# On some Ramanujan definite integrals: mathematical connections with $\phi$, $\zeta(2)$ and various parameters of Particle Physics


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## Abstract

*In this paper we have described and analyzed some Ramanujan definite integrals. Furthermore, we have obtained several mathematical connections between $\phi$, $\zeta(2)$ and various parameters of Particle Physics.*



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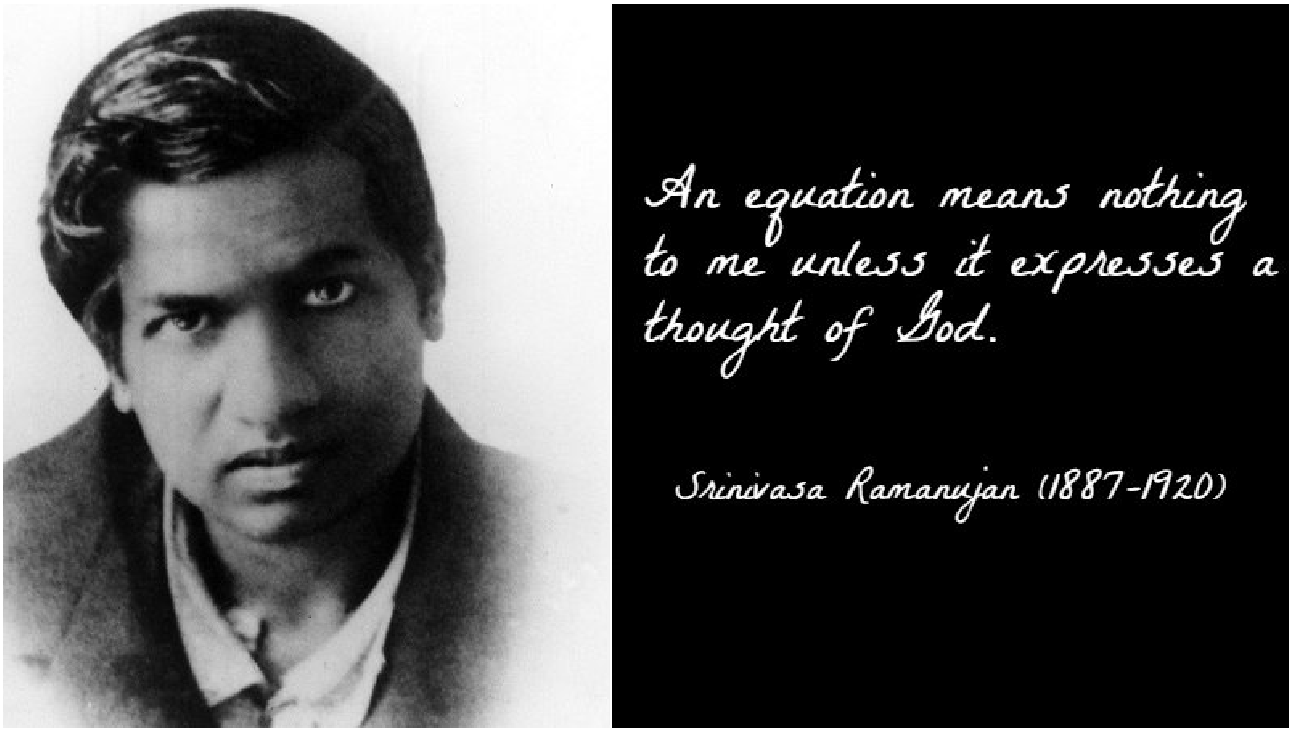


https://mobygeek.com/features/indian-mathematician-srinivasa-ramanujan-quotes-11012

**We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation**

From

**Some definite integrals** – *Srinivasa Ramanujan* - Messenger of Mathematics, XLIV, 1915, 10 – 18

We have that:

$$\int_0^\infty \frac{dx}{(1+x^2)(1+e^{-20\pi}x^2)(1+e^{-40\pi}x^2)\cdots}$$
$$= \pi^{\frac{3}{4}}\Gamma\left(\tfrac{3}{4}\right)\sqrt{5}\sqrt[4]{2}\tfrac{1}{2}(1+\sqrt[4]{5})^2\{\tfrac{1}{2}(1+\sqrt{5})\}\tfrac{5}{2}e^{-5\pi/4};$$

From the right-hand side, we obtain:

Pi^(3/4) gamma (3/4) sqrt5 * (2)^(1/4)*1/2(1+(5)^(1/4))^2 (1/2(1+sqrt5))*5/2*e^(-(5Pi)/4)

**Input:**

$$\pi^{3/4}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\,\sqrt[4]{2}\times\frac{1}{2}\left(1+\sqrt[4]{5}\right)^2\right)\left(\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\times\frac{5}{2}\,e^{-1/4\,(5\pi)}\right)$$

$\Gamma(x)$ is the gamma function

**Exact result:**

$$\frac{5}{4}\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)e^{-(5\pi)/4}\left(\frac{\pi}{2}\right)^{3/4}\Gamma\left(\frac{3}{4}\right)$$

**Decimal approximation:**

1.908000241453765777347491825531984948152566466070616872024...

1.90800024145376…

**Alternate forms:**

$$\frac{5}{3}\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)e^{-(5\pi)/4}\left(\frac{\pi}{2}\right)^{3/4}\frac{3}{4}!$$

$$\frac{5\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)e^{-(5\pi)/4}\,\pi^{7/4}}{4\sqrt[4]{2}\,\Gamma\left(\frac{1}{4}\right)}$$

$$\left(\frac{25}{2}+\frac{15\sqrt{5}}{2}+5\sqrt{\frac{5}{2}\left(5+3\sqrt{5}\right)}\right)e^{-(5\pi)/4}\left(\frac{\pi}{2}\right)^{3/4}\Gamma\left(\frac{3}{4}\right)$$

$n!$ is the factorial function

**Alternative representations:**

$$\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left((1+\sqrt{5})\,5\,e^{-1/4\,(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times2)\,2}=\frac{5\,G\left(1+\frac{3}{4}\right)\sqrt[4]{2}\;e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\,(1+\sqrt{5})}{8\,G\left(\frac{3}{4}\right)}$$

$$\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left((1+\sqrt{5})\,5\,e^{-1/4\,(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times2)\,2}=\frac{5}{8}\sqrt[4]{2}\;e^{-\log G(3/4)+\log G(1+3/4)}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\left(1+\sqrt{5}\right)$$

$$\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left((1+\sqrt{5})\,5\,e^{-1/4\,(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times2)\,2}=\frac{5}{8}\left(-1+\frac{3}{4}\right)!\sqrt[4]{2}\;e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\left(1+\sqrt{5}\right)$$

**Series representations:**

$$\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left((1+\sqrt{5})\,5\,e^{-1/4\,(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times2)\,2}=\frac{5}{3}\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)e^{-(5\pi)/4}\left(\frac{\pi}{2}\right)^{3/4}\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}\right)^k\Gamma^{(k)}(1)}{k!}$$

$$\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left((1+\sqrt{5})\,5\,e^{-1/4\,(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times2)\,2}=\frac{5}{4}\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)e^{-(5\pi)/4}\left(\frac{\pi}{2}\right)^{3/4}\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\quad\text{for } (z_0\notin\mathbb{Z}\text{ or } z_0>0)$$

$$\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4\,(5\,\pi)}\right)\right)\sqrt[4]{2}\,\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2} = \frac{5\sqrt{5}\,\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)e^{-(5\,\pi)/4}\left(\frac{\pi}{2}\right)^{3/4}}{4\sum_{k=1}^{\infty}\left(\frac{3}{4}\right)^k c_k}$$

$$\text{for}\left(c_1 = 1 \text{ and } c_2 = 1 \text{ and } c_k = \frac{\gamma\,c_{-1+k} + \sum_{j=1}^{-2+k}(-1)^{1+j+k}\,c_j\,\zeta(-j+k)}{-1+k}\right)$$

$$\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4\,(5\,\pi)}\right)\right)\sqrt[4]{2}\,\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2} = \frac{5\sqrt{5}\,\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)e^{-(5\,\pi)/4}\,\pi^{7/4}}{4\times 2^{3/4}\sum_{k=0}^{\infty}\left(\frac{3}{4}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}(-j+k)\pi+\pi\,z_0\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}}$$

## **Integral representations:**

$$\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4\,(5\,\pi)}\right)\right)\sqrt[4]{2}\,\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2} = \frac{5}{4}\sqrt{5}\,\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)e^{-(5\,\pi)/4}\left(\frac{\pi}{2}\right)^{3/4}\int_0^1\frac{1}{\sqrt[4]{\log\left(\frac{1}{t}\right)}}\,dt$$

$$\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4\,(5\,\pi)}\right)\right)\sqrt[4]{2}\,\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2} = \frac{5}{4}\sqrt{5}\,\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)e^{-(5\,\pi)/4}\left(\frac{\pi}{2}\right)^{3/4}\int_0^\infty\frac{e^{-t}}{\sqrt[4]{t}}\,dt$$

$$\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4\,(5\,\pi)}\right)\right)\sqrt[4]{2}\,\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2} = \frac{5}{4}\sqrt{5}\,\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)\exp\left(-\frac{5\,\pi}{4}+\int_0^1\frac{-1-\frac{3}{4}(-1+x)+x^{3/4}}{(-1+x)\log(x)}\,dx\right)\left(\frac{\pi}{2}\right)^{3/4}$$

Multiplying by 1/3, we obtain:

1/3 * (((Pi^(3/4) gamma (3/4) sqrt5 * (2)^(1/4)*1/2(1+(5)^(1/4))^2 (1/2(1+sqrt5))*5/2*e^(-(5Pi)/4))))

**Input:**

$$\frac{1}{3}\left(\pi^{3/4}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\,\sqrt[4]{2}\times\frac{1}{2}\left(1+\sqrt[4]{5}\right)^2\right)\left(\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\times\frac{5}{2}\,e^{-1/4\,(5\pi)}\right)\right)$$

$\Gamma(x)$ is the gamma function

**Exact result:**

$$\frac{5}{12}\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)e^{-(5\pi)/4}\left(\frac{\pi}{2}\right)^{3/4}\Gamma\left(\frac{3}{4}\right)$$

**Decimal approximation:**

0.6360000804845885924491639418439949827175221553568722906 74...

0.636000080484…

**Alternate forms:**

$$\frac{5}{9}\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)e^{-(5\pi)/4}\left(\frac{\pi}{2}\right)^{3/4}\frac{3}{4}!$$

$$\frac{5}{6}\left(5+5\sqrt[4]{5}+3\sqrt{5}+5^{3/4}\right)e^{-(5\pi)/4}\left(\frac{\pi}{2}\right)^{3/4}\Gamma\left(\frac{3}{4}\right)$$

$$\frac{5\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)e^{-(5\pi)/4}\,\pi^{7/4}}{12\sqrt[4]{2}\,\Gamma\left(\frac{1}{4}\right)}$$

$n!$ is the factorial function

**Alternative representations:**

$$\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4\,(5\pi)}\right)}{(2\,(2\times 2))\,3}=\frac{5\,G\left(1+\frac{3}{4}\right)\sqrt[4]{2}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\left(1+\sqrt{5}\right)}{24\,G\left(\frac{3}{4}\right)}$$

$$\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4\,(5\pi)}\right)}{(2\,(2\times 2))\,3}=\frac{5}{24}\sqrt[4]{2}\,e^{-\log G(3/4)+\log G(1+3/4)}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\left(1+\sqrt{5}\right)$$

$$\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)}{(2(2\times 2))\,3} = \frac{5}{24}\left(-1+\frac{3}{4}\right)!\,\sqrt[4]{2}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\left(1+\sqrt{5}\right)$$

## Series representations:

$$\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)}{(2(2\times 2))\,3} = \frac{5}{9}\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)e^{-(5\pi)/4}\left(\frac{\pi}{2}\right)^{3/4}\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}\right)^k\Gamma^{(k)}(1)}{k!}$$

$$\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)}{(2(2\times 2))\,3} = \frac{5}{12}\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)e^{-(5\pi)/4}\left(\frac{\pi}{2}\right)^{3/4}\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!} \quad \text{for } (z_0\notin\mathbb{Z} \text{ or } z_0>0)$$

$$\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)}{(2(2\times 2))\,3} = \frac{5\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)e^{-(5\pi)/4}\left(\frac{\pi}{2}\right)^{3/4}}{12\sum_{k=1}^{\infty}\left(\frac{3}{4}\right)^k c_k}$$

$$\text{for}\left(c_1=1 \text{ and } c_2=1 \text{ and } c_k=\frac{\gamma\,c_{-1+k}+\sum_{j=1}^{-2+k}(-1)^{1+j+k}c_j\,\zeta(-j+k)}{-1+k}\right)$$

$$\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)}{(2(2\times 2))\,3} = \frac{5\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)e^{-(5\pi)/4}\,\pi^{7/4}}{12\times 2^{3/4}\sum_{k=0}^{\infty}\left(\frac{3}{4}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\pi^{-j+k}\sin\left(\frac{1}{2}(-j+k)\pi+\pi z_0\right)\Gamma^{(j)}(1-z_0)}{j!(-j+k)!}}$$

## Integral representations:

$$\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)}{(2(2\times 2))\,3} = \frac{5}{12}\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)e^{-(5\pi)/4}\left(\frac{\pi}{2}\right)^{3/4}\int_0^1\frac{1}{\sqrt[4]{\log\left(\frac{1}{t}\right)}}\,dt$$

$$\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)}{(2(2\times 2))\,3} = \frac{5}{12}\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)e^{-(5\pi)/4}\left(\frac{\pi}{2}\right)^{3/4}\int_0^\infty \frac{e^{-t}}{\sqrt[4]{t}}\,dt$$

$$\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)}{(2(2\times 2))\,3} = \frac{5}{12}\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)\exp\left(-\frac{5\pi}{4}+\int_0^1 \frac{-1-\frac{3}{4}(-1+x)+x^{3/4}}{(-1+x)\log(x)}\,dx\right)\left(\frac{\pi}{2}\right)^{3/4}$$

Now, we have the following equation:

ln((sqrt7)/6 - 1/3)/ln((sqrt2)-1) - (log base 4 (8))

**Input:**

$$\frac{\log\left(\frac{\sqrt{7}}{6}-\frac{1}{3}\right)}{\log\left(\sqrt{2}-1\right)} - \log_4(8)$$

$\log(x)$ is the natural logarithm

$\log_b(x)$ is the base- $b$ logarithm

**Exact result:**

$$\frac{\log\left(\frac{\sqrt{7}}{6}-\frac{1}{3}\right)}{\log\left(\sqrt{2}-1\right)} - \frac{3}{2}$$

**Decimal approximation:**

1.029120816380225363261395009749721524645955376366807443980...

1.029120816…

**Alternate forms:**

$$\frac{1}{2}\left(\frac{\log\left(44+16\sqrt{7}\right)}{\sinh^{-1}(1)} - 3\right)$$

$$\frac{\log\left(\frac{1}{6}\left(\sqrt{7}-2\right)\right)}{\log\left(\sqrt{2}-1\right)} - \frac{3}{2}$$

$$-\frac{3}{2}-\frac{\log(6)}{\log(\sqrt{2}-1)}+\frac{\log(\sqrt{7}-2)}{\log(\sqrt{2}-1)}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

## Alternative representations:

$$\frac{\log\left(\frac{\sqrt{7}}{6}-\frac{1}{3}\right)}{\log(\sqrt{2}-1)}-\log_4(8)=-\frac{\log(8)}{\log(4)}+\frac{\log\left(-\frac{1}{3}+\frac{\sqrt{7}}{6}\right)}{\log(-1+\sqrt{2})}$$

$$\frac{\log\left(\frac{\sqrt{7}}{6}-\frac{1}{3}\right)}{\log(\sqrt{2}-1)}-\log_4(8)=-\log_4(8)+\frac{\log_e\left(-\frac{1}{3}+\frac{\sqrt{7}}{6}\right)}{\log_e(-1+\sqrt{2})}$$

$$\frac{\log\left(\frac{\sqrt{7}}{6}-\frac{1}{3}\right)}{\log(\sqrt{2}-1)}-\log_4(8)=-\log_4(8)+\frac{\log(a)\log_a\left(-\frac{1}{3}+\frac{\sqrt{7}}{6}\right)}{\log(a)\log_a(-1+\sqrt{2})}$$

## Series representations:

$$\frac{\log\left(\frac{\sqrt{7}}{6}-\frac{1}{3}\right)}{\log(\sqrt{2}-1)}-\log_4(8)=-\frac{3\sum_{k=1}^{\infty}\frac{(-1)^k\left(-2+\sqrt{2}\right)^k}{k}-2\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k\left(-8+\sqrt{7}\right)^k}{k}}{2\sum_{k=1}^{\infty}\frac{(-1)^k\left(-2+\sqrt{2}\right)^k}{k}}$$

$$\frac{\log\left(\frac{\sqrt{7}}{6}-\frac{1}{3}\right)}{\log(\sqrt{2}-1)}-\log_4(8)=\left(4\pi\left\lfloor\frac{\arg(-2+\sqrt{7}-6x)}{2\pi}\right\rfloor-6\pi\left\lfloor\frac{\arg(-1+\sqrt{2}-x)}{2\pi}\right\rfloor+i\log(x)+2i\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k(-2+\sqrt{7}-6x)^k x^{-k}}{k}-3i\sum_{k=1}^{\infty}\frac{(-1)^k(-1+\sqrt{2}-x)^k x^{-k}}{k}\right)\Bigg/\left(2\left(2\pi\left\lfloor\frac{\arg(-1+\sqrt{2}-x)}{2\pi}\right\rfloor-i\log(x)+i\sum_{k=1}^{\infty}\frac{(-1)^k(-1+\sqrt{2}-x)^k x^{-k}}{k}\right)\right)\quad\text{for } x<0$$

$$\frac{\log\left(\frac{\sqrt{7}}{6}-\frac{1}{3}\right)}{\log(\sqrt{2}-1)}-\log_4(8) = -\left(\left(2\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor - i\log(z_0) - 2i\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k\left(-2+\sqrt{7}-6z_0\right)^k z_0^{-k}}{k} + 3i\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\sqrt{2}-z_0\right)^k z_0^{-k}}{k}\right)\Big/\left(2\left(2\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor - i\log(z_0) + i\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\sqrt{2}-z_0\right)^k z_0^{-k}}{k}\right)\right)\right)$$

**Integral representation:**

$$\frac{\log\left(\frac{\sqrt{7}}{6}-\frac{1}{3}\right)}{\log(\sqrt{2}-1)}-\log_4(8) = -\frac{3\int_1^{-1+\sqrt{2}}\frac{1}{t}\,dt - 2\int_1^{\frac{1}{6}\left(-2+\sqrt{7}\right)}\frac{1}{t}\,dt}{2\int_1^{-1+\sqrt{2}}\frac{1}{t}\,dt}$$

We have, from the previous expression, the following ratio:

(((ln((sqrt7)/6 - 1/3)/ln((sqrt2)-1) - (log base 4 (8)))))*1/((((((1/3 * (((Pi^(3/4) gamma (3/4) sqrt5 * (2)^(1/4)*1/2(1+(5)^(1/4))^2 (1/2(1+sqrt5))*5/2*e^(-(5Pi)/4)))))))))

and obtain:

**Input:**

$$\left(\frac{\log\left(\frac{\sqrt{7}}{6}-\frac{1}{3}\right)}{\log(\sqrt{2}-1)}-\log_4(8)\right)\times \frac{1}{\frac{1}{3}\left(\pi^{3/4}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\,\sqrt[4]{2}\times\frac{1}{2}\left(1+\sqrt[4]{5}\right)^2\right)\left(\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\times\frac{5}{2}\,e^{-1/4\,(5\pi)}\right)\right)}$$

$\log(x)$ is the natural logarithm

$\log_b(x)$ is the base- $b$ logarithm

$\Gamma(x)$ is the gamma function

**Exact result:**

$$\frac{12\times 2^{3/4}\, e^{(5\pi)/4}\left(\frac{\log\left(\frac{\sqrt{7}}{6}-\frac{1}{3}\right)}{\log\left(\sqrt{2}-1\right)}-\frac{3}{2}\right)}{5\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)\pi^{3/4}\,\Gamma\left(\frac{3}{4}\right)}$$

**Decimal approximation:**

1.618114286394595483062760993633363693373736543739172837614...

