On some Ramanujan equations: mathematical connections with ϕ, $\zeta(2)$, and various parameters of Cosmology and Particle Physics. II

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Abstract

In this paper we have described and analyzed some Ramanujan equations. Furthermore, we have obtained various mathematical connections with ϕ, $\zeta(2)$, and several parameters of Cosmology and Particle Physics.

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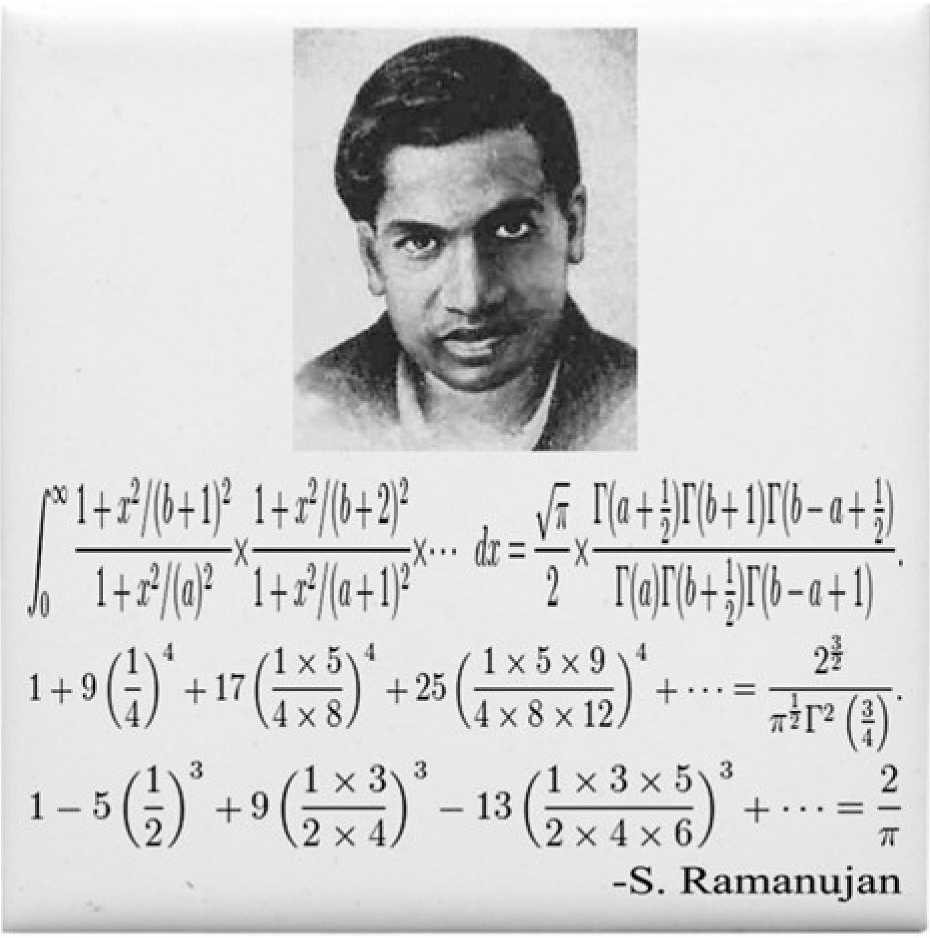

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RAMANUJAN

TWELVE LECTURES ON
SUBJECTS SUGGESTED BY HIS LIFE AND WORK

BY

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Sadleirian Professor of Pure Mathematics in the University of Cambridge

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1940

We have that:

(1.13) If $F(k) = 1 + \left(\frac{1}{2}\right)^2 k + \left(\frac{1.3}{2.4}\right)^2 k^2 + \dots$ and $F(1-k) = \surd(210)\,F(k)$, then

$$k = (\surd 2 - 1)^4 (2 - \surd 3)^2 (\surd 7 - \surd 6)^4 (8 - 3\surd 7)^2 (\surd 10 - 3)^4 \times (4 - \surd 15)^4 (\surd 15 - \surd 14)^2 (6 - \surd 35)^2.$$

