Analyzing some Ramanujan equations: mathematical connections with Prime Numbers Theory, ϕ, $\zeta(2)$ and various parameters of Particle Physics. II

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Abstract

In this paper we have described and analyzed some Ramanujan equations. We have obtained several mathematical connections between Prime Numbers Theory, ϕ, $\zeta(2)$ and various parameters of Particle Physics.

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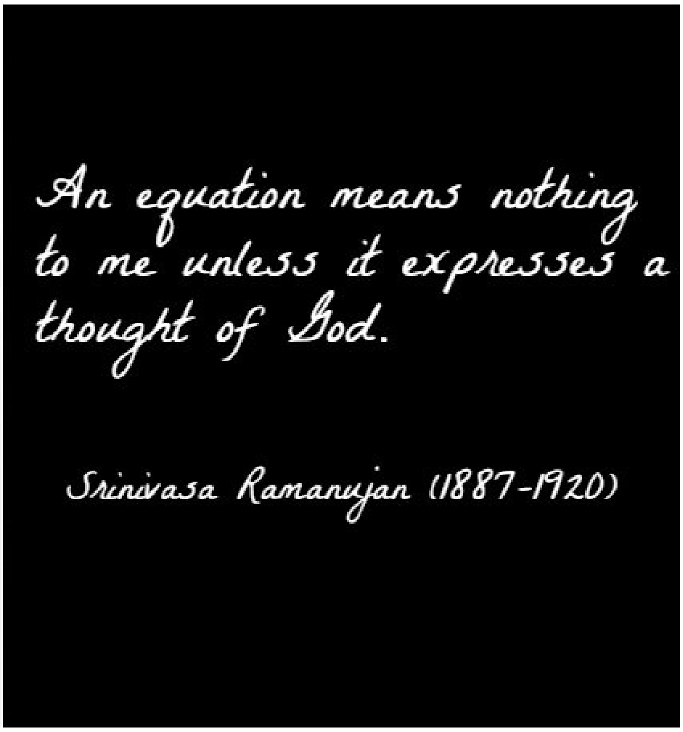


https://mobygeek.com/features/indian-mathematician-srinivasa-ramanujan-quotes-11012

From:

II

RAMANUJAN AND THE THEORY OF PRIME NUMBERS

16 *Jan.* 1913

We want to analyze the following Hardy's observations

Combining (2.9.6) and (2.10.5), we obtain

(2.10.6) $$\Phi(y) = e^{-y}\log 1 - e^{-2y}\log 2 + e^{-3y}\log 3 - \ldots.$$

Ramanujan now infers that

(2.10.7) $$\Phi(y) \to l,$$[1]

or

(2.10.8) $$\phi(y) - \phi(2y) + \phi(3y) - \ldots \to l,$$

for some l. He gives no reason, but the conclusion is correct and easily proved.[2] Up to this point his argument, though expressed in a less convenient notation than that which I have used, is quite sound.

Next, Ramanujan infers from (2.10.8) that

(2.10.9) $$\phi(y) \to l.$$

All that would be necessary, if he were aiming at the Prime Number Theorem only, would be the milder conclusion that

(2.10.10) $$\phi(y) = o\left(\frac{1}{y}\right),$$

and we may continue his argument as if he asserted no more than this. He then states that

(2.10.11) $$\phi_2(y) = \log 2 \sum_1^{\infty} 2^m e^{-2^m y} \sim \frac{1}{y},$$

and from (2.10.10) and (2.10.11) he deduces that

(2.10.12) $$\phi_1(y) = \phi(y) + \phi_2(y) \sim \frac{1}{y},$$

which is (2.8.3). What he actually says and professes to derive from (2.10.9) is that

(2.10.13) $$\phi_1(y) = \frac{1}{y} + O(1),$$

or at any rate

(2.10.14) $$\phi_1(y) = \frac{1}{y} + O(y^{-\delta})$$

for every positive δ.

Now (2.8.3) is true; it is, as I said, the half-way stage in the "Hardy-Littlewood" proof; and from (2.8.3) we can deduce the Prime Number Theorem in an "elementary" manner, that is to say by arguments which make no use of the notion of an analytic function of the complex variable.

[1] I use l for "a limit" (not necessarily the same in different contexts).

[2] For example, the series

$$\log 1 - \log 2 + \log 3 - \ldots$$

is summable $(C, 1)$. The sum is $-\frac{1}{2}\log\frac{1}{2}\pi$.

It follows that, if Ramanujan really had proved (2.10.12), he would have found an elementary proof of the Prime Number Theorem, a proof involving no function-theory at all. In particular, he would never have needed (2.7.6); and this is of course enough to convince any reader who knows the subject that the proof cannot possibly be correct. And in fact Ramanujan has deduced the true conclusion from two false propositions, the proposition (2.10.11), and the proposition that (2.10.8) implies (2.10.10).

2.11. I had better show the falsity of these propositions at once. In the first place, (2.10.8) does not imply (2.10.10), and still less (2.10.9). Suppose, for example, that

$$\chi(y) = y^{-1-ai}.$$

Then

$$\chi(y) - \chi(2y) + \chi(3y) - \ldots = y^{-1-ai}(1 - 2^{-1-ai} + 3^{-1-ai} - \ldots)$$
$$= (1 - 2^{-ai})\,\zeta(1+ai)\,y^{-1-ai},$$

which is 0 if

$$a = \frac{2k\pi}{\log 2};$$

but $y\chi(y)$ oscillates, in contradiction to Ramanujan's statement. It is true that $\chi(y)$ is not a power-series in e^{-y}, as is Ramanujan's $\phi_2(y)$, but we can find such series which mimic the behaviour of $\chi(y)$ as closely as we please, and the statement cannot be rehabilitated by any such reservation.

It is only natural that Ramanujan's argument should contain flaws like this, where his instincts misled him about the validity of difficult general theorems. There are true Tauberian theorems which have some superficial resemblance to the one which I have just refuted, and a good deal of experience and subtlety is needed to distinguish the true from the false. His second error is much more surprising, since one would have expected him to be right about the behaviour of a special function like $\phi_2(y)$.

He seems to have been deceived by an "integral analogy". The integral analogue of the series (2.10.11) is

$$\text{(2.11.1)} \qquad \log 2 \int_0^\infty 2^x e^{-2^x y}\,dx,$$

and

$$\int_0^\infty 2^x e^{-2^x y}\,dx = \frac{1}{\log 2}\int_1^\infty e^{-yz}\,dz = \frac{e^{-y}}{y \log 2} \sim \frac{1}{y\log 2},$$

so that (2.11.1) behaves in the manner which he attributes to (2.10.11). But (2.10.11) itself behaves differently, having "wobbles" of order $1/y$.

We can refute Ramanujan's assertion in numerous ways. In the first place, if (2.10.11) were true it would follow (by the Hardy-Littlewood Tauberian theorem) that

$$\sum_{2^n \leq x} 2^n \sim \frac{x}{\log 2}.$$

This is plainly false, since the series is practically doubled when x passes through a value 2^m.

A more direct argument is as follows. The function $\phi_2(y)$ satisfies the equation

$$\phi_2(y) - 2\phi_2(2y) = 2e^{-2y}\log 2.$$

It may also be verified at once that

$$\psi_2(y) = -\log 2 \sum_0^\infty \frac{(-1)^r y^r}{r!} \frac{2^{r+1}}{2^{r+1}-1}$$

satisfies

$$\psi_2(y) - 2\psi_2(2y) = 2e^{-2y}\log 2,$$

and therefore

$$h(y) = \phi_2(y) - \psi_2(y)$$

satisfies

$$h(y) - 2h(2y) = 0.$$

Also $y\,h(y)$ is not a constant.[1]

If now we write

$$y\,h(y) = H(\log y),$$

then

$$H(\log y) = H(\log y + \log 2),$$

so that H is periodic and not constant. Hence $y\,h(y)$ does not tend to a limit, nor does $y\phi_2(y)$.

Finally we can, if we please, exhibit the "wobbles" in a formula. We can prove that

$$(2.11.2)\quad \phi_2(y) = \frac{1}{y} - \log 2 \sum_0^\infty \frac{(-1)^r y^r}{r!} \frac{2^{r+1}}{2^{r+1}-1} - \frac{1}{y}\sum_{-\infty}^{\infty}{}' \Gamma\left(\frac{1+2k\pi i}{\log 2}\right) y^{-2k\pi i/\log 2},$$

where the dash excludes the value $k = 0$; and the last series shows the wobbles, of order $1/y$, explicitly. It converges rapidly, and the wobbles are small compared with the dominant term.

The formula (2.11.2) may be deduced by differentiation from the last formula on p. 283 of this paper (in which the sign of the last term should be changed).

Ramanujan, when I disputed the truth of his statement, produced the amended formula

$$\text{`}\phi_2(y) + \log 2\left(1 - \frac{y}{3.1!} + \frac{y^2}{7.2!} - \frac{y^3}{15.3!} + \dots\right) = \frac{1}{y} + F(y),$$

where

$$yF(y) = \cdot 0000098844 \cos\left(\frac{2\pi \log y}{\log 2} + \cdot 872811\right)$$

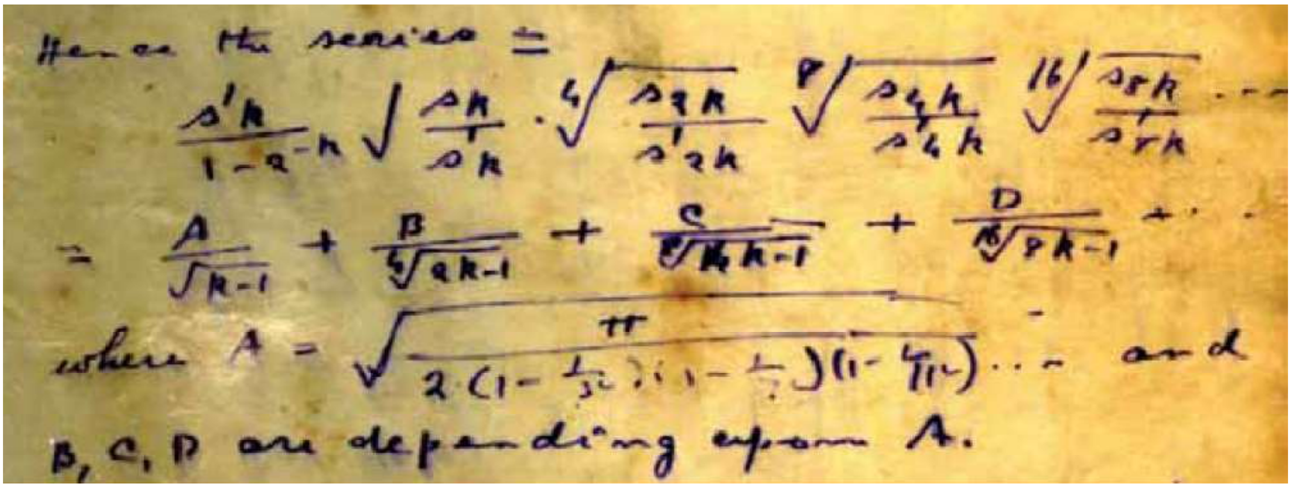
correct to 10 places of decimals'. This takes account explicitly of the terms in which $k = \pm 1$.

We have analyzed the mathematics described in some pages of Manuscript Book 3, which precede Ramanujan's formula and we have obtained interesting results, which we have shown in this paper

From:

Manuscript Book 3 of Srinivasa Ramanujan

Now, we have (page 4)


Hence the series = $\frac{s'k}{1-q^{-k}} \sqrt{\frac{sk}{s'k}} \cdot \sqrt[4]{\frac{s2k}{s'2k}} \sqrt[8]{\frac{s4k}{s'4k}} \sqrt[16]{\frac{s8k}{s'8k}} \cdots$

$= \frac{A}{\sqrt{k-1}} + \frac{B}{\sqrt[4]{2k-1}} + \frac{C}{\sqrt[8]{4k-1}} + \frac{D}{\sqrt[16]{8k-1}} + \cdots$

where $A = \sqrt{\frac{\pi}{2(1-\frac{1}{3^2})(1-\frac{1}{7^2})(1-\frac{1}{11^2})\cdots}}$ and B, C, D are depending upon A.

Thence:

sqrt[Pi/((((2(1-1/(3^2))(1-1/(7^2))(1-1/(11^2))))))]

Input:

$$\sqrt{\frac{\pi}{2\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{7^2}\right)\left(1-\frac{1}{11^2}\right)}}$$

Exact result:

$$\frac{77\sqrt{\frac{\pi}{10}}}{32}$$

Decimal approximation:

1.3487010114457515932717773356496802150836966584100947231 30...

1.3487010114457…..

Property:

$\frac{77\sqrt{\frac{\pi}{10}}}{32}$ is a transcendental number

All 2nd roots of (5929 π)/10240:

$\frac{77}{32}\sqrt{\frac{\pi}{10}}\, e^{0} \approx 1.3487$ (real, principal root)

$\frac{77}{32}\sqrt{\frac{\pi}{10}}\, e^{i\pi} \approx -1.3487$ (real root)

Series representations:

$$\sqrt{\frac{\pi}{2\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{7^2}\right)\left(1-\frac{1}{11^2}\right)}} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1+\frac{5929\pi}{10240}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{\frac{\pi}{2\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{7^2}\right)\left(1-\frac{1}{11^2}\right)}} = \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{5929\pi}{10240}-z_0\right)^k z_0^{-k}}{k!}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\sqrt{\frac{\pi}{2\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{7^2}\right)\left(1-\frac{1}{11^2}\right)}} = -\frac{\sum_{j=0}^{\infty} \text{Res}_{s=-j} \left(-1+\frac{5929\pi}{10240}\right)^{-s} \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}$$

From which, we obtain:

1+1/2 sqrt[Pi/((((2(1-1/(3^2))(1-1/(7^2))(1-1/(11^2))))))]

Input:

$$1+\frac{1}{2}\sqrt{\frac{\pi}{2\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{7^2}\right)\left(1-\frac{1}{11^2}\right)}}$$

Exact result:

$$1+\frac{77\sqrt{\frac{\pi}{10}}}{64}$$

Decimal approximation:

1.674350505722875796635888667824840107541848329205047361565...

1.6743505057…. result near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164.2696$ i.e. 1.65578...

Property:

$$1+\frac{77\sqrt{\frac{\pi}{10}}}{64} \text{ is a transcendental number}$$

Alternate forms:

$$\frac{1}{640}\left(640+77\sqrt{10\,\pi}\right)$$

$$\frac{64\sqrt{10}+77\sqrt{\pi}}{64\sqrt{10}}$$

Series representations:

$$1+\frac{1}{2}\sqrt{\frac{\pi}{2\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{7^2}\right)\left(1-\frac{1}{11^2}\right)}} = 1+\frac{1}{2}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-1+\frac{5929\,\pi}{10\,240}\right)^k\left(-\frac{1}{2}\right)_k}{k!}$$

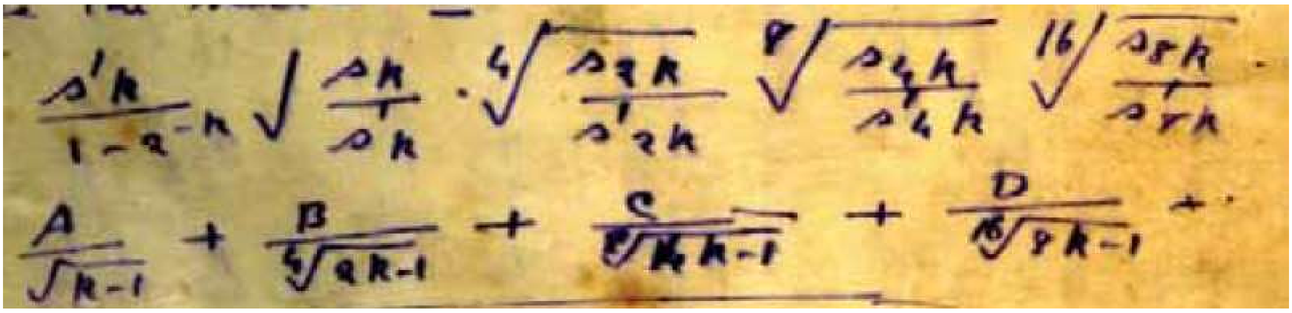
$$1+\frac{1}{2}\sqrt{\frac{\pi}{2\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{7^2}\right)\left(1-\frac{1}{11^2}\right)}} = 1+\frac{1}{2}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5929\,\pi}{10\,240}-z_0\right)^k z_0^{-k}}{k!}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$1+\frac{1}{2}\sqrt{\frac{\pi}{2\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{7^2}\right)\left(1-\frac{1}{11^2}\right)}} = 1+\frac{1}{2}\exp\left(i\,\pi\left\lfloor\frac{\arg\left(\frac{5929\,\pi}{10\,240}-x\right)}{2\,\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{5929\,\pi}{10\,240}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

for ($x \in \mathbb{R}$ and $x < 0$)

Now, from



For k = 2 and A = 1.3487010114457

1.3487010114457 / (sqrt(2-1)) + x/ ((2*2-1)^(1/4)) + y/((4*2-1)^(1/8)) + z/((8*2-1)^(1/16))

Input interpretation:

$$\frac{1.3487010114457}{\sqrt{2-1}} + \frac{x}{\sqrt[4]{2\times 2-1}} + \frac{y}{\sqrt[8]{4\times 2-1}} + \frac{z}{\sqrt[16]{8\times 2-1}}$$

Result:

$$\frac{x}{\sqrt[4]{3}} + \frac{y}{\sqrt[8]{7}} + \frac{z}{\sqrt[16]{15}} + 1.3487010114457$$

Geometric figure:

plane

Alternate forms:

$$1.9614\times 10^{-9}\,(3.87395\times 10^{8}\,x + 3.99758\times 10^{8}\,y + 4.30456\times 10^{8}\,z + 6.87622\times 10^{8})$$

$$0.7598356856516\,x + 0.7840842766892\,y + 0.8442951535105\,z + 1.3487010114457$$

$$0.7598356856516\,(1.0000000000000\,x + 1.0319129405153\,y + 1.1111549107969\,z + 1.7749903524064)$$

Real root:

$$z \approx -0.8999645236529\,x - 0.9286850379622\,y - 1.5974283469920$$

Root:

$$z \approx -0.8999645236529\,x - 0.9286850379622\,y - 1.5974283469920$$

Properties as a function:

Domain

$\mathbb{R}^3$

Range

$\mathbb{R}$ (all real numbers)

$\mathbb{R}$ is the set of real numbers

Root for the variable z:

$$z \approx 1.1844199221588\,(-0.7598356856516\,x - 0.7840842766892\,y - 1.3487010114457)$$

Partial derivatives:

$$\frac{\partial}{\partial x}\left(\frac{x}{\sqrt[4]{3}} + \frac{y}{\sqrt[8]{7}} + \frac{z}{\sqrt[16]{15}} + 1.3487010114457\right) = \frac{1}{\sqrt[4]{3}}$$

$$\frac{\partial}{\partial y}\left(\frac{x}{\sqrt[4]{3}} + \frac{y}{\sqrt[8]{7}} + \frac{z}{\sqrt[16]{15}} + 1.3487010114457\right) = \frac{1}{\sqrt[8]{7}}$$

$$\frac{\partial}{\partial z}\left(\frac{x}{\sqrt[4]{3}} + \frac{y}{\sqrt[8]{7}} + \frac{z}{\sqrt[16]{15}} + 1.3487010114457\right) = \frac{1}{\sqrt[16]{15}}$$

Indefinite integral:

$$\int\left(\frac{1.3487010114457}{\sqrt{2-1}} + \frac{x}{\sqrt[4]{2\times 2-1}} + \frac{y}{\sqrt[8]{4\times 2-1}} + \frac{z}{\sqrt[16]{8\times 2-1}}\right)dx = 0.37991784282580\,x^2 + 0.78408427668922\,x\,y + 0.84429515351053\,x\,z + 1.3487010114457\,x + \text{constant}$$

Definite integral over a sphere of radius R:

$$\iiint_{x^2+y^2+z^2<R^2}\left(\frac{x}{\sqrt[4]{3}} + \frac{y}{\sqrt[8]{7}} + \frac{z}{\sqrt[16]{15}} + 1.3487010114457\right)dx\,dy\,dz = 5.649425585929\,R^3$$

Definite integral over a cube of edge length 2 L:

$$\int_{-L}^{L}\int_{-L}^{L}\int_{-L}^{L}\left(1.3487010114457 + \frac{x}{\sqrt[4]{3}} + \frac{y}{\sqrt[8]{7}} + \frac{z}{\sqrt[16]{15}}\right)dz\,dy\,dx = 10.789608091566\,L^3$$

Thence:

B = -1.774990352407

C = 2.71918158991 × 10^-14

D = 4.3360531494978316297518598 × 10^-13

1.3487010114457 / (sqrt(2-1)) + (-1.774990352407)/ ((2*2-1)^(1/4)) + (2.71918158991e-14)/((4*2-1)^(1/8)) + (4.336053149497e-13)/((8*2-1)^(1/16))

Input interpretation:

$$\frac{1.3487010114457}{\sqrt{2-1}} - \frac{1.774990352407}{\sqrt[4]{2\times 2-1}} + \frac{2.71918158991\times 10^{-14}}{\sqrt[8]{4\times 2-1}} + \frac{4.336053149497\times 10^{-13}}{\sqrt[16]{8\times 2-1}}$$

Result:

$-4.73175... \times 10^{-14}$

$-4.73175...*10^{-14}$

From which:

-1/((((1.3487010114457 / (sqrt(2-1)) + (-1.774990352407)/ ((2*2-1)^(1/4)) + (2.71918158991e-14)/((4*2-1)^(1/8)) + (4.336053149497e-13)/((8*2-1)^(1/16))))))

Input interpretation:

$$-\frac{1}{\frac{1.3487010114457}{\sqrt{2-1}} - \frac{1.774990352407}{\sqrt[4]{2\times 2-1}} + \frac{2.71918158991\times 10^{-14}}{\sqrt[8]{4\times 2-1}} + \frac{4.336053149497\times 10^{-13}}{\sqrt[16]{8\times 2-1}}}$$

Result:

$\tilde{\infty}$

$\tilde{\infty}$ is complex infinity

Decimal approximation:

$2.1133844757525991136737819255680193283354353626113919... \times 10^{13}$

Decimal form:

21133844757525.991136737819255680193283354353626113919

21133844757525.99…..

