a stochastic interpretation on the Hans de Vries formular for the fine structure constant α

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Abstract

The formular for the fine structure constant α found by Hans de Vries is very elegant and accurate but there exists no explanation for it.In this paper i will show an stochastic interpretation of it.

Keywords: *fine structure constant; Sommerfeld;electromagnetism; electric charge*

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1. Introduction

In physics, the fine-structure constant, also known as the Sommerfeld constant, commonly denoted by α ,is a fundamental physical constant which quantifies the strength of the electromagnetic interaction between elementary charged particles.It is a dimensionless quantity, independent of the system of units used, which is related to the strength of the coupling of an elementary charge e with the electromagnetic field, by the formula $4\pi\varepsilon_0\hbar c\alpha = e^2$.
Its numerical value is ≈ 0.00729735.The constant was named by Arnold Sommerfeld, who introduced it in 1916 when extending the Bohr model of the atom.α quantified the gap in the fine structure of the spectral lines of the hydrogen atom,which had been measured precisely by Michelson and Morley in 1887.

1.1. The Hans de Vries formular

$$\alpha = \Gamma^2 . e^{-\frac{\pi^2}{2}} \tag{1}$$

$\alpha ... fine\ structure\ constant$

with

$$\Gamma = 1 + \frac{\alpha}{(2\pi)^0}(1 + \frac{\alpha}{(2\pi)^1}(1 + \frac{\alpha}{(2\pi)^2}(1 + ... \tag{2}$$

Someone can easy proof that the HdV formular is identical to

$$\alpha = \left[\sum_{n=0}^{\infty} \frac{\alpha^n}{(2\pi)^{\binom{n}{2}}}\right]^2 .e^{-\frac{\pi^2}{2}} \tag{3}$$

then

$$\sqrt{\alpha} = \sum_{n=0}^{\infty} \frac{\alpha^n}{(2\pi)^{\binom{n}{2}}} .e^{-\frac{\pi^2}{4}} = (1 + 1.\frac{\alpha}{(2\pi)^0} + 1.\frac{\alpha}{(2\pi)^0}.\frac{\alpha}{(2\pi)^1} + 1.\frac{\alpha}{(2\pi)^0}.\frac{\alpha}{(2\pi)^1}.\frac{\alpha}{(2\pi)^2} + ...).e^{-\frac{\pi^2}{4}} \tag{4}$$

Now we interprete the factor on the right in the formular $e^{-\frac{\pi^2}{4}}$ as expectationvalue of the wrapped normaldistribution which is

$$< z >= e^{i\mu - \frac{\sigma^2}{2}} = e^{-\frac{\pi^2}{4}} \tag{5}$$

for $\mu = 0 \; and \; \sigma = \frac{\pi}{\sqrt{2}}$

and the factor

$$(1 + 1.\frac{\alpha}{(2\pi)^0} + 1.\frac{\alpha}{(2\pi)^0}.\frac{\alpha}{(2\pi)^1} + 1.\frac{\alpha}{(2\pi)^0}.\frac{\alpha}{(2\pi)^1}.\frac{\alpha}{(2\pi)^2} + ...) \tag{6}$$

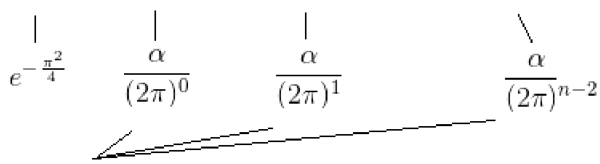
we interprete as a serial of conditional probabilities

Therefore we write formular (4) as follow

$$\sqrt{\alpha} = \sum_{n=1}^{\infty} P(A_1 \cap ... \cap A_n) = (1 + 1.\frac{\alpha}{(2\pi)^0} + 1.\frac{\alpha}{(2\pi)^0}.\frac{\alpha}{(2\pi)^1} + 1.\frac{\alpha}{(2\pi)^0}.\frac{\alpha}{(2\pi)^1}.\frac{\alpha}{(2\pi)^2} + ...).e^{-\frac{\pi^2}{4}} \tag{7}$$

with (see references)

$$P(A_1 \cap ... \cap A_n) = P(A_1).P(A_2 \mid A_1).P(A_3 \mid A_1 \cap A_2)...P(A_n \mid A_1 \cap ... \cap A_{n-1}) = \frac{\alpha^{n-1}}{(2\pi)^{\binom{n-1}{2}}}.e^{-\frac{\pi^2}{4}} \tag{8}$$



The denominator of $\frac{\alpha}{(2\pi)^i}$ is the i-dimensional "volumne" of a torus
therefore the factors $\frac{1}{(2\pi)^i}$ are normalisationfactors.

