The difference between a theory, a calculation, and an explanation

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Summary

In this paper, we try to show where and why quantum mechanics went wrong – and why and when the job of both the academic physicist as well as the would-be student of quantum mechanics turned into *calculating* rather than *explaining* what might or might not be *happening*. Modern quantum physicists effectively resemble econometrists modeling input-output relations: if they are lucky, they will get some kind of *mathematical description* of what goes in and what goes out, but the math does not tell them how stuff actually *happens*.

To show what an actual *explanation* might look like, we bring the *Zitterbewegung* electron model and our photon model together to provide a classical explanation of Compton scattering of photons by electrons so as to show what electron-photon interference might actually *be*: two electromagnetic oscillations interfering (classically) with each other. While developing the model, we also offer some reflections on the *nature* of the Uncertainty Principle.

Finally, we also offer a brief history of the bad ideas which led to the current mess in physics.

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The characteristics of a good theory

The science of physics probes reality *physically* as well as *logically*. Physical exploration involves experiments, measurements and discoveries. Logical progress occurs when a new theory or explanation replaces previously held beliefs and convictions. Such theory or explanation should be logically and mathematically consistent. The mathematical description, for example, should respect Occam's Razor—the *lex parsimoniae*: one should not multiply concepts without necessity. The need for new concepts or new principles – such as the conservation of strangeness, or postulating the existence of a new force or a new potential[1] – should, therefore, be continuously questioned.

When postulating the existence of the positron in 1928 – which directed experimental research to a search for it and which, about five years later, was effectively found to exist – Paul Dirac unknowingly added another condition for a good theory: all of the *degrees of freedom* in the mathematical description should map to a physical *reality*.

The mention of Dirac's *hole theory* makes us think we could, perhaps, usefully present the idea of energy states in a rather intuitive way before requesting the reader to wrestle his or her way through the nitty-gritty of this paper.

Positive and negative energy states

Dirac's positron hypothesis is often said to be based on a hole theory. Dirac himself, however, phrased his theory of antimatter in terms of *negative* energy states. It is probably useful to quote his original brief writing on it: "The wave equation for the electron admits of twice as many solutions as it ought to, half of them referring to states with negative values for the kinetic energy."[2]

The concept of *negative* energy is, of course, nothing special in physics—but *negative* energy will usually

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[1] We are, obviously, referring to the *invention* of the concept of strangeness by Murray Gell-Man and Kazuhiko Nishijima in the 1950s. Feynman's treatment of it in his 1963 *Lectures* on physics shows that the concept of strangeness – and the related conservation law – is a rather desperate assumption to explain the decay of K-mesons (kaons). Unfortunately, the concept of strangeness started a strange life of its own and would serve as the basis for the quark hypothesis which – for a reason I find even stranger than the concept of strangeness itself – was officially elevated to the status of *a scientific dogma* by the Nobel Prize Committee for Physics.

As for the invention of a new force or a new potential, we are, obviously, referring to the Yukawa potential. This hypothesis – which goes back to 1935 – might actually have been productive if it would have led to a genuine exploration of a stronger short-range force on an electric charge—or, if necessary, the invention of a *new* charge. Indeed, if the electromagnetic force acts on an electric charge, it would be more consistent to postulate some new charge – or some new wave equation, perhaps – matching the new force. Unfortunately, theorists took a whole different route. They invented a new *aether* theory instead: it is based on the medieval idea of messenger or *virtual* particles *mediating* forces.

[2] Paul A.M. Dirac, *Principles of Quantum Mechanics*, 4th revised edition (1958), section 73: *Theory of the positron*.

be associated with *potential* energy rather than with kinetic energy.[3] The question, therefore, becomes: how can kinetic energy be negative? Dirac was very much puzzled by it. Indeed, he refers to his wave equations as *equations of motion* and the question, therefore, becomes: how can motion be *negative*?

The answer is this: motion cannot be negative but its direction is that of the *force*, so its direction can be this or that which, using our right-hand rule or Minkowski's *metric signature* convention, we call positive or negative. We will explain this in more detail later but, ultimately, the reader should think of energy as *a force over some distance*. The motion may be linear but, as Dirac noted in his Nobel Lecture, "an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us."[4]

Dirac is talking about the *spin* property here, and spin is either in this or in the opposite direction. This explains Dirac's *negative* kinetic energy. It is *not* negative, really. It just appears because of a deep conceptual flaw in the mainstream interpretation of quantum mechanics. We will come back to this later, but the gist of the matter is this:

The mainstream interpretation of quantum mechanics does *not* integrate the concept of particle *spin* from the outset because it thinks of the + or – sign in front of the imaginary unit (*i*) in the elementary wavefunction ($a \cdot e^{-i \cdot \theta}$ or $a \cdot e^{+i \cdot \theta}$) as a mathematical convention only.

Indeed, most introductory courses in quantum mechanics will show that both $a \cdot e^{-i \cdot \theta} = a \cdot e^{-i \cdot (\omega t - kx)}$ and $a \cdot e^{+i \cdot \theta} = a \cdot e^{+i \cdot (\omega t - kx)}$ are acceptable waveforms for a particle that is propagating in a given direction (as opposed to, say, some real-valued sinusoid). One would then expect that the professors would proceed to provide some argument showing why one would be better than the other, or some discussion on why they might be different but, *no*! That is not the case. The professors usually conclude that "the choice is a matter of convention" and, that "happily, most physicists use the same convention."[5]

We are totally dumbfounded by this. All of the great physicists knew all particles – elementary or not –

[3] The idea is usually this: the potential energy of a charge at some large distance (infinity, for calculational purposes) is *defined* as zero. When bringing that charge closer to some other charge, work is done with or against the force. Its energy is, accordingly, negative or positive. The potential energy of an electron in an electron orbital, for example, is, obviously, *negative*.

[4] Paul A.M. Dirac, *Theory of electrons and positrons*, Nobel Lecture, 12 December 1933. Needless to say, we associate the *elementary* wavefunction with this motion. To be precise, we think the $a \cdot e^{-i \cdot \theta}$ or $a \cdot e^{+i \cdot \theta}$ function gives us the spatial position of the oscillating elementary *charge* at every point in time. In other words: the elementary wavefunction *represents* what Erwin Schrödinger referred to as the *Zitterbewegung* of the electron. The *Zitterbewegung* of an electron is a *physical* application of Wheeler's 'mass without mass' idea: all of the energy (and, therefore, all of the equivalent mass) of the electron is in the oscillatory motion of the *Zitterbewegung* (zbw) or *elementary charge*. This explains the dual wave-particle character of an electron and, as such, it also explains diffraction and the interference of an electron with itself when being forced through a single or double slit. As mentioned, we will come back to this later in the text and the reader should, therefore, just make a mental note of this and proceed his or her reading.

[5] In case you wonder, this is a quote from the MIT's edX course on quantum mechanics (8.01.1*x*). We quote this example for the same reason as why we use Feynman's *Lectures* as a standard reference: it's authoritative and, more importantly, also available online so the reader can check and explore for himself.

have spin.[6] All of them also knew it is spin that generates the magnetic moment. They should, therefore, have integrated the idea of spin in their mathematical description *from the outset*. The mistake is illustrated below.

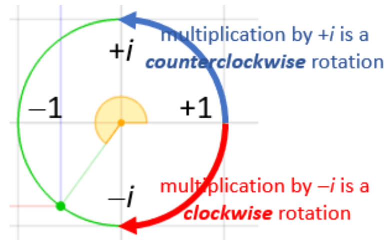

Figure 1: The meaning of +*i* and –*i*

It is a very subtle but fundamental blunder. Spin-zero particles do not exist.[7] All *real* particles – electrons, photons, anything – have spin, and spin (a shorthand for angular momentum) is always in one direction or the other: it is just the *magnitude* of the spin that differs. It is, therefore, completely odd that the plus (+) or the minus (–) sign of the imaginary unit (*i*) in the $a \cdot e^{\pm i\theta}$ function was *not* used to include the spin *direction* in the mathematical description from the start.

We do not hesitate to call it a blunder or a *mistake* rather than just an oddity because it is much more than just odd: it's plain *wrong*, indeed. Why? Because it has a lot of sad consequences. For example, this non-used *degree of freedom* in the mathematical description also leads to the *false* argument that the wavefunction of spin-½ particles has a 720-degree symmetry. Indeed, physicists treat –1 as a *common phase factor* in the argument of the wavefunction: they think we can just multiply a set of amplitudes – let's say *two* amplitudes, to focus our mind (think of a beam splitter or alternative paths here) – with –1 and get the same *states*.

We find it rather obvious that that is not necessarily the case: –1 is *not* necessarily a common phase factor. We should think of –1 as a complex number itself: the phase factor may be +π or, alternatively,

[6] Even neutrons have a magnetic moment and, therefore, spin! One should also note that photons – despite their spin-one property – cannot not come in a zero-spin state. When studying Feynman's *Lectures*, I found that to be one of the weirdest things: Feynman first spends several chapters on explaining spin-one particles to, then, in some obscure footnote suddenly write this: "The photon is a spin-one particle which has, however, no "zero" state." The issue is related to another glaring inconsistency: in the first three chapters of his *Lectures* on physics, he talks about adding wavefunctions and the basic rules of quantum mechanics, and it all happens with a plus sign. However, in his chapter on the theoretical distinction between bosons and fermions, he suddenly says we should be adding the amplitudes of fermions combine with a minus sign. In any case, we cracked our own jokes on the boson-fermion theory and so we should leave it at that.

We should add one note on the zero-spin state of helium, however: the electrons and protons inside the helium atom will *line up* in a way that minimizes energy. The helium atom, as a *composite* particle, may, therefore, be said to come in a zero-spin state. One may think of this situation as the exception which confirms the general rule which applies to *all* charged particles—including composite particles, neutral or not. [The latter idea – that of a neutral charged particle – obviously refers to a composite particle in which there are as many positive as negative electric charges.]

[7] We are obviously not talking about the zero-spin state of, say, a spin-one atom here. We talk about *elementary* particles only here.

$-\pi$. To put it simply, when going from +1 to -1, it matters how you get there – and vice versa – as illustrated below.[8]

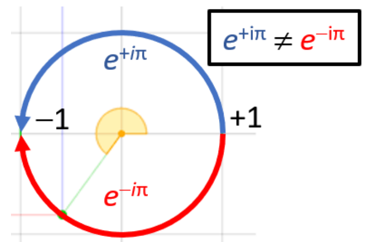

Figure 2: $e^{+i\pi} \neq e^{-i\pi}$

We will come back to all of this later – as part of a more detailed discussion of the meaning of quintessential concepts in quantum mechanics – so we will let this matter rest for the time being. Before we really get into the nitty-gritty, we'd just like to make another introductory remark on positive and negative energy states so as to make sure the reader has a bit of an intuitive grasp of our rather *physical* interpretation of what is usually presented as abstract concepts only.

Positive and negative energy wells

The reader should note that the property of the spin being *up* or *down* is always defined with reference to an external electric or magnetic field (or some combination thereof, of course).[9] The energy being positive or negative can, therefore, only be defined with reference to the direction of the magnetic field. Positrons should, therefore, not be thought of as holes. Similarly, we may note physicists always think of particles being trapped in a potential well with lower energy. However, a particle may well be trapped in a potential well with *higher* energy. This situation is illustrated in Feynman's very discussion of a two-state system, which we summarize by the illustrations below.[10]

[8] The quantum-mechanical argument is technical, and I did *not* reproduce it in this book. I encourage the reader to glance through it, though. See: *Euler's Wavefunction: The Double Life of – 1*. Note that the $e^{+i\pi} \neq e^{-i\pi}$ expression may look like *horror* to a mathematician! However, if he or she has a bit of a sense for geometry and the difference between identity and equivalence relations, there should be no surprise. If you're an amateur physicist, you should be excited, because it actually *is* the secret key to unlocking the so-called mystery of quantum mechanics. Remember Aquinas' warning: *quia parvus error in principio magnus est in fine*. A small error in the beginning can lead to great errors in the conclusions, and we sure think of this as an error in the beginning!

[9] Simple experiments – such as the Stern-Gerlach experiment – involve a magnetic or an electric field *only*. However, many experiments and apparatuses – such as the Penning trap, for example – will involve a combination of both.

[10] The illustration was taken from Feynman's *Lecture* on the ammonia maser. We gratefully acknowledge Michael Gottlieb and Rudolf Pfeiffer, who have worked for decades to get those *Lectures online* and, therefore, claim copyright on Feynman now.

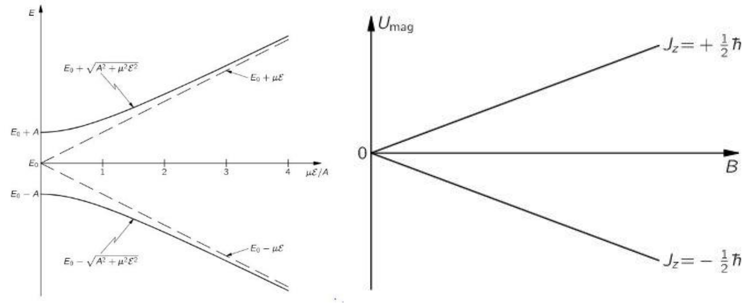

Figure 3: Two trivial examples of two-state systems in quantum mechanics

The illustration on the left shows the two possible energy states of an ammonia molecule (NH_4) in the context of a discussion of how an ammonia maser works – the ammonia molecule basically oscillates between two states that have an opposite *electric* dipole moment[11] – while, on the right, we have an illustration of the splitting of energy levels in a *magnetic* field. We might have added an illustration of the spin of a *proton* can flip between an *up* and a *down* state in an MRI apparatus[12] or other diagrams illustrating the basics of a *two-state system* in quantum mechanics, but we don't want to be too long here. Paraphrasing Feynman once again, we may say the point is this:

> No matter what the physics is originally—the ammonia molecule in the ammonia maser, laser technology, the fine or hyperfine splitting of the hydrogen spectrum, proton spin flipping in MRI technology, or whatever—you can translate it into a corresponding electron problem. So if we can solve the electron problem in general, we have solved all two-state problems. And we solved the electron problem. Everything else is, therefore, very easy to understand.[13]

In case you wonder what we want you to remember going to the next, it is this: don't think too much about the usual hocus-pocus involved in tunneling in and out of potential wells or walls and other

[11] The field that separates the two states is, therefore, *electric*. Other typical quantum-mechanical systems will typically involve *magnetic* dipoles which are, therefore, separated by magnetic fields.

[12] Indeed, most of us should be very familiar with the proton spin flipping mechanism because of its use in magnetic resonance imagining. I must assume that – if you have ever had an MRI – you also *googled* and watched one or more YouTube videos explaining the physics underpinning this amazing technology. If not, I warmly recommend you do so. It will give you a better intuitive grasp of the basics of quantum mechanics than *any* of the standard introductory courses. For starters, it gives you a great *engineering* feel for what might or might not be going on. MRI machines do not only manipulate the *direction* of the proton spin but – in order to *localize* the region that is being scanned, it also manipulates its *phase*. It is all very sophisticated but beautiful to watch. If anything, it shows how *real* spin – the motion of the elementary charge within a proton or an electron – actually is. You can *google* some videos for yourself but, in case you'd appreciate some advice here, we would single out the one from Doctor Klioze: https://www.youtube.com/watch?v=djAxjtN_7VE. It is almost half an hour long, but well worth spending the time! *Enjoy!*

[13] We refer the reader to section 7 of Chapter 10 of Feynman's *Lectures* on quantum mechanics for a comparison with Feynman's original quote. Back in 1963, Feynman modeled proton spin using the Pauli matrices, which is still very useful, of course!

quantum-mechanical *blah*. Once you have an intuitive understanding of what spin actually *is*, everything can be explained in very simple terms.[14]

The nature of antimatter

The reader may or may not find the introductory remarks above intuitive and self-evident, but the question which puzzled Dirac remains, of course: what is the *nature* of antimatter, really? We have no *definite* answer to that question but we do have a few thoughts that may amount to *some* answer, at least.

First, you should note that the suggestion of some physicists – Richard Feynman, in particular – that anti-particles may be particles travelling back in time is plain nonsense. Time has one direction only, and we will soon explain why.[15] But what is the alternative? As explained above, Dirac's suggestion that, somehow, antimatter may be defined by referring to the concept of *negative* energy states isn't much of an explanation.

The CODATA value[16] of the magnetic moment of an electron is $\mu \approx -9.2847647043(28)\times10^{-24}\ \text{J}\cdot\text{T}^{-1}$. Its theoretical value is $\mu = -q_e\hbar/2m$.[17] (its theoretical value). The minus sign is there because of the *negative* (elementary) charge of the electron. Indeed: q_e is the *elementary* charge that's associated with a proton or a *positron* – the latter being the electron's antimatter counterpart – and so it equal to *minus* the electron charge.

Now, consider a particular direction of the elementary current generating the magnetic moment. [In case you wonder: that's what spin actually *is*: it's an elementary *current*.[18]] It is then quite easy to see

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[14] We refer the reader to our papers for what we usually refer to as a *classical* explanation of various quantum-mechanical phenomena, including Mach-Zehnder interference and other previously mysterious experimental results. As for *tunneling* through a wall, think of this: when you are going to crash a projectile into a wall, it is not only its momentum that is going to matter. The *direction* of your projectile is going to matter at least as much, doesn't it? Taking a hit directly or sideways make a lot of difference, doesn't it? 😊

[15] We will do so in one of the next section(s). As for now, the reader may just want to note our argument is mathematical and philosophical: the very idea of motion includes the idea of moving in time, but in one direction only. A particle may go back to a place where it was *before*, or it may stay where it *was*, but if we'd allow it to go back *in time*, we would not be able to capture its motion in a well-behaved function. Well-behaved equations of motion imply the same particle may re-visit the same place in space, but not the same place *in time*. We do not need any complicated arguments in terms of entropy or – worse – imagined symmetries that can*not* exist *in reality*.

[16] The CODATA value is its *measured* value as published by the Committee on Data (CODATA) of the International Science Council. This Committee regularly makes informed decisions on what experiments count (or don't count) and, more importantly, to what degree of precision their results are supposed to be valid.

[17] You are, of course, aware of the so-called *anomaly* in the magnetic moment. It is a difference between the theoretical and measured value. The difference is of the order of Schwinger's factor ($\alpha/2\pi$). Mainstream physicists explain it using quantum field theory. We explain it by assuming the *Zitterbewegung* charge has a tiny but non-zero size itself.

[18] We are aware this may sound shocking to those who have been brainwashed in the old culture. If so, make the switch. It should not be difficult: a magnetic moment – *any* magnetic moment, really – is generated by a current. The magnetic moment of elementary particles is no exception.

that the magnetic moment of an electron ($\mu = -q_e\hbar/2m$) and that of a positron ($\mu = +q_e\hbar/2m$) would be opposite. We may, therefore, associate a particular direction of rotation with an angular frequency *vector* **ω** which – depending on the direction of the current – will be up or down with regard to the plane of rotation.[19] We can, therefore, associate this with the spin property, which is also up or down. We, therefore, have four possibilities[20]:

Matter-antimatter	**Spin up**	**Spin down**
Electron	$\mu_{-e} = -q_e\hbar/2m$	$\mu_{-e} = +q_e\hbar/2m$
Positron	$\mu_{+e} = +q_e\hbar/2m$	$\mu_{+e} = -q_e\hbar/2m$

Table 1: Occam's Razor: mathematical possibilities versus physical realities (1)

The table shows the ring current model also applies to antimatter. It also shows that antimatter has a different *spacetime* signature. Abusing Minkowski's notation, we may say the spacetime signature of an electron would be + + + + while that of a positron would be + – – –.[21]

Of course, we readily admit this cannot be the *complete* answer: the difference between a positive and a negative *Zitterbewegung* charge cannot be reduced to its direction of motion only! Why not? The answer is obvious: try to bring two electrons together, and then try to bring an electron and a positron together, and you will see two *very* different things happening. 😊 Hence, we obviously do have an unexplained *intrinsic* property here. Indeed, we need the idea of positive and negative charge here! Why? Because the magnetic moment *alone* does not allow one to distinguish between an electron with spin *up* and a positron with spin *down*.

