On some Ramanujan equations: mathematical connections with various topics concerning Prime Numbers Theory, ϕ, $\zeta(2)$ and several parameters of Particle Physics. III

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Abstract

In this paper we have described and analyzed some Ramanujan equations. We have obtained several mathematical connections between some topics concerning Prime Numbers Theory, ϕ, $\zeta(2)$ *and various parameters of Particle Physics.*

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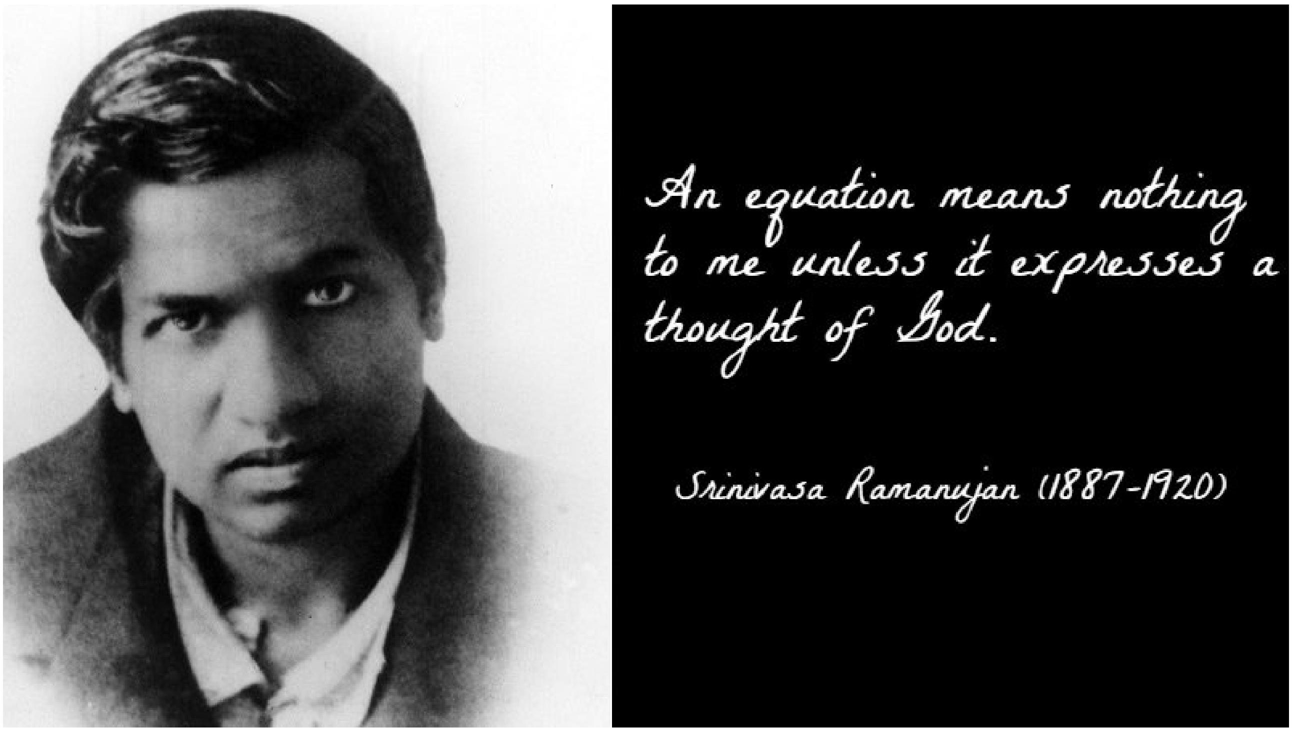
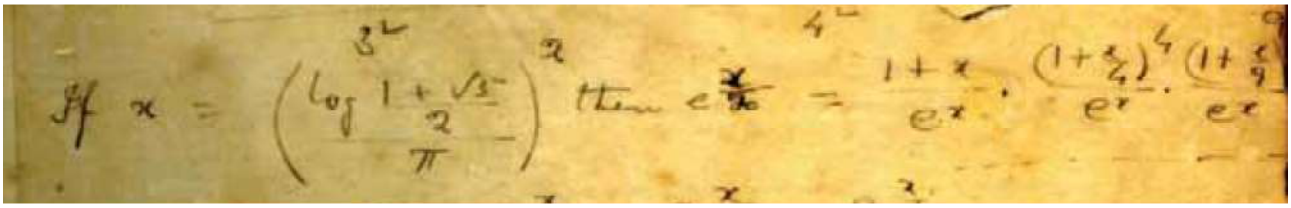


https://mobygeek.com/features/indian-mathematician-srinivasa-ramanujan-quotes-11012

We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation

We have already analyzed, in the part II of this paper, several Ramanujan formulas of the Manuscript Book 3.

(page 6):



$$\left(\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\times\frac{1}{\pi}\right)^2$$

Exact result:

$$\frac{\log^2\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)}{\pi^2}$$

Decimal approximation:

0.023462421710806909463112025130508650169194928065080959403...

0.0234624217108…..

Series representations:

$$\left(\frac{\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)}{\pi}\right)^2=\frac{\left(\sum_{k=1}^{\infty}\frac{\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^k}{k}\right)^2}{\pi^2}$$

$$\left(\frac{\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)}{\pi}\right)^2=\frac{\left(2 i \pi\left\lfloor\frac{\arg\left(1+\sqrt{5}-2x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(1+\sqrt{5}-2x\right)^k x^{-k}}{k}\right)^2}{\pi^2}$$

for $x < 0$

$$\left(\frac{\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)}{\pi}\right)^2=\frac{\left(2 i \pi\left\lfloor\frac{\arg\left(\frac{1}{2}\left(1+\sqrt{5}\right)-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(1+\sqrt{5}-2x\right)^k x^{-k}}{k}\right)^2}{\pi^2}$$

for $x < 0$

Integral representations:

$$\left(\frac{\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)}{\pi}\right)^2 = \frac{\left(\int_1^{\frac{1}{2}\left(1+\sqrt{5}\right)} \frac{1}{t}\,dt\right)^2}{\pi^2}$$

$$\left(\frac{\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)}{\pi}\right)^2 = -\frac{\left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-1+\frac{1}{2}\left(1+\sqrt{5}\right)\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2}{4\pi^4} \quad \text{for } -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

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$$\exp\left(\frac{0.0234624217108}{2}\right)$$

Result:

1.0118002913779…

1.0118002913779…

And the following final expression:

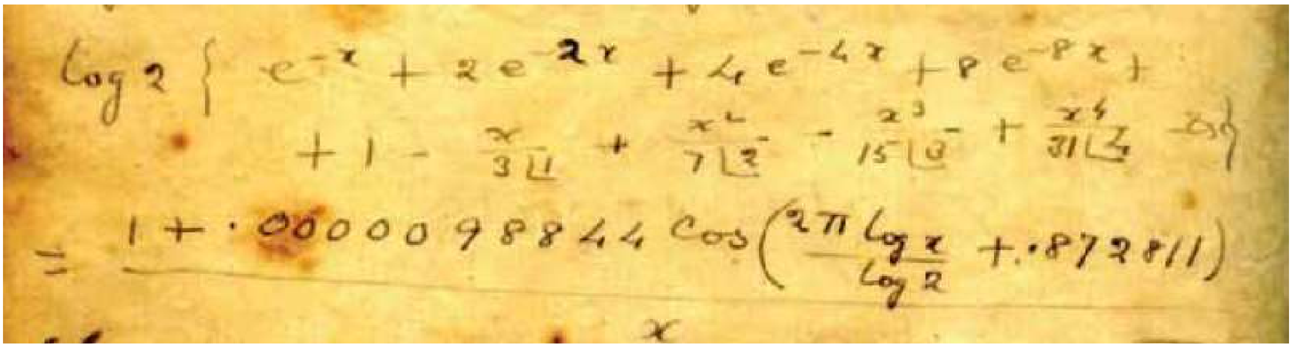
$$\frac{1+0.0234624217108}{\exp(0.0234624217108)} \times \frac{\left(1+\frac{1}{4}\right)^4}{\exp(0.0234624217108)} \times \frac{\left(1+\frac{2}{9}\right)^9}{\exp(0.0234624217108)}$$

Result:

14.17407524806…

14.17407524806…

From (page 7)



we obtain:

Input interpretation:

$$\frac{1}{x}\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(x)}{\log(2)}+0.872811\right)\right)=$$
$$\log(2)\left(e^{-x}+2e^{-2x}+4e^{-4x}+8e^{-8x}+\left(1-\frac{x}{(3\times1)!}+\frac{x^2}{(7\times2)!}-\frac{x^3}{(15\times3)!}+\frac{x^4}{(31\times4)!}\right)\right)$$

log(x) is the natural logarithm

n! is the factorial function

Result:

$$\frac{9.8844\times10^{-6}\cos\left(\frac{2\pi\log(x)}{\log(2)}+0.872811\right)+1}{x}=\left(x^4/\right.$$
$$1\,506\,141\,741\,511\,140\,879\,795\,014\,161\,993\,280\,686\,076\,322\,918\,971\,939\,407\,100\,785\,852\,066\,825\,250\,652\,908\,790\,935\,063\,463\,115\,967\,385\,069\,171\,243\,567\,440\,461\,925\,041\,295\,354\,731\,044\,782\,551\,067\,660\,468\,376\,444\,194\,611\,004\,520\,057\,054\,167\,040\,000\,000\,000\,000\,000\,000\,000\,000\,000 - x^3/$$
$$119\,622\,220\,865\,480\,194\,561\,963\,161\,495\,657\,715\,064\,383\,733\,760\,000\,000\,000+\frac{x^2}{87\,178\,291\,200}-\frac{x}{6}+8e^{-8x}+4e^{-4x}+2e^{-2x}+e^{-x}+1\left.\right)$$
$$\log(2)$$

Numerical solutions:

$x \approx 0.359463780075203...$

$x \approx 3.91211922690599...$

3.91211922690599…

Thence:

$$\frac{1}{3.9121192269}\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.9121192269)}{\log(2)}+0.872811\right)\right)$$

log(x) is the natural logarithm

Result:

0.25561790957241130435589321453504493118555391632711112658 8…

0.25561790957…

Series representations:

$$\frac{1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)}{3.91211922690000}=0.255615931417410+2.52661\times10^{-6}\sum_{k=0}^{\infty}\frac{(-1)^k\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)^{2k}}{(2k)!}$$

$$\frac{1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)}{3.91211922690000}=0.255615931417410-2.52661\times10^{-6}\sum_{k=0}^{\infty}\frac{(-1)^k\left(0.872811+\pi\left(-\frac{1}{2}+\frac{2\log(3.91211922690000)}{\log(2)}\right)\right)^{1+2k}}{(1+2k)!}$$

$$\frac{1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)}{3.91211922690000}=0.255615931417410+2.52661\times10^{-6}\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}-z_0\right)^k}{k!}$$

Integral representations:

$$\frac{1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)}{3.91211922690000}=0.255615931417410-2.52661\times10^{-6}\int_{\frac{\pi}{2}}^{0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}}\sin(t)\,dt$$

$$\frac{1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)}{3.91211922690000}=0.255618+\int_0^1\frac{1}{\log(2)}\left(-2.20525\times10^{-6}\log(2)-5.05322\times10^{-6}\,\pi\log(3.91211922690000)\right)\sin\left(t\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)dt$$

$$\frac{1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)}{3.91211922690000}=0.255615931417410+\frac{1.26331\times10^{-6}\sqrt{\pi}}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{s-\frac{(0.436406\log(2)+\pi\log(3.91211922690000))^2}{s\log^2(2)}}}{\sqrt{s}}\,ds \quad \text{for } \gamma>0$$

$$\frac{1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)}{3.91211922690000}=0.255615931417410+\frac{1.26331\times10^{-6}\sqrt{\pi}}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{4^s\,\Gamma(s)\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)^{-2s}}{\Gamma\left(\frac{1}{2}-s\right)}\,ds \quad \text{for } 0<\gamma<\frac{1}{2}$$

and:

$$\log(2)\left(\frac{1}{e^{3.912119}}+2e^{-2\times3.912119}+4e^{-4\times3.912119}+8e^{-8\times3.912119}+\left(1-\frac{3.912119}{(3\times1)!}+\frac{3.912119^2}{(7\times2)!}-\frac{3.912119^3}{(15\times3)!}+\frac{3.912119^4}{(31\times4)!}\right)\right)$$

$\log(x)$ is the natural logarithm

$n!$ is the factorial function

Result:

0.255617939182511498491080896013322516923560304390086680182...