1.61811428639…

**Alternate forms:**

$$-\frac{6\times 2^{3/4}\, e^{(5\pi)/4}\left(\log\left(\frac{1}{36}\left(11-4\sqrt{7}\right)\right)+3\sinh^{-1}(1)\right)}{5\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)\pi^{3/4}\sinh^{-1}(1)\,\Gamma\left(\frac{3}{4}\right)}$$

$$\frac{12\sqrt[4]{2}\, e^{(5\pi)/4}\left(\frac{\log\left(\frac{1}{6}\left(\sqrt{7}-2\right)\right)}{\log\left(\sqrt{2}-1\right)}-\frac{3}{2}\right)\Gamma\left(\frac{1}{4}\right)}{5\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)\pi^{7/4}}$$

$$\frac{3\times 2^{3/4}\sqrt[4]{161+72\sqrt{5}-12\sqrt{360+161\sqrt{5}}}\; e^{(5\pi)/4}\left(\frac{\log\left(\frac{1}{6}\left(\sqrt{7}-2\right)\right)}{\log\left(\sqrt{2}-1\right)}-\frac{3}{2}\right)}{5\sqrt{5}\,\pi^{3/4}\,\Gamma\left(\frac{3}{4}\right)}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

**Alternative representations:**

$$\frac{\frac{\log\left(\frac{\sqrt{7}}{6}-\frac{1}{3}\right)}{\log\left(\sqrt{2}-1\right)}-\log_4(8)}{\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)}{3(2(2\times 2))}}=\frac{-\frac{\log(8)}{\log(4)}+\frac{\log\left(-\frac{1}{3}+\frac{\sqrt{7}}{6}\right)}{\log\left(-1+\sqrt{2}\right)}}{\frac{5\,G\left(1+\frac{3}{4}\right)\sqrt[4]{2}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\left(1+\sqrt{5}\right)}{24\,G\left(\frac{3}{4}\right)}}$$

$$\frac{\frac{\log\left(\frac{\sqrt{7}}{6}-\frac{1}{3}\right)}{\log\left(\sqrt{2}-1\right)}-\log_4(8)}{\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)}{3(2(2\times 2))}}=\frac{-\log_4(8)+\frac{\log_e\left(-\frac{1}{3}+\frac{\sqrt{7}}{6}\right)}{\log_e\left(-1+\sqrt{2}\right)}}{\frac{5\,G\left(1+\frac{3}{4}\right)\sqrt[4]{2}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\left(1+\sqrt{5}\right)}{24\,G\left(\frac{3}{4}\right)}}$$

$$\frac{\frac{\log\left(\frac{\sqrt{7}}{6}-\frac{1}{3}\right)}{\log\left(\sqrt{2}-1\right)}-\log_4(8)}{\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)}{3(2(2\times 2))}} = \frac{-\log_4(8)+\frac{\log(a)\log_a\left(-\frac{1}{3}+\frac{\sqrt{7}}{6}\right)}{\log(a)\log_a\left(-1+\sqrt{2}\right)}}{\frac{5\,G\left(1+\frac{3}{4}\right)\sqrt[4]{2}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\left(1+\sqrt{5}\right)}{24\,G\left(\frac{3}{4}\right)}}$$

## Series representation:

$$\frac{\frac{\log\left(\frac{\sqrt{7}}{6}-\frac{1}{3}\right)}{\log\left(\sqrt{2}-1\right)}-\log_4(8)}{\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)}{3(2(2\times 2))}} =$$

$$-\frac{6\times 2^{3/4}\,e^{(5\pi)/4}\left(3\sum_{k=1}^{\infty}\frac{(-1)^k\left(-2+\sqrt{2}\right)^k}{k}-2\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k\left(-8+\sqrt{7}\right)^k}{k}\right)\sum_{k=1}^{\infty}\left(\frac{3}{4}\right)^k c_k}{5\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)\pi^{3/4}\sum_{k=1}^{\infty}\frac{(-1)^k\left(-2+\sqrt{2}\right)^k}{k}}$$

$$\text{for}\left(c_1=1\text{ and }c_2=1\text{ and }c_k=\frac{\gamma\,c_{-1+k}+\sum_{j=1}^{-2+k}(-1)^{1+j+k}c_j\,\zeta(-j+k)}{-1+k}\right)$$

## Integral representations:

$$\frac{\frac{\log\left(\frac{\sqrt{7}}{6}-\frac{1}{3}\right)}{\log\left(\sqrt{2}-1\right)}-\log_4(8)}{\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)}{3(2(2\times 2))}} =$$

$$\frac{12\times 2^{3/4}\exp\left(\frac{5\pi}{4}-\int_0^1\frac{-1-\frac{3}{4}(-1+x)+x^{3/4}}{(-1+x)\log(x)}\,dx\right)\left(-\frac{3}{2}+\frac{\log\left(-\frac{1}{3}+\frac{\sqrt{7}}{6}\right)}{\log\left(-1+\sqrt{2}\right)}\right)}{5\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)\pi^{3/4}}$$

$$\frac{\frac{\log\left(\frac{\sqrt{7}}{6}-\frac{1}{3}\right)}{\log\left(\sqrt{2}-1\right)}-\log_4(8)}{\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)}{3(2(2\times 2))}} =$$

$$\frac{12\times 2^{3/4}\,e^{(5\pi)/4}\left(-\frac{3}{2}+\frac{\log\left(-\frac{1}{3}+\frac{\sqrt{7}}{6}\right)}{\log\left(-1+\sqrt{2}\right)}\right)}{5\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)\pi^{3/4}\int_0^1\frac{1}{\sqrt[4]{\log\left(\frac{1}{t}\right)}}\,dt}$$

$$\frac{\frac{\log\left(\frac{\sqrt{7}}{6}-\frac{1}{3}\right)}{\log\left(\sqrt{2}-1\right)}-\log_4(8)}{\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)}{3(2(2\times 2))}} =$$

$$-\frac{6\,i\,2^{3/4}\,e^{(5\pi)/4}\left(-\frac{3}{2}+\frac{\log\left(\frac{1}{6}\left(-2+\sqrt{7}\right)\right)}{\log\left(-1+\sqrt{2}\right)}\right)}{5\sqrt{5}\left(1+\sqrt[4]{5}\right)^2\left(1+\sqrt{5}\right)\pi^{7/4}}\oint_L \frac{e^t}{t^{3/4}}\,dt$$

We have also:

(((Pi^(3/4) gamma (3/4) sqrt5 * (2)^(1/4)*1/2(1+(5)^(1/4))^2 (1/2(1+sqrt5))*5/2*e^(-(5Pi)/4))))^12-521-76-sqrt3

**Input:**

$$\left(\pi^{3/4}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\,\sqrt[4]{2}\times\frac{1}{2}\left(1+\sqrt[4]{5}\right)^2\right)\left(\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\times\frac{5}{2}\,e^{-1/4(5\pi)}\right)\right)^{12}-521-76-\sqrt{3}$$

$\Gamma(x)$ is the gamma function

**Exact result:**

$$-597-\sqrt{3}+\frac{3\,814\,697\,265\,625\left(1+\sqrt[4]{5}\right)^{24}\left(1+\sqrt{5}\right)^{12}e^{-15\pi}\,\pi^9\,\Gamma\left(\frac{3}{4}\right)^{12}}{8\,589\,934\,592}$$

**Decimal approximation:**

1729.043508310679280016280242272860168387258811938668910278...

1729.0435083…

**Alternate forms:**

$$\frac{3\,814\,697\,265\,625\left(1+\sqrt[4]{5}\right)^{24}\left(1+\sqrt{5}\right)^{12}e^{-15\pi}\,\pi^9\left(\frac{3}{4}!\right)^{12}}{272\,097\,792}-597-\sqrt{3}$$

$$-597-\sqrt{3}+\frac{3\,814\,697\,265\,625\left(1+\sqrt[4]{5}\right)^{24}\left(1+\sqrt{5}\right)^{12}e^{-15\pi}\,\pi^{21}}{134\,217\,728\,\Gamma\left(\frac{1}{4}\right)^{12}}$$

$$\frac{3\,814\,697\,265\,625\left(1+\sqrt[4]{5}\right)^{24}\left(1+\sqrt{5}\right)^{12}e^{-15\pi}\,\pi^9\left(\frac{3}{4}!\right)^{12}-272\,097\,792\left(597+\sqrt{3}\right)}{272\,097\,792}$$

$n!$ is the factorial function

## Alternative representations:

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^{12}-521-76-\sqrt{3}=$$
$$-597+\left(\frac{5}{8}\left(-1+\frac{3}{4}\right)!\sqrt[4]{2}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\left(1+\sqrt{5}\right)\right)^{12}-\sqrt{3}$$

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^{12}-521-76-\sqrt{3}=$$
$$-597+\left(\frac{5\,G\left(1+\frac{3}{4}\right)\sqrt[4]{2}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\left(1+\sqrt{5}\right)}{8\,G\left(\frac{3}{4}\right)}\right)^{12}-\sqrt{3}$$

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^{12}-521-76-\sqrt{3}=$$
$$-597+\left(\frac{5}{8}\,\Gamma\left(\frac{3}{4},0\right)\sqrt[4]{2}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\left(1+\sqrt{5}\right)\right)^{12}-\sqrt{3}$$

## Series representations:

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^{12}-521-76-\sqrt{3}=$$
$$-597-\sqrt{3}+\frac{3\,814\,697\,265\,625\left(1+\sqrt[4]{5}\right)^{24}\left(1+\sqrt{5}\right)^{12}e^{-15\pi}\,\pi^9\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}\right)^k\Gamma^{(k)}(1)}{k!}\right)^{12}}{272\,097\,792}$$

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^{12}-521-76-\sqrt{3}=$$
$$-597-\sqrt{3}+\frac{3\,814\,697\,265\,625\left(1+\sqrt[4]{5}\right)^{24}\left(1+\sqrt{5}\right)^{12}e^{-15\pi}\,\pi^9}{8\,589\,934\,592\left(\sum_{k=1}^{\infty}\left(\frac{3}{4}\right)^k c_k\right)^{12}}$$

$$\text{for}\left(c_1=1\text{ and }c_2=1\text{ and }c_k=\frac{\gamma\,c_{-1+k}+\sum_{j=1}^{-2+k}(-1)^{1+j+k}c_j\,\zeta(-j+k)}{-1+k}\right)$$

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^{12}-521-76-\sqrt{3} = -597-\sqrt{3}+\frac{3\,814\,697\,265\,625\left(1+\sqrt[4]{5}\right)^{24}\left(1+\sqrt{5}\right)^{12}e^{-15\pi}\pi^9\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)^{12}}{8\,589\,934\,592}$$

for ($z_0 \notin \mathbb{Z}$ or $z_0 > 0$)

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^{12}-521-76-\sqrt{3} = \left(e^{-15\pi}\left(254\,707\,645\,416\,259\,765\,625\,\pi^{21}+170\,333\,267\,211\,914\,062\,500\,\sqrt[4]{5}\,\pi^{21}+113\,908\,721\,923\,828\,125\,000\,\sqrt{5}\,\pi^{21}+76\,175\,354\,003\,906\,250\,000\times 5^{3/4}\,\pi^{21}-305\,664\,e^{15\pi}\left(\sum_{k=0}^{\infty}\left(\frac{3}{4}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)^{12}-512\sqrt{3}\,e^{15\pi}\left(\sum_{k=0}^{\infty}\left(\frac{3}{4}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)^{12}\right)\right)\Big/\left(512\left(\sum_{k=0}^{\infty}\left(\frac{3}{4}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)^{12}\right)$$

# Integral representations:

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^{12}-521-76-\sqrt{3} = -597-\sqrt{3}+\frac{3\,814\,697\,265\,625\left(1+\sqrt[4]{5}\right)^{24}\left(1+\sqrt{5}\right)^{12}e^{-15\pi}\pi^9\left(\int_0^1\frac{1}{\sqrt[4]{\log\left(\frac{1}{t}\right)}}\,dt\right)^{12}}{8\,589\,934\,592}$$

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^{12}-521-76-\sqrt{3} = -597-\sqrt{3}+\frac{3\,814\,697\,265\,625\left(1+\sqrt[4]{5}\right)^{24}\left(1+\sqrt{5}\right)^{12}\exp\left(-15\pi+12\int_0^1\frac{-1-\frac{3}{4}(-1+x)+x^{3/4}}{(-1+x)\log(x)}\,dx\right)\pi^9}{8\,589\,934\,592}$$

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^{12} - 521 - 76 - \sqrt{3} =$$

$$-597-\sqrt{3}+\frac{1}{8\,589\,934\,592}\,3\,814\,697\,265\,625\left(1+\sqrt[4]{5}\right)^{24}\left(1+\sqrt{5}\right)^{12}\exp\left(-9\,\gamma-15\,\pi+12\int_0^1\frac{-1+x^{3/4}-\log(x^{3/4})}{(-1+x)\log(x)}\,dx\right)\pi^9$$

(((Pi^(3/4) gamma (3/4) sqrt5 * (2)^(1/4)*1/2(1+(5)^(1/4))^2 (1/2(1+sqrt5))*5/2*e^(-(5Pi)/4))))^7+47

**Input:**

$$\left(\pi^{3/4}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\,\sqrt[4]{2}\times\frac{1}{2}\left(1+\sqrt[4]{5}\right)^2\right)\left(\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\times\frac{5}{2}\,e^{-1/4(5\pi)}\right)\right)^7+47$$

$\Gamma(x)$ is the gamma function

**Exact result:**

$$47+\frac{9\,765\,625\sqrt{5}\left(1+\sqrt[4]{5}\right)^{14}\left(1+\sqrt{5}\right)^7 e^{-(35\pi)/4}\,\pi^{21/4}\,\Gamma\left(\frac{3}{4}\right)^7}{524\,288\sqrt[4]{2}}$$

**Decimal approximation:**

139.0553380735384419814155621455777882239470604884875223801...