Is this good? We think it is. Is it *good enough*? We are not sure. Intrinsic properties need an explanation, don't they? Yes, most probably.[22] However, we are where we are right now and so we cannot add much to this discussion for the time being. Having said that, the reader will have to admit we were able to go way beyond Dirac's rather native *hole theory* here: in the electron with spin *up*, we have the same current as in the positron with spin *down*, but it is because we have *an opposite charge* (negative instead of positive) *spinning in the opposite direction* (up instead of down, or right versus left—whatever you want to call it). Hence, *negative* energy states have to do with the *direction* of the

[19] To determine what is up or down, one has to apply the ubiquitous right-hand rule.

[20] The use of the subscripts in the magnetic moment may be confusing, but shouldn't be: we use –e for an electron and +e for a positron. We do so to preserve the logic of denoting the (positive) elementary charge as q_e (without a + or a – in the subscript here).

[21] In case the reader wonders why we associate the + +++ signature with the *positron* rather than with the electron, the answer is: convention. Indeed, if I am not mistaken (which may or may not be the case), it is the + +++ *metric signature* which is the one which defines the usual righthand rule when dealing with the *direction* of electric currents and magnetic forces.

[22] If we should summarize our quest for truth in physics, we might say this was our drive: we wanted an explanation for these so-called *intrinsic* properties of elementary particles: their mass or energy, their *size*, and their magnetic moment. We think we have succeeded in finding that explanation. Having said that, we must admit the physicality of pair creation and annihilation still puzzles us deeply. We therefore think we greatly *reduced* the Mystery, but we sure did not *eliminate* it completely!

force.[23] Antimatter seems to have a different *metric* signature than matter.[24]

That is, for the time being, all we can say about it.[25] Let us get into the meat of the matter now—almost literally speaking! 😊

The Emperor wears no clothes: matter does *not* dissipate

We already mentioned how science progresses: it does so by probing reality *physically* as well as *logically*. Physical exploration involves experiments, measurements and discoveries. Logical progress occurs when a new theory or explanation replaces previously held beliefs and convictions.

We said such theory or explanation should be logically and mathematically consistent. We should add another requirement: it should also be applicable to a great variety of facts. Newton's force law[26] and theory of gravitation, Maxwell's equations[27], and Einstein's theory of relativity[28] are obvious examples of

[2323] If you find this statement confusing, try to think of it like this: the current that generates the magnetic moment in an electron versus a proton must be different: one carries a negative charge, and the other carries a positive charge. The *direction* of the *physical* current (the *motion* of the *zbw* charge, positive or negative) must, therefore, be opposite. The question then becomes: what distinguishes the positive and a negative *zbw* charge inside the *zbw* electron and positron? For the latter question, we have no answer: that is why we think of the property of the charge being positive or negative as being *intrinsic*. In other words, we have an explanation for the *direction* of motion and, therefore, for its fundamental *nature*. However, we have no explanation for the *nature* of the charge itself.

[24] We relate this to another proposition in the Annex to this paper: our interpretation of Planck's constant as a *vector* quantity—so we would write it as $\boldsymbol{h}$ rather than h. When thinking about natural units and imposing the condition that the magnetic and electric constant should be each other's reciprocal, we get the he $h^2 = 1$ condition (see the section on The relative strength of the electric and magnetic field in the Annex). This effectively corresponds to two different possibilities: $h = 1$ or $h = –1$. The $h = 1$ possibility corresponds to matter, while the $h = –1$ corresponds to antimatter. It is nothing but a reversal of spatial directions—both linear as well as rotational (or circular) directions.

[25] We offer some more *very* speculative thoughts on the nature of antimatter in our paper on *The Ring Current Model for Antimatter and Other Questions*, but we do not want to mention these here because we feel these ideas do not make all that much sense.

[26] Newton's force law is also referred to as the law of motion. The idea of a force and the idea of motion are inextricably linked: a force is that what changes the state of motion of an object, and mass is a measure of *inertia* or *resistance* to such change. A force changes the state of motion by acting on a *charge*. [Of course, you may think that, in many cases, a force does not necessarily change the state of motion of an object: it may also keep something together. As Feynman notes, "The Empire State Building swings less than one inch in the wind because the electrical forces hold every electron and proton more or less in its proper place." However, even this should be thought of as motion: Dirac, at least, referred to Schrödinger's equation as an equation of motion for the electrons.]

Historically, the idea of a force acting on a (positive or negative) *charge* came later: it is usually associated with Benjamin Franklin's experiments around 1750 and – more systematically, perhaps – the work of Charles-Augustin de Coulomb, who published three *memoirs* on electromagnetism in 1785. Newton's *Philosophiæ Naturalis Principia Mathematica* (Mathematical Principles of Natural Philosophy) were first published in 1687: scientific revolutions take time. After a hundred years or so, it seems the quantum-mechanical one hasn't ended yet.

[27] Maxwell's equations show the electric and magnetic forces are one electromagnetic force only.

[28] This is a reference to Einstein's special relativity theory, which may be said to correct Newton's law of motion so as to ensure consistency with Maxwell's equations, which show the velocity of light is *absolute*: it is always equal

such theories or explanations. However, **we do not count Heisenberg's quantum mechanics among such great theories or explanations.**

Why not? First, we think of it as an *approach* more than like a theory—some kind of problem-solving *algorithm*, if you want. Second, even as an approach or an algorithm, it would seem to be based on the wrong assumptions. Why? We will not ask you to think about some *thought experiment* here. No. We ask you to just think for yourself about the following *common-sense* remarks.

Prof. H. Pleijel, then Chairman of the Nobel Committee for Physics of the Royal Swedish Academy of Sciences, dutifully notes the following on the nature of the new 'matter waves' in the ceremonial speech for the 1933 Nobel Prize, which was awarded to Heisenberg for nothing less than "*the creation of quantum mechanics*"[29]:

> "Matter is formed or represented by a great number of this kind of waves which have somewhat different velocities of propagation and such phase that they combine at the point in question. Such a system of waves forms a crest which propagates itself with quite a different velocity from that of its component waves, this velocity being the so-called group velocity. Such a wave crest represents a material point which is thus either formed by it or connected with it, and is called a wave packet. […] As a result of this theory on is forced to the conclusion to conceive of matter as not being durable, or that it can have definite extension in space. The waves, which form the matter, travel, in fact, with different velocity and must, therefore, sooner or later separate. Matter changes form and extent in space. The picture which has been created, of matter being composed of unchangeable particles, must be modified."

This should sound very familiar to you. If so, you should ask yourself this question: **why is this everything *but* true?** Real-life particles – electrons or atoms traveling in space – don't dissipate. ***Matter does not change form and extent in space!***[30]

We trust the academic physicist will switch off and stop reading here—if he or she hasn't already. That's fine. We're not writing this paper for the academic physicist. We write it for *you*, so *you* should think about it. Show us *one* experiment in which matter obviously does change form and extent in space. Matter may *disintegrate*, of course, but that's something entirely *different*, and it only applies to

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to *c*—in the inertial as well as in the moving frame of reference. Einstein's general relativity theory explains gravity in terms of the geometry of spacetime. We do not consider gravity in this paper and, as such, we do not want to highlight this theory for the time being.

[29] To be precise, Heisenberg got a postponed prize from 1932. Erwin Schrödinger and Paul A.M. Dirac jointly got the 1933 prize. Prof. Pleijel acknowledges all three in the introduction of his speech: "This year's Nobel Prizes for Physics are dedicated to the new atomic physics. The prizes, which the Academy of Sciences has at its disposal, have namely been awarded to those men, Heisenberg, Schrödinger, and Dirac, who have created and developed the basic ideas of modern atomic physics."

[30] We will slightly *nuance* this statement later but we will not fundamentally alter it. We think of matter-particles as an electric charge in motion. The centripetal force that keeps the particle together is electromagnetic in nature, because it acts on a charge. Matter-particles, therefore, combine wave-particle duality. Of course, it makes a difference when this electromagnetic oscillation, and the electric charge, move through a slit or in free space. We will come back to this later. The point to note is: matter-particles do not dissipate. Feynman actually notes that at the very beginning of his *Lectures* on quantum mechanics, when describing the double-slit experiment for electrons: *"Electrons always arrive in identical lumps."*

composite non-stable particles which – for that reason – shouldn't be referred to as particles: call them resonances or *transients* instead!

Indeed, we refer to particles as particles *because* of their *stability* and *integrity*—and that integrity is defined by the Planck-Einstein relation: E = $h \cdot f$.[31] Nothing more. Nothing less. It's all we need as an explanation. A rather smart young man recently wrote this rather scathing criticism of his own profession—physics:

> "QED should be the quantized version of Maxwell's laws, but it is not that at all. QED is a simple addition to quantum mechanics that attempts to justify two experimental discrepancies in the Dirac equation: the Lamb shift and the anomalous magnetic moment of the electron.[32] The reality is that QED is a bunch of fudge factors, numerology, ignored infinities, hocus-pocus, manipulated calculations, illegitimate mathematics, incomprehensible theories, hidden data, biased experiments, miscalculations, suspicious coincidences, lies, arbitrary substitutions of infinite values and budgets of 600 million dollars to continue the game. Maybe it is time to consider alternative proposals. Winter is coming."[33]

Prof. Dr. John P. Ralston practices the same self-criticism in a recent publication: "*Quantum mechanics is the only subject in physics where teachers traditionally present haywire axioms they don't really believe, and regularly violate in research*."[34]

You think such *rebel speech* is unconvincing—*sacrilegious*, perhaps? Let us quote a proper Nobel Prize winner then.[35] Willis E. Lamb got a Nobel Prize in 1955 for, yes, the discovery of the Lamb shift in the

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[31] The interpretation of the Planck-Einstein relation is another matter, but one doesn't need matter-waves for it. The Planck-Einstein relation basically tells us particles have a unique frequency: a *clock speed* which is equal to the cycle time $T = 1/f = h/E = h/mc^2$. Combining this idea with the *reality* of the magnetic moment of elementary particles, one arrives at the conclusion the frequency must be equal to the frequency of the motion of the electric charge, which both Erwin Schrödinger as well as Paul A.M. Dirac paid much attention to. We will come back to this.

[32] The reader should note that we have very decent classical explanations for both. See our paper on the electron and the anomalous magnetic moment as well as our paper on the Lamb shift.

[33] Oliver Consa, *Something is Rotten in the State of QED*, February 2020.

[34] John P. Ralston, *How To Understand Quantum Mechanics*, 2017. We must add that, while we find Dr. Ralston's criticism of his own profession fairly enlightening, we do *not* think his own interpretation of quantum mechanics – yet another model, yes – is very convincing. Not at all, actually. :-/

[35] We could have quoted other *founding fathers* of quantum mechanics who ended up skeptical about the theory they had created (or Einstein, of course). Dirac, for example, was rather vocal about it. However, we opted to quote W.E. Lamb because he is, perhaps, one of the lesser known—together with John Stewart Bell. The case of J.S. Bell is very particular, because he was nominated for a Nobel Prize for the (in)famous No-Go Theorem, which tells us there are no hidden variables that can explain the interference in some kind of classical way? Bell's Theorem is what it is: a mathematical theorem. Nothing more. Nothing less. It, therefore, respects the GIGO principle: garbage in, garbage out. John Stewart Bell himself – a third-generation quantum physicist, we may say – did not like his own 'proof' and thought that some "radical conceptual renewal" might disprove his conclusions (John Stewart Bell, *Speakable and unspeakable in quantum mechanics*, pp. 169–172, Cambridge University Press, 1987). J.S. Bell kept exploring alternative theories – including Bohm's pilot wave theory, which is a hidden variables theory – until his death at a relatively young age (the year when he got nominated for the Nobel Prize, in fact).

spectrum of the hydrogen atom.[36] When he was well over 80, he wrote a rather remarkable paper on the (non)sense of the theoretical explanations of the phenomenon he had experimentally discovered.[37] The title of the paper is, significantly, *Anti-Photon*, and, in this paper, he politely blames Bohr rather than Heisenberg for all of these weird theories we are still struggling to understand:

> "The Complementarity Principle and the notion of wave-particle duality were introduced by N. Bohr in 1927. They reflect the fact that he mostly dealt with theoretical and philosophical concepts, and left the detailed work to postdoctoral assistants. It is very likely that Bohr never, by himself, made a significant quantum-mechanical calculation after the formulation of quantum mechanics in 1925-1926."[38]

In fact, it is hard to see what Heisenberg's matrix mechanics actually explain. Pretty much nothing, perhaps. Indeed, we may want to think of it as a *calculation* rather than as a theory. We may illustrate this by referring to Heisenberg's pioneering of the scattering matrix, which is now more generally referred to as the *S*-matrix in quantum mechanics.[39] Scattering is actually a somewhat misleading term because it may refer to *any* interaction process. As usual, we can rely on Richard Feynman for a more vivid description of what an *S*-matrix actually is:

> "The high-class theoretical physicist working in high-energy physics considers problems of the following general nature (because it's the way experiments are usually done). He starts with a couple of particles, like a proton and a proton, coming together from infinity. (In the lab, usually one particle is standing still, and the other comes from an accelerator that is practically at infinity on atomic level.) The things go crash and out come, say, two K-mesons, six π-mesons, and two neutrons in certain directions with certain momenta. What's the amplitude for this to happen? The mathematics looks like this: The φ-state specifies the spins and momenta of the incoming particles. The χ would be the question about what comes out. For instance, with what amplitude do you get the six mesons going in such-and-such directions, and the two neutrons going off in these directions, with their spins so-and-so. In other words, χ would be specified by giving all the momenta, and spins, and so on of the final products. Then the job of the theorist is

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[36] We should be precise: Willis Eugene Lamb Jr. got *half* of the 1955 Nobel Prize in Physics for his discovery. He had to share it with Polykarp Kusch. To be precise, Lamb got his half of the prize "for his discoveries concerning the fine structure of the hydrogen spectrum" (which is the Lamb shift), while Polykarp Kusch got it "for his precision determination of the magnetic moment of the electron" (the so-called anomaly in the magnetic moment). The reader can look it all up on the website of the Nobel Prize for Physics.

[37] One should, indeed, note that both Lamb as well as Kusch had *measured* the Lamb shift and the anomaly respectively, but both had left the explaining of it to (other) physicists—some more famous names you probably are more acquainted with, such as Julian Schwinger and Richard Feynman, who – together with Tomonaga – got a Nobel Prize in physics "for their fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles"—read: their *explanations* of this and other previously unexplained measurements.

[38] See: W.E. Lamb Jr., *Anti-photon*, Appl. Phys. B 60, 77-84 (1995).

[39] The *S* in *S*-matrix refers to scattering, not to the German *Spur* (the *trace* of a matrix) or some other term. I am inserting this rather pedantic note because part of the mystery of quantum mechanics is stuff like this: abbreviations or concepts that never get a proper definition or an accurate portrayal of the archaeology or genealogy of the idea. According to the Wikipedia article on it, a proper *S*-matrix was first introduced by John Archibald Wheeler in a 1937 article on the composite wavefunction for multi-particle nuclei.

to calculate the amplitude ⟨ χ | *S* | φ ⟩."[40]

The reader may or may not recognize the latter admonishment. The job of the theorist – or of the student in quantum mechanics – is to *calculate* rather than to *think* about what might or might not be *happening*.[41]

We will come back to this, and in much boring detail, I am afraid. Before we get into the nitty-gritty – and because many readers may not want to venture that far – we will first offer some preliminary remarks on the nature of the Uncertainty Principle.

The (Un)Certainty Principle

What is uncertainty in quantum mechanics? How does it relate to the philosophical concepts of freedom and determinism? When discussing the philosophical implications of the Uncertainty Principle in quantum mechanics, Richard Feynman notes that, "*from a practical point of view*", we have "indeterminability in classical mechanics as well."[42] He gives the example of water splashing over a dam:

> "If water falls over a dam, it splashes. If we stand nearby, every now and then a drop will land on our nose. This appears to be completely random, yet such a behavior would be predicted by purely classical laws. The exact position of all the drops depends upon the precise wigglings of the water before it goes over the dam. How? The tiniest irregularities are magnified in falling, so that we get complete randomness. Obviously, we cannot really predict the position of the drops unless we know the motion of the water absolutely exactly."

While it is, effectively, not *practically possible* to know the motion of the water "*absolutely exactly*", the deeper question is: is it *theoretically possible*? In classical mechanics, we assume it is. In quantum mechanics, we are supposed to assume it is *not*. Quoting Feynman once again, we may say that "the Uncertainty Principle *protects* quantum mechanics." Why should this be so?

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[40] See: *How states change with time*, Feynman's *Lectures*, Vol. III, Chapter 8, section 4.

[41] Lamb's remark on Niels Bohr leaving the actual calculations to postdocs may or may not reflect a remark of Feynman – I can't recall if it's in his *Lectures* or in his *Strange Theory of Light and Matter* – on the rather extraordinary amount of calculus one needs to calculate anything based on quantum field theory or Feynman's own path integral formulation of quantum mechanics and the fact that, therefore, such calculus is usually left to doctoral students who, in Feynman's words, have to "keep going at it for their PhD." We need to add a lot of the heavy lifting on the discovery of the Lamb shift was actually done by one of Lamb's own postdoc students, R. Retherford, who did not get much recognition for it—except for a brief mention in Lamb's Nobel Prize speech.

As for the calculations themselves, we may briefly mention that Stefano Laporta claims – in a 2017 article (other researchers have come up with similar articles in the meanwhile) – to have calculated 891 *four*-loop contributions to the anomalous magnetic moment, resulting in a precision of 'up to 1100 digits of precision' of the electron's *actual* g-ratio (slightly different from a theoretical value of 2, which is – in my not-so-humble view – simply just is what it is: a theoretical value based on mathematical idealizations). One gets an uncanny feeling here: if one has to calculate a zillion integrals all over space using 72 third-order diagrams to calculate the 12th digit in the anomalous magnetic moment, or 891 fourth-order diagrams to get the next level of precision, then there might something wrong with the theory—read: our mathematical idealizations. We refer to our own paper(s) for a more classical explanation of the anomalous magnetic moment.

[42] See: Feynman's Lectures, III-2-6.

There is no reason whatsoever to assume the uncertainty in classical mechanics is any different from the uncertainty in quantum mechanics. The uncertainty in quantum mechanics is not anything more than the *Ungenauigkeit*—*imprecision*, which is the term Heisenberg first used to describe the apparent randomness in our mathematical description of Nature—we encounter in classical mechanics as well. It is just an imprecision, indeed—as opposed to the weird metaphysical quality which Heisenberg would later claim it to be and which, without any precise definition, physicists now refer to as 'uncertainty'. The concept of 'uncertainty' effectively mixes two different connotations:

(1) an *imprecision* in our measurements that is due to the wave-particle nature of electrons and other elementary particles—a duality which the ring current model[43], combining the idea of charge and its motion, captures from the outset (as opposed to trying to cope with the duality in some wave equation or other *dual* mathematical descriptions); and
(2) a rapid *magnification* of what Feynman refers to as 'the tiniest irregularities' but which are, *in practice*, purely statistical.