0.25561793918…

Series representations:

$$\log(2)\left(e^{-3.91212}+2\,e^{-2\times 3.91212}+4\,e^{-4\times 3.91212}+8\,e^{-8\times 3.91212}+\left(1-\frac{3.91212}{(3\times 1)!}+\frac{3.91212^2}{(7\times 2)!}-\frac{3.91212^3}{(15\times 3)!}+\frac{3.91212^4}{(31\times 4)!}\right)\right)=$$
$$\log(2)+\frac{8\log(2)}{e^{31.297}}+\frac{4\log(2)}{e^{15.6485}}+\frac{2\log(2)}{e^{7.82424}}+\frac{\log(2)}{e^{3.91212}}-\frac{3.91212\log(2)}{\sum_{k=0}^{\infty}\frac{(3-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}+\frac{15.3047\log(2)}{\sum_{k=0}^{\infty}\frac{(14-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}-\frac{59.8737\log(2)}{\sum_{k=0}^{\infty}\frac{(45-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}+\frac{234.233\log(2)}{\sum_{k=0}^{\infty}\frac{(124-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}$$

for $((n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z}) \text{ and } n_0 \to 3 \text{ and } n_0 \to 14 \text{ and } n_0 \to 45 \text{ and } n_0 \to 124)$

$$\log(2)\left(e^{-3.91212}+2\,e^{-2\times 3.91212}+4\,e^{-4\times 3.91212}+8\,e^{-8\times 3.91212}+\left(1-\frac{3.91212}{(3\times 1)!}+\frac{3.91212^2}{(7\times 2)!}-\frac{3.91212^3}{(15\times 3)!}+\frac{3.91212^4}{(31\times 4)!}\right)\right)=$$
$$\left(2\,i\,\pi\left\lfloor\frac{\arg(2-x)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}}{k}\right)\left(1+\frac{8}{e^{31.297}}+\frac{4}{e^{15.6485}}+\frac{2}{e^{7.82424}}+\frac{1}{e^{3.91212}}-\frac{3.91212}{\sum_{k=0}^{\infty}\frac{(3-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}+\frac{15.3047}{\sum_{k=0}^{\infty}\frac{(14-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}-\frac{59.8737}{\sum_{k=0}^{\infty}\frac{(45-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}+\frac{234.233}{\sum_{k=0}^{\infty}\frac{(124-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}\right)$$

for $(x < 0 \text{ and } (n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z}) \text{ and } n_0 \to 3 \text{ and } n_0 \to 14 \text{ and } n_0 \to 45 \text{ and } n_0 \to 124)$

$$\log(2)\left(e^{-3.91212}+2\,e^{-2\times 3.91212}+4\,e^{-4\times 3.91212}+8\,e^{-8\times 3.91212}+\left(1-\frac{3.91212}{(3\times 1)!}+\frac{3.91212^2}{(7\times 2)!}-\frac{3.91212^3}{(15\times 3)!}+\frac{3.91212^4}{(31\times 4)!}\right)\right)=$$
$$\left(\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\,\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-z_0)^k\,z_0^{-k}}{k}\right)\left(1+\frac{8}{e^{31.297}}+\frac{4}{e^{15.6485}}+\frac{2}{e^{7.82424}}+\frac{1}{e^{3.91212}}-\frac{3.91212}{\sum_{k=0}^{\infty}\frac{(3-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}+\frac{15.3047}{\sum_{k=0}^{\infty}\frac{(14-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}-\frac{59.8737}{\sum_{k=0}^{\infty}\frac{(45-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}+\frac{234.233}{\sum_{k=0}^{\infty}\frac{(124-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}\right)$$

for $((n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z}) \text{ and } n_0 \to 3 \text{ and } n_0 \to 14 \text{ and } n_0 \to 45 \text{ and } n_0 \to 124)$

$$\log(2)\left(e^{-3.91212}+2\,e^{-2\times 3.91212}+4\,e^{-4\times 3.91212}+8\,e^{-8\times 3.91212}+\left(1-\frac{3.91212}{(3\times 1)!}+\frac{3.91212^2}{(7\times 2)!}-\frac{3.91212^3}{(15\times 3)!}+\frac{3.91212^4}{(31\times 4)!}\right)\right)=$$

$$\left(2\,i\,\pi\left\lfloor-\frac{-\pi+\arg\left(\frac{2}{z_0}\right)+\arg(z_0)}{2\,\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-z_0)^k\,z_0^{-k}}{k}\right)$$

$$\left(1+\frac{8}{e^{31.297}}+\frac{4}{e^{15.6485}}+\frac{2}{e^{7.82424}}+\frac{1}{e^{3.91212}}-\frac{3.91212}{\sum_{k=0}^{\infty}\frac{(3-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}+\right.$$

$$\left.\frac{15.3047}{\sum_{k=0}^{\infty}\frac{(14-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}-\frac{59.8737}{\sum_{k=0}^{\infty}\frac{(45-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}+\frac{234.233}{\sum_{k=0}^{\infty}\frac{(124-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}\right)$$

for $((n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z})$ and $n_0 \to 3$ and $n_0 \to 14$ and $n_0 \to 45$ and $n_0 \to 124)$

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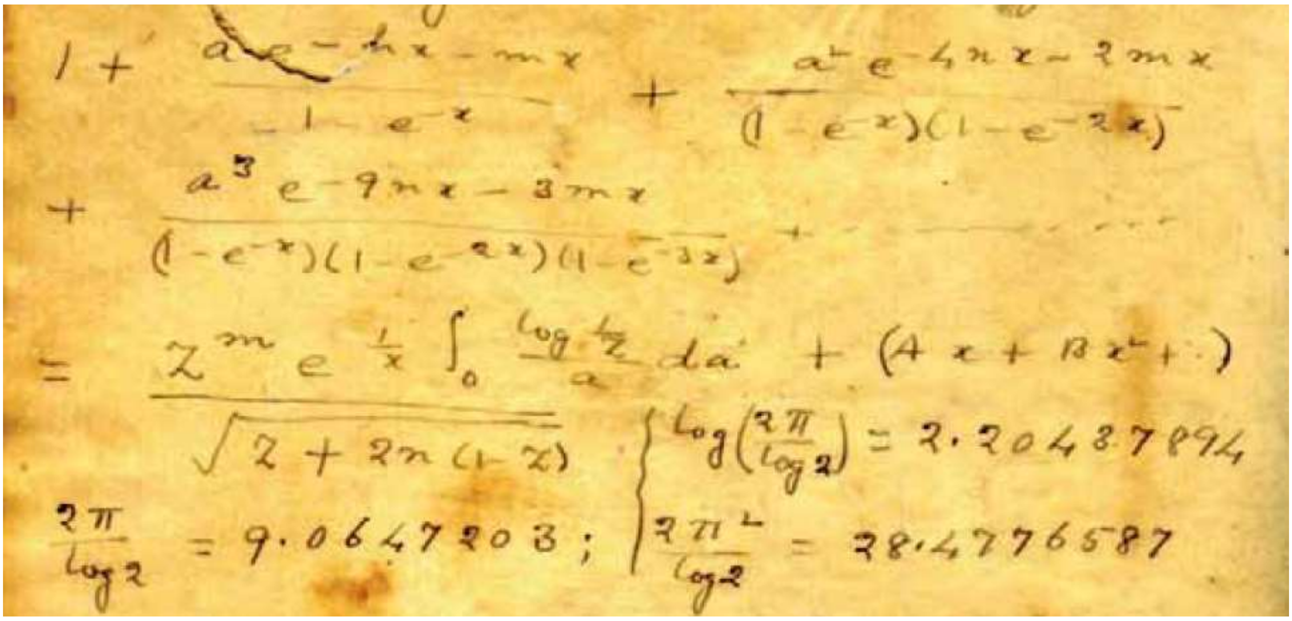


$$1+\frac{a\,e^{-nx-mx}}{1-e^{-x}}+\frac{a^2 e^{-4nx-2mx}}{(1-e^{-x})(1-e^{-2x})}+\frac{a^3 e^{-9nx-3mx}}{(1-e^{-x})(1-e^{-2x})(1-e^{-3x})}+\cdots$$

$$=\frac{z^m e^{\frac{1}{x}\int_0 \frac{\log^2 z}{a}\,da}}{\sqrt{z+2n(1-z)}}+(Ax+Bx^2+\cdots)$$

$$\log\left(\frac{2\pi}{\log 2}\right)=2.20437894$$

$$\frac{2\pi}{\log 2}=9.0647203;\quad \frac{2\pi^2}{\log 2}=28.4776587$$

we have:

$$\log\left(\frac{2\,\pi}{\log(2)}\right)\times\frac{2\,\pi^2}{\log(2)}\times\frac{2\,\pi}{\log(2)}$$

$\log(x)$ is the natural logarithm

$$\frac{4\,\pi^3\,\log\left(\frac{2\,\pi}{\log(2)}\right)}{\log^2(2)}$$

569.0456620556244658364918972442354124629248429568863086987...

569.0456620….. ≈ 569

where 569 is an Eisenstein prime numbers

Eisenstein prime numbers (from Wikipedia)

2, 5, 11, 17, 23, 29, 41, 47, 53, 59, 71, 83, 89, 101, 107, 113, 131, 137, 149, 167, 173, 179, 191, 197, 227, 233, 239, 251, 257, 263, 269, 281, 293, 311, 317, 347, 353, 359, 383, 389, 401, 419, 431, 443, 449, 461, 467, 479, 491, 503, 509, 521, 557, 563, **569**, 587…

In mathematics, an **Eisenstein prime** is an Eisenstein integer

$$z = a + b\omega, \quad \text{where} \quad \omega = e^{\frac{2\pi i}{3}},$$

that is irreducible (or equivalently prime) in the ring-theoretic sense: its only Eisenstein divisors are the units $\{\pm 1, \pm\omega, \pm\omega^2\}$, $a + b\omega$ itself and its associates.

The associates (unit multiples) and the complex conjugate of any Eisenstein prime are also prime.

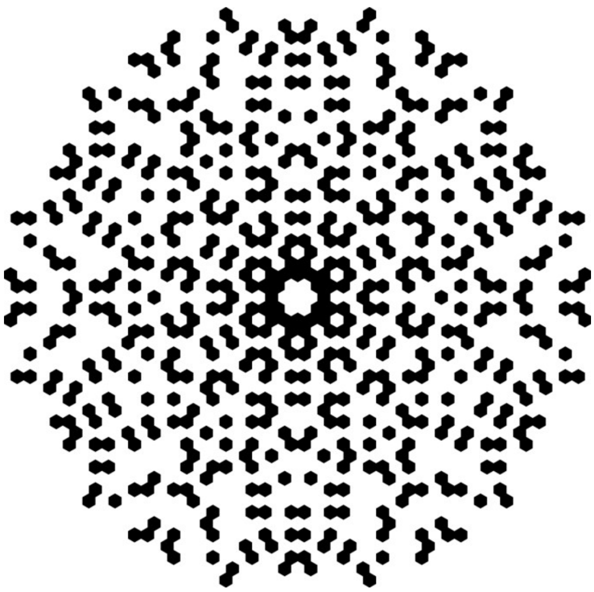
Furthermore:

137 + 131 + 113 + 101 + 47 + 29 + 11 = **569**

Or:

2 + 5 + 23 + 29 + 59 + 71 + 83 + 89 + 101 + 107 = **569**

Thence 569 is also the sum of several Eisenstein numbers



Eisenstein primes in a larger range

We have obtained, multiplying the two expressions

$$\frac{1}{3.9121192269}\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.9121192269)}{\log(2)}+0.872811\right)\right) \quad \text{(a)}$$

$$\log\left(\frac{2\pi}{\log(2)}\right)\times\frac{2\pi^2}{\log(2)}\times\frac{2\pi}{\log(2)} \quad \text{(b)}$$

$$\left(\frac{1}{3.9121192269}\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.9121192269)}{\log(2)}+0.872811\right)\right)\right)\left(\log\left(\frac{2\pi}{\log(2)}\right)\times\frac{2\pi^2}{\log(2)}\times\frac{2\pi}{\log(2)}\right)$$

$\log(x)$ is the natural logarithm

Result:

145.45826259...

