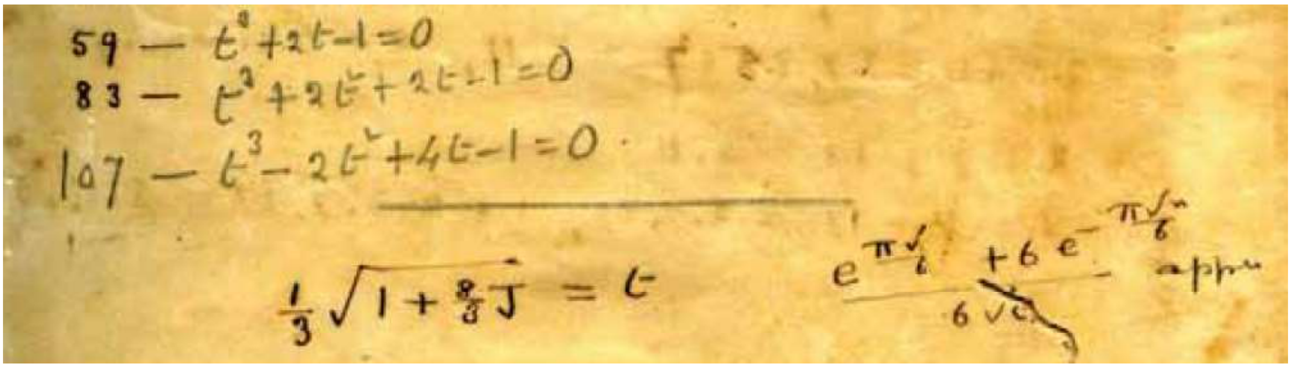


| | $8J+3$ | $64J^2-24J+9$ |
|---|---|---|
| 11 — | 11 | $7^2$ |
| 19 — | 27 | $27\cdot 17$ |
| 43 — | $27\cdot 3^2$ | $27\cdot 7^2\cdot 43$ |
| 67 — | $27\cdot 7^2$ | $27\cdot 31^2\cdot 67$ |
| 163 — | $27\cdot 77^2$ | $27\cdot 2413^2\cdot 163$ |

$$\sqrt[6]{2\sqrt{64J^2-24J+9}-(16J-3)} = t = \sqrt{3}\,x^{1/18}\frac{f(x^3)f(x)}{f^2(x^2)}$$

11 — $t-1=0$



59 — $t^3+2t-1=0$

83 — $t^3+2t^2+2t-1=0$

107 — $t^3-2t^2+4t-1=0$

$$\frac{1}{3}\sqrt{1+\frac{8}{3}J} = t$$

$$\frac{e^{\pi\sqrt{n}/\dots}+6e^{-\pi\sqrt{n}/6}}{6\sqrt{\dots}} \text{ approx}$$

Where 11, 59, 83 and 107 are Eisenstein numbers

8J + 3 = 59; J = 7

1/3*sqrt(1+(8*7)/3) = t

**Input:**

$$\frac{1}{3}\sqrt{1+\frac{8\times 7}{3}}$$

**Result:**

$$\frac{\sqrt{\frac{59}{3}}}{3}$$

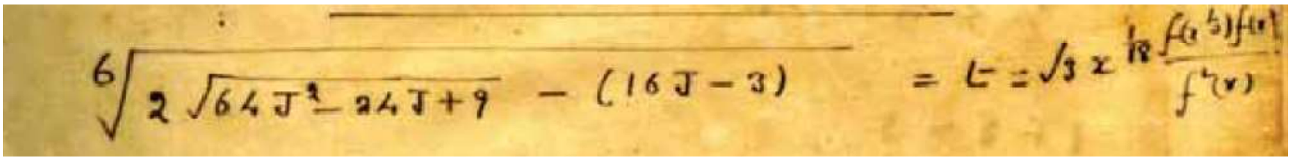
**Decimal approximation:**

1.4782371884055634138940034237914959205493936239706784810 63...

1.4782371884….

**Alternate form:**

$$\frac{\sqrt{177}}{9}$$



For J = 7, we obtain:

(((2*(64*7^2-24*7+9)^0.5 – (16*7-3))))^1/6

**Input:**

$$\sqrt[6]{2\sqrt{64\times 7^2-24\times 7+9}-(16\times 7-3)}$$

**Exact result:**

$$\sqrt[6]{2\sqrt{2977}-109}$$

**Decimal approximation:**

0.7059546293921263783647627318277918981577110045566313603 25...

0.705954629…

**Alternate form:**

$$\frac{\sqrt{3}}{\sqrt[6]{109+2\sqrt{2977}}}$$

**Minimal polynomial:**

$$x^{12}+218\,x^6-27$$

**All 6th roots of 2 sqrt(2977) - 109:**

$\sqrt[6]{2\sqrt{2977} - 109}\ e^0 \approx 0.71$ (real, principal root)

$\sqrt[6]{2\sqrt{2977} - 109}\ e^{(i\pi)/3} \approx 0.35 + 0.61\,i$

$\sqrt[6]{2\sqrt{2977} - 109}\ e^{(2i\pi)/3} \approx -0.35 + 0.61\,i$

$\sqrt[6]{2\sqrt{2977} - 109}\ e^{i\pi} \approx -0.71$ (real root)

$\sqrt[6]{2\sqrt{2977} - 109}\ e^{-(2i\pi)/3} \approx -0.35 - 0.61\,i$

From which:

1+5((((((2*(64*7^2-24*7+9)^0.5 – (16*7-3))))^1/6)))^6

**Input:**

$$1 + 5\left(\sqrt[6]{2\sqrt{64\times 7^2 - 24\times 7 + 9} - (16\times 7 - 3)}\right)^6$$

**Exact result:**

$$1 + 5\left(2\sqrt{2977} - 109\right)$$

**Decimal approximation:**

1.618914628149598572592813207617748386953015240529949686841...

1.618914628149…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

**Alternate forms:**

$$10\sqrt{2977} - 544$$

$$2\left(5\sqrt{2977} - 272\right)$$

**Minimal polynomial:**

$$x^2 + 1088\,x - 1764$$

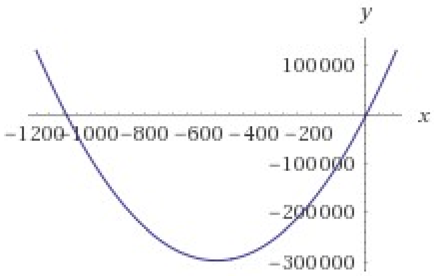
From the above minimal polynomial, we obtain:

-1764 + 1088 x + x^2

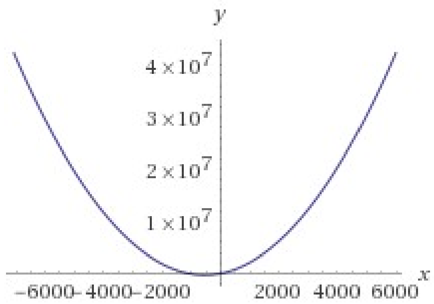
**Input:**

$-1764 + 1088 x + x^2$

**Plots:**




(x from −1199 to 110.7)




(x from −7072 to 5984)

**Geometric figure:**

parabola

**Alternate forms:**

$x(x + 1088) - 1764$

$(x + 544)^2 - 297700$

$-\left(-x + 10\sqrt{2977} - 544\right)\left(x + 10\sqrt{2977} + 544\right)$

**Roots:**

$x = -2\left(272 + 5\sqrt{2977}\right)$

$x = 10\sqrt{2977} - 544$

**Polynomial discriminant:**

$\Delta = 1190800$

**Properties as a real function:**

**Domain**

$\mathbb{R}$ (all real numbers)

**Range**

$\{y \in \mathbb{R} : y \geq -297700\}$

$\mathbb{R}$ is the set of real numbers

**Derivative:**

$$\frac{d}{dx}(-1764 + 1088 x + x^2) = 2 (x + 544)$$

**Indefinite integral:**

$$\int (-1764 + 1088 x + x^2) \, dx = \frac{x^3}{3} + 544 x^2 - 1764 x + \text{constant}$$

**Global minimum:**

$$\min\{-1764 + 1088 x + x^2\} = -297700 \text{ at } x = -544$$

**Definite integral:**

$$\int_{2\left(-272-5\sqrt{2977}\right)}^{\frac{882}{272+5\sqrt{2977}}} (-1764 + 1088 x + x^2) \, dx = -\frac{11908000 \sqrt{2977}}{3} \approx -2.16574 \times 10^8$$

**Definite integral area below the axis between the smallest and largest real roots:**

$$\int_{2\left(-272-5\sqrt{2977}\right)}^{\frac{882}{272+5\sqrt{2977}}} (-1764 + 1088 x + x^2) \, \theta(1764 - 1088 x - x^2) \, dx =$$

$$-\frac{11908000 (4411571645 + 80854448 \sqrt{2977})}{3 (272 + 5 \sqrt{2977})^3} \approx -2.16574 \times 10^8$$

from which:

1088 (-2 (272 + 5 sqrt(2977))) + (-2 (272 + 5 sqrt(2977)))^2

**Input:**

$$1088 \left(-2 \left(272 + 5 \sqrt{2977}\right)\right) + \left(-2 \left(272 + 5 \sqrt{2977}\right)\right)^2$$

**Result:**

1764

1764 result in the range of the mass of candidate “glueball” $f_0(1710)$ (“glueball” =1760 ± 15 MeV).

Performing the 15th root:

(((1088 (-2 (272 + 5 sqrt(2977))) + (-2 (272 + 5 sqrt(2977)))^2 )))^1/15

**Input:**

$$\sqrt[15]{1088\left(-2\left(272+5\sqrt{2977}\right)\right)+\left(-2\left(272+5\sqrt{2977}\right)\right)^2}$$

**Result:**

$$\sqrt[15]{4\left(272+5\sqrt{2977}\right)^2-2176\left(272+5\sqrt{2977}\right)}$$

**Decimal approximation:**

1.646012915965068680054939762967836675768640113228558037045...

$1.646012915965.... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$

**Alternate forms:**

$$42^{2/15}$$

$$2^{2/15}\sqrt[15]{\left(5\sqrt{2977}-272\right)\left(272+5\sqrt{2977}\right)}$$

**All 15th roots of 4 (272 + 5 sqrt(2977))^2 - 2176 (272 + 5 sqrt(2977)):**

$$\sqrt[15]{4\left(272+5\sqrt{2977}\right)^2-2176\left(272+5\sqrt{2977}\right)}\, e^0 \approx 1.65 \text{ (real, principal root)}$$

$$\sqrt[15]{4\left(272+5\sqrt{2977}\right)^2-2176\left(272+5\sqrt{2977}\right)}\, e^{(2 i \pi)/15} \approx 1.50+0.67 i$$

$$\sqrt[15]{4\left(272+5\sqrt{2977}\right)^2-2176\left(272+5\sqrt{2977}\right)}\, e^{(4 i \pi)/15} \approx 1.10+1.22 i$$

$$\sqrt[15]{4\left(272+5\sqrt{2977}\right)^2-2176\left(272+5\sqrt{2977}\right)}\, e^{(2 i \pi)/5} \approx 0.51+1.57 i$$

$$\sqrt[15]{4\left(272+5\sqrt{2977}\right)^2-2176\left(272+5\sqrt{2977}\right)}\, e^{(8 i \pi)/15} \approx -0.172+1.64 i$$

Now, we have:

$$\frac{3\sqrt{3}}{R^6} = \sqrt{8J+3} + \sqrt{2\sqrt{64J^2-24J+9} - 8J+6}$$

Sqrt(8*8+3)+(2(64*8^2-24*8+9)^0.5-8*8+6)^0.5

For J = 7, we obtain:

sqrt(59)+(2(64*7^2-24*7+9)^0.5 – 8*7+6)^0.5

**Input:**

$$\sqrt{59} + \sqrt{2\sqrt{64\times 7^2 - 24\times 7 + 9} - 8\times 7 + 6}$$

**Exact result:**

$$\sqrt{59} + \sqrt{2\sqrt{2977} - 50}$$

**Decimal approximation:**

15.370344856074216328586705213128343342850731332222280338 78...