139.055338073…

**Alternate forms:**

$$\frac{9\,765\,625\sqrt{5}\left(1+\sqrt[4]{5}\right)^{14}\left(1+\sqrt{5}\right)^7 e^{-(35\pi)/4}\,\pi^{21/4}\left(\frac{3}{4}!\right)^7}{69\,984\sqrt[4]{2}}+47$$

$$47+\frac{9\,765\,625\sqrt{5}\left(1+\sqrt[4]{5}\right)^{14}\left(1+\sqrt{5}\right)^7 e^{-(35\pi)/4}\,\pi^{49/4}}{32\,768\times 2^{3/4}\,\Gamma\left(\frac{1}{4}\right)^7}$$

$$47+\frac{9\,765\,625\left(92\,045+61\,555\sqrt[4]{5}+41\,163\sqrt{5}+27\,527\times 5^{3/4}\right)e^{-(35\pi)/4}\,\pi^{21/4}\,\Gamma\left(\frac{3}{4}\right)^7}{64\sqrt[4]{2}}$$

$n!$ is the factorial function

## Alternative representations:

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^7+47=$$
$$47+\left(\frac{5}{8}\left(-1+\frac{3}{4}\right)!\,\sqrt[4]{2}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\left(1+\sqrt{5}\right)\right)^7$$

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^7+47=$$
$$47+\left(\frac{5\,G\left(1+\frac{3}{4}\right)\sqrt[4]{2}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\left(1+\sqrt{5}\right)}{8\,G\left(\frac{3}{4}\right)}\right)^7$$

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^7+47=$$
$$47+\left(\frac{5}{8}\,\Gamma\left(\frac{3}{4},0\right)\sqrt[4]{2}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\left(1+\sqrt{5}\right)\right)^7$$

## Series representations:

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^7+47=$$
$$47+\frac{9\,765\,625\sqrt{5}\left(1+\sqrt[4]{5}\right)^{14}\left(1+\sqrt{5}\right)^7 e^{-(35\pi)/4}\,\pi^{21/4}\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}\right)^k\Gamma^{(k)}(1)}{k!}\right)^7}{69\,984\sqrt[4]{2}}$$

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^7+47=$$
$$47+\frac{9\,765\,625\sqrt{5}\left(1+\sqrt[4]{5}\right)^{14}\left(1+\sqrt{5}\right)^7 e^{-(35\pi)/4}\,\pi^{21/4}}{524\,288\sqrt[4]{2}\left(\sum_{k=1}^{\infty}\left(\frac{3}{4}\right)^k c_k\right)^7}$$
$$\text{for}\left(c_1=1\text{ and }c_2=1\text{ and }c_k=\frac{\gamma\,c_{-1+k}+\sum_{j=1}^{-2+k}(-1)^{1+j+k}c_j\,\zeta(-j+k)}{-1+k}\right)$$

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4\,(5\,\pi)}\right)\right)\sqrt[4]{2}\,\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^7+47=$$

$$47+\frac{9\,765\,625\sqrt{5}\,\left(1+\sqrt[4]{5}\right)^{14}\left(1+\sqrt{5}\right)^7 e^{-(35\,\pi)/4}\,\pi^{21/4}\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)^7}{524\,288\sqrt[4]{2}}$$

for $(z_0 \notin \mathbb{Z}$ or $z_0 > 0)$

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4\,(5\,\pi)}\right)\right)\sqrt[4]{2}\,\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^7+47=$$

$$47+\frac{9\,765\,625\sqrt{5}\,\left(1+\sqrt[4]{5}\right)^{14}\left(1+\sqrt{5}\right)^7 e^{-(35\,\pi)/4}\,\pi^{49/4}}{524\,288\sqrt[4]{2}\left(\sum_{k=0}^{\infty}\left(\frac{3}{4}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}(-j+k)\pi+\pi\,z_0\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)^7}$$

## Integral representations:

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4\,(5\,\pi)}\right)\right)\sqrt[4]{2}\,\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^7+47=$$

$$47+\frac{9\,765\,625\sqrt{5}\,\left(1+\sqrt[4]{5}\right)^{14}\left(1+\sqrt{5}\right)^7 e^{-(35\,\pi)/4}\,\pi^{21/4}\left(\int_0^1\frac{1}{\sqrt[4]{\log\left(\frac{1}{t}\right)}}\,dt\right)^7}{524\,288\sqrt[4]{2}}$$

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4\,(5\,\pi)}\right)\right)\sqrt[4]{2}\,\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^7+47=$$

$$47+\frac{9\,765\,625\sqrt{5}\,\left(1+\sqrt[4]{5}\right)^{14}\left(1+\sqrt{5}\right)^7 \exp\left(-\frac{35\,\pi}{4}+7\int_0^1\frac{-1-\frac{3}{4}(-1+x)+x^{3/4}}{(-1+x)\log(x)}\,dx\right)\pi^{21/4}}{524\,288\sqrt[4]{2}}$$

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4\,(5\,\pi)}\right)\right)\sqrt[4]{2}\,\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^7+47=$$

$$47+\frac{9\,765\,625\sqrt{5}\,\left(1+\sqrt[4]{5}\right)^{14}\left(1+\sqrt{5}\right)^7 e^{-(35\,\pi)/4}\,\pi^{21/4}\left(\int_0^{\infty}\frac{e^{-t}}{\sqrt[4]{t}}\,dt\right)^7}{524\,288\sqrt[4]{2}}$$

(((Pi^(3/4) gamma (3/4) sqrt5 * (2)^(1/4)*1/2(1+(5)^(1/4))^2 (1/2(1+sqrt5))*5/2*e^(-(5Pi)/4))))^7+34-2+golden ratio

**Input:**

$$\left(\pi^{3/4}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\,\sqrt[4]{2}\times\frac{1}{2}\left(1+\sqrt[4]{5}\right)^2\right)\left(\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\times\frac{5}{2}\,e^{-1/4\,(5\pi)}\right)\right)^7+34-2+\phi$$

$\Gamma(x)$ is the gamma function

$\phi$ is the golden ratio

**Exact result:**

$$\phi+32+\frac{9\,765\,625\,\sqrt{5}\,\left(1+\sqrt[4]{5}\right)^{14}\left(1+\sqrt{5}\right)^7 e^{-(35\pi)/4}\,\pi^{21/4}\,\Gamma\left(\frac{3}{4}\right)^7}{524\,288\,\sqrt[4]{2}}$$

**Decimal approximation:**

125.6733720622883368296201489799434263416673696682932852422...

125.6733720622...

**Alternate forms:**

$$\frac{9\,765\,625\,\sqrt{5}\,\left(1+\sqrt[4]{5}\right)^{14}\left(1+\sqrt{5}\right)^7 e^{-(35\pi)/4}\,\pi^{21/4}\left(\frac{3}{4}!\right)^7}{69\,984\,\sqrt[4]{2}}+\phi+32$$

$$\phi+32+\frac{9\,765\,625\,\sqrt{5}\,\left(1+\sqrt[4]{5}\right)^{14}\left(1+\sqrt{5}\right)^7 e^{-(35\pi)/4}\,\pi^{49/4}}{32\,768\times2^{3/4}\,\Gamma\left(\frac{1}{4}\right)^7}$$

$$\frac{1}{2}\left(65+\sqrt{5}\right)+\frac{9\,765\,625\left(92\,045+61\,555\,\sqrt[4]{5}+41\,163\,\sqrt{5}+27\,527\times5^{3/4}\right)e^{-(35\pi)/4}\,\pi^{21/4}\,\Gamma\left(\frac{3}{4}\right)^7}{64\,\sqrt[4]{2}}$$

$n!$ is the factorial function

**Alternative representations:**

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4\,(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times2)\,2}\right)^7+34-2+\phi=$$

$$32+\phi+\left(\frac{5}{8}\left(-1+\frac{3}{4}\right)!\,\sqrt[4]{2}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\left(1+\sqrt{5}\right)\right)^7$$

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\,\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^7+34-2+\phi=$$
$$32+\phi+\left(\frac{5\,G\left(1+\frac{3}{4}\right)\sqrt[4]{2}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\,\left(1+\sqrt{5}\right)}{8\,G\left(\frac{3}{4}\right)}\right)^7$$

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\,\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^7+34-2+\phi=$$
$$32+\phi+\left(\frac{5}{8}\,\Gamma\left(\frac{3}{4},0\right)\sqrt[4]{2}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\,\left(1+\sqrt{5}\right)\right)^7$$

**Series representations:**

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\,\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^7+34-2+\phi=$$
$$32+\phi+\frac{9\,765\,625\sqrt{5}\left(1+\sqrt[4]{5}\right)^{14}\left(1+\sqrt{5}\right)^7 e^{-(35\pi)/4}\,\pi^{21/4}\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}\right)^k\Gamma^{(k)}(1)}{k!}\right)^7}{69\,984\sqrt[4]{2}}$$

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\,\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^7+34-2+\phi=$$
$$32+\phi+\frac{9\,765\,625\sqrt{5}\left(1+\sqrt[4]{5}\right)^{14}\left(1+\sqrt{5}\right)^7 e^{-(35\pi)/4}\,\pi^{21/4}}{524\,288\sqrt[4]{2}\left(\sum_{k=1}^{\infty}\left(\frac{3}{4}\right)^k c_k\right)^7}$$
$$\text{for}\left(c_1=1\text{ and }c_2=1\text{ and }c_k=\frac{\gamma\,c_{-1+k}+\sum_{j=1}^{-2+k}(-1)^{1+j+k}\,c_j\,\zeta(-j+k)}{-1+k}\right)$$

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\,\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^7+34-2+\phi=$$
$$32+\phi+\frac{9\,765\,625\sqrt{5}\left(1+\sqrt[4]{5}\right)^{14}\left(1+\sqrt{5}\right)^7 e^{-(35\pi)/4}\,\pi^{21/4}\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)^7}{524\,288\sqrt[4]{2}}$$
$$\text{for }(z_0\notin\mathbb{Z}\text{ or }z_0>0)$$

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^7+34-2+\phi=$$

$$32+\phi+\frac{9\,765\,625\sqrt{5}\left(1+\sqrt[4]{5}\right)^{14}\left(1+\sqrt{5}\right)^7 e^{-(35\pi)/4}\,\pi^{49/4}}{524\,288\sqrt[4]{2}\left(\sum_{k=0}^{\infty}\left(\frac{3}{4}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}(-j+k)\pi+\pi z_0\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)^7}$$

## Integral representations:

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^7+34-2+\phi=$$

$$32+\phi+\frac{9\,765\,625\sqrt{5}\left(1+\sqrt[4]{5}\right)^{14}\left(1+\sqrt{5}\right)^7 e^{-(35\pi)/4}\,\pi^{21/4}\left(\int_0^1\frac{1}{\sqrt[4]{\log\left(\frac{1}{t}\right)}}\,dt\right)^7}{524\,288\sqrt[4]{2}}$$

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^7+34-2+\phi=32+\phi+$$

$$\frac{9\,765\,625\sqrt{5}\left(1+\sqrt[4]{5}\right)^{14}\left(1+\sqrt{5}\right)^7\exp\left(-\frac{35\pi}{4}+7\int_0^1\frac{-1-\frac{3}{4}(-1+x)+x^{3/4}}{(-1+x)\log(x)}\,dx\right)\pi^{21/4}}{524\,288\sqrt[4]{2}}$$

$$\left(\frac{\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2}{(2\times 2)\,2}\right)^7+34-2+\phi=$$

$$32+\phi+\frac{9\,765\,625\sqrt{5}\left(1+\sqrt[4]{5}\right)^{14}\left(1+\sqrt{5}\right)^7 e^{-(35\pi)/4}\,\pi^{21/4}\left(\int_0^{\infty}\frac{e^{-t}}{\sqrt[4]{t}}\,dt\right)^7}{524\,288\sqrt[4]{2}}$$

Now, we have that:

$$\int_0^\infty \frac{dx}{(1+x^2)(1+e^{-10\pi}x^2)(1+e^{-20x^2})\cdots}$$
$$= \frac{\pi}{2(1+e^{-5\pi}+e^{-15\pi}+e^{-30\pi}+\cdots)}$$
$$= \pi^{\frac{3}{4}}\Gamma\left(\tfrac{3}{4}\right)\sqrt{5}\sqrt[8]{2}\tfrac{1}{2}(1+\sqrt[4]{5})\{\tfrac{1}{2}(1+\sqrt{5})\}^{\frac{1}{2}}e^{-5\pi/8}.$$

we obtain:

Pi^(3/4) gamma (3/4) sqrt5 * (2)^(1/8)*1/2(1+(5)^(1/4))^2 (1/2(1+sqrt5))^0.5*e^(-(5Pi)/8)

**Input:**

$$\pi^{3/4}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\,\sqrt[8]{2}\times\frac{1}{2}\left(1+\sqrt[4]{5}\right)^2\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)$$

$\Gamma(x)$ is the gamma function

**Exact result:**

$$\frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\,e^{-(5\pi)/8}\,\pi^{3/4}\,\Gamma\left(\frac{3}{4}\right)}{2\times 2^{3/8}}$$

**Decimal approximation:**

3.919684108911248710336941241145699756460114771180718086101...

3.9196841089…

**Alternate forms:**

$$\frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\,e^{-(5\pi)/8}\,\pi^{7/4}}{2^{7/8}\,\Gamma\left(\frac{1}{4}\right)}$$

$$\frac{\frac{1}{3} \times 2^{5/8} \left(\sqrt{\frac{5}{2} - 5\,i} + \sqrt{\frac{5}{2} + 5\,i}\right)\left(1 + \sqrt[4]{5}\right)^2 e^{-(5\pi)/8}\, \pi^{3/4}\, \frac{3}{4}!}{\sqrt{5}\,\sqrt[8]{30\,247 + 13\,533\sqrt{5} + 24\sqrt{2\left(1\,589\,055 + 710\,647\sqrt{5}\right)}}\, e^{-(5\pi)/8}\, \pi^{3/4}\, \Gamma\left(\frac{3}{4}\right)}$$

$n!$ is the factorial function

**Alternative representations:**

$$\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\, e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2 = \frac{G\left(1+\frac{3}{4}\right)\sqrt[8]{2}\, e^{-(5\pi)/8}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,\sqrt{5}}{2\,G\left(\frac{3}{4}\right)}$$

$$\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\, e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2 = \frac{1}{2}\sqrt[8]{2}\, e^{-\log G(3/4)+\log G(1+3/4)}\, e^{-(5\pi)/8}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,\sqrt{5}$$

$$\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\, e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2 = \frac{1}{2}\left(-1+\frac{3}{4}\right)!\,\sqrt[8]{2}\, e^{-(5\pi)/8}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,\sqrt{5}$$

**Series representations:**

$$\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\, e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2 = \frac{1}{3} \times 2^{5/8}\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\, e^{-(5\pi)/8}\,\pi^{3/4}\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}\right)^k\Gamma^{(k)}(1)}{k!}$$

$$\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\, e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2 = \frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\, e^{-(5\pi)/8}\,\pi^{3/4}\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}}{2\times 2^{3/8}} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8\,(5\,\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2 = \frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\,e^{-(5\,\pi)/8}\,\pi^{3/4}}{2\times 2^{3/8}\sum_{k=1}^{\infty}\left(\frac{3}{4}\right)^k c_k}$$

$$\text{for}\left(c_1 = 1 \text{ and } c_2 = 1 \text{ and } c_k = \frac{\gamma\,c_{-1+k} + \sum_{j=1}^{-2+k}(-1)^{1+j+k}\,c_j\,\zeta(-j+k)}{-1+k}\right)$$

**Integral representations:**

$$\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8\,(5\,\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2 = \frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\,e^{-(5\,\pi)/8}\,\pi^{3/4}}{2\times 2^{3/8}}\int_0^1 \frac{1}{\sqrt[4]{\log\left(\frac{1}{t}\right)}}\,dt$$

$$\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8\,(5\,\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2 = \frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\,e^{-(5\,\pi)/8}\,\pi^{3/4}}{2\times 2^{3/8}}\int_0^\infty \frac{e^{-t}}{\sqrt[4]{t}}\,dt$$

$$\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8\,(5\,\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2 = \frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\,\exp\left(-\frac{5\pi}{8}+\int_0^1 \frac{-1-\frac{3}{4}(-1+x)+x^{3/4}}{(-1+x)\log(x)}\,dx\right)\pi^{3/4}}{2\times 2^{3/8}}$$

From which, multiplying by 1/6 and adding 1, we obtain:

1 +1/6((((Pi^(3/4) gamma (3/4) sqrt5 * (2)^(1/8)*1/2(1+(5)^(1/4))^2 (1/2(1+sqrt5))^0.5*e^(-(5Pi)/8)))))

**Input:**

$$1+\frac{1}{6}\left(\pi^{3/4}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\,\sqrt[8]{2}\times\frac{1}{2}\left(1+\sqrt[4]{5}\right)^2\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8\,(5\,\pi)}\right)\right)$$

$\Gamma(x)$ is the gamma function

**Exact result:**

$$1+\frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\ e^{-(5\pi)/8}\ \pi^{3/4}\ \Gamma\left(\frac{3}{4}\right)}{12\times 2^{3/8}}$$

**Decimal approximation:**

1.653280684818541451722823540190949959410019128530119681016...

1.653280684…. result very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164.2696$ i.e. 1.65578...

**Alternate forms:**

$$1+\frac{\sqrt{1+\sqrt{5}}\ \left(5+\sqrt{5}+2\times 5^{3/4}\right) e^{-(5\pi)/8}\ \pi^{3/4}\ \Gamma\left(\frac{3}{4}\right)}{12\times 2^{3/8}}$$

$$1+\frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\ e^{-(5\pi)/8}\ \pi^{7/4}}{6\times 2^{7/8}\ \Gamma\left(\frac{1}{4}\right)}$$

$$1+\frac{\left(\sqrt{\frac{5}{2}-5\,i}+\sqrt{\frac{5}{2}+5\,i}\right)\left(1+\sqrt[4]{5}\right)^2\ e^{-(5\pi)/8}\ \pi^{3/4}\ \frac{3}{4}!}{9\times 2^{3/8}}$$

$n!$ is the factorial function

**Alternative representations:**

$$1+\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ e^{-1/8(5\pi)}\right)}{2\times 6} = 1+\frac{G\left(1+\frac{3}{4}\right)\sqrt[8]{2}\ e^{-(5\pi)/8}\ \pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ \sqrt{5}}{2\times 6\ G\left(\frac{3}{4}\right)}$$

$$1+\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ e^{-1/8(5\pi)}\right)}{2\times 6} = 1+\frac{\sqrt[8]{2}\ e^{-\log G(3/4)+\log G(1+3/4)}\ e^{-(5\pi)/8}\ \pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ \sqrt{5}}{2\times 6}$$

$$1+\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ e^{-1/8(5\pi)}\right)}{2\times 6}=$$

$$1+\frac{\left(-1+\frac{3}{4}\right)!\ \sqrt[8]{2}\ e^{-(5\pi)/8}\ \pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ \sqrt{5}}{2\times 6}$$

**Series representations:**

$$1+\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ e^{-1/8(5\pi)}\right)}{2\times 6}=$$

$$1+\frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\ e^{-(5\pi)/8}\ \pi^{3/4}\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}\right)^k\Gamma^{(k)}(1)}{k!}}{9\times 2^{3/8}}$$

$$1+\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ e^{-1/8(5\pi)}\right)}{2\times 6}=$$

$$1+\frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\ e^{-(5\pi)/8}\ \pi^{3/4}\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}}{12\times 2^{3/8}}\quad \text{for } (z_0\notin\mathbb{Z} \text{ or } z_0>0)$$

$$1+\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ e^{-1/8(5\pi)}\right)}{2\times 6}=$$

$$1+\frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\ e^{-(5\pi)/8}\ \pi^{3/4}}{12\times 2^{3/8}\sum_{k=1}^{\infty}\left(\frac{3}{4}\right)^k c_k}$$

$$\text{for}\left(c_1=1 \text{ and } c_2=1 \text{ and } c_k=\frac{\gamma\, c_{-1+k}+\sum_{j=1}^{-2+k}(-1)^{1+j+k}\, c_j\, \zeta(-j+k)}{-1+k}\right)$$

$$1+\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)}{2\times 6}=$$

$$\left(e^{-(5\pi)/8}\left(5\times 2^{5/8}\sqrt{1+\sqrt{5}}\,\pi^{7/4}+2\times 2^{5/8}\times 5^{3/4}\sqrt{1+\sqrt{5}}\,\pi^{7/4}+2^{5/8}\sqrt{5\left(1+\sqrt{5}\right)}\,\pi^{7/4}+24\,e^{(5\pi)/8}\sum_{k=0}^{\infty}\left(\frac{3}{4}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)\right)\Bigg/\left(24\sum_{k=0}^{\infty}\left(\frac{3}{4}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)$$

## Integral representations:

$$1+\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)}{2\times 6}=1+\frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\,e^{-(5\pi)/8}\,\pi^{3/4}}{12\times 2^{3/8}}\int_0^1\frac{1}{\sqrt[4]{\log\left(\frac{1}{t}\right)}}\,dt$$

$$1+\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)}{2\times 6}=1+\frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\,e^{-(5\pi)/8}\,\pi^{3/4}}{12\times 2^{3/8}}\int_0^\infty\frac{e^{-t}}{\sqrt[4]{t}}\,dt$$

$$1+\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)}{2\times 6}=1+\frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\exp\left(-\frac{5\pi}{8}+\int_0^1\frac{-1-\frac{3}{4}(-1+x)+x^{3/4}}{(-1+x)\log(x)}\,dx\right)\pi^{3/4}}{12\times 2^{3/8}}$$

((1 +1/6((((Pi^(3/4) gamma (3/4) sqrt5 * (2)^(1/8)*1/2(1+(5)^(1/4))^2 (1/2(1+sqrt5))^0.5*e^(-(5Pi)/8))))))) - (34+1)/10^3

**Input:**

$$\left(1+\frac{1}{6}\left(\pi^{3/4}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\,\sqrt[8]{2}\times\frac{1}{2}\left(1+\sqrt[4]{5}\right)^2\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)\right)-\frac{34+1}{10^3}$$

$\Gamma(x)$ is the gamma function

**Exact result:**

$$\frac{193}{200}+\frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\,e^{-(5\pi)/8}\,\pi^{3/4}\,\Gamma\left(\frac{3}{4}\right)}{12\times 2^{3/8}}$$

**Decimal approximation:**

1.618280684818541451722823540190949959410019128530119681016...