145.4582….≈ **145** that is an Ulam number (see list below)

A002858	**Ulam numbers: a(1) = 1; a(2) = 2; for n>2, a(n) = least number > a(n-1) which is a unique sum of two distinct earlier terms. (Formerly M0557 N0201)**

1, 2, 3, 4, 6, 8, 11, 13, 16, 18, 26, 28, 36, 38, 47, 48, 53, 57, 62, 69, 72, 77, 82, 87, 97, 99, 102, 106, 114, 126, 131, 138, **145**, 148, 155, 175, 177, 180, 182, 189, 197, 206, 209, 219, 221, 236, 238, 241, 243, 253, 258, 260, 273, 282, 309, 316, 319, 324, 339…

Furthermore: 145 = 29 * 5 where 29 is a prime Lucas number and 5 is a Fibonacci prime number

Series representations:

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)}{(\log(2)\log(2))\,3.91211922690000} = \frac{1.02246372566964\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)} + \frac{0.0000101064\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)^{2k}}{(2k)!}}{\log^2(2)}$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)}{(\log(2)\log(2))\,3.91211922690000} = \frac{1.02246372566964\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)} - \frac{0.0000101064\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(0.872811+\pi\left(-\frac{1}{2}+\frac{2\log(3.91211922690000)}{\log(2)}\right)\right)^{1+2k}}{(1+2k)!}}{\log^2(2)}$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)}{(\log(2)\log(2))\,3.91211922690000} = \frac{1.02246372566964\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)} + \frac{0.0000101064\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}-z_0\right)^k}{k!}}{\log^2(2)}$$

Integral representations:

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)}{(\log(2)\log(2))\,3.91211922690000} =$$

$$-\frac{1}{\log^3(2)}\,0.0000202129\,\pi^3\left(-50\,585.3\log(2)+0.436406\log(2)+\pi\log(3.91211922690000)\int_0^1\sin\left(t\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)dt\right)\log\left(\frac{2\pi}{\log(2)}\right)$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)}{(\log(2)\log(2))\,3.91211922690000} =$$

$$\frac{1.02246372566964\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)}-\frac{0.0000101064\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)}\int_{\frac{\pi}{2}}^{0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}}\sin(t)\,dt$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)}{(\log(2)\log(2))\,3.91211922690000} =$$

$$-\frac{1}{\left(\int_1^2\frac{1}{t}\,dt\right)^2\log(2)}\,0.0000202129\,\pi^3\left(-50\,585.3\log(2)\int_1^{\frac{2\pi}{\log(2)}}\frac{1}{t}\,dt+2\log(2)\int_0^1\int_0^1\frac{\sin\left(\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)t_2\right)}{\log(2)+(2\pi-\log(2))\,t_1}\,dt_2\,dt_1\right)$$

While, from the division of the two expressions, we have obtained:

$$\frac{\log\left(\frac{2\pi}{\log(2)}\right)\times\frac{2\pi^2}{\log(2)}\times\frac{2\pi}{\log(2)}}{\frac{1}{3.9121192269}\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.9121192269)}{\log(2)}+0.872811\right)\right)}$$

$\log(x)$ is the natural logarithm

Result:

2226.1572478…

2226.1572478… ≈ 2226

We note that, for the following formula

a(n) = n*(5*n+1)

we obtain:

n(5*n+1) = 2226

Input:

$n(5n+1) = 2226$

$5n^2 + n = 2226$

Solutions:

$n = -\frac{106}{5}$

$n = 21$

21

Thence:

21(5*21+1)

where 5 and 21 are Fibonacci's number (21 is also equal to 3*7)

Input:

$21(5 \times 21 + 1)$

Result:

2226

2226

3*7(5*3*7+1) = 2226

With regard the two results 569.045662055 and 14.17407524806 , we obtain from the ratio and performing the 24th root:

(569.045662055 / 14.17407524806)*1/24

Input interpretation:

$$\frac{569.045662055}{14.17407524806} \times \frac{1}{24}$$

Result:

1.672788912433884801299828702959298950883894404190360741697...

1.67278891243….

Or, performing the 25th root:

(569.045662055 / 14.17407524806)*1/25

Input interpretation:

$$\frac{569.045662055}{14.17407524806} \times \frac{1}{25}$$

Result:

1.605877355936529409247835554840926992848538628022746312029...

1.605877355936…..

With the mean: 1.63933313418520710527……

We note also that

14.17407524806 * 5 = 70.8703762403 ≈ 71

where 5 and 71 are Eisenstein prime numbers

From the two results 2226.1572478 and 569.0456620 , we obtain also the following expression:

1+1/(2226.1572478 / 569.0456620)^1/3

Input interpretation:

$$1+\frac{1}{\sqrt[3]{\frac{2226.1572478}{569.0456620}}}$$

Result:

1.6346443620...

1.6346443620…

Also from the two results 569.0456620 and 145.45826259 , we obtain the following expression that is equal to:

1+1/(569.0456620 / 145.45826259)^1/3

Input interpretation:

$$1+\frac{1}{\sqrt[3]{\frac{569.0456620}{145.45826259}}}$$

Result:

1.6346443621...

1.6346443621….. practically the same previous result

We note that 2226 = 2237 – 11 or 2243 – 17 that are Eisenstein numbers, and that 145 = 137 + 8 = 137 + 5 + 3 , where 137 is a Eisenstein number, while 8 is a Fibonacci number (8 = 5 + 3 that are also prime numbers)

Now, we have

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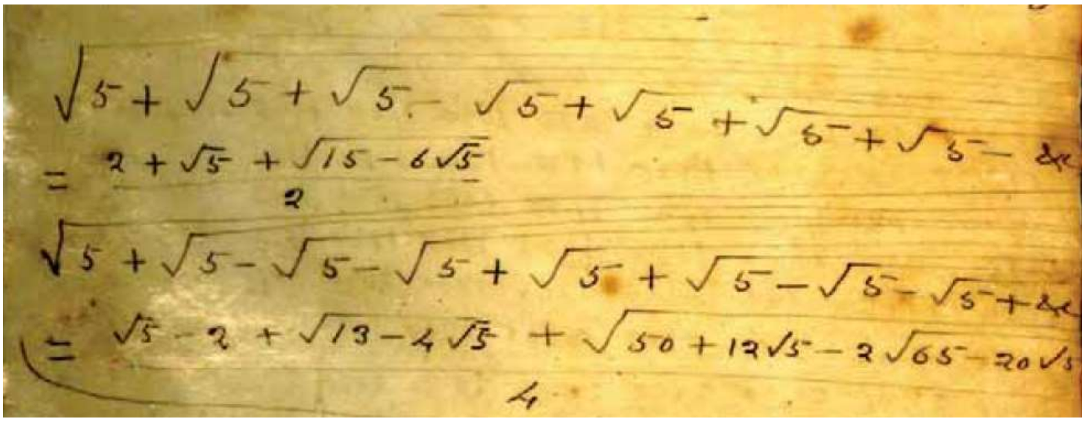


$\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5-\&c}}}}}}}$

$= \frac{2+\sqrt{5}+\sqrt{15-6\sqrt{5}}}{2}$

$\sqrt{5+\sqrt{5-\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5-\sqrt{5+\&c}}}}}}}}$

$= \frac{\sqrt{5}-2+\sqrt{13-4\sqrt{5}}+\sqrt{50+12\sqrt{5}-2\sqrt{65-20\sqrt{5}}}}{4}$

1/2((2+sqrt5+sqrt((15-6*sqrt5))))

Input:

$$\frac{1}{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)$$

Decimal approximation:

2.747238274932304333057465186134202826758163878776167987783...

2.7472382749…

Alternate forms:

$$\frac{1}{2}\left(\sqrt{15-6\sqrt{5}}+\sqrt{5}\right)+1$$

$$\frac{1}{2}\left(2+\sqrt{5}+\sqrt{3\left(5-2\sqrt{5}\right)}\right)$$

$$1+\frac{\sqrt{5}}{2}+\frac{1}{2}\sqrt{15-6\sqrt{5}}$$

Minimal polynomial:

$$x^4-4x^3-4x^2+31x-29$$

From which, performing the square root, we obtain:

sqrt(((1/2((2+sqrt5+sqrt((15-6*sqrt5)))))))

Input:

$$\sqrt{\frac{1}{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)}$$

Decimal approximation:

1.657479494573704924740483047406775190347623094018322205669...

1.6574794945...

Alternate forms:

$$\frac{1}{2}\sqrt{\left(\sqrt{15-6\sqrt{5}}+\sqrt{5}+2\right)2}$$

$$\sqrt{\frac{1}{2}\left(2+\sqrt{5}+\sqrt{3\left(5-2\sqrt{5}\right)}\right)}$$

Minimal polynomial:

$$x^8-4x^6-4x^4+31x^2-29$$

All 2nd roots of 1/2 (2 + sqrt(5) + sqrt(15 - 6 sqrt(5))):

$$\sqrt{\frac{1}{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)}\,e^0\approx 1.6575 \text{ (real, principal root)}$$

$$\sqrt{\frac{1}{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)}\,e^{i\pi}\approx -1.6575 \text{ (real root)}$$

Sqrt(5+sqrt(5+sqrt(5-sqrt(5+sqrt(5+sqrt(5+sqrt(5)))))))

Input:

$$\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}}$$

Decimal approximation:

2.747182586579282029689841249322497669365113151566810823790...

2.74718258657...

All 2nd roots of 5 + sqrt(5 + sqrt(5 - sqrt(5 + sqrt(5 + sqrt(5 + sqrt(5)))))):

$$\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}}\ e^{0} \approx 2.7472 \text{ (real, principal root)}$$

$$\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}}\ e^{i\pi} \approx -2.7472 \text{ (real root)}$$

From which:

sqrt((((Sqrt(5+sqrt(5+sqrt(5-sqrt(5+sqrt(5+sqrt(5+sqrt(5))))))))))

Input:

$$\sqrt{\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}}}$$

Result:

$$\sqrt[4]{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}}$$

Decimal approximation:

1.657462695380889116947746667056438163771988635326320818892...

1.65746269538… as above

All 2nd roots of sqrt(5 + sqrt(5 + sqrt(5 - sqrt(5 + sqrt(5 + sqrt(5 + sqrt(5))))))):

$$\sqrt[4]{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}}\ e^{0} \approx 1.65746 \text{ (real, principal root)}$$

$$\sqrt[4]{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}} \quad e^{i\pi} \approx -1.6575 \text{ (real root)}$$

We note that 5 is a prime number, an Eisenstein number and a Fibonacci number

We have also:

(((Sqrt(5+sqrt(5+sqrt(5-sqrt(5+sqrt(5+sqrt(5+sqrt(5))))))))))^5-17

where 17 is an Eisenstein number

Input:

$$\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}}^{\,5} - 17$$

Exact result:

$$\left(5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}\right)^{5/2} - 17$$

Decimal approximation:

139.4723571376543916181063119820527472777236517692678346916...

139.47235713765…

Alternate form:

$$-17+30\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}}+\sqrt{\left(5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}\right)\left(5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}\right)}+10\sqrt{\left(5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}\right)\left(5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}\right)}$$

(((Sqrt(5+sqrt(5+sqrt(5-sqrt(5+sqrt(5+sqrt(5+sqrt(5))))))))))^5-29-2

where 2 and 29 are Eisenstein numbers and Lucas numbers

Input:

$$\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}}^{\,5}-29-2$$

Exact result:

$$\left(5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}\right)^{5/2}-31$$

Decimal approximation:

125.47235713765439161810631198205274727772365176926783469 16...

125.4723571376…

Alternate form:

$$-31+30\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}}+\sqrt{\left(5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}\right)\left(5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}\right)}+10\sqrt{\left(5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}\right)\left(5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}\right)}$$

and also:

1/2*(29-2)((((((Sqrt(5+sqrt(5+sqrt(5-sqrt(5+sqrt(5+sqrt(5+sqrt(5))))))))))))^5-17-11)))-5-1/3

where 2, 5, 11, 17 and 29 are Eisenstein numbers

Input:

$$\frac{1}{2}(29-2)\left(\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}}^{\,5}-17-11\right)-5-\frac{1}{3}$$

Exact result:

$$\frac{27}{2}\left(\left(5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}\right)^{5/2}-28\right)-\frac{16}{3}$$

Decimal approximation:

1729.0434880250009535111018784243787549159359655517824350003...