15.37034485…

**Alternate forms:**

$$\sqrt{59} + \sqrt{2\left(\sqrt{2977} - 25\right)}$$

$$\sqrt{9 + 2\sqrt{2977} + 2\sqrt{118\left(\sqrt{2977} - 25\right)}}$$

**Minimal polynomial:**

$$x^8 - 36x^6 + 270x^4 - 11\,240\,180\,x^2 + 729$$

From which:

(3sqrt3)/R^6 = sqrt(59)+(2(64*7^2-24*7+9)^0.5 – 8*7+6)^0.5

**Input:**

$$\frac{3\sqrt{3}}{R^6} = \sqrt{59} + \sqrt{2\sqrt{64\times 7^2 - 24\times 7 + 9} - 8\times 7 + 6}$$

**Exact result:**

$$\frac{3\sqrt{3}}{R^6} = \sqrt{59} + \sqrt{2\sqrt{2977} - 50}$$

**Alternate forms:**

$$R^6 = \frac{9}{\sqrt{177} + \sqrt{6\left(\sqrt{2977} - 25\right)}} \quad \text{(for } R \neq 0\text{)}$$

$$\frac{3\sqrt{3}}{R^6} = \sqrt{59} + \sqrt{2\left(\sqrt{2977} - 25\right)}$$

$$\frac{3\sqrt{3}}{R^6} = \sqrt{9 + 2\sqrt{2977} + 2\sqrt{118\left(\sqrt{2977} - 25\right)}}$$

**Alternate form assuming R is positive:**

$$\left(\sqrt{59} + \sqrt{2\left(\sqrt{2977} - 25\right)}\right) R^6 = 3\sqrt{3} \quad \text{(for } R \neq 0\text{)}$$

**Real solutions:**

$$R = -\frac{\sqrt[4]{3}}{\sqrt[6]{\sqrt{59} + \sqrt{2\left(\sqrt{2977} - 25\right)}}}$$

$$R = \frac{\sqrt[4]{3}}{\sqrt[6]{\sqrt{59} + \sqrt{2\left(\sqrt{2977} - 25\right)}}}$$

$$R \approx -0.83464$$

$$R \approx 0.83464$$

**Complex solutions:**

$$R = -\frac{\sqrt[3]{-1}\,\sqrt[4]{3}}{\sqrt[6]{\sqrt{59} + \sqrt{2\left(\sqrt{2977} - 25\right)}}}$$

$$R = \frac{\sqrt[3]{-1}\ \sqrt[4]{3}}{\sqrt[6]{\sqrt{59} + \sqrt{2\left(\sqrt{2977} - 25\right)}}}$$

$$R = -\frac{(-1)^{2/3}\ \sqrt[4]{3}}{\sqrt[6]{\sqrt{59} + \sqrt{2\left(\sqrt{2977} - 25\right)}}}$$

$$R = \frac{(-1)^{2/3}\ \sqrt[4]{3}}{\sqrt[6]{\sqrt{59} + \sqrt{2\left(\sqrt{2977} - 25\right)}}}$$

From the real solution, we obtain:

3^(1/4)/(sqrt(59) + sqrt(2 (-25 + sqrt(2977))))^(1/6)

**Input:**

$$\frac{\sqrt[4]{3}}{\sqrt[6]{\sqrt{59} + \sqrt{2\left(-25 + \sqrt{2977}\right)}}}$$

**Decimal approximation:**

0.834640994290248724761345258121574722719561409707602735737...

0.83464099429…

**Alternate forms:**

$$\frac{\sqrt[4]{3}}{\sqrt[6]{\sqrt{2\sqrt{2977} - 50} + \sqrt{59}}}$$

$$\frac{\sqrt[12]{2810045 + 51502\sqrt{2977} - 2\sqrt{118\left(33459103991 + 613232113\sqrt{2977}\right)}}}{\sqrt[4]{3}}$$

**Minimal polynomial:**

$$27x^{48} - 11240180x^{36} + 7290x^{24} - 26244x^{12} + 19683$$

and, multiplying by 2:

2(((3^(1/4)/(sqrt(59) + sqrt(2 (-25 + sqrt(2977))))^(1/6))))

**Input:**

$$2 \times \frac{\sqrt[4]{3}}{\sqrt[6]{\sqrt{59} + \sqrt{2\left(-25 + \sqrt{2977}\right)}}}$$

**Result:**

$$\frac{2\sqrt[4]{3}}{\sqrt[6]{\sqrt{59} + \sqrt{2\left(\sqrt{2977} - 25\right)}}}$$

**Decimal approximation:**

1.669281988580497449522690516243149445439122819415205471475...

1.66928198858…. result very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164.2696$ i.e. 1.65578...

**Alternate forms:**

$$\frac{2\sqrt[4]{3}}{\sqrt[6]{\sqrt{2\sqrt{2977} - 50} + \sqrt{59}}}$$

$$\frac{2\sqrt[12]{2810045 + 51502\sqrt{2977} - 2\sqrt{118\left(33459103991 + 613232113\sqrt{2977}\right)}}}{\sqrt[4]{3}}$$

**Minimal polynomial:**

$$27x^{48} - 46039777280x^{36} + 122305904640x^{24} - 1803473947459584x^{12} + 5540271966595842048$$

From the expression

$$\sqrt{59} + \sqrt{2\sqrt{2977} - 50}$$

we obtain also the following result:

(((sqrt(59)+(2(64*7^2-24*7+9)^0.5 – 8*7+6)^0.5)))^3-Pi^2

**Input:**

$$\left(\sqrt{59}+\sqrt{2\sqrt{64\times 7^2-24\times 7+9}-8\times 7+6}\right)^3-\pi^2$$

**Exact result:**

$$\left(\sqrt{59}+\sqrt{2\sqrt{2977}-50}\right)^3-\pi^2$$

**Decimal approximation:**

3621.335957272373336630350392900948825056008218366173355690...

3621.3359572… result practically equal to the rest mass of double charmed Xi baryon 3621.40

**Property:**

$\left(\sqrt{59}+\sqrt{-50+2\sqrt{2977}}\right)^3-\pi^2$ is a transcendental number

**Alternate forms:**

$$\left(\sqrt{59}+\sqrt{2\left(\sqrt{2977}-25\right)}\right)^3-\pi^2$$

$$\sqrt{8434509-33754\sqrt{2977}+14\sqrt{1534\left(68789195+1388797\sqrt{2977}\right)}}-\pi^2$$

$$-91\sqrt{59}+6\sqrt{175643}+177\sqrt{2\sqrt{2977}-50}+\left(2\sqrt{2977}-50\right)^{3/2}-\pi^2$$

**Series representations:**

$$\left(\sqrt{59}+\sqrt{2\sqrt{64\times 7^2-24\times 7+9}-8\times 7+6}\right)^3-\pi^2=$$
$$-\pi^2+\left(\sqrt{2\left(-25+\sqrt{2977}\right)}+\sqrt{58}\sum_{k=0}^{\infty}58^{-k}\binom{\frac{1}{2}}{k}\right)^3$$

$$\left(\sqrt{59}+\sqrt{2\sqrt{64\times 7^2-24\times 7+9}-8\times 7+6}\right)^3-\pi^2 = -\pi^2+\left(\sqrt{2\left(-25+\sqrt{2977}\right)}+\sqrt{58}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{58}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^3$$

$$\left(\sqrt{59}+\sqrt{2\sqrt{64\times 7^2-24\times 7+9}-8\times 7+6}\right)^3-\pi^2 = -\pi^2+\left(\sqrt{2\left(-25+\sqrt{2977}\right)}+\frac{\sum_{j=0}^{\infty}\text{Res}_{s=-\frac{1}{2}+j}58^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)^3$$

and again, we obtain also:

1/2(((sqrt(59)+(2(64*7^2-24*7+9)^0.5 – 8*7+6)^0.5)))^3-89+golden ratio^2

**Input:**

$$\frac{1}{2}\left(\sqrt{59}+\sqrt{2\sqrt{64\times 7^2-24\times 7+9}-8\times 7+6}\right)^3-89+\phi^2$$

$\phi$ is the golden ratio

**Exact result:**

$$\phi^2-89+\frac{1}{2}\left(\sqrt{59}+\sqrt{2\sqrt{2977}-50}\right)^3$$

**Decimal approximation:**

1729.220814825481242472797028784778126213381268066512836020...

1729.220814825…

This result is very near to the mass of candidate glueball **$f_0$(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

**Alternate forms:**

$$\phi^2-89+\frac{1}{2}\left(\sqrt{59}+\sqrt{2\left(\sqrt{2977}-25\right)}\right)^3$$

$$-89+\frac{1}{4}\left(1+\sqrt{5}\right)^2+\frac{1}{2}\left(\sqrt{59}+\sqrt{2\sqrt{2977}-50}\right)^3$$

$$\phi^2+\frac{1}{2}\left(-178+\sqrt{2\left(\sqrt{2977}-25\right)}\left(127+2\sqrt{2977}\right)+\sqrt{59}\left(6\sqrt{2977}-91\right)\right)$$

**Minimal polynomial:**

$$65\,536\,x^{16}+91\,750\,400\,x^{15}-1\,045\,317\,419\,008\,x^{14}-1\,329\,686\,318\,284\,800\,x^{13}+7\,001\,336\,326\,135\,070\,720\,x^{12}+7\,884\,699\,866\,628\,960\,256\,000\,x^{11}-25\,215\,759\,577\,787\,522\,428\,477\,440\,x^{10}-24\,307\,291\,455\,852\,231\,520\,117\,760\,000\,x^9+51\,450\,768\,298\,020\,960\,583\,391\,016\,846\,848\,x^8+40\,564\,925\,206\,559\,826\,261\,748\,950\,192\,793\,600\,x^7-56\,062\,542\,019\,727\,571\,496\,307\,734\,567\,005\,678\,080\,x^6-33\,879\,179\,246\,147\,688\,195\,591\,356\,619\,668\,044\,992\,000\,x^5+24\,610\,228\,608\,670\,466\,448\,240\,019\,374\,677\,381\,930\,924\,480\,x^4+10\,388\,651\,443\,553\,019\,655\,836\,253\,492\,341\,955\,375\,623\,568\,000\,x^3+1\,422\,430\,569\,174\,349\,295\,355\,029\,596\,452\,414\,156\,889\,064\,849\,824\,x^2+84\,352\,511\,412\,881\,080\,058\,013\,283\,731\,292\,030\,562\,587\,963\,719\,200\,x+1\,860\,140\,008\,630\,568\,979\,467\,713\,027\,256\,858\,692\,614\,829\,601\,579\,201$$

**Series representations:**

$$\frac{1}{2}\left(\sqrt{59}+\sqrt{2\sqrt{64\times7^2-24\times7+9}-8\times7+6}\right)^3-89+\phi^2=-89+\phi^2+\frac{1}{2}\left(\sqrt{2\left(-25+\sqrt{2977}\right)}+\sqrt{58}\sum_{k=0}^{\infty}58^{-k}\binom{\frac{1}{2}}{k}\right)^3$$

$$\frac{1}{2}\left(\sqrt{59}+\sqrt{2\sqrt{64\times7^2-24\times7+9}-8\times7+6}\right)^3-89+\phi^2=-89+\phi^2+\frac{1}{2}\left(\sqrt{2\left(-25+\sqrt{2977}\right)}+\sqrt{58}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{58}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^3$$

$$\frac{1}{2}\left(\sqrt{59}+\sqrt{2\sqrt{64\times7^2-24\times7+9}-8\times7+6}\right)^3-89+\phi^2=-89+\phi^2+\frac{1}{2}\left(\sqrt{-50+2\sqrt{2977}}+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}58^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)^3$$

and:

(((sqrt(59)+(2(64*7^2-24*7+9)^0.5 – 8*7+6)^0.5)))^2-89-8

**Input:**

$$\left(\sqrt{59}+\sqrt{2\sqrt{64\times 7^2-24\times 7+9}-8\times 7+6}\right)^2-89-8$$

**Exact result:**

$$\left(\sqrt{59}+\sqrt{2\sqrt{2977}-50}\right)^2-97$$

**Decimal approximation:**

139.2475009946471218646532485086201213325930744944927347563...