This is, effectively, reflected in what Feynman refers to as the 'great equation of quantum mechanics'—the wave equation for an electron in free space:

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}=\frac{\mathrm{m}^2c^2}{h^2}\phi$$

The problem is: it is *not* a wave equation for an electron. This wavefunction does not *model* an electron: it models our rapidly increasing *uncertainty* about where it might be at a later point in time.

Why does it increase so rapidly? It's just scale: Planck's quantum of action can (also) be written as the product of momentum over some distance ($h = \mathrm{p}\cdot\lambda$)[44] and, in light of the very tiny distance scale (λ), the momentum (p) is, effectively, *relatively* large—which is why any imprecision in its *magnitude* or *direction* quickly translates into huge uncertainties concerning the actual *position* of the elementary particle that we are trying to describe.

We will revisit all of this in very much detail later. In order to do so, we must talk some more about what quantum mechanics is and isn't. Before we do so, we briefly want to think a rather weird and totally non-scientific question: what made Heisenberg change his thoughts about the *Ungenauigkeit* in *practical* experiments? **Why did a simple statement about the unavoidable interaction between the measurement apparatus and that what is being measured turn into a sweeping metaphysical statement?**

We are not sure. We just have this rather uncanny feeling all of these thought experiments are not about the Uncertainty Principle "protecting" quantum physics (we are quoting Feynman here): we feel it's about physicists protecting the Uncertainty Principle instead! Of course, the question then becomes: *why would they do this?* They are searching for the truth as well, don't they? They sure are.

We have no answer here, except this: geniuses like Heisenberg were – ultimately – religious too, and so they might have felt quantum-mechanical indeterminism (as opposed to the statistical indeterminism in

[43] We will say more about this later.

[44] The $h = \mathrm{E}\cdot\mathrm{T}$ and $h = \mathrm{p}\cdot\lambda$ are *complementary* but *equivalent* descriptions of the electron in the ring current model.

classical mechanics) was the very last place where God – or the Mystery, or whatever we cannot prove but happen to *believe* in – can hide.[45]

Let us turn back to a more scientific and rational discourse. What *are* those amplitudes, really?

The meaning of probability amplitudes and state vectors

The $\langle\, \chi \mid S \mid \varphi \,\rangle$ amplitude – which, according to Feynman, is all that matters in quantum physics – is a *probability amplitude* associated with the *in*-state φ and the *out*-state χ. These states are referred to as *state vectors*[46] and they are described in terms of *base states* which are written as $\langle\, i \mid$ and $\mid j \,\rangle$ for an *out*- and an *in*-state respectively. Hence, the *S*-matrix is actually written as $\langle\, i \mid S \mid j \,\rangle$ and, assuming we know what it is, we will be able to *resolve* and calculate the $\langle\, \chi \mid S \mid \varphi \,\rangle$ amplitude as $\Sigma_{ij} \langle\, \chi \mid i \,\rangle\langle\, i \mid S \mid j \,\rangle\langle\, j \mid \varphi \,\rangle$.

This may all look a bit mind-boggling and rightly so, because it actually *is* mind-boggling. Just *google*, for example the *S*-matrix for Compton scattering[47] and let me know how *inspiring* you think these so-called explanation of a *physical process* actually are. For me, they are not an explanation at all. I think it is way out of whack. It is like economists modeling input-output relations: we get a *mathematical description* of what goes in and what goes out but it doesn't tell us how stuff actually *happens*.[48] Furthermore, when the process involves the creation or annihilation of elementary or non-elementary particles (think of short-lived mesons here), even the math becomes ugly. Paraphrasing Dirac, one is then tempted to think that it may be "more important to have beauty in one's equations than to have them fit experiment."[49]

The question, of course, runs deeper than that: we can and should *not* doubt that the likes of Bohr, Pauli, Heisenberg, Schrödinger, Dirac did what we are all doing, and that is to, somehow, try to make sense of it all. Hence, I am sure that even Heisenberg did not initially think of his *interpretation* of these relations as some kind of surrender of reason—which is, effectively, what we think it actually amounts to: stating that it is a law of Nature that even experts cannot possibly understand Nature "the way they would like to", as Richard Feynman put it[50], relegates science to the realm of religious belief.

[45] In case the reader would be interested in our own convictions here, we will just say this: we think it's a ridiculous argument. If God is hiding Him- or Herself, He (or She) hides in much better places. Think of what we refer to as love here or – more mundane, perhaps – this strange order that comes out of disorder.

[46] Of course, these vectors are a bit different than the usual Cartesian coordinate vectors that you are used to but the idea of base vectors (base states) and combining or adding them remains the same. Physicists will solemnly talk of a Hilbert space but that is just *jargon* meant to impress you.

[47] I did this randomly just now and these two recent articles came out on top: https://arxiv.org/abs/1512.06681 and https://arxiv.org/abs/1810.11455. You may get others. This *input-output calculus* approach allows for endless refinements and, hence, endless research—which is always nice from an academic point of view, of course.

[48] The reader may be interested to know the author of this paper is an economist who has, effectively, dabbled in both macro- as well as micro-economic modeling and the training in statistics that comes with it. We are, therefore, very familiar with Marc Twain's 1907 quote: "There are three kinds of lies: lies, damned lies, and statistics."

[49] The quote is, apparently, from a May 1963 paper or article in *Scientific American*. We didn't analyze the context in which Dirac is supposed to have said this.

[50] See: Feynman's *Lectures*, III-1-1.

To be sure, the *accounting rules* and the *properties* of the *S*-matrix (as well as those of operator and Hamiltonian matrices) do incorporate some basic physical principles. One of them is the principle of *reversibility*, which is related to CPT-symmetry. We will say more about this later. As for now, the reader should note that the property of reversibility in *Nature* is logical but somewhat less straightforward than it appears to be at first. If we reverse *all* signs – the direction of time, the directions in space[51], and the sign of the charges – then, yes, *Nature* seems to respect the principle of reversibility.[52] However, we may see what physicists gravely refer to as symmetry-breaking if we only reverse signs of, say, the directions in space and the sign of the charge: some processes violate what is referred to as CP-symmetry, which involves swapping spatial directions and charges only.

We are not worried about that: if anything, it shows *physical* time has one direction only. However, because we wrote at length about these things elsewhere, we do not want to bore the reader with unnecessary philosophical reflections here.[53] We do want to highlight a few points here though, so as to give the reader a bit of an impression of what quantum-mechanical descriptions of physical processes by talking about amplitudes and states may actually *represent*.

The meaning of spin in quantum mechanics

We must now turn back to what we referred to as a deep conceptual flaw in the mainstream interpretation of quantum mechanics: the mainstream interpretation does *not* integrate the concept of particle *spin* from the outset because it thinks of the + or – sign in front of the imaginary unit (*i*) in the elementary wavefunction ($a \cdot e^{-i \cdot \theta}$ or $a \cdot e^{+i \cdot \theta}$) as a mathematical convention only. Indeed, most introductory courses in quantum mechanics will show that both $a \cdot e^{-i \cdot \theta} = a \cdot e^{-i \cdot (\omega t - kx)}$ and $a \cdot e^{+i \cdot \theta} = a \cdot e^{+i \cdot (\omega t - kx)}$ are acceptable waveforms for a particle that is propagating in a given direction (as opposed to, say, some real-valued sinusoid). Hence, one would, effectively, expect that the professors – mindful of *Occam's Razor* for a good theory – would then proceed to provide some argument showing why one would be better than the other, or some discussion on why they might be different but, *no*! That is not the case. The professors usually conclude that "the choice is a matter of convention" and, that "happily, most physicists use the same convention."[54]

We are really dumbfounded by this.[55] All of the great physicists knew all particles – elementary or not – have spin.[56] All of them also knew it is spin that generates the magnetic moment and that we should,

[51] We are talking parity-symmetry here: think of the mirror image of things, or of turning space inside out.

[52] As mentioned, we will make a critical note here later.

[53] See, for example, our paper on the physical meaning of the quantum-mechanical wavefunction (*Euler's wavefunction: the double life of –1*) and our blog post(s) on CPT-symmetry.

[54] In case you wonder, this is a quote from the MIT's edX course on quantum mechanics (8.01.1*x*). We quote this example for the same reason as why we use Feynman's *Lectures* as a standard reference: it's authoritative and, more importantly, also available online so the reader can check and explore for himself.

[55] The reader should *not* think we did not ask the question in the mentioned edX course on quantum mechanics. We did. We asked many other questions as well. However, we were effectively kindly instructed to "shut up and calculate" so as to prepare ourselves for the examination. We will be honest and admit we did not do the exam. We don't mind calculations. We do mind *mindless* calculations, however.

[56] Even photons – despite their spin-one property – cannot not come in a zero-spin state. When studying Feynman's *Lectures*, I found that to be one of the weirdest things: Feynman first spends several chapters on

therefore, consider the idea of thinking of elementary particles as a ring current. Indeed, the ring current model is not applicable to electrons (and positrons only). Indeed, Dirac starts his derivation of his famous wave equation for free electrons in his 1933 Nobel Prize speech with the following remark[57]:

> "This procedure is successful in the case of electrons and positrons. *It is to be hoped that in the future some such procedure will be found for the case of the other particles*. I should like here to outline the method for electrons and positrons, *showing how one can deduce the spin properties* of the electron, and then how one can infer the existence of positrons *with similar spin properties*."

This should suffice to remove any doubt in regard to the importance of the concept of spin in quantum mechanics: it is the lynchpin of everything. As mentioned, the introduction above serves as an introduction to Dirac's derivation of his wave equation for an electron in free space[58] – which we will *not* write down but which involves four operators, denoted as α_r p_r (r = 1, 2, 3) and $\alpha_0 mc$ respectively[59] – and says the following about it:

> "The new variables α_r which we have to introduce to get a relativistic wave equation linear in W[60], give rise to the spin of the electron. From the general principles of quantum mechanics one can easily deduce that these variables a give the electron a spin angular momentum of half a quantum and a magnetic moment of one Bohr magneton in the reverse direction to the angular momentum. These results are in agreement with experiment. They were, in fact, first obtained from the experimental evidence provided-by spectroscopy and afterwards confirmed by the theory."

He then continues by leaving us an excellent historical summary of the ring current or *Zitterbewegung* model of the electron[61]:

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explaining spin-one particles to, then, in some obscure footnote suddenly writes this: "The photon is a spin-one particle which has, however, no "zero" state." The issue is related to another glaring inconsistency: in the first three chapters of his *Lectures* on physics, he talks about adding wavefunctions and the basic rules of quantum mechanics, and it all happens with a plus sign. However, in his chapter on the theoretical distinction between bosons and fermions, he suddenly says we should be adding the amplitudes of fermions combine with a minus sign. In any case, we cracked our own jokes on the boson-fermion theory and so we should leave it at that.

[57] Needless to say, all *italics* in the quote are ours.

[58] The concept of free space should not confuse the reader: it makes abstraction of any external fields or forces, such as the force between a positively charged nucleus and the electron in some orbital.

[59] Dirac's wave equation can be written in various equivalent ways. We refer to Dirac's *Principles of Quantum Mechanics* or, for the reader who can follow Dirac's *very* succinct summary of it, the above-mentioned Nobel Prize speech.

[60] As mentioned, we don't want to get into the detail of (the derivation of) Dirac's equation but the reader should note Dirac only considers the *kinetic* energy of the electron which is, therefore, denoted as W rather than as $E = mc^2$. Our ring current model of an electron is more *holistic* in the sense that it integrates potential and kinetic energy.

[61] The term *Zitterbewegung* was, effectively, coined by Erwin Schrödinger in a 1930 paper which analyzed the solutions to Dirac's equation (Dirac had derived his equation in 1928 but the whole theoretical framework that accompanied it – his *Principles to Quantum Mechanics* – was also published in 1930 as well.

"The variables α also give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment."

We wrote about this elsewhere[62] and so we will not dwell on it much longer. The question here is: why did such great intellects *fail* to fully exploit the power of Euler's ubiquitous $\psi = a \cdot e^{i\theta}$ function? They knew about Occam's Razor as a criterion for a good scientific theory too, didn't they? We are not sure. Perhaps Schrödinger and Dirac were *too* obsessed by their wave equation – as opposed to the wavefunction that is its solution. Whatever the reason might have been, the fact is this: they did *not* integrate spin in their mathematical description—not from the outset, at least. The mistake is illustrated below.

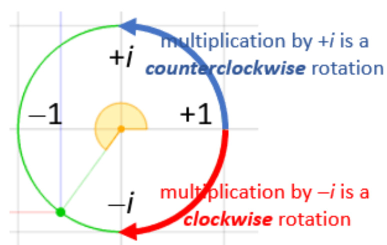

Figure 4: The meaning of +*i* and –*i*

It is a very subtle but fundamental blunder. Spin-zero particles do not exist.[63] All *real* particles – electrons, photons, anything – have spin, and spin (a shorthand for angular momentum) is always in one direction or the other: it is just the *magnitude* of the spin that differs. It is, therefore, completely odd that the plus (+) or the minus (–) sign of the imaginary unit (*i*) in the $a \cdot e^{\pm i\theta}$ function is *not* being used to include the spin *direction* in the mathematical description.

In fact, we call it a blunder because it is more than just odd: it's plain *wrong* because this non-used *degree of freedom* in the mathematical description leads to the *false* argument that the wavefunction of spin-½ particles have a 720-degree symmetry. Indeed, physicists treat –1 as a *common phase factor* in the argument of the wavefunction: they think we can just multiply a set of amplitudes – let's say *two* amplitudes, to focus our mind (think of a beam splitter or alternative paths here) – with –1 and get the same *states*. We find it rather obvious that that is not necessarily the case: –1 is *not* necessarily a common phase factor. We should think of –1 as a complex number itself: the phase factor may be +π or, alternatively, –π. To put it simply, when going from +1 to –1, it matters how you get there – and vice

[62] See our *Explanation of the Electron and Its Wavefunction*.

[63] We are obviously not talking about the zero-spin state of, say, a spin-one atom here. We talk about *elementary* particles only here.

versa – as illustrated below.[64]

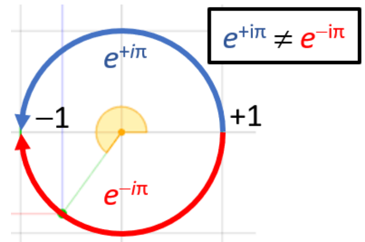

Figure 5: $e^{+i\pi} \neq e^{-i\pi}$

The reader may be very tired by now[65], so let us ask the primordial question for him or her: *what's the point here?* It is this: if we exploit the full descriptive power of Euler's function, then all weird symmetries disappear – and we just talk standard 360-degree symmetries in space. Also, weird mathematical conditions – such as the *Hermiticity* of quantum-mechanical operators – can easily be explained as embodying some common-sense *physical* law: energy and momentum conservation, for example, or *physical* reversibility: we need to be able to play the movie backwards.

Here we must note something funny with charges, however. Combining the + and – sign for the imaginary unit with the direction of travel, we get *four possible structures* for the wavefunction for an electron:

Spin and direction of travel	**Spin up (J = +ħ/2)**	**Spin down (J = –ħ/2)**
Positive *x*-direction	$\psi = \exp[i(kx-\omega t)]$	$\psi^* = \exp[-i(kx-\omega t)] = \exp[i(\omega t-kx)]$
Negative *x*-direction	$\chi = \exp[-i(kx+\omega t)] = \exp[i(\omega t-kx)]$	$\chi^* = \exp[i(kx+\omega t)]$

Table 2: Occam's Razor: mathematical possibilities versus physical realities (2)

However, this triggers the obvious question: how do we know it's an electron or a *negative* charge, as opposed to a positron, or a *positive* charge? Indeed, consider a particular direction of the elementary current generating the magnetic moment. It is then very easy to see that the magnetic moment of an electron ($\mu = -q_e\hbar/2m$) and that of a positron ($\mu = +q_e\hbar/2m$) would be opposite. We may, therefore, associate a particular direction of rotation with an angular frequency *vector* **ω** which – depending on the direction of the current – will be up or down with regard to the plane of rotation. We associate this with the spin property, which is also up or down. We, therefore, have another set of four possibilities reflecting one another:

[64] The quantum-mechanical argument is technical, and I did *not* reproduce it in this book. I encourage the reader to glance through it, though. See: *Euler's Wavefunction: The Double Life of – 1*. Note that the $e^{+i\pi} \neq e^{-i\pi}$ expression may look like *horror* to a mathematician! However, if he or she has a bit of a sense for geometry and the difference between identity and equivalence relations, there should be no surprise. If you're an amateur physicist, you should be excited, because it actually *is* the secret key to unlocking the so-called mystery of quantum mechanics. Remember Aquinas' warning: *quia parvus error in principio magnus est in fine*. A small error in the beginning can lead to great errors in the conclusions, and we sure think of this as an error in the beginning!

[65] We should also apologize for the (partial) overlap between this and our introductory discussions. We will try to rationalize and shorten the next version of this paper.

Matter-antimatter	**Spin up ($J = +\hbar/2$)**	**Spin down ($J = -\hbar/2$)**
Electron	$\mu_{e-} = -q_e\hbar/2m$	$\mu_{e-} = +q_e\hbar/2m$
Positron	$\mu_{e+} = +q_e\hbar/2m$	$\mu_{e+} = -q_e\hbar/2m$

Table 3: Electron versus positron spin

So how do we distinguish them? It is a deep philosophical question which we cannot fully answer for the time being. We can only offer a few thoughts here:

1. The idea of time reversal is a wonderful invention of our mind but we think it does *not* correspond to anything real. When charges are involved, you will see two like charges repel each other and, therefore, you will see them move away from each other. However, when playing the movie backward, you will know it the movie cannot be *real* because the same two charges now move *towards* each other. The funny thing is that a reversal of the sign of the charges doesn't help to fix the situation because, say, reversing the charge of two electrons now gives us two positrons moving towards each other, which doesn't make sense physically either!

The reader may think we did, perhaps, not think of P-symmetry: all left-handed things are now right-handed and vice versa, right? Yes, but that doesn't solve the problem here.

2. So what about CPT-symmetry then? Frankly, we think the concept of CPT-symmetry is not all that useful and we, therefore, wonder why physicists are so enthralled about it. We think the concept of motion itself implies that time can have one direction only. If it wouldn't, then we would not be able to describe trajectories in spacetime by a well-behaved function. In other words, the concept of motion itself would become meaningless. The diagrams below illustrate the point. The spacetime trajectory in the diagram on the right is not *kosher*, because our object travels back in time in not less than three sections of the graph. Spacetime trajectories – or, to put it more simply, *motions* – need to be described by well-defined functions: for every value of *t*, we should have one, and only one, value of *x*. The reverse, of course, is *not* true: a particle can travel back to where it was (or, if there is no motion, just stay where it is). Hence, it is easy to see that the concepts of motion and time are related to us using well-behaved functions to describe reality.[66]

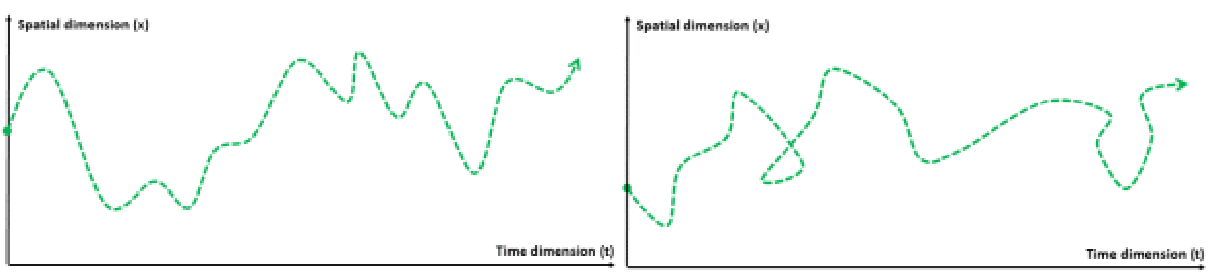

Figure 6: A well- and a not-well behaved trajectory in spacetime[67]

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[66] We wish the reader who would want to try using not-so-well-behaved functions to arrive at some kind of description of reality the best of luck.