1729.043488025…

Alternate forms:

$$\frac{1}{6}\left(81\left(5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}\right)^{5/2}-2300\right)$$

$$-\frac{1150}{3}+405\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}}+$$

$$\frac{27}{2}\sqrt{\left(5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}\right)\left(5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}\right)}+$$

$$135\sqrt{\left(5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}\right)\left(5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}\right)}$$

$$\frac{1}{6}\left(-2300+2430\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}}+\right.$$

$$81\sqrt{\left(5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}\right)\left(5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}\right)}+$$

$$\left.810\sqrt{\left(5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}\right)\left(5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}\right)}\right)$$

From which, performing the 15th root:

(((1/2*(29-2)((((((Sqrt(5+sqrt(5+sqrt(5-sqrt(5+sqrt(5+sqrt(5+sqrt(5))))))))))^5-17-11)))-5-1/3)))^1/15

Input:

$$\sqrt[15]{\frac{1}{2}(29-2)\left(\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}}^{\,5}-17-11\right)-5-\frac{1}{3}}$$

Exact result:

$$\sqrt[15]{\frac{27}{2}\left(\left(5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}\right)^{5/2}-28\right)-\frac{16}{3}}$$

Decimal approximation:

1.643817985079504951575779511111292878289677953724195845638...

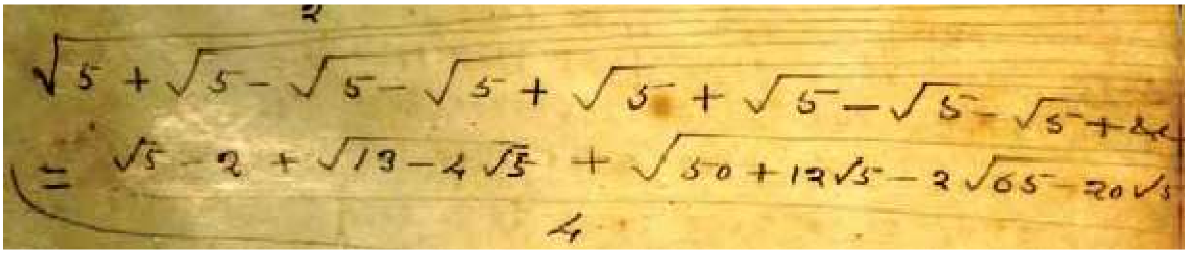
1.643817985...

Alternate forms:

$$\frac{1}{\sqrt[15]{\frac{6}{81\left(5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}\right)^{5/2}-2300}}}$$

$$1/\left(\left(6/\left(-2300+2430\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}}+81\sqrt{\left(5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}\right)\left(5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}\right)}+810\sqrt{\left(5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}\right)\left(5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}}}}\right)}\right)\right)\text{^}(1/15)\right)$$

Now:



$\sqrt{5+\sqrt{5-\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5-\sqrt{5+2\ldots}}}}}}}}$

$= \dfrac{\sqrt{5}-2+\sqrt{13-4\sqrt{5}}+\sqrt{50+12\sqrt{5}-2\sqrt{65-20\sqrt{5}}\ldots}}{4}$

Sqrt(5+sqrt(5-sqrt(5-sqrt(5+sqrt(5+sqrt(5-sqrt(5-sqrt(5))))))))

Input:

$$\sqrt{5+\sqrt{5-\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5-\sqrt{5}}}}}}}}$$

Decimal approximation:

2.621381929319906788437780567086780360374434770830522433285...

2.621381929319…..

Alternate form:

$$\sqrt{5+\sqrt{5-\sqrt{5-\sqrt{5+\sqrt{5+\frac{\sqrt{5\sqrt{2}+\sqrt{5-2\sqrt{5}}-\sqrt{5+2\sqrt{5}}}}{\sqrt[4]{2}}}}}}}$$

All 2nd roots of 5 + sqrt(5 - sqrt(5 - sqrt(5 + sqrt(5 + sqrt(5 - sqrt(5 - sqrt(5))))))):

$$\sqrt{5+\sqrt{5-\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5-\sqrt{5}}}}}}}}\; e^{0} \approx 2.6214 \text{ (real, principal root)}$$

$$\sqrt{5+\sqrt{5-\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5-\sqrt{5}}}}}}}}\; e^{i\pi} \approx -2.6214 \text{ (real root)}$$

And:

1/4(((sqrt5-2+sqrt(13-4sqrt5)+sqrt((50+12sqrt5-2sqrt((65-20sqrt5)))))))

Input:

$$\frac{1}{4}\left(\sqrt{5}-2+\sqrt{13-4\sqrt{5}}+\sqrt{50+12\sqrt{5}-2\sqrt{65-20\sqrt{5}}}\right)$$

Decimal approximation:

2.621408383075861505698495280612243127797970614721167679664...

2.6214083830758….

Alternate forms:

$$\frac{1}{4}\left(\sqrt{13-4\sqrt{5}}+\sqrt{5}+\sqrt{-2\sqrt{5\left(13-4\sqrt{5}\right)}+12\sqrt{5}+50}-2\right)$$

$$\frac{1}{4}\left(-2+\sqrt{5}+\sqrt{13-4\sqrt{5}}+\sqrt{2\left(25+6\sqrt{5}-\sqrt{5\left(13-4\sqrt{5}\right)}\right)}\right)$$

root of $x^8+4x^7-10x^6-54x^5+9x^4+226x^3+125x^2-301x-269$ near $x=2.62141$

Minimal polynomial:

$x^8+4x^7-10x^6-54x^5+9x^4+226x^3+125x^2-301x-269$

From which:

sqrt((((1/4(((sqrt5-2+sqrt(13-4sqrt5)+sqrt((50+12sqrt5-2sqrt((65-20sqrt5))))))))))

Input:

$$\sqrt{\frac{1}{4}\left(\sqrt{5}-2+\sqrt{13-4\sqrt{5}}+\sqrt{50+12\sqrt{5}-2\sqrt{65-20\sqrt{5}}}\right)}$$

Result:

$$\frac{1}{2}\sqrt{-2+\sqrt{5}+\sqrt{13-4\sqrt{5}}+\sqrt{50+12\sqrt{5}-2\sqrt{65-20\sqrt{5}}}}$$

Decimal approximation:

1.619076398159105247383508829602269202039776657295266862292...

1.6190763981591…

Alternate forms:

$$\frac{1}{2}\sqrt{\sqrt{13-4\sqrt{5}}+\sqrt{5}+\sqrt{-2\sqrt{5\left(13-4\sqrt{5}\right)}+12\sqrt{5}+50}-2}$$

$$\frac{1}{2}\sqrt{-2+\sqrt{5}+\sqrt{13-4\sqrt{5}}+\sqrt{2\left(25+6\sqrt{5}-\sqrt{5\left(13-4\sqrt{5}\right)}\right)}}$$

$$\sqrt{\text{root of } x^8 + 4x^7 - 10x^6 - 54x^5 + 9x^4 + 226x^3 + 125x^2 - 301x - 269 \text{ near } x = 2.62141}$$

Minimal polynomial:

$$x^{16} + 4x^{14} - 10x^{12} - 54x^{10} + 9x^8 + 226x^6 + 125x^4 - 301x^2 - 269$$

All 2nd roots of 1/4 (-2 + sqrt(5) + sqrt(13 - 4 sqrt(5)) + sqrt(50 + 12 sqrt(5) - 2 sqrt(65 - 20 sqrt(5)))):

$$\frac{1}{2}\sqrt{-2+\sqrt{5}+\sqrt{13-4\sqrt{5}}+\sqrt{50+12\sqrt{5}-2\sqrt{65-20\sqrt{5}}}}\; e^0 \approx 1.6191$$

(real, principal root)

$$\frac{1}{2}\sqrt{-2+\sqrt{5}+\sqrt{13-4\sqrt{5}}+\sqrt{50+12\sqrt{5}-2\sqrt{65-20\sqrt{5}}}}\; e^{i\pi} \approx -1.6191$$

(real root)

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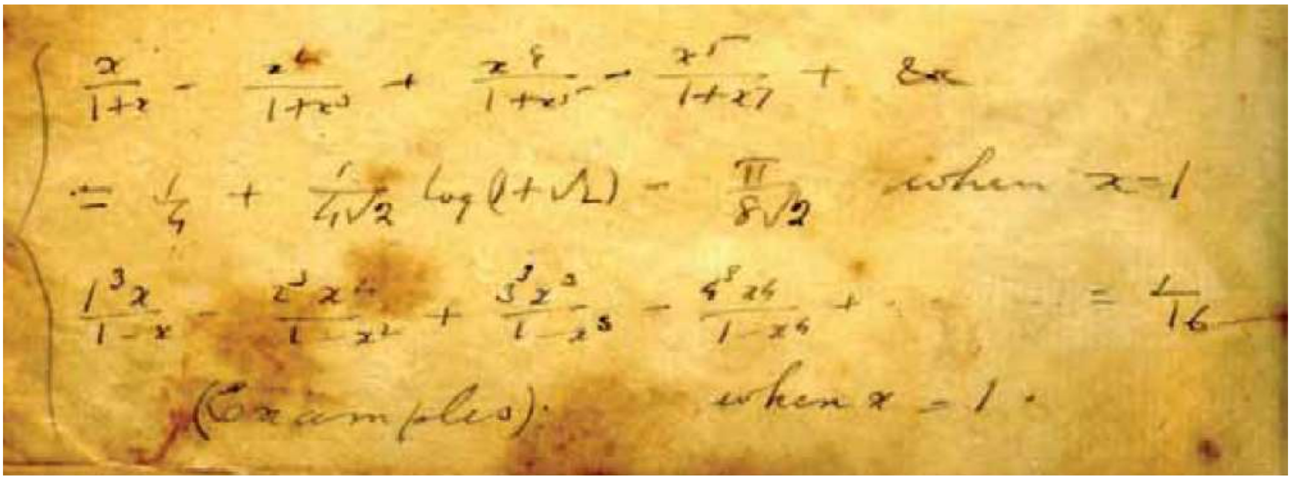

1/4+1/(4sqrt2)*ln(1+sqrt2) – Pi/(8sqrt2)

Input:

$$\frac{1}{4} + \frac{1}{4\sqrt{2}}\log\left(1+\sqrt{2}\right) - \frac{\pi}{8\sqrt{2}}$$

log(x) is the natural logarithm

Exact result:

$$\frac{1}{4} - \frac{\pi}{8\sqrt{2}} + \frac{\log(1+\sqrt{2})}{4\sqrt{2}}$$

Decimal approximation:

0.1281261264001597379100124581838486444992599725006611742 81...

0.1281261264…

Alternate forms:

$$\frac{1}{16}\left(4 - \sqrt{2}\,\pi + 2\sqrt{2}\,\sinh^{-1}(1)\right)$$

$$-\frac{\pi - 2\left(\sqrt{2} + \log(1+\sqrt{2})\right)}{8\sqrt{2}}$$

$$\frac{1}{16}\left(4 - \sqrt{2}\,\pi + 2\sqrt{2}\,\log\left(1+\sqrt{2}\right)\right)$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternative representations:

$$\frac{1}{4} + \frac{\log(1+\sqrt{2})}{4\sqrt{2}} - \frac{\pi}{8\sqrt{2}} = \frac{1}{4} + \frac{\log_e(1+\sqrt{2})}{4\sqrt{2}} - \frac{\pi}{8\sqrt{2}}$$

$$\frac{1}{4} + \frac{\log(1+\sqrt{2})}{4\sqrt{2}} - \frac{\pi}{8\sqrt{2}} = \frac{1}{4} + \frac{\log(a)\log_a(1+\sqrt{2})}{4\sqrt{2}} - \frac{\pi}{8\sqrt{2}}$$

$$\frac{1}{4} + \frac{\log(1+\sqrt{2})}{4\sqrt{2}} - \frac{\pi}{8\sqrt{2}} = \frac{1}{4} - \frac{\mathrm{Li}_1(-\sqrt{2})}{4\sqrt{2}} - \frac{\pi}{8\sqrt{2}}$$

Series representations:

$$\frac{1}{4} + \frac{\log(1+\sqrt{2})}{4\sqrt{2}} - \frac{\pi}{8\sqrt{2}} = \frac{1}{4} - \frac{\pi}{8\sqrt{2}} + \frac{\log(2)}{8\sqrt{2}} - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k 2^{-k/2}}{k}}{4\sqrt{2}}$$

$$\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}=$$
$$\frac{1}{16}\left(4-\sqrt{2}\,\pi+2\sqrt{2}\left(\log(z_0)+\left\lfloor\frac{\arg(1+\sqrt{2}-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(1+\sqrt{2}-z_0)^k\,z_0^{-k}}{k}\right)\right)$$

$$\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}=$$
$$\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{i\pi\left\lfloor\frac{\arg(1+\sqrt{2}-x)}{2\pi}\right\rfloor}{2\sqrt{2}}+\frac{\log(x)}{4\sqrt{2}}-\frac{\sum_{k=1}^{\infty}\frac{(-1)^k\,(1+\sqrt{2}-x)^k\,x^{-k}}{k}}{4\sqrt{2}}\quad\text{for } x<0$$

Integral representations:

$$\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}=\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{1}{4\sqrt{2}}\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt$$

$$\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}=\frac{1}{4}-\frac{\pi}{8\sqrt{2}}-\frac{i}{8\sqrt{2}\,\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^{-s/2}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds$$
for $-1<\gamma<0$

and

2(((1/4+1/(4sqrt2)*ln(1+sqrt2) – Pi/(8sqrt2))))

Input:

$$2\left(\frac{1}{4}+\frac{1}{4\sqrt{2}}\log\left(1+\sqrt{2}\right)-\frac{\pi}{8\sqrt{2}}\right)$$

log(x) is the natural logarithm

Exact result:

$$2\left(\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}\right)$$

Decimal approximation:

0.256252252800319475820024916367697288998519945001322348563...