139.247500994… result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternate forms:**

$$2\left(-44+\sqrt{2977}+\sqrt{118\left(\sqrt{2977}-25\right)}\right)$$

$$\left(\sqrt{59}+\sqrt{2\left(\sqrt{2977}-25\right)}\right)^2-97$$

$$-88+2\sqrt{2977}+2\sqrt{118\left(\sqrt{2977}-25\right)}$$

**Minimal polynomial:**

$$x^4+352x^3+46248x^2-8553280x-1032083248$$

(((sqrt(59)+(2(64*7^2-24*7+9)^0.5 – 8*7+6)^0.5)))^2-89-21-1/golden ratio

**Input:**

$$\left(\sqrt{59}+\sqrt{2\sqrt{64\times 7^2-24\times 7+9}-8\times 7+6}\right)^2-89-21-\frac{1}{\phi}$$

$\phi$ is the golden ratio

**Exact result:**

$$-\frac{1}{\phi}-110+\left(\sqrt{59}+\sqrt{2\sqrt{2977}-50}\right)^2$$

**Decimal approximation:**

125.62946700589722701644866167425448321487276531468697189 41...

125.629467005… result very near to the Higgs boson mass 125.18 GeV

**Alternate forms:**

$$-\frac{1}{\phi}-110+\left(\sqrt{59}+\sqrt{2\left(\sqrt{2977}-25\right)}\right)^2$$

$$-\frac{\left(101-2\sqrt{2977}-2\sqrt{118\left(\sqrt{2977}-25\right)}\right)\phi+1}{\phi}$$

$$-101+2\sqrt{2977}-\frac{2}{1+\sqrt{5}}+2\sqrt{59\left(2\sqrt{2977}-50\right)}$$

**Minimal polynomial:**

$$x^8+804x^7+282370x^6+34098392x^5-4424781597x^4-1782012614152x^3-84435418625638x^2+16339856255338860x+1279063200062376569$$

**Series representations:**

$$\left(\sqrt{59}+\sqrt{2\sqrt{64\times 7^2-24\times 7+9}-8\times 7+6}\right)^2-89-21-\frac{1}{\phi}=-110-\frac{1}{\phi}+\left(\sqrt{-50+2\sqrt{2977}}+\sqrt{58}\sum_{k=0}^{\infty}58^{-k}\binom{\frac{1}{2}}{k}\right)^2$$

$$\left(\sqrt{59}+\sqrt{2\sqrt{64\times 7^2-24\times 7+9}-8\times 7+6}\right)^2-89-21-\frac{1}{\phi}=-110-\frac{1}{\phi}+\left(\sqrt{-50+2\sqrt{2977}}+\sqrt{58}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{58}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

$$\left(\sqrt{59}+\sqrt{2\sqrt{64\times 7^2-24\times 7+9}-8\times 7+6}\right)^2-89-21-\frac{1}{\phi}=$$

$$-110-\frac{1}{\phi}+\left(\sqrt{-50+2\sqrt{2977}}+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}58^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)^2$$

For J = 8, from the previous expression, we obtain:

Sqrt(8*8+3)+(2(64*8^2-24*8+9)^0.5-8*8+6)^0.5

**Input:**

$$\sqrt{8\times 8+3}+\sqrt{2\sqrt{64\times 8^2-24\times 8+9}-8\times 8+6}$$

**Exact result:**

$$\sqrt{67}+\sqrt{2\sqrt{3913}-58}$$

**Decimal approximation:**

16.377297192110376663475926060954677975838225657159640058 4...

16.37729719…

**Alternate forms:**

$$\sqrt{67}+\sqrt{2\left(\sqrt{3913}-29\right)}$$

$$\sqrt{9+2\sqrt{3913}+2\sqrt{134\left(\sqrt{3913}-29\right)}}$$

**Minimal polynomial:**

$$x^8-36x^6+270x^4-16\,777\,972x^2+729$$

From which, we obtain:

2Pi(((Sqrt(8*8+3)+(2(64*8^2-24*8+9)^0.5-8*8+6)^0.5)))^2+41+2

where 2 and 41 are Eisenstein numbers

**Input:**

$$2\pi\left(\sqrt{8\times 8+3}+\sqrt{2\sqrt{64\times 8^2-24\times 8+9}-8\times 8+6}\right)^2+41+2$$

**Exact result:**

$$43+2\left(\sqrt{67}+\sqrt{2\sqrt{3913}-58}\right)^2\pi$$

**Decimal approximation:**

1728.249971556584427346322154853749569601787955976922400096...

1728.2499715…

This result is very near to the mass of candidate glueball **$f_0$(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

**Property:**

$43+2\left(\sqrt{67}+\sqrt{-58+2\sqrt{3913}}\right)^2\pi$ is a transcendental number

**Alternate forms:**

$$43+2\left(\sqrt{67}+\sqrt{2\left(\sqrt{3913}-29\right)}\right)^2\pi$$

$$43+18\pi+4\sqrt{3913}\,\pi+4\sqrt{134\left(\sqrt{3913}-29\right)}\,\pi$$

$$43+2\left(9+2\sqrt{3913}+2\sqrt{134\left(\sqrt{3913}-29\right)}\right)\pi$$

**Series representations:**

$$2\pi\left(\sqrt{8\times 8+3}+\sqrt{2\sqrt{64\times 8^2-24\times 8+9}-8\times 8+6}\right)^2+41+2 = 43+2\pi\left(\sqrt{2\left(-29+\sqrt{3913}\right)}+\sqrt{66}\sum_{k=0}^{\infty}66^{-k}\binom{\frac{1}{2}}{k}\right)^2$$

$$2\pi\left(\sqrt{8\times 8+3}+\sqrt{2\sqrt{64\times 8^2-24\times 8+9}-8\times 8+6}\right)^2+41+2 = 43+2\pi\left(\sqrt{2\left(-29+\sqrt{3913}\right)}+\sqrt{66}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{66}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

$$2\pi\left(\sqrt{8\times 8+3}+\sqrt{2\sqrt{64\times 8^2-24\times 8+9}-8\times 8+6}\right)^2+41+2 = 43+2\pi\left(\sqrt{-58+2\sqrt{3913}}+\frac{\sum_{j=0}^{\infty}\text{Res}_{s=-\frac{1}{2}+j}\,66^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)^2$$

and:

(((2Pi(((Sqrt(8*8+3)+(2(64*8^2-24*8+9)^0.5-8*8+6)^0.5)))^2+41+2)))^1/15

**Input:**

$$\sqrt[15]{2\pi\left(\sqrt{8\times 8+3}+\sqrt{2\sqrt{64\times 8^2-24\times 8+9}-8\times 8+6}\right)^2+41+2}$$

**Exact result:**

$$\sqrt[15]{43+2\left(\sqrt{67}+\sqrt{2\sqrt{3913}-58}\right)^2\pi}$$

**Decimal approximation:**

1.643767680731257693548923809974242826052063247729014573078...

$1.6437676807\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

**Property:**

$\sqrt[15]{43+2\left(\sqrt{67}+\sqrt{-58+2\sqrt{3913}}\right)^2\pi}$ is a transcendental number

**Alternate forms:**

$$\sqrt[15]{43+2\left(\sqrt{67}+\sqrt{2\left(\sqrt{3913}-29\right)}\right)^2\pi}$$

$$\sqrt[15]{43+18\pi+4\sqrt{3913}\,\pi+4\sqrt{134\left(\sqrt{3913}-29\right)}\,\pi}$$

$$\sqrt[15]{43+2\left(9+2\sqrt{3913}+2\sqrt{134\left(\sqrt{3913}-29\right)}\right)\pi}$$

**All 15th roots of 43 + 2 (sqrt(67) + sqrt(2 sqrt(3913) - 58))^2 π:**

$$\sqrt[15]{43+2\left(\sqrt{67}+\sqrt{2\sqrt{3913}-58}\right)^2\pi}\;e^0\approx 1.64377 \text{ (real, principal root)}$$

$$\sqrt[15]{43+2\left(\sqrt{67}+\sqrt{2\sqrt{3913}-58}\right)^2\pi}\;e^{(2i\pi)/15}\approx 1.50166+0.6686\,i$$

$$\sqrt[15]{43+2\left(\sqrt{67}+\sqrt{2\sqrt{3913}-58}\right)^2\pi}\;e^{(4i\pi)/15}\approx 1.0999+1.2216\,i$$

$$\sqrt[15]{43+2\left(\sqrt{67}+\sqrt{2\sqrt{3913}-58}\right)^2\pi}\;e^{(2i\pi)/5}\approx 0.5080+1.5633\,i$$

$$\sqrt[15]{43+2\left(\sqrt{67}+\sqrt{2\sqrt{3913}-58}\right)^2\pi}\;e^{(8i\pi)/15}\approx -0.17182+1.63476\,i$$

**Series representations:**

$$\sqrt[15]{2\pi\left(\sqrt{8\times 8+3}+\sqrt{2\sqrt{64\times 8^2-24\times 8+9}-8\times 8+6}\right)^2+41+2} = \sqrt[15]{43+2\pi\left(\sqrt{-58+2\sqrt{3913}}+\sqrt{66}\sum_{k=0}^{\infty}66^{-k}\binom{\frac{1}{2}}{k}\right)^2}$$

$$\sqrt[15]{2\pi\left(\sqrt{8\times 8+3}+\sqrt{2\sqrt{64\times 8^2-24\times 8+9}-8\times 8+6}\right)^2+41+2} = \sqrt[15]{43+2\pi\left(\sqrt{-58+2\sqrt{3913}}+\sqrt{66}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{66}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\sqrt[15]{2\pi\left(\sqrt{8\times 8+3}+\sqrt{2\sqrt{64\times 8^2-24\times 8+9}-8\times 8+6}\right)^2+41+2} = \sqrt[15]{43+2\pi\left(\sqrt{-58+2\sqrt{3913}}+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}66^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)^2}$$

**Integral representation:**

$$(1+z)^a = \frac{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0<\gamma<-\operatorname{Re}(a) \text{ and } |\arg(z)|<\pi)$$

and again:

8(((Sqrt(8*8+3)+(2(64*8^2-24*8+9)^0.5-8*8+6)^0.5)))-2Pi+1/golden ratio

**Input:**

$$8\left(\sqrt{8\times 8+3}+\sqrt{2\sqrt{64\times 8^2-24\times 8+9}-8\times 8+6}\right)-2\pi+\frac{1}{\phi}$$

$\phi$ is the golden ratio

**Exact result:**

$$\frac{1}{\phi} + 8\left(\sqrt{67} + \sqrt{2\sqrt{3913} - 58}\right) - 2\pi$$

**Decimal approximation:**

125.3532262184533217020600409165703747299965509067832632669...