[67] We actually do not like the concept of spacetime very much: time and space are *related* (through special and general relativity theory, to be precise) but they are *not* the same. Nor are they *similar*. We do, therefore, *not* think

We must now come back to the matter at hand, which is the meaning of conjugates in quantum physics.

The meaning of conjugating amplitudes, wavefunctions and matrices

The ubiquitous concept of a *state* in quantum mechanics is very abstract and general and, therefore, adds to the mystery of the quantum-mechanical description of things. We may start by noting that the concept of an initial and a final state – or an *in*- and an *out*-state – are very intimately linked: they correspond to the *bra* and the *ket* in Dirac's bra-ket notation which – we should not remind the reader – must be read *from right to left*. As such, we like to think the concepts of the initial and final state simply separate the present from the (possible) the future.

In practice, we will use *matrix mechanics* when states are *discrete*, while the wavefunction approach allows us to model how a particle may *evolve* which, in practice, usually means what and where we expect its momentum and position to be. One of Feynman's very first examples of a state is the *spin* state of *spin-one* atoms[68], which may be +1, 0 or –1. These are referred to as *base states*. Base states need to be independent, which is captured by the usual *Kronecker delta rule*:

$$\langle\, i \mid j \,\rangle = \delta_{ij}$$

This basically says a base state is a base state: $\langle\, i \mid j \,\rangle = 1$ if, and only if, $i = j$. Otherwise $\langle\, i \mid j \,\rangle = 0$. It obviously also means a particle can be in one state only at any point in time. The *evolution* from one state to another involves a *process* or – another oft-used terms in introductory courses – an *apparatus*, such as a *filter*, which may also be referred to as polarizer.[69] Now, the *amplitude* for a particle to go from state φ to state χ will be $\langle\, \chi \mid \varphi \,\rangle$. An amplitude is, of course, a *probability* amplitude here: it is a complex number or a *complex-valued* wavefunction. There is a most remarkable *law* in quantum mechanics, and it is this: **the amplitude to go from the *out*-state χ to the *in*-state φ is the (complex) conjugate of the amplitude to go from the *in*-state φ to the *out*-state χ** – and vice versa, of course! It is more easily written like this:

$$\langle\, \varphi \mid \chi \,\rangle = \langle\, \chi \mid \varphi \,\rangle^* \text{ and } \langle\, \chi \mid \varphi \,\rangle = \langle\, \varphi \mid \chi \,\rangle^*$$

Feynman *proves* this rule for a three-state system and a simple *filter* apparatus[70] but we find it easier to think of it as a simple consequence of this theoretical possibility of *reversibility*: a sign reversal of the imaginary unit *i*. Instead of +*i* (or, conversely, –*i*), we now use –*i* (or, conversely, +*i*) in our wavefunction.

It is probably easiest to show what this means in terms of actual *probabilities* – some easily understandable number between 0 and 1, that is – rather than in terms of probability *amplitudes*, which are complex-valued numbers evolving in space and in time. Indeed, we get the actual *probability* from

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that some 'kind of union of the two' will replace the separate concepts of space and time any time soon, despite Minkowski's stated expectations in this regard back in 1908. Grand statements and generalizations are not often useful in physics.

[68] While a photon is generally considered to be a spin-one particle, it does not have a zero state. In contrast, spin-one atoms are *composite* particles and, hence, its component particles may effectively line up in a way that produces a spin-zero state.

[69] We find the term somewhat confusing but a polarized beam is a beam of particles with the same spin.

[70] See: Feynman's Lectures, III-5-5 (*Interfering Amplitudes*) and III-5-6 (*The Machinery of Quantum Mechanics*).

taking the absolute square[71] of the complex-valued amplitude.

In our realist interpretation, we understand this quantum-mechanical rule in terms of mass or energy densities: we think of the oscillating or *Zitterbewegung* charge as passing more time here than there and, hence, the associated *energy density* will be higher here than there.[72] However, that interpretation does not matter here. We just want to show you that we can use this quantum-mechanical rule for calculating probabilities to derive the above-mentioned quantum-mechanical reversibility law. Indeed, the absolute square of a complex number is the product of the same number with its complex conjugate. We, therefore, get this[73]:

$$|\langle \varphi \mid \chi \rangle|^2 = \langle \varphi \mid \chi \rangle\langle \varphi \mid \chi \rangle^* = \langle \chi \mid \varphi \rangle^*\langle \chi \mid \varphi \rangle = \langle \chi \mid \varphi \rangle^*\langle \chi \mid \varphi \rangle = |\langle \chi \mid \varphi \rangle|^2$$

At this point, the reader may wonder what we want to say here. We are just saying that the quantum-mechanical $\langle \varphi \mid \chi \rangle = \langle \chi \mid \varphi \rangle^*$ rule only states physical processes should be reversible in space and in time, and that it amounts to stating that **the probability of going from the *out*-state χ to the *in*-state φ is the same as going from the *in*-state φ to the *out*-state χ**.

The question is: does this rule or law make sense? In other words, is it true, ***always*?** In mechanics – think of elastic collisions of particles or particles moving in force fields – it should be the case. However, when disintegration processes are involved, one should not expect this law to apply. Why not? Disintegration involves unstable systems moving to some kind of *stable* state. Energy, angular momentum, and linear momentum will be conserved, but that's about all we can say about it. Inventing new quantities to be conserved – such as strangeness[74] – or invoking other consequences of so-called symmetry-breaking processes does not add much explanatory power in our view.

[…]

Let us wrap up here—this *section* of this rather long and winding paper, at least! Let's summarize the basics of our discussion above: if we can go *from* state φ *to* state χ, then we must be able to go *back* to state φ *from* state χ in space and in time. This amounts to playing a movie backwards: an exploding suitcase bomb will, in practice, *not* suddenly un-explode and get back into the suitcase but, theoretically (read: making abstraction of arguments involving entropy and other *macroscopic* considerations), such

[71] The absolute square is, obviously, a shorthand for the absolute value of the square.

[72] You may think of this like follows. The energy in an oscillation – *any* physical oscillation – is always proportional to the *square* of the (maximum) amplitude of the oscillation, so we can write this: $E \propto a^2$. To be precise, the energy of an electron in our ring current model is equal to $E = m \cdot a^2 \cdot \omega^2$. We get this from (1) Einstein's mass-energy equivalence relation ($E = m \cdot c^2$) and (2) our interpretation of c (lightspeed) as a tangential velocity ($c = a \cdot \omega$). We think the mass of the electron is just the equivalent mass of the energy of the *Zitterbewegung* charge in its oscillation. We, therefore, think it makes sense to think that the probability of actually *finding* the charge here or there will be proportional to the energy density here and there.

[73] The reader should carefully note what is commutative and what not here!

[74] A careful analysis of the processes involving K-mesons in Feynman's rather enigmatic *Lecture* on it reveals that the quark hypothesis results from a rather unproductive approach to analyzing disintegration processes: inventing new quantities that are supposedly being conserved, such as strangeness, is… Well… As strange as it sounds. We, therefore, think the concept of quarks confuses rather than illuminates the search for a truthful theory of matter. See our introduction on *Smoking Gun Physics* in our general paper on the power of classical physics. This text is based on an earlier critical appraisal of the theory of virtual particles.

reversed processes may be *imagined*.[75] However, instead of imaging an exploding or un-exploding bomb, let us stick to the more abstract business of states and vectors and so we should try to think of an apparatus or a process that will be operating on some state $|\psi\rangle$ to produce some other state $|\varphi\rangle$. The Great Dirac taught us to write that like this:

$$\langle\varphi|A|\psi\rangle$$

We can now take the complex conjugate:

$$\langle\varphi|A|\psi\rangle^* = \langle\psi|A^\dagger|\varphi\rangle$$

$A^\dagger$ is, of course, the conjugate transpose of A – we write: $A^\dagger_{ij}=(A_{ji})^*$ – and we will call the operator (and the matrix) Hermitian if the conjugate transpose of this operator (or the matrix) gives us the same operator matrix, so that is if $A^\dagger = A$. Many quantum-mechanical operators are Hermitian, and we will also often *impose* that condition on the *S*-matrix. Why? Because you should think of an operator or an *S*-matrix as a symmetric apparatus or a reversible process. It's as simple as that. Hence, the Hermiticity condition amounts to a simple reversibility condition too!

Needless to say, we may be mistaken, of course! We, therefore, invite the professional reader to challenge this interpretation! The professional reader may also note we forgot to specify the second of the three basic rules of quantum math, and that's the resolution of amplitudes into base states—which is written like this:

$$\langle \chi \mid \varphi \rangle = \Sigma_{\text{all } i} \langle \chi \mid i \rangle\langle i \mid \varphi \rangle$$

This, too, can be shown to be true based on a thought experiment[76] or, else, one can use a much more abstract argument (as Dirac does as part of his formalization of the theoretical framework for working with amplitudes) and simply write:

$$| = \Sigma_{\text{all } i} \mid i \rangle\langle i \mid$$

This, then allows us to do what we did, and that's to write the $\langle \chi \mid S \mid \varphi \rangle$ amplitude as $\Sigma_{ij} \langle \chi \mid i \rangle\langle i \mid S \mid j \rangle\langle j \mid \varphi \rangle$.

Any case, this is all very abstract and, therefore, very tiring. Furthermore, it does not amount to what we think of as any *real* explanation for real-life physical processes: it merely *describes* them. As mentioned, this is more like economists modeling input-output relations: we get a *mathematical description* of what goes in and what goes out but it doesn't tell us how stuff actually *happens*. So how would or might a *real* explanation then look like? Let us illustrate this using the example of Compton scattering.

[75] This rather extreme example of before and after states is, perhaps, less extreme than it appears to be: the proton-proton collisions in CERN's LHC colliders are quite violent too, albeit at a much smaller scale. However, as mentioned, one should not necessarily expect that the *disintegration* processes involved in the creation of such *disequilibrium* situations would or should be reversible.

[76] Feynman provides such argument based on placing an open filter (no masks) in-between two other filters. See the above-mentioned reference to Feynman's *Lectures* (III-5-5).

A classical explanation of Compton scattering

Energy and momentum conservation

Compton scattering is referred to as inelastic because the frequency of the incoming and outgoing photon are different. The situation is illustrated below.

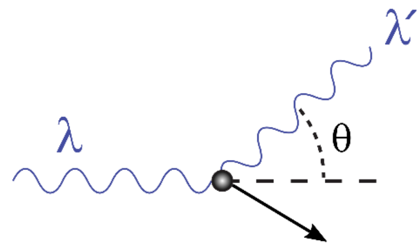

Figure 7: The scattering of a photon from an electron (Compton scattering)

The reader will be familiar with the formulas and, most probably, also with how these formulas can be derived from two very classical laws only: (1) energy conservation and (2) momentum conservation. Let us quickly do this to remind the reader of the elegance of classical reasoning:

1. The energy conservation law tells us that the total (relativistic) energy of the electron ($E = m_e c^2$) and the incoming photon must be equal to the total energy of the outgoing photon and the electron, which is now moving and, hence, includes the kinetic energy from its (linear) motion. We use a prime (') to designate variables measured after the interaction. Hence, $E_{e'}$ and $E_{\gamma'}$ are the energy of the moving electron (e') and the outgoing photon (γ') in the state after the event. We write:

$$\mathrm{E_e} + \mathrm{E_\gamma} = \mathrm{E_{e'}} + \mathrm{E_{\gamma'}}$$

We can now use (i) the mass-energy equivalence relation ($E = mc^2$), (ii) the Planck-Einstein relation for a photon ($E = h\cdot f$) and (iii) the relativistically correct relation ($E^2 - p^2c^2 = m^2c^4$) between energy and momentum for *any* particle – charged or non-charged, matter-particles or photons or whatever other distinction one would like to make[77] – to re-write this as[78]:

$$\mathrm{m_e}c^2 + hf = \sqrt{\mathrm{p_{e'}^2}c^2 + \mathrm{m_e^2}c^4} + hf' \Leftrightarrow \mathrm{p_{e'}^2}c^2 = (hf - hf' + \mathrm{m_e^2}c^4)^2 - \mathrm{m_e^2}c^4 \;(1)$$

2. This looks rather monstrous but things will fall into place soon enough because we will now derive another equation based on the momentum conservation law. Momentum is a vector, and so we have a

[77] This is, once again, a standard textbook equation but – if the reader would require a reminder of how this formula comes out of special relativity theory – we may refer him to the online *Lectures* of Richard Feynman. Chapters 15 and 16 offer a concise but comprehensive overview of the basics of relativity theory and section 5 of Chapter 6 gives the reader the formula he needs here. It should be noted that we dropped the 0 subscript for the rest mass or energy: $m_0 = m$. The *prime* symbol (') takes care of everything here and so you should carefully distinguish between primed and non-primed variables.

[78] We realize we are cutting some corners. We trust the reader will be able to *google* the various steps in-between.

vector equation here[79]:

$$\vec{p}_{\gamma} = \vec{p}_{\gamma'} + \vec{p}_{e'} \Leftrightarrow \vec{p}_{e'} = \vec{p}_{\gamma} - \vec{p}_{\gamma'}$$

For reasons that will be obvious later – it is just the usual thing: ensuring we can combine two equations into one, as we did with our formulas for the radius – we square this equation and multiply with Einstein's constant c^2 to get this[80]:

$$\vec{p}_{e'}^{2} = \vec{p}_{\gamma}^{2} + \vec{p}_{\gamma'}^{2} - 2\vec{p}_{\gamma}\vec{p}_{\gamma'} \Leftrightarrow p_{e'}^{2}c^2 = p_{\gamma}^{2}c^2 + p_{\gamma'}^{2}c^2 - 2(p_{\gamma}c)(p_{\gamma'}c)\cdot\cos\theta$$

$$\Leftrightarrow p_{e'}^{2}c^2 = h^2f^2 + h^2f'^2 - 2(hf)(hf')\cdot\cos\theta \ (2)$$

3. We can now combine equations (1) and (2):

$$p_{e'}^{2}c^2 = (Eq.1) = (Eq.2) = (hf - hf' + m_e^2c^4)^2 - m_e^2c^4 = h^2f^2 + h^2f'^2 - 2(hf)(hf')\cdot\cos\theta$$

The reader will be able to do the horrible stuff of actually squaring the expression between the brackets and verifying only cross-products remain. We get:

$$(hf - hf')m_e c^2 = h(f - f')m_e c^2 = h^2 ff'(1 - \cos\theta)$$

Multiplying both sides of the equation by the $1/hm_e ff'$ constant yields the formula we were looking for:

$$\frac{(f - f')m_e c}{f \cdot f'} = \frac{f m_e c - f'^{m_e} c}{f \cdot f'} = \frac{h}{m_e c}(1 - \cos\theta)$$

$$\Leftrightarrow \frac{c}{f'} - \frac{c}{f} = \lambda' - \lambda = \Delta f = \frac{h}{m_e c}(1 - \cos\theta)$$

The formulas allow us also to calculate the angle in which the electron is going to recoil. It is equal to:

$$\cot\left(\frac{\theta}{2}\right) = \left(1 + \frac{E_{\gamma}}{E_e}\right)\tan\ \varphi$$

The h/mc factor on the left-hand side of the right-hand side of the formula for the difference between the wavelengths is, effectively, a distance: about 2.426 *pico*meter (10^{-12} m). The $1 - \cos\theta$ factor goes from 0 to 2 as θ goes from 0 to π. Hence, the maximum difference between the two wavelengths is

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[79] We could have used **boldface** to denote vectors, but the calculations make the arrow notation more convenient here. So as to make sure our reader stays awake, we note that the objective of the step from the first to the second equation is to derive a formula for the (linear) momentum of the electron *after* the interaction. As mentioned, the linear momentum of the electron *before* the interaction is zero, because its (linear) velocity is zero: $\mathbf{p}_e = \mathbf{0}$.

[80] We do not want to sound disrespectful when referring to c^2 as Einstein's constant. It has a deep meaning, in fact. Einstein does not have any fundamental constant or unit named after him. Nor does Dirac. We think c^2 would be an appropriate 'Einstein constant'. Also, in light of Dirac's remarks on the nature of the strong force, we would suggest naming the unit of the strong charge after him. More to the point, note these steps – finally ! – incorporated the directional aspect we needed for the analysis. Note that we also use the rather obvious E = p*c* relation for photons in the transformation of formulas here.

about 4.85 pm. This corresponds, unsurprisingly, to *half* the (relativistic) energy of an electron.[81] Hence, a highly energetic photon could lose up to 255 keV.[82] That sounds enormous, but Compton scattering is usually done with highly energetic X- or gamma-rays.

Could we imagine that a photon loses all of its energy to the electron? No. We refer to Prof. Dr. Patrick LeClair's course notes on Compton scattering[83] for a very interesting and more detailed explanation of what actually happens to energies and frequencies, and what depends on what exactly. He shows that the electron's kinetic energy will always be a *fraction* of the incident photon's energy, and that fraction may approach but will never actually reach unity. In his words: "This means that there will always be some energy left over for a scattered photon. Put another way, it means that a stationary, free electron cannot absorb a photon! Scattering must occur. Absorption can only occur if the electron is bound to a nucleus."

What might *actually* be happening

A photon *interacts* with an electron, so we actually think of the photon as being briefly absorbed, before the electron emits another photon of *lower* energy. The energy difference between the incoming and outgoing photon then gets added to the *kinetic* energy of the electron according to the law we just derived:

$$\lambda' - \lambda = \Delta f = \frac{h}{\mathrm{m}c}(1 - \cos\theta)$$

Now, we think of the interference as a process during which – temporarily – an unstable wavicle is created. This unstable wavicle does *not* respect the integrity of Planck's quantum of action (E = h·*f*). The equilibrium situation is then re-established as the electron emits a new photon and moves away. Both the electron and the photon respect the integrity of Planck's quantum of action again and they are, therefore, *stable*.

The geometry of the whole thing is simple and difficult at the same time. There is, for example, also a formula for the angle of the *outgoing* photon, which uses the angle for the *incoming* photon, but that's all stuff which the reader can look up. The question is: how does this happen, *exactly*? And what determines the *plane* which is formed by the outgoing photon and the recoiling electron?

None of the standard textbooks will try to answer that question, because they don't think of electrons and photons as having some *internal structure* which may explain all of the formulas they get out of their arguments, which are based on the conservation of (1) energy and (2) linear and angular momentum. In contrast, we believe our geometrical models may not only show *why* but also *how* all of this happens. Let us walk over the basics of that.

[81] The energy is inversely proportional to the wavelength: E = h·*f* = h*c*/λ.