(sqrt2-1)^4(2-sqrt3)^2(sqrt7-sqrt6)^4(8-3sqrt7)^2(sqrt10-3)^4(4-sqrt15)^4(sqrt15-sqrt14)^2(6-sqrt35)^2

Input:

$$\left(\sqrt{2}-1\right)^4 \left(2-\sqrt{3}\right)^2 \left(\sqrt{7}-\sqrt{6}\right)^4 \left(8-3\sqrt{7}\right)^2 \left(\sqrt{10}-3\right)^4 \left(4-\sqrt{15}\right)^4 \left(\sqrt{15}-\sqrt{14}\right)^2 \left(6-\sqrt{35}\right)^2$$

Decimal approximation:

$2.7066257892455517275593316576258447509090535730406425\ldots \times 10^{-19}$

$2.706625789\ldots * 10^{-19}$

Alternate forms:

$$\left(17-12\sqrt{2}\right)\left(7-4\sqrt{3}\right)\left(337-52\sqrt{42}\right)\left(127-48\sqrt{7}\right)\left(721-228\sqrt{10}\right)\left(1921-496\sqrt{15}\right)\left(29-2\sqrt{210}\right)\left(71-12\sqrt{35}\right)$$

$$\left(\sqrt{2}-1\right)^4\left(7-4\sqrt{3}\right)\left(8-3\sqrt{7}\right)^2\left(\sqrt{10}-3\right)^4\left(\sqrt{15}-4\right)^4\left(\sqrt{35}-6\right)^2\left(337-52\sqrt{42}\right)\left(29-2\sqrt{210}\right)$$

$$\left(\sqrt{2}-1\right)^4\left(\sqrt{3}-2\right)^2\left(\sqrt{6}-\sqrt{7}\right)^4\left(3\sqrt{7}-8\right)^2\left(\sqrt{10}-3\right)^4\left(\sqrt{14}-\sqrt{15}\right)^2\left(\sqrt{15}-4\right)^4\left(\sqrt{35}-6\right)^2$$

We have also:

3*[-ln((((sqrt2-1)^4(2-sqrt3)^2(sqrt7-sqrt6)^4(8-3sqrt7)^2(sqrt10-3)^4(4-sqrt15)^4(sqrt15-sqrt14)^2(6-sqrt35)^2)))]-3

Input:

$$3\left(-\log\left(\left(\sqrt{2}-1\right)^4\left(2-\sqrt{3}\right)^2\left(\sqrt{7}-\sqrt{6}\right)^4\left(8-3\sqrt{7}\right)^2\left(\sqrt{10}-3\right)^4\left(4-\sqrt{15}\right)^4\left(\sqrt{15}-\sqrt{14}\right)^2\left(6-\sqrt{35}\right)^2\right)\right)-3$$

$\log(x)$ is the natural logarithm

Exact result:

$$-3-3\log\left(\left(\sqrt{2}-1\right)^4\left(2-\sqrt{3}\right)^2\left(8-3\sqrt{7}\right)^2\left(\sqrt{7}-\sqrt{6}\right)^4\left(\sqrt{10}-3\right)^4\left(4-\sqrt{15}\right)^4\left(\sqrt{15}-\sqrt{14}\right)^2\left(6-\sqrt{35}\right)^2\right)$$

Decimal approximation:

125.26024201197276225831920913839757565395112964883593403 44...

125.26024201…..

Property:

$$-3-3\log\left(\left(-1+\sqrt{2}\right)^4\left(2-\sqrt{3}\right)^2\left(8-3\sqrt{7}\right)^2\left(-\sqrt{6}+\sqrt{7}\right)^4\left(-3+\sqrt{10}\right)^4\left(4-\sqrt{15}\right)^4\left(-\sqrt{14}+\sqrt{15}\right)^2\left(6-\sqrt{35}\right)^2\right)$$ is a transcendental number