[-1/(((1.3487010114457 / (sqrt(2-1)) + (-1.774990352407)/ ((2*2-1)^(1/4)) + (2.71918158991e-14)/((4*2-1)^(1/8)) + (4.336053149497e-13)/((8*2-1)^(1/16)))))]^1/64

Input interpretation:

$$\sqrt[64]{-\frac{1}{\frac{1.3487010114457}{\sqrt{2-1}} - \frac{1.774990352407}{\sqrt[4]{2\times 2-1}} + \frac{2.71918158991\times 10^{-14}}{\sqrt[8]{4\times 2-1}} + \frac{4.336053149497\times 10^{-13}}{\sqrt[16]{8\times 2-1}}}}$$

Result:

$\tilde{\infty}$

$\tilde{\infty}$ is complex infinity

Decimal approximation:

1.615112540682022156252269667466251317803156934065705182647...

1.6151125406… result that is a good approximation to the value of the golden ratio 1.618033988749...

For B, C and D equal to 1, we obtain:

1.3487010114457 / (sqrt(2-1)) + 1/ ((2*2-1)^(1/4)) + 1/((4*2-1)^(1/8)) + 1/((8*2-1)^(1/16))

Input interpretation:

$$\frac{1.3487010114457}{\sqrt{2-1}} + \frac{1}{\sqrt[4]{2\times 2-1}} + \frac{1}{\sqrt[8]{4\times 2-1}} + \frac{1}{\sqrt[16]{8\times 2-1}}$$

Result:

3.7369161272970...

3.7369161272970…

From which:

1+1/(((1.3487010114457 / (sqrt(2-1)) + 1/ ((2*2-1)^(1/4)) + 1/((4*2-1)^(1/8)) + 1/((8*2-1)^(1/16)))))^1/3

Input interpretation:

$$1+\frac{1}{\sqrt[3]{\frac{1.3487010114457}{\sqrt{2-1}}+\frac{1}{\sqrt[4]{2\times2-1}}+\frac{1}{\sqrt[8]{4\times2-1}}+\frac{1}{\sqrt[16]{8\times2-1}}}}$$

Result:

1.64440991881989...

1.64440991881989…. ≈ $\zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Or for B = 1.3487010114457^(1/4) C = 1.3487010114457^(1/8)
D = 1.3487010114457^(1/16)

1.3487010114457 / (sqrt(2-1)) + 1.3487010114457^(1/4)/ ((2*2-1)^(1/4)) + 1.3487010114457^(1/8)/((4*2-1)^(1/8)) + 1.3487010114457^(1/16)/((8*2-1)^(1/16))

Input interpretation:

$$\frac{1.3487010114457}{\sqrt{2-1}}+\frac{\sqrt[4]{1.3487010114457}}{\sqrt[4]{2\times2-1}}+\frac{\sqrt[8]{1.3487010114457}}{\sqrt[8]{4\times2-1}}+\frac{\sqrt[16]{1.3487010114457}}{\sqrt[16]{8\times2-1}}$$

Result:

3.8417274648788...

3.8417274648788…

1+1/(((1.3487010114457 / (sqrt(2-1)) + 1.3487010114457^(1/4)/ ((2*2-1)^(1/4)) + 1.3487010114457^(1/8)/((4*2-1)^(1/8)) + 1.3487010114457^(1/16)/((8*2-1)^(1/16)))))^1/3

Input interpretation:

$$1+\frac{1}{\sqrt[3]{\frac{1.3487010114457}{\sqrt{2-1}}+\frac{\sqrt[4]{1.3487010114457}}{\sqrt[4]{2\times 2-1}}+\frac{\sqrt[8]{1.3487010114457}}{\sqrt[8]{4\times 2-1}}+\frac{\sqrt[16]{1.3487010114457}}{\sqrt[16]{8\times 2-1}}}}$$

Result:

1.63849546335453...

1.63849546335453…. ≈ $\zeta(2) = \frac{\pi^2}{6} = 1.644934$ ...

Now, we have that (page 4):

and we obtain:

sqrt((2(1-1/(3^2))(1-1/(7^2))(1-1/(11^2))(1-1/(19^2))))

Input:

$$\sqrt{2\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{7^2}\right)\left(1-\frac{1}{11^2}\right)\left(1-\frac{1}{19^2}\right)}$$

Exact result:

1.312371838687628161312371838687628161312371838687628161312...

1.3123718386….

Repeating decimal:

$1.\overline{312371838687628161}$ (period 18)

All 2nd roots of 3686400/2140369:

$\frac{1920 e^0}{1463} \approx 1.3124$ (real, principal root)

$\frac{1920 e^{i\pi}}{1463} \approx -1.3124$ (real root)

(1+1/7)(1+1/11)(1+1/19)

Input:

$$\left(1+\frac{1}{7}\right)\left(1+\frac{1}{11}\right)\left(1+\frac{1}{19}\right)$$

Exact result:

$$\frac{1920}{1463}$$

Decimal approximation:

1.312371838687628161312371838687628161312371838687628161312...

1.31237183868…

Repeating decimal:

$1.\overline{312371838687628161}$ (period 18)

1+1/2 sqrt((2(1-1/(3^2))(1-1/(7^2))(1-1/(11^2))(1-1/(19^2))))

Input:

$$1+\frac{1}{2}\sqrt{2\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{7^2}\right)\left(1-\frac{1}{11^2}\right)\left(1-\frac{1}{19^2}\right)}$$

Exact result:

$$\frac{2423}{1463}$$

Decimal approximation:

1.656185919343814080656185919343814080656185919343814080656...

Repeating decimal:

$1.\overline{656185919343814080}$ (period 18)

1.656185919343814080 result very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164.2696$ i.e. 1.65578...

(((sqrt((2(1-1/(3^2))(1-1/(7^2))(1-1/(11^2))(1-1/(19^2)))))))^28-322+29

Input:

$$\sqrt{2\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{7^2}\right)\left(1-\frac{1}{11^2}\right)\left(1-\frac{1}{19^2}\right)}^{28} - 322 + 29$$

Exact result:

73 184 043 198 562 322 621 718 913 147 652 150 738 960 708 612 223 275 749 249 704 733 219 890 585 694 827 165 030 058 747 / 42 348 802 231 187 137 787 060 717 408 722 876 276 440 616 678 015 751 752 984 202 821 775 117 454 966 460 187 610 721

Decimal approximation:

1728.125456749448178932121871333540855688072443303465291585...

1728.12545674….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

10^2(((sqrt((2(1-1/(3^2))(1-1/(7^2))(1-1/(11^2))(1-1/(19^2)))))))+8

Input:

$$10^2 \sqrt{2\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{7^2}\right)\left(1-\frac{1}{11^2}\right)\left(1-\frac{1}{19^2}\right)} + 8$$

Exact result:

$$\frac{203704}{1463}$$

Decimal approximation:

139.237183868762816131237183868762816131237183868762816131 2...

139.23718386….. result practically equal to the rest mass of Pion meson 139.57 MeV

10^2(((sqrt((2(1-1/(3^2))(1-1/(7^2))(1-1/(11^2))(1-1/(19^2)))))))-5-1/golden ratio

Input:

$$10^2 \sqrt{2\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{7^2}\right)\left(1-\frac{1}{11^2}\right)\left(1-\frac{1}{19^2}\right)} - 5 - \frac{1}{\phi}$$

ϕ is the golden ratio

Exact result:

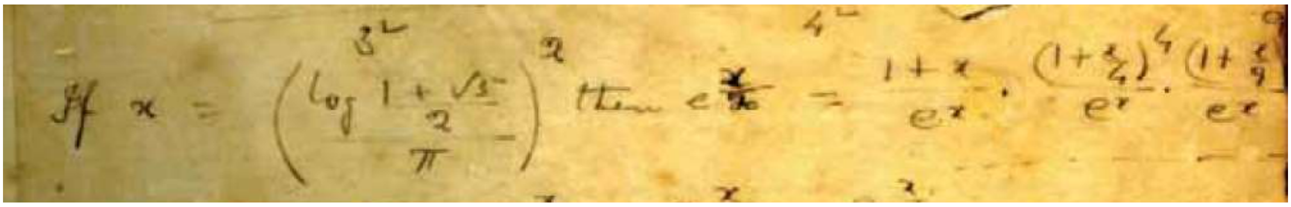
$$\frac{184685}{1463} - \frac{1}{\phi}$$

Decimal approximation:

125.6191498800129212830325970343971780135168746889570532691...

125.6191498….. result very near to the Higgs boson mass 125.18 GeV

Now, we have that (page 6):



(((ln[((1+(sqrt5))/2)]*1/Pi)))^2

Input:

$$\left(\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right) \times \frac{1}{\pi}\right)^2$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{\log^2\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)}{\pi^2}$$

Decimal approximation:

0.023462421710806909463112025130508650169194928065080959403...

0.0234624217108…..

Alternate forms:

$$\frac{\operatorname{csch}^{-1}(2)^2}{\pi^2}$$

$$\frac{\log^2\left(\frac{2}{1+\sqrt{5}}\right)}{\pi^2}$$

$$\frac{\left(\log\left(1+\sqrt{5}\right)-\log(2)\right)^2}{\pi^2}$$

$\operatorname{csch}^{-1}(x)$ is the inverse hyperbolic cosecant function

Alternative representations:

$$\left(\frac{\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)}{\pi}\right)^2 = \left(\frac{\log_e\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)}{\pi}\right)^2$$

$$\left(\frac{\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)}{\pi}\right)^2 = \left(\frac{\log(a)\log_a\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)}{\pi}\right)^2$$

$$\left(\frac{\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)}{\pi}\right)^2 = \left(-\frac{\operatorname{Li}_1\left(1+\frac{1}{2}\left(-1-\sqrt{5}\right)\right)}{\pi}\right)^2$$

Series representations:

$$\left(\frac{\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)}{\pi}\right)^2 = \frac{\left(\sum_{k=1}^{\infty}\frac{\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^k}{k}\right)^2}{\pi^2}$$

$$\left(\frac{\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)}{\pi}\right)^2 = \frac{\left(2 i \pi \left\lfloor \frac{\arg\left(1+\sqrt{5}-2x\right)}{2\pi}\right\rfloor + \log(x) - \sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k \left(1+\sqrt{5}-2x\right)^k x^{-k}}{k}\right)^2}{\pi^2} \quad \text{for } x < 0$$

$$\left(\frac{\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)}{\pi}\right)^2 = \frac{\left(2 i \pi \left\lfloor \frac{\arg\left(\frac{1}{2}\left(1+\sqrt{5}\right)-x\right)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(1+\sqrt{5}-2x\right)^k x^{-k}}{k}\right)^2}{\pi^2}$$

for $x < 0$

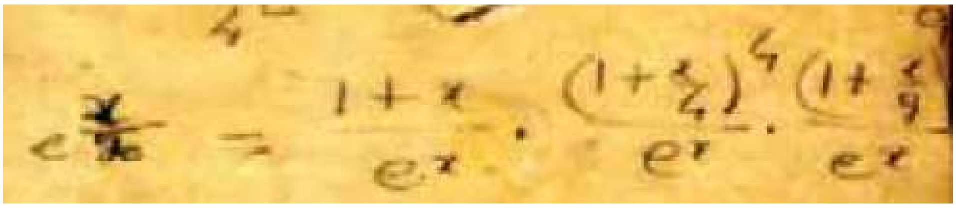
Integral representations:

$$\left(\frac{\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)}{\pi}\right)^2 = \frac{\left(\int_1^{\frac{1}{2}\left(1+\sqrt{5}\right)} \frac{1}{t}\, dt\right)^2}{\pi^2}$$

$$\left(\frac{\log\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)}{\pi}\right)^2 = -\frac{\left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-1+\frac{1}{2}\left(1+\sqrt{5}\right)\right)^{-s} \Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds\right)^2}{4\pi^4} \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

Page 6



Exp(0.0234624217108/2)

Input interpretation:

$$\exp\left(\frac{0.0234624217108}{2}\right)$$

Result:

1.0118002913779...

1.0118002913779…

(1+0.0234624217108)/(exp(0.0234624217108)) * (1+1/4)^4/(exp(0.0234624217108)) * (1+2/9)^9/(exp(0.0234624217108))

Input interpretation:

$$\frac{1+0.0234624217108}{\exp(0.0234624217108)} \times \frac{\left(1+\frac{1}{4}\right)^4}{\exp(0.0234624217108)} \times \frac{\left(1+\frac{2}{9}\right)^9}{\exp(0.0234624217108)}$$

Result:

14.17407524806...

14.17407524806…

1+1/((((1+0.0234624217108)/(exp(0.0234624217108)) * (1+1/4)^4/(exp(0.0234624217108)) * (1+2/9)^9/(exp(0.0234624217108)))))^1/6

Input interpretation:

$$1+\frac{1}{\sqrt[6]{\frac{1+0.0234624217108}{\exp(0.0234624217108)} \times \frac{\left(1+\frac{1}{4}\right)^4}{\exp(0.0234624217108)} \times \frac{\left(1+\frac{2}{9}\right)^9}{\exp(0.0234624217108)}}}$$

Result:

1.6428123484629...

1.6428123484629….. $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

123((((1+0.0234624217108)/(exp(0.0234624217108)) * (1+1/4)^4/(exp(0.0234624217108)) * (1+2/9)^9/(exp(0.0234624217108)))))-11-4+1/golden ratio

Input interpretation:

$$123\left(\frac{1+0.0234624217108}{\exp(0.0234624217108)} \times \frac{\left(1+\frac{1}{4}\right)^4}{\exp(0.0234624217108)} \times \frac{\left(1+\frac{2}{9}\right)^9}{\exp(0.0234624217108)}\right) - 11 - 4 + \frac{1}{\phi}$$

ϕ is the golden ratio

Result:

1729.029289500...

1729.0292895….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Pi^2((((1+0.0234624217108)/(exp(0.0234624217108)) * (1+1/4)^4/(exp(0.0234624217108)) * (1+2/9)^9/(exp(0.0234624217108)))))-1/golden ratio

Input interpretation:

$$\pi^2\left(\frac{1+0.0234624217108}{\exp(0.0234624217108)}\times\frac{\left(1+\frac{1}{4}\right)^4}{\exp(0.0234624217108)}\times\frac{\left(1+\frac{2}{9}\right)^9}{\exp(0.0234624217108)}\right)-\frac{1}{\phi}$$

ϕ is the golden ratio

Result:

139.2744814609...

139.2744814609…. result practically equal to the rest mass of Pion meson 139.57 MeV

Pi^2((((1+0.0234624217108)/(exp(0.0234624217108)) * (1+1/4)^4/(exp(0.0234624217108)) * (1+2/9)^9/(exp(0.0234624217108)))))-13-golden ratio

Input interpretation:

$$\pi^2\left(\frac{1+0.0234624217108}{\exp(0.0234624217108)}\times\frac{\left(1+\frac{1}{4}\right)^4}{\exp(0.0234624217108)}\times\frac{\left(1+\frac{2}{9}\right)^9}{\exp(0.0234624217108)}\right)-13-\phi$$

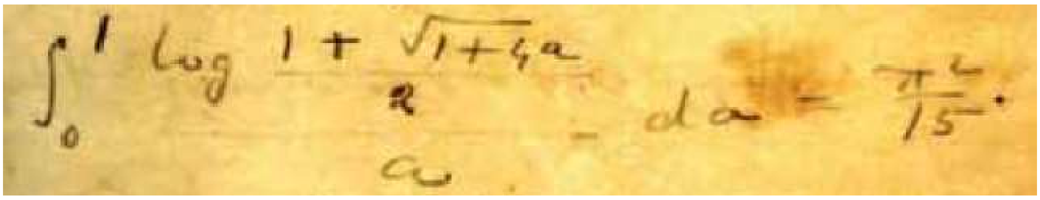
ϕ is the golden ratio

Result:

125.2744814609...