[82] The electron's rest energy is about 511 keV.

[83] See the *exposé* of Prof. Dr. Patrick R. LeClair on Compton scattering.

The Compton wavelength as a circumference

According to our ring current model of an electron, the wavefunction $\psi(\boldsymbol{x}, t) = \psi(x, y, z, t)$ describes the actual position of the pointlike *Zitterbewegung* charge in its oscillatory motion around some center.[84] We may therefore paraphrase Prof. Dr. Patrick LeClair and identify the Compton wavelength with a "*distance scale within which we can localize the electron in a particle-like sense*."[85] Why are we so sure of that?

We should first discuss the 2π factor. Indeed, we have *h*, not *ħ*, in the equation. Hence, should we think of the Compton *wavelength* or the Compton *radius* of an electron? 2π is a sizable factor – a factor equal to about 6.28 – so that is large enough to matter when discussing *size*.[86] Of course, we know it is the factor which relates the *circumference* of a circle with its radius but a wavelength is something linear, isn't it? If we should think of the wavelength of an electron, then what should we *imagine* it to be anyway? We all know that *Louis de Broglie* associated a wavelength with the *classical* momentum p = m*v* of a particle but, again, how should we *imagine* this wavelength? The *de Broglie* relation says $\lambda = h/\text{p} = h/\text{m}v$ goes to infinity (∞) for *v* going to 0 and m going to m_0 (the rest mass of the electron). Hence, the *de Broglie* and the *Compton* wavelength of an electron are very different concepts:

$$\lambda_C = h/\text{m}c \neq \lambda = h/\text{p} = h/\text{m}v$$

The illustration below (for which credit goes to an Italian group of *zbw* theorists[87]) helps to make an

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[84] We are often tempted to use a semicolon to separate the time variable from the space coordinates. Hence, we would prefer to write $\psi(\boldsymbol{x}, t) = \psi(x, y, z; t)$ instead of $\psi(\boldsymbol{x}, t) = \psi(x, y, z; t)$. The semicolon then functions as a *serial* comma. Of course, we are well aware of Minkowski's view on the relativity of space and time: "*Space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.*" However, we feel space and time are related but also very *separate* categories of our mind: perhaps some very advanced minds may claim they can effectively understand the Universe in terms of four-vector algebra, but we surely do *not*. We understand things in terms of motion, and any equation of motion implies the idea of a motion in space *and* in time—not in some 'kind of union of the two'. What I am saying is that the oft-used concept of spacetime is not something we can easily *imagine*: space and time are fundamentally different categories of the mind. We cannot go backward in time, for example. Not in *our* reality, at least. Why not? Because the $\boldsymbol{x} = (x, y, z) = (f_x(t), f_y(t), f_z(t) = f(t)$ – the fundamental equation of motion – would no longer be a proper function: a *physical* object can*not* be at two different places at the same point in time. It can *return to* a place where it was (or it can simply stay where it is), but it will then be there at a *different* point of time. That's why time goes by in one direction only. Why do we highlight this point? Because some physicists – including Feynman – seem to suggest we should think of antimatter as particles traveling back in time. We think that is plain nonsensical.

[85] See the reference above.

[86] The *nature* of a 2π factor is definitely very different from that of a 2 or 1/2 factor, which we often got when analyzing something using non-relativistic equations. A 2π factor is associated with something circular, so we need to explain *what circular feature*, and not in approximate but in *exact* terms—which is not easy in this particular case.

[87] Vassallo, G., Di Tommaso, A. O., and Celani, F, *The Zitterbewegung interpretation of quantum mechanics as theoretical framework for ultra-dense deuterium and low energy nuclear reactions*, in: Journal of Condensed Matter Nuclear Science, 2017, Vol 24, pp. 32-41. Don't worry about the rather weird distance scale (1×10^{-6} eV^{-1}). Time and distance can be expressed in *inverse* energy units when using so-called *natural units* ($c = \hbar = 1$). We are not very fond of using natural units because we think they may *hide* rather than clarify or simplify some of the more fundamental relations. Just note that 1×10^{-9} eV^{-1} = 1 GeV^{-1} ≈ 0.1975×10^{-15} m. As you can see, the *zbw* radius

interesting point. Think of the black circle (in the illustration on the left-hand side below) as circumscribing the *Zitterbewegung* of the pointlike charge. Think of it as the electron at rest: its *radius* is the radius of the oscillation of the oscillation of the *zbw* charge $a = ħ/mc$. Note that we actually do ask you to make abstraction of the two-dimensional *plane* of the oscillation of the *Zitterbewegung* (*zbw*) charge, which need not be perpendicular to the direction of motion of the electron as a whole: it can be in the same plane or, most likely, it may be *zittering* around itself. The point is this: we can introduce yet another definition of a wavelength here—the distance between two crests or two troughs of the wave, as shown in the illustration on the right-hand side.[88]

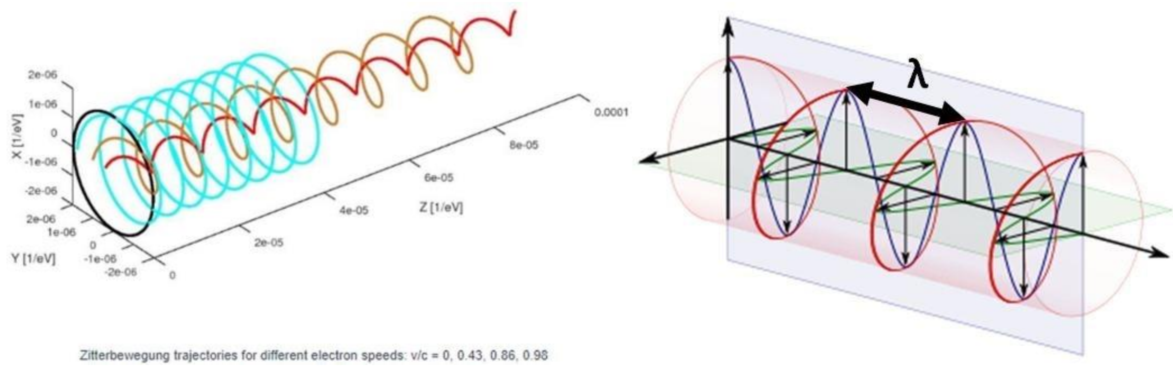

Figure 8: The wavelength(s) of an electron

We should now present a rather particular geometric property of the *Zitterbewegung* (*zbw*) motion: the $a = ħ/mc$ radius – the Compton radius of our electron – must decrease as the (classical) velocity of our electron increases. That is what is visualized in the illustration on the left-hand side (above). What happens here is quite easy to understand. If the tangential velocity remains equal to *c*, and the pointlike charge has to cover some horizontal distance as well, then the circumference of its rotational motion *must* decrease so it can cover the extra distance. How can we analyze this more precisely? This rather remarkable thing should be consistent with the use of the *relativistic* mass concept in our formula for the *zbw* radius *a*, which we write as:

$$a = \frac{\hbar}{\mathrm{m}c} = \frac{\lambda_C}{2\pi}$$

The λ_C is the *Compton* wavelength, so that's the circumference of the circular motion.[89] How can it decrease? If the electron moves, it will have some kinetic energy, which we must add to the *rest energy*. Hence, the mass m in the denominator (m*c*) increases and, because *ħ* and *c* are physical constants, *a*

(for $v = 0$) is of the order of $2{\times}10^{-6}$ eV^{-1} in the diagram, so that's about $0.4{\times}10^{-12}$ m, which is more or less in agreement with the Compton radius as calculated ($a_{v=0} \approx 0.386{\times}10^{-12}$ m).

[88] Because it is a wave in two or three dimensions, we cannot really say there are crests or troughs, but the terminology might help you with the interpretation of the rather particular geometry here, which is that of an Archimedes screw, but that's only because of the rather particular orientation of the *plane* of the *zbw* oscillation, which we ask you *not* to accept for granted: you should, instead, imagine the plane of oscillation itself is probably *not* stable: the (in)famous uncertainty in quantum mechanics may actually be related to our lack of knowledge in regard to the plane of the *zbw* oscillation: it may itself be *zittering* around.

[89] Needless to say, the *C* subscript stands for the *C* of Compton, not for the speed of light (*c*).

must decrease.[90] How does that work with the frequency? The frequency is proportional to the energy (E = $\hbar \cdot \omega$ = $h \cdot f$ = h/T) so the frequency – in whatever way you want to measure it – must also *increase*. The *cycle* time T, therefore, must *decrease*. What happens, really, is that we are stretching our Archimedes' screw, so to speak. It is quite easy to see that we get the following formula for our new λ wavelength:

$$\lambda = v \cdot \mathrm{T} = \frac{v}{f} = v \cdot \frac{h}{\mathrm{E}} = v \cdot \frac{h}{\mathrm{m}c^2} = \frac{v}{c} \cdot \frac{h}{\mathrm{m}c} = \beta \cdot \lambda_C$$

Can the (classical or linear) velocity go to c? In theory, yes, but, in practice, *no*. The m in the formula is not the mass of the *zbw* charge but the mass of the electron as a whole. That is *non*-zero for v = 0, unlike the rest mass of the *zbw* charge, which only acquires mass because of its motion. We calculated this *relativistic* mass of the *zbw* charge as equal to 1/2 of the electron (rest) mass. The point is this: we are *not* moving a zero-mass thing here. The energy that is, therefore, required to bring v up to c will be infinitely large: think of the enormous energies that are required to speed electrons up to *near-lightspeed* in accelerators.

The point is this: an electron does not become photon-like when moving at near-lightspeed velocities. However, we do see that the *circumference* of the circle that circumscribes the two-dimensional *zbw* oscillation of the *zbw* charges does seem to transform into some linear wavelength when v goes to c!

Of course, we immediately admit this still does not *explain* what's going on, *exactly*. It only shows we should not necessarily think of the Compton wavelength as a purely *linear* feature.

Another remark that we should make here is that, while we emphasize that we should not think of a photon as a *charge* travelling at lightspeed – it is *not*—and I mean *not at all*: photons do *not* carry charge[91] – the analysis above does relate the geometry of our *zbw* electron to the geometry of our photon model. Let us quickly introduce that now as part of a larger reflection of what may or may not be going on in photon-electron interactions.

Probing electrons with photons

The relation between what we think of as the *radius* of the *Zitterbewegung* oscillation of the electric charge (a = $\hbar/\mathrm{m_e}c$), the *Compton* wavelength of an electron (λ_C = $h/\mathrm{m_e}c$), and the *wavelength* of a photon (λ = c/f) is not very obvious. However, we should not be discouraged because we immediately note on thing, at least: the wavelength of a photon is the same as its Compton wavelength *and* its *de Broglie* wavelength. Furthermore, because v = c and, therefore, β = 1, it is also equal to that *third* wavelength we introduced above: λ = $\beta\lambda_C$. In short, the wavelength of a photon is its wavelength, so we write:

$$\lambda = \frac{c}{f} = \frac{ch}{\mathrm{E}} = \frac{ch}{\mathrm{m}c^2} = \frac{h}{\mathrm{m}c} = \frac{h}{\mathrm{p}}$$

But so what *is* that wavelength, *really*? Indeed, we should probably start by recognizing this: when

[90] We advise the reader to always think about proportional (y = kx) and inversely proportional (y = x/k) relations in our *exposé*, because they are not always intuitive.

[91] This is, in fact, the quintessential difference between matter-particles and energy carriers such as photons (for the electromagnetic force) and neutrinos (for the strong(er) force).

probing the *size* of an electron with photons, we had better have some idea of what a photon actually *is*. Indeed, almost any textbook will tell you that, because of the wave nature of light, there is a limitation on how close two spots can be and still be seen as two separate spots: that distance is *of the order of* the wavelength of the light that we are using.[92] There is a reason for that, of course, and it's got as much to do with the wave as with the particle nature of light. Light comes in lumps too: photons. These photons *pack* energy but they also pack one (natural) unit of *physical action* (h) or – what amounts to the same – one unit of angular momentum ($\hbar$).

Let us recall the basics of what we have actually presented a few times in previous papers already.[93] Angular momentum comes in units of $\hbar$. When analyzing the electron orbitals for the simplest of atoms (the one-proton hydrogen atom), this rule amounts to saying the electron orbitals are separated by a amount of *physical action* that is equal to $h = 2\pi\cdot\hbar$. Hence, when an electron jumps from one level to the next – say from the second to the first – then the atom will lose one unit of h. The photon that is emitted or absorbed will have to pack that somehow. It will also have to pack the related energy, which is given by the Rydberg formula:

$$\mathrm{E}_{n_2} - \mathrm{E}_{n_1} = -\frac{1}{{n_2}^2}\mathrm{E}_R + \frac{1}{{n_1}^2}\mathrm{E}_R = \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)\cdot \mathrm{E}_R = \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)\cdot \frac{\alpha^2 \mathrm{m}c^2}{2}$$

To focus our thinking, we considered the transition from the second to the first level, for which the $1/1^2 - 1/2^2$ factor is equal 0.75. Hence, the energy of the photon that is being emitted will be equal to $(0.75)\cdot\mathrm{E}_R \approx 10.2$ eV. Now, if the total action is equal to h, then the cycle time T can be calculated as:

$$\mathrm{E}\cdot\mathrm{T} = h \Leftrightarrow \mathrm{T} = \frac{h}{\mathrm{E}} \approx \frac{4.135\times10^{-15}\,\mathrm{eV}\cdot\mathrm{s}}{10.2\ \mathrm{eV}} \approx 0.4\times10^{-15}\ \mathrm{s}$$

This corresponds to a wavelength of $(3\times10^{8}\ \mathrm{m/s})\cdot(0.4\times10^{-15}\ \mathrm{s}) = 122$ nm, which is the wavelength of the light ($\lambda = c/f = c\cdot\mathrm{T} = \mathrm{h}\cdot c/\mathrm{E}$) that we would associate with this photon energy.[94] This rather simple calculation is sufficient to illustrate our photon model: we think of a photon as being *pointlike* but, at the same time, the Planck-Einstein relation tells us it packs *one wavelength*—or *one cycle*. The *integrity* of that cycle is associated with its energy (E) and its cycle time (T) or – alternatively – with its momentum (p) and its wavelength (λ), as evidenced in the $\mathrm{E}\cdot\mathrm{T} = \mathrm{p}\cdot\lambda = h$ relation:

$$\left.\begin{aligned} \mathrm{p} = \mathrm{m}c = \frac{\mathrm{E}}{c^2}c = \frac{\mathrm{E}}{c} \Leftrightarrow \mathrm{p}c = \mathrm{E} \\ c = f\lambda = \frac{\mathrm{E}}{h}\lambda = \Leftrightarrow \frac{hc}{\lambda} = \mathrm{E} \end{aligned}\right\} \Rightarrow \mathrm{p}\cdot c = \frac{h\cdot c}{\lambda} \Leftrightarrow \mathrm{p}\cdot\lambda = h = \mathrm{E}\cdot\mathrm{T}$$

So, yes, the equations above sort of vaguely tell us that if we think of measuring some distance or some time – as we do when we probe an electron with photons – that we will have to content ourselves with

[92] See, for example, Feynman's discussion of using photons to try to detect electrons as part of the two-slit experiment.

[93] See, for example, our paper on *Relativity, Light, and Photons*.

[94] Just so you can imagine what we are talking about, this is short-wave ultraviolet light (UV-C). It is the light that is used to purify water, food or even air. It kills or inactivate microorganisms by destroying nucleic acids and disrupting their DNA. It is, therefore, harmful. Fortunately, the ozone layer of our atmosphere blocks most of it.

measuring it in wavelength units (λ) or, equivalently, in cycle time units ($T = 1/f = \lambda/c$).

However, when probing electrons as part of Compton scattering processes, we are beating the usual game here: we are actually measuring an *effective radius of interference* not in terms of the wavelength of the light that we are using but in terms of the *difference* in the wavelength of the photon that goes in and comes out of the scattering process. That's what the Compton scattering formula tells us:

$$\lambda' - \lambda = \Delta f = \frac{h}{\mathrm{m}c}(1 - \cos\theta)$$

We already mentioned that the $1 - \cos\theta$ factor on the right-hand side of this equation goes from 0 to 2 as θ goes from 0 to π. Hence, the maximum possible change in the wavelength is equal to $2\lambda_C$, which we get from a head-on collision with the photon being scattered backwards at 180 degrees.[95] However, that doesn't answer the question in regard to the 2π factor: the $h/\mathrm{m}c$ factor is still the Compton *wavelength*, so that is 2π times the radius: $\lambda_C = 2\pi \cdot a = 2\pi\hbar/\mathrm{m}c$. In fact, mentioning the $2\lambda_C$ value for the *maximum* difference in wavelength introduces an *additional* factor 2.

Who ordered *that*? Let us advance a *possible* explanation: we are *not* saying it is *the* explanation, but it may be *an* explanation. It goes like follows.

The electron-photon excitation: a temporary spin-2 particle?

We think the energy of the incident photon – as an electromagnetic oscillation, that is – is temporarily absorbed by the electron and, hence, the electron is, therefore, in an excited state, which is a state of non-equilibrium. As it returns to equilibrium, the electron emits some of the excess energy as a new photon, and the remainder gives the electron as a whole some additional momentum.

How should we model this? One intriguing possibility is that the electron radius becomes larger because it must now incorporate *two* units of *h* or, when talking angular momentum, two units of ħ. So we should, perhaps, re-do our calculation of the Compton radius of our electron as follows:

$$\left.\begin{array}{c}\left.\begin{array}{r}\mathrm{E} = \mathrm{m}c^2 \\ \mathrm{E} = 2hf = 2\hbar\omega\end{array}\right\} \Rightarrow \mathrm{m}c^2 = 2\hbar\omega \\ c = a\omega \Leftrightarrow a = \dfrac{c}{\omega} \Leftrightarrow \omega = \dfrac{c}{a}\end{array}\right\} \Rightarrow \mathrm{m}a^2\omega^2 = 2\hbar\omega \Longrightarrow \mathrm{m}\frac{c^2}{\omega^2}\omega^2 = \frac{2\hbar c}{a} \Leftrightarrow a = \frac{2\hbar}{\mathrm{m}c}$$

This may actually work—as long as we remember the energy and mass factors here are the *combined* energies and masses of the electron *and* the photon. Of course, this triggers the next question: what's the typical energy or mass of the incoming photon? To demonstrate the Compton shift, Compton used X-ray photons with an energy of about 17 keV, so that's about 3.3% of the energy of the electron, which is equal to 511 keV. For practical purposes, we may say the photon doesn't change the energy of the electron very much but, of course, it is significant enough to cause a significant *change* in the state of

[95] The calculation of the angle of the *outgoing* photon involves a different formula, which the reader can also look up from any standard course. See, for example, Prof. Dr. Patrick R. LeClair's lecture on Compton scattering, which we referenced already. The reader should note that the $1 - \cos\theta$ is equal to -1 for ±π, and that there is *no* change in wavelength for θ = 0, which is when the photon goes straight through, in which case there is no scattering.

motion of the electron.[96]

Hence, the excited state of an electron may involve a *larger* radius—twice as large, *approximately*. Do we think it explains the above-mentioned factor 2? We will let the reader think for himself here as we haven't made our mind up on it yet.