0.2562522528…..

Alternate forms:

$$\frac{1}{8}\left(4-\sqrt{2}\,\pi+2\sqrt{2}\,\sinh^{-1}(1)\right)$$

$$-\frac{\pi-2\left(\sqrt{2}+\log\left(1+\sqrt{2}\right)\right)}{4\sqrt{2}}$$

$$\frac{1}{8}\left(4-\sqrt{2}\,\pi+2\sqrt{2}\,\log\left(1+\sqrt{2}\right)\right)$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternative representations:

$$2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)=2\left(\frac{1}{4}+\frac{\log_e(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)$$

$$2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)=2\left(\frac{1}{4}+\frac{\log(a)\log_a(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)$$

$$2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)=2\left(\frac{1}{4}-\frac{\mathrm{Li}_1(-\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)$$

Series representations:

$$2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)=\frac{1}{2}-\frac{\pi}{4\sqrt{2}}+\frac{\log(2)}{4\sqrt{2}}-\frac{\sum_{k=1}^{\infty}\frac{(-1)^k\,2^{-k/2}}{k}}{2\sqrt{2}}$$

$$2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)=$$
$$\frac{1}{2}-\frac{\pi}{4\sqrt{2}}+\frac{i\,\pi\left\lfloor\frac{\arg\left(1+\sqrt{2}-x\right)}{2\pi}\right\rfloor}{\sqrt{2}}+\frac{\log(x)}{2\sqrt{2}}-\frac{\sum_{k=1}^{\infty}\frac{(-1)^k\left(1+\sqrt{2}-x\right)^k x^{-k}}{k}}{2\sqrt{2}}\quad\text{for } x<0$$

$$2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)=$$
$$\frac{1}{8}\left(4-\sqrt{2}\,\pi+2\sqrt{2}\left(\log(z_0)+\left\lfloor\frac{\arg(1+\sqrt{2}-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(1+\sqrt{2}-z_0\right)^k z_0^{-k}}{k}\right)\right)$$

Integral representations:

$$2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)=\frac{1}{2}-\frac{\pi}{4\sqrt{2}}+\frac{1}{2\sqrt{2}}\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt$$

$$2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)=\frac{1}{2}-\frac{\pi}{4\sqrt{2}}-\frac{i}{4\sqrt{2}\,\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^{-s/2}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds$$

for $-1<\gamma<0$

Where 0.2562522528….. is a value very near to the result of a previous expression

$$\frac{1}{3.9121192269}\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.9121192269)}{\log(2)}+0.872811\right)\right)$$

Result:

0.255617909572411304355893214535044931185553916327111126588...

0.25561790957…

Multiplying by the Eisenstein number 569 the above expression, we obtain:

569(((2(((1/4+1/(4sqrt2)*ln(1+sqrt2) – Pi/(8sqrt2)))))))

Input:

$$569\left(2\left(\frac{1}{4}+\frac{1}{4\sqrt{2}}\log\left(1+\sqrt{2}\right)-\frac{\pi}{8\sqrt{2}}\right)\right)$$

log(x) is the natural logarithm

Exact result:

$$1138\left(\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}\right)$$

Decimal approximation:

145.8075318433817817415941774132197574401578487057524163327...

145.80753184…. ≈ 146 = 89 + 55 + 2 that are Fibonacci numbers (89 and 2 are also Eisenstein numbers

Alternate forms:

$$-\frac{569}{8}\left(-4+\sqrt{2}\,\pi-2\sqrt{2}\,\sinh^{-1}(1)\right)$$

$$-\frac{569\left(\pi - 2\left(\sqrt{2} + \log\left(1 + \sqrt{2}\right)\right)\right)}{4\sqrt{2}}$$

$$-\frac{569}{8}\left(-4 + \sqrt{2}\,\pi - 2\sqrt{2}\,\log\left(1 + \sqrt{2}\right)\right)$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternative representations:

$$569 \times 2\left(\frac{1}{4} + \frac{\log\left(1 + \sqrt{2}\right)}{4\sqrt{2}} - \frac{\pi}{8\sqrt{2}}\right) = 1138\left(\frac{1}{4} + \frac{\log_e\left(1 + \sqrt{2}\right)}{4\sqrt{2}} - \frac{\pi}{8\sqrt{2}}\right)$$

$$569 \times 2\left(\frac{1}{4} + \frac{\log\left(1 + \sqrt{2}\right)}{4\sqrt{2}} - \frac{\pi}{8\sqrt{2}}\right) = 1138\left(\frac{1}{4} + \frac{\log(a)\log_a\left(1 + \sqrt{2}\right)}{4\sqrt{2}} - \frac{\pi}{8\sqrt{2}}\right)$$

$$569 \times 2\left(\frac{1}{4} + \frac{\log\left(1 + \sqrt{2}\right)}{4\sqrt{2}} - \frac{\pi}{8\sqrt{2}}\right) = 1138\left(\frac{1}{4} - \frac{\mathrm{Li}_1\left(-\sqrt{2}\right)}{4\sqrt{2}} - \frac{\pi}{8\sqrt{2}}\right)$$

Series representations:

$$569 \times 2\left(\frac{1}{4} + \frac{\log\left(1 + \sqrt{2}\right)}{4\sqrt{2}} - \frac{\pi}{8\sqrt{2}}\right) = \frac{569}{2} - \frac{569\,\pi}{4\sqrt{2}} + \frac{569\log(2)}{4\sqrt{2}} - \frac{569\sum_{k=1}^{\infty}\frac{(-1)^k\,2^{-k/2}}{k}}{2\sqrt{2}}$$

$$569 \times 2\left(\frac{1}{4} + \frac{\log\left(1 + \sqrt{2}\right)}{4\sqrt{2}} - \frac{\pi}{8\sqrt{2}}\right) = \frac{569}{8}\left(4 - \sqrt{2}\,\pi + 2\sqrt{2}\left(\log(z_0) + \left\lfloor\frac{\arg\left(1 + \sqrt{2} - z_0\right)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty}\frac{(-1)^k\left(1 + \sqrt{2} - z_0\right)^k z_0^{-k}}{k}\right)\right)$$

$$569 \times 2\left(\frac{1}{4} + \frac{\log\left(1 + \sqrt{2}\right)}{4\sqrt{2}} - \frac{\pi}{8\sqrt{2}}\right) = \frac{569}{2} - \frac{569\,\pi}{4\sqrt{2}} + \frac{569\,i\,\pi\left\lfloor\frac{\arg\left(1+\sqrt{2}-x\right)}{2\pi}\right\rfloor}{\sqrt{2}} + \frac{569\log(x)}{2\sqrt{2}} - \frac{569\sum_{k=1}^{\infty}\frac{(-1)^k\left(1+\sqrt{2}-x\right)^k x^{-k}}{k}}{2\sqrt{2}} \quad \text{for } x < 0$$

Integral representations:

$$569 \times 2\left(\frac{1}{4} + \frac{\log\left(1 + \sqrt{2}\right)}{4\sqrt{2}} - \frac{\pi}{8\sqrt{2}}\right) = \frac{569}{2} - \frac{569\,\pi}{4\sqrt{2}} + \frac{569}{2\sqrt{2}}\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt$$

$$569 \times 2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)=$$
$$\frac{569}{2}-\frac{569\pi}{4\sqrt{2}}-\frac{569 i}{4\sqrt{2}\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^{-s/2}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

While, from the ratio between 569 and the expression, we obtain:

569*1/(((2(((1/4+1/(4sqrt2)*ln(1+sqrt2) – Pi/(8sqrt2)))))))

Input:

$$569 \times \frac{1}{2\left(\frac{1}{4}+\frac{1}{4\sqrt{2}}\log(1+\sqrt{2})-\frac{\pi}{8\sqrt{2}}\right)}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{569}{2\left(\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}\right)}$$

Decimal approximation:

2220.468283817915430533877252738516260486086056101841398030...

2220.4682838….≈ 2220 = 2207 + 11 + 2 that are Lucas numbers or 2220 = 2237 – 17 that are Eisenstein numbers

Alternate forms:

$$\frac{4552}{4-\sqrt{2}\,\pi+2\sqrt{2}\,\sinh^{-1}(1)}$$

$$-\frac{2276\sqrt{2}}{\pi-2(\sqrt{2}+\log(1+\sqrt{2}))}$$

$$-\frac{4552}{-4+\sqrt{2}\,\pi-2\sqrt{2}\,\log(1+\sqrt{2})}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternative representations:

$$\frac{569}{2\left(\frac{1}{4}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}=\frac{569}{2\left(\frac{1}{4}+\frac{\log_e\left(1+\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}$$

$$\frac{569}{2\left(\frac{1}{4}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}=\frac{569}{2\left(\frac{1}{4}+\frac{\log(a)\log_a\left(1+\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}$$

$$\frac{569}{2\left(\frac{1}{4}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}=\frac{569}{2\left(\frac{1}{4}-\frac{\mathrm{Li}_1\left(-\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}$$

Series representations:

$$\frac{569}{2\left(\frac{1}{4}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}=-\frac{4552}{-4+\sqrt{2}\,\pi-\sqrt{2}\,\log(2)+2\sqrt{2}\sum_{k=1}^{\infty}\frac{(-1)^k\,2^{-k/2}}{k}}$$

$$\frac{569}{2\left(\frac{1}{4}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}=$$
$$\frac{4552}{4-\sqrt{2}\,\pi+2\sqrt{2}\left(2\,i\,\pi\left\lfloor\frac{\arg\left(1+\sqrt{2}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(1+\sqrt{2}-x\right)^k x^{-k}}{k}\right)}\quad\text{for } x<0$$

$$\frac{569}{2\left(\frac{1}{4}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}=$$
$$\frac{569}{2\left(\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{2\,i\,\pi\left\lfloor\frac{\arg\left(1+\sqrt{2}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(1+\sqrt{2}-x\right)^k x^{-k}}{k}}{4\sqrt{2}}\right)}\quad\text{for } x<0$$

Integral representation:

$$\frac{569}{2\left(\frac{1}{4}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}=-\frac{4552\pi}{\pi\left(-4+\sqrt{2}\,\pi\right)+i\sqrt{2}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^{-s/2}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}$$

for $-1<\gamma<0$

Performing the 16[th] root of the last expression, we obtain:

(((569*1/(((2(((1/4+1/(4sqrt2)*ln(1+sqrt2) – Pi/(8sqrt2)))))))))^1/16

Input:

$$\sqrt[16]{569\times\frac{1}{2\left(\frac{1}{4}+\frac{1}{4\sqrt{2}}\log\left(1+\sqrt{2}\right)-\frac{\pi}{8\sqrt{2}}\right)}}$$

log(x) is the natural logarithm

Exact result:

$$\sqrt[16]{\frac{569}{2\left(\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}\right)}}$$

Decimal approximation:

1.618649382419098765844707055865436577665848131975553460909...