125.353226218… result very near to the Higgs boson mass 125.18 GeV

**Property:**

$8\left(\sqrt{67} + \sqrt{-58 + 2\sqrt{3913}}\right) + \frac{1}{\phi} - 2\pi$ is a transcendental number

**Alternate forms:**

$$\frac{1}{\phi} + 8\sqrt{67} + 8\sqrt{2\left(\sqrt{3913} - 29\right)} - 2\pi$$

$$\frac{1 - 2\left(\pi - 4\left(\sqrt{67} + \sqrt{2(\sqrt{3913} - 29)}\right)\right)\phi}{\phi}$$

$$8\sqrt{67} + \frac{2}{1 + \sqrt{5}} + 8\sqrt{2\sqrt{3913} - 58} - 2\pi$$

**Series representations:**

$$8\left(\sqrt{8 \times 8 + 3} + \sqrt{2\sqrt{64 \times 8^2 - 24 \times 8 + 9} - 8 \times 8 + 6}\right) - 2\pi + \frac{1}{\phi} = 8\sqrt{2\left(-29 + \sqrt{3913}\right)} + \frac{1}{\phi} - 2\pi + 8\sqrt{66}\sum_{k=0}^{\infty} 66^{-k}\binom{\frac{1}{2}}{k}$$

$$8\left(\sqrt{8 \times 8 + 3} + \sqrt{2\sqrt{64 \times 8^2 - 24 \times 8 + 9} - 8 \times 8 + 6}\right) - 2\pi + \frac{1}{\phi} = 8\sqrt{2\left(-29 + \sqrt{3913}\right)} + \frac{1}{\phi} - 2\pi + 8\sqrt{66}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{66}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$8\left(\sqrt{8\times8+3}+\sqrt{2\sqrt{64\times8^2-24\times8+9}-8\times8+6}\right)-2\pi+\frac{1}{\phi}=$$

$$8\sqrt{2\left(-29+\sqrt{3913}\right)}+\frac{1}{\phi}-2\pi+\frac{4\sum_{j=0}^{\infty}\text{Res}_{s=-\frac{1}{2}+j}66^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}$$

8(((Sqrt(8*8+3)+(2(64*8^2-24*8+9)^0.5-8*8+6)^0.5)))+8+1/golden ratio

**Input:**

$$8\left(\sqrt{8\times8+3}+\sqrt{2\sqrt{64\times8^2-24\times8+9}-8\times8+6}\right)+8+\frac{1}{\phi}$$

$\phi$ is the golden ratio

**Exact result:**

$$\frac{1}{\phi}+8+8\left(\sqrt{67}+\sqrt{2\sqrt{3913}-58}\right)$$

**Decimal approximation:**

139.6364115256329081789853276831293804983908897055334749089...

139.636411525… result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternate forms:**

$$\frac{1}{\phi}+8+8\left(\sqrt{67}+\sqrt{2\left(\sqrt{3913}-29\right)}\right)$$

$$\frac{8\left(1+\sqrt{67}+\sqrt{2(\sqrt{3913}-29)}\right)\phi+1}{\phi}$$

$$8+8\sqrt{67}+\frac{2}{1+\sqrt{5}}+8\sqrt{2\sqrt{3913}-58}$$

**Minimal polynomial:**

$$x^{16} - 120 x^{15} + 2132 x^{14} + 248640 x^{13} - 10330694 x^{12} - 73922040 x^{11} - 8786794779216 x^{10} + 659624958286200 x^9 - 2144180716247613 x^8 - 762065699022225720 x^7 + 16223656731692065968 x^6 + 10367284038395248440 x^5 + 19340574211806681723907834 x^4 - 580281156613767249791987520 x^3 + 6479962253147408820035452436 x^2 - 31915762103137296028298228040 x + 58509618544637394415853759041$$

**Series representations:**

$$8\left(\sqrt{8\times 8+3}+\sqrt{2\sqrt{64\times 8^2-24\times 8+9}-8\times 8+6}\right)+8+\frac{1}{\phi} = 8+8\sqrt{2\left(-29+\sqrt{3913}\right)}+\frac{1}{\phi}+8\sqrt{66}\sum_{k=0}^{\infty} 66^{-k}\binom{\frac{1}{2}}{k}$$

$$8\left(\sqrt{8\times 8+3}+\sqrt{2\sqrt{64\times 8^2-24\times 8+9}-8\times 8+6}\right)+8+\frac{1}{\phi} = 8+8\sqrt{2\left(-29+\sqrt{3913}\right)}+\frac{1}{\phi}+8\sqrt{66}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{66}\right)^k\left(-\frac{1}{2}\right)_k}{k!}$$

$$8\left(\sqrt{8\times 8+3}+\sqrt{2\sqrt{64\times 8^2-24\times 8+9}-8\times 8+6}\right)+8+\frac{1}{\phi} = 8+8\sqrt{2\left(-29+\sqrt{3913}\right)}+\frac{1}{\phi}+\frac{4\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 66^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}$$

Now, we have that:

$$(2.11.2)\quad \phi_2(y)=\frac{1}{y}-\log 2\sum_{0}^{\infty}\frac{(-1)^r y^r}{r!}\frac{2^{r+1}}{2^{r+1}-1}-\frac{1}{y}\sum_{-\infty}^{\infty}{}'\,\Gamma\left(\frac{1+2k\pi i}{\log 2}\right)y^{-2k\pi i/\log 2},$$

and

$$\phi_2(y)+\log 2\left(1-\frac{y}{3.1!}+\frac{y^2}{7.2!}-\frac{y^3}{15.3!}+\dots\right)=\frac{1}{y}+F(y),$$

where
$$yF(y) = \cdot 0000098844\cos\left(\frac{2\pi\log y}{\log 2}+\cdot 872811\right)$$

We have obtained (see part II):

1/3.9121192269((1+0.0000098844 cos((2Pi*ln(3.9121192269))/ln2+0.872811)))

**Input interpretation:**

$$\frac{1}{3.9121192269}\left(1+9.8844\times 10^{-6}\cos\left(\frac{2\pi\log(3.9121192269)}{\log(2)}+0.872811\right)\right)$$

log(x) is the natural logarithm

**Result:**

0.255617909572411304355893214535044931185553916327111126588...

0.25561790957…

For y = 3.9121192269 , r = 2 and k = 1 , we obtain:

1/3.9121192269 – ln(2) (3.9121192269^2)/2! 2^3/(2^3-1) -1/3.9121192269 gamma (((1+2*Pi)/(ln(2)))) 3.9121192269^(((2Pi)/(ln(2))))

**Input interpretation:**

$$\frac{1}{3.9121192269}-\log(2)\times\frac{3.9121192269^2}{2!}\times\frac{2^3}{2^3-1}-\frac{1}{3.9121192269}\Gamma\left(\frac{1+2\pi}{\log(2)}\right)\times 3.9121192269^{(2\pi)/\log(2)}$$

log(x) is the natural logarithm

n! is the factorial function

Γ(x) is the gamma function

**Result:**

$-6.908634196...\times 10^{10}$

$-6.9086374196...*10^{10}$

## Alternative representations:

$$\frac{1}{3.91211922690000} - \frac{(3.91211922690000^2 \log(2))\, 2^3}{2!\,(2^3-1)} - \frac{\Gamma\left(\frac{1+2\pi}{\log(2)}\right) 3.91211922690000^{(2\pi)/\log(2)}}{3.91211922690000} = \frac{1}{3.91211922690000} - \frac{G\left(1+\frac{1+2\pi}{\log(2)}\right) 3.91211922690000^{(2\pi)/\log(2)}}{3.91211922690000\, G\left(\frac{1+2\pi}{\log(2)}\right)} - \frac{8 \log(2)\, 3.91211922690000^2}{7\,(1!! \times 2!!)}$$

$$\frac{1}{3.91211922690000} - \frac{(3.91211922690000^2 \log(2))\, 2^3}{2!\,(2^3-1)} - \frac{\Gamma\left(\frac{1+2\pi}{\log(2)}\right) 3.91211922690000^{(2\pi)/\log(2)}}{3.91211922690000} = \frac{1}{3.91211922690000} - \frac{3.91211922690000^{(2\pi)/\log_e(2)} \exp\left(-\log G\left(\frac{1+2\pi}{\log(2)}\right) + \log G\left(1+\frac{1+2\pi}{\log(2)}\right)\right)}{3.91211922690000} - \frac{8 \log_e(2)\, 3.91211922690000^2}{7\,(1!! \times 2!!)}$$

$$\frac{1}{3.91211922690000} - \frac{(3.91211922690000^2 \log(2))\, 2^3}{2!\,(2^3-1)} - \frac{\Gamma\left(\frac{1+2\pi}{\log(2)}\right) 3.91211922690000^{(2\pi)/\log(2)}}{3.91211922690000} = \frac{1}{3.91211922690000} - \frac{3.91211922690000^{(2\pi)/(\log(a)\log_a(2))} \exp\left(-\log G\left(\frac{1+2\pi}{\log(2)}\right) + \log G\left(1+\frac{1+2\pi}{\log(2)}\right)\right)}{3.91211922690000} - \frac{8 \log(a) \log_a(2)\, 3.91211922690000^2}{7\,(1!! \times 2!!)}$$

## Integral representations:

$$\frac{1}{3.91211922690000} - \frac{(3.91211922690000^2 \log(2))\, 2^3}{2!\,(2^3-1)} - \frac{\Gamma\left(\frac{1+2\pi}{\log(2)}\right) 3.91211922690000^{(2\pi)/\log(2)}}{3.91211922690000} = 0.255615931417410 - \frac{17.4910592519779 \log(2)}{\int_0^\infty e^{-t}\, t^2\, dt} - \frac{0.511231862834819\, e^{(2.72815845786415\pi)/\log(2)}\, i\pi}{\oint_L e^t\, t^{-(1+2\pi)/\log(2)}\, dt}$$

$$\frac{1}{3.91211922690000} - \frac{(3.91211922690000^2 \log(2))\, 2^3}{2!\,(2^3-1)} - \frac{\Gamma\left(\frac{1+2\pi}{\log(2)}\right) 3.91211922690000^{(2\pi)/\log(2)}}{3.91211922690000} = -\left(\left(0.5112318628348 \left(1.0000000000000\, e^{(2.72815845786415\pi)/\left(\int_1^2 \frac{1}{t}\,dt\right)}\, i\,\pi \int_0^1 \log^2\left(\frac{1}{t}\right) dt + 23.715028907407 \oint_L e^t\, t^{-(1+2\pi)/\log(2)}\, dt - 1.0000000000000 \oint_L e^t\, t^{-(1+2\pi)/\log(2)}\, dt\right)\right) \Big/ \left(2 \oint_L e^t\, t^{-(1+2\pi)/\log(2)}\, dt\right)\right)$$

$$\frac{1}{3.91211922690000} - \frac{(3.91211922690000^2 \log(2))\, 2^3}{2!\,(2^3-1)} - \frac{\Gamma\left(\frac{1+2\pi}{\log(2)}\right) 3.91211922690000^{(2\pi)/\log(2)}}{3.91211922690000} = -\left(\left(0.5112318628348 \left(1.0000000000000\, e^{(2.72815845786415\pi)/\left(\int_1^2 \frac{1}{t}\,dt\right)}\, i\,\pi \int_0^\infty e^{-t}\, t^2\, dt + 23.715028907407 \oint_L e^t\, t^{-(1+2\pi)/\log(2)}\, dt - 0.50000000000000\,(2 - e^{-\infty}\,(\infty + 2\,\infty + 2)) \oint_L e^t\, t^{-(1+2\pi)/\log(2)}\, dt\right)\right) \Big/ \left((2 - e^{-\infty}\,(\infty + 2\,\infty + 2)) \oint_L e^t\, t^{-(1+2\pi)/\log(2)}\, dt\right)\right)$$

Performing the 52[th] root of the above expression, we obtain:

(((-(1/3.9121192269 – ln(2) (3.9121192269^2)/2! 2^3/(2^3-1) -1/3.9121192269 gamma (((1+2*Pi)/(ln(2)))) 3.9121192269^(((2Pi)/(ln(2))))))))^1/52

**Input interpretation:**

$$\left(-\left(\frac{1}{3.9121192269} - \log(2) \times \frac{3.9121192269^2}{2!} \times \frac{2^3}{2^3-1} - \frac{1}{3.9121192269}\,\Gamma\left(\frac{1+2\pi}{\log(2)}\right) \times 3.9121192269^{(2\pi)/\log(2)}\right)\right) \verb|^| (1/52)$$

log(x) is the natural logarithm

n! is the factorial function

$\Gamma(x)$ is the gamma function

**Result:**

1.6160316045...

