On the mathematical connections between ϕ, $\zeta(2)$, some Ramanujan equations and various parameters of String Theory Mathematics and Particle Physics.

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Abstract

In this paper we have described and analyzed some Ramanujan equations. Furthermore, we have obtained several mathematical connections between ϕ, $\zeta(2)$, and various parameters of String Theory Mathematics and Particle Physics.

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https://link.springer.com/chapter/10.1007/978-81-322-0767-2_12

https://www.sciencephoto.com/media/228058/view/indian-mathematician-srinivasa-ramanujan

We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation

From

Simple geodesics and Weil-Petersson volumes of moduli spaces of bordered Riemann surfaces - *Maryam Mirzakhani*

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Oblatum 19-VII-2005 & 17-VII-2006 - Published online: 12 October 2006 – © Springer-Verlag 2006

6. Polynomial behavior of the Weil-Petersson volume

In this section we use the recursive formula for the volumes of moduli spaces stated in Sect. 5 to establish the following result:

Theorem 6.1. *The function $V_{g,n}(L)$ is a polynomial in $L_1^2, \ldots, L_n^2$, namely:*

$$V_{g,n}(L) = \sum_{\substack{\alpha \\ |\alpha| \leq 3g-3+n}} C_\alpha \cdot L^{2\alpha},$$

where $C_\alpha > 0$ lies in $\pi^{6g-6+2n-|2\alpha|} \cdot \mathbb{Q}$.

Calculation of $V_{1,1}(L)$. First, we elaborate on the main idea of the calculation of the volume polynomials through an example when $g = n = 1$. In this case, Theorem 4.2 for a hyperbolic surface of genus one with one geodesic boundary component implies that for any $X \in \mathcal{T}(S_{1,1}, L)$

$$\sum_{\gamma} \mathcal{D}(L, \ell_\gamma(X), \ell_\gamma(X)) = L,$$

where the sum is over all non-peripheral simple closed curves γ on $S_{1,1}$. By Lemma 3.2, we have

$$\frac{\partial}{\partial L} \mathcal{D}(L, x, x) = \frac{1}{1 + e^{x - \frac{L}{2}}} + \frac{1}{1 + e^{x + \frac{L}{2}}}.$$

Integrating over $\mathcal{M}_{1,1}(L)$, as in the calculation of $\mathrm{Vol}(\mathcal{M}_{1,1})$ in the Introduction, we get:

$$L \cdot V_{1,1}(L) = \int_0^\infty x \; \mathcal{D}(L, x, x) \; dx.$$

So we have

$$\frac{\partial}{\partial L} L \cdot V_{1,1}(L) = \int_0^\infty x \cdot \left(\frac{1}{1 + e^{x + \frac{L}{2}}} + \frac{1}{1 + e^{x - \frac{L}{2}}} \right) dx.$$

By setting $y_1 = x + L/2$ and $y_2 = x - L/2$, we get

$$\int_0^\infty x \cdot \left(\frac{1}{1+e^{x+\frac{L}{2}}} + \frac{1}{1+e^{x-\frac{L}{2}}} \right) dx$$

$$= \int_{L/2}^\infty \frac{y_1 - L/2}{1+e^{y_1}} dy_1 + \int_{-L/2}^\infty \frac{y_2 + L/2}{1+e^{y_2}} dy_2$$

$$= 2\int_0^\infty \frac{y}{1+e^y} dy + \int_0^{L/2} \frac{y - L/2}{1+e^y} dy + \int_0^{L/2} \frac{y + L/2}{1+e^y} dy$$

$$= \frac{\pi^2}{6} + \int_0^{L/2} (y - L/2) \left(\frac{1}{1+e^y} + \frac{1}{1+e^{-y}} \right) dy = \frac{\pi^2}{6} + \frac{L^2}{8}.$$

Since we have

$$\frac{1}{1+e^y} + \frac{1}{1+e^{-y}} = 1.$$

Therefore, we have:

$$V_{1,1}(L) = \frac{L^2}{24} + \frac{\pi^2}{6}. \quad (6.1)$$

If L = 8, we obtain:

[((64/24+(pi^2)/6))]

Input:

$$\frac{64}{24} + \frac{\pi^2}{6}$$

Result:

$$\frac{8}{3} + \frac{\pi^2}{6}$$

Decimal approximation:

4.311600733514893103139081833312691855885616567873465104402...

4.3116007335….

Property:

$\frac{8}{3} + \frac{\pi^2}{6}$ is a transcendental number

Alternate form:

$$\frac{1}{6}(16 + \pi^2)$$

Alternative representations:

$$\frac{64}{24} + \frac{\pi^2}{6} = \frac{64}{24} + \frac{1}{6}(180\,°)^2$$

$$\frac{64}{24} + \frac{\pi^2}{6} = \frac{64}{24} + \zeta(2)$$

$$\frac{64}{24} + \frac{\pi^2}{6} = \frac{64}{24} + \frac{1}{6}(-i\log(-1))^2$$

Series representations:

$$\frac{64}{24} + \frac{\pi^2}{6} = \frac{8}{3} + \sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$\frac{64}{24} + \frac{\pi^2}{6} = \frac{8}{3} - 2\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$

$$\frac{64}{24} + \frac{\pi^2}{6} = \frac{8}{3} + \frac{4}{3}\sum_{k=0}^{\infty} \frac{1}{(1+2k)^2}$$

Integral representations:

$$\frac{64}{24} + \frac{\pi^2}{6} = \frac{8}{3} + \frac{8}{3}\left(\int_0^1 \sqrt{1-t^2}\, dt\right)^2$$

$$\frac{64}{24} + \frac{\pi^2}{6} = \frac{8}{3} + \frac{2}{3}\left(\int_0^\infty \frac{1}{1+t^2}\, dt\right)^2$$

$$\frac{64}{24}+\frac{\pi^2}{6}=\frac{8}{3}+\frac{2}{3}\left(\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

Raising this expression to the fourth power, multiplying by 1/3, adding 11 and subtracting the conjugate of the golden ratio, we obtain:

1/3 [((64/24+(pi^2)/6))]^4 + 11 - 1/golden ratio

Input:

$$\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4+11-\frac{1}{\phi}$$

ϕ is the golden ratio

Result:

$$-\frac{1}{\phi}+11+\frac{1}{3}\left(\frac{8}{3}+\frac{\pi^2}{6}\right)^4$$

Decimal approximation:

125.5767709856754789129114809252751927477015848403346660096...

125.576770985….

Property:

$11-\frac{1}{\phi}+\frac{1}{3}\left(\frac{8}{3}+\frac{\pi^2}{6}\right)^4$ is a transcendental number

Alternate forms:

$$\frac{110\,248-1944\sqrt{5}+16\,384\,\pi^2+1536\,\pi^4+64\,\pi^6+\pi^8}{3888}$$

$$-\frac{1}{\phi}+11+\frac{(16+\pi^2)^4}{3888}$$

$$11-\frac{2}{1+\sqrt{5}}+\frac{1}{3}\left(\frac{8}{3}+\frac{\pi^2}{6}\right)^4$$

Alternative representations:

$$\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4+11-\frac{1}{\phi}=11--\frac{1}{2\cos(216°)}+\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4$$

$$\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4+11-\frac{1}{\phi}=11-\frac{1}{2\cos\left(\frac{\pi}{5}\right)}+\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4$$

$$\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4+11-\frac{1}{\phi}=11--\frac{1}{2\cos(216°)}+\frac{1}{3}\left(\frac{64}{24}+\frac{1}{6}(180°)^2\right)^4$$

Series representations:

$$\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4+11-\frac{1}{\phi}=11-\frac{1}{\phi}+\frac{1}{3}\left(\frac{8}{3}+\sum_{k=1}^{\infty}\frac{1}{k^2}\right)^4$$

$$\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4+11-\frac{1}{\phi}=11-\frac{1}{\phi}+\frac{256}{243}\left(2+\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}\right)^4$$

$$\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4+11-\frac{1}{\phi}=11-\frac{1}{\phi}+\frac{1}{3}\left(\frac{8}{3}-2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}\right)^4$$

Integral representations:

$$\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4+11-\frac{1}{\phi}=11-\frac{1}{\phi}+\frac{16}{243}\left(4+\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^2\right)^4$$

$$\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4+11-\frac{1}{\phi}=11-\frac{1}{\phi}+\frac{16}{243}\left(4+\left(\int_0^{1}\frac{1}{\sqrt{1-t^2}}\,dt\right)^2\right)^4$$

$$\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4+11-\frac{1}{\phi}=11-\frac{1}{\phi}+\frac{4096}{243}\left(1+\left(\int_0^{1}\sqrt{1-t^2}\,dt\right)^2\right)^4$$

Raising this expression to the fourth power, multiplying by 1/3, adding 21, adding the square of golden ratio and 1, we obtain:

1/3 [((64/24+(pi^2)/6))]^4 + 21 + golden ratio^2+1

Input:

$$\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4+21+\phi^2+1$$

ϕ is the golden ratio

Result:

$$\phi^2+22+\frac{1}{3}\left(\frac{8}{3}+\frac{\pi^2}{6}\right)^4$$

Decimal approximation:

139.8128389631752686093206545940064689831422031999461917339...

139.8128389631…..

Property:

$22+\phi^2+\frac{1}{3}\left(\frac{8}{3}+\frac{\pi^2}{6}\right)^4$ is a transcendental number

Alternate forms:

$$\frac{156\,904+1944\sqrt{5}+16\,384\,\pi^2+1536\,\pi^4+64\,\pi^6+\pi^8}{3888}$$

$$\phi^2+22+\frac{(16+\pi^2)^4}{3888}$$

$$\phi^2+\frac{151\,072+16\,384\,\pi^2+1536\,\pi^4+64\,\pi^6+\pi^8}{3888}$$

Alternative representations:

$$\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4+21+\phi^2+1=22+(-2\cos(216\,°))^2+\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4$$

$$\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4+21+\phi^2+1=22+\left(2\cos\left(\frac{\pi}{5}\right)\right)^2+\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4$$

$$\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4+21+\phi^2+1=22+(-2\cos(216\,°))^2+\frac{1}{3}\left(\frac{64}{24}+\frac{1}{6}\,(180\,°)^2\right)^4$$

Series representations:

$$\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4+21+\phi^2+1=22+\phi^2+\frac{1}{3}\left(\frac{8}{3}+\sum_{k=1}^{\infty}\frac{1}{k^2}\right)^4$$

$$\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4+21+\phi^2+1=22+\phi^2+\frac{256}{243}\left(2+\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}\right)^4$$

$$\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4+21+\phi^2+1=22+\phi^2+\frac{1}{3}\left(\frac{8}{3}-2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}\right)^4$$

Integral representations:

$$\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4+21+\phi^2+1=22+\phi^2+\frac{16}{243}\left(4+\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^2\right)^4$$

$$\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4+21+\phi^2+1=22+\phi^2+\frac{16}{243}\left(4+\left(\int_0^{1}\frac{1}{\sqrt{1-t^2}}\,dt\right)^2\right)^4$$

$$\frac{1}{3}\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^4+21+\phi^2+1=22+\phi^2+\frac{4096}{243}\left(1+\left(\int_0^{1}\sqrt{1-t^2}\,dt\right)^2\right)^4$$

Raising this expression to the fifth power, adding 256, subtracting 16 and subtracting 1, (note that √256 = 16) we get:

[((64/24+(pi^2)/6))^5+256-16]-1

Input:

$$\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1$$

Result:

$$239+\left(\frac{8}{3}+\frac{\pi^2}{6}\right)^5$$

Decimal approximation:

1729.0220168745124950830306769568459505247279215293387077 22...