Alternate forms:

$$-3\left(1+\log\left(\left(\sqrt{2}-1\right)^4\left(7-4\sqrt{3}\right)\left(8-3\sqrt{7}\right)^2\left(\sqrt{10}-3\right)^4\left(\sqrt{15}-4\right)^4\left(\sqrt{35}-6\right)^2\left(337-52\sqrt{42}\right)\left(29-2\sqrt{210}\right)\right)\right)$$

$$-3-12\log\left(\left(\sqrt{2}-1\right)\left(\sqrt{10}-3\right)\right)-3\log\left(\left(\sqrt{3}-2\right)^2\left(\sqrt{6}-\sqrt{7}\right)^4\left(3\sqrt{7}-8\right)^2\left(\sqrt{14}-\sqrt{15}\right)^2\left(\sqrt{15}-4\right)^4\left(\sqrt{35}-6\right)^2\right)$$

$$-3\left(1+\log\left(\left(\sqrt{2}-1\right)^4\left(2-\sqrt{3}\right)^2\left(8-3\sqrt{7}\right)^2\left(\sqrt{7}-\sqrt{6}\right)^4\left(\sqrt{10}-3\right)^4\left(4-\sqrt{15}\right)^4\left(\sqrt{15}-\sqrt{14}\right)^2\left(6-\sqrt{35}\right)^2\right)\right)$$

Alternative representations:

$$3(-1)\log\left(\left(\sqrt{2}-1\right)^4\left(2-\sqrt{3}\right)^2\left(\sqrt{7}-\sqrt{6}\right)^4\left(8-3\sqrt{7}\right)^2\left(\sqrt{10}-3\right)^4\left(4-\sqrt{15}\right)^4\left(\sqrt{15}-\sqrt{14}\right)^2\left(6-\sqrt{35}\right)^2\right)-3=$$
$$-3-3\log_e\left(\left(-1+\sqrt{2}\right)^4\left(2-\sqrt{3}\right)^2\left(8-3\sqrt{7}\right)^2\left(-\sqrt{6}+\sqrt{7}\right)^4\left(-3+\sqrt{10}\right)^4\left(4-\sqrt{15}\right)^4\left(-\sqrt{14}+\sqrt{15}\right)^2\left(6-\sqrt{35}\right)^2\right)$$

$$3(-1)\log\left(\left(\sqrt{2}-1\right)^4\left(2-\sqrt{3}\right)^2\left(\sqrt{7}-\sqrt{6}\right)^4\left(8-3\sqrt{7}\right)^2\left(\sqrt{10}-3\right)^4\left(4-\sqrt{15}\right)^4\left(\sqrt{15}-\sqrt{14}\right)^2\left(6-\sqrt{35}\right)^2\right)-3=$$
$$-3-3\log(a)\log_a\left(\left(-1+\sqrt{2}\right)^4\left(2-\sqrt{3}\right)^2\left(8-3\sqrt{7}\right)^2\left(-\sqrt{6}+\sqrt{7}\right)^4\left(-3+\sqrt{10}\right)^4\left(4-\sqrt{15}\right)^4\left(-\sqrt{14}+\sqrt{15}\right)^2\left(6-\sqrt{35}\right)^2\right)$$

$$3(-1)\log\left(\left(\sqrt{2}-1\right)^4\left(2-\sqrt{3}\right)^2\left(\sqrt{7}-\sqrt{6}\right)^4\left(8-3\sqrt{7}\right)^2\left(\sqrt{10}-3\right)^4\left(4-\sqrt{15}\right)^4\left(\sqrt{15}-\sqrt{14}\right)^2\left(6-\sqrt{35}\right)^2\right)-3=$$
$$-3+3\,\mathrm{Li}_1\left(1-\left(-1+\sqrt{2}\right)^4\left(2-\sqrt{3}\right)^2\left(8-3\sqrt{7}\right)^2\left(-\sqrt{6}+\sqrt{7}\right)^4\left(-3+\sqrt{10}\right)^4\left(4-\sqrt{15}\right)^4\left(-\sqrt{14}+\sqrt{15}\right)^2\left(6-\sqrt{35}\right)^2\right)$$

3*[-ln((((sqrt2-1)^4(2-sqrt3)^2(sqrt7-sqrt6)^4(8-3sqrt7)^2(sqrt10-3)^4(4-sqrt15)^4(sqrt15-sqrt14)^2(6-sqrt35)^2)))]+11