1.6186493824…

Alternate forms:

$$2^{3/16}\sqrt[16]{\frac{569}{4-\sqrt{2}\,\pi+2\sqrt{2}\,\sinh^{-1}(1)}}$$

$$2^{5/32}\sqrt[16]{\frac{569}{2\left(\sqrt{2}+\log\left(1+\sqrt{2}\right)\right)-\pi}}$$

$$2^{3/16}\sqrt[16]{\frac{569}{4-\sqrt{2}\,\pi+2\sqrt{2}\,\log\left(1+\sqrt{2}\right)}}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

All 16th roots of 569/(2 (1/4 - π/(8 sqrt(2)) + log(1 + sqrt(2))/(4 sqrt(2)))):

$$e^{0}\sqrt[16]{\frac{569}{2\left(\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}\right)}}\approx 1.6186 \text{ (real, principal root)}$$

$$e^{(i\pi)/8}\sqrt[16]{\frac{569}{2\left(\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}\right)}}\approx 1.4954+0.6194\,i$$

$$e^{(i\pi)/4}\sqrt[16]{\frac{569}{2\left(\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}\right)}}\approx 1.1446+1.1446\,i$$

$$e^{(3i\pi)/8}\sqrt[16]{\frac{569}{2\left(\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}\right)}}\approx 0.6194+1.4954\,i$$

$$e^{(i\pi)/2}\sqrt[16]{\frac{569}{2\left(\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}\right)}}\approx 1.6186\,i$$

Alternative representations:

$$\sqrt[16]{\frac{569}{2\left(\frac{1}{4}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}}=\sqrt[16]{\frac{569}{2\left(\frac{1}{4}+\frac{\log_e\left(1+\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}}$$

$$\sqrt[16]{\frac{569}{2\left(\frac{1}{4}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}}=\sqrt[16]{\frac{569}{2\left(\frac{1}{4}+\frac{\log(a)\log_a\left(1+\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}}$$

$$\sqrt[16]{\frac{569}{2\left(\frac{1}{4}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}}=\sqrt[16]{\frac{569}{2\left(\frac{1}{4}-\frac{\text{Li}_1\left(-\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}}$$

Series representations:

$$\sqrt[16]{\frac{569}{2\left(\frac{1}{4}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}} = 2^{3/16}\sqrt[16]{569}\sqrt[16]{\frac{1}{4-\sqrt{2}\,\pi+\sqrt{2}\,\log(2)-2\sqrt{2}\sum_{k=1}^{\infty}\frac{2^{-k/2}\,e^{i k \pi}}{k}}}$$

$$\sqrt[16]{\frac{569}{2\left(\frac{1}{4}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}} = \sqrt[16]{\frac{569}{2}}\sqrt[16]{\frac{1}{\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\frac{\log(2)}{2}-\sum_{k=1}^{\infty}\frac{(-1)^k\,2^{-k/2}}{k}}{4\sqrt{2}}}}$$

$$\sqrt[16]{\frac{569}{2\left(\frac{1}{4}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}} = \sqrt[16]{\frac{569}{2}}\sqrt[16]{\frac{1}{\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{2 i \pi\left\lfloor\frac{\arg\left(1+\sqrt{2}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(1+\sqrt{2}-x\right)^k x^{-k}}{k}}{4\sqrt{2}}}} \quad \text{for } x<0$$

Integral representations:

$$\sqrt[16]{\frac{569}{2\left(\frac{1}{4}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}} = 2^{3/16}\sqrt[16]{569}\sqrt[16]{\frac{1}{4-\sqrt{2}\,\pi+2\sqrt{2}\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt}}$$

$$\sqrt[16]{\frac{569}{2\left(\frac{1}{4}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}} = 2^{3/16}\sqrt[16]{569\,\pi}\sqrt[16]{\frac{1}{\pi\left(4-\sqrt{2}\,\pi\right)-i\sqrt{2}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^{-s/2}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}} \quad \text{for } -1<\gamma<0$$

$$\sqrt[16]{\frac{569}{2\left(\frac{1}{4}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}} = \sqrt[16]{\frac{569}{2}}\sqrt[16]{\frac{1}{\frac{1}{4}-\frac{\pi}{8\sqrt{2}}-\frac{i}{8\sqrt{2}\,\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^{-s/2}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}} \quad \text{for } -1<\gamma<0$$

Furthermore, subtracting 491, that is also an Eisenstein number, and 2/5 to the expression

$$\frac{569}{2\left(\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}\right)}$$

we obtain:

(((569*1/(((2(((1/4+1/(4sqrt2)*ln(1+sqrt2) – Pi/(8sqrt2)))))))))-491-2/5

Input:

$$569\times\frac{1}{2\left(\frac{1}{4}+\frac{1}{4\sqrt{2}}\log\left(1+\sqrt{2}\right)-\frac{\pi}{8\sqrt{2}}\right)}-491-\frac{2}{5}$$

log(x) is the natural logarithm

Exact result:

$$\frac{569}{2\left(\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\right)}{4\sqrt{2}}\right)}-\frac{2457}{5}$$

Decimal approximation:

1729.068283817915430533877252738516260486086056101841398030...

1729.068283817…

Alternate forms:

$$\frac{4552}{4-\sqrt{2}\,\pi+2\sqrt{2}\,\sinh^{-1}(1)}-\frac{2457}{5}$$

$$-\frac{2457}{5}-\frac{2276\sqrt{2}}{\pi-2\left(\sqrt{2}+\log\left(1+\sqrt{2}\right)\right)}$$

$$\frac{4552}{4-\sqrt{2}\,\pi+2\sqrt{2}\,\log\left(1+\sqrt{2}\right)}-\frac{2457}{5}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternative representations:

$$\frac{569}{2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}-491-\frac{2}{5}=-491-\frac{2}{5}+\frac{569}{2\left(\frac{1}{4}+\frac{\log_e(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}$$

$$\frac{569}{2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}-491-\frac{2}{5}=-491-\frac{2}{5}+\frac{569}{2\left(\frac{1}{4}+\frac{\log(a)\log_a(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}$$

$$\frac{569}{2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}-491-\frac{2}{5}=-491-\frac{2}{5}+\frac{569}{2\left(\frac{1}{4}-\frac{\text{Li}_1(-\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}$$

Series representations:

$$\frac{569}{2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}-491-\frac{2}{5}=-\frac{2457}{5}-\frac{4552}{-4+\sqrt{2}\,\pi-\sqrt{2}\,\log(2)+2\sqrt{2}\sum_{k=1}^{\infty}\frac{(-1)^k\,2^{-k/2}}{k}}$$

$$\frac{569}{2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}-491-\frac{2}{5}=-\frac{2457}{5}+\frac{4552}{4-\sqrt{2}\,\pi+2\sqrt{2}\left(2\,i\,\pi\left\lfloor\frac{\arg(1+\sqrt{2}-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(1+\sqrt{2}-x)^k\,x^{-k}}{k}\right)}\quad\text{for } x<0$$

$$\frac{569}{2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}-491-\frac{2}{5}=-\frac{2457}{5}+\frac{569}{2\left(\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{2\,i\,\pi\left\lfloor\frac{\arg(1+\sqrt{2}-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(1+\sqrt{2}-x)^k\,x^{-k}}{k}}{4\sqrt{2}}\right)}\quad\text{for } x<0$$

Integral representations:

$$\frac{569}{2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}-491-\frac{2}{5}=-\frac{2457}{5}+\frac{4552}{4-\sqrt{2}\,\pi+2\sqrt{2}\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt}$$

$$\frac{569}{2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}-491-\frac{2}{5}=$$
$$-\frac{2457}{5}-\frac{4552\pi}{\pi\left(-4+\sqrt{2}\,\pi\right)+i\sqrt{2}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^{-s/2}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1<\gamma<0$$

$$\frac{569}{2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)}-491-\frac{2}{5}=$$
$$-\frac{2457}{5}+\frac{569}{2\left(\frac{1}{4}-\frac{\pi}{8\sqrt{2}}-\frac{i}{8\sqrt{2}\,\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^{-s/2}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)} \quad \text{for } -1<\gamma<0$$

From the previous expression

$$569\left(2\left(\frac{1}{4}+\frac{1}{4\sqrt{2}}\log\left(1+\sqrt{2}\right)-\frac{\pi}{8\sqrt{2}}\right)\right)$$

$$1138\left(\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}\right)$$

We obtain also:

569(((2(((1/4+1/(4sqrt2)*ln(1+sqrt2) – Pi/(8sqrt2))))))) - 7 + 1

where 1 and 7 are Lucas numbers

Input:

$$569\left(2\left(\frac{1}{4}+\frac{1}{4\sqrt{2}}\log\left(1+\sqrt{2}\right)-\frac{\pi}{8\sqrt{2}}\right)\right)-7+1$$

$\log(x)$ is the natural logarithm

Exact result:

$$1138\left(\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}\right)-6$$

Decimal approximation:

139.80753184338178174159417741321975744015784870575241633327...

139.807531843…

Alternate forms:

$$\frac{1}{8}\left(2228-569\sqrt{2}\,\pi+1138\sqrt{2}\,\sinh^{-1}(1)\right)$$

$$\frac{1}{8}\left(2228-569\sqrt{2}\,\pi+1138\sqrt{2}\,\log\left(1+\sqrt{2}\right)\right)$$

$$\frac{557}{2}-\frac{569\pi}{4\sqrt{2}}+\frac{569\log(1+\sqrt{2})}{2\sqrt{2}}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternative representations:

$$569\times 2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)-7+1=-6+1138\left(\frac{1}{4}+\frac{\log_e(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)$$

$$569\times 2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)-7+1=$$
$$-6+1138\left(\frac{1}{4}+\frac{\log(a)\log_a(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)$$

$$569\times 2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)-7+1=-6+1138\left(\frac{1}{4}-\frac{\mathrm{Li}_1(-\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)$$

Series representations:

$$569\times 2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)-7+1=$$
$$\frac{557}{2}-\frac{569\pi}{4\sqrt{2}}+\frac{569\log(2)}{4\sqrt{2}}-\frac{569\sum_{k=1}^{\infty}\frac{(-1)^k 2^{-k/2}}{k}}{2\sqrt{2}}$$

$$569\times 2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)-7+1=\frac{557}{2}-\frac{569\pi}{4\sqrt{2}}+$$
$$\frac{569\,i\,\pi\left\lfloor\frac{\arg\left(1+\sqrt{2}-x\right)}{2\pi}\right\rfloor}{\sqrt{2}}+\frac{569\log(x)}{2\sqrt{2}}-\frac{569\sum_{k=1}^{\infty}\frac{(-1)^k\left(1+\sqrt{2}-x\right)^k x^{-k}}{k}}{2\sqrt{2}}\quad\text{for } x<0$$

$$569\times 2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)-7+1 = \frac{557}{2}-\frac{569\pi}{4\sqrt{2}}+\frac{569\left\lfloor\frac{\arg(1+\sqrt{2}-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)}{2\sqrt{2}}+\frac{569\log(z_0)}{2\sqrt{2}}+\frac{569\left\lfloor\frac{\arg(1+\sqrt{2}-z_0)}{2\pi}\right\rfloor\log(z_0)}{2\sqrt{2}}-\frac{569\sum_{k=1}^{\infty}\frac{(-1)^k(1+\sqrt{2}-z_0)^k z_0^{-k}}{k}}{2\sqrt{2}}$$

Integral representations:

$$569\times 2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)-7+1 = \frac{557}{2}-\frac{569\pi}{4\sqrt{2}}+\frac{569}{2\sqrt{2}}\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt$$

$$569\times 2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)-7+1 = \frac{557}{2}-\frac{569\pi}{4\sqrt{2}}-\frac{569\,i}{4\sqrt{2}\,\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^{-s/2}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

569(((2(((1/4+1/(4sqrt2)*ln(1+sqrt2) – Pi/(8sqrt2)))))))-18-2

Input:

$$569\left(2\left(\frac{1}{4}+\frac{1}{4\sqrt{2}}\log\left(1+\sqrt{2}\right)-\frac{\pi}{8\sqrt{2}}\right)\right)-18-2$$

log(x) is the natural logarithm

Exact result:

$$1138\left(\frac{1}{4}-\frac{\pi}{8\sqrt{2}}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}\right)-20$$

Decimal approximation:

125.80753184338178174159417741321975744015784870575241633327...