125.2744814609… result very near to the Higgs boson mass 125.18 GeV

From (page 6)



integrate log(((((1+sqrt(1+4a))/2))))/a da, 0..1

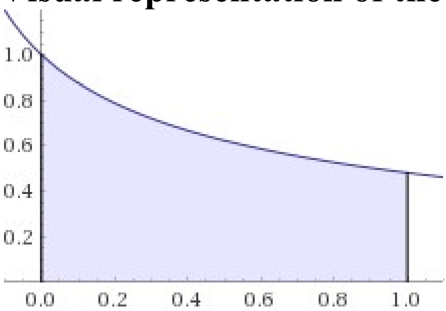
Definite integral:

$$\int_0^1 \frac{\log\left(\frac{1}{2}\left(1+\sqrt{1+4a}\right)\right)}{a}\, da = \frac{\pi^2}{15} \approx 0.657974$$

0.657974

log(x) is the natural logarithm

Visual representation of the integral:



Indefinite integral:

$$\int \frac{\log\left(\frac{1}{2}\left(1+\sqrt{1+4a}\right)\right)}{a}\, da = \text{Li}_2\left(\frac{1}{2}\left(\sqrt{4a+1}+1\right)\right) + \frac{1}{2}\left(2\log\left(1-\sqrt{4a+1}\right)+\log\left(\frac{1}{8}\left(\sqrt{4a+1}+1\right)\right)\right)\log\left(\frac{1}{2}\left(\sqrt{4a+1}+1\right)\right) + \text{constant}$$

(assuming a complex-valued logarithm)

$\text{Li}_n(x)$ is the polylogarithm function and:

1+(((integrate log(((((1+sqrt(1+4a))/2))))/a da, 0..1)))

Input:

$$1+\int_0^1 \frac{\log\left(\frac{1}{2}\left(1+\sqrt{1+4a}\right)\right)}{a}\,da$$

log(x) is the natural logarithm

Computation result:

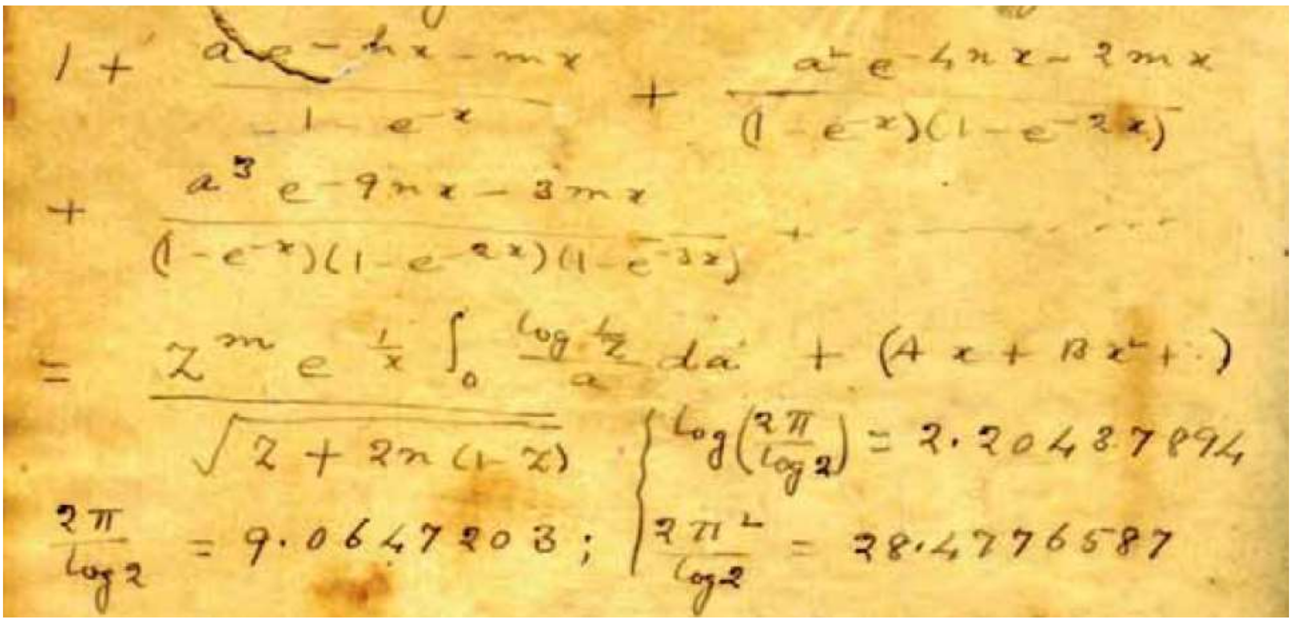
$$1+\int_0^1 \frac{\log\left(\frac{1}{2}\left(1+\sqrt{1+4a}\right)\right)}{a}\,da = 1.65797$$

Decimal approximation:

1.657973626739290574588966066658410075687579960482719375094...

1.6579736267…. result very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164.2696$ i.e. 1.65578...

From (page 7)



$$1+\frac{a e^{-nx-mx}}{1-e^{-x}}+\frac{a^2 e^{-4nx-2mx}}{(1-e^{-x})(1-e^{-2x})}+\frac{a^3 e^{-9nx-3mx}}{(1-e^{-x})(1-e^{-2x})(1-e^{-3x})}+\cdots$$

$$=\frac{z^m e^{\frac{1}{x}\int_0^{a}\frac{\log z}{a}da}}{\sqrt{z+2n(1-z)}}+(Ax+Bx^2+\cdots)$$

$$\log\left(\frac{2\pi}{\log 2}\right)=2.2048\,7894$$

$$\frac{2\pi}{\log 2}=9.0647203;\quad \frac{2\pi^2}{\log 2}=28.4776587$$

we have:

ln((2Pi)/(ln2))

Input:

$$\log\left(\frac{2\pi}{\log(2)}\right)$$

$\log(x)$ is the natural logarithm

Decimal approximation:

2.204389986991009810573098631043904749177058395112672088687...

2.20438998699…

Alternate form:

$$\log(2) + \log(\pi) - \log(\log(2))$$

Alternative representations:

$$\log\left(\frac{2\pi}{\log(2)}\right) = \log_e\left(\frac{2\pi}{\log(2)}\right)$$

$$\log\left(\frac{2\pi}{\log(2)}\right) = \log(a)\log_a\left(\frac{2\pi}{\log(2)}\right)$$

$$\log\left(\frac{2\pi}{\log(2)}\right) = -\text{Li}_1\left(1 - \frac{2\pi}{\log(2)}\right)$$

Series representations:

$$\log\left(\frac{2\pi}{\log(2)}\right) = \log\left(-1 + \frac{2\pi}{\log(2)}\right) - \sum_{k=1}^{\infty} \frac{\left(-\frac{\log(2)}{2\pi - \log(2)}\right)^k}{k}$$

$$\log\left(\frac{2\pi}{\log(2)}\right) = 2 i \pi \left\lfloor \frac{\arg\left(-x + \frac{2\pi}{\log(2)}\right)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} \left(-x + \frac{2\pi}{\log(2)}\right)^k}{k} \quad \text{for } x < 0$$

$$\log\left(\frac{2\pi}{\log(2)}\right) = 2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{2\pi}{\log(2)} - z_0\right)^k z_0^{-k}}{k}$$

Integral representations:

$$\log\left(\frac{2\pi}{\log(2)}\right) = \int_1^{\frac{2\pi}{\log(2)}} \frac{1}{t}\, dt$$

$$\log\left(\frac{2\pi}{\log(2)}\right) = -\frac{i}{2\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\,\Gamma(1+s)\left(-1+\frac{2\pi}{\log(2)}\right)^{-s}}{\Gamma(1-s)}\, ds \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

(2Pi^2)/ln2

Input:

$$\frac{2\pi^2}{\log(2)}$$

$\log(x)$ is the natural logarithm

Decimal approximation:

28.477658649975010867721351422733690893640556875329304062 90...

28.47765864…

Alternative representations:

$$\frac{2\pi^2}{\log(2)} = \frac{2\pi^2}{\log_e(2)}$$

$$\frac{2\pi^2}{\log(2)} = \frac{2\pi^2}{\log(a)\log_a(2)}$$

$$\frac{2\pi^2}{\log(2)} = \frac{2\pi^2}{2\coth^{-1}(3)}$$

Series representations:

$$\frac{2\pi^2}{\log(2)} = \frac{2\pi^2}{2 i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{2\pi^2}{\log(2)} = \frac{2\pi^2}{\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

$$\frac{2\pi^2}{\log(2)} = \frac{2\pi^2}{2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{2\pi^2}{\log(2)} = \frac{2\pi^2}{\int_1^2 \frac{1}{t}\, dt}$$

$$\frac{2\pi^2}{\log(2)} = \frac{4 i \pi^3}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds} \quad \text{for } -1 < \gamma < 0$$

(2Pi)/ln2

Input:

$$\frac{2\pi}{\log(2)}$$

$\log(x)$ is the natural logarithm

Decimal approximation:

9.064720283654387619255365891433333620343722935447591168372...

9.06472028…

Alternative representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a)\log_a(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\coth^{-1}(3)}$$

Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_1^2 \frac{1}{t}\, dt}$$

$$\frac{2\pi}{\log(2)} = \frac{4 i \pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)}\, ds} \quad \text{for } -1 < \gamma < 0$$

Summing the various results, we obtain:

((ln((2Pi)/(ln2)))) + (((2Pi^2)/ln2)) + (((2Pi)/ln2))

Input:

$$\log\left(\frac{2\pi}{\log(2)}\right) + \frac{2\pi^2}{\log(2)} + \frac{2\pi}{\log(2)}$$

$\log(x)$ is the natural logarithm

Decimal approximation:

39.74676892062040829754981594521092926316133820588956731996...

39.74676892…

Alternate forms:

$$\frac{2\pi(1+\pi)}{\log(2)} + \log\left(\frac{2\pi}{\log(2)}\right)$$

$$\frac{2\pi + 2\pi^2 + \log(2)\log\left(\frac{2\pi}{\log(2)}\right)}{\log(2)}$$

$$\frac{2\pi + 2\pi^2 + \log(2)\log(\pi) - \log(2)(\log(\log(2)) - \log(2))}{\log(2)}$$

Alternative representations:

$$\log\left(\frac{2\pi}{\log(2)}\right)+\frac{2\pi^2}{\log(2)}+\frac{2\pi}{\log(2)}=\log_e\left(\frac{2\pi}{\log(2)}\right)+\frac{2\pi}{\log_e(2)}+\frac{2\pi^2}{\log_e(2)}$$

$$\log\left(\frac{2\pi}{\log(2)}\right)+\frac{2\pi^2}{\log(2)}+\frac{2\pi}{\log(2)}=\log(a)\log_a\left(\frac{2\pi}{\log(2)}\right)+\frac{2\pi}{\log(a)\log_a(2)}+\frac{2\pi^2}{\log(a)\log_a(2)}$$

$$\log\left(\frac{2\pi}{\log(2)}\right)+\frac{2\pi^2}{\log(2)}+\frac{2\pi}{\log(2)}=-\text{Li}_1\left(1-\frac{2\pi}{\log(2)}\right)+-\frac{2\pi}{\text{Li}_1(-1)}+-\frac{2\pi^2}{\text{Li}_1(-1)}$$

Series representations:

$$\log\left(\frac{2\pi}{\log(2)}\right)+\frac{2\pi^2}{\log(2)}+\frac{2\pi}{\log(2)}=$$
$$\left\lfloor\frac{\arg\left(\frac{2\pi}{\log(2)}-z_0\right)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg\left(\frac{2\pi}{\log(2)}-z_0\right)}{2\pi}\right\rfloor\log(z_0)+$$
$$\frac{2\pi}{\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}}+$$
$$\frac{2\pi^2}{\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}}-$$
$$\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{2\pi}{\log(2)}-z_0\right)^k z_0^{-k}}{k}$$

$$\log\left(\frac{2\pi}{\log(2)}\right)+\frac{2\pi^2}{\log(2)}+\frac{2\pi}{\log(2)} = \left(-2\pi-2\pi^2+4\pi^2\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\left\lfloor\frac{\arg\left(-x+\frac{2\pi}{\log(2)}\right)}{2\pi}\right\rfloor-2i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\log(x)-2i\pi\left\lfloor\frac{\arg\left(-x+\frac{2\pi}{\log(2)}\right)}{2\pi}\right\rfloor\log(x)-\log^2(x)+2i\pi\left\lfloor\frac{\arg\left(-x+\frac{2\pi}{\log(2)}\right)}{2\pi}\right\rfloor\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}+\log(x)\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}+2i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\sum_{k=1}^{\infty}\frac{(-1)^k x^{-k}\left(-x+\frac{2\pi}{\log(2)}\right)^k}{k}+\log(x)\sum_{k=1}^{\infty}\frac{(-1)^k x^{-k}\left(-x+\frac{2\pi}{\log(2)}\right)^k}{k}-\sum_{k_1=1}^{\infty}\sum_{k_2=1}^{\infty}\frac{(-1)^{k_1+k_2}(2-x)^{k_1}x^{-k_1-k_2}\left(-x+\frac{2\pi}{\log(2)}\right)^{k_2}}{k_1 k_2}\right)\Big/\left(-2i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor-\log(x)+\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}\right) \quad \text{for } x<0$$

$$\log\left(\frac{2\pi}{\log(2)}\right)+\frac{2\pi^2}{\log(2)}+\frac{2\pi}{\log(2)} = \left(-2\pi-2\pi^2+4\pi^2\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor^2-4i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor\log(z_0)-\log^2(z_0)+2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}+\log(z_0)\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}+2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{2\pi}{\log(2)}-z_0\right)^k z_0^{-k}}{k}+\log(z_0)\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{2\pi}{\log(2)}-z_0\right)^k z_0^{-k}}{k}-\sum_{k_1=1}^{\infty}\sum_{k_2=1}^{\infty}\frac{(-1)^{k_1+k_2}(2-z_0)^{k_1}\left(\frac{2\pi}{\log(2)}-z_0\right)^{k_2}z_0^{-k_1-k_2}}{k_1 k_2}\right)\Big/\left(-2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor-\log(z_0)+\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)$$

Integral representations:

$$\log\left(\frac{2\pi}{\log(2)}\right)+\frac{2\pi^2}{\log(2)}+\frac{2\pi}{\log(2)}=\frac{2\pi+2\pi^2+\log(2)\int_0^1\int_0^1\frac{1}{(1+t_1)(\log(2)+(2\pi-\log(2))t_2)}\,dt_2\,dt_1}{\int_1^2\frac{1}{t}\,dt}$$

$$\log\left(\frac{2\pi}{\log(2)}\right)+\frac{2\pi^2}{\log(2)}+\frac{2\pi}{\log(2)}=$$
$$\frac{i\left(8\pi^3+8\pi^4-\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)\left(-1+\frac{2\pi}{\log(2)}\right)^{-s}}{\Gamma(1-s)}\,ds\right)}{2\pi\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\ \text{for}$$
$$-1<\gamma<0$$

Performing the following algebraic sum, we obtain:

-((ln((2Pi)/(ln2)))) + (((2Pi^2)/ln2)) - (((2Pi)/ln2))

Input:

$$-\log\left(\frac{2\pi}{\log(2)}\right)+\frac{2\pi^2}{\log(2)}-\frac{2\pi}{\log(2)}$$

log(x) is the natural logarithm

Decimal approximation:

17.20854837932961343789288690025645252411977554476904080584...

17.208548379…

Alternate forms:

$$\frac{2(\pi-1)\pi}{\log(2)}-\log(2\pi)+\log(\log(2))$$

$$\frac{2(\pi-1)\pi}{\log(2)}-\log\left(\frac{2\pi}{\log(2)}\right)$$

$$\frac{-2\pi+2\pi^2-\log(2)\log\left(\frac{2\pi}{\log(2)}\right)}{\log(2)}$$

Alternative representations:

$$-\log\left(\frac{2\pi}{\log(2)}\right)+\frac{2\pi^2}{\log(2)}-\frac{2\pi}{\log(2)} = -\log(a)\log_a\left(\frac{2\pi}{\log(2)}\right)-\frac{2\pi}{\log(a)\log_a(2)}+\frac{2\pi^2}{\log(a)\log_a(2)}$$

$$-\log\left(\frac{2\pi}{\log(2)}\right)+\frac{2\pi^2}{\log(2)}-\frac{2\pi}{\log(2)} = -\log_e\left(\frac{2\pi}{\log(2)}\right)-\frac{2\pi}{\log_e(2)}+\frac{2\pi^2}{\log_e(2)}$$

$$-\log\left(\frac{2\pi}{\log(2)}\right)+\frac{2\pi^2}{\log(2)}-\frac{2\pi}{\log(2)} = \text{Li}_1\left(1-\frac{2\pi}{\log(2)}\right)--\frac{2\pi}{\text{Li}_1(-1)}+-\frac{2\pi^2}{\text{Li}_1(-1)}$$

Series representations:

$$-\log\left(\frac{2\pi}{\log(2)}\right)+\frac{2\pi^2}{\log(2)}-\frac{2\pi}{\log(2)} =$$

$$-\left\lfloor\frac{\arg\left(\frac{2\pi}{\log(2)}-z_0\right)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)-\log(z_0)-\left\lfloor\frac{\arg\left(\frac{2\pi}{\log(2)}-z_0\right)}{2\pi}\right\rfloor\log(z_0)-$$

$$\frac{2\pi}{\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}}+$$

$$\frac{2\pi^2}{\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}}+$$

$$\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{2\pi}{\log(2)}-z_0\right)^k z_0^{-k}}{k}$$

$$-\log\left(\frac{2\pi}{\log(2)}\right)+\frac{2\pi^2}{\log(2)}-\frac{2\pi}{\log(2)} = \left(-2\pi+2\pi^2+4\pi^2\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\left\lfloor\frac{\arg\left(-x+\frac{2\pi}{\log(2)}\right)}{2\pi}\right\rfloor-2i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\log(x)-\right.$$
$$2i\pi\left\lfloor\frac{\arg\left(-x+\frac{2\pi}{\log(2)}\right)}{2\pi}\right\rfloor\log(x)-\log^2(x)+$$
$$2i\pi\left\lfloor\frac{\arg\left(-x+\frac{2\pi}{\log(2)}\right)}{2\pi}\right\rfloor\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}}{k}+$$
$$\log(x)\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}}{k}+2i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\sum_{k=1}^{\infty}\frac{(-1)^k\,x^{-k}\left(-x+\frac{2\pi}{\log(2)}\right)^k}{k}+$$
$$\log(x)\sum_{k=1}^{\infty}\frac{(-1)^k\,x^{-k}\left(-x+\frac{2\pi}{\log(2)}\right)^k}{k}-$$
$$\left.\sum_{k_1=1}^{\infty}\sum_{k_2=1}^{\infty}\frac{(-1)^{k_1+k_2}\,(2-x)^{k_1}\,x^{-k_1-k_2}\left(-x+\frac{2\pi}{\log(2)}\right)^{k_2}}{k_1\,k_2}\right)\Big/$$
$$\left(2i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}}{k}\right)\quad \text{for } x<0$$

$$-\log\left(\frac{2\pi}{\log(2)}\right)+\frac{2\pi^2}{\log(2)}-\frac{2\pi}{\log(2)} = \left(-2\pi+2\pi^2+4\pi^2\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor^2-4i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor\log(z_0)-\right.$$
$$\log^2(z_0)+2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-z_0)^k\,z_0^{-k}}{k}+$$
$$\log(z_0)\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-z_0)^k\,z_0^{-k}}{k}+2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor$$
$$\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{2\pi}{\log(2)}-z_0\right)^k z_0^{-k}}{k}+\log(z_0)\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{2\pi}{\log(2)}-z_0\right)^k z_0^{-k}}{k}-$$
$$\left.\sum_{k_1=1}^{\infty}\sum_{k_2=1}^{\infty}\frac{(-1)^{k_1+k_2}\,(2-z_0)^{k_1}\left(\frac{2\pi}{\log(2)}-z_0\right)^{k_2} z_0^{-k_1-k_2}}{k_1\,k_2}\right)\Big/$$
$$\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-z_0)^k\,z_0^{-k}}{k}\right)$$

Integral representations:

$$-\log\left(\frac{2\pi}{\log(2)}\right)+\frac{2\pi^2}{\log(2)}-\frac{2\pi}{\log(2)}=\frac{-2\pi+2\pi^2+\log(2)\int_0^1\int_0^1\frac{1}{(1+t_1)(\log(2)+(2\pi-\log(2))t_2)}\,dt_2\,dt_1}{\int_1^2\frac{1}{t}\,dt}$$

$$-\log\left(\frac{2\pi}{\log(2)}\right)+\frac{2\pi^2}{\log(2)}-\frac{2\pi}{\log(2)}=\frac{i\left(-8\pi^3+8\pi^4+\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)\left(-1+\frac{2\pi}{\log(2)}\right)^{-s}}{\Gamma(1-s)}\,ds\right)}{2\pi\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\quad\text{for}\quad -1<\gamma<0$$

Multiplying the results, we obtain:

((ln((2Pi)/(ln2)))) (((2Pi^2)/ln2)) (((2Pi)/ln2))

Input:

$$\log\left(\frac{2\pi}{\log(2)}\right)\times\frac{2\pi^2}{\log(2)}\times\frac{2\pi}{\log(2)}$$

log(x) is the natural logarithm

Exact result:

$$\frac{4\pi^3\log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)}$$

Decimal approximation:

569.04566205562446583649189724423541246292484295688630869 87...