Input:

$$3\left(-\log\left(\left(\sqrt{2}-1\right)^4\left(2-\sqrt{3}\right)^2\left(\sqrt{7}-\sqrt{6}\right)^4\left(8-3\sqrt{7}\right)^2\left(\sqrt{10}-3\right)^4\left(4-\sqrt{15}\right)^4\left(\sqrt{15}-\sqrt{14}\right)^2\left(6-\sqrt{35}\right)^2\right)\right)+11$$

log(x) is the natural logarithm

Exact result:

$$11-3\log\left(\left(\sqrt{2}-1\right)^4\left(2-\sqrt{3}\right)^2\left(8-3\sqrt{7}\right)^2\left(\sqrt{7}-\sqrt{6}\right)^4\left(\sqrt{10}-3\right)^4\left(4-\sqrt{15}\right)^4\left(\sqrt{15}-\sqrt{14}\right)^2\left(6-\sqrt{35}\right)^2\right)$$

Decimal approximation:

139.2602420119727622583192091383975756539511296488359340344...

139.26024201…..

Property:

$$11-3\log\left(\left(-1+\sqrt{2}\right)^4\left(2-\sqrt{3}\right)^2\left(8-3\sqrt{7}\right)^2\left(-\sqrt{6}+\sqrt{7}\right)^4\left(-3+\sqrt{10}\right)^4\left(4-\sqrt{15}\right)^4\left(-\sqrt{14}+\sqrt{15}\right)^2\left(6-\sqrt{35}\right)^2\right)$$ is a transcendental number

Alternate forms:

$$11-3\log\left(\left(\sqrt{2}-1\right)^4\left(7-4\sqrt{3}\right)\left(8-3\sqrt{7}\right)^2\left(\sqrt{10}-3\right)^4\left(\sqrt{15}-4\right)^4\left(\sqrt{35}-6\right)^2\left(337-52\sqrt{42}\right)\left(29-2\sqrt{210}\right)\right)$$

$$11-12\log\left(\sqrt{2}-1\right)-3\log\left(\left(\sqrt{3}-2\right)^2\right)-3\log\left(\left(\sqrt{6}-\sqrt{7}\right)^4\right)-3\log\left(\left(3\sqrt{7}-8\right)^2\right)-12\log\left(\sqrt{10}-3\right)-3\log\left(\left(\sqrt{14}-\sqrt{15}\right)^2\right)-3\log\left(\left(\sqrt{15}-4\right)^4\right)-3\log\left(\left(\sqrt{35}-6\right)^2\right)$$

$$11-3\left(4\log\left(\sqrt{2}-1\right)+2\log\left(2-\sqrt{3}\right)+2\log\left(8-3\sqrt{7}\right)+4\log\left(\sqrt{7}-\sqrt{6}\right)+4\log\left(\sqrt{10}-3\right)+4\log\left(4-\sqrt{15}\right)+2\log\left(\sqrt{15}-\sqrt{14}\right)+2\log\left(6-\sqrt{35}\right)\right)$$

Alternative representations:

$$3(-1)\log\left(\left(\sqrt{2}-1\right)^4\left(2-\sqrt{3}\right)^2\left(\sqrt{7}-\sqrt{6}\right)^4\left(8-3\sqrt{7}\right)^2\left(\sqrt{10}-3\right)^4\left(4-\sqrt{15}\right)^4\left(\sqrt{15}-\sqrt{14}\right)^2\left(6-\sqrt{35}\right)^2\right)+11 = 11-3\log_e\left(\left(-1+\sqrt{2}\right)^4\left(2-\sqrt{3}\right)^2\left(8-3\sqrt{7}\right)^2\left(-\sqrt{6}+\sqrt{7}\right)^4\left(-3+\sqrt{10}\right)^4\left(4-\sqrt{15}\right)^4\left(-\sqrt{14}+\sqrt{15}\right)^2\left(6-\sqrt{35}\right)^2\right)$$