125.807531843…

Alternate forms:

$$\frac{1}{8}\left(2116-569\sqrt{2}\,\pi+1138\sqrt{2}\,\sinh^{-1}(1)\right)$$

$$\frac{1}{8}\left(2116-569\sqrt{2}\,\pi+1138\sqrt{2}\,\log\left(1+\sqrt{2}\right)\right)$$

$$\frac{529}{2}-\frac{569\,\pi}{4\sqrt{2}}+\frac{569\log(1+\sqrt{2})}{2\sqrt{2}}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternative representations:

$$569\times 2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)-18-2=-20+1138\left(\frac{1}{4}+\frac{\log_e(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)$$

$$569\times 2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)-18-2=$$
$$-20+1138\left(\frac{1}{4}+\frac{\log(a)\log_a(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)$$

$$569\times 2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)-18-2=-20+1138\left(\frac{1}{4}-\frac{\mathrm{Li}_1(-\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)$$

Series representations:

$$569\times 2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)-18-2=$$
$$\frac{529}{2}-\frac{569\,\pi}{4\sqrt{2}}+\frac{569\log(2)}{4\sqrt{2}}-\frac{569\sum_{k=1}^{\infty}\frac{(-1)^k\,2^{-k/2}}{k}}{2\sqrt{2}}$$

$$569\times 2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)-18-2=\frac{529}{2}-\frac{569\,\pi}{4\sqrt{2}}+$$
$$\frac{569\,i\,\pi\left\lfloor\frac{\arg\left(1+\sqrt{2}-x\right)}{2\pi}\right\rfloor}{\sqrt{2}}+\frac{569\log(x)}{2\sqrt{2}}-\frac{569\sum_{k=1}^{\infty}\frac{(-1)^k\left(1+\sqrt{2}-x\right)^k x^{-k}}{k}}{2\sqrt{2}}\quad\text{for } x<0$$

$$569\times 2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)-18-2=$$
$$\frac{529}{2}-\frac{569\,\pi}{4\sqrt{2}}+\frac{569\left\lfloor\frac{\arg\left(1+\sqrt{2}-z_0\right)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)}{2\sqrt{2}}+\frac{569\log(z_0)}{2\sqrt{2}}+$$
$$\frac{569\left\lfloor\frac{\arg\left(1+\sqrt{2}-z_0\right)}{2\pi}\right\rfloor\log(z_0)}{2\sqrt{2}}-\frac{569\sum_{k=1}^{\infty}\frac{(-1)^k\left(1+\sqrt{2}-z_0\right)^k z_0^{-k}}{k}}{2\sqrt{2}}$$

Integral representations:

$$569\times 2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)-18-2=\frac{529}{2}-\frac{569\pi}{4\sqrt{2}}+\frac{569}{2\sqrt{2}}\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt$$

$$569\times 2\left(\frac{1}{4}+\frac{\log(1+\sqrt{2})}{4\sqrt{2}}-\frac{\pi}{8\sqrt{2}}\right)-18-2=$$
$$\frac{529}{2}-\frac{569\pi}{4\sqrt{2}}-\frac{569\,i}{4\sqrt{2}\,\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^{-s/2}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

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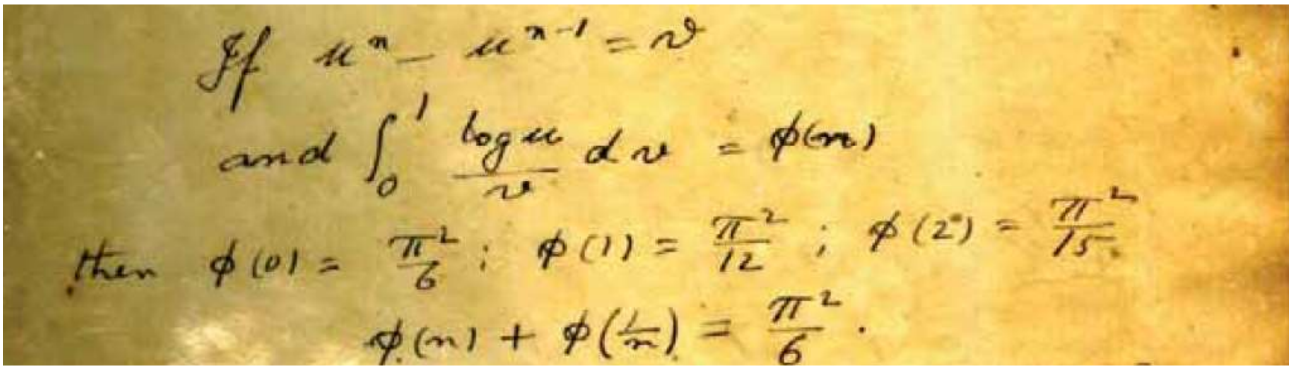

If $u^n - u^{n-1} = v$

and $\int_0^1 \frac{\log u}{v}\,dv = \phi(n)$

then $\phi(0) = \frac{\pi^2}{6}$; $\phi(1) = \frac{\pi^2}{12}$; $\phi(2) = \frac{\pi^2}{15}$.

$\phi(n) + \phi\left(\frac{1}{n}\right) = \frac{\pi^2}{6}$.

We perform the following integrals:

(((integrate (ln (2)/v)dv, v = 1..4.646)))1/x = (Pi^2)/15

Input:

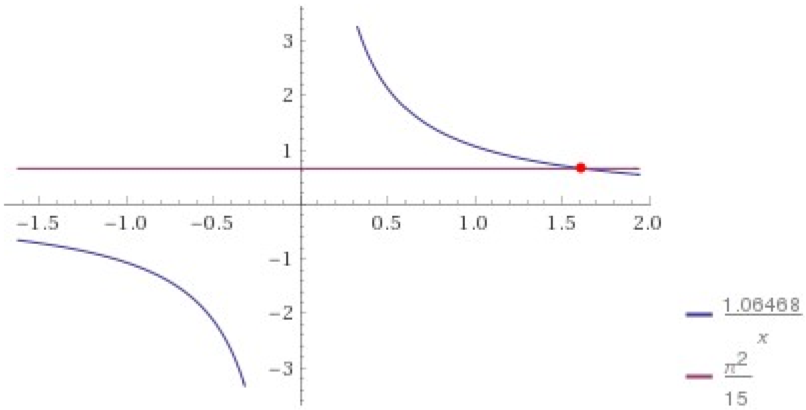
$$\left(\int_1^{4.646}\frac{\log(2)}{v}\,dv\right)\times\frac{1}{x}=\frac{\pi^2}{15}$$

log(x) is the natural logarithm

Result:

$$\frac{1.06468}{x}=\frac{\pi^2}{15}$$

Plot:



Alternate form assuming x is real:

$$\frac{1.61812}{x} = 1$$

Alternate form assuming x is positive:

$x = 1.61812$

Solution:

$x \approx 1.61812$

1.61812

(((integrate (ln (2)/v)dv, v = 1..6.821)))1/x = (Pi^2)/12

Input:

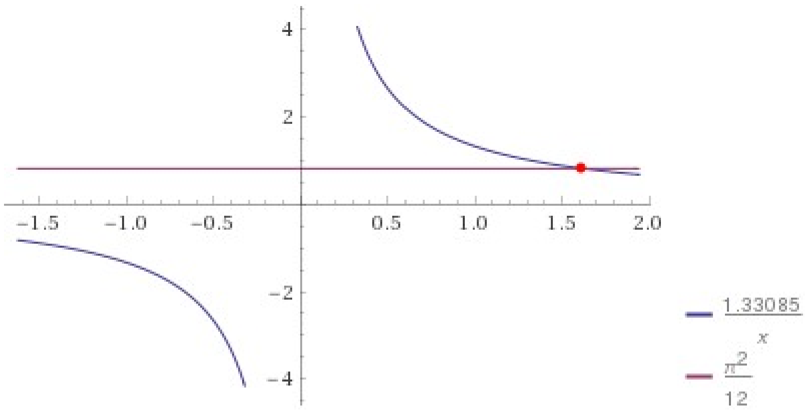
$$\frac{1}{x}\int_{1}^{6.821} \frac{\log(2)}{v}\, dv = \frac{\pi^2}{12}$$

log(x) is the natural logarithm

Result:

$$\frac{1.33085}{x} = \frac{\pi^2}{12}$$

Plot:



Alternate form assuming x is real:

$$\frac{1.61812}{x} = 1$$

Alternate form assuming x is positive:

$x = 1.61812$

Solution:

$x \approx 1.61812$

1.61812

(((integrate (ln (2)/v)dv, v = 1..46.526)))1/x = (Pi^2)/6

Input interpretation:

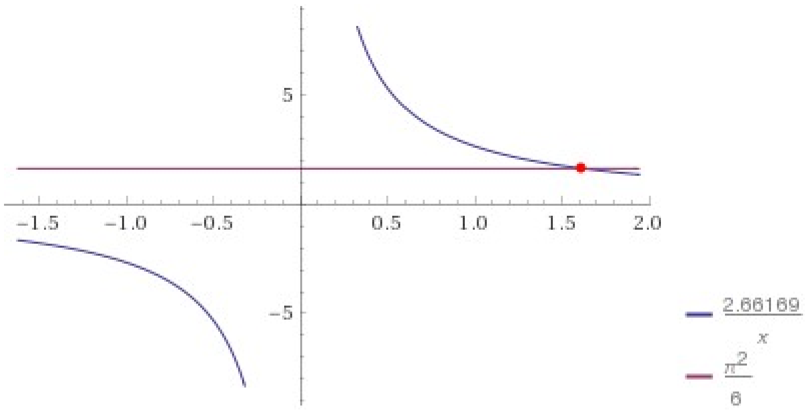
$$\left(\int_1^{46.526} \frac{\log(2)}{v}\, dv\right) \times \frac{1}{x} = \frac{\pi^2}{6}$$

log(x) is the natural logarithm

Result:

$$\frac{2.66169}{x} = \frac{\pi^2}{6}$$

Plot:



Alternate form assuming x is real:

$$\frac{1.61812}{x} = 1$$

Alternate form assuming x is positive:

$x = 1.61812$

Solution:

$x \approx 1.61812$

1.61812

All the results are equal to 1.61812 that is a very good approximation to the value of the golden ratio. We note that, from the previous expression

$$\frac{1}{3.9121192269}\left(1+9.8844\times 10^{-6}\cos\left(\frac{2\pi\log(3.9121192269)}{\log(2)}+0.872811\right)\right)$$

log(x) is the natural logarithm

Result:

0.255617909572411304355893214535044931185553916327111126588...

0.25561790957…

And from the values of the integration intervals 4.646 , 6.821 and 46.526 , we obtain:

1+sqrt(((0.25561790957(46.526* 1/6.821* 1/4.646))))

Input interpretation:

$$1+\sqrt{0.25561790957\left(46.526\times\frac{1}{6.821}\times\frac{1}{4.646}\right)}$$

Result:

1.612604059122715331292196377098680518900209981339939623299...

1.6126040591…

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We have:

exp((sqrt130*Pi)/4)

Input:

$$\exp\left(\frac{1}{4}\left(\sqrt{130}\,\pi\right)\right)$$

Exact result:

$$e^{1/2\sqrt{65/2}\,\pi}$$

Decimal approximation:

7745.883719183247888245403932864313029838541931800183836941...

7745.88371918…

Property:

$e^{1/2\sqrt{65/2}\,\pi}$ is a transcendental number

Series representations:

$$e^{\left(\sqrt{130}\,\pi\right)/4}=e^{1/4\,\pi\sqrt{129}\,\sum_{k=0}^{\infty}129^{-k}\binom{1/2}{k}}$$

$$e^{\left(\sqrt{130}\,\pi\right)/4}=\exp\left(\frac{1}{4}\,\pi\sqrt{129}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{129}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{(\sqrt{130}\,\pi)/4} = \exp\left(\frac{\pi \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j}\, 129^{-s}\, \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{8\sqrt{\pi}}\right)$$

Integral representation:

$$(1+z)^a = \frac{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\, d\,s}{(2\,\pi\,i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

And:

12(323+40sqrt65)

Input:

$$12\left(323 + 40\sqrt{65}\right)$$

Decimal approximation:

7745.883719183303833135974350545810142944510226692115226209...