1.6160316045…. result that is a good approximation to the value of the golden ratio 1.618033988749...

and:

27*(((-(1/3.9121192269 – ln(2) (3.9121192269^2)/2! 2^3/(2^3-1) -1/3.9121192269 gamma (((1+2*Pi)/(ln(2)))) 3.9121192269^(((2Pi)/(ln(2)))))))^1/6-1/2

**Input interpretation:**

$$27\left(-\left(\frac{1}{3.9121192269}-\log(2)\times\frac{3.9121192269^2}{2!}\times\frac{2^3}{2^3-1}-\frac{1}{3.9121192269}\,\Gamma\left(\frac{1+2\pi}{\log(2)}\right)\times 3.9121192269^{(2\pi)/\log(2)}\right)\right)\hat{}\,(1/6)-\frac{1}{2}$$

$\log(x)$ is the natural logarithm

$n!$ is the factorial function

$\Gamma(x)$ is the gamma function

**Result:**

1729.034105...

1729.034105…

This result is very near to the mass of candidate glueball **$f_0$(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

With regard 27 (From Wikipedia):

*"The fundamental group of the complex form, compact real form, or any algebraic version of $E_6$ is the cyclic group **Z**/3**Z**, and its outer automorphism group is the cyclic group **Z**/2**Z**. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, $E_6$ plays a role in some grand unified theories".*

**Alternative representations:**

$$27\left(-\left(\frac{1}{3.91211922690000}-\frac{(3.91211922690000^2\log(2))\,2^3}{2!\,(2^3-1)}-\frac{\Gamma\left(\frac{1+2\pi}{\log(2)}\right)3.91211922690000^{(2\pi)/\log(2)}}{3.91211922690000}\right)\right)\text{^}(1/6)-\frac{1}{2}=$$

$$-\frac{1}{2}+27\left(-\frac{1}{3.91211922690000}+\frac{G\left(1+\frac{1+2\pi}{\log(2)}\right)3.91211922690000^{(2\pi)/\log(2)}}{3.91211922690000\,G\left(\frac{1+2\pi}{\log(2)}\right)}+\frac{8\log(2)\,3.91211922690000^2}{7\,(1!!\times2!!)}\right)\text{^}(1/6)$$

$$27\left(-\left(\frac{1}{3.91211922690000}-\frac{(3.91211922690000^2\log(2))\,2^3}{2!\,(2^3-1)}-\frac{\Gamma\left(\frac{1+2\pi}{\log(2)}\right)3.91211922690000^{(2\pi)/\log(2)}}{3.91211922690000}\right)\right)\text{^}(1/6)-\frac{1}{2}=-\frac{1}{2}+27\left(-\frac{1}{3.91211922690000}+\frac{3.91211922690000^{(2\pi)/\log_e(2)}\exp\left(-\log G\left(\frac{1+2\pi}{\log(2)}\right)+\log G\left(1+\frac{1+2\pi}{\log(2)}\right)\right)}{3.91211922690000}+\frac{8\log_e(2)\,3.91211922690000^2}{7\,(1!!\times2!!)}\right)\text{^}(1/6)$$

$$27\left(-\left(\frac{1}{3.91211922690000}-\frac{(3.91211922690000^2\log(2))\,2^3}{2!\,(2^3-1)}-\frac{\Gamma\left(\frac{1+2\pi}{\log(2)}\right)3.91211922690000^{(2\pi)/\log(2)}}{3.91211922690000}\right)\right)\text{^}(1/6)-\frac{1}{2}=$$

$$-\frac{1}{2}+27\left(-\frac{1}{3.91211922690000}+\frac{(1)_{-1+\frac{1+2\pi}{\log(2)}}\,3.91211922690000^{(2\pi)/(\log(a)\log_a(2))}}{3.91211922690000}+\frac{8\log(a)\log_a(2)\,3.91211922690000^2}{7\,(1!!\times2!!)}\right)\text{^}(1/6)$$

## Integral representations:

$$27\left(-\left(\frac{1}{3.91211922690000}-\frac{\left(3.91211922690000^2\log(2)\right)2^3}{2!\left(2^3-1\right)}-\frac{\Gamma\left(\frac{1+2\pi}{\log(2)}\right)3.91211922690000^{(2\pi)/\log(2)}}{3.91211922690000}\right)\right)\text{^}(1/6)-\frac{1}{2}=$$

$$-\frac{1}{2}+27\left(-0.255615931417410+\frac{17.4910592519779\log(2)}{\int_0^\infty e^{-t}t^2\,dt}+\frac{0.511231862834819\,e^{(2.72815845786415\pi)/\log(2)}\,i\pi}{\oint_L e^t\,t^{-(1+2\pi)/\log(2)}\,dt}\right)\text{^}(1/6)$$

$$27\left(-\left(\frac{1}{3.91211922690000}-\frac{\left(3.91211922690000^2\log(2)\right)2^3}{2!\left(2^3-1\right)}-\frac{\Gamma\left(\frac{1+2\pi}{\log(2)}\right)3.91211922690000^{(2\pi)/\log(2)}}{3.91211922690000}\right)\right)\text{^}(1/6)-\frac{1}{2}=$$

$$\frac{1}{2}\left(-1+54\left(-0.255615931417410+\frac{17.4910592519779\int_1^2\frac{1}{t}\,dt}{\int_0^1\log^2\left(\frac{1}{t}\right)dt}+\frac{0.511231862834819\,e^{(2.72815845786415\pi)/\left(\int_1^2\frac{1}{t}\,dt\right)}\,i\pi}{\oint_L e^t\,t^{-(1+2\pi)/\log(2)}\,dt}\right)\text{^}(1/6)\right)$$

$$27\left(-\left(\frac{1}{3.91211922690000}-\frac{\left(3.91211922690000^2\log(2)\right)2^3}{2!\left(2^3-1\right)}-\frac{\Gamma\left(\frac{1+2\pi}{\log(2)}\right)3.91211922690000^{(2\pi)/\log(2)}}{3.91211922690000}\right)\right)\text{^}(1/6)-\frac{1}{2}=$$

$$\frac{1}{2}\left(-1+54\left(-0.255615931417410+\frac{17.4910592519779\int_1^2\frac{1}{t}\,dt}{\int_0^\infty e^{-t}t^2\,dt}+\frac{0.511231862834819\times3.91211922690000^{(2\pi)/\left(\int_1^2\frac{1}{t}\,dt\right)}\,i\pi}{\oint_L e^t\,t^{-(1+2\pi)/\log(2)}\,dt}\right)\text{^}(1/6)\right)$$

Performing the 15th root, we obtain:

((((27*(((-(1/3.9121192269 – ln(2) (3.9121192269^2)/2! 2^3/(2^3-1) - 1/3.9121192269 gamma (((1+2*Pi)/(ln(2)))) 3.9121192269^(((2Pi)/(ln(2))))))))^1/6-1/2))))^1/15

**Input interpretation:**

$$\left(27\left(-\left(\frac{1}{3.9121192269} - \log(2)\times\frac{3.9121192269^2}{2!}\times\frac{2^3}{2^3-1} - \frac{1}{3.9121192269}\,\Gamma\left(\frac{1+2\pi}{\log(2)}\right)\times 3.9121192269^{(2\pi)/\log(2)}\right)\right)\char`^(1/6) - \frac{1}{2}\right)\char`^(1/15)$$

$\log(x)$ is the natural logarithm

$n!$ is the factorial function

$\Gamma(x)$ is the gamma function

**Result:**

1.64381739040...

$1.64381739040\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

or, remaining 2πi in the previous expression:

1/3.9121192269 – ln(2) (3.9121192269^2)/2! 2^3/(2^3-1) -1/3.9121192269 gamma (((1+2*Pi*i)/(ln(2)))) 3.9121192269^(((2Pi*i)/(ln(2)))))

**Input interpretation:**

$$\frac{1}{3.9121192269} - \log(2)\times\frac{3.9121192269^2}{2!}\times\frac{2^3}{2^3-1} - \frac{1}{3.9121192269}\,\Gamma\left(\frac{1+2\pi i}{\log(2)}\right)\times 3.9121192269^{(2\pi i)/\log(2)}$$

**Result:**

$-5.8063263442\ldots + 1.3497831003\ldots\times 10^{-6}\,i$

(using the principal branch of the logarithm for complex exponentiation)

**Polar coordinates:**

$r = 5.806326344$ (radius), $\theta = 179.99998668058419°$ (angle)

5.806326344

## Alternative representations:

$$\frac{1}{3.91211922690000} - \frac{(3.91211922690000^2 \log(2))\, 2^3}{2!\,(2^3 - 1)} - \frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right) 3.91211922690000^{(2(\pi i))/\log(2)}}{3.91211922690000} = \frac{1}{3.91211922690000} - \frac{G\left(1 + \frac{1+2 i \pi}{\log(2)}\right) 3.91211922690000^{(2 i \pi)/\log(2)}}{3.91211922690000\, G\left(\frac{1+2 i \pi}{\log(2)}\right)} - \frac{8 \log(2)\, 3.91211922690000^2}{7\,(1!! \times 2!!)}$$

$$\frac{1}{3.91211922690000} - \frac{(3.91211922690000^2 \log(2))\, 2^3}{2!\,(2^3 - 1)} - \frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right) 3.91211922690000^{(2(\pi i))/\log(2)}}{3.91211922690000} = \frac{1}{3.91211922690000} - \frac{3.91211922690000^{(2 i \pi)/\log_e(2)} \exp\left(-\log G\left(\frac{1+2 i \pi}{\log(2)}\right) + \log G\left(1 + \frac{1+2 i \pi}{\log(2)}\right)\right)}{3.91211922690000} - \frac{8 \log_e(2)\, 3.91211922690000^2}{7\,(1!! \times 2!!)}$$

$$\frac{1}{3.91211922690000} - \frac{(3.91211922690000^2 \log(2))\, 2^3}{2!\,(2^3 - 1)} - \frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right) 3.91211922690000^{(2(\pi i))/\log(2)}}{3.91211922690000} = \frac{1}{3.91211922690000} - \frac{G\left(1 + \frac{1+2 i \pi}{\log(2)}\right) 3.91211922690000^{(2 i \pi)/\log_e(2)}}{3.91211922690000\, G\left(\frac{1+2 i \pi}{\log(2)}\right)} - \frac{8 \log_e(2)\, 3.91211922690000^2}{7\,(1!! \times 2!!)}$$