$$3(-1)\log\left(\left(\sqrt{2}-1\right)^4\left(2-\sqrt{3}\right)^2\left(\sqrt{7}-\sqrt{6}\right)^4\left(8-3\sqrt{7}\right)^2\left(\sqrt{10}-3\right)^4\left(4-\sqrt{15}\right)^4\left(\sqrt{15}-\sqrt{14}\right)^2\left(6-\sqrt{35}\right)^2\right)+11 = 11-3\log(a)\log_a\left(\left(-1+\sqrt{2}\right)^4\left(2-\sqrt{3}\right)^2\left(8-3\sqrt{7}\right)^2\left(-\sqrt{6}+\sqrt{7}\right)^4\left(-3+\sqrt{10}\right)^4\left(4-\sqrt{15}\right)^4\left(-\sqrt{14}+\sqrt{15}\right)^2\left(6-\sqrt{35}\right)^2\right)$$

$$3(-1)\log\left(\left(\sqrt{2}-1\right)^4\left(2-\sqrt{3}\right)^2\left(\sqrt{7}-\sqrt{6}\right)^4\left(8-3\sqrt{7}\right)^2\left(\sqrt{10}-3\right)^4\left(4-\sqrt{15}\right)^4\left(\sqrt{15}-\sqrt{14}\right)^2\left(6-\sqrt{35}\right)^2\right)+11 = 11+3\,\text{Li}_1\left(1-\left(-1+\sqrt{2}\right)^4\left(2-\sqrt{3}\right)^2\left(8-3\sqrt{7}\right)^2\left(-\sqrt{6}+\sqrt{7}\right)^4\left(-3+\sqrt{10}\right)^4\left(4-\sqrt{15}\right)^4\left(-\sqrt{14}+\sqrt{15}\right)^2\left(6-\sqrt{35}\right)^2\right)$$

-5/2+27*1/2(((3*[-ln((((sqrt2-1)^4(2-sqrt3)^2(sqrt7-sqrt6)^4(8-3sqrt7)^2(sqrt10-3)^4(4-sqrt15)^4(sqrt15-sqrt14)^2(6-sqrt35)^2)))])))

Input:

$$-\frac{5}{2}+27\times\frac{1}{2}\left(3\left(-\log\left(\left(\sqrt{2}-1\right)^4\left(2-\sqrt{3}\right)^2\left(\sqrt{7}-\sqrt{6}\right)^4\left(8-3\sqrt{7}\right)^2\left(\sqrt{10}-3\right)^4\left(4-\sqrt{15}\right)^4\left(\sqrt{15}-\sqrt{14}\right)^2\left(6-\sqrt{35}\right)^2\right)\right)\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$-\frac{5}{2}-\frac{81}{2}\log\left(\left(\sqrt{2}-1\right)^4\left(2-\sqrt{3}\right)^2\left(8-3\sqrt{7}\right)^2\left(\sqrt{7}-\sqrt{6}\right)^4\left(\sqrt{10}-3\right)^4\left(4-\sqrt{15}\right)^4\left(\sqrt{15}-\sqrt{14}\right)^2\left(6-\sqrt{35}\right)^2\right)$$

Decimal approximation:

1729.0132671616322904873093233683672713283402502592851094 65...

1729.01326716…..

Property:

$$-\frac{5}{2}-\frac{81}{2}\log\left(\left(-1+\sqrt{2}\right)^4\left(2-\sqrt{3}\right)^2\left(8-3\sqrt{7}\right)^2\left(-\sqrt{6}+\sqrt{7}\right)^4\left(-3+\sqrt{10}\right)^4\left(4-\sqrt{15}\right)^4\left(-\sqrt{14}+\sqrt{15}\right)^2\left(6-\sqrt{35}\right)^2\right) \text{ is a transcendental number}$$