To understand the HdV formular we have to understand the following two questions.

1) What isA_1, A_2, A_3...

2) Why do we have those values for the probabilities $P(A_1 \cap ... \cap A_n)$

1.1.1. Understanding question 1) what is $A_1, A_2, ...$

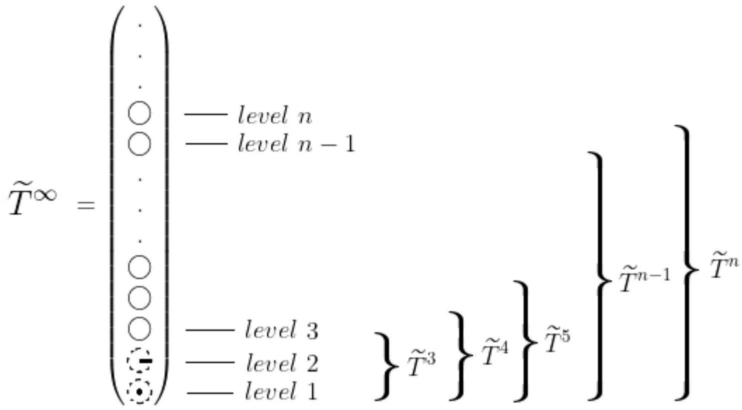
Normally a n-dimensional torus is defined as (see references)

$$\mathbb{T}^n = \mathbb{S}^1 \times ... \times \mathbb{S}^1 = (\mathbb{S}^1)^n \tag{9}$$

But in our formular we have two denominators which have the dimension of a point and a line. Therefore we define a torus as

$$\tilde{\mathbb{T}}^n = \{0\} \times [0,1] \times \mathbb{S}^1 \times ... \times = \mathbb{S}^1 = \{0\} \times [0,1] \times (\mathbb{S}^1)^{n-2} \tag{10}$$

The infinite Torus $\tilde{\mathbb{T}}^\infty$ then can be seen as an infinite ladder.

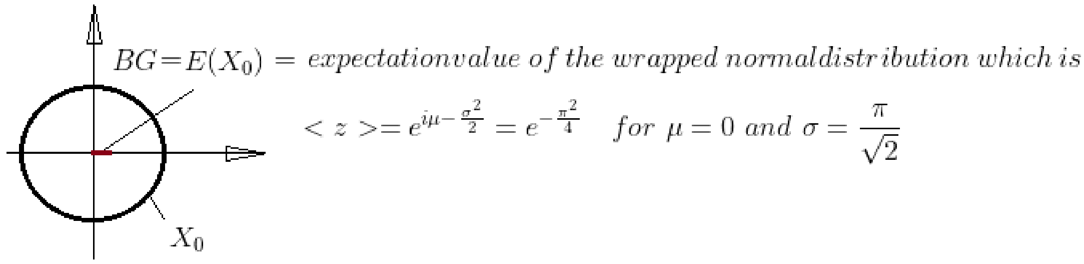


With this geometrical picture we can explain our probability sum in equation (7)
P(absorbing or emitting a photon) $=P(\pm\gamma)=\sqrt{\alpha}$ is given by the different levels of the $\tilde{\mathbb{T}}^\infty$.
A photon is emitted when we climb down from one level to the prior level.Or a photon is absorbed when we climb up on the torusladder one step from one level to the next.
Our Events$A_1, A_2, ...$ are then