## Integral representations:

$$\frac{1}{3.91211922690000} - \frac{(3.91211922690000^2 \log(2))\, 2^3}{2!\,(2^3 - 1)} - \frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right) 3.91211922690000^{(2(\pi i))/\log(2)}}{3.91211922690000} = 0.255615931417410 - \frac{17.4910592519779 \log(2)}{\int_0^\infty e^{-t}\, t^2\, dt} - \frac{0.511231862834819\, e^{(2.72815845786415\, i \pi)/\log(2)}\, \pi\, \mathcal{A}}{\oint_L e^t\, t^{-(1+2 i \pi)/\log(2)}\, dt}$$

$$\frac{1}{3.91211922690000} - \frac{(3.91211922690000^2 \log(2))\, 2^3}{2!\,(2^3 - 1)} - \frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right) 3.91211922690000^{(2(\pi i))/\log(2)}}{3.91211922690000} =$$

$$-\left(\left(0.5112318628348\left(1.0000000000000\, e^{(2.72815845786415\, i\pi)/\left(\int_1^2 \frac{1}{t}\, dt\right)}\, \pi\, \mathcal{A} \int_0^1 \log^2\left(\frac{1}{t}\right) dt + 23.715028907407 \oint_L e^t\, t^{-(1+2 i\pi)/\log(2)}\, dt - 1.0000000000000 \oint_L e^t\, t^{-(1+2 i\pi)/\log(2)}\, dt\right)\right) \Big/ \left(2 \oint_L e^t\, t^{-(1+2 i\pi)/\log(2)}\, dt\right)\right)$$

$$\frac{1}{3.91211922690000} - \frac{(3.91211922690000^2 \log(2))\, 2^3}{2!\,(2^3 - 1)} - \frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right) 3.91211922690000^{(2(\pi i))/\log(2)}}{3.91211922690000} =$$

$$-\left(\left(0.5112318628348\left(1.0000000000000 \times 3.91211922690000^{(2 i\pi)/\left(\int_1^2 \frac{1}{t}\, dt\right)}\, \pi\, \mathcal{A} \int_0^\infty e^{-t}\, t^2\, dt + 23.715028907407 \oint_L e^t\, t^{-(1+2 i\pi)/\log(2)}\, dt - 0.50000000000000\,(2 - e^{-\infty}\,(\infty + 2\,\infty + 2)) \oint_L e^t\, t^{-(1+2 i\pi)/\log(2)}\, dt\right)\right) \Big/ \left(\left(2 - e^{-\infty}\,(\infty + 2\,\infty + 2)\right) \oint_L e^t\, t^{-(1+2 i\pi)/\log(2)}\, dt\right)\right)$$

$$\frac{1}{3.9121192269} - \log(2) \times \frac{3.9121192269^2}{2!} \times \frac{2^3}{2^3 - 1} - \frac{1}{3.9121192269}\, \Gamma\left(\frac{1 + 2\pi}{\log(2)}\right) \times 3.9121192269^{(2\pi)/\log(2)}$$

$$\frac{1}{3.9121192269} - \log(2) \times \frac{3.9121192269^2}{2!} \times \frac{2^3}{2^3 - 1} - \frac{1}{3.9121192269}\, \Gamma\left(\frac{1 + 2\pi i}{\log(2)}\right) \times 3.9121192269^{(2\pi i)/\log(2)}$$

From which:

1.093364164+(((([1/(((1/3.9121192269 – ln(2) (3.9121192269^2)/2! 2^3/(2^3-1) - 1/3.9121192269 gamma (((1+2*Pi*i)/(ln(2)))) 3.9121192269^(((2Pi*i)/(ln(2)))))))^1/4]))))

where 1.093364164 is the Hausdorff dimension of Rauzy fractal boundary r (Boundary of the Rauzy fractal)

**Input interpretation:**

$$1.093364164 + 1\Big/\left(\left(\frac{1}{3.9121192269} - \log(2)\times\frac{3.9121192269^2}{2!}\times\frac{2^3}{2^3-1} - \frac{1}{3.9121192269}\,\Gamma\left(\frac{1+2\pi i}{\log(2)}\right)\times 3.9121192269^{(2\pi i)/\log(2)}\right)\hat{}\,(1/4)\right)$$

log(x) is the natural logarithm

n! is the factorial function

Γ(x) is the gamma function

i is the imaginary unit

**Result:**

$1.548886490\ldots - 0.4555222726\ldots i$

(using the principal branch of the logarithm for complex exponentiation)

**Polar coordinates:**

$r = 1.61448$ (radius), $\theta = -16.3884°$ (angle)

1.61448 result that is a good approximation to the value of the golden ratio 1.618033988749...

## Alternative representations:

$$1.09336+\frac{1}{\sqrt[4]{\frac{1}{3.91211922690000}-\frac{\left(3.91211922690000^2\log(2)\right)2^3}{2!\left(2^3-1\right)}-\frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right)3.91211922690000^{(2(\pi i))/\log(2)}}{3.91211922690000}}}$$

$$=1.09336+\frac{1}{\sqrt[4]{\frac{1}{3.91211922690000}-\frac{G\left(1+\frac{1+2i\pi}{\log(2)}\right)3.91211922690000^{(2i\pi)/\log(2)}}{3.91211922690000\,G\left(\frac{1+2i\pi}{\log(2)}\right)}-\frac{8\log(2)\,3.91211922690000^2}{7(1!!\times 2!!)}}}$$

$$1.09336+\frac{1}{\sqrt[4]{\frac{1}{3.91211922690000}-\frac{\left(3.91211922690000^2\log(2)\right)2^3}{2!\left(2^3-1\right)}-\frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right)3.91211922690000^{(2(\pi i))/\log(2)}}{3.91211922690000}}}$$

$$=1.09336+1\Big/\left(\left(\frac{1}{3.91211922690000}-\frac{(1)_{-1+\frac{1+2i\pi}{\log(2)}}\,3.91211922690000^{(2i\pi)/(\log(a)\log_a(2))}}{3.91211922690000}-\frac{8\log(a)\log_a(2)\,3.91211922690000^2}{7(1!!\times 2!!)}\right)^\wedge(1/4)\right)$$

$$1.09336+\frac{1}{\sqrt[4]{\frac{1}{3.91211922690000}-\frac{\left(3.91211922690000^2\log(2)\right)2^3}{2!\left(2^3-1\right)}-\frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right)3.91211922690000^{(2(\pi i))/\log(2)}}{3.91211922690000}}}$$

$$=1.09336+1\Big/\left(\left(\frac{1}{3.91211922690000}-\frac{3.91211922690000^{(2i\pi)/\log_e(2)}\exp\left(-\log G\left(\frac{1+2i\pi}{\log(2)}\right)+\log G\left(1+\frac{1+2i\pi}{\log(2)}\right)\right)}{3.91211922690000}-\frac{8\log_e(2)\,3.91211922690000^2}{7(1!!\times 2!!)}\right)^\wedge(1/4)\right)$$

## Integral representations:

$$1.09336 + \frac{1}{\sqrt[4]{\frac{1}{3.91211922690000} - \frac{(3.91211922690000^2 \log(2)) 2^3}{2!(2^3-1)} - \frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right) 3.91211922690000^{(2(\pi i))/\log(2)}}{3.91211922690000}}}$$

$$= 1.09336 + 1 \Big/ \left(\left(0.255615931417410 - \frac{17.4910592519779 \log(2)}{\int_0^\infty e^{-t} t^2 \, dt} - \frac{0.511231862834819 \, e^{(2.72815845786415 i \pi)/\log(2)} \pi \mathcal{A}}{\oint_L e^t \, t^{-(1+2 i \pi)/\log(2)} \, dt}\right) \text{^} (1/4)\right)$$

$$1.09336 + \frac{1}{\sqrt[4]{\frac{1}{3.91211922690000} - \frac{(3.91211922690000^2 \log(2)) 2^3}{2!(2^3-1)} - \frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right) 3.91211922690000^{(2(\pi i))/\log(2)}}{3.91211922690000}}}$$

$$= 1.09336 + 1 \Big/ \left(\left(0.255615931417410 - \frac{17.4910592519779 \int_1^2 \frac{1}{t} \, dt}{\int_0^1 \log^2\left(\frac{1}{t}\right) dt} - \frac{0.511231862834819 \, e^{(2.72815845786415 i \pi)/\left(\int_1^2 \frac{1}{t} dt\right)} \pi \mathcal{A}}{\oint_L e^t \, t^{-(1+2 i \pi)/\log(2)} \, dt}\right) \text{^} (1/4)\right)$$

$$1.09336 + \frac{1}{\sqrt[4]{\frac{1}{3.91211922690000} - \frac{(3.91211922690000^2 \log(2)) 2^3}{2!(2^3-1)} - \frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right) 3.91211922690000^{(2(\pi i))/\log(2)}}{3.91211922690000}}}$$

$$= 1.09336 + 1 \Big/ \left(\left(0.255615931417410 - \frac{17.4910592519779 \int_1^2 \frac{1}{t} \, dt}{\int_0^\infty e^{-t} t^2 \, dt} - \frac{0.511231862834819 \times 3.91211922690000^{(2 i \pi)/\left(\int_1^2 \frac{1}{t} dt\right)} \pi \mathcal{A}}{\oint_L e^t \, t^{-(1+2 i \pi)/\log(2)} \, dt}\right) \text{^} (1/4)\right)$$

and:

4[(((1/3.9121192269 – ln(2) (3.9121192269^2)/2! 2^3/(2^3-1) -1/3.9121192269 gamma (((1+2*Pi*i)/(ln(2)))) 3.9121192269^(((2Pi*i)/(ln(2)))))))]^2+5

**Input interpretation:**

$$4\left(\frac{1}{3.9121192269}-\log(2)\times\frac{3.9121192269^2}{2!}\times\frac{2^3}{2^3-1}-\frac{1}{3.9121192269}\,\Gamma\left(\frac{1+2\pi i}{\log(2)}\right)\times 3.9121192269^{(2\pi i)/\log(2)}\right)^2+5$$

**Result:**

139.8537025... −
0.00006269824940... $i$

(using the principal branch of the logarithm for complex exponentiation)

**Polar coordinates:**

$r = 139.85370246$ (radius), $\theta = -0.0000256864496°$ (angle)

139.85370246 result practically equal to the rest mass of Pion meson 139.57 MeV

**Alternative representations:**

$$4\left(\frac{1}{3.91211922690000}-\frac{(3.91211922690000^2\log(2))\,2^3}{2!\,(2^3-1)}-\frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right)3.91211922690000^{(2(\pi i))/\log(2)}}{3.91211922690000}\right)^2+5=$$

$$5+4\left(\frac{1}{3.91211922690000}-\frac{G\left(1+\frac{1+2i\pi}{\log(2)}\right)3.91211922690000^{(2i\pi)/\log(2)}}{3.91211922690000\,G\left(\frac{1+2i\pi}{\log(2)}\right)}-\frac{8\log(2)\,3.91211922690000^2}{7\,(1!!\times 2!!)}\right)^2$$

$$4\left(\frac{1}{3.91211922690000}-\frac{(3.91211922690000^2\log(2))\,2^3}{2!\,(2^3-1)}-\frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right)3.91211922690000^{(2(\pi i))/\log(2)}}{3.91211922690000}\right)^2+5=$$

$$5+4\left(\frac{1}{3.91211922690000}-\frac{(1)_{-1+\frac{1+2i\pi}{\log(2)}}\,3.91211922690000^{(2i\pi)/(\log(a)\log_a(2))}}{3.91211922690000}-\frac{8\log(a)\log_a(2)\,3.91211922690000^2}{7\,(1!!\times 2!!)}\right)^2$$

$$4\left(\frac{1}{3.91211922690000}-\frac{(3.91211922690000^2\log(2))\,2^3}{2!\,(2^3-1)}-\frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right)3.91211922690000^{(2(\pi i))/\log(2)}}{3.91211922690000}\right)^2+$$

$$5=5+4\left(\frac{1}{3.91211922690000}-\frac{3.91211922690000^{(2i\pi)/\log_e(2)}\exp\left(-\log G\left(\frac{1+2i\pi}{\log(2)}\right)+\log G\left(1+\frac{1+2i\pi}{\log(2)}\right)\right)}{3.91211922690000}-\frac{8\log_e(2)\,3.91211922690000^2}{7\,(1!!\times 2!!)}\right)^2$$