Alternate forms:

$$\frac{1}{2}\left(-5-81\log\left(\left(\sqrt{2}-1\right)^4\left(7-4\sqrt{3}\right)\left(8-3\sqrt{7}\right)^2\left(\sqrt{10}-3\right)^4\left(\sqrt{15}-4\right)^4\left(\sqrt{35}-6\right)^2\left(337-52\sqrt{42}\right)\left(29-2\sqrt{210}\right)\right)\right)$$

$$-\frac{5}{2}-162\log\left(\left(\sqrt{2}-1\right)\left(\sqrt{10}-3\right)\right)-\frac{81}{2}\log\left(\left(\sqrt{3}-2\right)^2\left(\sqrt{6}-\sqrt{7}\right)^4\left(3\sqrt{7}-8\right)^2\left(\sqrt{14}-\sqrt{15}\right)^2\left(\sqrt{15}-4\right)^4\left(\sqrt{35}-6\right)^2\right)$$

$$\frac{1}{2}\left(-5-81\log\left(\left(\sqrt{2}-1\right)^4\left(2-\sqrt{3}\right)^2\left(8-3\sqrt{7}\right)^2\left(\sqrt{7}-\sqrt{6}\right)^4\left(\sqrt{10}-3\right)^4\left(4-\sqrt{15}\right)^4\left(\sqrt{15}-\sqrt{14}\right)^2\left(6-\sqrt{35}\right)^2\right)\right)$$

Alternative representations:

$$-\frac{5}{2}+\frac{27}{2}\left(3\left(-\log\left(\left(\sqrt{2}-1\right)^4\left(2-\sqrt{3}\right)^2\left(\sqrt{7}-\sqrt{6}\right)^4\left(8-3\sqrt{7}\right)^2\left(\sqrt{10}-3\right)^4\left(4-\sqrt{15}\right)^4\left(\sqrt{15}-\sqrt{14}\right)^2\left(6-\sqrt{35}\right)^2\right)\right)\right)=$$
$$-\frac{5}{2}-\frac{81}{2}\log_e\left(\left(-1+\sqrt{2}\right)^4\left(2-\sqrt{3}\right)^2\left(8-3\sqrt{7}\right)^2\left(-\sqrt{6}+\sqrt{7}\right)^4\left(-3+\sqrt{10}\right)^4\left(4-\sqrt{15}\right)^4\left(-\sqrt{14}+\sqrt{15}\right)^2\left(6-\sqrt{35}\right)^2\right)$$

$$-\frac{5}{2}+\frac{27}{2}\left(3\left(-\log\left(\left(\sqrt{2}-1\right)^4\left(2-\sqrt{3}\right)^2\left(\sqrt{7}-\sqrt{6}\right)^4\left(8-3\sqrt{7}\right)^2\left(\sqrt{10}-3\right)^4\left(4-\sqrt{15}\right)^4\left(\sqrt{15}-\sqrt{14}\right)^2\left(6-\sqrt{35}\right)^2\right)\right)\right)=$$
$$-\frac{5}{2}-\frac{81}{2}\log(a)\log_a\left(\left(-1+\sqrt{2}\right)^4\left(2-\sqrt{3}\right)^2\left(8-3\sqrt{7}\right)^2\left(-\sqrt{6}+\sqrt{7}\right)^4\left(-3+\sqrt{10}\right)^4\left(4-\sqrt{15}\right)^4\left(-\sqrt{14}+\sqrt{15}\right)^2\left(6-\sqrt{35}\right)^2\right)$$

$$-\frac{5}{2}+\frac{27}{2}\left(3\left(-\log\left(\left(\sqrt{2}-1\right)^4\left(2-\sqrt{3}\right)^2\left(\sqrt{7}-\sqrt{6}\right)^4\left(8-3\sqrt{7}\right)^2\left(\sqrt{10}-3\right)^4\left(4-\sqrt{15}\right)^4\left(\sqrt{15}-\sqrt{14}\right)^2\left(6-\sqrt{35}\right)^2\right)\right)\right)=-\frac{5}{2}+\frac{81}{2}\,\mathrm{Li}_1\left(1-\left(-1+\sqrt{2}\right)^4\left(2-\sqrt{3}\right)^2\left(8-3\sqrt{7}\right)^2\left(-\sqrt{6}+\sqrt{7}\right)^4\left(-3+\sqrt{10}\right)^4\left(4-\sqrt{15}\right)^4\left(-\sqrt{14}+\sqrt{15}\right)^2\left(6-\sqrt{35}\right)^2\right)$$