1.618280684…

**Alternate forms:**

$$\frac{1}{600}\left(579+25\times 2^{5/8}\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\,e^{-(5\pi)/8}\,\pi^{3/4}\,\Gamma\left(\frac{3}{4}\right)\right)$$

$$\frac{193}{200}+\frac{\sqrt{1+\sqrt{5}}\,\left(5+\sqrt{5}+2\times 5^{3/4}\right)e^{-(5\pi)/8}\,\pi^{3/4}\,\Gamma\left(\frac{3}{4}\right)}{12\times 2^{3/8}}$$

$$\frac{193}{200}+\frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\,e^{-(5\pi)/8}\,\pi^{7/4}}{6\times 2^{7/8}\,\Gamma\left(\frac{1}{4}\right)}$$

**Alternative representations:**

$$\left(1+\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)}{2\times 6}\right)-\frac{34+1}{10^3}=$$

$$1-\frac{35}{10^3}+\frac{G\left(1+\frac{3}{4}\right)\sqrt[8]{2}\,e^{-(5\pi)/8}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,\sqrt{5}}{2\times 6\,G\left(\frac{3}{4}\right)}$$

$$\left(1+\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ e^{-1/8(5\pi)}\right)}{2\times 6}\right)-\frac{34+1}{10^3}=$$

$$1-\frac{35}{10^3}+\frac{\sqrt[8]{2}\ e^{-\log G(3/4)+\log G(1+3/4)}\ e^{-(5\pi)/8}\ \pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ \sqrt{5}}{2\times 6}$$

$$\left(1+\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ e^{-1/8(5\pi)}\right)}{2\times 6}\right)-\frac{34+1}{10^3}=$$

$$1-\frac{35}{10^3}+\frac{\left(-1+\frac{3}{4}\right)!\ \sqrt[8]{2}\ e^{-(5\pi)/8}\ \pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ \sqrt{5}}{2\times 6}$$

## Series representations:

$$\left(1+\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ e^{-1/8(5\pi)}\right)}{2\times 6}\right)-\frac{34+1}{10^3}=$$

$$\frac{193}{200}+\frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\ e^{-(5\pi)/8}\ \pi^{3/4}\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}\right)^k\Gamma^{(k)}(1)}{k!}}{9\times 2^{3/8}}$$

$$\left(1+\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ e^{-1/8(5\pi)}\right)}{2\times 6}\right)-\frac{34+1}{10^3}=$$

$$\frac{193}{200}+\frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\ e^{-(5\pi)/8}\ \pi^{3/4}\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}}{12\times 2^{3/8}}$$

for $(z_0 \notin \mathbb{Z}$ or $z_0 > 0)$

$$\left(1+\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)}{2\times 6}\right)-\frac{34+1}{10^3}=$$

$$\frac{193}{200}+\frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\,e^{-(5\pi)/8}\,\pi^{3/4}}{12\times 2^{3/8}\sum_{k=1}^{\infty}\left(\frac{3}{4}\right)^k c_k}$$

$$\text{for}\left(c_1=1\text{ and }c_2=1\text{ and }c_k=\frac{\gamma\,c_{-1+k}+\sum_{j=1}^{-2+k}(-1)^{1+j+k}\,c_j\,\zeta(-j+k)}{-1+k}\right)$$

$$\left(1+\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)}{2\times 6}\right)-\frac{34+1}{10^3}=$$

$$\left(e^{-(5\pi)/8}\left(125\times 2^{5/8}\sqrt{1+\sqrt{5}}\,\pi^{7/4}+50\times 2^{5/8}\times 5^{3/4}\sqrt{1+\sqrt{5}}\,\pi^{7/4}+25\times 2^{5/8}\sqrt{5\left(1+\sqrt{5}\right)}\,\pi^{7/4}+579\,e^{(5\pi)/8}\sum_{k=0}^{\infty}\left(\frac{3}{4}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)\right)\Bigg/\left(600\sum_{k=0}^{\infty}\left(\frac{3}{4}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)$$

## Integral representations:

$$\left(1+\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)}{2\times 6}\right)-\frac{34+1}{10^3}=$$

$$\frac{193}{200}+\frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\,e^{-(5\pi)/8}\,\pi^{3/4}}{12\times 2^{3/8}}\int_0^1\frac{1}{\sqrt[4]{\log\left(\frac{1}{t}\right)}}\,dt$$

$$\left(1+\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ e^{-1/8(5\pi)}\right)}{2\times 6}\right)-\frac{34+1}{10^3}=$$

$$\frac{193}{200}+\frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\ e^{-(5\pi)/8}\ \pi^{3/4}}{12\times 2^{3/8}}\int_0^\infty \frac{e^{-t}}{\sqrt[4]{t}}\,dt$$

$$\left(1+\frac{\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ e^{-1/8(5\pi)}\right)}{2\times 6}\right)-\frac{34+1}{10^3}=$$

$$\frac{193}{200}+\frac{\left(1+\sqrt[4]{5}\right)^2\sqrt{5\left(1+\sqrt{5}\right)}\ \exp\left(-\frac{5\pi}{8}+\int_0^1\frac{-1-\frac{3}{4}(-1+x)+x^{3/4}}{(-1+x)\log(x)}\,dx\right)\pi^{3/4}}{12\times 2^{3/8}}$$

We obtain also:

**Input:**

$$\left(\pi^{3/4}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\ \sqrt[8]{2}\times\frac{1}{2}\left(1+\sqrt[4]{5}\right)^2\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ e^{-1/8(5\pi)}\right)\right)^4-89-21-\frac{1}{\phi}$$

$\Gamma(x)$ is the gamma function

$\phi$ is the golden ratio

**Exact result:**

$$-\frac{1}{\phi}-110+\frac{25\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2 e^{-(5\pi)/2}\,\pi^3\,\Gamma\left(\frac{3}{4}\right)^4}{32\sqrt{2}}$$

**Decimal approximation:**

125.4321117445795591686972378223077926910362282788865076243...

125.432111744…

**Alternate forms:**

$$-\frac{1}{\phi}-110+\frac{25\left(1+\sqrt[4]{5}\right)^8\left(3+\sqrt{5}\right)e^{-(5\pi)/2}\,\pi^7}{4\sqrt{2}\,\Gamma\left(\frac{1}{4}\right)^4}$$

$$\frac{100}{81}\sqrt{2}\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2 e^{-(5\pi)/2}\pi^3\left(\frac{3}{4}!\right)^4-\frac{1}{\phi}-110$$

$$\frac{200\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2 e^{-(5\pi)/2}\pi^3\left(\frac{3}{4}!\right)^4\phi-81\sqrt{2}\,(110\phi+1)}{81\sqrt{2}\,\phi}$$

$n!$ is the factorial function

## Alternative representations:

$$\left(\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)^4-89-21-\frac{1}{\phi}=$$
$$-110-\frac{1}{\phi}+\left(\frac{1}{2}\left(-1+\frac{3}{4}\right)!\sqrt[8]{2}\,e^{-(5\pi)/8}\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\sqrt{5}\right)^4$$

$$\left(\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)^4-89-21-\frac{1}{\phi}=$$
$$-110-\frac{1}{\phi}+\left(\frac{G\left(1+\frac{3}{4}\right)\sqrt[8]{2}\,e^{-(5\pi)/8}\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\sqrt{5}}{2\,G\left(\frac{3}{4}\right)}\right)^4$$

$$\left(\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)^4-89-21-\frac{1}{\phi}=$$
$$-110-\frac{1}{\phi}+\left(\frac{1}{2}\Gamma\left(\frac{3}{4},0\right)\sqrt[8]{2}\,e^{-(5\pi)/8}\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\sqrt{5}\right)^4$$

## Series representations:

$$\left(\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)^4-89-21-\frac{1}{\phi}=$$
$$-110-\frac{1}{\phi}+\frac{200}{81}\sqrt{2}\left(1+\sqrt[4]{5}\right)^8\left(3+\sqrt{5}\right)e^{-(5\pi)/2}\pi^3\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}\right)^k\Gamma^{(k)}(1)}{k!}\right)^4$$

$$\left(\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)^4-89-21-\frac{1}{\phi}=$$
$$-110-\frac{1}{\phi}+\frac{100}{81}\sqrt{2}\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2 e^{-(5\pi)/2}\pi^3\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}\right)^k\Gamma^{(k)}(1)}{k!}\right)^4$$

$$\left(\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)^4-89-21-\frac{1}{\phi}=$$
$$-110-\frac{1}{\phi}+\frac{25\left(1+\sqrt[4]{5}\right)^8\left(3+\sqrt{5}\right)e^{-(5\pi)/2}\,\pi^3}{16\sqrt{2}\left(\sum_{k=1}^{\infty}\left(\frac{3}{4}\right)^k c_k\right)^4}$$
$$\text{for}\left(c_1=1\text{ and }c_2=1\text{ and }c_k=\frac{\gamma\,c_{-1+k}+\sum_{j=1}^{-2+k}(-1)^{1+j+k}\,c_j\,\zeta(-j+k)}{-1+k}\right)$$

**Integral representations:**

$$\left(\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)^4-89-21-\frac{1}{\phi}=$$
$$-110-\frac{1}{\phi}+\frac{25\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2e^{-(5\pi)/2}\,\pi^3\left(\int_0^1\frac{1}{\sqrt[4]{\log\left(\frac{1}{t}\right)}}\,dt\right)^4}{32\sqrt{2}}$$

$$\left(\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)^4-89-21-\frac{1}{\phi}=$$
$$-110-\frac{1}{\phi}+\frac{25\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2\exp\left(-\frac{5\pi}{2}+4\int_0^1\frac{-1-\frac{3}{4}(-1+x)+x^{3/4}}{(-1+x)\log(x)}\,dx\right)\pi^3}{32\sqrt{2}}$$

$$\left(\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)^4-89-21-\frac{1}{\phi}=$$
$$-110-\frac{1}{\phi}+\frac{25\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2\exp\left(-3\gamma-\frac{5\pi}{2}+4\int_0^1\frac{-1+x^{3/4}-\log\left(x^{3/4}\right)}{(-1+x)\log(x)}\,dx\right)\pi^3}{32\sqrt{2}}$$

(((Pi^(3/4) gamma (3/4) sqrt5 * (2)^(1/8)*1/2(1+(5)^(1/4))^2 (1/2(1+sqrt5))^0.5*e^(-(5Pi)/8))))^4-89-8

**Input:**

$$\left(\pi^{3/4}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\,\sqrt[8]{2}\times\frac{1}{2}\left(1+\sqrt[4]{5}\right)^2\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)^4-89-8$$

$\Gamma(x)$ is the gamma function

**Exact result:**

$$\frac{25\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2 e^{-(5\pi)/2}\pi^3\,\Gamma\left(\frac{3}{4}\right)^4}{32\sqrt{2}}-97$$

**Decimal approximation:**

139.05014573332945401690182465667343080875653745869227048 64...

139.0501457…

**Alternate forms:**

$$\frac{25\left(1+\sqrt[4]{5}\right)^8\left(3+\sqrt{5}\right)e^{-(5\pi)/2}\pi^7}{4\sqrt{2}\,\Gamma\left(\frac{1}{4}\right)^4}-97$$

$$\frac{100}{81}\sqrt{2}\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2 e^{-(5\pi)/2}\pi^3\left(\frac{3}{4}!\right)^4-97$$

$$\frac{25\left(123+84\sqrt[4]{5}+55\sqrt{5}+36\times 5^{3/4}\right)e^{-(5\pi)/2}\pi^3\,\Gamma\left(\frac{3}{4}\right)^4}{\sqrt{2}}-97$$

$n!$ is the factorial function

**Alternative representations:**

$$\left(\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)^4-89-8=$$
$$-97+\left(\frac{1}{2}\left(-1+\frac{3}{4}\right)!\,\sqrt[8]{2}\,e^{-(5\pi)/8}\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\sqrt{5}\right)^4$$

$$\left(\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)^4-89-8=$$
$$-97+\left(\frac{G\left(1+\frac{3}{4}\right)\sqrt[8]{2}\,e^{-(5\pi)/8}\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\sqrt{5}}{2\,G\left(\frac{3}{4}\right)}\right)^4$$

$$\left(\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)^4-89-8=$$
$$-97+\left(\frac{1}{2}\Gamma\left(\frac{3}{4},0\right)\sqrt[8]{2}\,e^{-(5\pi)/8}\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\sqrt{5}\right)^4$$

## Series representations:

$$\left(\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)^4-89-8=$$
$$-97+\frac{200}{81}\sqrt{2}\left(1+\sqrt[4]{5}\right)^8\left(3+\sqrt{5}\right)e^{-(5\pi)/2}\,\pi^3\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}\right)^k\Gamma^{(k)}(1)}{k!}\right)^4$$

$$\left(\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)^4-89-8=$$
$$-97+\frac{100}{81}\sqrt{2}\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2e^{-(5\pi)/2}\,\pi^3\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}\right)^k\Gamma^{(k)}(1)}{k!}\right)^4$$

$$\left(\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)^4-89-8=$$
$$-97+\frac{25\left(1+\sqrt[4]{5}\right)^8\left(3+\sqrt{5}\right)e^{-(5\pi)/2}\,\pi^3}{16\sqrt{2}\left(\sum_{k=1}^{\infty}\left(\frac{3}{4}\right)^k c_k\right)^4}$$
$$\text{for}\left(c_1=1\text{ and }c_2=1\text{ and }c_k=\frac{\gamma\,c_{-1+k}+\sum_{j=1}^{-2+k}(-1)^{1+j+k}\,c_j\,\zeta(-j+k)}{-1+k}\right)$$

## Integral representations:

$$\left(\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)^4-89-8=$$
$$-97+\frac{25\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2e^{-(5\pi)/2}\,\pi^3\left(\int_0^1\frac{1}{\sqrt[4]{\log\left(\frac{1}{t}\right)}}\,dt\right)^4}{32\sqrt{2}}$$

$$\left(\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)^4-89-8=$$
$$-97+\frac{25\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2\exp\left(-\frac{5\pi}{2}+4\int_0^1\frac{-1-\frac{3}{4}(-1+x)+x^{3/4}}{(-1+x)\log(x)}\,dx\right)\pi^3}{32\sqrt{2}}$$

$$\left(\frac{1}{2}\left(\pi^{3/4}\sqrt{5}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)^4-89-8=$$

$$-97+\frac{25\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2\exp\left(-3\gamma-\frac{5\pi}{2}+4\int_0^1\frac{-1+x^{3/4}-\log\left(x^{3/4}\right)}{(-1+x)\log(x)}\,dx\right)\pi^3}{32\sqrt{2}}$$

27*1/2((((((Pi^(3/4) gamma (3/4) sqrt5 * (2)^(1/8)*1/2(1+(5)^(1/4))^2 (1/2(1+sqrt5))^0.5*e^(-(5Pi)/8))))^4-89-21+2)))

**Input:**

$$27\times\frac{1}{2}\left(\left(\pi^{3/4}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\,\sqrt[8]{2}\times\frac{1}{2}\left(1+\sqrt[4]{5}\right)^2\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)^4-89-21+2\right)$$

$\Gamma(x)$ is the gamma function

**Exact result:**

$$\frac{27}{2}\left(\frac{25\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2 e^{-(5\pi)/2}\,\pi^3\,\Gamma\left(\frac{3}{4}\right)^4}{32\sqrt{2}}-108\right)$$

**Decimal approximation:**

1728.676967399947629228174632865091315918213255692345651567...