4[(((1/3.9121192269 – ln(2) (3.9121192269^2)/2! 2^3/(2^3-1) -1/3.9121192269 gamma (((1+2*Pi*i)/(ln(2)))) 3.9121192269^(((2Pi*i)/(ln(2)))))))]^2+2-11-1/golden ratio

**Input interpretation:**

$$4\left(\frac{1}{3.9121192269}-\log(2)\times\frac{3.9121192269^2}{2!}\times\frac{2^3}{2^3-1}-\frac{1}{3.9121192269}\,\Gamma\left(\frac{1+2\pi i}{\log(2)}\right)\times 3.9121192269^{(2\pi i)/\log(2)}\right)^2+2-11-\frac{1}{\phi}$$

$\log(x)$ is the natural logarithm

$n!$ is the factorial function

$\Gamma(x)$ is the gamma function

$i$ is the imaginary unit

$\phi$ is the golden ratio

**Result:**

125.2356685... −
0.00006269824940... $i$

(using the principal branch of the logarithm for complex exponentiation)

**Polar coordinates:**

$r = 125.23566847$ (radius), $\theta = -0.0000286846800°$ (angle)

125.32566847 result very near to the Higgs boson mass 125.18 GeV

**Alternative representations:**

$$4\left(\frac{1}{3.91211922690000} - \frac{(3.91211922690000^2 \log(2))\, 2^3}{2!\,(2^3-1)} - \frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right) 3.91211922690000^{(2(\pi i))/\log(2)}}{3.91211922690000}\right)^2 + 2 - 11 - \frac{1}{\phi} = -9 - \frac{1}{\phi} + 4\left(\frac{1}{3.91211922690000} - \frac{G\left(1+\frac{1+2 i\pi}{\log(2)}\right) 3.91211922690000^{(2 i\pi)/\log(2)}}{3.91211922690000\, G\left(\frac{1+2 i\pi}{\log(2)}\right)} - \frac{8\log(2)\, 3.91211922690000^2}{7\,(1!!\times 2!!)}\right)^2$$

$$4\left(\frac{1}{3.91211922690000} - \frac{(3.91211922690000^2 \log(2))\, 2^3}{2!\,(2^3-1)} - \frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right) 3.91211922690000^{(2(\pi i))/\log(2)}}{3.91211922690000}\right)^2 + 2 - 11 - \frac{1}{\phi} = -9 - \frac{1}{\phi} + 4\left(\frac{1}{3.91211922690000} - \frac{(1)_{-1+\frac{1+2 i\pi}{\log(2)}}\, 3.91211922690000^{(2 i\pi)/(\log(a)\log_a(2))}}{3.91211922690000} - \frac{8\log(a)\log_a(2)\, 3.91211922690000^2}{7\,(1!!\times 2!!)}\right)^2$$

$$4\left(\frac{1}{3.91211922690000} - \frac{(3.91211922690000^2 \log(2))\, 2^3}{2!\,(2^3-1)} - \frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right) 3.91211922690000^{(2(\pi i))/\log(2)}}{3.91211922690000}\right)^2 + 2 - 11 - \frac{1}{\phi} = -9 - \frac{1}{\phi} + 4\left(\frac{1}{3.91211922690000} - \frac{3.91211922690000^{(2 i \pi)/\log_e(2)} \exp\left(-\log G\left(\frac{1+2 i \pi}{\log(2)}\right) + \log G\left(1 + \frac{1+2 i \pi}{\log(2)}\right)\right)}{3.91211922690000} - \frac{8 \log_e(2)\, 3.91211922690000^2}{7\,(1!! \times 2!!)}\right)^2$$

## Integral representations:

$$4\left(\frac{1}{3.91211922690000} - \frac{(3.91211922690000^2 \log(2))\, 2^3}{2!\,(2^3-1)} - \frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right) 3.91211922690000^{(2(\pi i))/\log(2)}}{3.91211922690000}\right)^2 + 2 - 11 - \frac{1}{\phi} = -9 - \frac{1}{\phi} + 4\left(0.255615931417410 - \frac{17.4910592519779 \log(2)}{\int_0^\infty e^{-t}\, t^2\, dt} - \frac{0.511231862834819\, e^{(2.72815845786415 i \pi)/\log(2)}\, \pi\, \mathcal{A}}{\oint_L e^t\, t^{-(1+2 i \pi)/\log(2)}\, dt}\right)^2$$

$$4\left(\frac{1}{3.91211922690000}-\frac{(3.91211922690000^2\log(2))\,2^3}{2!\,(2^3-1)}-\frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right)3.91211922690000^{(2(\pi i))/\log(2)}}{3.91211922690000}\right)^2+2-11-\frac{1}{\phi}=$$

$$\Bigg(1.04543207031\Bigg(1.0000000000000\times 3.91211922690000^{(4i\pi)/\left(\int_1^2\frac{1}{t}dt\right)}\,\phi\,\pi^2\,\mathcal{A}^2\left(\int_0^1\log^2\left(\frac{1}{t}\right)dt\right)^2+2\int_0^1\int_0^1\frac{\log^2\left(\frac{1}{t_2}\right)}{1+t_1}\,dt_2\,dt_1-$$
$$(19.921943263324-4.066882473590\,i)\,\mathcal{A}\oint_L e^t\,t^{-(1+2i\pi)/\log(2)}\,dt+$$
$$1170.5673183162\,\phi\left(\int_1^2\frac{1}{t}\,dt\right)^2\left(\oint_L e^t\,t^{-(1+2i\pi)/\log(2)}\,dt\right)^2-$$
$$0.9565423028425\left(\int_0^1\log^2\left(\frac{1}{t}\right)dt\right)^2\left(\oint_L e^t\,t^{-(1+2i\pi)/\log(2)}\,dt\right)^2-$$
$$8.358880725583\,\phi\left(\int_0^1\log^2\left(\frac{1}{t}\right)dt\right)^2\left(\oint_L e^t\,t^{-(1+2i\pi)/\log(2)}\,dt\right)^2\Bigg)\Bigg)\Big/$$
$$\left(\phi\left(\int_0^1\log^2\left(\frac{1}{t}\right)dt\right)^2\left(\oint_L e^t\,t^{-(1+2i\pi)/\log(2)}\,dt\right)^2\right)$$

$$4\left(\frac{1}{3.91211922690000}-\frac{(3.91211922690000^2\log(2))\,2^3}{2!\,(2^3-1)}-\frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right)3.91211922690000^{(2(\pi i))/\log(2)}}{3.91211922690000}\right)^2+$$
$$2-11-\frac{1}{\phi}=-8.738641982422441-\frac{1}{\phi}+$$
$$\frac{1.04543207031024\times 3.91211922690000^{(4i\pi)/\left(\int_1^2\frac{1}{t}dt\right)}\pi^2\mathcal{A}^2}{\left(\oint_L e^t\,t^{-(1+2i\pi)/\log(2)}\,dt\right)^2}-$$
$$\frac{1.04543207031024\times 3.91211922690000^{(2i\pi)/\left(\int_1^2\frac{1}{t}dt\right)}\pi\,\mathcal{A}}{\oint_L e^t\,t^{-(1+2i\pi)/\log(2)}\,dt}+$$
$$\frac{1223.74861502481\left(\int_1^2\frac{1}{t}\,dt\right)^2}{\left(\int_1^\infty e^{-t}\,t^2\,dt+\sum_{k=0}^\infty\frac{(-1)^k}{(3+k)k!}\right)^2}-\frac{35.767947217371\int_1^2\frac{1}{t}\,dt}{\int_1^\infty e^{-t}\,t^2\,dt+\sum_{k=0}^\infty\frac{(-1)^k}{(3+k)k!}}+$$
$$\frac{71.535894434743\times 3.91211922690000^{(2i\pi)/\left(\int_1^2\frac{1}{t}dt\right)}\pi\,\mathcal{A}\int_1^2\frac{1}{t}\,dt}{2-e^{-\infty}(\infty+2\infty+2)+\frac{(-1)^\infty\,{}_2F_2(1,\infty+4;\infty+2,\infty+5;-1)}{(\infty+4)(\infty+1)!}\oint_L e^t\,t^{-(1+2i\pi)/\log(2)}\,dt}$$

and again:

27*1/2*(((4[(((1/3.9121192269 – ln(2) (3.9121192269^2)/2! 2^3/(2^3-1) - 1/3.9121192269 gamma (((1+2*Pi*i)/(ln(2)))) 3.9121192269^(((2Pi*i)/(ln(2)))))))]^2+2-8-1/golden ratio)))-2

**Input interpretation:**

$$27\times\frac{1}{2}\left(4\left(\frac{1}{3.9121192269}-\log(2)\times\frac{3.9121192269^2}{2!}\times\frac{2^3}{2^3-1}-\frac{1}{3.9121192269}\Gamma\left(\frac{1+2\pi i}{\log(2)}\right)\times 3.9121192269^{(2\pi i)/\log(2)}\right)^2+2-8-\frac{1}{\phi}\right)-2$$

log(x) is the natural logarithm

n! is the factorial function

Γ(x) is the gamma function

i is the imaginary unit

ϕ is the golden ratio

**Result:**

1729.181524... −
0.0008464263668... i

(using the principal branch of the logarithm for complex exponentiation)

**Polar coordinates:**

$r = 1729.1815244$ (radius), $\theta = -0.0000280460193°$ (angle)

1729.1815244

This result is very near to the mass of candidate glueball **$f_0$(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

## Alternative representations:

$$\frac{27}{2}\left(4\left(\frac{1}{3.91211922690000}-\frac{3.91211922690000^2\log(2)\,2^3}{2!\,(2^3-1)}-\frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right)3.91211922690000^{(2\pi i)/\log(2)}}{3.91211922690000}\right)^2+2-8-\frac{1}{\phi}\right)-2=-2+\frac{27}{2}\left(-6-\frac{1}{\phi}+4\left(\frac{1}{3.91211922690000}-\frac{G\left(1+\frac{1+2i\pi}{\log(2)}\right)3.91211922690000^{(2i\pi)/\log(2)}}{3.91211922690000\,G\left(\frac{1+2i\pi}{\log(2)}\right)}-\frac{8\log(2)\,3.91211922690000^2}{7\,(1!!\times 2!!)}\right)^2\right)$$

$$\frac{27}{2}\left(4\left(\frac{1}{3.91211922690000}-\frac{3.91211922690000^2\log(2)\,2^3}{2!\,(2^3-1)}-\frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right)3.91211922690000^{(2\pi i)/\log(2)}}{3.91211922690000}\right)^2+2-8-\frac{1}{\phi}\right)-2=-2+\frac{27}{2}\left(-6-\frac{1}{\phi}+4\left(\frac{1}{3.91211922690000}-\frac{(1)_{-1+\frac{1+2i\pi}{\log(2)}}\,3.91211922690000^{(2i\pi)/(\log(a)\log_a(2))}}{3.91211922690000}-\frac{8\log(a)\log_a(2)\,3.91211922690000^2}{7\,(1!!\times 2!!)}\right)^2\right)$$

$$\frac{27}{2}\left(4\left(\frac{1}{3.91211922690000}-\frac{3.91211922690000^2\log(2)\,2^3}{2!\,(2^3-1)}-\frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right)3.91211922690000^{(2\pi i)/\log(2)}}{3.91211922690000}\right)^2+2-8-\frac{1}{\phi}\right)-2=-2+\frac{27}{2}\left(-6-\frac{1}{\phi}+4\left(\frac{1}{3.91211922690000}-\left(3.91211922690000^{(2i\pi)/\log_e(2)}\exp\left(-\log G\left(\frac{1+2i\pi}{\log(2)}\right)+\log G\left(1+\frac{1+2i\pi}{\log(2)}\right)\right)\right)\Big/3.91211922690000-\frac{8\log_e(2)\,3.91211922690000^2}{7\,(1!!\times 2!!)}\right)^2\right)$$