569.0456620…..

Alternate forms:

$$\frac{4\pi^3(\log(2)+\log(\pi)-\log(\log(2)))}{\log^2(2)}$$

$$\frac{4\pi^3\log(\pi)}{\log^2(2)}-\frac{4\pi^3\log(\log(2))}{\log^2(2)}+\frac{4\pi^3}{\log(2)}$$

Alternative representations:

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi)\right)(2\pi^2)}{\log(2)\log(2)}=4\pi\log_e\left(\frac{2\pi}{\log(2)}\right)\pi^2\left(\frac{1}{\log_e(2)}\right)^2$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi)\right)(2\pi^2)}{\log(2)\log(2)}=4\pi\log(a)\log_a\left(\frac{2\pi}{\log(2)}\right)\pi^2\left(\frac{1}{\log(a)\log_a(2)}\right)^2$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi)\right)(2\pi^2)}{\log(2)\log(2)}=-4\pi\,\mathrm{Li}_1\left(1-\frac{2\pi}{\log(2)}\right)\pi^2\left(-\frac{1}{\mathrm{Li}_1(-1)}\right)^2$$

Series representations:

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi)\right)(2\pi^2)}{\log(2)\log(2)}=\frac{4\pi^3\left(-2i\pi\left\lfloor\frac{\arg\left(-x+\frac{2\pi}{\log(2)}\right)}{2\pi}\right\rfloor-\log(x)+\sum_{k=1}^{\infty}\frac{(-1)^k x^{-k}\left(-x+\frac{2\pi}{\log(2)}\right)^k}{k}\right)}{\left(2\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor-i\log(x)+i\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}\right)^2}\quad\text{for } x<0$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi)\right)(2\pi^2)}{\log(2)\log(2)}=\frac{4\pi^3\left(-2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor-\log(z_0)+\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{2\pi}{\log(2)}-z_0\right)^k z_0^{-k}}{k}\right)}{\left(2\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor-i\log(z_0)+i\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^2}$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi)\right)(2\pi^2)}{\log(2)\log(2)}=\frac{4\pi^3\left(\left\lfloor\frac{\arg\left(\frac{2\pi}{\log(2)}-z_0\right)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg\left(\frac{2\pi}{\log(2)}-z_0\right)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{2\pi}{\log(2)}-z_0\right)^k z_0^{-k}}{k}\right)}{\left(\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^2}$$

Integral representations:

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi)\right)(2\pi^2)}{\log(2)\log(2)} = \frac{4\pi^3 \int_1^{\frac{2\pi}{\log(2)}} \frac{1}{t}\,dt}{\left(\int_1^2 \frac{1}{t}\,dt\right)^2}$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi)\right)(2\pi^2)}{\log(2)\log(2)} = \frac{8 i \pi^4 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\,\Gamma(1+s)\left(-1+\frac{2\pi}{\log(2)}\right)^{-s}}{\Gamma(1-s)}\,ds}{\left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2} \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

Where 569 is:

569 is a prime number.

Properties:

569 is an odd number.

569 has a representation as a sum of 2 squares:

$569 = 13^2 + 20^2$

569 and 571 form a twin prime pair.

569 is the hypotenuse of a primitive Pythagorean triple:

$569^2 = 231^2 + 520^2$

569 has the representation $569 = 2^9 + 57$.

Eisenstein prime numbers (from Wikipedia)

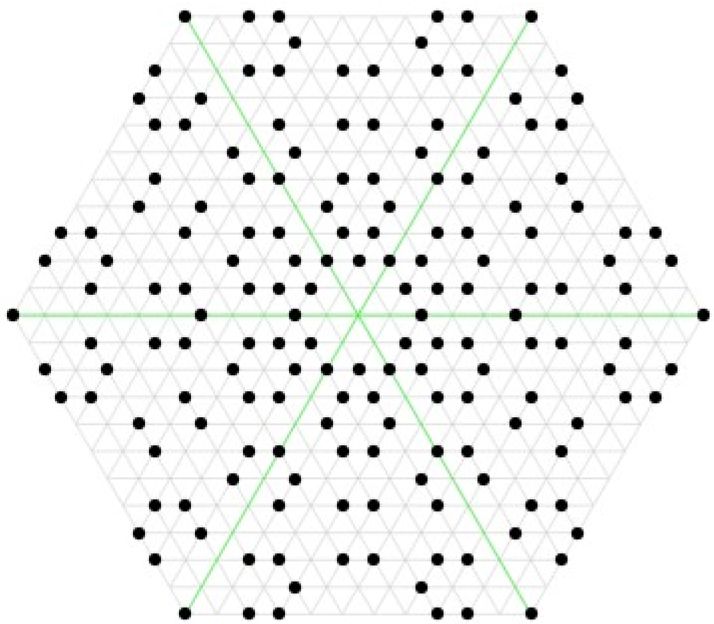
2, 5, 11, 17, 23, 29, 41, 47, 53, 59, 71, 83, 89, 101, 107, 113, 131, 137, 149, 167, 173, 179, 191, 197, 227, 233, 239, 251, 257, 263, 269, 281, 293, 311, 317, 347, 353, 359, 383, 389, 401, 419, 431, 443, 449, 461, 467, 479, 491, 503, 509, 521, 557, 563, **569**, 587…

In mathematics, an **Eisenstein prime** is an Eisenstein integer

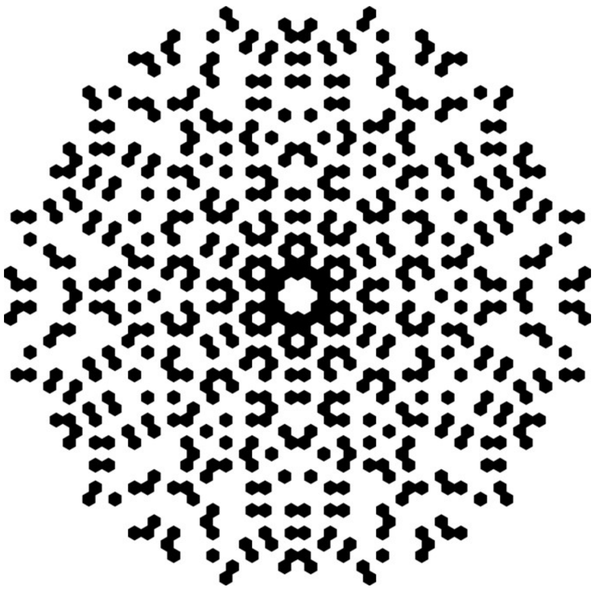
$$z = a + b\omega, \quad \text{where} \quad \omega = e^{\frac{2\pi i}{3}},$$

that is irreducible (or equivalently prime) in the ring-theoretic sense: its only Eisenstein divisors are the units $\{\pm 1, \pm\omega, \pm\omega^2\}$, $a + b\omega$ itself and its associates.

The associates (unit multiples) and the complex conjugate of any Eisenstein prime are also prime.



Small Eisenstein primes. Those on the green axes are associate to a natural prime of the form $3n - 1$. All others have an absolute value squared equal to a natural prime.



Eisenstein primes in a larger range

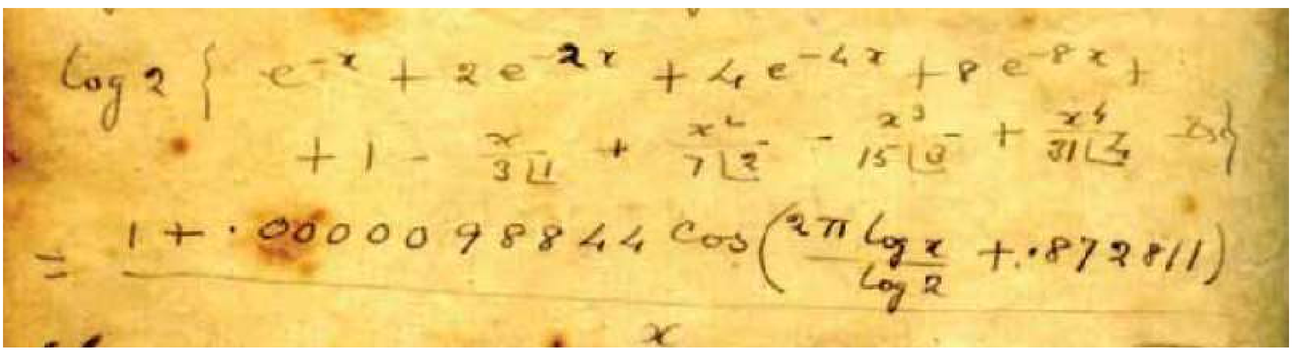
We note that the figures above recall fractal geometry, which seems to be connected to the distribution of prime numbers

Now, we have:

$$\phi_2(y) + \log 2\left(1 - \frac{y}{3.1!} + \frac{y^2}{7.2!} - \frac{y^3}{15.3!} + \dots\right) = \frac{1}{y} + F(y),$$

where
$$yF(y) = \cdot 0000098844 \cos\left(\frac{2\pi \log y}{\log 2} + \cdot 872811\right)$$

and from Ramanujan formula of the Manuscript Book 3



$$\log 2\left\{e^{-x} + 2e^{-2x} + 4e^{-4x} + 8e^{-8x} + \dots + 1 - \frac{x}{3\lfloor 1} + \frac{x^2}{7\lfloor 2} - \frac{x^3}{15\lfloor 3} + \frac{x^4}{31\lfloor 4} - \&c\right\} = \frac{1 + \cdot 0000098844 \cos\left(2\pi\frac{\log x}{\log 2} + \cdot 872811\right)}{x}$$

we obtain:

1/x((1+0.0000098844 cos((2Pi*ln(x))/ln2+0.872811))) = ln 2 ((((e^(-x)+2e^(-2x)+4e^(-4x)+8e^(-8x)+((1-x/(3*1)!+x^2/(7*2)!-x^3/(15*3)!+x^4/(31*4)!))))))

Input interpretation:

$$\frac{1}{x}\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(x)}{\log(2)}+0.872811\right)\right)=\log(2)\left(e^{-x}+2e^{-2x}+4e^{-4x}+8e^{-8x}+\left(1-\frac{x}{(3\times1)!}+\frac{x^2}{(7\times2)!}-\frac{x^3}{(15\times3)!}+\frac{x^4}{(31\times4)!}\right)\right)$$

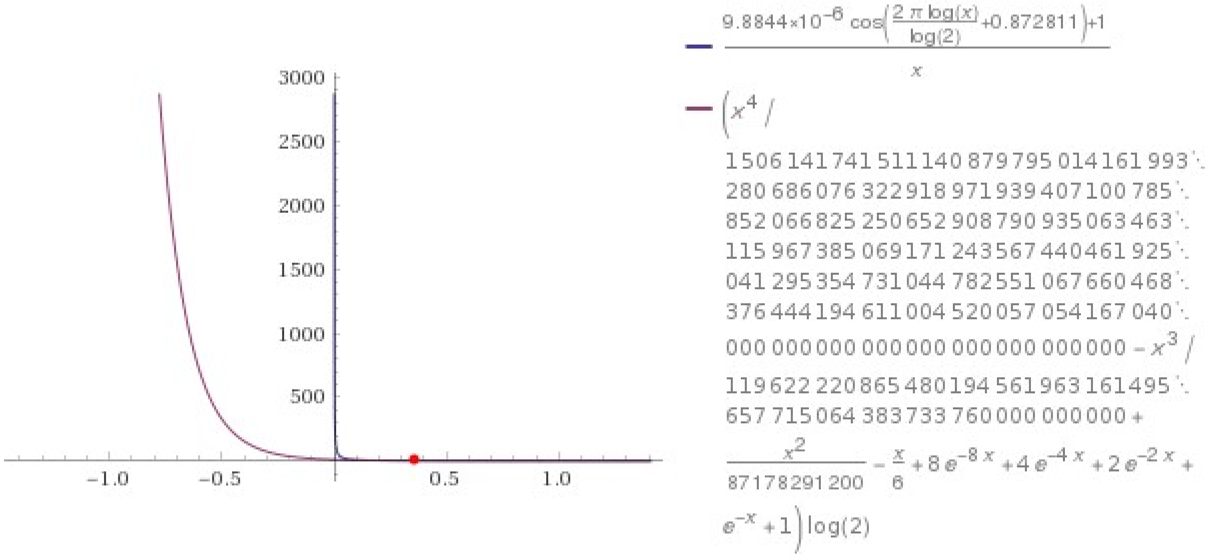
log(x) is the natural logarithm

n! is the factorial function

Result:

$$\frac{9.8844\times10^{-6}\cos\left(\frac{2\pi\log(x)}{\log(2)}+0.872811\right)+1}{x}=\left(x^4/1506141741511140879795014161993280686076322918971939407100785852066825250652908790935063463115967385069171243567440461925041295354731044782551067660468376444194611004520057054167040000000000000000000000000000000-x^3/119622220865480194561963161495657715064383733760000000000+\frac{x^2}{87178291200}-\frac{x}{6}+8e^{-8x}+4e^{-4x}+2e^{-2x}+e^{-x}+1\right)\log(2)$$

Plot:



Numerical solutions:

$x \approx 0.359463780075203\ldots$

$x \approx 3.91211922690599\ldots$

3.91211922690599…

1/3.9121192269((1+0.0000098844 cos((2Pi*ln(3.9121192269))/ln2+0.872811)))

Input interpretation:

$$\frac{1}{3.9121192269}\left(1+9.8844\times10^{-6}\cos\left(\frac{2\,\pi\log(3.9121192269)}{\log(2)}+0.872811\right)\right)$$

log(x) is the natural logarithm

Result:

0.25561790957241130435589321453504493118555391632711112658 8…

0.25561790957…

Addition formulas:

$$\frac{1+9.8844\times10^{-6}\cos\left(\frac{2\,\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)}{3.91211922690000}=0.255615931417410+$$
$$2.52661\times10^{-6}\cos(0.872811)\cos\left(-\frac{2\,\pi\log(3.91211922690000)}{\log(2)}\right)+$$
$$2.52661\times10^{-6}\sin(0.872811)\sin\left(-\frac{2\,\pi\log(3.91211922690000)}{\log(2)}\right)$$

$$\frac{1 + 9.8844\times 10^{-6} \cos\left(\frac{2\pi \log(3.91211922690000)}{\log(2)} + 0.872811\right)}{3.91211922690000} = 0.255616 + 2.52661\times 10^{-6} \cos(0.872811) \cos\left(\frac{2\pi \log(3.91211922690000)}{\log(2)}\right) - 2.52661\times 10^{-6} \sin(0.872811) \sin\left(\frac{2\pi \log(3.91211922690000)}{\log(2)}\right)$$

$$\frac{1 + 9.8844\times 10^{-6} \cos\left(\frac{2\pi \log(3.91211922690000)}{\log(2)} + 0.872811\right)}{3.91211922690000} = 0.255616 + 2.52661\times 10^{-6} \cosh\left(-\frac{2 i \pi \log(3.91211922690000)}{\log(2)}\right) \cos(0.872811) - 2.52661\times 10^{-6}\, i \sinh\left(-\frac{2 i \pi \log(3.91211922690000)}{\log(2)}\right) \sin(0.872811)$$

$$\frac{1 + 9.8844\times 10^{-6} \cos\left(\frac{2\pi \log(3.91211922690000)}{\log(2)} + 0.872811\right)}{3.91211922690000} = 0.255615931417410 + 2.52661\times 10^{-6} \cosh\left(\frac{2 i \pi \log(3.91211922690000)}{\log(2)}\right) \cos(0.872811) + 2.52661\times 10^{-6}\, i \sinh\left(\frac{2 i \pi \log(3.91211922690000)}{\log(2)}\right) \sin(0.872811)$$

Alternative representations:

$$\frac{1 + 9.8844\times 10^{-6} \cos\left(\frac{2\pi \log(3.91211922690000)}{\log(2)} + 0.872811\right)}{3.91211922690000} = \frac{1 + 9.8844\times 10^{-6} \cosh\left(i \left(0.872811 + \frac{2\pi \log(3.91211922690000)}{\log(2)}\right)\right)}{3.91211922690000}$$

$$\frac{1 + 9.8844\times 10^{-6} \cos\left(\frac{2\pi \log(3.91211922690000)}{\log(2)} + 0.872811\right)}{3.91211922690000} = \frac{1 + 9.8844\times 10^{-6} \cosh\left(-i \left(0.872811 + \frac{2\pi \log(3.91211922690000)}{\log(2)}\right)\right)}{3.91211922690000}$$

$$\frac{1 + 9.8844\times 10^{-6} \cos\left(\frac{2\pi \log(3.91211922690000)}{\log(2)} + 0.872811\right)}{3.91211922690000} = \frac{1}{3.91211922690000} \left(1 + 4.9422\times 10^{-6} \left(e^{-i (0.872811+(2\pi \log(3.91211922690000))/\log(2))} + e^{i (0.872811+(2\pi \log(3.91211922690000))/\log(2))}\right)\right)$$

Series representations:

$$\frac{1+9.8844\times 10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)}{3.91211922690000}=0.255615931417410+2.52661\times 10^{-6}\sum_{k=0}^{\infty}\frac{(-1)^k\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)^{2k}}{(2k)!}$$

$$\frac{1+9.8844\times 10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)}{3.91211922690000}=0.255615931417410-2.52661\times 10^{-6}\sum_{k=0}^{\infty}\frac{(-1)^k\left(0.872811+\pi\left(-\frac{1}{2}+\frac{2\log(3.91211922690000)}{\log(2)}\right)\right)^{1+2k}}{(1+2k)!}$$

$$\frac{1+9.8844\times 10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)}{3.91211922690000}=0.255615931417410+2.52661\times 10^{-6}\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}-z_0\right)^k}{k!}$$

Integral representations:

$$\frac{1+9.8844\times 10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)}{3.91211922690000}=0.255615931417410-2.52661\times 10^{-6}\int_{\frac{\pi}{2}}^{0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}}\sin(t)\,dt$$

$$\frac{1+9.8844\times 10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)}{3.91211922690000}=0.255618+\int_0^1\frac{1}{\log(2)}\left(-2.20525\times 10^{-6}\log(2)-5.05322\times 10^{-6}\,\pi\log(3.91211922690000)\right)\sin\left(t\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)dt$$

$$\frac{1+9.8844\times 10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)}{3.91211922690000}=0.255615931417410+\frac{1.26331\times 10^{-6}\sqrt{\pi}}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{s-\frac{(0.436406\log(2)+\pi\log(3.91211922690000))^2}{s\log^2(2)}}}{\sqrt{s}}\,ds \quad \text{for } \gamma>0$$

$$\frac{1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)}{3.91211922690000} = 0.255615931417410 + \frac{1.26331\times10^{-6}\sqrt{\pi}}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{4^s\,\Gamma(s)\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)^{-2s}}{\Gamma\left(\frac{1}{2}-s\right)}\,ds \quad \text{for } 0<\gamma<\frac{1}{2}$$

and:

ln 2 ((((e^(-3.912119)+2e^(-2*3.912119)+4e^(-4*3.912119)+8e^(-8*3.912119)+((1-(3.912119)/(3*1)!+(3.912119)^2/(7*2)!-(3.912119)^3/(15*3)!+(3.912119)^4/(31*4)!))))))