1729.02201687…

Property:

$239+\left(\frac{8}{3}+\frac{\pi^2}{6}\right)^5$ is a transcendental number

Alternate forms:

$$\frac{2\,907\,040+327\,680\,\pi^2+40\,960\,\pi^4+2560\,\pi^6+80\,\pi^8+\pi^{10}}{7776}$$

$$\frac{90\,845}{243}+\frac{10\,240\,\pi^2}{243}+\frac{1280\,\pi^4}{243}+\frac{80\,\pi^6}{243}+\frac{5\,\pi^8}{486}+\frac{\pi^{10}}{7776}$$

Alternative representations:

$$\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1=239+\left(\frac{64}{24}+\zeta(2)\right)^5$$

$$\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1=239+\left(\frac{64}{24}+\frac{1}{6}\,(180\,°)^2\right)^5$$

$$\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1=239+\left(\frac{64}{24}+\frac{1}{6}\,(-i\log(-1))^2\right)^5$$

Series representations:

$$\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1=239+\left(\frac{8}{3}+\sum_{k=1}^{\infty}\frac{1}{k^2}\right)^5$$

$$\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1=239+\left(\frac{8}{3}-2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}\right)^5$$

$$\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1=239+\left(\frac{8}{3}+\frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2\,k)^2}\right)^5$$

Integral representations:

$$\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1=239+\left(\frac{8}{3}+\frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2\right)^5$$

$$\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1=239+\left(\frac{8}{3}+\frac{2}{3}\left(\int_0^\infty\frac{1}{1+t^2}\,dt\right)^2\right)^5$$

$$\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1=239+\left(\frac{8}{3}+\frac{2}{3}\left(\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt\right)^2\right)^5$$

Performing the 15th root, we obtain:

((([((64/24+(pi^2)/6))^5+256-16]-1)))^1/15

Input:

$$\sqrt[15]{\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1}$$

Exact result:

$$\sqrt[15]{239+\left(\frac{8}{3}+\frac{\pi^2}{6}\right)^5}$$

Decimal approximation:

1.643816624216572614601097529396669902370674257653073517851...

1.643816624216….

Property:

$\sqrt[15]{239+\left(\frac{8}{3}+\frac{\pi^2}{6}\right)^5}$ is a transcendental number

Alternate form:

$$\frac{\sqrt[15]{2907040+327680\,\pi^2+40960\,\pi^4+2560\,\pi^6+80\,\pi^8+\pi^{10}}}{\sqrt[3]{6}}$$

All 15th roots of 239 + (8/3 + π^2/6)^5:

$$\sqrt[15]{239+\left(\frac{8}{3}+\frac{\pi^2}{6}\right)^5}\ e^0 \approx 1.64382 \text{ (real, principal root)}$$

$$\sqrt[15]{239+\left(\frac{8}{3}+\frac{\pi^2}{6}\right)^5}\ e^{(2 i \pi)/15} \approx 1.5017+0.6686 i$$

$$\sqrt[15]{239+\left(\frac{8}{3}+\frac{\pi^2}{6}\right)^5}\ e^{(4 i \pi)/15} \approx 1.0999+1.2216 i$$

$$\sqrt[15]{239+\left(\frac{8}{3}+\frac{\pi^2}{6}\right)^5}\ e^{(2 i \pi)/5} \approx 0.5080+1.5634 i$$

$$\sqrt[15]{239+\left(\frac{8}{3}+\frac{\pi^2}{6}\right)^5}\ e^{(8 i \pi)/15} \approx -0.17183+1.63481 i$$

Alternative representations:

$$\sqrt[15]{\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1} = \sqrt[15]{239+\left(\frac{64}{24}+\zeta(2)\right)^5}$$

$$\sqrt[15]{\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1} = \sqrt[15]{239+\left(\frac{64}{24}+\frac{1}{6}\,(180\,°)^2\right)^5}$$

$$\sqrt[15]{\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1} = \sqrt[15]{239+\left(\frac{64}{24}+\frac{1}{6}\cos^{-1}(-1)^2\right)^5}$$

Series representations:

$$\sqrt[15]{\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1} = \sqrt[15]{239+\left(\frac{8}{3}+\sum_{k=1}^{\infty}\frac{1}{k^2}\right)^5}$$

$$\sqrt[15]{\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1} = \sqrt[15]{239+\left(\frac{8}{3}-2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}\right)^5}$$

$$\sqrt[15]{\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1} = \sqrt[15]{239+\left(\frac{8}{3}+\frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}\right)^5}$$

Integral representations:

$$\sqrt[15]{\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1} = \sqrt[15]{239+\left(\frac{8}{3}+\frac{2}{3}\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^2\right)^5}$$

$$\sqrt[15]{\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1} = \sqrt[15]{239+\left(\frac{8}{3}+\frac{8}{3}\left(\int_0^{1}\sqrt{1-t^2}\,dt\right)^2\right)^5}$$

$$\sqrt[15]{\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1} = \sqrt[15]{239+\left(\frac{8}{3}+\frac{2}{3}\left(\int_0^{\infty}\frac{\sin(t)}{t}\,dt\right)^2\right)^5}$$

Performing the 15th root and subtracting $(21+5)/10^3$, we obtain:

((([((64/24+(pi^2)/6))^5+256-16]-1)))^1/15 - (21+5)1/10^3

Input:

$$\sqrt[15]{\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1} - (21+5)\times\frac{1}{10^3}$$

Exact result:

$$\sqrt[15]{239+\left(\frac{8}{3}+\frac{\pi^2}{6}\right)^5} - \frac{13}{500}$$

Decimal approximation:

1.617816624216572614601097529396669902370674257653073517851...

1.6178166242165….

Property:

$$-\frac{13}{500}+\sqrt[15]{239+\left(\frac{8}{3}+\frac{\pi^2}{6}\right)^5} \text{ is a transcendental number}$$

Alternate forms:

$$\frac{250\times 6^{2/3}\sqrt[15]{2907040+327680\pi^2+40960\pi^4+2560\pi^6+80\pi^8+\pi^{10}}-39}{1500}$$

$$\frac{500\sqrt[15]{2907040+327680\pi^2+40960\pi^4+2560\pi^6+80\pi^8+\pi^{10}}-13\sqrt[3]{6}}{500\sqrt[3]{6}}$$

Alternative representations:

$$\sqrt[15]{\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1}-\frac{21+5}{10^3}=-\frac{26}{10^3}+\sqrt[15]{239+\left(\frac{64}{24}+\zeta(2)\right)^5}$$

$$\sqrt[15]{\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1}-\frac{21+5}{10^3}=-\frac{26}{10^3}+\sqrt[15]{239+\left(\frac{64}{24}+\frac{1}{6}(180\,°)^2\right)^5}$$

$$\sqrt[15]{\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1}-\frac{21+5}{10^3}=-\frac{26}{10^3}+\sqrt[15]{239+\left(\frac{64}{24}+\frac{1}{6}\cos^{-1}(-1)^2\right)^5}$$

Series representations:

$$\sqrt[15]{\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1}-\frac{21+5}{10^3}=-\frac{13}{500}+\sqrt[15]{239+\left(\frac{8}{3}+\sum_{k=1}^{\infty}\frac{1}{k^2}\right)^5}$$

$$\sqrt[15]{\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1}-\frac{21+5}{10^3}=-\frac{13}{500}+\sqrt[15]{239+\left(\frac{8}{3}-2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}\right)^5}$$

$$\sqrt[15]{\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1}-\frac{21+5}{10^3}=-\frac{13}{500}+\sqrt[15]{239+\left(\frac{8}{3}+\frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}\right)^5}$$

Integral representations:

$$\sqrt[15]{\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1}-\frac{21+5}{10^3}=-\frac{13}{500}+\sqrt[15]{239+\left(\frac{8}{3}+\frac{2}{3}\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^2\right)^5}$$

$$\sqrt[15]{\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1}-\frac{21+5}{10^3}=$$
$$-\frac{13}{500}+\sqrt[15]{239+\left(\frac{8}{3}+\frac{8}{3}\left(\int_0^1 \sqrt{1-t^2}\,dt\right)^2\right)^5}$$

$$\sqrt[15]{\left(\left(\frac{64}{24}+\frac{\pi^2}{6}\right)^5+256-16\right)-1}-\frac{21+5}{10^3}=-\frac{13}{500}+\sqrt[15]{239+\left(\frac{8}{3}+\frac{2}{3}\left(\int_0^\infty \frac{\sin(t)}{t}\,dt\right)^2\right)^5}$$

Now, we have that:

Lemma 6.2. *For any $k \geq 0$, we have*

$$\frac{F_{2k+1}(t)}{(2k+1)!}=\sum_{i=0}^{k+1}\zeta(2i)\ (2^{2i+1}-4)\frac{t^{2k+2-2i}}{(2k+2-2i)!}.$$

Therefore, $F_{2k+1}(t)$ is a polynomial in t^2 of degree $k+1$, and the coefficient of $t^{2k+2-2i}$ lies in $\pi^{2i}\cdot\mathbb{Q}_{>0}$.

$$\frac{F_{2k+1}(t)}{(2k+1)!}=\frac{1}{(2k+1)!}\int_0^\infty x^{2k+1}\cdot\left(\frac{1}{1+e^{(x+t)/2}}+\frac{1}{1+e^{(x-t)/2}}\right)dx$$
$$=\frac{t^{2k+2}}{(2k+2)!}+\sum_{i=1}^{k+1}\frac{t^{2k+2-2i}}{(2k+2-2i)!}\ \zeta(2i)\ (2^{2i+1}-4).$$

□

For k = 0 and t = 2, we obtain:

(2^2) / 2! + sum((((((2^(2-2i)*zeta(2i) (2^(2i+1)-4)))/(2-2i)!)))), i = 1..1

Input interpretation:

$$\frac{2^2}{2!} + \sum_{i=1}^{1} \frac{2^{2-2i}\,\zeta(2i)\left(2^{2i+1}-4\right)}{(2-2i)!}$$

$n!$ is the factorial function

$\zeta(s)$ is the Riemann zeta function

Result:

$$2 + \frac{2\pi^2}{3} \approx 8.57974$$

8.57974

Alternate form:

$$\frac{2}{3}\left(3+\pi^2\right)$$

For k = 1, we obtain:

(2^4) / 4! + sum((((((2^(4-2i)*zeta(2i) (2^(2i+1)-4)))/(4-2i)!)))), i = 1..1

Input interpretation:

$$\frac{2^4}{4!} + \sum_{i=1}^{1} \frac{2^{4-2i}\,\zeta(2i)\left(2^{2i+1}-4\right)}{(4-2i)!}$$

$n!$ is the factorial function

$\zeta(s)$ is the Riemann zeta function

Result:

$$\frac{2}{3} + \frac{4\pi^2}{3} \approx 13.8261$$

13.8261

Alternate form:

$$\frac{2}{3}\left(1+2\pi^2\right)$$

From the above expression, performing the root of 2e and subtracting $3/10^3$, we obtain:

((((2^4) / 4! + sum((((((2^(4-2i)*zeta(2i) (2^(2i+1)-4)))/(4-2i)!)))), i = 1..1)))^1/(2e) - 3/10^3

Input interpretation:

$$\sqrt[2e]{\frac{2^4}{4!}+\sum_{i=1}^{1}\frac{2^{4-2i}\,\zeta(2i)\left(2^{2i+1}-4\right)}{(4-2i)!}}-\frac{3}{10^3}$$

$n!$ is the factorial function

$\zeta(s)$ is the Riemann zeta function

Result:

$$\sqrt[2e]{\frac{2}{3}+\frac{4\pi^2}{3}}-\frac{3}{1000}\approx 1.61814$$

1.61814

Alternate forms:

$$\frac{1000\sqrt[2e]{\frac{2}{3}+\frac{4\pi^2}{3}}-3}{1000}$$

$$\frac{125\times 2^{3+1/(2e)}\sqrt[2e]{\frac{1}{3}\left(1+2\pi^2\right)}-3}{1000}$$

Raising to the second power, the above expression, subtracting 55 and adding π, we obtain:

((((2^4) / 4! + sum((((((2^(4-2i)*zeta(2i) (2^(2i+1)-4)))/(4-2i)!)))), i = 1..1)))^2-55+Pi

Input interpretation:

$$\left(\frac{2^4}{4!}+\sum_{i=1}^{1}\frac{2^{4-2i}\,\zeta(2i)\left(2^{2i+1}-4\right)}{(4-2i)!}\right)^2-55+\pi$$

$n!$ is the factorial function

$\zeta(s)$ is the Riemann zeta function

Result:

$$-55+\pi+\left(\frac{2}{3}+\frac{4\pi^2}{3}\right)^2 \approx 139.304$$

139.304

Alternate forms:

$$-55+\pi+\frac{4}{9}(1+2\pi^2)^2$$

$$-\frac{491}{9}+\pi+\frac{16\pi^2}{9}+\frac{16\pi^4}{9}$$

$$\frac{1}{9}(-491+9\pi+16\pi^2+16\pi^4)$$

And subtracting 55 and 13, and adding 2, we obtain:

((((2^4) / 4! + sum((((((2^(4-2i)*zeta(2i) (2^(2i+1)-4)))/(4-2i)!)))), i = 1..1)))^2-55-13+2

Input interpretation:

$$\left(\frac{2^4}{4!}+\sum_{i=1}^{1}\frac{2^{4-2i}\,\zeta(2i)\,(2^{2i+1}-4)}{(4-2i)!}\right)^2-55-13+2$$

$n!$ is the factorial function

$\zeta(s)$ is the Riemann zeta function

Result:

$$\left(\frac{2}{3}+\frac{4\pi^2}{3}\right)^2-66 \approx 125.162$$

125.162

Alternate forms:

$$\frac{4}{9}(1+2\pi^2)^2-66$$

$$\frac{2}{9}(-295+8\pi^2+8\pi^4)$$

$$-\frac{590}{9}+\frac{16\pi^2}{9}+\frac{16\pi^4}{9}$$

From this expression, subtracting 55 and 8, multiplying by 27*1/2 and, in conclusion, subtracting the golden ratio, we obtain:

27*1/2(((((((2^4) / 4! + sum((((((2^(4-2i)*zeta(2i) (2^(2i+1)-4)))/(4-2i)!)))), i = 1..1)))^2-55-8)))-golden ratio

Input interpretation:

$$27\times\frac{1}{2}\left(\left(\frac{2^4}{4!}+\sum_{i=1}^{1}\frac{2^{4-2i}\,\zeta(2i)\left(2^{2i+1}-4\right)}{(4-2i)!}\right)^2-55-8\right)-\phi$$

$n!$ is the factorial function

$\zeta(s)$ is the Riemann zeta function

ϕ is the golden ratio

Result:

$$\frac{27}{2}\left(\left(\frac{2}{3}+\frac{4\pi^2}{3}\right)^2-63\right)-\phi\approx 1728.57$$

1728.57

Alternate forms:

$$-\phi-\frac{1689}{2}+24\left(\pi^2+\pi^4\right)$$

$$-\phi-\frac{1689}{2}+24\pi^2+24\pi^4$$

$$\frac{1}{2}\left(-2\phi-1689+48\pi^2+48\pi^4\right)$$

And performing the 15th root, we have:

((((27*1/2(((((((2^4) / 4! + sum((((((2^(4-2i)*zeta(2i) (2^(2i+1)-4)))/(4-2i)!)))), i = 1..1)))^2-55-8)))-golden ratio))))^1/15

Input interpretation:

$$\sqrt[15]{27\times\frac{1}{2}\left(\left(\frac{2^4}{4!}+\sum_{i=1}^{1}\frac{2^{4-2i}\,\zeta(2i)\left(2^{2i+1}-4\right)}{(4-2i)!}\right)^2-55-8\right)-\phi}$$

$n!$ is the factorial function

$\zeta(s)$ is the Riemann zeta function

ϕ is the golden ratio

Result:

$$\sqrt[15]{\frac{27}{2}\left(\left(\frac{2}{3}+\frac{4\pi^2}{3}\right)^2-63\right)-\phi} \approx 1.64379$$

1.64379

Alternate forms:

$$\sqrt[15]{-\phi-\frac{1689}{2}+24\left(\pi^2+\pi^4\right)}$$

$$\sqrt[15]{-\phi-\frac{1689}{2}+24\pi^2+24\pi^4}$$

$$\sqrt[15]{\frac{1}{2}\left(3\left(-563+16\pi^2+16\pi^4\right)-2\phi\right)}$$

Now, we have that:

Lemma 3.1. *The functions $\mathcal{D}$ and $\mathcal{R}$ are given by*

$$\mathcal{D}(x,y,z) = 2\ \log\left(\frac{e^{\frac{x}{2}}+e^{\frac{y+z}{2}}}{e^{\frac{-x}{2}}+e^{\frac{y+z}{2}}}\right), \tag{3.1}$$

and

$$\mathcal{R}(x,y,z) = x - \log\left(\frac{\cosh\left(\frac{y}{2}\right)+\cosh\left(\frac{x+z}{2}\right)}{\cosh\left(\frac{y}{2}\right)+\cosh\left(\frac{x-z}{2}\right)}\right). \tag{3.2}$$

For x = 2, y = 3 and z = 5, we obtain:

2 ln(((e+e^4)/(e^(-1)+e^4)))

Input:

$$2\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)$$

log(x) is the natural logarithm

Decimal approximation:

0.083744006169247980285018464244095845653978939727576587189...

0.083744006…

Alternate forms:

$$2\log\left(\frac{e(e+e^4)}{1+e^5}\right)$$

$$2\left(\log(e+e^4)-\log\left(\frac{1}{e}+e^4\right)\right)$$

$$4+2\log(1-e+e^2)-2\log(1-e+e^2-e^3+e^4)$$

Alternative representations:

$$2\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)=2\log_e\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)$$

$$2\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)=2\log(a)\log_a\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)$$

$$2\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)=-2\,\text{Li}_1\left(1-\frac{e+e^4}{\frac{1}{e}+e^4}\right)$$

Series representations:

$$2\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)=-2\sum_{k=1}^{\infty}\frac{\left(1-\frac{e(e+e^4)}{1+e^5}\right)^k}{k}$$

$$2\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)=4i\pi\left\lfloor\frac{\arg\left(\frac{e+e^4}{\frac{1}{e}+e^4}-x\right)}{2\pi}\right\rfloor+2\log(x)-2\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{e(e+e^4)}{1+e^5}-x\right)^k x^{-k}}{k}\quad\text{for } x<0$$

$$2\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)=4i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+2\log(z_0)-2\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{e(e+e^4)}{1+e^5}-z_0\right)^k z_0^{-k}}{k}$$

Integral representations:

$$2\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right) = 2\int_1^{\frac{e(e+e^4)}{1+e^5}} \frac{1}{t}\,dt$$

$$2\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right) = -\frac{i}{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-1+\frac{e(e+e^4)}{1+e^5}\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

2-ln((((cosh(3/2)+cosh(7/3))/(cosh(3/2)+cosh(-3/2)))))

Input:

$$2-\log\left(\frac{\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)}{\cosh\left(\frac{3}{2}\right)+\cosh\left(-\frac{3}{2}\right)}\right)$$

cosh(x) is the hyperbolic cosine function

log(x) is the natural logarithm

Exact result:

$$2-\log\left(\frac{1}{2}\left(\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)\right)\operatorname{sech}\left(\frac{3}{2}\right)\right)$$

sech(x) is the hyperbolic secant function

Decimal approximation:

1.526109777220024207286759335081623699469682143162123428905...

1.52610977722…

Alternate forms:

$$2+\log\left(\frac{2}{1+\cosh\left(\frac{7}{3}\right)\operatorname{sech}\left(\frac{3}{2}\right)}\right)$$

$$2-\log\left(\frac{1}{2}\left(1+\cosh\left(\frac{7}{3}\right)\operatorname{sech}\left(\frac{3}{2}\right)\right)\right)$$

$$2+\log(2)-\log\left(\left(\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)\right)\operatorname{sech}\left(\frac{3}{2}\right)\right)$$

Alternative representations:

$$2-\log\left(\frac{\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)}{\cosh\left(\frac{3}{2}\right)+\cosh\left(-\frac{3}{2}\right)}\right)=2-\log\left(\frac{\frac{1}{2}\left(\frac{1}{e^{3/2}}+e^{3/2}\right)+\frac{1}{2}\left(\frac{1}{e^{7/3}}+e^{7/3}\right)}{\frac{1}{e^{3/2}}+e^{3/2}}\right)$$

$$2-\log\left(\frac{\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)}{\cosh\left(\frac{3}{2}\right)+\cosh\left(-\frac{3}{2}\right)}\right)=2-\log_e\left(\frac{\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)}{\cosh\left(-\frac{3}{2}\right)+\cosh\left(\frac{3}{2}\right)}\right)$$

$$2-\log\left(\frac{\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)}{\cosh\left(\frac{3}{2}\right)+\cosh\left(-\frac{3}{2}\right)}\right)=2-\log(a)\log_a\left(\frac{\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)}{\cosh\left(-\frac{3}{2}\right)+\cosh\left(\frac{3}{2}\right)}\right)$$

Series representation:

$$2-\log\left(\frac{\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)}{\cosh\left(\frac{3}{2}\right)+\cosh\left(-\frac{3}{2}\right)}\right)=2+\sum_{k=1}^{\infty}\frac{\left(\frac{1}{2}\left(1-\cosh\left(\frac{7}{3}\right)\operatorname{sech}\left(\frac{3}{2}\right)\right)\right)^k}{k}$$

Integral representations:

$$2-\log\left(\frac{\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)}{\cosh\left(\frac{3}{2}\right)+\cosh\left(-\frac{3}{2}\right)}\right)=2-\int_1^{\frac{1}{2}\left(1+\cosh\left(\frac{7}{3}\right)\operatorname{sech}\left(\frac{3}{2}\right)\right)}\frac{1}{t}\,dt$$

$$2-\log\left(\frac{\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)}{\cosh\left(\frac{3}{2}\right)+\cosh\left(-\frac{3}{2}\right)}\right)=$$
$$2-\log\left(\frac{\left(\int_0^{\infty}\frac{t^{(3i)/\pi}}{1+t^2}\,dt\right)\left(2+\int_0^1\left(\frac{3}{2}\sinh\left(\frac{3t}{2}\right)+\frac{7}{3}\sinh\left(\frac{7t}{3}\right)\right)dt\right)}{\pi}\right)$$

$$2-\log\left(\frac{\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)}{\cosh\left(\frac{3}{2}\right)+\cosh\left(-\frac{3}{2}\right)}\right)=$$
$$2-\log\left(-\frac{i\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{9/(16s)+s}+e^{49/(36s)+s}}{\sqrt{s}}\,ds\right)\int_0^{\infty}\frac{t^{(3i)/\pi}}{1+t^2}\,dt}{2\pi^{3/2}}\right)\quad \text{for } \gamma>0$$

From the sum of the two results, adding $8/10^3$, we obtain:

(((2-ln((((cosh(3/2)+cosh(7/3))/(cosh(3/2)+cosh(-3/2)))))))) + (((2 ln(((e+e^4)/(e^(-1)+e^4)))))) + 8/10^3

Input:

$$\left(2-\log\left(\frac{\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)}{\cosh\left(\frac{3}{2}\right)+\cosh\left(-\frac{3}{2}\right)}\right)\right)+2\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)+\frac{8}{10^3}$$

$\cosh(x)$ is the hyperbolic cosine function

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{251}{125}+2\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)-\log\left(\frac{1}{2}\left(\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)\right)\operatorname{sech}\left(\frac{3}{2}\right)\right)$$

$\operatorname{sech}(x)$ is the hyperbolic secant function

Decimal approximation:

1.617853783389272187571777799325719545123661082889700016095...