Planck's quantum of action as a vector

What is uncertainty in quantum mechanics? How does it relate to the philosophical concepts of freedom and determinism? We already briefly discussed this, but let us walk over the basics once more. Richard Feynman notes that, "*from a practical point of view*", we have "indeterminability in classical mechanics as well."[97] He gives the example of water splashing over a dam:

> "If water falls over a dam, it splashes. If we stand nearby, every now and then a drop will land on our nose. This appears to be completely random, yet such a behavior would be predicted by purely classical laws. The exact position of all the drops depends upon the precise wigglings of the water before it goes over the dam. How? The tiniest irregularities are magnified in falling, so that we get complete randomness. Obviously, we cannot really predict the position of the drops unless we know the motion of the water absolutely exactly."

While it is, effectively, not *practically possible* to know the motion of the water "*absolutely exactly*", the deeper question is: is it *theoretically possible*? In classical mechanics, we assume it is. In quantum mechanics, we assume it is *not*. Quoting Feynman once again, we may say that "the Uncertainty Principle *protects* quantum mechanics." Why should this be so?

We think it's plain nonsense: there is no reason whatsoever to assume the uncertainty in classical mechanics in any different from the uncertainty in quantum mechanics. The uncertainty in quantum mechanics is not anything more than the *Ungenauigkeit*—*imprecision*, which is the term Heisenberg first used to describe the apparent randomness in our mathematical description of Nature[98]—we encounter in classical mechanics as well. It is just an imprecision, indeed—as opposed to the weird metaphysical quality which Heisenberg would later claim it to be[99] and which, without any precise

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[96] Arthur Compton actually did *not* fire photons into free electrons but into electron shells bound into atoms: it is only because the *binding* energy between the nucleus and the orbital electron is much lower than the energy of the X-ray photons that one could think of the electrons as being free. In fact, the experiment knocked them out of their orbitals!

[97] See: [Feynman's Lectures, III-2-6](#).

[98] We gratefully acknowledge [the Wikipedia article on Heisenberg](#) for this information. It states that it was effectively in Copenhagen – in 1927, to be precise – that Heisenberg developed his uncertainty principle but that – in the letter he wrote to Wolfgang Pauli to describe it – he used the term *Ungenauigkeit* rather than *Ungewissheit* or *Unbestimmtheit* (indefiniteness). The reader should note that German was probably more important than English or French as a language of science back then.

[99] We think the term 'indefiniteness' (*Ungewissheit* or *Unbestimmtheit* in German) might better capture this metaphysical quality. When saying this, we hastily add we do *not* believe such metaphysical quality applies to quantum mechanics: probing electrons with photons is a complicated process but we do *not* believe it is *fundamentally* different from, say, a blind man probing some object by taking it in his hands and trying to *feel* it or – if you prefer a more accurate metaphor – carefully scanning some object by firing spherical elastic balls at it and carefully keeping track of how they bounce back.

definition, physicists now refer to as 'uncertainty'. The concept of 'uncertainty' effectively mixes two different connotations:

(3) an *imprecision* in our measurements that is due to the wave-particle nature of electrons and other elementary particles—a duality which the ring current model, combining the idea of charge and its motion, captures from the outset (as opposed to trying to cope with the duality in some wave equation or other *dual* mathematical descriptions); and
(4) a rapid *magnification* of what Feynman refers to as 'the tiniest irregularities' but which are, *in practice*, purely statistical.

This is reflected in what Feynman refers to as the 'great equation of quantum mechanics'—the wave

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Needless to say, we do acknowledge the force is not a simple *contact* force here: the *structure* of the electromagnetic force is complicated, as evidenced from the fact that Volume II of Feynman's *Lectures* is the most voluminous of all three, even if all it does is go through the *consequences* of Maxwell's set of equations. In fact, Feynman shows that, when using four-vector notation, Maxwell's equations reduce to two only:

$$\Box^2 A_\mu = \frac{1}{\epsilon_0} j_\mu$$

$$\nabla_\mu j_\mu = 0$$

The A_μ and j_μ are four-*vectors* – A_μ combines the classical electric potential and the (magnetic) vector potential $\boldsymbol{A}$, so $A_\mu = (\phi, \boldsymbol{A})$ – and so I should probably use **boldface** rather than *straight face* (yes, I just invented that term right here and now). If you are reading this footnote, it means you are *damn* serious in trying to figure this out—so, yes, please do email me if you have some ideas here. I promise I *will* reply. The point that I want to make here is this: the first relation above models the *dynamics* of Maxwell's equations. The second just states the principle of charge conservation. Combine that with the idea of various *modes* of an oscillation in space and in time, and… Well… That's as close as I got to modelling the Great Mystery in Physics.

The implications of this 're-write' of Maxwell's equations are indeed quite deep. From the first equation, we get this:

$$\frac{j_\mu}{A_\mu} = \Box^2 \epsilon_0$$

The relation is, obviously, *extremely beautiful*: the *ratio* of two *physical* (but four-dimensional) realities equals the product of a mathematical operator and a (one-dimensional) physical (scalar) constant. I honestly don't have a clue about what it might mean, but it *must* be a very importantly relation. Why? Because the electric constant ε_0 is related to lightspeed *and* the fine-structure constant. We will let you *google* those relationships. Again, if you feel you make sense of them, please do not hesitate to email us: I think about them pretty much *all of the time*. Feynman had to get his Nobel Prize and, therefore, probably deceives us here and there. However, in Vol. II, he is a genius at work and, for me, I think the *depth* of his equations is quite obvious here:

Remember that Eqs. (25.21) could be deduced from Maxwell's equations only if we imposed the gauge condition

$$\frac{\partial \phi}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{A} = 0, \tag{25.23}$$

which just says $\nabla_\mu A_\mu = 0$; the gauge condition says that the divergence of the four-vector A_μ is zero. This condition is called the *Lorenz condition*. It is very convenient because it is an invariant condition and therefore Maxwell's equations stay in the form of Eq. (25.22) for all frames.

equation for an electron in free space[100] [101]:

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2}=\frac{\mathrm{m}^2c^2}{h^2}\phi=\frac{1}{\lambda_C^2}\phi$$

We replaced the m^2c^2/h^2 by the Compton wavelength $\lambda_C = h/\mathrm{m}c$ in the formula above because we effectively do think as the *distance scale within which we can localize the electron in a particle-like sense*. The *dissipation* which this wave equation models is, therefore, not the dissipation of the electron itself but, effectively, our rapidly increasing *uncertainty* about where it might be at a later point in time.

This, of course, triggers the question: why would it increase so rapidly? The answer is simple. It's just scale: Planck's quantum of action can (also) be written as the product of momentum over some distance ($h = \mathrm{p}\cdot\lambda$)[102] and, in light of the very tiny distance scale (λ), the momentum (p) is, effectively, *relatively* large—which is why any imprecision in its *magnitude* or *direction* quickly translates into huge uncertainties concerning the actual *position* of the elementary particle that we are trying to describe.

This is clear enough[103], but the smart reader will, of course, immediately ask for more clarity: do we think the uncertainty is in the *direction* or in the *magnitude* of the momentum? This is an *essential* question, indeed! If the uncertainty is purely statistical, then what's the *hidden variable*? We think the hidden variable may be in the motion of the plane of rotation of the *Zitterbewegung* charge. Hence, we think the statistical uncertainty has to do with the *direction* rather than the magnitude of the momentum. We may, perhaps, focus the idea of the reader with the illustration below. It shows the *precessional* motion of the plane of rotation of a spinning top. It reminds us of the *vector* quality of the angular momentum. We should, therefore, probably write Planck's quantum as a vector: $\boldsymbol{h}$ (or, to use the right unit here, $\boldsymbol{\hbar}$) instead of h (or $\hbar$)—just like we would use **L** (the **boldface** to denote a vector) rather than L for angular momentum. Look at the wobbling angular momentum of the spinning top below, and then think of the plane of rotation of our pointlike charge.

[100] This is just one of the many expressions of it. We took it from Feynman's *Lectures*, section I-48-6. Feynman did *not* substitute m^2c^2/h^2 for $1/\lambda_C^2$ in his presentation. We must, therefore, assume he did not think of any possible association or relation with the Compton wavelength and/or *radius* of an electron.

[101] Needless to say, free space means we don't think of any electromagnetic field or any other force field here. Schrödinger adapted this equation to include a central force field: a hydrogen nucleus (a proton) attracting the electrons around it. It is good to note that Dirac refers to such wave equations as the *equations of motion* of an electron, because that's what they are.

[102] The $h = \mathrm{E}\cdot\mathrm{T}$ and $h = \mathrm{p}\cdot\lambda$ are *complementary* but *equivalent* descriptions of the electron in the ring current model.

[103] The reader will immediately recognize this is just the same mechanism as the (statistical) mechanism which makes chaos theory comprehensible and, therefore, rational and *logically true*.

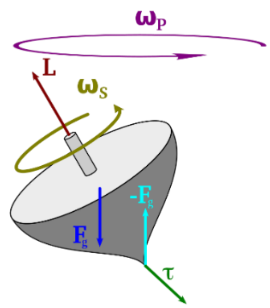

Figure 9: A wobbling plane of rotation[104]

Of course, we should remind ourselves we only have regularity in this motion because of the force field, which is gravitational in the example of the spinning top (it's the $\mathbf{F}_g$ vector). In quantum mechanics, the force field would be some *magnetic* field: think of the Stern-Gerlach experiment here.[105] The question then becomes: what's the motion in the *absence* of a magnetic field?

We do not have the answer to that question but we do *not* believe that motion would be entirely *random*: one would expect that *interactions* with other charged and non-charged particles (think of the interaction with photons which we just described) would not only result in some change of the linear state of motion of an electron but in changes in the *direction* of its angular momentum as well. These are, of course, assumptions we may not be able to test because the *measurement* itself – the probing with photons, or the measurement of the angular momentum by applying some magnetic field – would cause such changes themselves! This is, in fact, what Heisenberg's *Ungenauigkeit* is all about![106]

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[104] The illustration is taken from Wikimedia Commons under the CC-BY-SA 2.5 license so I need to acknowledge the author: Xavier Snelgrove.

[105] The reader will know that – in order to *separate* particles based on their magnetic moment being *up* or *down* – the magnetic field cannot be *homogenous*: a field *gradient* is necessary so as to produce an actual *force* on the particle. We can, therefore, not *prove* that an electron – or whatever other elementary particle with spin – will effectively *line up* in a homogenous magnetic field as well. In fact, we should note something which may surprise the reader: an actual Stern-Gerlach experiment has actually *never* been done with electrons—or with other *charged* particles. Stern-Gerlach experiments are always done with electrically neutral particles, such as potassium atoms (see, for example, this rather typical lab experiment for students) or, in the original experiment, silver atoms. There is a very practical reason for this, of course: it's because any electric charge in the magnetic field in the Stern-Gerlach apparatus would be subject to a Lorentz force which would be much larger than the force resulting from the magnetic moment. At the same time, I find it a rather weird state of affairs: most, if not all, of mainstream quantum theory hinges on the assumption that an electron has *two* spin states only (*up* versus *down*) but, at the same time, *no experiment has actually verified this*. It is *not* that no one ever suggested it. H. Batelaan, T. J. Gay, and J. J. Schwendiman, for example, wrote a rather intriguing *letter* to the *Physical Review* journal in 1997, explaining in very much detail how the Stern-Gerlach experiment could be modified to also split an electron beam based on the magnetic moment being up or down. Their proposal is based, in fact, on a proposal of the French physicist Léon Brillouin. Brillouin was an active participant in the 1927 Solvay Conference and his proposal, which he published in 1928 (L. Brillouin, Proc. Natl. Acad. Sci. U.S.A. 14, 755, 1928), can be traced straight back to the discussions there. In light of the venerable history of this proposal, it is *very* strange no one has followed up on it. One would think this should be a higher priority than the new US$600m accelerator project which Dr. Consa talks about in his scathing review of the current state of affairs in the field of experimental research.

[106] We may usefully remind the reader Heisenberg's very first arguments were entirely based on this kind of *physical* reasoning. Think of Heisenberg's thought experiment involving a microscope using gamma rays instead of

Hence, *in the absence of a magnetic field* (read: for a particle *in free* space), we would not be able to ascertain the *direction* of Planck's quantum – written as a vector (***h*** or, in its reduced form, ***ħ***) but, while we would think of it as being fairly *random*, we would not doubt its direction is the result of the motion of the plane of rotation of the *Zitterbewegung* charge which, itself, is the result of previous interactions and events. Such *randomness* would, therefore, be random but it would *not* be mysterious!

So how would we write the Uncertainty Principle then? Simple, we just write ***h*** and **p** as vectors. We get the $\mathbf{p}\cdot\lambda = \boldsymbol{h}$ expression from the Planck-Einstein relation and we just apply it to the ring current geometry (with λ the *de Broglie* wavelength[107]). We must then introduce the idea of a non-precise *direction* of ***h*** so we can write something like this:

$$\Delta\boldsymbol{h} = \Delta(\mathbf{p}\cdot\lambda)$$

Plain *vector* algebra – and remembering Δ***h*** denotes an uncertainty in the *direction* of the ***h*** and **p** vectors, not of its magnitude – then gives us the expression you are used to—Heisenberg's (in)famous Uncertainty Principle:

$$h = \Delta p\cdot\Delta\lambda$$

That's all what it is. By way of conclusion, we may, perhaps, add one more remark: why are we so sure the uncertainty must be in the *direction* of the angular momentum of the electron, rather than in its magnitude?

The honest answer is: we can*not* be sure. We are basically just siding with Einstein's instinct here: the velocity of light is the velocity of light, Planck's quantum of action is Planck's quantum action, and the classical velocity of the electron is the classical velocity of the electron. Hence, if we assume the Compton radius of the electron and the motion of the charge inside is *defined* by *h* and *c*, and if we assume the linear motion of the same charge is defined by the classical (linear) velocity of the electron (*v*), then there is no room for uncertainty! The uncertainty must, therefore, be in the wobbling of ***h***. Nothing more. Nothing less.[108]

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optical light: it is *very* obvious Heisenberg does *not* think of uncertainty as something metaphysical here! Instead, he plainly relates the uncertainty to the *interaction* between the observer – the *act of probing* or measuring – and what is being observed or measured.

[107] For a *geometric* interpretation of the *de Broglie* wavelength, we refer to our discussion of the Compton wavelength and what happens to it when the *Zitterbewegung* charge inside of the electron is also subjected to some linear motion.

[108] We are very much aware of our rather arrogant tone here. I am happy to get other opinions here. To be sure, I would treat them as being philosophical rather than scientific—as philosophical as Einstein's or our own intuition here. However, that should not prevent one from having a healthy logical discussion on scientific matters.

Conclusions

The professional physicist will point out that (1) the lines above were *definitely unscientific* and (2) despite our claim we would give you a *real* explanation of Compton scattering, we failed to produce all of the *details* of the process. Indeed, how does it *work*, *exactly*?

We effectively did not work out all of the details, but we do have some very strong *clues* here on how a *definitive* explanation – including *all* relevant variables, including the mentioned *plane* formed by the outgoing photon and the recoiling electron – may look like. The *plane* of the *Zitterbewegung* oscillation, for example, must – without any doubt – play a crucial role in determining the angles of the incoming as well as the outgoing photon.

Also, it is quite obvious that the *circular motion* of the *zbw* charge must explain the π or 2π factor. It is *not* a size factor: the size of the electron is of the order of the Compton *radius* (*a*) but, because of the circular motion, it is actually the *circumference* of the motion (λ = 2π*a*) that must enter the Compton scattering formula. The image of a hand sling throwing a stone comes to mind, of course—but that's probably too simplistic: throwing some mass out by converting circular to linear motion is easy enough to imagine, but here we're not talking mass.

At the same time, our models are all 'mass without mass' models and so they show that energy is, ultimately, motion: an oscillating force on a charge in case of the electron – or, in case of the photon, an oscillating electromagnetic field. The motion that's associated with an electron is circular, while the motion of the oscillating electromagnetic field is linear. As such, we may – figuratively speaking – say that 'circular' motion is being converted into 'linear' motion, and vice versa.

In terms of the 'mechanics' of what might actually be going on when a photon is absorbed or emitted by an electron, that's all what we can give you for the time being ! You'll have to admit we can't quite show you what's going on inside of the box, but we did open it at least, didn't we?

Just in case you think we didn't, we invite you – once again – to google the papers explaining the *S*-matrix of Compton scattering processes.[109] We are sure you will prefer to re-read our paper and – who knows? – start doing some pretty calculations yourself!

There is, perhaps, one question we still need to answer. Why and when did things go wrong in the history of scientific thought here?

Now, I have not had the time to do much historical research here, but I do have a few clues, which I will sum up in some separate concluding remarks.

[109] As mentioned in the introduction, we rather randomly googled the following two papers: https://arxiv.org/abs/1512.06681 and https://arxiv.org/abs/1810.11455. You may get others. This *input-output calculus* approach allows for endless refinements and, hence, endless research—which is always nice from an academic point of view, of course.

A brief history of bad ideas

Dr. Oliver Consa traces all of the nonsense in modern physics back to the Shelter Island (1947), Pocono (1948) and Oldstone (1949) Conferences, about which he writes the following:

> "After the end of World War II, American physicists organized a series of three transcendent conferences for the development of modern physics: Shelter Island (1947), Pocono (1948) and Oldstone (1949). These conferences were intended to be a continuation of the mythical Solvay conferences. But, after World War II, the world had changed. The launch of the atomic bombs in Hiroshima and Nagasaki (1945), followed by the immediate surrender of Japan, made the Manhattan Project scientists true war heroes. Physicists were no longer a group of harmless intellectuals; they had become the powerful holders of the secrets of the atomic bomb."[110]

Secrets that could not be kept, of course. The gatekeepers did their best, however. Julius Robert Oppenheimer – father of the atomic bomb and prominent pacifist at the same time – was, effectively, one of them.

The Solvay Conferences actually continued after World War II, and Niels Bohr and Robert Oppenheimer pretty much dominated the very first post-WW II Solvay Conference, which was held in 1948. Bohr does so by providing the introductory lecture '*On the Notions of Causality and Complementarity*'[111], while Oppenheimer's '*Electron Theory*' sets the tone for subsequent Solvay Conferences—most notably the one that would consecrate quantum field theory (QFT), which was held 13 years later (1961).[112]

Significantly, Paul Dirac is pretty much the only one asking Oppenheimer critical questions. As for Albert Einstein, I find it rather strange that – despite him being a member of the scientific committee[113] – he actually hardly interferes in discussions. It makes me think he had actually lost interest in the development of quantum theory.

Let us go back to the discussions between Dirac and Oppenheimer on the '*Electron Theory*' paper in 1948. It may effectively mark one of the watershed moment: the point where things went wrong. In fact, both Oppenheimer and Dirac make rather historical mistakes here:

1. Robert Oppenheimer uses perturbation theory to arrive at some kind of 'new' model of an electron, based on Schwinger's new quantum field theory (QFT)—which, as we now know, do not lead anywhere.

2. Paul Dirac then challenges the use of these theories making the following sensible comment:

> "All the infinities that are continually bothering us arise when we use a perturbation method,

[110] Oliver Consa, *Something is rotten in the state of QED*, February 2020.

[111] See the 1948 Solvay Conference report on the ULB's digital archives.

[112] Institut international de physique Solvay (1962), *La théorie quantique des champs: douzième Conseil de physique*, held at the Free University of Brussels, 9 to 14 October 1961.

[113] Einstein was a member of the Solvay scientific committee from the very first conference (1911) – representing, in typical style, a country (Austria, *not* Germany) rather than an institution or just being a member in some personal capacity – till 1948. He was *not* a member of the 1951 scientific committee. The reason might well be age or a lack of interest, of course: Einstein was 72 years in 1951, and would die four years later (1955).