1728.6769673…

With regard 27 (From Wikipedia):

*"The fundamental group of the complex form, compact real form, or any algebraic version of $E_6$ is the cyclic group **Z**/3**Z**, and its outer automorphism group is the cyclic group **Z**/2**Z**. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, $E_6$ plays a role in some grand unified theories".*

**Alternate forms:**

$$\frac{675\left(1+\sqrt[4]{5}\right)^8\left(3+\sqrt{5}\right)e^{-(5\pi)/2}\,\pi^7}{8\sqrt{2}\,\Gamma\left(\frac{1}{4}\right)^4}-1458$$

$$\frac{50}{3}\sqrt{2}\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2 e^{-(5\pi)/2}\pi^3\left(\frac{3}{4}!\right)^4-1458$$

$$\frac{675\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2 e^{-(5\pi)/2}\pi^3\,\Gamma\left(\frac{3}{4}\right)^4}{64\sqrt{2}}-1458$$

$n!$ is the factorial function

## Alternative representations:

$$\frac{27}{2}\left(\left(\frac{1}{2}\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)^4-89-21+2\right)=$$
$$\frac{27}{2}\left(-108+\left(\frac{1}{2}\left(-1+\frac{3}{4}\right)!\,\sqrt[8]{2}\,e^{-(5\pi)/8}\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\sqrt{5}\right)^4\right)$$

$$\frac{27}{2}\left(\left(\frac{1}{2}\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)^4-89-21+2\right)=$$
$$\frac{27}{2}\left(-108+\left(\frac{G\left(1+\frac{3}{4}\right)\sqrt[8]{2}\,e^{-(5\pi)/8}\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\sqrt{5}}{2\,G\left(\frac{3}{4}\right)}\right)^4\right)$$

$$\frac{27}{2}\left(\left(\frac{1}{2}\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)^4-89-21+2\right)=$$
$$\frac{27}{2}\left(-108+\left(\frac{1}{2}\Gamma\left(\frac{3}{4},0\right)\sqrt[8]{2}\,e^{-(5\pi)/8}\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\sqrt{5}\right)^4\right)$$

## Series representations:

$$\frac{27}{2}\left(\left(\frac{1}{2}\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)^4-89-21+2\right)=$$
$$-1458+\frac{100}{3}\sqrt{2}\left(1+\sqrt[4]{5}\right)^8\left(3+\sqrt{5}\right)e^{-(5\pi)/2}\pi^3\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}\right)^k\Gamma^{(k)}(1)}{k!}\right)^4$$

$$\frac{27}{2}\left(\left(\frac{1}{2}\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)^4-89-21+2\right)=$$
$$\frac{27}{2}\left(-108+\frac{100}{81}\sqrt{2}\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2 e^{-(5\pi)/2}\pi^3\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}\right)^k\Gamma^{(k)}(1)}{k!}\right)^4\right)$$

$$\frac{27}{2}\left(\left(\frac{1}{2}\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ e^{-1/8(5\pi)}\right)\right)^4-89-21+2\right)=$$
$$-1458+\frac{675\left(1+\sqrt[4]{5}\right)^8\left(3+\sqrt{5}\right)e^{-(5\pi)/2}\pi^3}{32\sqrt{2}\left(\sum_{k=1}^{\infty}\left(\frac{3}{4}\right)^k c_k\right)^4}$$
$$\text{for}\left(c_1=1\text{ and }c_2=1\text{ and }c_k=\frac{\gamma c_{-1+k}+\sum_{j=1}^{-2+k}(-1)^{1+j+k}c_j\,\zeta(-j+k)}{-1+k}\right)$$

**Integral representations:**

$$\frac{27}{2}\left(\left(\frac{1}{2}\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ e^{-1/8(5\pi)}\right)\right)^4-89-21+2\right)=$$
$$\frac{27}{2}\left(-108+\frac{25\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2 e^{-(5\pi)/2}\pi^3\left(\int_0^1\frac{1}{\sqrt[4]{\log\left(\frac{1}{t}\right)}}\,dt\right)^4}{32\sqrt{2}}\right)$$

$$\frac{27}{2}\left(\left(\frac{1}{2}\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ e^{-1/8(5\pi)}\right)\right)^4-89-21+2\right)=$$
$$\frac{27}{2}\left(-108+\frac{25\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2\exp\left(-\frac{5\pi}{2}+4\int_0^1\frac{-1-\frac{3}{4}(-1+x)+x^{3/4}}{(-1+x)\log(x)}\,dx\right)\pi^3}{32\sqrt{2}}\right)$$

$$\frac{27}{2}\left(\left(\frac{1}{2}\pi^{3/4}\sqrt{5}\left(\sqrt[8]{2}\left(1+\sqrt[4]{5}\right)^2\right)\Gamma\left(\frac{3}{4}\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\ e^{-1/8(5\pi)}\right)\right)^4-89-21+2\right)=$$
$$\frac{27}{2}\left(-108+\frac{25\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2\exp\left(-3\gamma-\frac{5\pi}{2}+4\int_0^1\frac{-1+x^{3/4}-\log\left(x^{3/4}\right)}{(-1+x)\log(x)}\,dx\right)\pi^3}{32\sqrt{2}}\right)$$

and:

(((27*1/2((((((Pi^(3/4) gamma (3/4) sqrt5 * (2)^(1/8)*1/2(1+(5)^(1/4))^2 (1/2(1+sqrt5))^0.5*e^(-(5Pi)/8))))^4-89-21+2))))))^1/15

**Input:**

$$\left(27\times\frac{1}{2}\left(\left(\pi^{3/4}\,\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\,\sqrt[8]{2}\times\frac{1}{2}\left(1+\sqrt[4]{5}\right)^2\right)\left(\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,e^{-1/8(5\pi)}\right)\right)^4-89-21+2\right)\right)\text{^}(1/15)$$

$\Gamma(x)$ is the gamma function

**Exact result:**

$$\frac{\sqrt[5]{3}}{\sqrt[15]{\dfrac{2}{\dfrac{25\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2 e^{-(5\pi)/2}\,\pi^3\,\Gamma\left(\frac{3}{4}\right)^4}{32\sqrt{2}}-108}}}$$

**Decimal approximation:**

1.64379475247302277189234450616634367004757599252167265633...

1.643794752...

**Alternate forms:**

$$\frac{\sqrt[5]{3}}{\sqrt[15]{\dfrac{2}{\dfrac{25\left(1+\sqrt[4]{5}\right)^8\left(3+\sqrt{5}\right)e^{-(5\pi)/2}\,\pi^7}{4\sqrt{2}\,\Gamma\left(\frac{1}{4}\right)^4}-108}}}$$

$$\frac{\sqrt[5]{3}}{\sqrt[15]{\dfrac{2}{\dfrac{25\left(1+\sqrt[4]{5}\right)^8\left(3+\sqrt{5}\right)e^{-(5\pi)/2}\,\pi^3\,\Gamma\left(\frac{3}{4}\right)^4}{16\sqrt{2}}-108}}}$$

$$\frac{\sqrt[30]{2}\,\sqrt[15]{50\left(1+\sqrt[4]{5}\right)^8\left(1+\sqrt{5}\right)^2 e^{-(5\pi)/2}\,\pi^3\left(\frac{3}{4}!\right)^4-2187\sqrt{2}}}{\sqrt[15]{3}}$$

Subtracting the two results, we obtain after some calculations:

1+1/(((Pi*1/(((3.91968410891124871-[((Pi^(3/4) gamma (3/4) sqrt5 (2)^(1/4)*1/2(1+(5)^(1/4))^2 (1/2(1+sqrt5))5/2*e^(-(5Pi)/4)))])))))

**Input interpretation:**

$$1+\frac{1}{\pi\times\frac{1}{3.91968410891124871-\pi^{3/4}\,\Gamma\left(\frac{3}{4}\right)\sqrt{5}\left(\sqrt[4]{2}\times\frac{1}{2}\left(1+\sqrt[4]{5}\right)^2\right)\left(\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\times\frac{5}{2}\,e^{-1/4\,(5\pi)}\right)}}$$

$\Gamma(x)$ is the gamma function

**Result:**

1.64033886288815923...

1.64033886…

**Alternative representations:**

$$1+\frac{1}{\frac{\pi}{3.919684108911248710000-\frac{\pi^{3/4}\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\left(\left(1+\sqrt{5}\right)5\,e^{-1/4\,(5\pi)}\right)\right)}{2\,(2\times 2)}}}=1+\frac{1}{\frac{\pi}{3.919684108911248710000-\frac{5}{8}\left(-1+\frac{3}{4}\right)!\,\sqrt[4]{2}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\left(1+\sqrt{5}\right)}}$$

$$1+\frac{1}{\frac{\pi}{3.919684108911248710000-\frac{\pi^{3/4}\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\left(\left(1+\sqrt{5}\right)5\,e^{-1/4\,(5\pi)}\right)\right)}{2\,(2\times 2)}}}=1+\frac{1}{\frac{\pi}{3.919684108911248710000-\frac{5}{8}\,\Gamma\left(\frac{3}{4},0\right)\sqrt[4]{2}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\left(1+\sqrt{5}\right)}}$$

$$1+\frac{1}{\frac{\pi}{3.919684108911248710000-\frac{\pi^{3/4}\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\left(\left(1+\sqrt{5}\right)5\,e^{-1/4\,(5\pi)}\right)\right)}{2\,(2\times 2)}}}=1+\frac{1}{\frac{\pi}{3.919684108911248710000-\frac{5}{8}\,(1)_{-1+\frac{3}{4}}\,\sqrt[4]{2}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\left(1+\sqrt{5}\right)}}$$

## Series representations:

$$1+\frac{1}{\frac{\pi}{3.91968410891124871000-\frac{\pi^{3/4}\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\left(\left(1+\sqrt{5}\right)5e^{-1/4(5\pi)}\right)\right)}{2(2\times 2)}}}=$$

$$\frac{1}{\pi}1.00000000000000000000\,e^{-(5\pi)/4}\left(3.91968410891124871000\,e^{(5\pi)/4}+1.00000000000000000000\,e^{(5\pi)/4}\pi-4.62807117722066311081\,\pi^{3/4}\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\Gamma\left(\frac{3}{4}\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}-4.62807117722066311081\,\pi^{3/4}\exp^2\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\Gamma\left(\frac{3}{4}\right)\sqrt{x}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2\right)\text{ for }(x\in\mathbb{R}\text{ and }x<0)$$

$$1+\frac{1}{\frac{\pi}{3.91968410891124871000-\frac{\pi^{3/4}\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\left(\left(1+\sqrt{5}\right)5e^{-1/4(5\pi)}\right)\right)}{2(2\times 2)}}}=$$

$$\frac{1}{\pi}1.00000000000000000000\,e^{-(5\pi)/4}\left(3.91968410891124871000\,e^{(5\pi)/4}+1.00000000000000000000\,e^{(5\pi)/4}\pi-6.17076156962755081441\,\pi^{3/4}\sqrt{4}^2\left(\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^2\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}\right)^k\Gamma^{(k)}(1)}{k!}-6.17076156962755081441\,\pi^{3/4}\sqrt{4}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{3^{k_2}\times 4^{-k_1-k_2}\binom{\frac{1}{2}}{k_1}\Gamma^{(k_2)}(1)}{k_2!}\right)$$

$$1+\frac{1}{\frac{\pi}{3.91968410891124871000 0-\frac{\pi^{3/4}\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\left(\left(1+\sqrt{5}\right)5e^{-1/4(5\pi)}\right)\right)}{2(2\times 2)}}}=$$

$$\frac{1}{\pi}\,1.00000000000000000000\,e^{-(5\pi)/4}\left(3.9196841089112487100\,e^{(5\pi)/4}+1.00000000000000000000\,e^{(5\pi)/4}\,\pi-4.6280711772206631108\,\pi^{3/4}\sqrt{4}^{2}\left(\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^{2}\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}-4.6280711772206631108\,\pi^{3/4}\sqrt{4}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{4^{-k_1}\binom{\frac{1}{2}}{k_1}\left(\frac{3}{4}-z_0\right)^{k_2}\Gamma^{(k_2)}(z_0)}{k_2!}\right)\text{ for }(z_0\notin\mathbb{Z}\text{ or }z_0>0)$$

## Integral representations:

$$1+\frac{1}{\frac{\pi}{3.919684108911248710000-\frac{\pi^{3/4}\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\left(\left(1+\sqrt{5}\right)5e^{-1/4(5\pi)}\right)\right)}{2(2\times 2)}}}=$$

$$\frac{1}{\pi}\left(3.919684108911248710000+\pi+e^{-(5\pi)/4}\,\pi^{3/4}\left(-4.628071177220663110808-4.628071177220663110808\sqrt{5}\right)\sqrt{5}\int_0^{\infty}\frac{\mathcal{A}^{-t}}{\sqrt[4]{t}}\,dt\right)$$

$$1+\frac{1}{\frac{\pi}{3.919684108911248710000-\frac{\pi^{3/4}\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\left(\left(1+\sqrt{5}\right)5e^{-1/4(5\pi)}\right)\right)}{2(2\times 2)}}}=$$

$$\frac{1}{\pi}\left(3.919684108911248710000+\pi+e^{-(5\pi)/4}\,\pi^{3/4}\left(-4.628071177220663110808-4.628071177220663110808\sqrt{5}\right)\sqrt{5}\int_0^{1}\frac{1}{\sqrt[4]{\log\left(\frac{1}{t}\right)}}\,dt\right)$$

$$1 + \frac{1}{\frac{\pi}{3.9196841089112487100000 - \frac{\pi^{3/4}\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2 \Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\left(\left(1+\sqrt{5}\right)5 e^{-1/4(5\pi)}\right)\right)}{2(2\times 2)}}} =$$

$$\frac{1}{\pi}\Big(3.9196841089112487100000 + \pi + e^{-(5\pi)/4}\pi^{3/4}\csc\left(\frac{3\pi}{8}\right)\left(-4.628071177220663110808 - 4.628071177220663110808\sqrt{5}\right)\sqrt{5}\int_0^\infty \frac{\sin(t)}{\sqrt[4]{t}}\,dt\Big)$$

Now, we have that:

e/3

**Input:**

$$\frac{e}{3}$$

**Decimal approximation:**

0.906093942819681745120095823784220832585749031233319858322...

0.9060939428… result very near to the range 0.910-0.918. We know that α’ is the Regge slope (string tension). With regard the Omega mesons, the values are:

| | | | | |
|---|---|---|---|---|
| $\omega$ | 6 | $m_{u/d} = 0 - 60$ | $0.910 - 0.918$ | |
| $\omega/\omega_3$ | $5+3$ | $m_{u/d} = 255 - 390$ | $0.988 - 1.18$ | |
| $\omega/\omega_3$ | $5+3$ | $m_{u/d} = 240 - 345$ | $0.937 - 1.000$ | |
| $\Psi$ | 3 | $m_c = 1500$ | $0.979$ | $-0.09$ |

**Property:**

$\frac{e}{3}$ is a transcendental number

**Continued fraction:**

$$\cfrac{1}{1+\cfrac{1}{9+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{5+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{26+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{5+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{44+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\cdots}}}}}}}}}}}}}}}}}}}}}}}$$

**Alternative representation:**

$$\frac{e}{3} = \frac{\exp(z)}{3} \quad \text{for } z = 1$$

**Series representations:**

$$\frac{e}{3} = \frac{1}{3}\sum_{k=0}^{\infty}\frac{1}{k!}$$

$$\frac{e}{3} = \frac{1}{6}\sum_{k=0}^{\infty}\frac{1+k}{k!}$$

$$\frac{e}{3} = \frac{\sum_{k=0}^{\infty}\frac{-1+k+z}{k!}}{3\,z}$$

From the two results, we have also:

1/10^52(((3.91968410891124871- [((Pi^(3/4) gamma (3/4) sqrt5 (2)^(1/4)*1/2(1+(5)^(1/4))^2 (1/2(1+sqrt5))5/2*e^(-(5Pi)/4)))] - (e/3))))

(we have multiplied by $1/10^{52}$)

**Input interpretation:**

$$\frac{1}{10^{52}}\left(3.91968410891124871 - \pi^{3/4}\,\Gamma\left(\frac{3}{4}\right)\sqrt{5}\left(\sqrt[4]{2}\times\frac{1}{2}\left(1+\sqrt[4]{5}\right)^2\right)\left(\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\times\frac{5}{2}\,e^{-1/4(5\pi)}\right) - \frac{e}{3}\right)$$

$\Gamma(x)$ is the gamma function

**Result:**

$1.1055899246378012\ldots \times 10^{-52}$

1.1055899…*10$^{-52}$ result practically equal to the value of Cosmological Constant

**Alternative representations:**

$$\frac{3.919684108911248710000 - \frac{\pi^{3/4}\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)}{2(2\times 2)} - \frac{e}{3}}{10^{52}} = \frac{3.919684108911248710000 - \frac{e}{3} - \frac{5\,G\left(1+\frac{3}{4}\right)\sqrt[4]{2}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\left(1+\sqrt{5}\right)}{8\,G\left(\frac{3}{4}\right)}}{10^{52}}$$

$$\frac{3.919684108911248710000 - \frac{\pi^{3/4}\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)}{2(2\times 2)} - \frac{e}{3}}{10^{52}} = \frac{1}{10^{52}}\left(3.919684108911248710000 - \frac{e}{3} - \frac{5}{8}\sqrt[4]{2}\,e^{-\log G(3/4)+\log G(1+3/4)}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\left(1+\sqrt{5}\right)\right)$$

$$\frac{3.919684108911248710000 - \frac{\pi^{3/4}\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)}{2(2\times 2)} - \frac{e}{3}}{10^{52}} = \frac{1}{10^{52}}\left(3.919684108911248710000 - \frac{e}{3} - \frac{5}{8}\left(-1+\frac{3}{4}\right)!\,\sqrt[4]{2}\,e^{-(5\pi)/4}\,\pi^{3/4}\left(1+\sqrt[4]{5}\right)^2\sqrt{5}\left(1+\sqrt{5}\right)\right)$$