1.617853783389...

Alternate forms:

$$\frac{501}{125}+\log(2)+2\log\left(\frac{e+e^4}{1+e^5}\right)-\log\left(1+\cosh\left(\frac{7}{3}\right)\operatorname{sech}\left(\frac{3}{2}\right)\right)$$

$$\frac{251}{125}+2\log\left(\frac{e\left(e+e^4\right)}{1+e^5}\right)-\log\left(\frac{1}{2}\left(1+\cosh\left(\frac{7}{3}\right)\operatorname{sech}\left(\frac{3}{2}\right)\right)\right)$$

$$\frac{1}{125}\left(251+250\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)-125\log\left(\frac{1}{2}\left(\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)\right)\operatorname{sech}\left(\frac{3}{2}\right)\right)\right)$$

Alternative representations:

$$\left(2-\log\left(\frac{\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)}{\cosh\left(\frac{3}{2}\right)+\cosh\left(-\frac{3}{2}\right)}\right)\right)+2\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)+\frac{8}{10^3}=$$
$$2-\log\left(\frac{\frac{1}{2}\left(\frac{1}{e^{3/2}}+e^{3/2}\right)+\frac{1}{2}\left(\frac{1}{e^{7/3}}+e^{7/3}\right)}{\frac{1}{e^{3/2}}+e^{3/2}}\right)+2\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)+\frac{8}{10^3}$$

$$\left(2-\log\left(\frac{\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)}{\cosh\left(\frac{3}{2}\right)+\cosh\left(-\frac{3}{2}\right)}\right)\right)+2\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)+\frac{8}{10^3}=$$
$$2-\log(a)\log_a\left(\frac{\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)}{\cosh\left(-\frac{3}{2}\right)+\cosh\left(\frac{3}{2}\right)}\right)+2\log(a)\log_a\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)+\frac{8}{10^3}$$

$$\left(2-\log\left(\frac{\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)}{\cosh\left(\frac{3}{2}\right)+\cosh\left(-\frac{3}{2}\right)}\right)\right)+2\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)+\frac{8}{10^3}=$$
$$2-\log_e\left(\frac{\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)}{\cosh\left(-\frac{3}{2}\right)+\cosh\left(\frac{3}{2}\right)}\right)+2\log_e\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)+\frac{8}{10^3}$$

Series representations:

$$\left(2-\log\left(\frac{\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)}{\cosh\left(\frac{3}{2}\right)+\cosh\left(-\frac{3}{2}\right)}\right)\right)+2\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)+\frac{8}{10^3}=$$
$$\frac{251}{125}+\sum_{k=1}^{\infty}\frac{-2\left(1-\frac{e\left(e+e^4\right)}{1+e^5}\right)^k+\left(\frac{1}{2}\left(1-\cosh\left(\frac{7}{3}\right)\operatorname{sech}\left(\frac{3}{2}\right)\right)\right)^k}{k}$$

$$\left(2-\log\left(\frac{\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)}{\cosh\left(\frac{3}{2}\right)+\cosh\left(-\frac{3}{2}\right)}\right)\right)+2\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)+\frac{8}{10^3}=$$
$$\frac{251}{125}+\sum_{k=1}^{\infty}\left(\frac{2(-1)^{-1+k}\left(-1+\frac{e+e^4}{\frac{1}{e}+e^4}\right)^k}{k}+\frac{\left(1-\frac{1}{2}\left(\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)\right)\operatorname{sech}\left(\frac{3}{2}\right)\right)^k}{k}\right)$$

Integral representations:

$$\left(2-\log\left(\frac{\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)}{\cosh\left(\frac{3}{2}\right)+\cosh\left(-\frac{3}{2}\right)}\right)\right)+2\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)+\frac{8}{10^3}=\frac{1}{125}\left(251+250\log\left(\frac{e\left(e+e^4\right)}{1+e^5}\right)-\right.$$
$$\left.125\log\left(\frac{\left(\int_0^\infty\frac{t^{(3i)/\pi}}{1+t^2}\,dt\right)\left(2+\int_0^1\left(\frac{3}{2}\sinh\left(\frac{3t}{2}\right)+\frac{7}{3}\sinh\left(\frac{7t}{3}\right)\right)dt\right)}{\pi}\right)\right)$$

$$\left(2-\log\left(\frac{\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)}{\cosh\left(\frac{3}{2}\right)+\cosh\left(-\frac{3}{2}\right)}\right)\right)+2\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)+\frac{8}{10^3}=\frac{1}{125}\left(251+250\log\left(\frac{e\left(e+e^4\right)}{1+e^5}\right)-125\log\left(-\frac{i\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{9/(16s)+s}+e^{49/(36s)+s}}{\sqrt{s}}\,ds\right)\int_0^\infty\frac{t^{(3i)/\pi}}{1+t^2}\,dt}{2\pi^{3/2}}\right)\right)\quad\text{for }\gamma>0$$

$$\left(2-\log\left(\frac{\cosh\left(\frac{3}{2}\right)+\cosh\left(\frac{7}{3}\right)}{\cosh\left(\frac{3}{2}\right)+\cosh\left(-\frac{3}{2}\right)}\right)\right)+2\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)+\frac{8}{10^3}=$$

$$\frac{251}{125}+\int_1^{\frac{e(e+e^4)}{1+e^5}}\left(\frac{2}{t}-\left(\left(1-\frac{e\left(e+e^4\right)}{1+e^5}\right)\left(-1+\frac{1}{2}\left(1+\cosh\left(\frac{7}{3}\right)\operatorname{sech}\left(\frac{3}{2}\right)\right)\right)\right)\Big/\left(\left(-1+\frac{e\left(e+e^4\right)}{1+e^5}\right)\left(-\frac{e\left(e+e^4\right)}{1+e^5}+t+\frac{1}{2}\left(1+\cosh\left(\frac{7}{3}\right)\operatorname{sech}\left(\frac{3}{2}\right)\right)-\frac{1}{2}t\left(1+\cosh\left(\frac{7}{3}\right)\operatorname{sech}\left(\frac{3}{2}\right)\right)\right)\right)\right)dt$$

With regard the first expression,

$$2\log\left(\frac{e+e^4}{\frac{1}{e}+e^4}\right)$$

0.083744006169247980285018464244095845653978939727576587189...

0.083744006169….

we have that:

MOCK THETA ORDER 6

We have the following mock theta function:
(https://en.wikipedia.org/wiki/Mock_modular_form#Order_6)

$$\sigma(q)=\sum_{n\geq 0}\frac{q^{(n+1)(n+2)/2}(-q;q)_n}{(q;q^2)_{n+1}}$$

That is:

(A053271 sequence OEIS)

Sum_{n >= 0} q^((n+1)(n+2)/2) (1+q)(1+q^2)...(1+q^n)/((1-q)(1-q^3)...(1-q^(2n+1)))

We have that:

sum q^((n+1)(n+2)/2) (1+q)(1+q^2)(1+q^n)))/((1-q)(1-q^3)(1-q^(2n+1))), n = 0 to k

Input interpretation:

$$\sum_{n=0}^{k} \frac{q^{1/2 (n+1)(n+2)} (1+q)(1+q^2)(1+q^n)}{(1-q)(1-q^3)(1-q^{2n+1})}$$

Result:

$$\sum_{n=0}^{k} \frac{q^{1/2 (n+1)(n+2)} (1+q)(1+q^2)(1+q^n)}{(1-q)(1-q^3)(1-q^{2n+1})}$$

For q = 0.498 and n = 2, we develop the above formula in the following way:

((0.498^((2+1)(2+2)/2) (1+0.498)(1+0.498^2)(1+0.498^2)))/((1-0.498)(1-0.498^3)(1-0.498^(2*2+1)))

Input:

$$\frac{0.498^{(2+1)\times(2+2)/2} (1+0.498)(1+0.498^2)(1+0.498^2)}{(1-0.498)(1-0.498^3)(1-0.498^{2\times 2+1})}$$

Result:

0.083440629509793437950061797866197269356504655927671224018...

0.0834406295….

From

Volumes And Random Matrices
Edward Witten - arXiv:2004.05183v1 [math.SG] 10 Apr 2020

We have that:

In the present example, there is only one fixed point in the U(1) action on diffS^1/PSL(2,$\mathbb{R}$) or diffS^1/U(1). The product over eigenvalues at this fixed point becomes formally $\prod_{n=2}^{\infty} n/2\pi\beta$ in the example of diffS^1/PSL(2,$\mathbb{R}$) (or $\prod_{n=1}^{\infty} n/2\pi\beta$ in the other example). This infinite product is treated with (for example) ζ-function regularization. For diffS^1/PSL(2,$\mathbb{R}$), the result is

$$Z(\beta) = \frac{C}{4\pi^{1/2}\beta^{3/2}} \exp(\pi^2/\beta), \tag{2.3}$$

where the constant C, which has been normalized for later convenience, depends on the regularization and so is considered inessential, but the rest is "universal." (This problem was first studied

From (2.3),

$$Z(\beta) = \frac{C}{4\pi^{1/2}\beta^{3/2}} \exp(\pi^2/\beta)$$

For C = (55+8+1/5) and β = 6, we obtain:

(55+8+1/5)Exp(((pi^2)/6))/((4sqrtPi*6^1.5))

Input:

$$\left(55+8+\frac{1}{5}\right)\times\frac{\exp\left(\frac{\pi^2}{6}\right)}{4\sqrt{\pi}\times 6^{1.5}}$$

Result:

3.142252376458037833491922039433261022484035813805735580881...