A_1...absorbing a photon by climbing up to level 1 from vacuum or emitting a photon by climbing down from level 1 to vacuum.
A_2...absorbing a photon by climbing up to level 2 from level 1 or emitting a photon by climbing down from level 2 to level 1
and so on.
$\Rightarrow$ understood question 1) what is $A_1, A_2, ...$

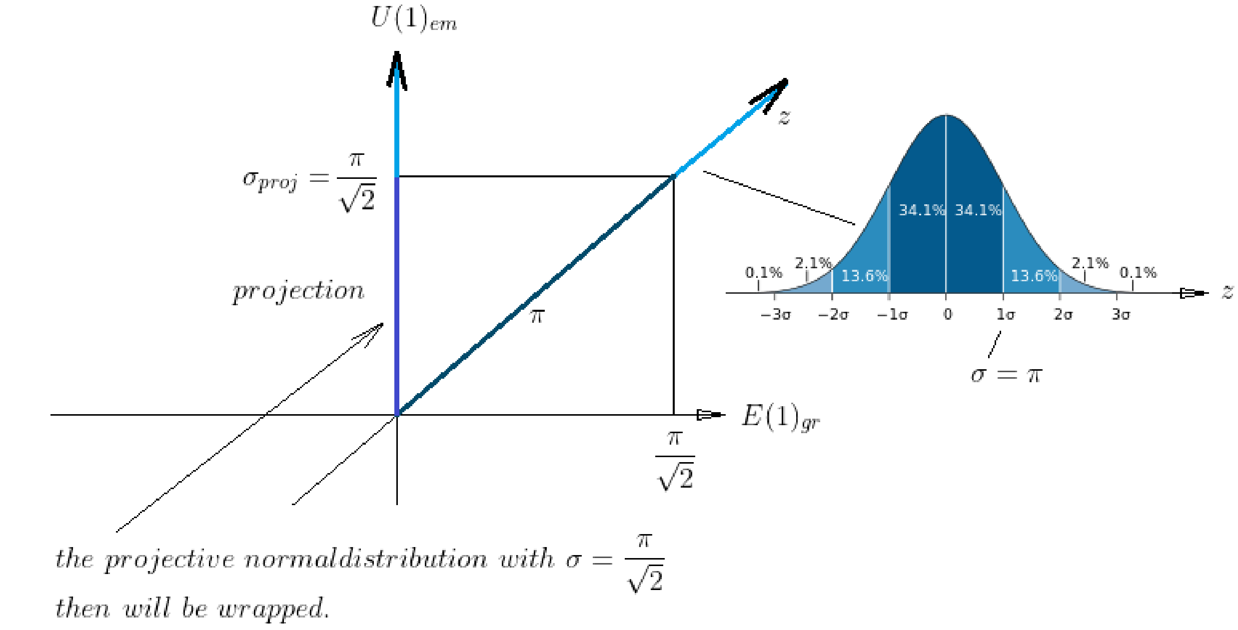
1.1.2. Understanding question 2) values for $P(A_1 \cap ... \cap A_n)$

We call the factor $e^{-\frac{\pi^2}{4}}$ the Basic-Generator of the electromagnetic field (short BG).
Explanation and visualisation of the Basic-Generator BG.