## Series representations:

$$\frac{3.919684108911248710000 - \frac{\pi^{3/4}\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)}{2(2\times 2)} - \frac{e}{3}}{10^{52}} =$$

$$1.0000000000000000000\,e^{-(5\pi)/4}\left(3.9196841089112487100 0\times 10^{-52}\,e^{(5\pi)/4} - 3.3333333333333333333\times 10^{-53}\,e^{1+(5\pi)/4} - 4.62807117722066311081\times 10^{-52}\,\pi^{3/4}\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\Gamma\left(\frac{3}{4}\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!} - 4.62807117722066311081\times 10^{-52}\,\pi^{3/4}\exp^2\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\Gamma\left(\frac{3}{4}\right)\sqrt{x}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{3.919684108911248710000 - \frac{\pi^{3/4}\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\left(\left(1+\sqrt{5}\right)5\,e^{-1/4(5\pi)}\right)\right)}{2(2\times 2)} - \frac{e}{3}}{10^{52}} =$$

$$1.0000000000000000000\,e^{-(5\pi)/4}\left(3.91968410891124871000\times 10^{-52}\,e^{(5\pi)/4} - 3.3333333333333333333\times 10^{-53}\,e^{1+(5\pi)/4} - 6.17076156962755081441\times 10^{-52}\,\pi^{3/4}\sqrt{4}^2\left(\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^2\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}\right)^k\Gamma^{(k)}(1)}{k!} - 6.17076156962755081441\times 10^{-52}\,\pi^{3/4}\sqrt{4}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{3^{k_2}\times 4^{-k_1-k_2}\binom{\frac{1}{2}}{k_1}\Gamma^{(k_2)}(1)}{k_2!}\right)$$

$$\frac{3.919684108911248710000 - \frac{\pi^{3/4}\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2 \Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\left(\left(1+\sqrt{5}\right)5\, e^{-1/4(5\pi)}\right)\right)}{2(2\times 2)} - \frac{e}{3}}{10^{52}} =$$

$$1.0000000000000000000\, e^{-(5\pi)/4} \left(3.9196841089112487100 0\times 10^{-52}\, e^{(5\pi)/4} - 3.3333333333333333333\times 10^{-53}\, e^{1+(5\pi)/4} - 4.62807117722066311081\times 10^{-52}\, \pi^{3/4}\sqrt{4}^{2}\left(\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)^2 \sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}-z_0\right)^k \Gamma^{(k)}(z_0)}{k!} - 4.62807117722066311081\times 10^{-52}\, \pi^{3/4}\sqrt{4}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{4^{-k_1}\binom{\frac{1}{2}}{k_1}\left(\frac{3}{4}-z_0\right)^{k_2}\Gamma^{(k_2)}(z_0)}{k_2!}\right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

**Integral representations:**

$$\frac{3.919684108911248710000 - \frac{\pi^{3/4}\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2 \Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\left(\left(1+\sqrt{5}\right)5\, e^{-1/4(5\pi)}\right)\right)}{2(2\times 2)} - \frac{e}{3}}{10^{52}} =$$

$$3.9196841089112487100 0\times 10^{-52} - 3.3333333333333333333\times 10^{-53}\, e + e^{-(5\pi)/4}\pi^{3/4}\left(-4.628071177220663110808\times 10^{-52} - 4.628071177220663110808\times 10^{-52}\sqrt{5}\right)\sqrt{5}\int_0^{\infty}\frac{\mathcal{A}^{-t}}{\sqrt[4]{t}}\,dt$$

$$\frac{3.919684108911248710000 - \frac{\pi^{3/4}\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2 \Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\left(\left(1+\sqrt{5}\right)5\, e^{-1/4(5\pi)}\right)\right)}{2(2\times 2)} - \frac{e}{3}}{10^{52}} =$$

$$3.9196841089112487100 0\times 10^{-52} - 3.3333333333333333333\times 10^{-53}\, e + e^{-(5\pi)/4}\pi^{3/4}\left(-4.628071177220663110808\times 10^{-52} - 4.628071177220663110808\times 10^{-52}\sqrt{5}\right)\sqrt{5}\int_0^{1}\frac{1}{\sqrt[4]{\log\left(\frac{1}{t}\right)}}\,dt$$

$$\frac{3.919684108911248710000 - \frac{\pi^{3/4}\sqrt[4]{2}\left(1+\sqrt[4]{5}\right)^2\Gamma\left(\frac{3}{4}\right)\left(\sqrt{5}\left(\left(1+\sqrt{5}\right)5e^{-1/4(5\pi)}\right)\right)}{2(2\times 2)} - \frac{e}{3}}{10^{52}} =$$

$$3.91968410891124871000\times 10^{-52} - 3.33333333333333333333\times 10^{-53}\,e + e^{-(5\pi)/4}\pi^{3/4}\csc\left(\frac{3\pi}{8}\right)\left(-4.628071177220663110808\times 10^{-52} - 4.628071177220663110808\times 10^{-52}\sqrt{5}\right)\sqrt{5}\int_0^\infty \frac{\sin(t)}{\sqrt[4]{t}}\,dt$$

log(x) is the natural logarithm

csc(x) is the cosecant function

From:

**Some definite integrals** – *Srinivasa Ramanujan*
Journal of the Indian Mathematical Society, XI, 1919, 81 – 87

We have:

$$\int_0^\infty \frac{\cos \pi t x}{\cosh \pi x} e^{-\pi(t+i)x^2} dx = \frac{1+i}{2\sqrt{2}} e^{-\frac{1}{4}\pi t}\left\{1 - \frac{i}{\sqrt{(t+i)}}\right\}$$

$$\int_0^\infty \frac{\sin \pi t x}{\sinh \pi x} e^{-\pi(t+i)x^2} dx = \frac{1}{2} - \frac{1+i}{2\sqrt{2}} \cdot \frac{e^{-\frac{1}{4}\pi t}}{\sqrt{(t+i)}},$$

From

$$\int_0^\infty \frac{\cos \pi t x}{\cosh \pi x} e^{-\pi(t+i)x^2} dx = \frac{1+i}{2\sqrt{2}} e^{-\frac{1}{4}\pi t}\left\{1 - \frac{i}{\sqrt{(t+i)}}\right\}$$

we obtain:

(1+i)/(2sqrt2) *exp(-1/4*Pi*2/3i)* 1-i/(sqrt(2/3i+i))

**Input:**

$$\frac{1+i}{2\sqrt{2}}\exp\left(-\frac{1}{4}\pi\times\frac{2}{3}i\right)\times 1-\frac{i}{\sqrt{\frac{2}{3}i+i}}$$

$i$ is the imaginary unit

**Exact result:**

$$-\sqrt[4]{-1}\sqrt{\frac{3}{5}}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)e^{-(i\pi)/6}}{\sqrt{2}}$$

**Decimal approximation:**

$-0.064759644360631970082098182936353450135792275493780979 0\ldots -$
$0.41831303495390573228252036398877796977821024433801799 7\ldots i$

**Polar coordinates:**

$r\approx 0.423296$ (radius), $\theta\approx -98.8002°$ (angle)

0.423296

**Alternate forms:**

$$\left(-\frac{1}{20}-\frac{i}{20}\right)\left(5(-1)^{5/6}\sqrt{2}+2\sqrt{30}\right)$$

$$-\sqrt[4]{-1}\sqrt{\frac{3}{5}}-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{5/6}}{\sqrt{2}}$$

$$\left(-\frac{1}{20}-\frac{i}{20}\right)(-1)^{5/6}\left(5\sqrt{2}-(2-2i)(-1)^{5/12}\sqrt{15}\right)$$

**Series representations:**

$$\frac{\exp\left(-\frac{i(\pi 2)}{4\times 3}\right)(1+i)}{2\sqrt{2}}-\frac{i}{\sqrt{\frac{2i}{3}+i}}=$$

$$\left(-2i\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}+\exp\left(-\frac{i\pi}{6}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!}+\right.$$
$$\left. i\exp\left(-\frac{i\pi}{6}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\sqrt{z_0}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!}\right)$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\le 0$))

$$\frac{\exp\left(-\frac{i(\pi 2)}{4\times 3}\right)(1+i)}{2\sqrt{2}} - \frac{i}{\sqrt{\frac{2i}{3}+i}} = \left(-2\,i\exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!} + \exp\left(-\frac{i\pi}{6}\right)\exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{5i}{3}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!} + i\exp\left(-\frac{i\pi}{6}\right)\exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{5i}{3}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right) \Big/ \left(2\exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{5i}{3}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right) \quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\exp\left(-\frac{i(\pi 2)}{4\times 3}\right)(1+i)}{2\sqrt{2}} - \frac{i}{\sqrt{\frac{2i}{3}+i}} = -\left(\left(\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor-1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor} z_0^{-1/2-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor-1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor} \left(2\,i\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!} - \exp\left(-\frac{i\pi}{6}\right)\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor} z_0^{1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!} - i\left(\exp\left(-\frac{i\pi}{6}\right)\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor} z_0^{1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!}\right)\right)\right)\Big/\left(2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!}\right)\right)$$

sqrt15 *(((((1+i)/(2sqrt2) *exp(-1/4*Pi*2/3i)* 1-i/(sqrt(2/3i+i))))))

where 15 = 5*3 (Fibonacci numbers)

**Input:**

$$\sqrt{15}\left(\frac{1+i}{2\sqrt{2}}\exp\left(-\frac{1}{4}\pi\times\frac{2}{3}i\right)\times 1-\frac{i}{\sqrt{\frac{2}{3}i+i}}\right)$$

$i$ is the imaginary unit

**Exact result:**

$$\sqrt{15}\left(-\sqrt[4]{-1}\sqrt{\frac{3}{5}}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)e^{-(i\pi)/6}}{\sqrt{2}}\right)$$

**Decimal approximation:**

$-0.2508130241150426818817357786512750002687437670709867319\ldots -$
$1.6201194178779579655241602356532803351506055045659448 67\ldots i$

**Polar coordinates:**

$r \approx 1.63942$ (radius), $\theta \approx -98.8002°$ (angle)

1.63942

**Alternate forms:**

$$\frac{1}{4}\left(-12\sqrt[4]{-1}-(1+i)(-1)^{5/6}\sqrt{30}\right)$$

$$-3\sqrt[4]{-1}-\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{5/6}\sqrt{\frac{15}{2}}$$

$$\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{15}{2}}\left((-1+i)\sqrt[4]{-1}\sqrt{\frac{6}{5}}+e^{-(i\pi)/6}\right)$$

**Expanded form:**

$$-3\sqrt[4]{-1}+\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{\frac{15}{2}}e^{-(i\pi)/6}$$

## Series representations:

$$\sqrt{15}\left(\frac{\exp\left(-\frac{i(\pi 2)}{4\times 3}\right)(1+i)}{2\sqrt{2}}-\frac{i}{\sqrt{\frac{2i}{3}+i}}\right)=\left(\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(15-z_0)^k z_0^{-k}}{k!}\right)\left(-2i\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}+\exp\left(-\frac{i\pi}{6}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!}+i\exp\left(-\frac{i\pi}{6}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!}\right)\right)\Big/\left(2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!}\right)$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0$))

$$\sqrt{15}\left(\frac{\exp\left(-\frac{i(\pi 2)}{4\times 3}\right)(1+i)}{2\sqrt{2}}-\frac{i}{\sqrt{\frac{2i}{3}+i}}\right)=\left(\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(15-x)}{2\pi}\right\rfloor\right)\left(\sum_{k=0}^{\infty}\frac{(-1)^k(15-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\left(-2i\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+\exp\left(-\frac{i\pi}{6}\right)\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{5i}{3}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+i\exp\left(-\frac{i\pi}{6}\right)\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{5i}{3}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\Big/\left(2\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{5i}{3}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$

for ($x\in\mathbb{R}$ and $x<0$)

$$\sqrt{15}\left(\frac{\exp\left(-\frac{i(\pi 2)}{4\times 3}\right)(1+i)}{2\sqrt{2}}-\frac{i}{\sqrt{\frac{2i}{3}+i}}\right)=$$

$$-\left(\left(\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor+1/2\lfloor\arg(15-z_0)/(2\pi)\rfloor-1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor} z_0^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor+1/2\lfloor\arg(15-z_0)/(2\pi)\rfloor-1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor} \left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(15-z_0)^k z_0^{-k}}{k!}\right) \left(2i\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!} - \exp\left(-\frac{i\pi}{6}\right)\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor} z_0^{1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor} \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!} - i\left(\exp\left(-\frac{i\pi}{6}\right)\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor} z_0^{1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!}\right)\right)\right)\right) \Big/ \left(2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!}\right)$$

From

$$\int_0^\infty \frac{\sin\pi tx}{\sinh\pi x}e^{-\pi(t+i)x^2}dx = \frac{1}{2}-\frac{1+i}{2\sqrt{2}}\cdot\frac{e^{-\frac{1}{4}\pi t}}{\sqrt{(t+i)}}$$

we obtain:

((((1/2-(1+i)/(2sqrt2) *exp(-1/4*Pi*2/3i)* 1/(sqrt(2/3i+i))))))

**Input:**

$$\frac{1}{2}-\frac{1+i}{2\sqrt{2}}\exp\left(-\frac{1}{4}\pi\times\frac{2}{3}i\right)\times\frac{1}{\sqrt{\frac{2}{3}i+i}}$$

$i$ is the imaginary unit

**Exact result:**

$$\frac{1}{2}+\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sqrt{\frac{3}{10}}\ e^{-(i\pi)/6}$$

**Decimal approximation:**

0.1645898033750315455386239496903085646839072460582711413... +
0.1936491673103708442589632699891199805416460852645795413 2... $i$

**Polar coordinates:**

$r \approx 0.254145$ (radius), $\theta \approx 49.6375°$ (angle)

0.254145

**Alternate forms:**

$$\frac{1}{2}+\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{7/12}\sqrt{\frac{3}{10}}$$

$$\frac{1}{2}-\frac{1}{2}\sqrt{\frac{3}{5}}\ e^{-(i\pi)/6}$$

$$\left(\frac{1}{20}+\frac{i}{20}\right)\left((-1)^{7/12}\sqrt{30}+(5-5\,i)\right)$$

**Series representations:**

$$\frac{1}{2}-\frac{(1+i)\exp\left(-\frac{\pi\,2\,i}{4\times3}\right)}{\left(2\sqrt{2}\right)\sqrt{\frac{2\,i}{3}+i}}=-\left(\left(\left(\exp\left(-\frac{i\,\pi}{6}\right)+i\exp\left(-\frac{i\,\pi}{6}\right)-\sqrt{z_0}^{\,2}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5\,i}{3}-z_0\right)^{k_2}z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\right/\left(2\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5\,i}{3}-z_0\right)^k z_0^{-k}}{k!}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\frac{1}{2} - \frac{(1+i)\exp\left(-\frac{\pi 2 i}{4\times 3}\right)}{(2\sqrt{2})\sqrt{\frac{2i}{3}+i}} =$$

$$-\left(\left(\exp\left(-\frac{i\pi}{6}\right) + i\exp\left(-\frac{i\pi}{6}\right) - \exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}^2 \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}(2-x)^{k_1}\left(\frac{5i}{3}-x\right)^{k_2} x^{-k_1-k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}}{k_1!\,k_2!}\right)\middle/ \left(2\exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}^2 \left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{5i}{3}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right) \quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{1}{2} - \frac{(1+i)\exp\left(-\frac{\pi 2 i}{4\times 3}\right)}{(2\sqrt{2})\sqrt{\frac{2i}{3}+i}} =$$

$$\left(\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor-1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor} z_0^{-1-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor-1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor} \left(-\exp\left(-\frac{i\pi}{6}\right) - i\exp\left(-\frac{i\pi}{6}\right) + \left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor+1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor} z_0^{1+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor+1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor} \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5i}{3}-z_0\right)^{k_2} z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\right)\middle/ \left(2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!}\right)$$

sqrt42 ((((1/2-(1+i)/(2sqrt2) *exp(-1/4*Pi*2/3i)* 1/(sqrt(2/3i+i))))))

where 42 = 21*2 (Fibonacci numbers)

**Input:**

$$\sqrt{42}\left(\frac{1}{2}-\frac{1+i}{2\sqrt{2}}\exp\left(-\frac{1}{4}\pi\times\frac{2}{3}i\right)\times\frac{1}{\sqrt{\frac{2}{3}i+i}}\right)$$

$i$ is the imaginary unit

**Exact result:**

$$\sqrt{42}\left(\frac{1}{2}+\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sqrt{\frac{3}{10}}\,e^{-(i\pi)/6}\right)$$

**Decimal approximation:**

1.06666383727551432942518409446853257038158222231532731155... +
1.25499003980111332196725803867778123408922305394800829971... $i$

**Polar coordinates:**

$r \approx 1.64705$ (radius), $\theta \approx 49.6375°$ (angle)

1.64705

**Alternate forms:**

$$\sqrt{\frac{21}{2}}-3\sqrt{\frac{7}{10}}\,e^{-(i\pi)/6}$$

$$\sqrt{\frac{21}{2}}+\left(\frac{3}{2}+\frac{3i}{2}\right)(-1)^{7/12}\sqrt{\frac{7}{5}}$$

$$\frac{1}{10}\left(5\sqrt{42}+(3+3i)(-1)^{7/12}\sqrt{35}\right)$$

**Expanded form:**

$$\sqrt{\frac{21}{2}}+\left(\frac{3}{2}+\frac{3i}{2}\right)(-1)^{3/4}\sqrt{\frac{7}{5}}\,e^{-(i\pi)/6}$$

**Series representations:**

$$\sqrt{42}\left(\frac{1}{2}-\frac{(1+i)\exp\left(-\frac{\pi 2 i}{4\times 3}\right)}{\left(2\sqrt{2}\right)\sqrt{\frac{2i}{3}+i}}\right) =$$

$$\left(\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (42-z_0)^k z_0^{-k}}{k!}\right)\left(-\exp\left(-\frac{i\pi}{6}\right)-i\exp\left(-\frac{i\pi}{6}\right)+\sqrt{z_0}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5i}{3}-z_0\right)^{k_2} z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\right) \Big/ \left(2\sqrt{z_0}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!}\right)$$

for (not $(z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0)$)

$$\sqrt{42}\left(\frac{1}{2}-\frac{(1+i)\exp\left(-\frac{\pi 2 i}{4\times 3}\right)}{\left(2\sqrt{2}\right)\sqrt{\frac{2i}{3}+i}}\right) =$$

$$\left(\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(42-x)}{2\pi}\right\rfloor\right)\left(\sum_{k=0}^{\infty}\frac{(-1)^k (42-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\left(-\exp\left(-\frac{i\pi}{6}\right)-i\exp\left(-\frac{i\pi}{6}\right)+\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}(2-x)^{k_1}\left(\frac{5i}{3}-x\right)^{k_2}x^{-k_1-k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}}{k_1!\,k_2!}\right)\right) \Big/ \left(2\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\left(\sum_{k=0}^{\infty}\frac{(-1)^k (2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{5i}{3}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\sqrt{42}\left(\frac{1}{2}-\frac{(1+i)\exp\left(-\frac{\pi 2 i}{4\times 3}\right)}{(2\sqrt{2})\sqrt{\frac{2i}{3}+i}}\right)=$$

$$\left(\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor+1/2\lfloor\arg(42-z_0)/(2\pi)\rfloor-1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor}\right.$$

$$z_0^{-1/2-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor+1/2\lfloor\arg(42-z_0)/(2\pi)\rfloor-1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor}$$

$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(42-z_0)^k z_0^{-k}}{k!}\right)$$

$$\left(-\exp\left(-\frac{i\pi}{6}\right)-i\exp\left(-\frac{i\pi}{6}\right)+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor+1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor}\right.$$

$$z_0^{1+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor+1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor}$$

$$\left.\left.\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5i}{3}-z_0\right)^{k_2}z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\right)\Big/$$

$$\left(2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!}\right)$$

From the division of the two expression, we obtain:

((((1+i)/(2sqrt2) *exp(-1/4*Pi*2/3i)* 1-i/(sqrt(2/3i+i))))) / ((((1/2-(1+i)/(2sqrt2) *exp(-1/4*Pi*2/3i)* 1/(sqrt(2/3i+i))))))

**Input:**

$$\frac{\frac{1+i}{2\sqrt{2}}\exp\left(-\frac{1}{4}\pi\times\frac{2}{3}i\right)\times 1-\frac{i}{\sqrt{\frac{2}{3}i+i}}}{\frac{1}{2}-\frac{1+i}{2\sqrt{2}}\exp\left(-\frac{1}{4}\pi\times\frac{2}{3}i\right)\times\frac{1}{\sqrt{\frac{2}{3}i+i}}}$$

$i$ is the imaginary unit

**Exact result:**

$$\frac{-\sqrt[4]{-1}\sqrt{\frac{3}{5}}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)e^{-(i\pi)/6}}{\sqrt{2}}}{\frac{1}{2}+\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sqrt{\frac{3}{10}}\,e^{-(i\pi)/6}}$$

**Decimal approximation:**

$-1.419182955142515415807239815743524251402100521473711361\ldots - 0.871800284629623353006971646612583719041152089342363070\ldots\, i$

**Polar coordinates:**

$r \approx 1.66557$ (radius), $\theta \approx -148.438°$ (angle)

1.66557 result very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164.2696$ i.e. 1.65578...