3.1422523764… ≈ π

Series representations:

$$\frac{\left(55+8+\frac{1}{5}\right)\exp\left(\frac{\pi^2}{6}\right)}{4\sqrt{\pi}\,6^{1.5}} = \frac{1.07505\exp\left(\frac{\pi^2}{6}\right)}{\sqrt{-1+\pi}\sum_{k=0}^{\infty}(-1+\pi)^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{\left(55+8+\frac{1}{5}\right)\exp\left(\frac{\pi^2}{6}\right)}{4\sqrt{\pi}\,6^{1.5}} = \frac{1.07505\exp\left(\frac{\pi^2}{6}\right)}{\sqrt{-1+\pi}\sum_{k=0}^{\infty}\frac{(-1)^k(-1+\pi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{\left(55+8+\frac{1}{5}\right)\exp\left(\frac{\pi^2}{6}\right)}{4\sqrt{\pi}\,6^{1.5}} = \frac{2.15011\exp\left(\frac{\pi^2}{6}\right)\sqrt{\pi}}{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}(-1+\pi)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}$$

For C = (29+4), we obtain:

(29+4)Exp(((pi^2)/6))/((4sqrtPi*6^1.5))

Input:

$$(29+4)\times\frac{\exp\left(\frac{\pi^2}{6}\right)}{4\sqrt{\pi}\times 6^{1.5}}$$

Result:

1.640733044669545071285339039577493888322360472398564464700...

1.6407330446…

Series representations:

$$\frac{(29+4)\exp\left(\frac{\pi^2}{6}\right)}{4\sqrt{\pi}\,6^{1.5}}=\frac{0.561341\exp\left(\frac{\pi^2}{6}\right)}{\sqrt{-1+\pi}\sum_{k=0}^{\infty}(-1+\pi)^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{(29+4)\exp\left(\frac{\pi^2}{6}\right)}{4\sqrt{\pi}\,6^{1.5}}=\frac{0.561341\exp\left(\frac{\pi^2}{6}\right)}{\sqrt{-1+\pi}\sum_{k=0}^{\infty}\frac{(-1)^k(-1+\pi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{(29+4)\exp\left(\frac{\pi^2}{6}\right)}{4\sqrt{\pi}\,6^{1.5}}=\frac{1.12268\exp\left(\frac{\pi^2}{6}\right)\sqrt{\pi}}{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}(-1+\pi)^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}$$

For C = 29 + √4π

(29 + sqrt(4Pi))Exp(((pi^2)/6))/((4sqrtPi*6^1.5))

Input:

$$\left(29+\sqrt{4\pi}\right)\times\frac{\exp\left(\frac{\pi^2}{6}\right)}{4\sqrt{\pi}\times 6^{1.5}}$$

Result:

1.618106227335810347966765631923611986785142404932836804964...

1.6181062273…

Series representations:

$$\frac{(29+\sqrt{4\pi})\exp\left(\frac{\pi^2}{6}\right)}{4\sqrt{\pi}\,6^{1.5}} = \frac{0.0170103\exp\left(\frac{\pi^2}{6}\right)\left(29+\sqrt{-1+4\pi}\sum_{k=0}^{\infty}(-1+4\pi)^{-k}\binom{\frac{1}{2}}{k}\right)}{\sqrt{-1+\pi}\sum_{k=0}^{\infty}(-1+\pi)^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{(29+\sqrt{4\pi})\exp\left(\frac{\pi^2}{6}\right)}{4\sqrt{\pi}\,6^{1.5}} = \frac{0.0170103\exp\left(\frac{\pi^2}{6}\right)\left(29+\sqrt{-1+4\pi}\sum_{k=0}^{\infty}\frac{(-1)^k(-1+4\pi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\sqrt{-1+\pi}\sum_{k=0}^{\infty}\frac{(-1)^k(-1+\pi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{(29+\sqrt{4\pi})\exp\left(\frac{\pi^2}{6}\right)}{4\sqrt{\pi}\,6^{1.5}} = \frac{0.0170103\exp\left(\frac{\pi^2}{6}\right)\left(29+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(4\pi-z_0)^k z_0^{-k}}{k!}\right)}{\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\pi-z_0)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

From

$$Z(\beta) = \int_0^{\infty} \mathrm{d}E\rho(E)\exp(-\beta E), \tag{2.4}$$

$$\rho(E) = \frac{C}{4\pi^2}\sinh(2\pi\sqrt{E}). \tag{2.5}$$

From

Modular equations and approximations to π – *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

we have the following Ramanujan's equation:

$$\frac{5\sqrt{5}}{2\pi\sqrt{3}} = 1 + 12\frac{1}{2}\frac{1}{6}\frac{5}{6}\left(\frac{4}{125}\right) + 23\frac{1\cdot 3}{2\cdot 4}\frac{1\cdot 7}{6\cdot 12}\frac{5\cdot 11}{6\cdot 12}\left(\frac{4}{125}\right)^2 + \cdots,$$

that is:

(5sqrt5) / (2Pi*sqrt3)

Input:

$$\frac{5\sqrt{5}}{2\pi\sqrt{3}}$$

Result:

$$\frac{5\sqrt{\frac{5}{3}}}{2\pi}$$

Decimal approximation:

1.027340740102499675941615157239129241668605901250790303864...

1.02734074010…

Property:

$\frac{5\sqrt{\frac{5}{3}}}{2\pi}$ is a transcendental number

Series representations:

$$\frac{5\sqrt{5}}{2\pi\sqrt{3}} = \frac{5\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}}{2\pi\sqrt{2}\sum_{k=0}^{\infty} 2^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{5\sqrt{5}}{2\pi\sqrt{3}} = \frac{5\sqrt{4}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}{2\pi\sqrt{2}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{5\sqrt{5}}{2\pi\sqrt{3}} = \frac{5\sum_{k=0}^{\infty} \frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}{2\pi\sum_{k=0}^{\infty} \frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}} \quad \text{for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

that is about equal to:

1+12*5/72*(4/125)+23*3/8*7/72*55/72*(4/125)^2

Input:

$$1+12\times\frac{5}{72}\times\frac{4}{125}+23\times\frac{3}{8}\times\frac{7}{72}\times\frac{55}{72}\left(\frac{4}{125}\right)^2$$

Exact result:

$$\frac{2773771}{2700000}$$

Decimal approximation:

1.027322592592592592592592592592592592592592592592592592592...

1.0273225925…

We have that:

(2Pi*sqrt3) * 1.02734074010249

Input interpretation:

$$\left(2\pi\sqrt{3}\right)\times 1.02734074010249$$

Result:

11.1803398874988...

11.1803398874988…

Series representations:

$$1.02734074010249000\times 2\left(\pi\sqrt{3}\right)=2.05468148020498000\,\pi\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}$$

$$1.02734074010249000\times 2\left(\pi\sqrt{3}\right)=2.05468148020498000\,\pi\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}$$

$$1.02734074010249000\times 2\left(\pi\sqrt{3}\right)=\frac{1.02734074010249000\,\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}$$

and:

(5sqrt5) / 1.02734074010249

Input interpretation:

$$\frac{5\sqrt{5}}{1.02734074010249}$$

Result:

10.8827961854054...

10.8827961854054…

Input interpretation:

10.8827961854054

Possible closed forms:

$2\sqrt{3}\,\pi \approx 10.88279618540530710$

$6\sqrt{2\,\zeta(2)} \approx 10.88279618540530710$

We have also:

(2Pi*x) * 1.02734074010249 = 11.1803398874988

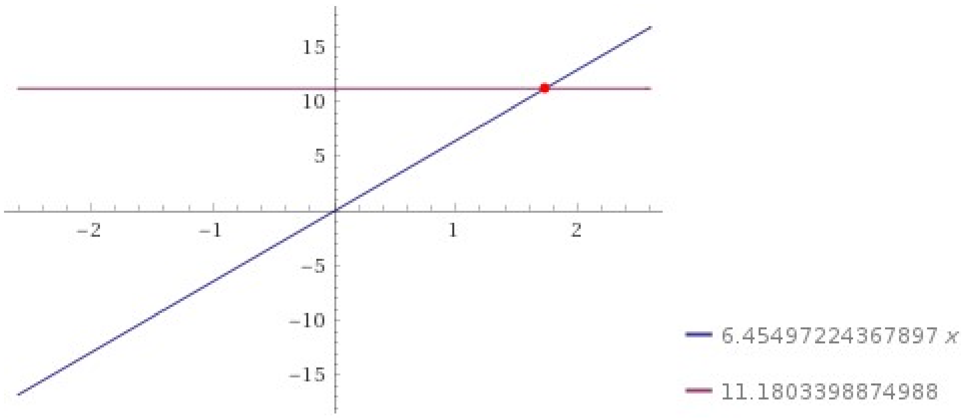
Input interpretation:

$(2\,\pi\,x)\times 1.02734074010249 = 11.1803398874988$

Result:

$6.45497224367897\,x = 11.1803398874988$

Plot:



Alternate form:

$6.45497224367897\,x - 11.1803398874988 = 0$

Solution:

$x \approx 1.73205080756887$

1.73205080756887 = √3

Input interpretation:

1.73205080756887

$\sqrt{3} \approx 1.73205080756887 72935$

Thence, for E = 3 and C = 5, from

$$\rho(E) = \frac{C}{4\pi^2}\sinh(2\pi\sqrt{E}).$$

we obtain:

((5 * sinh(2Pi*sqrt3)) / (4Pi^2)

Input:

$$\frac{5\sinh(2\pi\sqrt{3})}{4\pi^2}$$

sinh(x) is the hyperbolic sine function

Decimal approximation:

3372.240988545132176088366472858596594196602040150285416838...

3372.2409885

Alternate forms:

$$\frac{5\sinh(\sqrt{3}\,\pi)\cosh(\sqrt{3}\,\pi)}{2\pi^2}$$

$$\frac{5e^{2\sqrt{3}\,\pi}}{8\pi^2} - \frac{5e^{-2\sqrt{3}\,\pi}}{8\pi^2}$$

cosh(x) is the hyperbolic cosine function

Alternative representations:

$$\frac{5\sinh(2\pi\sqrt{3})}{4\pi^2} = \frac{5}{\operatorname{csch}(2\pi\sqrt{3})(4\pi^2)}$$

$$\frac{5\sinh(2\pi\sqrt{3})}{4\pi^2} = \frac{5\left(-e^{-2\pi\sqrt{3}} + e^{2\pi\sqrt{3}}\right)}{2(4\pi^2)}$$

$$\frac{5\sinh(2\pi\sqrt{3})}{4\pi^2} = -\frac{5i}{\csc(2i\pi\sqrt{3})(4\pi^2)}$$

Series representations:

$$\frac{5\sinh(2\pi\sqrt{3})}{4\pi^2} = \frac{5\sum_{k=0}^{\infty} I_{1+2k}(2\sqrt{3}\,\pi)}{2\pi^2}$$

$$\frac{5\sinh(2\pi\sqrt{3})}{4\pi^2} = \frac{5\sum_{k=0}^{\infty} \frac{3^{1/2+k}(2\pi)^{1+2k}}{(1+2k)!}}{4\pi^2}$$

$$\frac{5\sinh(2\pi\sqrt{3})}{4\pi^2} = \frac{5i\sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\left(-i+4\sqrt{3}\right)\pi\right)^{2k}}{(2k)!}}{4\pi^2}$$