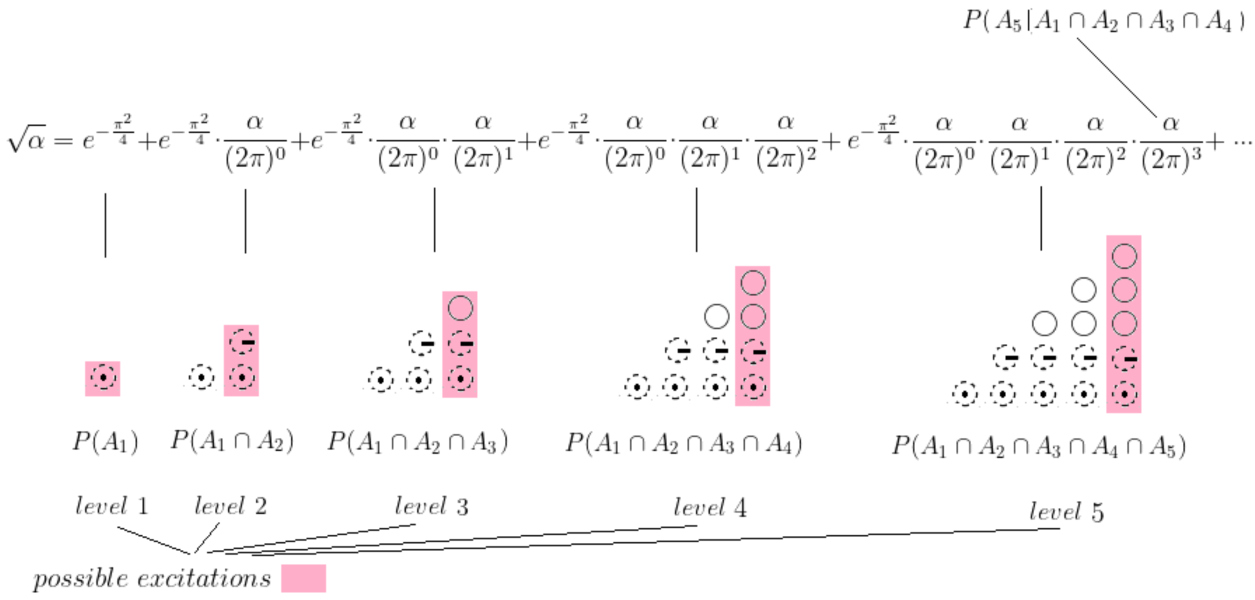
$$We\ write\ E(X_0) = e^{-\frac{\pi^2}{4}} \quad X_0 = \left\{ x \mid x = e^{i\theta} \quad , 0 \leqslant \theta < 2\pi \right\}$$

The factor $\frac{\pi}{\sqrt{2}}$ comes of a projection of the normaldistribution with standard deviation $\sigma = \pi$.



Schematic representation of the formular and its probabilities

The probability to absorb a photon is:

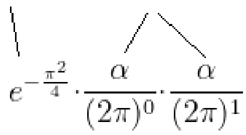


How to understand the single components the multipliers of $P(A_1 \cap ... \cap A_n)$? .
We want to restrict our examinations on the case of absorbing a photon.
For that we have to understand that a photon can only be absorbed if another source is emitting a photon.
We have two different types of possible sources.
1) the vacuum
2) a second particle with electrical charge

We show for example the multipliers of $P(A_1 \cap A_2 \cap A_3)$.
As we have seen on equation (8)

$$P(A_1 \cap A_2 \cap A_3) = P(A_1).P(A_2 \mid A_1).P(A_3 \mid A_1 \cap A_2) = e^{-\frac{\pi^2}{4}}.\frac{\alpha}{(2\pi)^0}.\frac{\alpha}{(2\pi)^1} \tag{11}$$

We have multipliers without α and with α



The first without α has the vacuum as source for emitting a photon and the one with α has another electromagnetic charged particle as source for emitting a photon.
Therefore we have the picture of source and sink in the HdV formular (1).
We observe the sink and say that we have the vacuum and another electromagnetic charged particle as possible sources.
What does the multipliers with α express?
To understand this we split the multipliers in two components