Input interpretation:

$$\log(2)\left(\frac{1}{e^{3.912119}}+2\,e^{-2\times3.912119}+4\,e^{-4\times3.912119}+8\,e^{-8\times3.912119}+\left(1-\frac{3.912119}{(3\times1)!}+\frac{3.912119^2}{(7\times2)!}-\frac{3.912119^3}{(15\times3)!}+\frac{3.912119^4}{(31\times4)!}\right)\right)$$

log(x) is the natural logarithm

n! is the factorial function

Result:

0.2556179391825114984910808960133225169235603043900866680182...

0.25561793918…

Alternative representations:

$$\log(2)\left(e^{-3.91212}+2\,e^{-2\times3.91212}+4\,e^{-4\times3.91212}+8\,e^{-8\times3.91212}+\left(1-\frac{3.91212}{(3\times1)!}+\frac{3.91212^2}{(7\times2)!}-\frac{3.91212^3}{(15\times3)!}+\frac{3.91212^4}{(31\times4)!}\right)\right) = \log(a)\log_a(2)\left(1+\frac{8}{e^{31.297}}+\frac{4}{e^{15.6485}}+\frac{2}{e^{7.82424}}+\frac{1}{e^{3.91212}}-\frac{3.91212}{\Gamma(4)}+\frac{3.91212^2}{\Gamma(15)}-\frac{3.91212^3}{\Gamma(46)}+\frac{3.91212^4}{\Gamma(125)}\right)$$

$$\log(2)\left(e^{-3.91212}+2\,e^{-2\times 3.91212}+4\,e^{-4\times 3.91212}+8\,e^{-8\times 3.91212}+\left(1-\frac{3.91212}{(3\times 1)!}+\frac{3.91212^2}{(7\times 2)!}-\frac{3.91212^3}{(15\times 3)!}+\frac{3.91212^4}{(31\times 4)!}\right)\right)=$$
$$\log_e(2)\left(1+\frac{8}{e^{31.297}}+\frac{4}{e^{15.6485}}+\frac{2}{e^{7.82424}}+\frac{1}{e^{3.91212}}-\frac{3.91212}{(1)_3}+\frac{3.91212^2}{(1)_{14}}-\frac{3.91212^3}{(1)_{45}}+\frac{3.91212^4}{(1)_{124}}\right)$$

$$\log(2)\left(e^{-3.91212}+2\,e^{-2\times 3.91212}+4\,e^{-4\times 3.91212}+8\,e^{-8\times 3.91212}+\left(1-\frac{3.91212}{(3\times 1)!}+\frac{3.91212^2}{(7\times 2)!}-\frac{3.91212^3}{(15\times 3)!}+\frac{3.91212^4}{(31\times 4)!}\right)\right)=$$
$$\log(a)\log_a(2)\left(1+\frac{8}{e^{31.297}}+\frac{4}{e^{15.6485}}+\frac{2}{e^{7.82424}}+\frac{1}{e^{3.91212}}-\frac{3.91212}{(1)_3}+\frac{3.91212^2}{(1)_{14}}-\frac{3.91212^3}{(1)_{45}}+\frac{3.91212^4}{(1)_{124}}\right)$$

Series representations:

$$\log(2)\left(e^{-3.91212}+2\,e^{-2\times 3.91212}+4\,e^{-4\times 3.91212}+8\,e^{-8\times 3.91212}+\left(1-\frac{3.91212}{(3\times 1)!}+\frac{3.91212^2}{(7\times 2)!}-\frac{3.91212^3}{(15\times 3)!}+\frac{3.91212^4}{(31\times 4)!}\right)\right)=$$
$$\log(2)+\frac{8\log(2)}{e^{31.297}}+\frac{4\log(2)}{e^{15.6485}}+\frac{2\log(2)}{e^{7.82424}}+\frac{\log(2)}{e^{3.91212}}-\frac{3.91212\log(2)}{\sum_{k=0}^{\infty}\frac{(3-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}+\frac{15.3047\log(2)}{\sum_{k=0}^{\infty}\frac{(14-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}-\frac{59.8737\log(2)}{\sum_{k=0}^{\infty}\frac{(45-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}+\frac{234.233\log(2)}{\sum_{k=0}^{\infty}\frac{(124-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}$$

for $((n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z}) \text{ and } n_0 \to 3 \text{ and } n_0 \to 14 \text{ and } n_0 \to 45 \text{ and } n_0 \to 124)$

$$\log(2)\left(e^{-3.91212}+2\,e^{-2\times 3.91212}+4\,e^{-4\times 3.91212}+8\,e^{-8\times 3.91212}+\left(1-\frac{3.91212}{(3\times 1)!}+\frac{3.91212^2}{(7\times 2)!}-\frac{3.91212^3}{(15\times 3)!}+\frac{3.91212^4}{(31\times 4)!}\right)\right)=$$
$$\left(2\,i\,\pi\left\lfloor\frac{\arg(2-x)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}}{k}\right)$$
$$\left(1+\frac{8}{e^{31.297}}+\frac{4}{e^{15.6485}}+\frac{2}{e^{7.82424}}+\frac{1}{e^{3.91212}}-\frac{3.91212}{\sum_{k=0}^{\infty}\frac{(3-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}+\frac{15.3047}{\sum_{k=0}^{\infty}\frac{(14-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}-\frac{59.8737}{\sum_{k=0}^{\infty}\frac{(45-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}+\frac{234.233}{\sum_{k=0}^{\infty}\frac{(124-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}\right)$$

for ($x < 0$ and ($n_0 \geq 0$ or $n_0 \notin \mathbb{Z}$) and $n_0 \to 3$ and $n_0 \to 14$ and $n_0 \to 45$ and $n_0 \to 124$)

$$\log(2)\left(e^{-3.91212}+2\,e^{-2\times 3.91212}+4\,e^{-4\times 3.91212}+8\,e^{-8\times 3.91212}+\left(1-\frac{3.91212}{(3\times 1)!}+\frac{3.91212^2}{(7\times 2)!}-\frac{3.91212^3}{(15\times 3)!}+\frac{3.91212^4}{(31\times 4)!}\right)\right)=$$
$$\left(\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\,\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-z_0)^k\,z_0^{-k}}{k}\right)$$
$$\left(1+\frac{8}{e^{31.297}}+\frac{4}{e^{15.6485}}+\frac{2}{e^{7.82424}}+\frac{1}{e^{3.91212}}-\frac{3.91212}{\sum_{k=0}^{\infty}\frac{(3-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}+\frac{15.3047}{\sum_{k=0}^{\infty}\frac{(14-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}-\frac{59.8737}{\sum_{k=0}^{\infty}\frac{(45-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}+\frac{234.233}{\sum_{k=0}^{\infty}\frac{(124-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}\right)$$

for (($n_0 \geq 0$ or $n_0 \notin \mathbb{Z}$) and $n_0 \to 3$ and $n_0 \to 14$ and $n_0 \to 45$ and $n_0 \to 124$)

$$\log(2)\left(e^{-3.91212}+2\,e^{-2\times 3.91212}+4\,e^{-4\times 3.91212}+8\,e^{-8\times 3.91212}+\left(1-\frac{3.91212}{(3\times 1)!}+\frac{3.91212^2}{(7\times 2)!}-\frac{3.91212^3}{(15\times 3)!}+\frac{3.91212^4}{(31\times 4)!}\right)\right)=$$
$$\left(2\,i\,\pi\left\lfloor-\frac{-\pi+\arg\left(\frac{2}{z_0}\right)+\arg(z_0)}{2\,\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-z_0)^k\,z_0^{-k}}{k}\right)$$
$$\left(1+\frac{8}{e^{31.297}}+\frac{4}{e^{15.6485}}+\frac{2}{e^{7.82424}}+\frac{1}{e^{3.91212}}-\frac{3.91212}{\sum_{k=0}^{\infty}\frac{(3-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}+\frac{15.3047}{\sum_{k=0}^{\infty}\frac{(14-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}-\frac{59.8737}{\sum_{k=0}^{\infty}\frac{(45-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}+\frac{234.233}{\sum_{k=0}^{\infty}\frac{(124-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}\right)$$

for (($n_0 \geq 0$ or $n_0 \notin \mathbb{Z}$) and $n_0 \to 3$ and $n_0 \to 14$ and $n_0 \to 45$ and $n_0 \to 124$)

Now, from the previous expression

$$\log\left(\frac{2\pi}{\log(2)}\right) \times \frac{2\pi^2}{\log(2)} \times \frac{2\pi}{\log(2)}$$

log(x) is the natural logarithm

$$\frac{4\pi^3 \log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)}$$

569.04566205562446583649189724423541246292484295688630869 87...

569.0456620….. ≈ 569

We obtain, multiplying the two expression:

(((1/3.9121192269((1+0.0000098844 cos((2Pi*ln(3.9121192269))/ln2+0.872811)))))) * (((((ln((2Pi)/(ln2)))) * (((2Pi^2)/ln2)) * (((2Pi)/ln2)))))

Input interpretation:

$$\left(\frac{1}{3.9121192269}\left(1 + 9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.9121192269)}{\log(2)} + 0.872811\right)\right)\right)$$
$$\left(\log\left(\frac{2\pi}{\log(2)}\right) \times \frac{2\pi^2}{\log(2)} \times \frac{2\pi}{\log(2)}\right)$$

log(x) is the natural logarithm

Result:

145.45826259...

145.4582…≈ 145 that is an Ulam number (see list below)

A002858	Ulam numbers: a(1) = 1; a(2) = 2; for n>2, a(n) = least number > a(n-1) which is a unique sum of two distinct earlier terms. (Formerly M0557 N0201)

1, 2, 3, 4, 6, 8, 11, 13, 16, 18, 26, 28, 36, 38, 47, 48, 53, 57, 62, 69, 72, 77, 82, 87, 97, 99, 102, 106, 114, 126, 131, 138, **145**, 148, 155, 175, 177, 180, 182, 189, 197, 206, 209, 219, 221, 236, 238, 241, 243, 253, 258, 260, 273, 282, 309, 316, 319, 324, 339…

Furthermore: 145 = 29 * 5 where 29 is a prime Lucas number and 5 is a Fibonacci prime number

Addition formulas:

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)}{(\log(2)\log(2))\,3.91211922690000} = \frac{1}{\log^2(2)}1.02246372566964\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)\left(1+9.8844\times10^{-6}\cos(0.872811)\cos\left(-\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)+9.8844\times10^{-6}\sin(0.872811)\sin\left(-\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)}{(\log(2)\log(2))\,3.91211922690000} = \frac{1}{\log^2(2)}\,0.0000101064\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)\left(101170.+\cos(0.872811)\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)-\sin(0.872811)\sin\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)}{(\log(2)\log(2))\,3.91211922690000} = \frac{1}{\log^2(2)}1.02246372566964\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)\left(1+9.8844\times10^{-6}\cosh\left(\frac{2i\pi\log(3.91211922690000)}{\log(2)}\right)\cos(0.872811)+9.8844\times10^{-6}\,i\sinh\left(\frac{2i\pi\log(3.91211922690000)}{\log(2)}\right)\sin(0.872811)\right)$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)}{(\log(2)\log(2))\,3.91211922690000} = \frac{1}{\log^2(2)}\,0.0000101064\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)\left(101170.+\cosh\left(-\frac{2i\pi\log(3.91211922690000)}{\log(2)}\right)\cos(0.872811)-i\left(\sinh\left(-\frac{2i\pi\log(3.91211922690000)}{\log(2)}\right)\sin(0.872811)\right)\right)$$

Alternative representations:

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)}{(\log(2)\log(2))\,3.91211922690000} = \frac{1}{3.91211922690000}\,4\pi\left(1+9.8844\times10^{-6}\cosh\left(-i\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)\right)\log\left(\frac{2\pi}{\log(2)}\right)\pi^2\left(\frac{1}{\log(2)}\right)^2$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)}{(\log(2)\log(2))\,3.91211922690000} = \frac{1}{3.91211922690000}\,4\pi\log\left(\frac{2\pi}{\log(2)}\right)\left(1+4.9422\times10^{-6}\left(e^{-i(0.872811+(2\pi\log(3.91211922690000))/\log(2))}+e^{i(0.872811+(2\pi\log(3.91211922690000))/\log(2))}\right)\right)\pi^2\left(\frac{1}{\log(2)}\right)^2$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)}{(\log(2)\log(2))\,3.91211922690000} = \frac{1}{3.91211922690000}\,4\pi\left(1+9.8844\times10^{-6}\cosh\left(i\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)\right)\log_e\left(\frac{2\pi}{\log(2)}\right)\pi^2\left(\frac{1}{\log_e(2)}\right)^2$$

Series representations:

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)}{(\log(2)\log(2))\,3.91211922690000} = \frac{1.02246372566964\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)} + \frac{0.0000101064\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)^{2k}}{(2k)!}}{\log^2(2)}$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)}{(\log(2)\log(2))\,3.91211922690000} =$$

$$\frac{1.02246372566964\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)} - \frac{0.0000101064\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(0.872811+\pi\left(-\frac{1}{2}+\frac{2\log(3.91211922690000)}{\log(2)}\right)\right)^{1+2k}}{(1+2k)!}}{\log^2(2)}$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)}{(\log(2)\log(2))\,3.91211922690000} =$$

$$\frac{1.02246372566964\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)} + \frac{0.0000101064\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}-z_0\right)^k}{k!}}{\log^2(2)}$$

Integral representations:

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)}{(\log(2)\log(2))\,3.91211922690000} =$$

$$-\frac{1}{\log^3(2)}\,0.0000202129\,\pi^3\left(-50585.3\log(2)+0.436406\log(2)+\pi\log(3.91211922690000)\int_0^1\sin\left(t\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)dt\right)\log\left(\frac{2\pi}{\log(2)}\right)$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)}{(\log(2)\log(2))\,3.91211922690000} =$$

$$\frac{1.02246372566964\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)} - \frac{0.0000101064\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)}\int_{\frac{\pi}{2}}^{0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}}\sin(t)\,dt$$

$$\frac{\left(\log\left(\frac{2\pi}{\log(2)}\right)(2\pi^2)(2\pi)\right)\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)}{(\log(2)\log(2))\,3.91211922690000} =$$

$$-\frac{1}{\left(\int_1^2 \frac{1}{t}\,dt\right)^2 \log(2)}\,0.0000202129\,\pi^3 \left(-50\,585.3\log(2)\int_1^{\frac{2\pi}{\log(2)}} \frac{1}{t}\,dt + 2\log(2)\int_0^1\int_0^1 \frac{\sin\left(\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)t_2\right)}{\log(2)+(2\pi-\log(2))\,t_1}\,dt_2\,dt_1\right)$$

While, from the division of the two expression, we obtain:

(((((ln((2Pi)/(ln2)))) * (((2Pi^2)/ln2)) * (((2Pi)/ln2))))) / (((1/3.9121192269((1+0.0000098844 cos((2Pi*ln(3.9121192269))/ln2+0.872811))))))

Input interpretation:

$$\frac{\log\left(\frac{2\pi}{\log(2)}\right)\times\frac{2\pi^2}{\log(2)}\times\frac{2\pi}{\log(2)}}{\frac{1}{3.9121192269}\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.9121192269)}{\log(2)}+0.872811\right)\right)}$$

log(x) is the natural logarithm

Result:

2226.1572478…

2226.1572478… ≈ 2226

We note that, for the following formula

a(n) = n*(5*n+1)

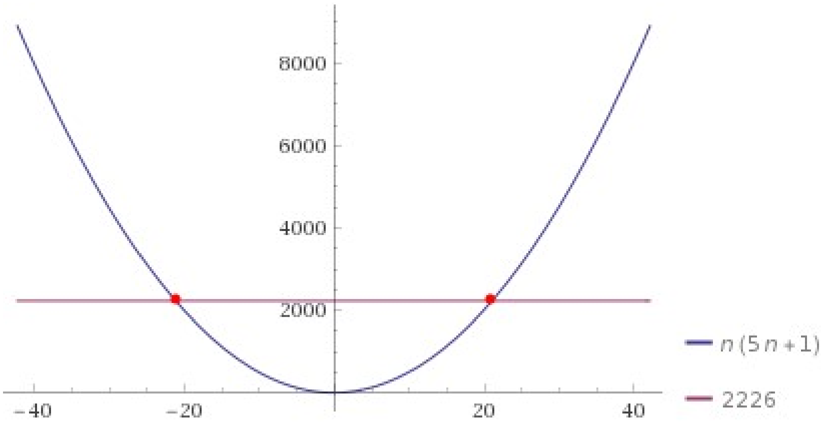
we obtain:

n(5*n+1) = 2226

Input:

$n(5n+1) = 2226$

Plot:



Alternate forms:

$5n^2 + n = 2226$

$5n^2 + n - 2226 = 0$

Solutions:

$n = -\frac{106}{5}$

$n = 21$

21

Thence:

21(5*21+1)

where 5 and 21 are Fibonacci's number (21 is also equal to 3*7)

Input:

$21(5 \times 21 + 1)$

Result:

2226

2226

3*7(5*3*7+1) = 2226

Addition formulas:

$$\frac{\log\left(\frac{2\pi}{\log(2)}\right)\left(\left(2\pi^2\right)(2\pi)\right)}{\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(\log(2)\log(2))}{3.91211922690000}} =$$

$$\left(15.6484769076000\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)\right)\Big/ \left(\log^2(2)\left(1+9.8844\times10^{-6}\cos(0.872811)\cos\left(-\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)+ 9.8844\times10^{-6}\sin(0.872811)\sin\left(-\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)\right)$$

$$\frac{\log\left(\frac{2\pi}{\log(2)}\right)\left(\left(2\pi^2\right)(2\pi)\right)}{\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(\log(2)\log(2))}{3.91211922690000}} =$$

$$\left(1.58315\times10^6\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)\right)\Big/ \left(\log^2(2)\left(101\,170.+\cos(0.872811)\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)- \sin(0.872811)\sin\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)\right)$$

$$\frac{\log\left(\frac{2\pi}{\log(2)}\right)\left(\left(2\pi^2\right)(2\pi)\right)}{\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(\log(2)\log(2))}{3.91211922690000}} =$$

$$\left(15.6484769076000\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)\right)\Big/ \left(\log^2(2)\left(1+9.8844\times10^{-6}\cosh\left(\frac{2 i\pi\log(3.91211922690000)}{\log(2)}\right)\cos(0.872811)+ 9.8844\times10^{-6}\, i\sinh\left(\frac{2 i\pi\log(3.91211922690000)}{\log(2)}\right)\sin(0.872811)\right)\right)$$

$$\frac{\log\left(\frac{2\pi}{\log(2)}\right)\left(\left(2\pi^2\right)(2\pi)\right)}{\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(\log(2)\log(2))}{3.91211922690000}} =$$

$$\left(1.58315\times10^6\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)\right)\Big/ \left(\log^2(2)\left(101\,170.+\cosh\left(-\frac{2 i\pi\log(3.91211922690000)}{\log(2)}\right)\cos(0.872811)- i\left(\sinh\left(-\frac{2 i\pi\log(3.91211922690000)}{\log(2)}\right)\sin(0.872811)\right)\right)\right)$$