**Alternate forms:**

$$\frac{-5(-1)^{5/6}\sqrt{2}-2\sqrt{30}}{(-1)^{7/12}\sqrt{30}+(5-5i)}$$

$$-\frac{(1-i)\left(-5i+(1+i)(-1)^{5/12}\sqrt{30}\right)}{\sqrt{2}\left(5\sqrt[6]{-1}-\sqrt{15}\right)}$$

$$-\frac{(-1)^{5/6}\left((2+2i)(-1)^{5/12}\sqrt{15}-5i\sqrt{2}\right)}{\sqrt[12]{-1}\sqrt{30}+(-5-5i)}$$

**Expanded form:**

$$\frac{\left(\frac{1}{2}+\frac{i}{2}\right)e^{-(i\pi)/6}}{\sqrt{2}\left(\frac{1}{2}+\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sqrt{\frac{3}{10}}\,e^{-(i\pi)/6}\right)}-\frac{\sqrt[4]{-1}\sqrt{\frac{3}{5}}}{\frac{1}{2}+\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sqrt{\frac{3}{10}}\,e^{-(i\pi)/6}}$$

## Series representations:

$$\frac{\frac{\exp\left(-\frac{i(\pi 2)}{4\times 3}\right)(1+i)}{2\sqrt{2}}-\frac{i}{\sqrt{\frac{2i}{3}+i}}}{\frac{1}{2}-\frac{(1+i)\exp\left(-\frac{\pi 2 i}{4\times 3}\right)}{\left(2\sqrt{2}\right)\sqrt{\frac{2i}{3}+i}}}=\left(\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{1+k}\left(-\frac{1}{2}\right)_k\left(2i(2-z_0)^k-(1+i)\exp\left(-\frac{i\pi}{6}\right)\left(\frac{5i}{3}-z_0\right)^k\right)z_0^{-k}}{k!}\right)\Big/\left(-\exp\left(-\frac{i\pi}{6}\right)-i\exp\left(-\frac{i\pi}{6}\right)+\sqrt{z_0}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5i}{3}-z_0\right)^{k_2}z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{\frac{\exp\left(-\frac{i(\pi 2)}{4\times 3}\right)(1+i)}{2\sqrt{2}}-\frac{i}{\sqrt{\frac{2i}{3}+i}}}{\frac{1}{2}-\frac{(1+i)\exp\left(-\frac{\pi 2 i}{4\times 3}\right)}{\left(2\sqrt{2}\right)\sqrt{\frac{2i}{3}+i}}}=\left(\sqrt{x}\sum_{k=0}^{\infty}\frac{1}{k!}(-1)^{1+k}x^{-k}\left(2i(2-x)^k\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)-(1+i)\left(\frac{5i}{3}-x\right)^k\exp\left(-\frac{i\pi}{6}\right)\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\right)\left(-\frac{1}{2}\right)_k\right)\Big/\left(-\exp\left(-\frac{i\pi}{6}\right)-i\exp\left(-\frac{i\pi}{6}\right)+\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}(2-x)^{k_1}\left(\frac{5i}{3}-x\right)^{k_2}x^{-k_1-k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}}{k_1!\,k_2!}\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{\frac{\exp\left(-\frac{i(\pi 2)}{4\times 3}\right)(1+i)}{2\sqrt{2}}-\frac{i}{\sqrt{\frac{2i}{3}+i}}}{\frac{1}{2}-\frac{(1+i)\exp\left(-\frac{\pi 2 i}{4\times 3}\right)}{\left(2\sqrt{2}\right)\sqrt{\frac{2i}{3}+i}}} =$$

$$-\left(\left(\sqrt{x}\left(-2\,i\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.$$
$$\exp\left(-\frac{i\pi}{6}\right)\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{5i}{3}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+$$
$$\left.\left.i\exp\left(-\frac{i\pi}{6}\right)\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{5i}{3}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\Big/$$
$$\left(\left(\exp\left(-\frac{i\pi}{6}\right)+i\exp\left(-\frac{i\pi}{6}\right)-\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\right)\right.$$
$$\left.\left.\sqrt{x}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}(2-x)^{k_1}\left(\frac{5i}{3}-x\right)^{k_2}x^{-k_1-k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}}{k_1!\,k_2!}\right)\right)$$

for $(x\in\mathbb{R}$ and $x<0)$

$$\frac{\frac{\exp\left(-\frac{i(\pi 2)}{4\times 3}\right)(1+i)}{2\sqrt{2}} - \frac{i}{\sqrt{\frac{2i}{3}+i}}}{\frac{1}{2} - \frac{(1+i)\exp\left(-\frac{\pi 2 i}{4\times 3}\right)}{\left(2\sqrt{2}\right)\sqrt{\frac{2i}{3}+i}}} =$$

$$\left(\sqrt{z_0}\left(2i\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} - \exp\left(-\frac{i\pi}{6}\right)\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor} z_0^{1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k\left(-\frac{1}{2}\right)_k \left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!} - i\left(\exp\left(-\frac{i\pi}{6}\right)\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor} z_0^{1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k\left(-\frac{1}{2}\right)_k \left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!}\right)\right)\right) \Big/$$

$$\left(\exp\left(-\frac{i\pi}{6}\right) + i\exp\left(-\frac{i\pi}{6}\right) - \left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor + 1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor} z_0^{1+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor+1/2\left\lfloor\arg\left(\frac{5i}{3}-z_0\right)/(2\pi)\right\rfloor} \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5i}{3}-z_0\right)^{k_2} z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)$$

From which:

((((1+i)/(2sqrt2) *exp(-1/4*Pi*2/3i)* 1-i/(sqrt(2/3i+i))))) / ((((1/2-(1+i)/(2sqrt2) *exp(-1/4*Pi*2/3i)* 1/(sqrt(2/3i+i)))))) + ((2*47)/10^3)i

where 47 is a Lucas number

**Input:**

$$\frac{\frac{1+i}{2\sqrt{2}}\exp\left(-\frac{1}{4}\pi\times\frac{2}{3}i\right)\times 1 - \frac{i}{\sqrt{\frac{2}{3}i+i}}}{\frac{1}{2} - \frac{1+i}{2\sqrt{2}}\exp\left(-\frac{1}{4}\pi\times\frac{2}{3}i\right)\times\frac{1}{\sqrt{\frac{2}{3}i+i}}} + \frac{2\times 47}{10^3}i$$

$i$ is the imaginary unit

**Exact result:**

$$\frac{47 i}{500}+\frac{-\sqrt[4]{-1}\sqrt{\frac{3}{5}}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)e^{-(i\pi)/6}}{\sqrt{2}}}{\frac{1}{2}+\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sqrt{\frac{3}{10}}\,e^{-(i\pi)/6}}$$

**Decimal approximation:**

- 1.419182955142515415807239815743524251402100521473711361... -
0.7778002846296233530069716466125837190411520893423630700... $i$

**Polar coordinates:**

$r \approx 1.61835$ (radius), $\theta \approx -151.275°$ (angle)

1.61835

**Alternate forms:**

$$\frac{(235+235 i)-2500(-1)^{5/6}\sqrt{2}-1000\sqrt{30}-47\sqrt[12]{-1}\sqrt{30}}{500\left((-1)^{7/12}\sqrt{30}+(5-5 i)\right)}$$

$$-\frac{(-2500-2500 i)-235(-1)^{2/3}\sqrt{2}+47 i\sqrt{30}+1000(-1)^{5/12}\sqrt{30}}{500\sqrt{2}\left(5\sqrt[6]{-1}-\sqrt{15}\right)}$$

$$-\left(\left((-1)^{5/6}\left((235-235 i)\sqrt[6]{-1}-2500 i\sqrt{2}+(1000+1000 i)(-1)^{5/12}\sqrt{15}+47(-1)^{3/4}\sqrt{30}\right)\right)\Big/\left(500\left(\sqrt[12]{-1}\sqrt{30}+(-5-5 i)\right)\right)\right)$$

**Expanded form:**

$$\frac{47 i}{500}-\frac{\sqrt[4]{-1}\sqrt{\frac{3}{5}}}{\frac{1}{2}+\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sqrt{\frac{3}{10}}\,e^{-(i\pi)/6}}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)e^{-(i\pi)/6}}{\sqrt{2}\left(\frac{1}{2}+\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sqrt{\frac{3}{10}}\,e^{-(i\pi)/6}\right)}$$

**Series representations:**

$$\frac{\frac{\exp\left(-\frac{i(\pi 2)}{4\times 3}\right)(1+i)}{2\sqrt{2}}-\frac{i}{\sqrt{\frac{2i}{3}+i}}}{\frac{1}{2}-\frac{(1+i)\exp\left(-\frac{\pi 2 i}{4\times 3}\right)}{\left(2\sqrt{2}\right)\sqrt{\frac{2i}{3}+i}}}+\frac{i(2\times 47)}{10^3}=\left(47\,i\exp\left(-\frac{i\pi}{6}\right)+47\,i^2\exp\left(-\frac{i\pi}{6}\right)-\right.$$

$$500\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{1+k}\left(-\frac{1}{2}\right)_k\left(2\,i\,(2-z_0)^k-(1+i)\exp\left(-\frac{i\pi}{6}\right)\left(\frac{5i}{3}-z_0\right)^k\right)z_0^{-k}}{k!}-$$

$$\left.47\,i\sqrt{z_0}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5i}{3}-z_0\right)^{k_2}z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\Bigg/$$

$$\left(500\left(\exp\left(-\frac{i\pi}{6}\right)+i\exp\left(-\frac{i\pi}{6}\right)-\right.\right.$$

$$\left.\left.\sqrt{z_0}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5i}{3}-z_0\right)^{k_2}z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{\frac{\exp\left(-\frac{i(\pi 2)}{4\times 3}\right)(1+i)}{2\sqrt{2}} - \frac{i}{\sqrt{\frac{2i}{3}+i}}}{\frac{1}{2} - \frac{(1+i)\exp\left(-\frac{\pi 2 i}{4\times 3}\right)}{\left(2\sqrt{2}\right)\sqrt{\frac{2i}{3}+i}}} + \frac{i(2\times 47)}{10^3} =$$

$$\left(47\,i\exp\left(-\frac{i\pi}{6}\right) + 47\,i^2\exp\left(-\frac{i\pi}{6}\right) + 1000\,i\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!} - \right.$$
$$500\exp\left(-\frac{i\pi}{6}\right)\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!} -$$
$$500\,i\exp\left(-\frac{i\pi}{6}\right)\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!} -$$
$$\left.47\,i\sqrt{z_0}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5i}{3}-z_0\right)^{k_2} z_0^{-k_1-k_2}}{k_1!\,k_2!}\right) /$$
$$\left(500\left(\exp\left(-\frac{i\pi}{6}\right) + i\exp\left(-\frac{i\pi}{6}\right) - \right.\right.$$
$$\left.\left.\sqrt{z_0}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5i}{3}-z_0\right)^{k_2} z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Furthermore, we obtain also:

987 [(((((1+i)/(2sqrt2) *exp(-1/4*Pi*2/3i)* 1-i/(sqrt(2/3i+i))))) / ((((1/2-(1+i)/(2sqrt2) *exp(-1/4*Pi*2/3i)* 1/(sqrt(2/3i+i))))))]-(144+8)i

**Input:**

$$987\times\frac{\frac{1+i}{2\sqrt{2}}\exp\left(-\frac{1}{4}\pi\times\frac{2}{3}i\right)\times 1 - \frac{i}{\sqrt{\frac{2}{3}i+i}}}{\frac{1}{2} - \frac{1+i}{2\sqrt{2}}\exp\left(-\frac{1}{4}\pi\times\frac{2}{3}i\right)\times\frac{1}{\sqrt{\frac{2}{3}i+i}}} - (144+8)\,i$$

$i$ is the imaginary unit

**Exact result:**

$$-152\,i+\frac{987\left(-\sqrt[4]{-1}\,\sqrt{\frac{3}{5}}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)e^{-(i\pi)/6}}{\sqrt{2}}\right)}{\frac{1}{2}+\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sqrt{\frac{3}{10}}\,e^{-(i\pi)/6}}$$

**Decimal approximation:**

$-1400.733576725662715401745698138858436133873214694553113\ldots - 1012.466880929438249417881015206620130693617112180912350 1\ldots\, i$

**Polar coordinates:**

$r \approx 1728.34$ (radius), $\theta \approx -144.14°$ (angle)

1728.34

**Alternate forms:**

$$-\frac{(760+760\,i)+4935\,(-1)^{5/6}\sqrt{2}+1974\sqrt{30}-152\,\sqrt[12]{-1}\,\sqrt{30}}{(-1)^{7/12}\sqrt{30}+(5-5\,i)}$$

$$-\frac{1}{\sqrt{2}\left(5\sqrt[6]{-1}-\sqrt{15}\right)}(1-i)\left(-4935\,i-(380-380\,i)\sqrt[6]{-1}\,\sqrt{2}+(76-76\,i)\sqrt{30}+(987+987\,i)(-1)^{5/12}\sqrt{30}\right)$$

$$-\left(\left((-1)^{5/6}\left((-760-760\,i)\sqrt[6]{-1}+4935\sqrt{2}-(1974-1974\,i)(-1)^{5/12}\sqrt{15}+152\sqrt[4]{-1}\,\sqrt{30}\right)\right)\Big/\left((-1)^{7/12}\sqrt{30}+(5-5\,i)\right)\right)$$

**Expanded form:**

$$-152\,i-\frac{987\sqrt[4]{-1}\,\sqrt{\frac{3}{5}}}{\frac{1}{2}+\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sqrt{\frac{3}{10}}\,e^{-(i\pi)/6}}+\frac{\left(\frac{987}{2}+\frac{987\,i}{2}\right)e^{-(i\pi)/6}}{\sqrt{2}\left(\frac{1}{2}+\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sqrt{\frac{3}{10}}\,e^{-(i\pi)/6}\right)}$$