Integral representations:

$$\frac{5\sinh(2\pi\sqrt{3})}{4\pi^2} = \frac{5\sqrt{3}}{2\pi}\int_0^1 \cosh\left(2\sqrt{3}\,\pi t\right)dt$$

$$\frac{5\sinh(2\pi\sqrt{3})}{4\pi^2} = -\frac{5i\sqrt{3}}{8\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{(3\pi^2)/s+s}}{s^{3/2}}\,ds \quad \text{for } \gamma > 0$$

Thence, from

$$Z(\beta) = \int_0^{\infty} \mathrm{d}E\rho(E)\exp(-\beta E),$$

For E = 3, β = 6 and ρ(E) = 3372.2409885, we obtain:

Integrate (3372.2409885*exp(-6*3))dx, x = 0..12038

Definite integral:

$$\int_0^{12038} 3372.2409885 \exp(-6\times 3)\, dx = 0.61826159154$$

0.61826159154

From which:

1 + Integrate (3372.2409885*exp(-6*3))dx, x = 0..12038

Input interpretation:

$$1+\int_0^{12038} 3372.2409885 \exp(-6\times 3)\, dx$$

Result:

1.61826159154

Computation result:

$$1+\int_0^{12038} 3372.2409885 \exp(-6\times 3)\, dx = 1.61826$$

1.61826

From

$$Z_U(\beta) = \int_0^\infty \mathrm{d}E\, \frac{e^S}{4\pi^2} \sinh(2\pi\sqrt{E}) e^{-\beta E}.$$

For E = 3, e^S = C = 5 and β = 6, we obtain:

integrate(((5/(4Pi^2)*sinh(2Pi*sqrt3)*e^(-6*3))))dx, x=0..12038

Definite integral:

$$\int_0^{12038} \frac{5 \sinh(2 \pi \sqrt{3}) e^{-6 \times 3}}{4 \pi^2} dx = \frac{30\,095 \sinh(2 \sqrt{3} \pi)}{2 e^{18} \pi^2} \approx 0.61826$$

sinh(x) is the hyperbolic sine function

0.61826 as above

From the formula of coefficients of the '5th order' mock theta function $\psi_1(q)$:
(A053261 OEIS Sequence)

sqrt(golden ratio) * exp(Pi*sqrt(n/15)) / (2*5^(1/4)*sqrt(n)) , for n = 258, we obtain:

sqrt(golden ratio) * exp(Pi*sqrt(258/15)) / (2*5^(1/4)*sqrt(258)) – 23

where 23 is a Sophie Germain prime number

Input:

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{258}{15}}\right)}{2 \sqrt[4]{5} \sqrt{258}} - 23$$

ϕ is the golden ratio

Exact result:

$$\frac{e^{\sqrt{86/5}\,\pi} \sqrt{\frac{\phi}{258}}}{2 \sqrt[4]{5}} - 23$$

Decimal approximation:

12037.727897975493845530836815818616753786155456383769840630...

12037.72789…. result very near to the value b of the integration interval [a, b] = [0,..12038]

Property:

$$-23 + \frac{e^{\sqrt{86/5}\,\pi} \sqrt{\frac{\phi}{258}}}{2 \sqrt[4]{5}}$$ is a transcendental number

Alternate forms:

$$\frac{1}{4} \sqrt{\frac{1}{645} \left(5 + \sqrt{5}\right)} e^{\sqrt{86/5}\,\pi} - 23$$

$$\frac{\sqrt{\frac{1}{129}\left(1+\sqrt{5}\right) e^{\sqrt{86/5}\,\pi}}}{4\sqrt[4]{5}} - 23$$

$$\frac{5^{3/4}\sqrt{129\left(1+\sqrt{5}\right) e^{\sqrt{86/5}\,\pi}} - 59\,340}{2580}$$

Series representations:

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{258}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{258}} - 23 = \left(-230\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (258-z_0)^k z_0^{-k}}{k!} + 5^{3/4}\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{86}{5}-z_0\right)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right) \Big/ \left(10\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(258-z_0)^k z_0^{-k}}{k!}\right) \text{ for (not } (z_0\in\mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{258}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{258}} - 23 = \left(-230\exp\left(i\pi\left\lfloor\frac{\arg(258-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(258-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!} + 5^{3/4}\exp\left(i\pi\left\lfloor\frac{\arg(\phi-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{86}{5}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{86}{5}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(\phi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right) \Big/ \left(10\exp\left(i\pi\left\lfloor\frac{\arg(258-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(258-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$

for $(x\in\mathbb{R}$ and $x<0)$

$$\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{258}{15}}\right)}{2\sqrt[4]{5}\sqrt{258}}-23=$$

$$\left(\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(258-z_0)/(2\pi)\rfloor} z_0^{-1/2\lfloor\arg(258-z_0)/(2\pi)\rfloor}\left(-230\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(258-z_0)/(2\pi)\rfloor}\right.\right.$$

$$z_0^{1/2\lfloor\arg(258-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(258-z_0)^k z_0^{-k}}{k!}+$$

$$5^{3/4}\exp\left(\pi\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{86}{5}-z_0\right)/(2\pi)\right\rfloor} z_0^{1/2\left(1+\left\lfloor\arg\left(\frac{86}{5}-z_0\right)/(2\pi)\right\rfloor\right)}\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{86}{5}-z_0\right)^k z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor} z_0^{1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}$$

$$\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right)\right)\Bigg/\left(10\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(258-z_0)^k z_0^{-k}}{k!}\right)$$

In number theory, a prime number p is a **Sophie Germain prime** if $2p + 1$ is also prime. The number $2p + 1$ associated with a Sophie Germain prime is called a safe prime. For example, 11 is a Sophie Germain prime and $2 \times 11 + 1 = 23$ is its associated safe prime

From

$$y^2 = -\frac{1}{16\pi^2}\sinh^2(2\pi\sqrt{E}).$$

For E = 3, we obtain:

-1/(16Pi^2) sinh^2(2Pi*sqrt3)

Input:

$$-\frac{\sinh^2(2\pi\sqrt{3})}{16\pi^2}$$

sinh(x) is the hyperbolic sine function

Decimal approximation:

$-4.4894893154690608912354299907820811434721458939776049\ldots\times 10^6$

$-4.4894893154\ldots*10^6$

Alternate forms:

$$\frac{1-\cosh(4\sqrt{3}\,\pi)}{32\,\pi^2}$$

$$-\frac{\sinh^2(\sqrt{3}\,\pi)\cosh^2(\sqrt{3}\,\pi)}{4\,\pi^2}$$

$$\frac{1}{32\,\pi^2}-\frac{e^{-4\sqrt{3}\,\pi}}{64\,\pi^2}-\frac{e^{4\sqrt{3}\,\pi}}{64\,\pi^2}$$

$\cosh(x)$ is the hyperbolic cosine function

Alternative representations:

$$\frac{\sinh^2(2\,\pi\sqrt{3})(-1)}{16\,\pi^2}=-\frac{\left(\frac{1}{\operatorname{csch}(2\,\pi\sqrt{3})}\right)^2}{16\,\pi^2}$$

$$\frac{\sinh^2(2\,\pi\sqrt{3})(-1)}{16\,\pi^2}=-\frac{\left(-\frac{i}{\csc(2\,i\,\pi\sqrt{3})}\right)^2}{16\,\pi^2}$$

$$\frac{\sinh^2(2\,\pi\sqrt{3})(-1)}{16\,\pi^2}=-\frac{\left(\frac{1}{2}\left(-e^{-2\,\pi\sqrt{3}}+e^{2\,\pi\sqrt{3}}\right)\right)^2}{16\,\pi^2}$$

Series representations:

$$\frac{\sinh^2(2\,\pi\sqrt{3})(-1)}{16\,\pi^2}=-\frac{\sum_{k=1}^{\infty}\frac{2^{-1+4\,k}\times 3^k\,\pi^{2\,k}}{(2\,k)!}}{16\,\pi^2}$$

$$\frac{\sinh^2(2\,\pi\sqrt{3})(-1)}{16\,\pi^2}=-\frac{3}{4}\sum_{k=0}^{0}(-1)^k\,(-1+k)^2\binom{2}{k}\,{}_1F_2\left(1;\,\frac{3}{2},\,2;\,12\,(-1+k)^2\,\pi^2\right)$$

$$\frac{\sinh^2(2\,\pi\sqrt{3})(-1)}{16\,\pi^2}=\frac{1}{16\,\pi^2}+\frac{\sum_{k=1}^{\infty}\frac{((-i+4\sqrt{3})\pi)^{2\,k}}{2\,(2\,k)!}}{16\,\pi^2}$$

From which, performing the square root and changing the sign, we obtain:

(((1/(16Pi^2) sinh^2(2Pi*sqrt3))))^1/2

Input:

$$\sqrt{\frac{1}{16\pi^2}\sinh^2\left(2\pi\sqrt{3}\right)}$$

sinh(x) is the hyperbolic sine function

Exact result:

$$\frac{\sinh(2\sqrt{3}\,\pi)}{4\pi}$$

Decimal approximation:

2118.841503149553867364722757070314017002145983582492700860...

2118.8415031… result very near to the rest mass of strange D meson 2112.3

Alternate forms:

$$\frac{\sinh(\sqrt{3}\,\pi)\cosh(\sqrt{3}\,\pi)}{2\pi}$$

$$\frac{e^{2\sqrt{3}\,\pi}}{8\pi}-\frac{e^{-2\sqrt{3}\,\pi}}{8\pi}$$

cosh(x) is the hyperbolic cosine function

All 2nd roots of (sinh^2(2 sqrt(3) π))/(16 π^2):

$$\frac{e^0\sinh(2\sqrt{3}\,\pi)}{4\pi}\approx 2119. \text{ (real, principal root)}$$

$$\frac{e^{i\pi}\sinh(2\sqrt{3}\,\pi)}{4\pi}\approx -2119. \text{ (real root)}$$

Alternative representations:

$$\sqrt{\frac{\sinh^2(2\pi\sqrt{3})}{16\pi^2}}=\sqrt{\frac{\left(\frac{1}{\operatorname{csch}(2\pi\sqrt{3})}\right)^2}{16\pi^2}}$$

$$\sqrt{\frac{\sinh^2(2\pi\sqrt{3})}{16\pi^2}} = \sqrt{\frac{\left(-\frac{i}{\csc(2i\pi\sqrt{3})}\right)^2}{16\pi^2}}$$

$$\sqrt{\frac{\sinh^2(2\pi\sqrt{3})}{16\pi^2}} = \sqrt{\frac{\left(i\cos\left(\frac{\pi}{2}+2i\pi\sqrt{3}\right)\right)^2}{16\pi^2}}$$

Series representations:

$$\sqrt{\frac{\sinh^2(2\pi\sqrt{3})}{16\pi^2}} = \frac{\sum_{k=0}^{\infty} I_{1+2k}(2\sqrt{3}\,\pi)}{2\pi}$$

$$\sqrt{\frac{\sinh^2(2\pi\sqrt{3})}{16\pi^2}} = \frac{\sum_{k=0}^{\infty} \frac{3^{1/2+k}(2\pi)^{1+2k}}{(1+2k)!}}{4\pi}$$

$$\sqrt{\frac{\sinh^2(2\pi\sqrt{3})}{16\pi^2}} = \frac{i\sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\left(-i+4\sqrt{3}\right)\pi\right)^{2k}}{(2k)!}}{4\pi}$$

Integral representations:

$$\sqrt{\frac{\sinh^2(2\pi\sqrt{3})}{16\pi^2}} = \frac{\sqrt{3}}{2}\int_0^1 \cosh\left(2\sqrt{3}\,\pi t\right)dt$$

$$\sqrt{\frac{\sinh^2(2\pi\sqrt{3})}{16\pi^2}} = -\frac{1}{8}\,i\sqrt{\frac{3}{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{(3\pi^2)/s+s}}{s^{3/2}}\,ds \quad \text{for } \gamma > 0$$

From the formula of coefficients of the '5th order' mock theta function $\psi_1(q)$:
(A053261 OEIS Sequence)

sqrt(golden ratio) * exp(Pi*sqrt(n/15)) / (2*5^(1/4)*sqrt(n)) , for n = 188, we obtain

sqrt(golden ratio) * exp(Pi*sqrt(188/15)) / (2*5^(1/4)*sqrt(188)) + 21

where 21 is a Fibonacci number

Input:

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{188}{15}}\right)}{2 \sqrt[4]{5} \sqrt{188}} + 21$$

ϕ is the golden ratio

Exact result:

$$\frac{e^{2 \sqrt{47/15} \pi} \sqrt{\frac{\phi}{47}}}{4 \sqrt[4]{5}} + 21$$

Decimal approximation:

2119.305446447894919679480418487556286678292648424815301112...