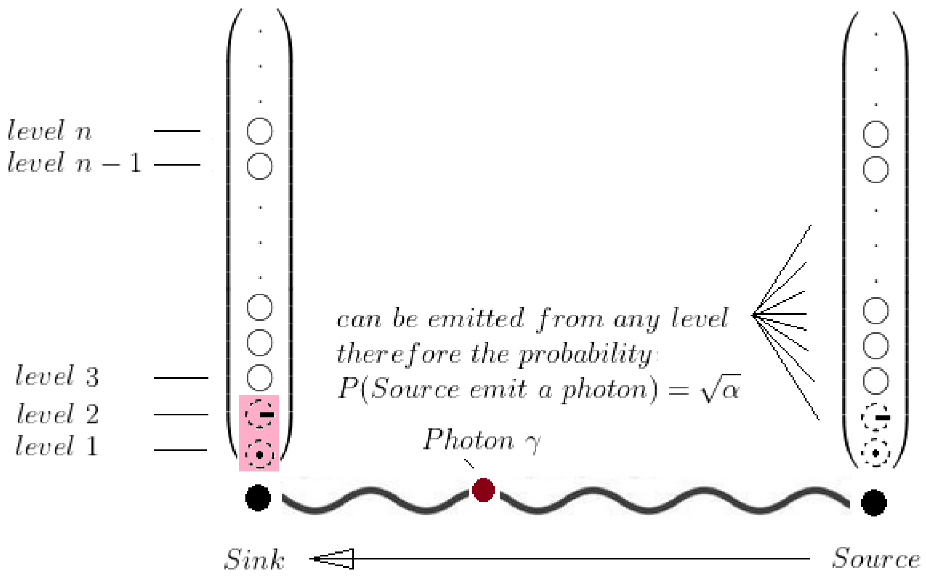
$$\frac{\alpha}{(2\pi)^i} = \frac{\sqrt{\alpha}}{(2\pi)^i}.\sqrt{\alpha} \tag{12}$$

The first component is the probability that the sink absorbs a photon if the level=i+1 is already reached and the second component is the probability that the source emits a photon.
It does not matter on which level the source emits the photon so the probability $= \sqrt{\alpha}$.
How can we understand the probability $\frac{\sqrt{\alpha}}{(2\pi)^i}$ the sink absorbs a photon on level=i+1?
For the probability that the source emits a photon we can write

$$\sqrt{\alpha} = \frac{\sqrt{\alpha}}{1} = \frac{\sqrt{\alpha}}{volumne(\tilde{\mathbb{T}}^{i+2})}.(2\pi)^i \tag{13}$$

Now the sink absorbs the photon on a volumne $(2\pi)^i$ of the torus $\tilde{\mathbb{T}}^{i+2})$ which is $(2\pi)^i$ times bigger as volumne=1 so the probability is getting smaller to $\frac{\sqrt{\alpha}}{(2\pi)^i}$.

Schematic representation



the sink is in the state of level 2 before absorbing the photon.

after absorbing the photon the state is level 3 

The probability to reach level 2 is $P(A_1 \cap A_2) = e^{-\frac{\pi^2}{4}} \cdot \frac{\alpha}{(2\pi)^0}$
to climb from level 2 to level 3 the source must
1) emit a photon by probability $\sqrt{\alpha}$ *and*
2) the sink must absorb the photon by probability $\cdot \frac{\sqrt{\alpha}}{(2\pi)^1}$

Therefore the probability to change the excitation from level 2 to level 3 is $\cdot \frac{\sqrt{\alpha}}{(2\pi)^1} \cdot \sqrt{\alpha}$
This explains the factor

$$P(A_1 \cap A_2 \cap A_3) = e^{-\frac{\pi^2}{4}} \cdot \frac{\alpha}{(2\pi)^0} \cdot \boxed{\frac{\alpha}{(2\pi)^1}} = e^{-\frac{\pi^2}{4}} \cdot \frac{\alpha}{(2\pi)^0} \boxed{\frac{\sqrt{\alpha}}{(2\pi)^1}\sqrt{\alpha}}$$

$\Rightarrow$ understood question 2)

Last but not least the value for the Finestructure Constant by the Hans de Vries formular.
I have cutted the sum on n= 100 and calculated the result by iteration.

$$\alpha = \left[\sum_{n=0}^{\infty} \frac{\alpha^n}{(2\pi)^{\binom{n}{2}}} \right]^2 .e^{-\frac{\pi^2}{2}}$$

$$\alpha \approx 0,0072973525686 \approx \frac{1}{137,035\ 999\ 096}$$

$Value\ for\ \alpha\ by\ Wikipedia$

$$\alpha = 0,0072973525693(11)$$

The calculated value by the HdV formular fits very good to the empirical measurements.

2. Conclusion

I found a realistic interpretation for the Hans de Vries formular by stochastic arguments.
Some questions still leave open but the work can be seen as first step into deeper understanding.
Its exciting to understand the source for the electrical charge and maybe an important physically open question is answered by the HdV formular and this interpretation.

References

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