Alternative representations:

$$\frac{\log\left(\frac{2\pi}{\log(2)}\right)\left((2\pi^2)(2\pi)\right)}{\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(\log(2)\log(2))}{3.91211922690000}} = \frac{4\pi\log\left(\frac{2\pi}{\log(2)}\right)\pi^2\left(\frac{1}{\log(2)}\right)^2}{\frac{1+9.8844\times10^{-6}\cosh\left(-i\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)}{3.91211922690000}}$$

$$\frac{\log\left(\frac{2\pi}{\log(2)}\right)\left((2\pi^2)(2\pi)\right)}{\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(\log(2)\log(2))}{3.91211922690000}} = \frac{4\pi\log_e\left(\frac{2\pi}{\log(2)}\right)\pi^2\left(\frac{1}{\log_e(2)}\right)^2}{\frac{1+9.8844\times10^{-6}\cosh\left(i\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)}{3.91211922690000}}$$

$$\frac{\log\left(\frac{2\pi}{\log(2)}\right)\left((2\pi^2)(2\pi)\right)}{\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(\log(2)\log(2))}{3.91211922690000}} = \frac{4\pi\log_e\left(\frac{2\pi}{\log(2)}\right)\pi^2\left(\frac{1}{\log_e(2)}\right)^2}{\frac{1+9.8844\times10^{-6}\cosh\left(-i\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)}{3.91211922690000}}$$

Series representations:

$$\frac{\log\left(\frac{2\pi}{\log(2)}\right)\left((2\pi^2)(2\pi)\right)}{\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(\log(2)\log(2))}{3.91211922690000}} = \frac{15.6484769076000\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)\left(1+9.8844\times10^{-6}\sum_{k=0}^{\infty}\frac{(-1)^k\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)^{2k}}{(2k)!}\right)}$$

$$\frac{\log\left(\frac{2\pi}{\log(2)}\right)\left(\left(2\pi^2\right)(2\pi)\right)}{\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(\log(2)\log(2))}{3.91211922690000}} =$$

$$\frac{15.6484769076000\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)\left(1-9.8844\times10^{-6}\sum_{k=0}^{\infty}\frac{(-1)^k\left(0.872811+\pi\left(-\frac{1}{2}+\frac{2\log(3.91211922690000)}{\log(2)}\right)\right)^{1+2k}}{(1+2k)!}\right)}$$

$$\frac{\log\left(\frac{2\pi}{\log(2)}\right)\left(\left(2\pi^2\right)(2\pi)\right)}{\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(\log(2)\log(2))}{3.91211922690000}} =$$

$$\frac{15.6484769076000\,\pi^3\log\left(\frac{2\pi}{\log(2)}\right)}{\log^2(2)\left(1+9.8844\times10^{-6}\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}-z_0\right)^k}{k!}\right)}$$

Integral representations:

$$\frac{\log\left(\frac{2\pi}{\log(2)}\right)\left(\left(2\pi^2\right)(2\pi)\right)}{\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(\log(2)\log(2))}{3.91211922690000}} =$$

$$\frac{15.6485\,i\,\pi^4\int_1^{\frac{2\pi}{\log(2)}}\frac{1}{t}\,dt}{\left(\int_1^2\frac{1}{t}\,dt\right)^2\left(i\pi+4.9422\times10^{-6}\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{s-\frac{(0.436406\log(2)+\pi\log(3.91211922690000))^2}{s\log^2(2)}}}{\sqrt{s}}\,ds\right)}$$

for $\gamma > 0$

$$\frac{\log\left(\frac{2\pi}{\log(2)}\right)\left(\left(2\pi^2\right)(2\pi)\right)}{\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(\log(2)\log(2))}{3.91211922690000}} =$$

$$\left(15.6485\,i\,\pi^4\int_1^{\frac{2\pi}{\log(2)}}\frac{1}{t}\,dt\right)\Big/\left(\left(\int_1^2\frac{1}{t}\,dt\right)^2\left(i\pi+4.9422\times10^{-6}\sqrt{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{4^s\,\Gamma(s)\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)^{-2s}}{\Gamma\left(\frac{1}{2}-s\right)}\,ds\right)\right)$$

for $0 < \gamma < \frac{1}{2}$

$$\frac{\log\left(\frac{2\pi}{\log(2)}\right)\left((2\pi^2)(2\pi)\right)}{\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(\log(2)\log(2))}{3.91211922690000}} = -\frac{3.1663\times10^6\, i\,\pi^4 \int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)\left(-1+\frac{2\pi}{\log(2)}\right)^{-s}}{\Gamma(1-s)}\,ds}{\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2\left(-101\,170. + \int_{\frac{\pi}{2}}^{0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}}\sin(t)\,dt\right)} \text{ for } -1<\gamma<0$$

$$\frac{\log\left(\frac{2\pi}{\log(2)}\right)\left((2\pi^2)(2\pi)\right)}{\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(\log(2)\log(2))}{3.91211922690000}} =$$

$$-\left(\left(1.58315\times10^6\, i\,\pi^4\log(2)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)\left(-1+\frac{2\pi}{\log(2)}\right)^{-s}}{\Gamma(1-s)}\,ds\right)\Big/\left(\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2\left(-50\,585.3\log(2)+0.436406\log(2)\int_0^1\sin\left(t\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)dt + \pi\log(3.91211922690000)\int_0^1\sin\left(t\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)dt\right)\right)\right) \text{ for } -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

From the previous expression

$$\frac{1}{3.9121192269}\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.9121192269)}{\log(2)}+0.872811\right)\right)$$

we have also:

exp(((1/3.9121192269((1+0.0000098844 cos((2Pi*ln(3.9121192269))/ln2+0.872811))))))

Input interpretation:

$$\exp\left(\frac{1}{3.9121192269}\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.9121192269)}{\log(2)}+0.872811\right)\right)\right)$$

log(x) is the natural logarithm

Result:

1.2912592559...

1.2912592559…

From which:

[exp(((1/3.9121192269((1+0.0000098844 cos((2Pi*ln(3.9121192269))/ln2+0.872811))))))]^2

Input interpretation:

$$\exp^2\left(\frac{1}{3.9121192269}\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.9121192269)}{\log(2)}+0.872811\right)\right)\right)$$

log(x) is the natural logarithm

Result:

1.6673504659...

1.6673504659…. result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

exp(((ln 2 ((((e^(-3.912119)+2e^(-2*3.912119)+4e^(-4*3.912119)+8e^(-8*3.912119)+((1-(3.912119)/(3*1)!+(3.912119)^2/(7*2)!-(3.912119)^3/(15*3)!+(3.912119)^4/(31*4)!))))))))

Input interpretation:

$$\exp\left(\log(2)\left(\frac{1}{e^{3.912119}}+2\,e^{-2\times3.912119}+4\,e^{-4\times3.912119}+8\,e^{-8\times3.912119}+\left(1-\frac{3.912119}{(3\times1)!}+\frac{3.912119^2}{(7\times2)!}-\frac{3.912119^3}{(15\times3)!}+\frac{3.912119^4}{(31\times4)!}\right)\right)\right)$$

log(x) is the natural logarithm

n! is the factorial function

Result:

1.291259294103344007276534431276392693222406494617387989981...

1.2912592941…

from which:

[exp(((ln 2 ((((e^(-3.912119)+2e^(-2*3.912119)+4e^(-4*3.912119)+8e^(-8*3.912119)+((1-(3.912119)/(3*1)!+(3.912119)^2/(7*2)!-(3.912119)^3/(15*3)!+(3.912119)^4/(31*4)!)))))))))]^2

Input interpretation:

$$\exp^2\left(\log(2)\left(\frac{1}{e^{3.912119}}+2\,e^{-2\times3.912119}+4\,e^{-4\times3.912119}+8\,e^{-8\times3.912119}+\left(1-\frac{3.912119}{(3\times1)!}+\frac{3.912119^2}{(7\times2)!}-\frac{3.912119^3}{(15\times3)!}+\frac{3.912119^4}{(31\times4)!}\right)\right)\right)$$

log(x) is the natural logarithm

n! is the factorial function

Result:

1.667350564608266255760737408833265097313854772968934844189...

1.6673505646…. as above

and again:

(55 ζ(3)^2)/(12 π log^3(2)) * ln 2 ((((e^(-3.912119)+2e^(-2*3.912119)+4e^(-4*3.912119)+8e^(-8*3.912119)+((1-(3.912119)/(3*1)!+(3.912119)^2/(7*2)!-(3.912119)^3/(15*3)!+(3.912119)^4/(31*4)!))))))

Input interpretation:

$$\frac{55\,\zeta(3)^2}{12\,\pi\log^3(2)}\log(2)\left(\frac{1}{e^{3.912119}}+2\,e^{-2\times3.912119}+4\,e^{-4\times3.912119}+8\,e^{-8\times3.912119}+\left(1-\frac{3.912119}{(3\times1)!}+\frac{3.912119^2}{(7\times2)!}-\frac{3.912119^3}{(15\times3)!}+\frac{3.912119^4}{(31\times4)!}\right)\right)$$

ζ(s) is the Riemann zeta function

log(x) is the natural logarithm

n! is the factorial function

Result:

1.618067260266940620501253089062700418522840473420406067097...

1.618067260266…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternative representations:

$$\frac{1}{12\pi\log^3(2)}\left(\log(2)\left(e^{-3.91212}+2e^{-2\times 3.91212}+4e^{-4\times 3.91212}+8e^{-8\times 3.91212}+\left(1-\frac{3.91212}{(3\times 1)!}+\frac{3.91212^2}{(7\times 2)!}-\frac{3.91212^3}{(15\times 3)!}+\frac{3.91212^4}{(31\times 4)!}\right)\right)\right)(55\zeta(3)^2)=\frac{1}{12\pi\log^3(2)}55\log(2)\left(1+\frac{8}{e^{31.297}}+\frac{4}{e^{15.6485}}+\frac{2}{e^{7.82424}}+\frac{1}{e^{3.91212}}-\frac{3.91212}{2!!\times 3!!}+\frac{3.91212^2}{13!!\times 14!!}-\frac{3.91212^3}{44!!\times 45!!}+\frac{3.91212^4}{123!!\times 124!!}\right)\zeta(3,1)^2$$

$$\frac{1}{12\pi\log^3(2)}\left(\log(2)\left(e^{-3.91212}+2e^{-2\times 3.91212}+4e^{-4\times 3.91212}+8e^{-8\times 3.91212}+\left(1-\frac{3.91212}{(3\times 1)!}+\frac{3.91212^2}{(7\times 2)!}-\frac{3.91212^3}{(15\times 3)!}+\frac{3.91212^4}{(31\times 4)!}\right)\right)\right)(55\zeta(3)^2)=\frac{1}{12\pi(\log(a)\log_a(2))^3}55\log(a)\log_a(2)\left(1+\frac{8}{e^{31.297}}+\frac{4}{e^{15.6485}}+\frac{2}{e^{7.82424}}+\frac{1}{e^{3.91212}}-\frac{3.91212}{(1)_3}+\frac{3.91212^2}{(1)_{14}}-\frac{3.91212^3}{(1)_{45}}+\frac{3.91212^4}{(1)_{124}}\right)\zeta(3,1)^2$$

$$\frac{1}{12\pi\log^3(2)}\left(\log(2)\left(e^{-3.91212}+2e^{-2\times 3.91212}+4e^{-4\times 3.91212}+8e^{-8\times 3.91212}+\left(1-\frac{3.91212}{(3\times 1)!}+\frac{3.91212^2}{(7\times 2)!}-\frac{3.91212^3}{(15\times 3)!}+\frac{3.91212^4}{(31\times 4)!}\right)\right)\right)(55\zeta(3)^2)=\frac{1}{12\pi\log_e^3(2)}55\log_e(2)\left(1+\frac{8}{e^{31.297}}+\frac{4}{e^{15.6485}}+\frac{2}{e^{7.82424}}+\frac{1}{e^{3.91212}}-\frac{3.91212}{2!!\times 3!!}+\frac{3.91212^2}{13!!\times 14!!}-\frac{3.91212^3}{44!!\times 45!!}+\frac{3.91212^4}{123!!\times 124!!}\right)\zeta(3,1)^2$$

Integral representations:

$$\frac{1}{12\pi\log^3(2)}\left(\log(2)\left(e^{-3.91212}+2e^{-2\times3.91212}+4e^{-4\times3.91212}+8e^{-8\times3.91212}+\left(1-\frac{3.91212}{(3\times1)!}+\frac{3.91212^2}{(7\times2)!}-\frac{3.91212^3}{(15\times3)!}+\frac{3.91212^4}{(31\times4)!}\right)\right)\right)(55\,\zeta(3)^2)=$$
$$\frac{1}{108\pi(1!)^2\log^2(2)}55\left(1+\frac{8}{e^{31.297}}+\frac{4}{e^{15.6485}}+\frac{2}{e^{7.82424}}+\frac{1}{e^{3.91212}}-\frac{3.91212}{\int_0^\infty t^3\,\mathcal{A}^{-t}\,dt}+\frac{15.3047}{\int_0^\infty t^{14}\,\mathcal{A}^{-t}\,dt}-\frac{59.8737}{\int_0^\infty t^{45}\,\mathcal{A}^{-t}\,dt}+\frac{234.233}{\int_0^\infty t^{124}\,\mathcal{A}^{-t}\,dt}\right)\left(\int_0^1\frac{\log^3(1-t^2)}{t^3}\,dt\right)^2$$

$$\frac{1}{12\pi\log^3(2)}\left(\log(2)\left(e^{-3.91212}+2e^{-2\times3.91212}+4e^{-4\times3.91212}+8e^{-8\times3.91212}+\left(1-\frac{3.91212}{(3\times1)!}+\frac{3.91212^2}{(7\times2)!}-\frac{3.91212^3}{(15\times3)!}+\frac{3.91212^4}{(31\times4)!}\right)\right)\right)(55\,\zeta(3)^2)=$$
$$\frac{1}{108\pi(1!)^2\left(\int_1^2\frac{1}{t}\,dt\right)^2}55\left(1+\frac{8}{e^{31.297}}+\frac{4}{e^{15.6485}}+\frac{2}{e^{7.82424}}+\frac{1}{e^{3.91212}}-\frac{3.91212}{\int_0^1\log^3\left(\frac{1}{t}\right)dt}+\frac{15.3047}{\int_0^1\log^{14}\left(\frac{1}{t}\right)dt}-\frac{59.8737}{\int_0^1\log^{45}\left(\frac{1}{t}\right)dt}+\frac{234.233}{\int_0^1\log^{124}\left(\frac{1}{t}\right)dt}\right)\left(\int_0^1\frac{\log^3(1-t^2)}{t^3}\,dt\right)^2$$

$$\frac{1}{12\pi\log^3(2)}\left(\log(2)\left(e^{-3.91212}+2e^{-2\times3.91212}+4e^{-4\times3.91212}+8e^{-8\times3.91212}+\left(1-\frac{3.91212}{(3\times1)!}+\frac{3.91212^2}{(7\times2)!}-\frac{3.91212^3}{(15\times3)!}+\frac{3.91212^4}{(31\times4)!}\right)\right)\right)(55\,\zeta(3)^2)=$$
$$\frac{1}{108\pi(1!)^2\left(\int_1^2\frac{1}{t}\,dt\right)^2}55\left(1+\frac{8}{e^{31.297}}+\frac{4}{e^{15.6485}}+\frac{2}{e^{7.82424}}+\frac{1}{e^{3.91212}}-\frac{3.91212}{\int_0^\infty t^3\,\mathcal{A}^{-t}\,dt}+\frac{15.3047}{\int_0^\infty t^{14}\,\mathcal{A}^{-t}\,dt}-\frac{59.8737}{\int_0^\infty t^{45}\,\mathcal{A}^{-t}\,dt}+\frac{234.233}{\int_0^\infty t^{124}\,\mathcal{A}^{-t}\,dt}\right)\left(\int_0^1\frac{\log^3(1-t^2)}{t^3}\,dt\right)^2$$

(55 ζ(3)^2)/(12 π log^3(2)) * (((1/3.9121192269((1+0.0000098844 cos((2Pi*ln(3.9121192269))/ln2+0.872811))))))

Input interpretation:

$$\frac{55\,\zeta(3)^2}{12\pi\log^3(2)}\left(\frac{1}{3.9121192269}\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.9121192269)}{\log(2)}+0.872811\right)\right)\right)$$

$\zeta(s)$ is the Riemann zeta function

log(x) is the natural logarithm

Result:

1.6180670728...

1.6180670728…. as above

Addition formulas:

$$\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)\left(55\zeta(3)^2\right)}{3.91211922690000\left(12\pi\log^3(2)\right)} = \frac{1}{\pi\log^3(2)}1.17157301899646\left(1+9.8844\times10^{-6}\cos(0.872811)\cos\left(-\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)+9.8844\times10^{-6}\sin(0.872811)\sin\left(-\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)\zeta(3)^2$$

$$\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)\left(55\zeta(3)^2\right)}{3.91211922690000\left(12\pi\log^3(2)\right)} = \frac{1}{\pi\log^3(2)}0.0000115803\left(101170.+\cos(0.872811)\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)-\sin(0.872811)\sin\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)\zeta(3)^2$$

$$\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)\left(55\zeta(3)^2\right)}{3.91211922690000\left(12\pi\log^3(2)\right)} = \frac{1}{\pi\log^3(2)}1.17157301899646\left(1+9.8844\times10^{-6}\cosh\left(\frac{2i\pi\log(3.91211922690000)}{\log(2)}\right)\cos(0.872811)+9.8844\times10^{-6}\,i\sinh\left(\frac{2i\pi\log(3.91211922690000)}{\log(2)}\right)\sin(0.872811)\right)\zeta(3)^2$$

$$\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)\left(55\zeta(3)^2\right)}{3.91211922690000\left(12\pi\log^3(2)\right)} = \frac{1}{\pi\log^3(2)}0.0000115803\left(101170.+\cosh\left(-\frac{2i\pi\log(3.91211922690000)}{\log(2)}\right)\cos(0.872811)-i\left(\sinh\left(-\frac{2i\pi\log(3.91211922690000)}{\log(2)}\right)\sin(0.872811)\right)\right)\zeta(3)^2$$

Alternative representations:

$$\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)\left(55\,\zeta(3)^2\right)}{3.91211922690000\left(12\pi\log^3(2)\right)}=$$
$$\left(55\left(1+4.9422\times10^{-6}\left(e^{-i\,(0.872811+(2\pi\log(3.91211922690000))/\log(2))}+e^{i\,(0.872811+(2\pi\log(3.91211922690000))/\log(2))}\right)\right)\zeta(3,1)^2\right)\Big/\left(3.91211922690000\left(12\pi\log^3(2)\right)\right)$$

$$\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)\left(55\,\zeta(3)^2\right)}{3.91211922690000\left(12\pi\log^3(2)\right)}=$$
$$\frac{55\left(1+9.8844\times10^{-6}\cosh\left(-i\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)\right)\zeta(3,1)^2}{3.91211922690000\left(12\pi\left(\log(a)\log_a(2)\right)^3\right)}$$

$$\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)\left(55\,\zeta(3)^2\right)}{3.91211922690000\left(12\pi\log^3(2)\right)}=$$
$$\left(55\left(1+4.9422\times10^{-6}\left(e^{-i\,(0.872811+(2\pi\log(3.91211922690000))/\log(2))}+e^{i\,(0.872811+(2\pi\log(3.91211922690000))/\log(2))}\right)\right)\zeta(3,1)^2\right)\Big/\left(3.91211922690000\left(12\pi\log_e^3(2)\right)\right)$$

Note that:

From the expression Γ(1/4) /(2π^3/4), that is one of four values found by Ramanujan concerning the Dedekind eta function, we obtain:

Γ(1/4) /(2π^(3/4))

Input:

$$\frac{\Gamma\left(\frac{1}{4}\right)}{2\,\pi^{3/4}}$$

$\Gamma(x)$ is the gamma function

Decimal approximation:

0.768225422326056659002594179576180644517866914464805014676...