2119.305446… as above

Property:

$$21 + \frac{e^{2 \sqrt{47/15} \pi} \sqrt{\frac{\phi}{47}}}{4 \sqrt[4]{5}} \text{ is a transcendental number}$$

Alternate forms:

$$21 + \frac{1}{4} \sqrt{\frac{1}{470} \left(5 + \sqrt{5}\right)} e^{2 \sqrt{47/15} \pi}$$

$$21 + \frac{\sqrt{\frac{1}{94} (1 + \sqrt{5}) e^{2 \sqrt{47/15} \pi}}}{4 \sqrt[4]{5}}$$

$$\frac{39480 + 5^{3/4} \sqrt{94 (1 + \sqrt{5})} e^{2 \sqrt{47/15} \pi}}{1880}$$

Series representations:

$$\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{188}{15}}\right)}{2\sqrt[4]{5}\sqrt{188}}+21=\left(210\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(188-z_0)^k z_0^{-k}}{k!}+5^{3/4}\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{188}{15}-z_0\right)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right)\Bigg/\left(10\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(188-z_0)^k z_0^{-k}}{k!}\right)$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0$))

$$\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{188}{15}}\right)}{2\sqrt[4]{5}\sqrt{188}}+21=\left(210\exp\left(i\pi\left\lfloor\frac{\arg(188-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(188-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+5^{3/4}\exp\left(i\pi\left\lfloor\frac{\arg(\phi-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{188}{15}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{188}{15}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(\phi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Bigg/\left(10\exp\left(i\pi\left\lfloor\frac{\arg(188-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(188-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$

for ($x\in\mathbb{R}$ and $x<0$)

$$\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{188}{15}}\right)}{2\sqrt[4]{5}\sqrt{188}}+21=\left(\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(188-z_0)/(2\pi)\rfloor}z_0^{-1/2\lfloor\arg(188-z_0)/(2\pi)\rfloor}\left(210\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(188-z_0)/(2\pi)\rfloor}z_0^{1/2\lfloor\arg(188-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(188-z_0)^k z_0^{-k}}{k!}+5^{3/4}\exp\left(\pi\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{188}{15}-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2\left(1+\left\lfloor\arg\left(\frac{188}{15}-z_0\right)/(2\pi)\right\rfloor\right)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{188}{15}-z_0\right)^k z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}z_0^{1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right)\right)\Bigg/\left(10\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(188-z_0)^k z_0^{-k}}{k!}\right)$$

We have that:

$$Z_{\widehat{U}}(\beta) = \int_0^\infty dE\, e^{-\beta E} \widehat{\rho}(E), \quad \widehat{\rho}(E) = \frac{e^S}{\pi} \frac{\sqrt{2}\cosh(2\pi\sqrt{E})}{\sqrt{E}}.$$

And

$$y^2 = -\frac{2}{E}\cosh^2(2\pi\sqrt{E}).$$

For E = 3, e^S = C = 5 , β = 6 from

$$\widehat{\rho}(E) = \frac{e^S}{\pi} \frac{\sqrt{2}\cosh(2\pi\sqrt{E})}{\sqrt{E}}.$$

we obtain:

(5sqrt2)/Pi * (cosh(2Pi*sqrt3))/(sqrt3)

Input:

$$\frac{5\sqrt{2}}{\pi} \times \frac{\cosh(2\pi\sqrt{3})}{\sqrt{3}}$$

cosh(x) is the hyperbolic cosine function

Exact result:

$$\frac{5\sqrt{\frac{2}{3}}\cosh(2\sqrt{3}\pi)}{\pi}$$

Decimal approximation:

34600.53688139009158971676001026757414931924207990618583219...

34600.53688139…

Alternate forms:

$$\frac{5e^{-2\sqrt{3}\pi}}{\sqrt{6}\pi} + \frac{5e^{2\sqrt{3}\pi}}{\sqrt{6}\pi}$$

$$\frac{5\sqrt{\frac{2}{3}}\sinh^2(\sqrt{3}\pi)}{\pi} + \frac{5\sqrt{\frac{2}{3}}\cosh^2(\sqrt{3}\pi)}{\pi}$$

$$\frac{5\sqrt{\frac{2}{3}}\left(\cosh\left(\sqrt{3}\,\pi\right)-i\sinh\left(\sqrt{3}\,\pi\right)\right)\left(\cosh\left(\sqrt{3}\,\pi\right)+i\sinh\left(\sqrt{3}\,\pi\right)\right)}{\pi}$$

$\sinh(x)$ is the hyperbolic sine function

Alternative representations:

$$\frac{\cosh\left(2\pi\sqrt{3}\right)\left(5\sqrt{2}\right)}{\sqrt{3}\,\pi}=\frac{5\cos\left(-2i\pi\sqrt{3}\right)\sqrt{2}}{\pi\sqrt{3}}$$

$$\frac{\cosh\left(2\pi\sqrt{3}\right)\left(5\sqrt{2}\right)}{\sqrt{3}\,\pi}=\frac{5\cos\left(2i\pi\sqrt{3}\right)\sqrt{2}}{\pi\sqrt{3}}$$

$$\frac{\cosh\left(2\pi\sqrt{3}\right)\left(5\sqrt{2}\right)}{\sqrt{3}\,\pi}=\frac{5\left(e^{-2\pi\sqrt{3}}+e^{2\pi\sqrt{3}}\right)\sqrt{2}}{2\pi\sqrt{3}}$$

Series representations:

$$\frac{\cosh\left(2\pi\sqrt{3}\right)\left(5\sqrt{2}\right)}{\sqrt{3}\,\pi}=\frac{5\sqrt{\frac{2}{3}}\sum_{k=0}^{\infty}\frac{12^k\pi^{2k}}{(2k)!}}{\pi}$$

$$\frac{\cosh\left(2\pi\sqrt{3}\right)\left(5\sqrt{2}\right)}{\sqrt{3}\,\pi}=\frac{5i\sqrt{\frac{2}{3}}\sum_{k=0}^{\infty}\frac{\left(\frac{1}{2}\left(-i+4\sqrt{3}\right)\pi\right)^{1+2k}}{(1+2k)!}}{\pi}$$

$$\frac{\cosh\left(2\pi\sqrt{3}\right)\left(5\sqrt{2}\right)}{\sqrt{3}\,\pi}=-\frac{5\sqrt{\frac{2}{3}}\sum_{k=0}^{\infty}I_{2k}(2)\,T_{2k}\left(\sqrt{3}\,\pi\right)\left(-2+\delta_k\right)}{\pi}$$

Integral representations:

$$\frac{\cosh\left(2\pi\sqrt{3}\right)\left(5\sqrt{2}\right)}{\sqrt{3}\,\pi}=\frac{5\sqrt{\frac{2}{3}}}{\pi}+10\sqrt{2}\int_0^1\sinh\left(2\sqrt{3}\,\pi t\right)dt$$

$$\frac{\cosh\left(2\pi\sqrt{3}\right)\left(5\sqrt{2}\right)}{\sqrt{3}\,\pi}=\frac{5\sqrt{\frac{2}{3}}}{\pi}\int_{\frac{i\pi}{2}}^{2\sqrt{3}\,\pi}\sinh(t)\,dt$$

$$\frac{\cosh(2\pi\sqrt{3})(5\sqrt{2})}{\sqrt{3}\,\pi} = -\frac{5\,i}{\sqrt{6}\,\pi^{3/2}} \int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{e^{(3\pi^2)/s+s}}{\sqrt{s}}\,ds \quad \text{for } \gamma > 0$$

From

$$Z_{\widehat{U}}(\beta) = \int_0^\infty \mathrm{d}E\, e^{-\beta E} \widehat{\rho}(E),$$

we obtain:

integrate(((e^(-6*3) (5sqrt2)/Pi * (cosh(2Pi*sqrt3))/(sqrt3))))dx, x=0..3071

Definite integral:

$$\int_0^{3071} \frac{e^{-6\times 3}(5\sqrt{2})\cosh(2\pi\sqrt{3})}{\pi\sqrt{3}}\,dx = \frac{15355\sqrt{\frac{2}{3}}\cosh(2\sqrt{3}\,\pi)}{e^{18}\,\pi} \approx 1.61831$$

cosh(x) is the hyperbolic cosine function

1.61831

We note that from From the formula of coefficients of the '5th order' mock theta function ψ_1(q): (A053261 OEIS Sequence)

sqrt(golden ratio) * exp(Pi*sqrt(n/15)) / (2*5^(1/4)*sqrt(n)) , for n = 202, we obtain

sqrt(golden ratio) * exp(Pi*sqrt(202/15)) / (2*5^(1/4)*sqrt(202)) + 29+2

where 29 and 2 are Lucas numbers

sqrt(golden ratio) * exp(Pi*sqrt(202/15)) / (2*5^(1/4)*sqrt(202)) + 29+2

Input:

$$\sqrt{\phi} \times \frac{\exp\left(\pi\sqrt{\frac{202}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{202}} + 29 + 2$$

ϕ is the golden ratio

Exact result:

$$\frac{e^{\sqrt{202/15}\,\pi}\sqrt{\frac{\phi}{202}}}{2\sqrt[4]{5}}+31$$

Decimal approximation:

3071.126834588784407296370866918273577586687151996630308269...

3071.126834588...