## Integral representations:

$$\frac{27}{2}\left(4\left(\frac{1}{3.91211922690000} - \frac{3.91211922690000^2 \log(2)\, 2^3}{2!\,(2^3-1)} - \frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right) 3.91211922690000^{(2\pi i)/\log(2)}}{3.91211922690000}\right)^2 + 2 - 8 - \frac{1}{\phi}\right) - 2 =$$

$$-2 + \frac{27}{2}\left(-6 - \frac{1}{\phi} + 4\left(0.255615931417410 - \frac{17.4910592519779 \log(2)}{\int_0^\infty e^{-t}\, t^2\, dt} - \frac{0.511231862834819 \times 3.91211922690000^{(2 i\pi)/\log(2)}\, \pi\, \mathcal{A}}{\oint_L e^t\, t^{-(1+2 i\pi)/\log(2)}\, dt}\right)^2\right)$$

$$\frac{27}{2}\left(4\left(\frac{1}{3.91211922690000} - \frac{3.91211922690000^2 \log(2)\, 2^3}{2!\,(2^3-1)} - \frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right) 3.91211922690000^{(2\pi i)/\log(2)}}{3.91211922690000}\right)^2 + 2 - 8 - \frac{1}{\phi}\right) - 2 =$$

$$\Bigg(14.113332949\Bigg(1.0000000000000 \times 3.91211922690000^{(4 i\pi)/\left(\int_1^2 \frac{1}{t} dt\right)}\, \phi\, \pi^2\, \mathcal{A}^2 \left(\int_0^1 \log^2\left(\frac{1}{t}\right) dt\right)^2 + 2 \int_0^1 \int_0^1 \frac{\log^2\left(\frac{1}{t_2}\right)}{1+t_1}\, dt_2\, dt_1 - (19.921943263324 - 4.066882473590\, i)\, \mathcal{A} \oint_L e^t\, t^{-(1+2 i\pi)/\log(2)}\, dt + 1170.5673183162\, \phi \left(\int_1^2 \frac{1}{t}\, dt\right)^2 \left(\oint_L e^t\, t^{-(1+2 i\pi)/\log(2)}\, dt\right)^2 - 0.9565423028425 \left(\int_0^1 \log^2\left(\frac{1}{t}\right) dt\right)^2 \left(\oint_L e^t\, t^{-(1+2 i\pi)/\log(2)}\, dt\right)^2 - 5.630963787847\, \phi \left(\int_0^1 \log^2\left(\frac{1}{t}\right) dt\right)^2 \left(\oint_L e^t\, t^{-(1+2 i\pi)/\log(2)}\, dt\right)^2\Bigg)\Bigg) \Bigg/ \left(\phi \left(\int_0^1 \log^2\left(\frac{1}{t}\right) dt\right)^2 \left(\oint_L e^t\, t^{-(1+2 i\pi)/\log(2)}\, dt\right)^2\right)$$

$$\frac{27}{2}\left(4\left(\frac{1}{3.91211922690000}-\frac{3.91211922690000^2\log(2)\,2^3}{2!\,(2^3-1)}-\frac{\Gamma\left(\frac{1+2\pi i}{\log(2)}\right)3.91211922690000^{(2\pi i)/\log(2)}}{3.91211922690000}\right)^2+2-8-\frac{1}{\phi}\right)-2=-79.47166676270295-\frac{27}{2\phi}+$$

$$\frac{14.1133329491882\times 3.91211922690000^{(4i\pi)/\left(\int_1^2\frac{1}{t}dt\right)}\pi^2\mathcal{A}^2}{\left(\oint_L e^t\,t^{-(1+2i\pi)/\log(2)}\,dt\right)^2}-\frac{14.1133329491882\times 3.91211922690000^{(2i\pi)/\left(\int_1^2\frac{1}{t}dt\right)}\pi\mathcal{A}}{\oint_L e^t\,t^{-(1+2i\pi)/\log(2)}\,dt}+$$

$$\frac{16520.6063028349\left(\int_1^2\frac{1}{t}\,dt\right)^2}{\left(\int_1^\infty e^{-t}\,t^2\,dt+\sum_{k=0}^\infty\frac{(-1)^k}{(3+k)k!}\right)^2}-\frac{482.86728743451\int_1^2\frac{1}{t}\,dt}{\int_1^\infty e^{-t}\,t^2\,dt+\sum_{k=0}^\infty\frac{(-1)^k}{(3+k)k!}}+$$

$$\frac{965.73457486903\times 3.91211922690000^{(2i\pi)/\left(\int_1^2\frac{1}{t}dt\right)}\pi\mathcal{A}\int_1^2\frac{1}{t}\,dt}{\left(2-e^{-\infty}(\infty+2\infty+2)+\frac{(-1)^\infty\,{}_2F_2(1,\infty+4;\infty+2,\infty+5;-1)}{(\infty+4)(\infty+1)!}\right)\oint_L e^t\,t^{-(1+2i\pi)/\log(2)}\,dt}$$

Performing the 15th root, we obtain:

((((27*1/2*(((4[(((1/3.9121192269 – ln(2) (3.9121192269^2)/2! 2^3/(2^3-1) - 1/3.9121192269 gamma (((1+2*Pi*i)/(ln(2)))) 3.9121192269^(((2Pi*i)/(ln(2))))))))]^2+2-8-1/golden ratio)))-2))))^1/15

**Input interpretation:**

$$\left(27\times\frac{1}{2}\left(4\left(\frac{1}{3.9121192269}-\log(2)\times\frac{3.9121192269^2}{2!}\times\frac{2^3}{2^3-1}-\frac{1}{3.9121192269}\Gamma\left(\frac{1+2\pi i}{\log(2)}\right)\times 3.9121192269^{(2\pi i)/\log(2)}\right)^2+2-8-\frac{1}{\phi}\right)-2\right)\hat{}\,(1/15)$$

log(x) is the natural logarithm

n! is the factorial function

$\Gamma(x)$ is the gamma function

$i$ is the imaginary unit

$\phi$ is the golden ratio

**Result:**

1.64382673359... −
5.36430394076... × $10^{-8}$ $i$

(using the principal branch of the logarithm for complex exponentiation)

**Polar coordinates:**

$r = 1.64382673359$ (radius), $\theta = -1.870 \times 10^{-6}°$ (angle)

$1.64382673359 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$

**Observations**

Figs.

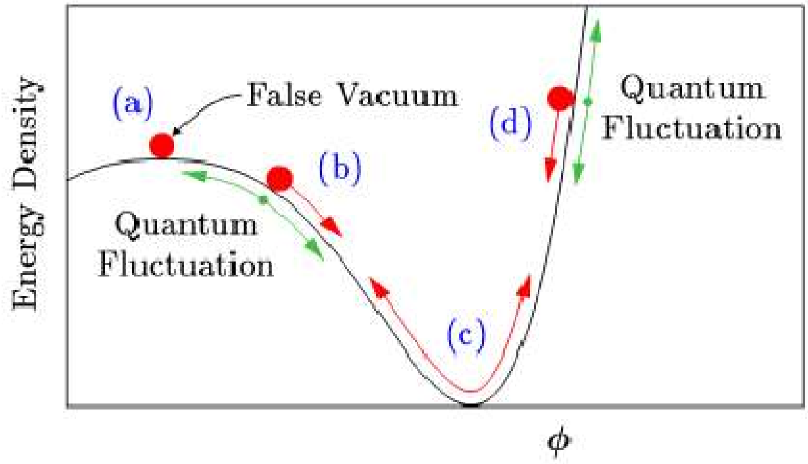
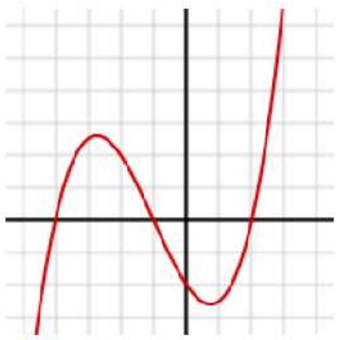



FIG. 1: In simple inflationary models, the universe at early times is dominated by the potential energy density of a scalar field, $\phi$. Red arrows show the classical motion of $\phi$. When $\phi$ is near region (a), the energy density will remain nearly constant, $\rho \cong \rho_f$, even as the universe expands. Moreover, cosmic expansion acts like a frictional drag, slowing the motion of $\phi$: Even near regions (b) and (d), $\phi$ behaves more like a marble moving in a bowl of molasses, slowly creeping down the side of its potential, rather than like a marble sliding down the inside of a polished bowl. During this period of "slow roll," $\rho$ remains nearly constant. Only after $\phi$ has slid most of the way down its potential will it begin to oscillate around its minimum, as in region (c), ending inflation.



Graph of a cubic function with 3 real roots (where the curve crosses the horizontal axis at $y = 0$). The case shown has two critical points. Here the function is $f(x) = (x^3 + 3x^2 - 6x - 8)/4$.

The ratio between $M_0$ and q

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3})\, M_s}{2}.$$

i.e. the gravitating mass $M_0$ and the Wheelerian mass q of the wormhole, is equal to:

$$\frac{\sqrt{3\,(2.17049 \times 10^{37})^2 - 0.001^2}}{\frac{1}{2}\,((3\sqrt{3})\,(4.2 \times 10^6 \times 1.9891 \times 10^{30}))}$$

1.732050787905194420703947625671018160083566548802082460520...

1.7320507879

$1.7320507879 \approx \sqrt{3}$ that is the ratio between the gravitating mass $M_0$ and the Wheelerian mass q of the wormhole

We note that:

$$\left(-\frac{1}{2} + \frac{i}{2}\sqrt{3}\right) - \left(-\frac{1}{2} - \frac{i}{2}\sqrt{3}\right)$$

$i$ is the imaginary unit

$i\sqrt{3}$

1.732050807568877293527446341505872366942805253810380628055... $i$

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

1.73205

This result is very near to the ratio between $M_0$ and q, that is equal to 1.7320507879 $\approx \sqrt{3}$

With regard $\sqrt{3}$, we note that is a fundamental value of the formula structure that we need to calculate a Cubic Equation

We have that the previous result

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) = i\sqrt{3} =$$

$= 1.7320508075688772935274463415058723669428052538103806280558...\, i$

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

can be related with:

$$u^2(-u)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)+v^2(-v)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)=q$$

Considering:

$$(-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q$$

$= i\sqrt{3} = 1.7320508075688772935274463415058723669428052538103806280558...\, i$

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

Thence:

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) \Rightarrow$$

$$\Rightarrow (-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q=1.73205\approx\sqrt{3}$$

**We observe how the graph above, concerning the cubic function, is very similar to the graph that represent the scalar field (in red). It is possible to hypothesize that cubic functions and cubic equations, with their roots, are connected to the scalar field.**

*From:*
*https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA*

*Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.*

*Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.*

*Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.*
*In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.*

*From Wikipedia*

*In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field $\phi$ and a Dirac field $\psi$. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.*

*Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV*

*Note that:*

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= \qquad\qquad 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

*Thence:*

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

*And*

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

*That are connected with 64, 128, 256, 512, 1024 and 4096 = $64^2$*

*(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)*

*All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.*

*In mathematics, the Fibonacci numbers, commonly denoted $F_n$, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the*

*golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.*
*Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences*

*The beginning of the sequence is thus:*

*0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...*

*The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.*

*The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.*

*The sequence of Lucas numbers is:*

*2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……*

*All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.*

*A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:*

*2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).*

*In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.*[1] *That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies*[3] *- golden spirals are one special case of these logarithmic spirals*

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **$f_0$(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

**We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.**

## References

**Manuscript Book 3 of Srinivasa Ramanujan**