## Series representations:

$$\frac{987\left(\frac{(1+i)\exp\left(-\frac{i(\pi 2)}{4\times 3}\right)}{2\sqrt{2}}-\frac{i}{\sqrt{\frac{2i}{3}+i}}\right)}{\frac{1}{2}-\frac{(1+i)\exp\left(-\frac{\pi 2 i}{4\times 3}\right)}{\left(2\sqrt{2}\right)\sqrt{\frac{2i}{3}+i}}}-i(144+8)=$$

$$-\left(\left(152\,i\exp\left(-\frac{i\pi}{6}\right)+152\,i^2\exp\left(-\frac{i\pi}{6}\right)+987\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{1+k}\left(-\frac{1}{2}\right)_k\left(2\,i\,(2-z_0)^k-(1+i)\exp\left(-\frac{i\pi}{6}\right)\left(\frac{5i}{3}-z_0\right)^k\right)z_0^{-k}}{k!}-152\,i\sqrt{z_0}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5i}{3}-z_0\right)^{k_2}z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\Bigg/\left(\exp\left(-\frac{i\pi}{6}\right)+i\exp\left(-\frac{i\pi}{6}\right)-\sqrt{z_0}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5i}{3}-z_0\right)^{k_2}z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\right)$$

for (not ($z_0\in\mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\frac{987\left(\frac{(1+i)\exp\left(-\frac{i(\pi 2)}{4\times 3}\right)}{2\sqrt{2}}-\frac{i}{\sqrt{\frac{2i}{3}+i}}\right)}{\frac{1}{2}-\frac{(1+i)\exp\left(-\frac{\pi 2 i}{4\times 3}\right)}{\left(2\sqrt{2}\right)\sqrt{\frac{2i}{3}+i}}}-i(144+8)=$$

$$-\left(\left(152\,i\exp\left(-\frac{i\pi}{6}\right)+152\,i^2\exp\left(-\frac{i\pi}{6}\right)-1974\,i\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}+\right.\right.$$

$$987\exp\left(-\frac{i\pi}{6}\right)\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!}+$$

$$987\,i\exp\left(-\frac{i\pi}{6}\right)\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!}-152\,i\sqrt{z_0}^{\,2}$$

$$\left.\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5i}{3}-z_0\right)^{k_2}z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\Big/$$

$$\left(\exp\left(-\frac{i\pi}{6}\right)+i\exp\left(-\frac{i\pi}{6}\right)-\sqrt{z_0}^{\,2}\right.$$

$$\left.\left.\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5i}{3}-z_0\right)^{k_2}z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{987\left(\frac{(1+i)\exp\left(-\frac{i(\pi 2)}{4\times 3}\right)}{2\sqrt{2}}-\frac{i}{\sqrt{\frac{2i}{3}+i}}\right)}{\frac{1}{2}-\frac{(1+i)\exp\left(-\frac{\pi 2 i}{4\times 3}\right)}{\left(2\sqrt{2}\right)\sqrt{\frac{2i}{3}+i}}}-i(144+8)=$$

$$-\left(\left(152\,i\exp\left(-\frac{i\pi}{6}\right)+152\,i^2\exp\left(-\frac{i\pi}{6}\right)+987\sqrt{x}\right.\right.$$

$$\sum_{k=0}^{\infty}\frac{1}{k!}(-1)^{1+k}x^{-k}\left(2\,i\,(2-x)^k\exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)-\right.$$

$$\left.(1+i)\left(\frac{5i}{3}-x\right)^k\exp\left(-\frac{i\pi}{6}\right)\exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\right)\left(-\frac{1}{2}\right)_k-$$

$$152\,i\exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}^2$$

$$\left.\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}(2-x)^{k_1}\left(\frac{5i}{3}-x\right)^{k_2}x^{-k_1-k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}}{k_1!\,k_2!}\right)\Big/$$

$$\left(\exp\left(-\frac{i\pi}{6}\right)+i\exp\left(-\frac{i\pi}{6}\right)-\exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\right.$$

$$\left.\left.\sqrt{x}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}(2-x)^{k_1}\left(\frac{5i}{3}-x\right)^{k_2}x^{-k_1-k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}}{k_1!\,k_2!}\right)\right)$$

for ($x\in\mathbb{R}$ and $x<0$)

$$\frac{\frac{\exp\left(-\frac{i(\pi 2)}{4\times 3}\right)(1+i)}{2\sqrt{2}} - \frac{i}{\sqrt{\frac{2i}{3}+i}}}{\frac{1}{2} - \frac{(1+i)\exp\left(-\frac{\pi 2 i}{4\times 3}\right)}{\left(2\sqrt{2}\right)\sqrt{\frac{2i}{3}+i}}} + \frac{i\,(2\times 47)}{10^3} = \Bigg(47\,i\exp\left(-\frac{i\pi}{6}\right) + 47\,i^2\exp\left(-\frac{i\pi}{6}\right) -$$

$$500\sqrt{x}\sum_{k=0}^{\infty}\frac{1}{k!}(-1)^{1+k}x^{-k}\Bigg(2\,i\,(2-x)^k\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right) -$$

$$(1+i)\left(\frac{5i}{3}-x\right)^k\exp\left(-\frac{i\pi}{6}\right)\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\Bigg)\left(-\frac{1}{2}\right)_k -$$

$$47\,i\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}^2$$

$$\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}(2-x)^{k_1}\left(\frac{5i}{3}-x\right)^{k_2}x^{-k_1-k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}}{k_1!\,k_2!}\Bigg)\Big/$$

$$\Bigg(500\Bigg(\exp\left(-\frac{i\pi}{6}\right) + i\exp\left(-\frac{i\pi}{6}\right) - \exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)$$

$$\sqrt{x}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}(2-x)^{k_1}\left(\frac{5i}{3}-x\right)^{k_2}x^{-k_1-k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}}{k_1!\,k_2!}\Bigg)\Bigg)$$

for ($x \in \mathbb{R}$ and $x < 0$)

and:

89*[(((((1+i)/(2sqrt2) *exp(-1/4*Pi*2/3i)* 1-i/(sqrt(2/3i+i))))) / ((((1/2-(1+i)/(2sqrt2) *exp(-1/4*Pi*2/3i)* 1/(sqrt(2/3i+i))))))]+18i

**Input:**

$$89\times\frac{\frac{1+i}{2\sqrt{2}}\exp\left(-\frac{1}{4}\pi\times\frac{2}{3}i\right)\times 1 - \frac{i}{\sqrt{\frac{2}{3}i+i}}}{\frac{1}{2} - \frac{1+i}{2\sqrt{2}}\exp\left(-\frac{1}{4}\pi\times\frac{2}{3}i\right)\times\frac{1}{\sqrt{\frac{2}{3}i+i}}} + 18\,i$$

$i$ is the imaginary unit

**Exact result:**

$$18\,i+\frac{89\left(-\sqrt[4]{-1}\,\sqrt{\frac{3}{5}}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)e^{-(i\pi)/6}}{\sqrt{2}}\right)}{\frac{1}{2}+\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sqrt{\frac{3}{10}}\;e^{-(i\pi)/6}}$$

**Decimal approximation:**

−126.30728300768387200684434360117365837478694641116031115... −
59.590225332036478417620476548519950994662535951470313235... $i$

**Polar coordinates:**

$r \approx 139.659$ (radius), $\theta \approx -154.743°$ (angle)

139.659

**Alternate forms:**

$$\frac{(90+90\,i)-445\,(-1)^{5/6}\sqrt{2}-178\sqrt{30}-18\sqrt[12]{-1}\,\sqrt{30}}{(-1)^{7/12}\sqrt{30}+(5-5\,i)}$$

$$-\frac{(1-i)\left(-445\,i+(45-45\,i)\sqrt[6]{-1}\,\sqrt{2}-(9-9\,i)\sqrt{30}+(89+89\,i)(-1)^{5/12}\sqrt{30}\right)}{\sqrt{2}\left(5\sqrt[6]{-1}-\sqrt{15}\right)}$$

$$-\frac{(-1)^{5/6}\left((90-90\,i)\sqrt[6]{-1}-445\,i\sqrt{2}+(178+178\,i)(-1)^{5/12}\sqrt{15}+18\,(-1)^{3/4}\sqrt{30}\right)}{\sqrt[12]{-1}\,\sqrt{30}+(-5-5\,i)}$$

**Expanded form:**

$$18\,i-\frac{89\sqrt[4]{-1}\,\sqrt{\frac{3}{5}}}{\frac{1}{2}+\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sqrt{\frac{3}{10}}\;e^{-(i\pi)/6}}+\frac{\left(\frac{89}{2}+\frac{89\,i}{2}\right)e^{-(i\pi)/6}}{\sqrt{2}\left(\frac{1}{2}+\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sqrt{\frac{3}{10}}\;e^{-(i\pi)/6}\right)}$$

**Series representations:**

$$\frac{89\left(\frac{(1+i)\exp\left(-\frac{i(\pi 2)}{4\times 3}\right)}{2\sqrt{2}}-\frac{i}{\sqrt{\frac{2i}{3}+i}}\right)}{\frac{1}{2}-\frac{(1+i)\exp\left(-\frac{\pi 2 i}{4\times 3}\right)}{\left(2\sqrt{2}\right)\sqrt{\frac{2i}{3}+i}}}+18\,i=\left(18\,i\exp\left(-\frac{i\pi}{6}\right)+18\,i^2\exp\left(-\frac{i\pi}{6}\right)-89\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{1+k}\left(-\frac{1}{2}\right)_k\left(2\,i\,(2-z_0)^k-(1+i)\exp\left(-\frac{i\pi}{6}\right)\left(\frac{5i}{3}-z_0\right)^k\right)z_0^{-k}}{k!}-18\,i\sqrt{z_0}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5i}{3}-z_0\right)^{k_2}z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\Big/\left(\exp\left(-\frac{i\pi}{6}\right)+i\exp\left(-\frac{i\pi}{6}\right)-\sqrt{z_0}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5i}{3}-z_0\right)^{k_2}z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{89\left(\frac{(1+i)\exp\left(-\frac{i(\pi 2)}{4\times 3}\right)}{2\sqrt{2}}-\frac{i}{\sqrt{\frac{2i}{3}+i}}\right)}{\frac{1}{2}-\frac{(1+i)\exp\left(-\frac{\pi 2 i}{4\times 3}\right)}{\left(2\sqrt{2}\right)\sqrt{\frac{2i}{3}+i}}}+18i=$$

$$\left(18i\exp\left(-\frac{i\pi}{6}\right)+18i^2\exp\left(-\frac{i\pi}{6}\right)+178i\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}-\right.$$

$$89\exp\left(-\frac{i\pi}{6}\right)\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!}-$$

$$89i\exp\left(-\frac{i\pi}{6}\right)\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!}-$$

$$\left.18i\sqrt{z_0}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5i}{3}-z_0\right)^{k_2}z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\Big/$$

$$\left(\exp\left(-\frac{i\pi}{6}\right)+i\exp\left(-\frac{i\pi}{6}\right)-\right.$$

$$\left.\sqrt{z_0}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5i}{3}-z_0\right)^{k_2}z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{89\left(\frac{(1+i)\exp\left(-\frac{i(\pi 2)}{4\times 3}\right)}{2\sqrt{2}}-\frac{i}{\sqrt{\frac{2i}{3}+i}}\right)}{\frac{1}{2}-\frac{(1+i)\exp\left(-\frac{\pi 2 i}{4\times 3}\right)}{(2\sqrt{2})\sqrt{\frac{2i}{3}+i}}}+18i=\left(18i\exp\left(-\frac{i\pi}{6}\right)+18i^2\exp\left(-\frac{i\pi}{6}\right)-\right.$$

$$89\sqrt{x}\sum_{k=0}^{\infty}\frac{1}{k!}(-1)^{1+k}x^{-k}\left(2i(2-x)^k\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)-\right.$$

$$\left.(1+i)\left(\frac{5i}{3}-x\right)^k\exp\left(-\frac{i\pi}{6}\right)\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\right)\left(-\frac{1}{2}\right)_k-$$

$$18i\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}^2$$

$$\left.\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}(2-x)^{k_1}\left(\frac{5i}{3}-x\right)^{k_2}x^{-k_1-k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}}{k_1!k_2!}\right)\Big/$$

$$\left(\exp\left(-\frac{i\pi}{6}\right)+i\exp\left(-\frac{i\pi}{6}\right)-\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\right.$$

$$\left.\sqrt{x}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}(2-x)^{k_1}\left(\frac{5i}{3}-x\right)^{k_2}x^{-k_1-k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}}{k_1!k_2!}\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

89*[((((1+i)/(2sqrt2) *exp(-1/4*Pi*2/3i)* 1-i/(sqrt(2/3i+i))))) / ((((1/2-(1+i)/(2sqrt2) *exp(-1/4*Pi*2/3i)* 1/(sqrt(2/3i+i))))))]+76i

**Input:**

$$89\times\frac{\frac{1+i}{2\sqrt{2}}\exp\left(-\frac{1}{4}\pi\times\frac{2}{3}i\right)\times 1-\frac{i}{\sqrt{\frac{2}{3}i+i}}}{\frac{1}{2}-\frac{1+i}{2\sqrt{2}}\exp\left(-\frac{1}{4}\pi\times\frac{2}{3}i\right)\times\frac{1}{\sqrt{\frac{2}{3}i+i}}}+76i$$

$i$ is the imaginary unit

**Exact result:**

$$76i+\frac{89\left(-\sqrt[4]{-1}\sqrt{\frac{3}{5}}+\frac{\left(\frac{1}{2}+\frac{i}{2}\right)e^{-(i\pi)/6}}{\sqrt{2}}\right)}{\frac{1}{2}+\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sqrt{\frac{3}{10}}\,e^{-(i\pi)/6}}$$

## Decimal approximation:

$-126.307283007683872006844343601173658374786946411160311 15\ldots - 1.5902253320364784176204765485199509946625359514703132357\ldots i$

## Polar coordinates:

$r \approx 126.317$ (radius), $\theta \approx -179.279°$ (angle)

126.317

## Alternate forms:

$$\frac{(380+380 i) - 445 (-1)^{5/6} \sqrt{2} - 178 \sqrt{30} - 76 \sqrt[12]{-1} \sqrt{30}}{(-1)^{7/12} \sqrt{30} + (5-5 i)}$$

$$-\frac{(1-i)\left(-445 i + (190-190 i) \sqrt[6]{-1} \sqrt{2} - (38-38 i) \sqrt{30} + (89+89 i)(-1)^{5/12} \sqrt{30}\right)}{\sqrt{2}\left(5 \sqrt[6]{-1} - \sqrt{15}\right)}$$

$$-\frac{(-1)^{5/6}\left((380-380 i) \sqrt[6]{-1} - 445 i \sqrt{2} + (178+178 i)(-1)^{5/12} \sqrt{15} + 76 (-1)^{3/4} \sqrt{30}\right)}{\sqrt[12]{-1} \sqrt{30} + (-5-5 i)}$$

## Expanded form:

$$76 i - \frac{89 \sqrt[4]{-1} \sqrt{\frac{3}{5}}}{\frac{1}{2} + \left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \sqrt{\frac{3}{10}}\, e^{-(i \pi)/6}} + \frac{\left(\frac{89}{2} + \frac{89 i}{2}\right) e^{-(i \pi)/6}}{\sqrt{2}\left(\frac{1}{2} + \left(\frac{1}{2} + \frac{i}{2}\right)(-1)^{3/4} \sqrt{\frac{3}{10}}\, e^{-(i \pi)/6}\right)}$$

**Series representations:**

$$\frac{89\left(\frac{(1+i)\exp\left(-\frac{i(\pi 2)}{4\times 3}\right)}{2\sqrt{2}}-\frac{i}{\sqrt{\frac{2i}{3}+i}}\right)}{\frac{1}{2}-\frac{(1+i)\exp\left(-\frac{\pi 2 i}{4\times 3}\right)}{\left(2\sqrt{2}\right)\sqrt{\frac{2i}{3}+i}}}+76\,i=\left(76\,i\exp\left(-\frac{i\pi}{6}\right)+76\,i^2\exp\left(-\frac{i\pi}{6}\right)-89\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{1+k}\left(-\frac{1}{2}\right)_k\left(2\,i\,(2-z_0)^k-(1+i)\exp\left(-\frac{i\pi}{6}\right)\left(\frac{5i}{3}-z_0\right)^k\right)z_0^{-k}}{k!}-76\,i\sqrt{z_0}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5i}{3}-z_0\right)^{k_2}z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\Big/\left(\exp\left(-\frac{i\pi}{6}\right)+i\exp\left(-\frac{i\pi}{6}\right)-\sqrt{z_0}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5i}{3}-z_0\right)^{k_2}z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0$))

$$\frac{89\left(\frac{(1+i)\exp\left(-\frac{i(\pi 2)}{4\times 3}\right)}{2\sqrt{2}}-\frac{i}{\sqrt{\frac{2i}{3}+i}}\right)}{\frac{1}{2}-\frac{(1+i)\exp\left(-\frac{\pi 2 i}{4\times 3}\right)}{\left(2\sqrt{2}\right)\sqrt{\frac{2i}{3}+i}}}+76i=$$

$$\left(76i\exp\left(-\frac{i\pi}{6}\right)+76i^2\exp\left(-\frac{i\pi}{6}\right)+178i\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}-\right.$$

$$89\exp\left(-\frac{i\pi}{6}\right)\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!}-$$

$$89i\exp\left(-\frac{i\pi}{6}\right)\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{5i}{3}-z_0\right)^k z_0^{-k}}{k!}-$$

$$\left.76i\sqrt{z_0}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5i}{3}-z_0\right)^{k_2}z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\Big/$$

$$\left(\exp\left(-\frac{i\pi}{6}\right)+i\exp\left(-\frac{i\pi}{6}\right)-\right.$$

$$\left.\sqrt{z_0}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}(2-z_0)^{k_1}\left(\frac{5i}{3}-z_0\right)^{k_2}z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\frac{89\left(\frac{(1+i)\exp\left(-\frac{i(\pi\,2)}{4\times 3}\right)}{2\sqrt{2}}-\frac{i}{\sqrt{\frac{2i}{3}+i}}\right)}{\frac{1}{2}-\frac{(1+i)\exp\left(-\frac{\pi\,2\,i}{4\times 3}\right)}{\left(2\sqrt{2}\right)\sqrt{\frac{2i}{3}+i}}}+76\,i=\left(76\,i\exp\left(-\frac{i\pi}{6}\right)+76\,i^2\exp\left(-\frac{i\pi}{6}\right)-\right.$$

$$89\sqrt{x}\sum_{k=0}^{\infty}\frac{1}{k!}(-1)^{1+k}x^{-k}\left(2\,i\,(2-x)^k\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)-\right.$$

$$\left.(1+i)\left(\frac{5\,i}{3}-x\right)^k\exp\left(-\frac{i\pi}{6}\right)\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\right)\left(-\frac{1}{2}\right)_k-$$

$$76\,i\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}^2$$

$$\left.\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}(2-x)^{k_1}\left(\frac{5i}{3}-x\right)^{k_2}x^{-k_1-k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}}{k_1!\,k_2!}\right)\Big/$$

$$\left(\exp\left(-\frac{i\pi}{6}\right)+i\exp\left(-\frac{i\pi}{6}\right)-\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg\left(\frac{5i}{3}-x\right)}{2\pi}\right\rfloor\right)\right.$$

$$\left.\sqrt{x}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}(2-x)^{k_1}\left(\frac{5i}{3}-x\right)^{k_2}x^{-k_1-k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}}{k_1!\,k_2!}\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

**Observations**

*From:*
*https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA*

*Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.*

*Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.*

*Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.*
*In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.*

*From Wikipedia*

*In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field $\phi$ and a Dirac field $\psi$. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.*

*Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson:*

*125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV*

*Note that:*

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64 g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276 e^{-\pi\sqrt{22}} - \cdots, \\ 64 g_{22}^{-24} &= 4096 e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372 e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

*Thence:*

$$64 g_{22}^{-24} = 4096 e^{-\pi\sqrt{22}} + \cdots$$

*And*

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372 e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

*That are connected with 64, 128, 256, 512, 1024 and 4096 = $64^2$*

*(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)*

*All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.*

*In mathematics, the Fibonacci numbers, commonly denoted $F_n$, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.*
*Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences*

*The beginning of the sequence is thus:*

*0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...*

*The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.*

*The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.*

*The sequence of Lucas numbers is:*

*2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……*

*All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.*

*A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:*

*2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).*

*In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals*

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **$f_0$(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

**We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.**

**References**


**Some definite integrals** – *Srinivasa Ramanujan* - Messenger of Mathematics, XLIV, 1915, 10 – 18

**Some definite integrals** – *Srinivasa Ramanujan*
Journal of the Indian Mathematical Society, XI, 1919, 81 – 87