> when we try to expand the solution of the wave equation as a power series in the electron charge. Suppose we look at the equations without using a perturbation method, then there is no reason to believe that infinities would occur. The problem, to solve the equations without using perturbation methods, is of course very difficult mathematically, but it can be done in some simple cases. For example, for a single electron by itself one can work out very easily the solutions without using perturbation methods and one gets solutions without infinities."[114]

At the same time, he is too stubbornly foolish himself[115]: he simply keeps defending his un-defendable electron equation which, of course, does not lead anywhere either:

> "We would not get infinities if we solve the wave equations without using a perturbation method [but] if we look at the solutions which we obtain in this way, we meet another difficulty: namely we have the run-away electrons appearing. Most of the terms in our wave functions will correspond to electrons which are running away[116], in the sense we have discussed yesterday and cannot correspond to anything physical. Thus nearly all the terms in the wave functions have to be discarded, according to present ideas. Only a small part of the wave function has a physical meaning."

It is, however, *very* weird that Dirac does *not* follow through on his own conclusion:

> "*Only a small part of the wave function has a physical meaning. We now have the problem of picking out that very small physical part of the exact solution of the wave equation.*"

It is the ring current or *Zitterbewegung* electron, of course! It is that trivial solution that Schrödinger had found, and which Dirac mentioned very prominently in his 1933 Nobel Prize lecture[117]. The other part of the solution(s) is (are), effectively, bizarre oscillations which Dirac here refers to as 'run-away electrons'.

Dirac just loved his own wavefunction too much to jettison it. "Kill your darlings" is a common piece of advice given by unexperienced writers. Dirac should have thought about it, but so he did not. Here again, the Nobel Prize Committee for Physics may actually have consecrated bad theory that, as a result of the award, became difficult – if not impossible – to backtrack on.

That is my take on it but we will, of course, let you think about this for yourself. 😊

Jean Louis Van Belle, 25 April 2020

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[114] See pp. 282-283 of the report of the 1948 Solvay Conference, *Discussion du rapport de Mr. Oppenheimer.*

[115] I think it is significant that Paul A.M. Dirac was no longer invited nor even mentioned in the 1951 Solvay Conference. Niels Bohr and Wolfgang Pauli made subsequent remarks that clearly show Paul Dirac was the only one who actually dared to challenge what had, by then, become mainstream theory.

[116] See our own remarks in the beginning of this paper on this major problem of the wave equation: actual matter-particles do *not* dissipate.

[117] See p. 17 of this paper.

Annex: Light, photons and the Mach-Zehnder interference experiment

We did not say all that much about photons in this paper. We only gave you the basic model: a photon as a one-cycle oscillation. The photon model may look simpler than the electron model but statements like that are probably nothing but a matter of taste. Electromagnetic oscillations are rather complicated. Let us, therefore, add a few remarks to this paper so as to illustrate this complexity.

Polarization states based on the electric field vector only

You have probably seen an illustration like the one below a zillion times already: it shows linear, elliptical, and circular polarization states for light—and for a photon, because we think of a photon as an electromagnetic oscillation too.

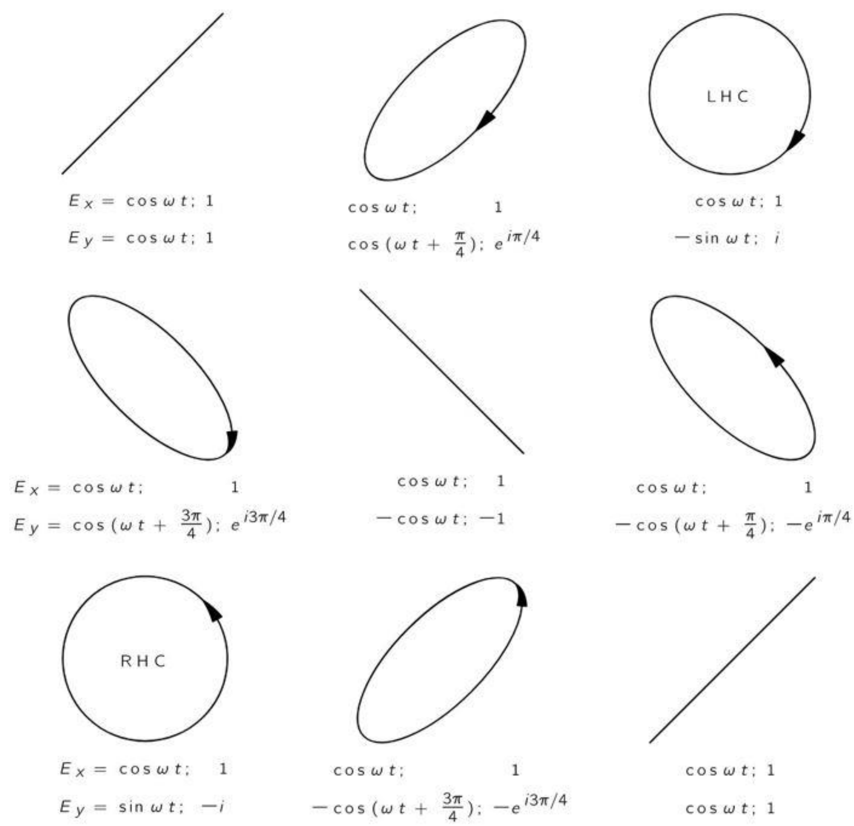

Figure 10: Linear, elliptical, and circular polarization states (electric field vector only)

So what is wrong with this image, which we borrowed from Feynman's *Lectures*?[118] Nothing much, except that Feynman makes the same mistake here as many professors explaining polarization states of photons: he only thinks of the *electric* field vector ***E***. There is a magnetic field vector ***B*** too, whose

[118] We, once again, gratefully acknowledge Caltech (and Michael Gottlieb and Rudolf Pfeiffer, of course) for making Feynman's *Lectures* available online as a common reference for amateur physicists. This illustration was taken from Feynman's *Lecture* on the polarization of light. Note that, interestingly, Feynman expresses the E_x and E_y components in both real as well as complex notation. I am not sure, but it suggests Feynman knew he was presenting one aspect of the electromagnetic oscillation only.

direction in space will always be *perpendicular* to $\boldsymbol{E}$ as well as to the velocity vector $\boldsymbol{c}$.[119] In fact, we may also write $\boldsymbol{B}$ as $\boldsymbol{B} = i \cdot \boldsymbol{E}/c$ or $-i \cdot \boldsymbol{E}/c$[120].

We are not sure why physicists make that mistake. There may be two reasons:

1. The *strength* of the magnetic field is $1/c$ times that of the electric field, so they may think that is negligibly small.
2. There may be a magnetic field but Lorentz force law tells us there is no force if there is no charge. Hence, as long as the force does 't have a charge to play with (think of an electron here), we should – perhaps – not consider the magnetic field.

Let us tackle the second consideration first. It is nonsensical because the magnetic field has a reality of its own and the magnetic effect should, therefore, always be considered. To be precise, because of the absence of a charge and, therefore, some velocity $\boldsymbol{v}$ we can use in the $\mathbf{F} = q\mathbf{E} + q\boldsymbol{v}\times\mathbf{B}$ formula for the Lorentz force, we should actually switch to a relativistically correct mathematical framework (read: four-vector notation) and, as we do so, talk about the magnetic vector potential instead of the magnetic field. Because we just want to make some basic remarks here, we will not dwell on this and just refer the reader to more advanced reading material here.[121] The point is: the second reason is not a good reason because it is *false*.

The first reason is nonsensical as well but the argument has to be somewhat more subtle here: it has got to do with scales and natural units. Let us say a few words about that in a separate sub-section so as to provide for some structure here.

The relative strength of the electric and magnetic field

We can, effectively, only talk about the relative strength of the electric and magnetic field in terms of natural time and distance units. Why? Think of this: if we *change* our time and distance units so as to ensure we measure c as being equal to unity ($c = 1$), then the $B = E/c$ formula for the *magnitudes* of the field *vectors* $\boldsymbol{E}$ and $\boldsymbol{B}$ reduces to $B = E$.

We should, of course, immediately add that simplifications like this are always quite tricky because the *physical* m/s dimension of c continues to matter even if the *numerical* value of c is 1. Indeed, the *electric* field strength is measured in newton per coulomb (N/C), but the magnetic field strength is, effectively, expressed in units of (N/C)/(m/s) = (N·s)/(C·m) = T (*tesla*). Having said that, you immediately see what

[119] You may not have seen a *vector* notation for c but just think of it as a *velocity* – the velocity of the photon, of course – and then you will appreciate both the magnitude as well as the *direction* must be taken into account in a *physical* analysis of what might or might not be happening.

[120] The + or – sign depends on our orientation, which determines whether we should rotate $\boldsymbol{E}/c$ clockwise or counter-clockwise to obtain $\boldsymbol{B}$. Needless to say, we use **boldface** letters to represent geometric vectors – the electric ($\boldsymbol{E}$) and magnetic field vectors ($\boldsymbol{B}$), in this case. There is a risk of confusion between the energy E and the electric field E because we use the same symbols but the context (and our italics (E instead of E), if we do not forget them) should make clear what is what.

[121] We did a few blog posts on this, but these stuck rather closely to Feynman's treatment of the matter, so we may as well direct the reader to Feynman's *Lectures* here too.

happens: we can no longer say one field is more important than the other. Both are equally relevant and, therefore, need to be part of the analysis. So what are we saying here?

We are actually saying a *linear* polarization state is not really linear. We need to think of the combined *electromagnetic* field of a photon making its way through space. Of course, you may say natural time and distance units are just what they are – natural units – and you may wonder why we would want to work with them. I have no *definite* answer to that question, but think of this. We may not only want to think of natural time and distance units but also of some kind of natural unit for the force, the energy or the (linear or angular) momentum. Force, energy, linear and angular momentum are currently expressed in N (newton), N·m, N·s, and N·m·s, respectively. This shows that, if we already have a new unit for distance and time (let us write those units as m_γ and s_γ respectively), we will just need to calculate a new unit for the force *too*. We will write that unit as N_γ for the time being and we will calculate it based on…

Well... Based on what?

It should be based on some *physical* law, obviously—a law such as the one we used to define natural units for time and distance, which is this: the speed of light is always equal to *c*, *regardless of the frame of reference*. It is obvious we can – I'd say, *should* – use the Planck-Einstein relation now. If we have the wavelength and the cycle time for a photon – *any* photon, really (a highly energetic gamma ray photon, a visible light photon, a low-frequency radiowave photon or whatever) – then we know the following relations will *always* be true—and *also* regardless of the frame of reference:

$$h = \mathrm{E}_\gamma \cdot \mathrm{T}_\gamma = \mathrm{E}_\gamma \cdot \frac{\lambda_\gamma}{c} = \mathrm{p}_\gamma \cdot c \cdot \frac{\lambda_\gamma}{c} = \mathrm{p}_\gamma \cdot \lambda_\gamma = \mathrm{F}_\gamma \cdot \lambda_\gamma \cdot \mathrm{T}_\gamma$$

You will say: what *is* that F_γ force in that $h = F_\gamma \cdot \lambda_\gamma \cdot T_\gamma$ relation? However, you should not worry too much about that now: think of it as the force that comes with the idea of an oscillating electromagnetic *field*[122] and just note the relation(s) above become(s) self-evident when dividing *h* by *h* and doing some substitution[123]:

$$\frac{h}{h} = \frac{\mathrm{p}_\gamma \cdot \lambda_\gamma}{\mathrm{E}_\gamma \cdot \mathrm{T}_\gamma} = \frac{\mathrm{p}_\gamma}{\mathrm{E}_\gamma} \cdot \frac{\lambda_\gamma}{\mathrm{T}_\gamma} = \frac{1}{c} \cdot c = 1$$

The point is this: if you are defining natural time and distance units based on the *absolute* velocity of light, then you should also define a natural force unit based on the Planck-Einstein relation. If you do so

[122] The electric and/or magnetic field are usually defined in terms of a force *per unit charge*. Indeed, we will say the field strength is so many *newton* per *coulomb*, with an added complication for the magnetic field because of the mentioned second per meter (s/m) factor: the *physical* dimension of the magnetic field, which we write as [***B***], is equal to [***B***] = (N/C)/(m/s) = (N/C)·(m/s).

[123] You may want to note the p = E/*c* relation becomes self-evident when using the mass-equivalence relation E/m = c^2 and writing p as p = m*c* = (E/c^2)·*c* = E/*c*. Also, because light and, therefore, photons travel at the speed of light, the λ/T = *c* relation is equally self-evident. The reader may sometimes wonder why we would be writing simple things like this, but we hope these small digressions help in understanding what we are trying to understand. 😊

– i.e. if you do re-define time, distance, and force units so as to make sure the *numerical* value of *c* and *h* are equal to unity[124] – then you also get natural units for energy and (linear or angular) momentum.

This should suffice to show that the idea of the *relative* strength of the electric and magnetic field does not make much sense. However, we want to add an interesting argument here. You may or may not know that we can *define* the electric and magnetic constants ε_0 and μ_0 in terms of the fine-structure constant α, the lightspeed *c* and Planck's quantum of action *h*:

$$\left.\begin{matrix} \alpha = \dfrac{q_e^2}{2\varepsilon_0 hc} \Leftrightarrow \varepsilon_0 = \dfrac{q_e^2}{2\alpha hc} \\ \mu_0 = \dfrac{1}{\varepsilon_0 c^2} \end{matrix}\right\} \Leftrightarrow \left\{\begin{matrix} \varepsilon_0 = \dfrac{q_e^2}{2\alpha hc} \\ \mu_0 = \dfrac{2\alpha h}{q_e^2 c} \end{matrix}\right.$$

It is now easy to see that the electric and magnetic constants become each other's reciprocal if, *and only if*, we use a natural unit for *h* as well:

$$\mu_0 = \frac{1}{\varepsilon_0} \Leftrightarrow \frac{2\alpha h}{q_e^2 c} = \frac{q_e^2}{2\alpha hc} \Leftrightarrow \frac{c}{h} = hc \Leftrightarrow \frac{c}{h} = hc \Leftrightarrow \frac{c}{c} = 1 = h^2 \Leftrightarrow h = \pm 1$$

We may have lost the reader by now so what's the point? The point is this: you should think of a *linearly* polarized photon as a *two*-dimensional oscillation as well. The oscillating *electric* field vector is just one of the two aspects of the *electromagnetic* oscillation and, hence, you should always think of the *magnetic* field vector too! When you do that, the *hocus-pocus* of quantum-mechanical interference and diffraction experiments should disappear.[125]

Note: The $h^2 = 1$ condition effectively corresponds to two different possibilities: $h = 1$ or $h = -1$. This is in line with our interpretation of *h* as a vector quantity (***h*** instead of *h*). The $h = 1$ possibility corresponds to matter, while the $h = -1$ corresponds to antimatter. It is nothing but a reversal of spatial directions—both linear as well as rotational (or circular) directions.

The geometry of a polarized photon

To describe the *exact* geometry of a linearly, circularly, or elliptically polarized photon – taking into account *both* the *electric* as well as the *magnetic* field – one needs some more sophisticated geometric formulas. We did *not* work this out fully, but the following remarks may encourage the reader to explore the *exact* geometry of a photon somewhat further.

The geometry of the ellipse (or ellipsoid) relates the geometry of the circle (or the sphere) to the geometry of a (wave)length.[126] We will want to think of an electromagnetic field vector rotating around the *direction* of wave propagation here so we will limit ourselves to a two-dimensional analysis here.

[124] We would like to mention once again you should not forget about the *physical* dimension of the *c* and *h* constants. Writing $c = h = 1$ is, basically, meaningless because of the *physical* dimension of these constants: *c* is a velocity (expressed in m/s or m_γ/s_γ or whatever time and distance unit you'd want to choose), while *h* is expressed in units of *physical action*: force times distance times time.

[125] See, for example, our classical explanation of one-photon Mach-Zehnder interference.

[126] An ellipse is a shape in two dimensions only, while an ellipsoid is a shape in three dimensions. Think of the difference between a circle and a sphere here.

Have a look at the a, b, and c lengths in the illustration below. If b goes to zero, a goes to c. Also, it is easy to see that the two focal points (F_2 and F_1) will become the start and end point of a line.

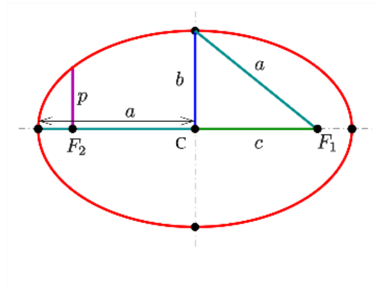

Figure 11: The *latus rectum* formula: $a \cdot p = b^2$

Conversely, if we let the two focal points approach each other, we will get a perfect circle when they coincide. The length p then becomes the radius of the circle, and the *latus rectum* formula becomes a $a \cdot p = b^2 = a \cdot b = b^2 = a^2$. We can now relate the *circumference* of a circle to a linear distance. Just think of *flattening* a circle by *stretching* the ellipse, as illustrated below[127].

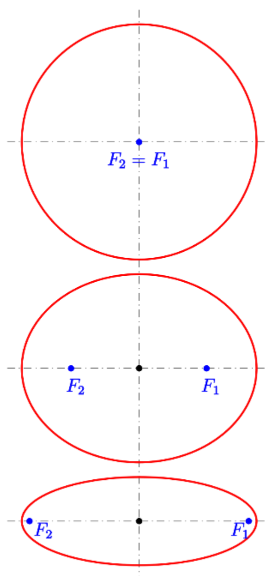

Figure 12: Flattening the circle by stretching the ellipse

If we denote the *radius* of our circle by r, then it is easy to see the circumference of the 'linear ellipse' will be equal to $2\pi r$. It is equally obvious that the *length* of the line will then be equal to *half* of the circumference of the circle (πr). In other words, when the *eccentricity* of the ellipse ($e = c/a$[128]) goes

[127] These illustrations were provided under the Creative Commons CC BY-SA 4.0 license so we acknowledge the source and/or the author here.

[128] The symbol e obviously does *not* refer to Euler's number here, and the c here is just the distance from the center to the focal point(s).

from 0 to 1, the length of the circular arc πr – think of it as a *circular* length[129] – becomes a simple *linear* length.[130]

What is the point here? As mentioned, we did *not* work this out fully. We just wanted to draw attention to the fact that most analyses of the polarization state of a photon are rather simplistic because they forget about the magnetic field vector. Nothing more, nothing less—for the time being, at least![131]

Let us conclude this Annex by a short reflection on a question which few people seem to care about: if light – or photons – may interfere constructively or destructively, what happens with the energy? The question is quite obvious for the case of *destructive* interference. Indeed, what happens there? Does the energy just disappear as two photons *annihilate* each other? Maybe. Maybe not. Let us think about that in a separate section.

What happens to the energy when there is destructive interference?

It is an interesting question: if two light beams are *exactly* 180 degrees *out* of phase, we get *destructive* interference and the resulting beam should have *zero* energy. Hence, the question is: where is the energy of the incoming beams now? The energy conservation principle tells us it should be *some*where, right? Right.

We did not spend too much time on this but, when *googling*, we found one good discussion on the Physics StackExchange site, one very good video on MIT's OpenCourseWare, and one very good paper written by a former Princeton faculty member on it. These, taken together, give us a pretty reasonable answer to the question: photons – or the two *halves* of a photon in a one-photon Mach-Zehnder interference experiment[132] – do not destroy each other when coming together at the detection

[129] However, it seems that this is a non-existing term. Everyone seems to go through the trouble of writing such length as 'the length of the circular arc.'