Property:

$$31+\frac{e^{\sqrt{202/15}\,\pi}\sqrt{\frac{\phi}{202}}}{2\sqrt[4]{5}} \text{ is a transcendental number}$$

Alternate forms:

$$31+\frac{1}{4}\sqrt{\frac{1}{505}\left(5+\sqrt{5}\right)}\,e^{\sqrt{202/15}\,\pi}$$

$$31+\frac{\sqrt{\frac{1}{101}\left(1+\sqrt{5}\right)}\,e^{\sqrt{202/15}\,\pi}}{4\sqrt[4]{5}}$$

$$\frac{62620+5^{3/4}\sqrt{101\left(1+\sqrt{5}\right)}\,e^{\sqrt{202/15}\,\pi}}{2020}$$

Series representations:

$$\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{202}{15}}\right)}{2\sqrt[4]{5}\sqrt{202}}+29+2=\left(310\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(202-z_0)^k z_0^{-k}}{k!}+5^{3/4}\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{202}{15}-z_0\right)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right)\Bigg/\left(10\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(202-z_0)^k z_0^{-k}}{k!}\right) \text{ for } (\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$$

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{202}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{202}}+29+2=$$

$$\left(310\exp\left(i\pi\left\lfloor\frac{\arg(202-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(202-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$5^{3/4}\exp\left(i\pi\left\lfloor\frac{\arg(\phi-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{202}{15}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\right.$$
$$\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{202}{15}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(\phi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Big/$$
$$\left(10\exp\left(i\pi\left\lfloor\frac{\arg(202-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(202-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\quad\text{for } (x\in\mathbb{R}\text{ and } x<0)$$

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{202}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{202}}+29+2=\left(\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(202-z_0)/(2\pi)\rfloor}z_0^{-1/2\lfloor\arg(202-z_0)/(2\pi)\rfloor}\right.$$
$$\left(310\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(202-z_0)/(2\pi)\rfloor}z_0^{1/2\lfloor\arg(202-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(202-z_0)^k z_0^{-k}}{k!}+\right.$$
$$5^{3/4}\exp\left(\pi\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{202}{15}-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2\left(1+\left\lfloor\arg\left(\frac{202}{15}-z_0\right)/(2\pi)\right\rfloor\right)}\right.$$
$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{202}{15}-z_0\right)^k z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}$$
$$\left.\left.z_0^{1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right)\right)\Big/$$
$$\left(10\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(202-z_0)^k z_0^{-k}}{k!}\right)$$

From

$$y^2=-\frac{2}{E}\cosh^2(2\pi\sqrt{E}).$$

for E = 3, we obtain:

-2/3 cosh^2(2Pi*sqrt3)

Input:

$$-\frac{2}{3}\cosh^2\left(2\pi\sqrt{3}\right)$$

cosh(x) is the hyperbolic cosine function

Decimal approximation:

$$-4.7263449140370235003708482021277800276916979474964075\ldots\times 10^8$$

$-4.726344914\ldots*10^8$

Property:

$-\frac{2}{3}\cosh^2\left(2\sqrt{3}\,\pi\right)$ is a transcendental number

Alternate forms:

$$\frac{1}{3}\left(-1-\cosh\left(4\sqrt{3}\,\pi\right)\right)$$

$$-\frac{1}{3}-\frac{1}{6}e^{-4\sqrt{3}\,\pi}-\frac{1}{6}e^{4\sqrt{3}\,\pi}$$

$$-\frac{2}{3}\left(\cosh\left(\sqrt{3}\,\pi\right)-i\sinh\left(\sqrt{3}\,\pi\right)\right)^2\left(\cosh\left(\sqrt{3}\,\pi\right)+i\sinh\left(\sqrt{3}\,\pi\right)\right)^2$$

sinh(x) is the hyperbolic sine function

Alternative representations:

$$\frac{1}{3}\cosh^2\left(2\pi\sqrt{3}\right)(-2)=-\frac{2}{3}\cos^2\left(-2i\pi\sqrt{3}\right)$$

$$\frac{1}{3}\cosh^2\left(2\pi\sqrt{3}\right)(-2)=-\frac{2}{3}\cos^2\left(2i\pi\sqrt{3}\right)$$

$$\frac{1}{3}\cosh^2\left(2\pi\sqrt{3}\right)(-2)=-\frac{2}{3}\left(\frac{1}{\sec\left(2i\pi\sqrt{3}\right)}\right)^2$$

Series representations:

$$\frac{1}{3}\cosh^2\left(2\pi\sqrt{3}\right)(-2)=-\frac{2}{3}-\frac{2}{3}\sum_{k=1}^{\infty}\frac{2^{-1+4k}\times 3^k\,\pi^{2k}}{(2k)!}$$

$$\frac{1}{3}\cosh^2\left(2\pi\sqrt{3}\right)(-2) = -\frac{2}{3}\left(\sum_{k=0}^{\infty}\frac{12^k\pi^{2k}}{(2k)!}\right)^2$$

$$\frac{1}{3}\cosh^2\left(2\pi\sqrt{3}\right)(-2) = \frac{1}{3}\sum_{k=1}^{\infty}\frac{\left(\left(-i+4\sqrt{3}\right)\pi\right)^{2k}}{(2k)!}$$

From which, performing the cubic root, changing the sign and adding π, we obtain:

(((2/3 cosh^2(2Pi*sqrt3))))^1/3 + Pi

Input:

$$\sqrt[3]{\frac{2}{3}\cosh^2\left(2\pi\sqrt{3}\right)} + \pi$$

cosh(x) is the hyperbolic cosine function

Exact result:

$$\pi + \sqrt[3]{\frac{2}{3}}\cosh^{2/3}\left(2\sqrt{3}\pi\right)$$

Decimal approximation:

782.0895999494976341560228342054072222192274793800076281565...

782.08959994… result practically equal to the rest mass of Omega meson 782.65

Alternate forms:

$$\frac{1}{3}\left(3\pi + \sqrt[3]{2}\left(3\cosh\left(2\sqrt{3}\pi\right)\right)^{2/3}\right)$$

$$\frac{\left(e^{-2\sqrt{3}\pi} + e^{2\sqrt{3}\pi}\right)^{2/3}}{\sqrt[3]{6}} + \pi$$

$$\frac{2^{2/3}\pi + \frac{2\cosh^{2/3}\left(2\sqrt{3}\pi\right)}{\sqrt[3]{3}}}{2^{2/3}}$$

Alternative representations:

$$\sqrt[3]{\frac{1}{3}\cosh^2\left(2\pi\sqrt{3}\right)2} + \pi = \pi + \sqrt[3]{\frac{2}{3}\cos^2\left(-2i\pi\sqrt{3}\right)}$$

$$\sqrt[3]{\frac{1}{3}\cosh^2\left(2\pi\sqrt{3}\right)2} + \pi = \pi + \sqrt[3]{\frac{2}{3}\cos^2\left(2i\pi\sqrt{3}\right)}$$

$$\sqrt[3]{\frac{1}{3}\cosh^2\left(2\pi\sqrt{3}\right)2} + \pi = \pi + \sqrt[3]{\frac{2}{3}\left(\frac{1}{\sec(2i\pi\sqrt{3})}\right)^2}$$

Observations

Figs.

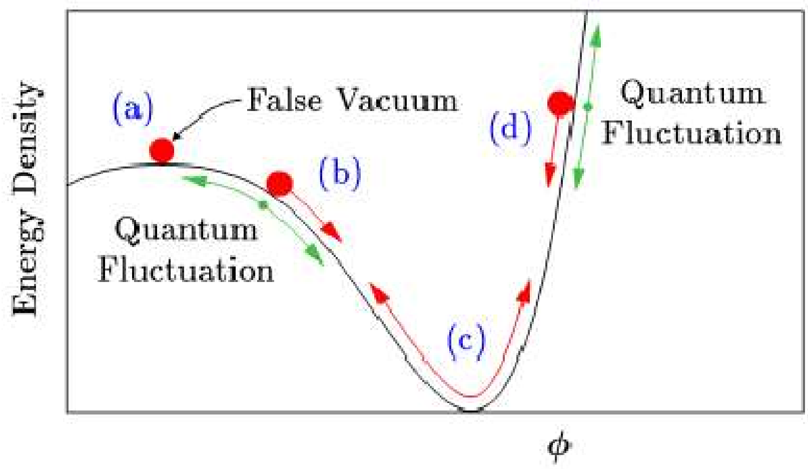
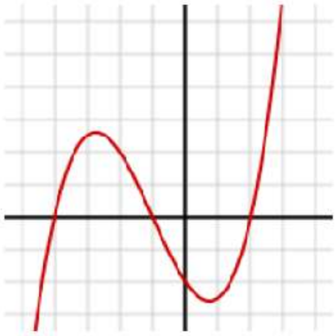


FIG. 1: In simple inflationary models, the universe at early times is dominated by the potential energy density of a scalar field, ϕ. Red arrows show the classical motion of ϕ. When ϕ is near region (a), the energy density will remain nearly constant, $\rho \cong \rho_f$, even as the universe expands. Moreover, cosmic expansion acts like a frictional drag, slowing the motion of ϕ: Even near regions (b) and (d), ϕ behaves more like a marble moving in a bowl of molasses, slowly creeping down the side of its potential, rather than like a marble sliding down the inside of a polished bowl. During this period of "slow roll," ρ remains nearly constant. Only after ϕ has slid most of the way down its potential will it begin to oscillate around its minimum, as in region (c), ending inflation.



Graph of a cubic function with 3 real roots (where the curve crosses the horizontal axis at $y = 0$). The case shown has two critical points. Here the function is $f(x) = (x^3 + 3x^2 - 6x - 8)/4$.

The ratio between M_0 and q

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3})\, M_s}{2}.$$

i.e. the gravitating mass M_0 and the Wheelerian mass q of the wormhole, is equal to:

$$\frac{\sqrt{3\,(2.17049 \times 10^{37})^2 - 0.001^2}}{\frac{1}{2}\,((3\sqrt{3})\,(4.2 \times 10^6 \times 1.9891 \times 10^{30}))}$$

1.732050787905194420703947625671018160083566548802082460520...

1.7320507879

$1.7320507879 \approx \sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q of the wormhole

We note that:

$$\left(-\frac{1}{2} + \frac{i}{2}\sqrt{3}\right) - \left(-\frac{1}{2} - \frac{i}{2}\sqrt{3}\right)$$

i is the imaginary unit

$i\sqrt{3}$

$1.7320508075688772935274463415058723669428052538103806280 55\ldots i$

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

1.73205

This result is very near to the ratio between M_0 and q, that is equal to 1.7320507879 $\approx \sqrt{3}$

With regard $\sqrt{3}$, we note that is a fundamental value of the formula structure that we need to calculate a Cubic Equation

We have that the previous result

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) = i\sqrt{3} =$$

$$= 1.732050807568877293527446341505872366942805253810380628055\ldots i$$

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

can be related with:

$$u^2(-u)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)+v^2(-v)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)=q$$

Considering:

$$(-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q$$

$$= i\sqrt{3} = 1.732050807568877293527446341505872366942805253810380628055\ldots i$$

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

Thence:

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) \Rightarrow$$

$$\Rightarrow (-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q=1.73205\approx\sqrt{3}$$

We observe how the graph above, concerning the cubic function, is very similar to the graph that represent the scalar field (in red). It is possible to hypothesize that cubic functions and cubic equations, with their roots, are connected to the scalar field.

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.
In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa *interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.*

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64 g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276 e^{-\pi\sqrt{22}} - \cdots, \\ 64 g_{22}^{-24} &= 4096 e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372 e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64 g_{22}^{-24} = 4096 e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372 e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

References

Simple geodesics and Weil-Petersson volumes of moduli spaces of bordered Riemann surfaces - *Maryam Mirzakhani*
Department of Mathematics, Princeton University, Fine Hall,Washington Road, Princeton, NJ 08544, USA (e-mail: mmirzakh@math.princeton.edu)
Oblatum 19-VII-2005 & 17-VII-2006 - Published online: 12 October 2006 – © Springer-Verlag 2006

Volumes And Random Matrices
Edward Witten - arXiv:2004.05183v1 [math.SG] 10 Apr 2020

Modular equations and approximations to π – *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372