7745.88371918…

Alternate form:

$$3876 + 480\sqrt{65}$$

Minimal polynomial:

$$x^2 - 7752\,x + 47\,376$$

From which:

1/4[exp((sqrt130*Pi)/4)]-199-7-3/2

Input:

$$\frac{1}{4}\exp\left(\frac{1}{4}\left(\sqrt{130}\,\pi\right)\right) - 199 - 7 - \frac{3}{2}$$

Exact result:

$$\frac{1}{4}\,e^{1/2\sqrt{65/2}\,\pi} - \frac{415}{2}$$

Decimal approximation:

1728.970929795811972061350983216078257459635482950045959235...

1728.9709297…

Property:

$-\frac{415}{2}+\frac{1}{4}e^{1/2\sqrt{65/2}\,\pi}$ is a transcendental number

Alternate form:

$\frac{1}{4}\left(e^{1/2\sqrt{65/2}\,\pi}-830\right)$

Series representations:

$$\frac{1}{4}\exp\left(\frac{\sqrt{130}\,\pi}{4}\right)-199-7-\frac{3}{2}=-\frac{415}{2}+\frac{1}{4}\exp\left(\frac{1}{4}\pi\sqrt{129}\sum_{k=0}^{\infty}129^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{1}{4}\exp\left(\frac{\sqrt{130}\,\pi}{4}\right)-199-7-\frac{3}{2}=-\frac{415}{2}+\frac{1}{4}\exp\left(\frac{1}{4}\pi\sqrt{129}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{129}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{1}{4}\exp\left(\frac{\sqrt{130}\,\pi}{4}\right)-199-7-\frac{3}{2}=-\frac{415}{2}+\frac{1}{4}\exp\left(\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}129^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{8\sqrt{\pi}}\right)$$

Performing the 18th root, we obtain:

(((exp((sqrt130*Pi)/4))))^1/18

Input:

$\sqrt[18]{\exp\left(\frac{1}{4}\left(\sqrt{130}\,\pi\right)\right)}$

Exact result:

$e^{1/36\sqrt{65/2}\,\pi}$

Decimal approximation:

1.644597018183361755096803362658528241004892838006401991537...

1.64459701818…

Property:

$e^{1/36\sqrt{65/2}\,\pi}$ is a transcendental number

All 18th roots of e^(1/2 sqrt(65/2) π):

$e^{1/36\sqrt{65/2}\,\pi}\, e^0 \approx 1.6446$ (real, principal root)

$e^{1/36\sqrt{65/2}\,\pi}\, e^{(i\pi)/9} \approx 1.5454 + 0.5625\, i$

$e^{1/36\sqrt{65/2}\,\pi}\, e^{(2 i\pi)/9} \approx 1.2598 + 1.0571\, i$

$e^{1/36\sqrt{65/2}\,\pi}\, e^{(i\pi)/3} \approx 0.8223 + 1.4243\, i$

$e^{1/36\sqrt{65/2}\,\pi}\, e^{(4 i\pi)/9} \approx 0.28558 + 1.6196\, i$

Series representations:

$$\sqrt[18]{\exp\left(\frac{\sqrt{130}\,\pi}{4}\right)} = \sqrt[18]{\exp\left(\frac{1}{4}\pi\sqrt{129}\sum_{k=0}^{\infty} 129^{-k}\binom{\frac{1}{2}}{k}\right)}$$

$$\sqrt[18]{\exp\left(\frac{\sqrt{130}\,\pi}{4}\right)} = \sqrt[18]{\exp\left(\frac{1}{4}\pi\sqrt{129}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{129}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}$$

$$\sqrt[18]{\exp\left(\frac{\sqrt{130}\,\pi}{4}\right)} = \sqrt[18]{\exp\left(\frac{\pi\sum_{j=0}^{\infty}\text{Res}_{s=-\frac{1}{2}+j}\, 129^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{8\sqrt{\pi}}\right)}$$

Integral representation:

$$(1+z)^a = \frac{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

Now, with these three previous results: 2226.1572478; 569.0456620; 145.4582 performing some calculations with the above expression

$$\exp\left(\frac{1}{4}\left(\sqrt{130}\,\pi\right)\right)$$

We obtain:

1+1/((((((exp((sqrt130*Pi)/4))))*1/(2226.1572478))))^1/3

Input interpretation:

$$1+\frac{1}{\sqrt[3]{\exp\left(\frac{1}{4}\left(\sqrt{130}\,\pi\right)\right)\times\frac{1}{2226.1572478}}}$$

Result:

1.65992556817...

1.65992556817…

Series representations:

$$1+\frac{1}{\sqrt[3]{\frac{\exp\left(\frac{\sqrt{130}\,\pi}{4}\right)}{2226.15724780000}}}=1+\frac{13.05725681405340}{\sqrt[3]{\exp\left(\frac{1}{4}\pi\sqrt{129}\sum_{k=0}^{\infty}129^{-k}\binom{\frac{1}{2}}{k}\right)}}$$

$$1+\frac{1}{\sqrt[3]{\frac{\exp\left(\frac{\sqrt{130}\,\pi}{4}\right)}{2226.15724780000}}}=1+\frac{13.05725681405340}{\sqrt[3]{\exp\left(\frac{1}{4}\pi\sqrt{129}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{129}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}}$$

$$1+\frac{1}{\sqrt[3]{\frac{\exp\left(\frac{\sqrt{130}\,\pi}{4}\right)}{2226.15724780000}}}=1+\frac{13.05725681405340}{\sqrt[3]{\exp\left(\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}129^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{8\sqrt{\pi}}\right)}}$$

1+1/((((((exp((sqrt130*Pi)/4))))*1/569.0456620)))^1/6

Input interpretation:

$$1+\frac{1}{\sqrt[6]{\exp\left(\frac{1}{4}\left(\sqrt{130}\,\pi\right)\right)\times\frac{1}{569.0456620}}}$$

Result:

1.6471615264...

1.6471615264…

Series representations:

$$1+\frac{1}{\sqrt[6]{\frac{\exp\left(\frac{\sqrt{130}\,\pi}{4}\right)}{569.046}}}=1+\frac{2.87867}{\sqrt[6]{\exp\left(\frac{1}{4}\pi\sqrt{129}\sum_{k=0}^{\infty}129^{-k}\binom{\frac{1}{2}}{k}\right)}}$$

$$1+\frac{1}{\sqrt[6]{\frac{\exp\left(\frac{\sqrt{130}\,\pi}{4}\right)}{569.046}}}=1+\frac{2.87867}{\sqrt[6]{\exp\left(\frac{1}{4}\pi\sqrt{129}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{129}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}}$$

$$1+\frac{1}{\sqrt[6]{\frac{\exp\left(\frac{\sqrt{130}\,\pi}{4}\right)}{569.046}}}=1+\frac{2.87867}{\sqrt[6]{\exp\left(\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}129^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{8\sqrt{\pi}}\right)}}$$

((((((exp((sqrt130*Pi)/4))))*1/145.45826259)))

Input interpretation:

$$\exp\left(\frac{1}{4}\left(\sqrt{130}\,\pi\right)\right)\times\frac{1}{145.45826259}$$

Result:

53.251589709...

53.251589709…. ≈ 53 (Eiseinstein number)

Series representations:

$$\frac{\exp\left(\frac{\sqrt{130}\,\pi}{4}\right)}{145.458262590000} = 0.00687482431175930 \exp\left(\frac{1}{4}\,\pi\,\sqrt{129}\,\sum_{k=0}^{\infty} 129^{-k} \binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(\frac{\sqrt{130}\,\pi}{4}\right)}{145.458262590000} = 0.00687482431175930 \exp\left(\frac{1}{4}\,\pi\,\sqrt{129}\,\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{129}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp\left(\frac{\sqrt{130}\,\pi}{4}\right)}{145.458262590000} = 0.00687482431175930 \exp\left(\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 129^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{8\sqrt{\pi}}\right)$$

((((((exp((sqrt130*Pi)/4))))*1/145.45826259)))^1/8

Input interpretation:

$$\sqrt[8]{\exp\left(\frac{1}{4}\left(\sqrt{130}\,\pi\right)\right)\times\frac{1}{145.45826259}}$$

Result:

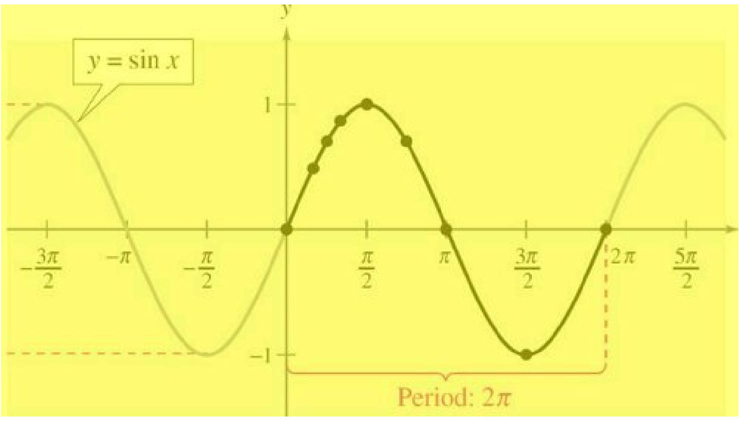
1.64358274098…

1.64358274098…

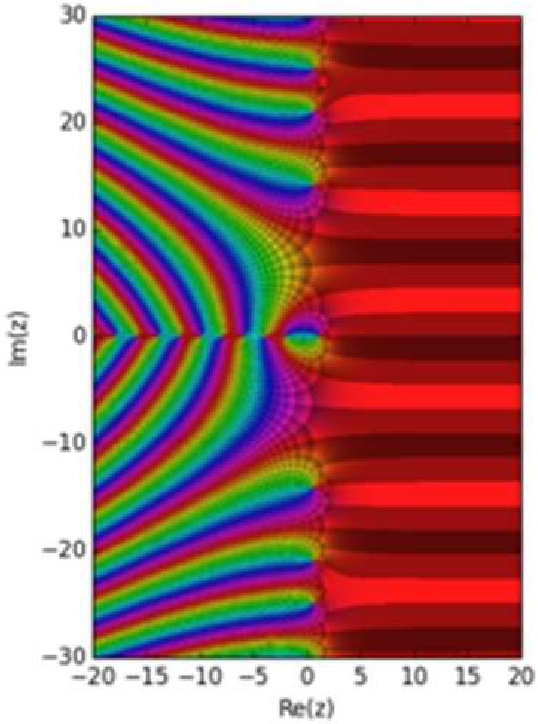
Appendix

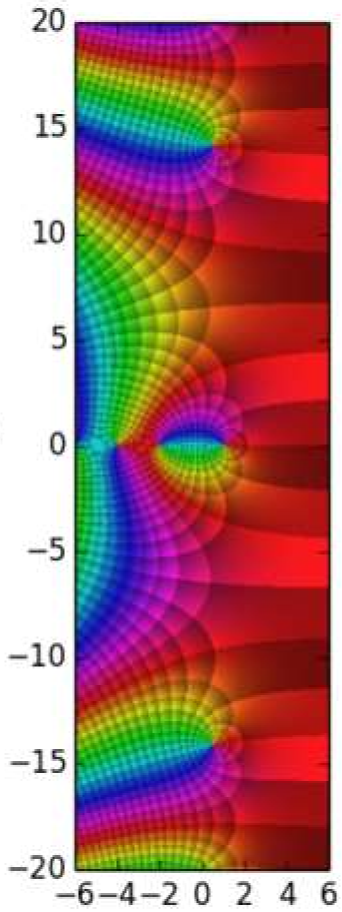
Basel Problem $\zeta(2) = \pi^2 / 6$

https://mathtuition88.com/2014/04/10/the-basel-problem/



From Wikipedia - The pole at z=1, and two zeros on the critical line





https://going-postal.com/2019/01/the-basel-problem/

From Wikipedia

Specific values	
At zero	$-\frac{1}{2}$
Limit to $+\infty$	1
Value at 2	$\frac{\pi^2}{6}$
Value at -1	$-\frac{1}{12}$
Value at -2	0

Continued fraction of ζ(2):

$$1 + \cfrac{1}{1 + \cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{2+\cfrac{1}{4+\cfrac{1}{7+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{10+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{15+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{\cdots}}}}}}}}}}}}}}}}}}}}}}$$

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.
In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the *golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.*

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the *second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.*

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:
2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] *That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies*[3] *- golden spirals are one special case of these logarithmic spirals*

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to ζ(2) = $\frac{\pi^2}{6} = 1.644934\ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

References

Manuscript Book 3 of Srinivasa Ramanujan