[130] We may mention a rather weird mathematical fact here: there is no easy formula for the circumference (or *perimeter* as it is often referred to) of an ellipse. See the Wikipedia entry on it or this very interesting paper of Prof. Dr. Paul Abbott.

[131] The next version of this paper may hold some more clues! 😊

[132] Note the tricky question at the end of the video: "How is it that we get constructive interference in one beam and destructive interference in another beam, when the two paths are the same in the two cases?" Fortunately, we get a quick answer from one of the comments on the video: reflecting off of the back of a beam splitter does not introduce a phase shift, while reflecting off of mirrors, and the front side of a beam splitter, will introduce a 180 degree (i.e. a half wavelength) phase shift. It took me quite a while to think this through, and I am actually still not finished with it. For me, it just confirms the tricky *physics* of these mirrors and beam splitters in these so-called one-photon Mach-Zehnder experiments is what explains the equally tricky results and conclusions. We think an entirely *classical* explanation of the interference of a photon with itself should be possible, and that it is likely based on the fact that beam splitters also result in linear polarization. However, we admit we did not have the time, energy, and courage so far to fully work out the details of such explanation. We invite the interested reader to further engage and – if possible – work out such fully-fledged solution him- or herself.

point—whether it be a screen or a photon detector.[133] They just bounce back[134] or – if there is no screen – (continue to) travel along happily.

The other side of the coin (or the question, I should say) is also interesting: if there is *constructive* interference, we should get *twice* the energy when the beams are in phase. Why? We illustrated this before[135] but let us, for the convenience of the reader, quickly review the basics here. They are this:

1. These interference experiments – one photon at a time or – more likely because easier to do in a student lab environment – always involve a beam splitter. Such beam splitter does not only split the beam into two: it also changes its *shape*—we get two *linearly* polarized beam.[136] Hence, you should think of two *linearly* polarized beams, which we will refer to as beam *a* and *b,* respectively. These waves then arrive at the detector – the screen or the single-photon detector[137] – with a phase difference – unless the apparatus has been set up to ensure the distances along both paths are exactly the same. Hence, the general case is that we would describe *a* by $cos(\omega\cdot t - k\cdot x) = cos(\theta)$ and *b* by $cos(\theta + \Delta)$ respectively.

In the classical analysis, the difference in phase (Δ) will be there because of a difference of the path lengths[138] and the recombined wavefunction will be equal to the same cosine function, but with argument $\theta + \Delta/2$, multiplied by an *envelope* equal to $2\cdot\cos(\Delta/2)$. We know this sounds rather monstrous but we are just applying basic calculus here.[139] Indeed, to make a long story short, we can just write this:

$$\cos(\theta) + \cos(\theta + \Delta) = 2\cdot\cos(\theta + \Delta/2)\cdot\cos(\Delta/2)$$

---

[133] If it is a screen, we require a *photon* beam so as to provide sufficient energy to excite the electrons of the atoms that make up the screen, so we can literally *see* the effect as these excitation states go back to normal and re-emit photons of the same frequency as the incoming photons in all directions, including the line of sight formed by our eye and the screen. One-photon Mach-Zehnder interference experiments obviously involve a much more delicate experimental set-up involving one-photon detectors!

[134] Again, if it is a screen – and we cannot *see* the effect – then we just have a dark spot on the screen: there is no interaction here. In other words, the photon(s) must effectively bounce back.

[135] See the above-mentioned reference to [our paper on one-photon Mach-Zehnder interference](). Alternatively, we may also refer to Chapter XV of [our 2019 manuscript on quantum mechanics]().

[136] You can check various sources here but [the Wikipedia article on beam splitters]() is quite easy to read and quite complete: most polarizing beam splitters – including those in student labs – use birefringent materials to split light into two beams of *orthogonal polarization states*.

[137] If we have *one* photon only, then our wave reduces to *one cycle* of the electromagnetic oscillation only. It is, therefore, hard to talk about a *beam*. However, the principle of interference is the same: the *principle of superposition* tells us all we have to do is to add the fields. If the field direction is the same, we get constructive interference. If it is opposite, we get destructive interference.

[138] Feynman's path integral approach to quantum mechanics allows photons (or probability amplitudes, we should say) to travel somewhat slower or faster than *c*, but that should not bother us here.

[139] We are just applying the formula for the sum of two cosines here. If we would add sines, we would get $\sin(\theta) + \sin(\theta + \Delta) = 2\cdot\sin(\theta + \Delta/2)\cdot\cos(\Delta/2)$. Hence, we get the same envelope: $2\cdot\cos(\Delta/2)$.

2. What this tells us, is that we always get a recombined beam with the same frequency. However, when the phase difference between the two incoming beams is small, its amplitude is going to be much larger. To be precise, it is going to be twice the amplitude of the incoming beams for $\Delta = 0$.

In contrast, if the two beams are out of phase, the amplitude is going to be much smaller. To be equally precise as above, it is going to be zero if the two waves are 180 degrees out of phase ($\Delta = \pi$), as shown below.

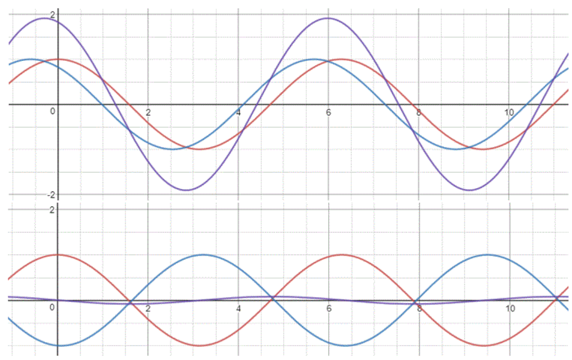

Figure 13: Constructive and destructive interference for linearly polarized beams

3. That may make *prima facie* sense but, in fact, it does not. Why not? Because we know the energy of an oscillation is proportional to the *square* of the amplitude. Hence, twice the amplitude means *four* times the energy, and zero amplitude means zero energy. The energy conservation law is being violated: photons are being multiplied or, conversely, are being destroyed.

Let us be *very* explicit about the energy calculation here. In order to do so, let us show you the set-up of a typical Mach-Zehnder interference experiment:

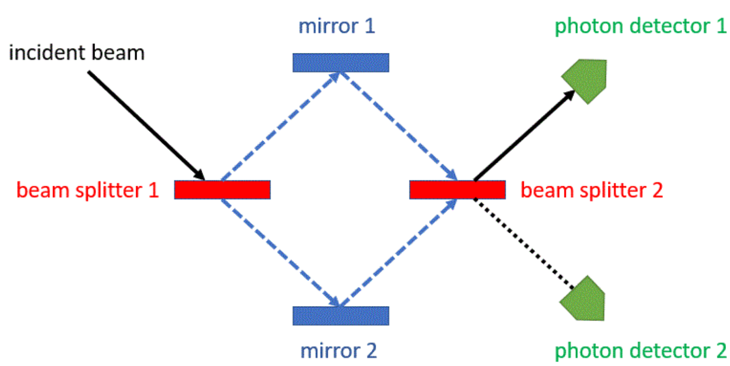

Figure 14: The Mach-Zehnder interferometer

So what is going on here? Before the photon enters the beam splitter, we have one wavefunction: the photon. When it goes through, we have two probability amplitudes[140] that – somehow – recombine and

[140] We already referred to our paper(s) or, alternatively, chapter VIII of our manuscript for a geometric interpretation of the wavefunction of a photon. When analyzing interference in this context, the wavefunction

interfere with each other. What we want to do here is to explain this classically. So let us look at the apparatus. We have two beam splitters (BS1 and BS2) and two perfect mirrors (M1 and M2). An incident beam coming from the left is split at BS1 and recombines at BS2, which sends two outgoing beams to the photon detectors D0 and D1. More importantly, by playing with the distances between the beam splitters and the mirrors, the interferometer can be set up to produce a precise interference effect which ensures all the light goes into D0. Alternatively, the setup may be altered to ensure all the light goes into D1.

Let us now discuss the energy calculations.

4. The energy conservation principle tells us that, when the incoming beam splits up at BS1, the energy of the *a* and *b* beam will be split in half too.[141] We know the energy is given by (or, to be precise, proportional to) the square of the amplitude (let us denote this amplitude by *A*).[142] Hence, if we normalize or standardize the energy of the incoming beam by equating it to one, then we will want the energy of the two individual beams to add up to $A^2 = 1^2 = 1$, then the (maximum) amplitude of the *a* and *b* beams must be 1/√2 of the amplitude of the original beam, and our formula becomes:

$$(1/\sqrt{2})\cdot\cos(\theta) + (1/\sqrt{2})\cdot\cos(\theta + \Delta) = (2/\sqrt{2})\cdot\cos(\theta + \Delta/2)\cdot\cos(\Delta/2)$$

This reduces to $(2/\sqrt{2})\cdot\cos(\theta)$ for $\Delta = 0$. Hence, we still get *twice* the energy – $(2/\sqrt{2})^2$ equals 2 – when the beams are in phase and zero energy when the two beams are 180 degrees out of phase.

This does *not* make sense. We must have made *some* mistake in our modeling here but *what* mistake, *exactly*?

5. The only mistake we might have made is in the assumption that the *direction* of the (linear) polarization of the two beams is the same. They are not: the two linear waves must be orthogonal to each other, and that is why we cannot just add the amplitudes of the *a* and *b* beams—because they have different directions.

All makes sense now. If the *a* and *b* beams – after being split from the original beam – are linearly polarized, then the angle between the axes of polarization should be equal to 90 degrees to ensure that the two oscillations are independent. We can then add them like we would add the two parts of a complex number. Remembering the geometric interpretation of the imaginary unit as a counterclockwise rotation, we can then write the sum of our a and b beams as:

$$(1/\sqrt{2})\cdot\cos(\theta) + i\cdot(1/\sqrt{2})\cdot\cos(\theta + \Delta) = (1/\sqrt{2})\cdot[\cos(\theta) + i\cdot\cos(\theta + \Delta)]$$

What can we do with this? Not all that much, except noting that we can write the *cos*(θ + Δ) as a sine for $\Delta = \pm\,\pi/2$. To be precise, we get:

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concept gives way to the concept of a probability amplitude which we associate with a possible *path* rather than a particle.

[141] Again, the term 'beam' may not seem to be appropriate in the context of *one photon* only, but think of it as a *one-cycle* beam. 😊

[142] If we would reason in terms of average energies, we would have to apply a 1/2 factor because the average of the $\sin^2\theta$ and $\cos^2\theta$ over a cycle is equal to 1/2.

$$(1/\surd 2)\cdot\cos(\theta) + i\cdot(1/\surd 2)\cdot\cos(\theta + \pi/2) = (1/\surd 2)\cdot(\cos\theta - i\cdot\sin\theta) = (1/\surd 2)\cdot e^{-i\cdot\theta}$$

$$(1/\surd 2)\cdot\cos(\theta) + i\cdot(1/\surd 2)\cdot\cos(\theta - \pi/2) = (1/\surd 2)\cdot(\cos\theta + i\cdot\cos\theta) = (1/\surd 2)\cdot e^{i\cdot\theta}$$

This gives us the classical explanation we were looking for:

1. The incoming photon is circularly polarized (left- or right-handed).
2. The first beam splitter splits our photon into two linearly polarized waves.
3. The mirrors reflect those waves and the second beam splitter recombines the two linear waves back into a circularly polarized wave.
4. The positive or negative interference then explains the binary outcome of the Mach-Zehnder experiment – at the level of a photon – *in classical terms*.

6. What about the energy equation now? In other words, *what about that 1/√2 factor now?* If the $e^{-i\cdot\theta}$ and $e^{i\cdot\theta}$ wavefunctions can, effectively, be interpreted geometrically as a *physical* oscillation in *two* dimensions – which is, effectively, our interpretation of the wavefunction[143] – then then each of the two (independent) oscillations will pack one half of the energy of the wave. Hence, if such *circularly* polarized wave splits into two *linearly* polarized waves, then the two linearly polarized waves will effectively, pack half of the energy without any need for us to think their (maximum) amplitude should be adjusted.

Let us be *precise* here. If we think of the ***x***-direction as the direction of the incident beam in the Mach-Zehnder experiment, and we would want to also think of rotations in the ***xz***-plane, then we need to need to introduce some new convention here. Let us introduce *another* imaginary unit, which we will denote by *j*, and which will represent a 90-degree counterclockwise rotation in the ***xz***-plane.[144] We then get the following *classical* explanation for the results of the one-photon Mach-Zehnder experiment:

Photon polarization	**At BS1**	**At mirror**	**At BS2**	**Final result**
RHC	Photon ($e^{i\cdot\theta} = \cos\theta + i\cdot\sin\theta$) is split into two linearly polarized beams: Upper beam (vertical oscillation) = $j\cdot\sin\theta$ Lower beam (horizontal oscillation) = $\cos\theta$	The vertical oscillation gets rotated clockwise and becomes $-j\cdot j\cdot\sin\theta = -j^2\cdot\sin\theta = \sin\theta$ The horizontal oscillation is not affected and is still represented by $\cos\theta$	Photon is recombined. The upper beam gets rotated counter-clockwise and becomes $j\cdot\sin\theta$. The lower beam is still represented by $\cos\theta$	The photon wavefunction is given by $\cos\theta + j\cdot\sin\theta = e^{+j\cdot\theta}$. This is an RHC photon travelling in the *xz*-plane but rotated over 90 degrees.

[143] We can assign the physical dimension of the electric field (force per unit charge, N/C) to the two perpendicular oscillations.

[144] This convention may make the reader think of the quaternion theory but we are thinking more of simple Euler angles here: *i* is a (counterclockwise) rotation around the *x*-axis, and *j* is a rotation around the *y*-axis.

LHC	Photon ($e^{-i\cdot\theta}$ = $cos\theta$ – $i\cdot sin\theta$) is split into two linearly polarized beams: Upper beam (vertical oscillation) = $-j\cdot\sin\theta$ Lower beam (horizontal oscillation) = $\cos\theta$	The vertical oscillation gets rotated clockwise and becomes $(-j)\cdot(-j)\cdot\sin\theta$ = = $j^2\cdot\sin\theta = -\sin\theta$ The horizontal oscillation is not affected and is still represented by $\cos\theta$	Photon is recombined. The upper beam gets rotated counter-clockwise and becomes $-j\cdot\sin\theta$. The lower beam is still represented by $\cos\theta$	The photon wavefunction is given by $\cos\theta - j\cdot\sin\theta = e^{-j\cdot\theta}$. This is an LHC photon travelling in the *xz*-plane but rotated over 90 degrees.

Of course, we may also set up the apparatus with different path lengths, in which case the two linearly polarized beams will be out of phase when arriving at BS1. Let us assume the phase shift is equal to Δ = 180° = π. This amounts to putting a minus sign in front of *either* the sine *or* the cosine function. Why? Because of the $cos(\theta \pm \pi) = -cos\theta$ and $sin(\theta \pm \pi) = -sin\theta$ identities. Let us assume the distance along the upper path is longer and, hence, that the phase shift affects the sine function.[145] In that case, the sequence of events might be like this:

Photon polarization	At BS1	At mirror	At BS2	Final result
RHC	Photon ($e^{i\cdot\theta}$ = $cos\theta$ + $i\cdot sin\theta$) is split into two linearly polarized beams: Upper beam (vertical oscillation) = $j\cdot\sin\theta$ Lower beam (horizontal oscillation) = $\cos\theta$	The vertical oscillation gets rotated clockwise and becomes $-j\cdot j\cdot\sin\theta$ = $-j^2\cdot\sin\theta = \sin\theta$ The horizontal oscillation is not affected and is still represented by $\cos\theta$	Photon is recombined. The upper beam gets rotated counter-clockwise and – because of the longer distance – becomes $j\cdot\sin(\theta + \pi) = -j\cdot\sin\theta$. The lower beam is still represented by $\cos\theta$	The photon wavefunction is given by $\cos\theta - j\cdot\sin\theta = e^{-j\cdot\theta}$. This is an LHC photon travelling in the *xz*-plane but rotated over 90 degrees.
LHC	Photon ($e^{-i\cdot\theta}$ = $cos\theta$ – $i\cdot sin\theta$) is split into two linearly polarized beams: Upper beam (vertical oscillation) = $-j\cdot\sin\theta$ Lower beam (horizontal oscillation) = $\cos\theta$	The vertical oscillation gets rotated clockwise and becomes $(-j)\cdot(-j)\cdot\sin\theta$ = = $j^2\cdot\sin\theta = -\sin\theta$ The horizontal oscillation is not affected and is still represented by $\cos\theta$	Photon is recombined. The upper beam gets rotated counter-clockwise and – because of the longer distance – becomes $-j\cdot\sin(\theta + \pi) = +j\cdot\sin\theta$. The lower beam is still represented by $\cos\theta$	The photon wavefunction is given by $\cos\theta + j\cdot\sin\theta = e^{+j\cdot\theta}$. This is an RHC photon travelling in the *xz*-plane but rotated over 90 degrees.

145 The reader can easily work out the math for the opposite case (longer length of the lower path).

What happens when the difference between the phases of the two beams is not equal to 0 or 180 degrees? What if it is some random value in-between? Do we get an elliptically polarized wave or some other nice result? Denoting the phase shift as Δ, we can write:

$$\cos\theta + j\cdot\sin(\theta + \Delta) = \cos\theta + j\cdot(\sin\theta\cdot\cos\Delta + \cos\theta\cdot\sin\Delta)$$

However, this is also just a circularly polarized wave, but with a random phase shift between the horizontal and vertical component of the wave, as shown below. Of course, for the special values $\Delta = 0$ and $\Delta = \pi$, we get $\cos\theta + j\cdot\sin\theta$ and $\cos\theta - j\cdot\sin\theta$ once more.

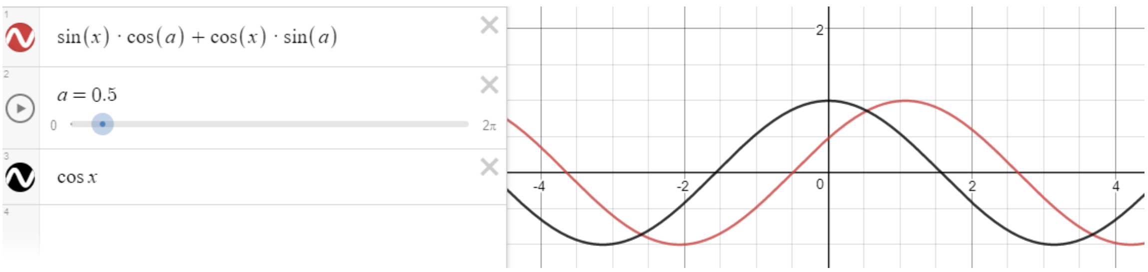

Figure 15: Random phase shift between two waves

Mystery solved? We think so, but we invite you to re-do the geometry and the related math. If anything, we hope the argument shows you Bell's No-Go Theorem should *not* prevent you from trying to go *every*where. 😊

PS: The above argument suggests two energy states are, effectively, separated by Planck's quantum of action—for an electron as well as for a photon. That is what the $E_n = n\cdot h\cdot f$ relation suggests. If so, we have a rather natural explanation for the *half*-photon explaining one-photon Mach-Zehnder interference here: the half-photon is something *real*. 😊