0.768225422326…

Alternate forms:

$$\frac{2\times \frac{1}{4}!}{\pi^{3/4}}$$

$$\frac{2\,\Gamma\left(\frac{5}{4}\right)}{\pi^{3/4}}$$

$$\sqrt{\frac{2\,(2+\sqrt{2})\,K\left(\frac{\left(-2-2\sqrt{2}\right)^2}{\left(4+2\sqrt{2}\right)^2}\right)}{(4+2\sqrt{2})\,\pi}}$$

$n!$ is the factorial function

$K(m)$ is the complete elliptic integral of the first kind with parameter $m = k^2$

Alternative representations:

$$\frac{\Gamma\left(\frac{1}{4}\right)}{2\,\pi^{3/4}} = \frac{G\left(1+\frac{1}{4}\right)}{G\left(\frac{1}{4}\right)\left(2\,\pi^{3/4}\right)}$$

$$\frac{\Gamma\left(\frac{1}{4}\right)}{2\,\pi^{3/4}} = \frac{e^{-\log G(1/4)+\log G(1+1/4)}}{2\,\pi^{3/4}}$$

$$\frac{\Gamma\left(\frac{1}{4}\right)}{2\,\pi^{3/4}} = \frac{\left(-1+\frac{1}{4}\right)!}{2\,\pi^{3/4}}$$

Series representations:

$$\frac{\Gamma\left(\frac{1}{4}\right)}{2\,\pi^{3/4}} = \frac{2\sum_{k=0}^{\infty}\frac{4^{-k}\,\Gamma^{(k)}(1)}{k!}}{\pi^{3/4}}$$

$$\frac{\Gamma\left(\frac{1}{4}\right)}{2\,\pi^{3/4}} = \frac{1}{2\,\pi^{3/4}\sum_{k=1}^{\infty}4^{-k}\,c_k}$$

$$\text{for}\left(c_1 = 1 \text{ and } c_2 = 1 \text{ and } c_k = \frac{\gamma\,c_{-1+k} + \sum_{j=1}^{-2+k}(-1)^{1+j+k}\,c_j\,\zeta(-j+k)}{-1+k}\right)$$

$$\frac{\Gamma\left(\frac{1}{4}\right)}{2\,\pi^{3/4}} = \frac{\sum_{k=0}^{\infty}\frac{\left(\frac{1}{4}-z_0\right)^k\,\Gamma^{(k)}(z_0)}{k!}}{2\,\pi^{3/4}} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$\frac{\Gamma\left(\frac{1}{4}\right)}{2\,\pi^{3/4}} = \frac{\sqrt[4]{\pi}}{2\sum_{k=0}^{\infty}\left(\frac{1}{4}-z_0\right)^k \sum_{j=0}^{k} \frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}(-j+k)\pi+\pi z_0\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}}$$

Integral representations:

$$\frac{\Gamma\left(\frac{1}{4}\right)}{2\,\pi^{3/4}} = \frac{1}{2\,\pi^{3/4}}\int_0^1 \frac{1}{\log^{3/4}\left(\frac{1}{t}\right)}\,dt$$

$$\frac{\Gamma\left(\frac{1}{4}\right)}{2\,\pi^{3/4}} = \frac{1}{2\,\pi^{3/4}}\int_0^{\infty} \frac{e^{-t}}{t^{3/4}}\,dt$$

$$\frac{\Gamma\left(\frac{1}{4}\right)}{2\,\pi^{3/4}} = \frac{e^{\int_0^1 \frac{-\frac{3}{4}+\sqrt[4]{x}-\frac{x}{4}}{(-1+x)\log(x)}\,dx}}{2\,\pi^{3/4}}$$

Dividing by 3, we obtain:

1/3* ((Γ(1/4) /(2π^(3/4))))

Input:

$$\frac{1}{3}\times\frac{\Gamma\left(\frac{1}{4}\right)}{2\,\pi^{3/4}}$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{\Gamma\left(\frac{1}{4}\right)}{6\,\pi^{3/4}}$$

Decimal approximation:

0.256075140775352219667531393192060214839288971488268338225…

0.256075140775…

Alternate forms:

$$\frac{2\times\frac{1}{4}!}{3\,\pi^{3/4}}$$

$$\frac{1}{3}\sqrt{\frac{2\,(2+\sqrt{2})\,K\left(\frac{\left(-2-2\sqrt{2}\right)^2}{\left(4+2\sqrt{2}\right)^2}\right)}{(4+2\sqrt{2})\,\pi}}$$

$n!$ is the factorial function

$K(m)$ is the complete elliptic integral of the first kind with parameter $m=k^2$

Alternative representations:

$$\frac{\Gamma\left(\frac{1}{4}\right)}{\left(2\,\pi^{3/4}\right)3}=\frac{G\left(1+\frac{1}{4}\right)}{3\,G\left(\frac{1}{4}\right)\left(2\,\pi^{3/4}\right)}$$

$$\frac{\Gamma\left(\frac{1}{4}\right)}{\left(2\,\pi^{3/4}\right)3}=\frac{e^{-\log G(1/4)+\log G(1+1/4)}}{3\left(2\,\pi^{3/4}\right)}$$

$$\frac{\Gamma\left(\frac{1}{4}\right)}{\left(2\,\pi^{3/4}\right)3}=\frac{\left(-1+\frac{1}{4}\right)!}{3\left(2\,\pi^{3/4}\right)}$$

Series representations:

$$\frac{\Gamma\left(\frac{1}{4}\right)}{\left(2\,\pi^{3/4}\right)3}=\frac{2\sum_{k=0}^{\infty}\frac{4^{-k}\,\Gamma^{(k)}(1)}{k!}}{3\,\pi^{3/4}}$$

$$\frac{\Gamma\left(\frac{1}{4}\right)}{\left(2\,\pi^{3/4}\right)3}=\frac{1}{6\,\pi^{3/4}\sum_{k=1}^{\infty}4^{-k}\,c_k}$$

$$\text{for}\left(c_1=1\text{ and }c_2=1\text{ and }c_k=\frac{\gamma\,c_{-1+k}+\sum_{j=1}^{-2+k}(-1)^{1+j+k}\,c_j\,\zeta(-j+k)}{-1+k}\right)$$

$$\frac{\Gamma\left(\frac{1}{4}\right)}{\left(2\,\pi^{3/4}\right)3}=\frac{\sum_{k=0}^{\infty}\frac{\left(\frac{1}{4}-z_0\right)^k\,\Gamma^{(k)}(z_0)}{k!}}{6\,\pi^{3/4}}\quad\text{for }(z_0\notin\mathbb{Z}\text{ or }z_0>0)$$

$$\frac{\Gamma\left(\frac{1}{4}\right)}{\left(2\pi^{3/4}\right)3} = \frac{\sqrt[4]{\pi}}{6\sum_{k=0}^{\infty}\left(\frac{1}{4}-z_0\right)^k \sum_{j=0}^{k} \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2}(-j+k)\pi+\pi z_0\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}}$$

Integral representations:

$$\frac{\Gamma\left(\frac{1}{4}\right)}{\left(2\pi^{3/4}\right)3} = \frac{1}{6\pi^{3/4}}\int_0^1 \frac{1}{\log^{3/4}\left(\frac{1}{t}\right)}\,dt$$

$$\frac{\Gamma\left(\frac{1}{4}\right)}{\left(2\pi^{3/4}\right)3} = \frac{1}{6\pi^{3/4}}\int_0^{\infty} \frac{e^{-t}}{t^{3/4}}\,dt$$

$$\frac{\Gamma\left(\frac{1}{4}\right)}{\left(2\pi^{3/4}\right)3} = \frac{e^{\int_0^1 \frac{-\frac{3}{4}+\sqrt[4]{x}-\frac{x}{4}}{(-1+x)\log(x)}\,dx}}{6\pi^{3/4}}$$

From the **Landau-Ramanujan constant**

Definition:

Let $S(x)$ denote the number of positive integers not exceeding x which can be expressed as a sum of two squares (i.e., those $n \le x$ such that the sum of squares function $r_2(n) > 0$). For example, the first few positive integers that can be expressed as a sum of squares are

$$1 = 0^2 + 1^2$$
$$2 = 1^2 + 1^2$$
$$4 = 0^2 + 2^2$$
$$5 = 1^2 + 2^2$$
$$8 = 2^2 + 2^2$$

(OEIS A001481), so $S(1) = 1$, $S(2) = 2$, $S(4) = 3$, $S(5) = 4$, $S(8) = 5$, and so on. Then

$$\lim_{x\to\infty} \frac{\sqrt{\ln x}}{x} S(x) = K,$$

as proved by Landau, where K is a constant. Ramanujan independently stated the theorem in the slightly different form that the number of numbers between A and x which are either squares of sums of two squares is

$$S(x) = K\int_A^x \frac{dt}{\sqrt{\ln t}} + \theta(x),$$

where $K \approx 0.764$ and $\theta(x)$ is very small compared with the previous integral.

Decimal form:

0.76422365358922066299069873125009232811679054139 34

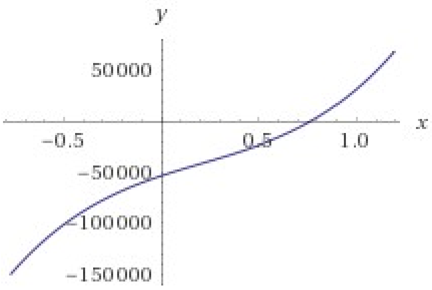
0.764223653589… that we can write as:

57125 x^3 - 35804 x^2 + 63607 x – 53196

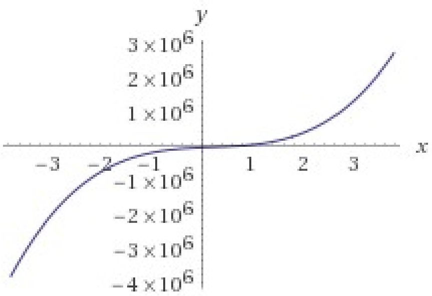
Input:

$57125 x^3 - 35804 x^2 + 63607 x - 53196$

Plots:



(x from −0.8 to 1.2)



(x from −3.8 to 3.8)

Alternate forms:

$x (x (57125 x - 35804) + 63607) - 53196$

$$57125 \left(x - \frac{35804}{171375}\right)^3 + \frac{9618723209 \left(x - \frac{35804}{171375}\right)}{171375} - \frac{3607937920338928}{88108171875}$$

$$
-\Bigg(\bigg(\Big(-171\,375\,\sqrt[3]{1\,803\,968\,960\,169\,464+171\,375\sqrt{141\,107\,003\,466\,442\,869\,369}}\,x+\left(1\,803\,968\,960\,169\,464+171\,375\sqrt{141\,107\,003\,466\,442\,869\,369}\right)^{2/3}+35\,804\,\sqrt[3]{1\,803\,968\,960\,169\,464+171\,375\sqrt{141\,107\,003\,466\,442\,869\,369}}-9\,618\,723\,209\Big)\Big(171\,375\left(1\,803\,968\,960\,169\,464+171\,375\sqrt{141\,107\,003\,466\,442\,869\,369}\right)^{2/3}x^2-71\,608\left(1\,803\,968\,960\,169\,464+171\,375\sqrt{141\,107\,003\,466\,442\,869\,369}\right)^{2/3}x-9\,618\,723\,209\,\sqrt[3]{1\,803\,968\,960\,169\,464+171\,375\sqrt{141\,107\,003\,466\,442\,869\,369}}\,x+171\,375\sqrt{141\,107\,003\,466\,442\,869\,369}\,x+1\,803\,968\,960\,169\,464\,x+63\,607\left(1\,803\,968\,960\,169\,464+171\,375\sqrt{141\,107\,003\,466\,442\,869\,369}\right)^{2/3}+\sqrt{141\,107\,003\,466\,442\,869\,369}\,\sqrt[3]{1\,803\,968\,960\,169\,464+171\,375\sqrt{141\,107\,003\,466\,442\,869\,369}}+12\,536\,004\,236\,\sqrt[3]{1\,803\,968\,960\,169\,464+171\,375\sqrt{141\,107\,003\,466\,442\,869\,369}}-35\,804\sqrt{141\,107\,003\,466\,442\,869\,369}+162\,979\,031\,489\,119\Big)\bigg)\Big/\left(514\,125\left(1\,803\,968\,960\,169\,464+171\,375\sqrt{141\,107\,003\,466\,442\,869\,369}\right)\right)\Bigg)
$$

Real root:

$x \approx 0.764223653589221$

0.764223653589221

Complex roots:

$x \approx -0.06873 - 1.10172\,i$

$x \approx -0.06873 + 1.10172\,i$

Polynomial discriminant:

$\Delta = -188\,142\,671\,288\,590\,492\,492$

Properties as a real function:

Domain

$\mathbb{R}$ (all real numbers)

Range

$\mathbb{R}$ (all real numbers)

Bijectivity

bijective from its domain to $\mathbb{R}$

$\mathbb{R}$ is the set of real numbers

Derivative:

$$\frac{d}{dx}(57125 x^3 - 35804 x^2 + 63607 x - 53196) = 171375 x^2 - 71608 x + 63607$$

Indefinite integral:

$$\int (-53196 + 63607 x - 35804 x^2 + 57125 x^3)\, dx = \frac{57125 x^4}{4} - \frac{35804 x^3}{3} + \frac{63607 x^2}{2} - 53196 x + \text{constant}$$

From the previous expression,

$57125 x^3 - 35804 x^2 + 63607 x - 53196$

we obtain also:

(57125 (3x)^3 - 35804 (3x)^2 + 63607 (3x) – 53196)

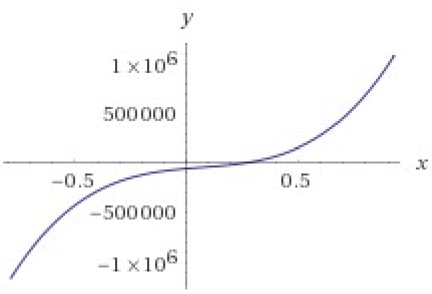
Input:

$57125 (3 x)^3 - 35804 (3 x)^2 + 63607 (3 x) - 53196$

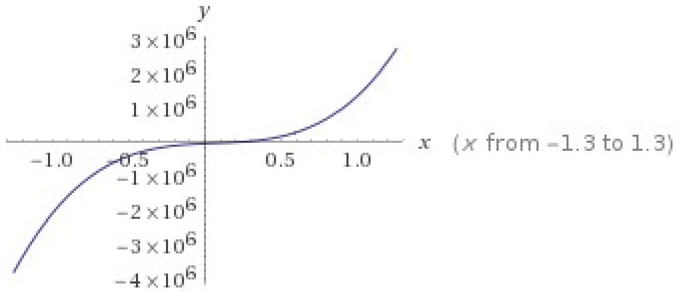
Result:

$1542375 x^3 - 322236 x^2 + 190821 x - 53196$

Plots:



(x from −0.8 to 0.9)



Alternate forms:

$3(514125x^3 - 107412x^2 + 63607x - 17732)$

$x(x(1542375x - 322236) + 190821) - 53196$

$3x(3x(171375x - 35804) + 63607) - 53196$

Real root:

$x \approx 0.25474$

Real root:

$x \approx 0.254741217863074$

0.254741217863074

Complex roots:

$x \approx -0.02291 - 0.36724 i$

$x \approx -0.02291 + 0.36724 i$

Polynomial discriminant:

$\Delta = -137156007369382469026668$

Properties as a real function:

Domain

$\mathbb{R}$ (all real numbers)

Range

$\mathbb{R}$ (all real numbers)

Bijectivity

bijective from its domain to $\mathbb{R}$

$\mathbb{R}$ is the set of real numbers

Derivative:

$$\frac{d}{dx}(57125\,(3\,x)^3 - 35804\,(3\,x)^2 + 63607\,(3\,x) - 53196) = 4627125\,x^2 - 644472\,x + 190821$$

Indefinite integral:

$$\int(-53196 + 190821\,x - 322236\,x^2 + 1542375\,x^3)\,dx = \frac{1542375\,x^4}{4} - 107412\,x^3 + \frac{190821\,x^2}{2} - 53196\,x + \text{constant}$$

From the mean of the two results, we can write the following expression:

(57125 ((4/(lnPi))x-0.256075140775/2)^3 - 35804 ((4/(lnPi))x-0.256075140775/2)^2 + 63607 ((4/(lnPi))x-0.256075140775/2) – 53196)

Input interpretation:

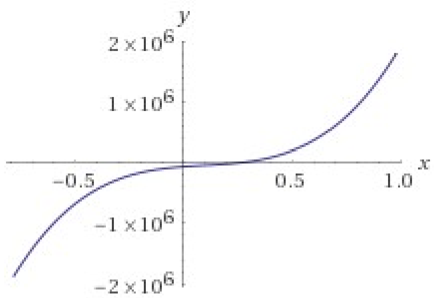
$$57125\left(\frac{4}{\log(\pi)}\,x - \frac{0.256075140775}{2}\right)^3 - 35804\left(\frac{4}{\log(\pi)}\,x - \frac{0.256075140775}{2}\right)^2 + 63607\left(\frac{4}{\log(\pi)}\,x - \frac{0.256075140775}{2}\right) - 53196$$

$\log(x)$ is the natural logarithm

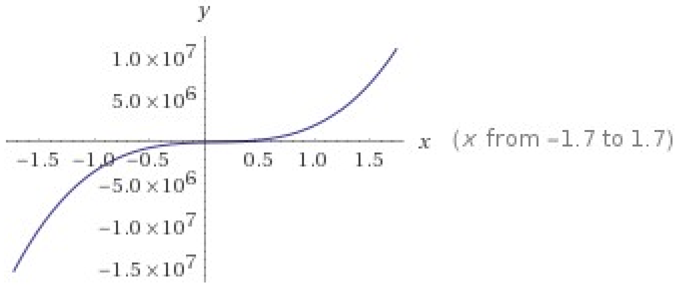
Result:

$$57125\left(\frac{4\,x}{\log(\pi)} - 0.128037570388\right)^3 - 35804\left(\frac{4\,x}{\log(\pi)} - 0.128037570388\right)^2 + 63607\left(\frac{4\,x}{\log(\pi)} - 0.128037570388\right) - 53196$$

Plots:



(x from -0.8 to 1)



Alternate forms:

$$\frac{4(914000x^3 - 264416.2963823x^2 + 99047.0380042x - 23268.55949802)}{\log^3(\pi)}$$

$$\frac{3656000x^3}{\log^3(\pi)} - 705081.21705x^2 + 264114.606615x - 62046.94821914$$

$$x(x(2.43723341263\times10^6 x - 705081.21705) + 264114.606615) - 62046.948219$$

Expanded form:

$$\frac{3656000x^3}{\log^3(\pi)} - 267916.112219x^2 - \frac{572864x^2}{\log^2(\pi)} + 41854.3134702x + \frac{254428x}{\log(\pi)} - 62046.94821914$$

Real root:

$x \approx 0.25534952227$

0.25534952227

Complex roots:

$x = 0.0169731034 - 0.3152940349i$

$x = 0.0169731034 + 0.3152940349i$

Polynomial discriminant:

$\Delta = -3.424751988\times10^{23}$

Properties as a real function:

Domain

$\mathbb{R}$ (all real numbers)

Range

$\mathbb{R}$ (all real numbers)

Bijectivity

bijective from its domain to $\mathbb{R}$

$\mathbb{R}$ is the set of real numbers

Derivative:

$$\frac{d}{dx}\left(57125\left(\frac{4x}{\log(\pi)} - 0.128037570388\right)^3 - 35804\left(\frac{4x}{\log(\pi)} - 0.128037570388\right)^2 + 63607\left(\frac{4x}{\log(\pi)} - 0.128037570388\right) - 53196\right) =$$

$$7.31170023788\times10^6 x^2 - 1.41016243411\times10^6 x + 264114.606615$$

Indefinite integral:

$$\int\left(57125\left(\frac{4x}{\log(\pi)} - \frac{0.256075140775}{2}\right)^3 - 35804\left(\frac{4x}{\log(\pi)} - \frac{0.256075140775}{2}\right)^2 + 63607\left(\frac{4x}{\log(\pi)} - \frac{0.256075140775}{2}\right) - 53196\right)dx =$$

$$609308.35316 x^4 - 235027.072351 x^3 + 132057.303307 x^2 - 62046.948219 x + \text{constant}$$

And multiplying, from the expression that is equal to 569, we obtain:

(57125 ((1/569.045662)(4/(lnPi))x-0.256075140775/2)^3 - 35804 ((1/569.045662)(4/(lnPi))x-0.256075140775/2)^2 + 63607 (((1/569.045662)4/(lnPi))x-0.256075140775/2) – 53196)

Input interpretation:

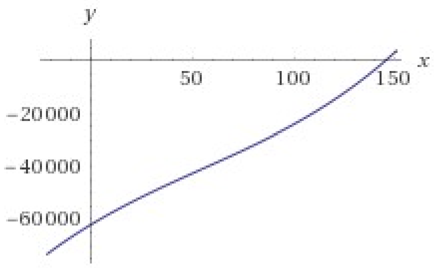
$$57125\left(\frac{1}{569.045662}\times\frac{4}{\log(\pi)}x - \frac{0.256075140775}{2}\right)^3 - 35804\left(\frac{1}{569.045662}\times\frac{4}{\log(\pi)}x - \frac{0.256075140775}{2}\right)^2 + 63607\left(\left(\frac{1}{569.045662}\times\frac{4}{\log(\pi)}\right)x - \frac{0.256075140775}{2}\right) - 53196$$

$\log(x)$ is the natural logarithm

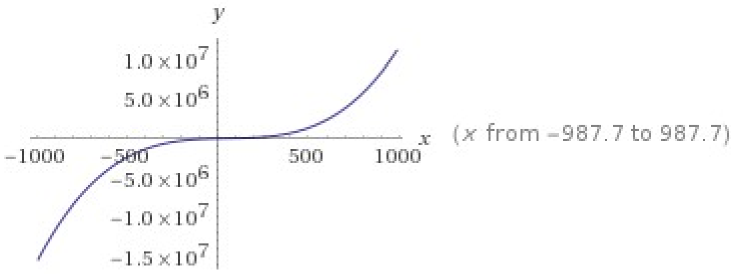
Result:

$$57125(0.00614059x - 0.128037570388)^3 - 35804(0.00614059x - 0.128037570388)^2 + 63607(0.00614059x - 0.128037570388) - 53196$$

Plots:



(x from -21.5 to 150.4)



Alternate forms:

$x\,((0.0132268\,x - 2.17743)\,x + 464.136) - 62\,046.9$

$5.69033 \times 10^{-47} \left(2.32444 \times 10^{44}\, x^3 - 3.82655 \times 10^{46}\, x^2 + 8.15658 \times 10^{48}\, x - 1.09039 \times 10^{51}\right)$

$0.0132268\,x^3 - 2.17743\,x^2 + 464.136\,x - 62\,046.9$

Real root:

$x \approx 145.306$

$145.306 \approx 145$

Complex roots:

$x = 9.65847 - 179.417\,i$

$x = 9.65847 + 179.417\,i$

Polynomial discriminant:

$\Delta = -1.00866 \times 10^7$

Properties as a real function:

Domain

$\mathbb{R}$ (all real numbers)

Range

$\mathbb{R}$ (all real numbers)

Bijectivity

bijective from its domain to $\mathbb{R}$

$\mathbb{R}$ is the set of real numbers

Derivative:

$$\frac{d}{dx}\left(57\,125\,(0.00614059\,x - 0.128037570388)^3 - 35\,804\,(0.00614059\,x - 0.128037570388)^2 + 63\,607\,(0.00614059\,x - 0.128037570388) - 53\,196\right) = 0.0396805\,x^2 - 4.35487\,x + 464.136$$

Indefinite integral:

$$\int \left(57125 \left(\frac{4x}{569.045662 \log(\pi)} - \frac{0.256075140775}{2}\right)^3 - 35804 \left(\frac{4x}{569.045662 \log(\pi)} - \frac{0.256075140775}{2}\right)^2 + 63607 \left(\frac{4x}{569.045662 \log(\pi)} - \frac{0.256075140775}{2}\right) - 53196\right) dx = 0.00330671 x^4 - 0.725811 x^3 + 232.068 x^2 - 62046.9 x + \text{constant}$$

Now, we observe that:

0.25561790957^1/3.28477980525949

Input interpretation:

$$\sqrt[3.28477980525949]{0.25561790957}$$

Result:

0.66016181584686933680251808663541772183900552942696877985 8...

Input interpretation:

0.6601618158468693

0.6601618158468693

Rational approximation:

$$\frac{50865831}{77050550}$$

Possible closed forms:

$\Pi_2 \approx 0.66016181584686957392$

root of $9408 x^3 - 6374 x^2 - 224 x + 219$ near $x = 0.660162$ ≈ 0.66016181584686930 5956

$$\frac{14745063 \pi}{70169132} \approx 0.66016181584686949609$$

Π_2 is the twin primes constant

Or:

0.25561790957^(1/2 sqrt(1/310 (-2572 - 2444 e + 3088 π - 535 log(2))))

Input interpretation:

$$0.25561790957^{1/2\sqrt{1/310\,(-2572-2444\,e+3088\,\pi-535\log(2))}}$$

log(x) is the natural logarithm

Result:

0.66016181585...

0.66016181585….. = Twin Prime Constant

Alternative representations:

$$0.255617909570000^{1/2\sqrt{1/310\,(-2572-2444\,e+3088\,\pi-535\log(2))}} = 0.255617909570000^{1/2\sqrt{1/310\,(-2572-2444\,e+3088\,\pi-535\log_e(2))}}$$

$$0.255617909570000^{1/2\sqrt{1/310\,(-2572-2444\,e+3088\,\pi-535\log(2))}} = 0.255617909570000^{1/2\sqrt{1/310\,(-2572-2444\,e+3088\,\pi-535\log(a)\log_a(2))}}$$

$$0.255617909570000^{1/2\sqrt{1/310\,(-2572-2444\,e+3088\,\pi-535\log(2))}} = 0.255617909570000^{1/2\sqrt{1/310\left(-2572-2444\,e+3088\,\pi-1070\coth^{-1}(3)\right)}}$$

Series representations:

$$0.255617909570000^{1/2\sqrt{1/310\,(-2572-2444\,e+3088\,\pi-535\log(2))}} = 0.255617909570000^{1/2\sum_{k=0}^{\infty}\left(\left(-\frac{1}{310}\right)^k(-2882-2444\,e+3088\,\pi-535\log(2))^k\left(-\frac{1}{2}\right)_k\right)/k!}$$

$$0.255617909570000^{1/2\sqrt{1/310\,(-2572-2444\,e+3088\,\pi-535\log(2))}} = 0.25561790957000^{1/2\sqrt{1/310\left(-2572-2444\,e+3088\,\pi-535\left(2\,i\,\pi\left\lfloor\arg(2-x)/(2\,\pi)\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\left((-1)^k\,(2-x)^k\,x^{-k}\right)/k\right)\right)}}$$

for $x < 0$

$$0.255617909570000^{1/2\sqrt{1/310(-2572-2444e+3088\pi-535\log(2))}} =$$

$$0.25561790957000^{\cdot}.$$

$$0^{1/2\sqrt{1/310\left(-2572-2444e+3088\pi-535\left(\log(z_0)+\lfloor\arg(2-z_0)/(2\pi)\rfloor(\log(1/z_0)+\log(z_0))-\sum_{k=1}^{\infty}\left((-1)^k(2-z_0)^k z_0^{-k}\right)/k\right)\right)}}$$

Integral representations:

$$0.255617909570000^{1/2\sqrt{1/310(-2572-2444e+3088\pi-535\log(2))}} =$$

$$0.255617909570000^{1/2\sqrt{1/310\left(-4(643+611e-772\pi)-535\int_1^2 1/t\,dt\right)}}$$

$$0.255617909570000^{1/2\sqrt{1/310(-2572-2444e+3088\pi-535\log(2))}} =$$

$$0.255617909570000^{1/2\sqrt{1/310\left(-4(643+611e-772\pi)-535/(2i\pi)\int_{-i\infty+\gamma}^{i\infty+\gamma}\left(\Gamma(-s)^2\Gamma(1+s)\right)/\Gamma(1-s)\,ds\right)}}$$

for $-1 < \gamma < 0$

$\Gamma(x)$ is the gamma function

We have:

4/3 - (19 ζ(2))/29

Input:

$$\frac{4}{3} - \frac{19\,\zeta(2)}{29}$$

$\zeta(s)$ is the Riemann zeta function

Exact result:

$$\frac{4}{3} - \frac{19\pi^2}{174}$$

Decimal approximation:

0.2556179102258746335755440862204202783278144325426722879 20...

0.2556179102258746……

Property:

$\frac{4}{3} - \frac{19\pi^2}{174}$ is a transcendental number

Alternate form:

$$\frac{1}{174}\left(232 - 19\pi^2\right)$$

Alternative representations:

$$\frac{4}{3}-\frac{19\,\zeta(2)}{29}=\frac{4}{3}-\frac{19\,\zeta(2,1)}{29}$$

$$\frac{4}{3}-\frac{19\,\zeta(2)}{29}=\frac{4}{3}+\frac{19\,\text{Li}_2(-1)}{\frac{29}{2}}$$

$$\frac{4}{3}-\frac{19\,\zeta(2)}{29}=\frac{4}{3}-\frac{19\,S_{1,1}(1)}{29}$$

$\zeta(s, a)$ is the generalized Riemann zeta function

Series representations:

$$\frac{4}{3}-\frac{19\,\zeta(2)}{29}=\frac{4}{3}-\frac{19}{29}\sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{4}{3}-\frac{19\,\zeta(2)}{29}=\frac{4}{3}+\frac{38}{29}\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{4}{3}-\frac{19\,\zeta(2)}{29}=\frac{4}{3}-\frac{76}{87}\sum_{k=0}^{\infty}\frac{1}{(1+2\,k)^2}$$

Integral representations:

$$\frac{4}{3}-\frac{19\,\zeta(2)}{29}=\frac{4}{3}-\frac{152}{87}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{4}{3}-\frac{19\,\zeta(2)}{29}=\frac{4}{3}-\frac{38}{87}\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{4}{3}-\frac{19\,\zeta(2)}{29}=\frac{4}{3}-\frac{38}{87}\left(\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

From which:

4/3 - (19(x))/29 = 0.25561791022587463357554408

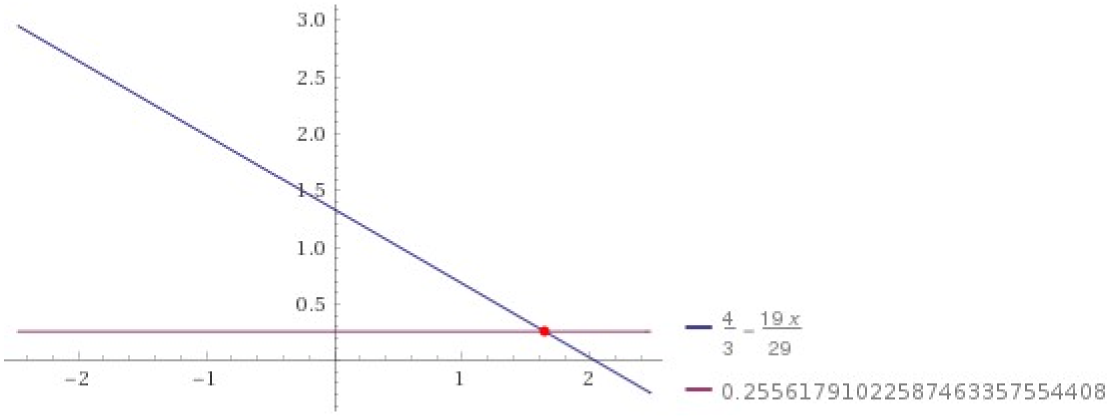
Input interpretation:

$$\frac{4}{3} - \frac{19x}{29} = 0.25561791022587463357554408$$

Result:

$$\frac{4}{3} - \frac{19x}{29} = 0.25561791022587463357554408$$

Plot:



Alternate forms:

$$1.07771542310745869975778925 - \frac{19x}{29} = 0$$

$$\frac{1}{87}(116 - 57x) = 0.25561791022587463357554408$$

Solution:

$x \approx 1.64493406684822643647241518$

1.64493406684822643647241518

Rational approximation:

$$\frac{14989248869061}{9112370623937} = 1 + \frac{5876878245124}{9112370623937}$$

Possible closed forms:

$$\frac{\pi^2}{6} \approx 1.64493406684822643647241516664 6$$

$$\zeta(2) \approx 1.644934066848226436472415166646$$

$$\frac{1}{24\mathcal{P}_A^2} \approx 1.644934066848226436472415166646$$

$\zeta(2)$ is zeta of 2

$\mathcal{P}_A$ is Plouffe's A-constant

We note that, multiplying the expression

1/3.9121192269((1+0.0000098844 cos((2Pi*ln(3.9121192269))/ln2+0.872811)))

by

(1+log base 2 (3)), that is equal to the Hausdorff dimension of Octahedron fractal 2.5849, we obtain:

1/3.9121192269((1+0.0000098844 cos((2Pi*ln(3.9121192269))/ln2+0.872811)))*(1+log base 2 (3))

Input interpretation:

$$\frac{1}{3.9121192269}\left(1+9.8844\times 10^{-6}\cos\left(\frac{2\pi\log(3.9121192269)}{\log(2)}+0.872811\right)\right)(1+\log_2(3))$$

log(x) is the natural logarithm

$\log_b(x)$ is the base- b logarithm

Result:

0.66076271076...

0.66076271076…. result that is very near to the Twin Prime Constant 0.66016181585…..

Addition formulas:

$$\frac{\left(1+9.8844\times 10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(1+\log_2(3))}{3.91211922690000} = 0.255615931417410\,(1+\log_2(3))\left(1+9.8844\times 10^{-6}\cos(0.872811)\cos\left(-\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)+9.8844\times 10^{-6}\sin(0.872811)\sin\left(-\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)$$

$$\frac{\left(1+9.8844\times 10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(1+\log_2(3))}{3.91211922690000} = 2.52661\times 10^{-6}\,(1+\log_2(3))\left(101170.+\cos(0.872811)\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)-\sin(0.872811)\sin\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)$$

$$\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(1+\log_2(3))}{3.91211922690000} =$$
$$0.255615931417410\,(1+\log_2(3))\left(1+9.8844\times10^{-6}\cosh\left(\frac{2\,i\,\pi\log(3.91211922690000)}{\log(2)}\right)\cos(0.872811)+9.8844\times10^{-6}\,i\sinh\left(\frac{2\,i\,\pi\log(3.91211922690000)}{\log(2)}\right)\sin(0.872811)\right)$$

$$\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(1+\log_2(3))}{3.91211922690000} =$$
$$2.52661\times10^{-6}\,(1+\log_2(3))\left(101170.+\cosh\left(-\frac{2\,i\,\pi\log(3.91211922690000)}{\log(2)}\right)\cos(0.872811)-i\left(\sinh\left(-\frac{2\,i\,\pi\log(3.91211922690000)}{\log(2)}\right)\sin(0.872811)\right)\right)$$

Alternative representations:

$$\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(1+\log_2(3))}{3.91211922690000} = \frac{\left(1+9.8844\times10^{-6}\cosh\left(i\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)\right)\left(1+\frac{\log(3)}{\log(2)}\right)}{3.91211922690000}$$

$$\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(1+\log_2(3))}{3.91211922690000} = \frac{\left(1+9.8844\times10^{-6}\cosh\left(i\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)\right)(1+\log_2(3))}{3.91211922690000}$$

$$\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(1+\log_2(3))}{3.91211922690000} = \frac{\left(1+9.8844\times10^{-6}\cosh\left(-i\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)\right)\left(1+\frac{\log(3)}{\log(2)}\right)}{3.91211922690000}$$

Series representations:

$$\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(1+\log_2(3))}{3.91211922690000} =$$
$$0.255615931417410\,(1+\log_2(3))\left(1+9.8844\times10^{-6}\sum_{k=0}^{\infty}\frac{(-1)^k\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)^{2k}}{(2k)!}\right)$$

$$\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(1+\log_2(3))}{3.91211922690000}=$$
$$0.255615931417410\,(1+\log_2(3))\left(1+9.8844\times10^{-6}J_0\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)+0.0000197688\sum_{k=1}^{\infty}(-1)^k J_{2k}\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)$$

Integral representations:

$$\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(1+\log_2(3))}{3.91211922690000}=$$
$$0.255615931417410\left(1-9.8844\times10^{-6}\int_{\frac{\pi}{2}}^{0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}}\sin(t)\,dt\right)(1+\log_2(3))$$

$$\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(1+\log_2(3))}{3.91211922690000}=$$
$$0.255615931417410\left(1.00001+-8.62721\times10^{-6}-\frac{0.0000197688\,\pi\log(3.91211922690000)}{\log(2)}\int_0^1\sin\left(t\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)\right)dt\right)(1+\log_2(3))$$

$$\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(1+\log_2(3))}{3.91211922690000}=$$
$$0.255615931417410\,(1+\log_2(3))\left(1+\frac{4.9422\times10^{-6}\sqrt{\pi}}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{s-\frac{(0.436406\log(2)+\pi\log(3.91211922690000))^2}{s\log^2(2)}}}{\sqrt{s}}\,ds\right)\quad\text{for }\gamma>0$$

$$\frac{\left(1+9.8844\times10^{-6}\cos\left(\frac{2\pi\log(3.91211922690000)}{\log(2)}+0.872811\right)\right)(1+\log_2(3))}{3.91211922690000}=$$
$$0.255615931417410\,(1+\log_2(3))\left(1+\frac{4.9422\times10^{-6}\sqrt{\pi}}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{4^s\,\Gamma(s)\left(0.872811+\frac{2\pi\log(3.91211922690000)}{\log(2)}\right)^{-2s}}{\Gamma\left(\frac{1}{2}-s\right)}\,ds\right)$$
$$\text{for }0<\gamma<\frac{1}{2}$$

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.
In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64 g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276 e^{-\pi\sqrt{22}} - \cdots, \\ 64 g_{22}^{-24} &= 4096 e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372 e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64 g_{22}^{-24} = 4096 e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372 e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the *golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.*

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the *second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.*

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:
2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

References

II

RAMANUJAN AND THE THEORY OF PRIME NUMBERS

16 *Jan.* 1913

Manuscript Book 3 of Srinivasa Ramanujan