((-5/2+27*1/2(((3*[-ln((((sqrt2-1)^4(2-sqrt3)^2(sqrt7-sqrt6)^4(8-3sqrt7)^2(sqrt10-3)^4(4-sqrt15)^4(sqrt15-sqrt14)^2(6-sqrt35)^2))))]))))))^1/15

Input:

$$\left(-\frac{5}{2}+27\times\frac{1}{2}\left(3\left(-\log\left(\left(\sqrt{2}-1\right)^4\left(2-\sqrt{3}\right)^2\left(\sqrt{7}-\sqrt{6}\right)^4\left(8-3\sqrt{7}\right)^2\left(\sqrt{10}-3\right)^4\left(4-\sqrt{15}\right)^4\left(\sqrt{15}-\sqrt{14}\right)^2\left(6-\sqrt{35}\right)^2\right)\right)\right)\right)\text{^}(1/15)$$

log(x) is the natural logarithm

Exact result:

$$\left(-\frac{5}{2}-\frac{81}{2}\log\left(\left(\sqrt{2}-1\right)^4\left(2-\sqrt{3}\right)^2\left(8-3\sqrt{7}\right)^2\left(\sqrt{7}-\sqrt{6}\right)^4\left(\sqrt{10}-3\right)^4\left(4-\sqrt{15}\right)^4\left(\sqrt{15}-\sqrt{14}\right)^2\left(6-\sqrt{35}\right)^2\right)\right)\text{^}(1/15)$$

Decimal approximation:

1.643816069646520274323907216427386661097336879014745829281...

1.64381606964……

Property:

$$\left(-\frac{5}{2}-\frac{81}{2}\log\left(\left(-1+\sqrt{2}\right)^4\left(2-\sqrt{3}\right)^2\left(8-3\sqrt{7}\right)^2\left(-\sqrt{6}+\sqrt{7}\right)^4\left(-3+\sqrt{10}\right)^4\left(4-\sqrt{15}\right)^4\left(-\sqrt{14}+\sqrt{15}\right)^2\left(6-\sqrt{35}\right)^2\right)\right)\text{^}(1/15)$$ is a transcendental number

Alternate forms:

$$\frac{1}{\sqrt[15]{-\frac{2}{5+81\log\left(\left(\sqrt{2}-1\right)^4\left(7-4\sqrt{3}\right)\left(8-3\sqrt{7}\right)^2\left(\sqrt{10}-3\right)^4\left(\sqrt{15}-4\right)^4\left(\sqrt{35}-6\right)^2\left(337-52\sqrt{42}\right)\left(29-2\sqrt{210}\right)\right)}}}$$

$$1/\left(\left(2/\left(-5-324\log\left(\left(\sqrt{2}-1\right)\left(\sqrt{10}-3\right)\right)-81\log\left(\left(\sqrt{3}-2\right)^2\left(\sqrt{6}-\sqrt{7}\right)^4\left(3\sqrt{7}-8\right)^2\left(\sqrt{14}-\sqrt{15}\right)^2\left(\sqrt{15}-4\right)^4\left(\sqrt{35}-6\right)^2\right)\right)\right)\text{^}(1/15)\right)$$

$$\left(-\frac{5}{2}-\frac{81}{2}\left(4\log\left(\sqrt{2}-1\right)+2\log\left(2-\sqrt{3}\right)+2\log\left(8-3\sqrt{7}\right)+4\log\left(\sqrt{7}-\sqrt{6}\right)+4\log\left(\sqrt{10}-3\right)+4\log\left(4-\sqrt{15}\right)+2\log\left(\sqrt{15}-\sqrt{14}\right)+2\log\left(6-\sqrt{35}\right)\right)\right)\hat{}\,(1/15)$$

And:

sqrt210 * (((1+1/4*(2.706625789245e-19)+(9/64)*(2.706625789245e-19)^2)))

Input interpretation:

$$\sqrt{210}\left(1+\frac{1}{4}\times 2.706625789245\times 10^{-19}+\frac{9}{64}\left(2.706625789245\times 10^{-19}\right)^2\right)$$

Result:

14.491376746189438574699232507742...

14.491376…

Pi/(55/2+2/Pi) (((sqrt210 * (((1+1/4*(2.706625789245e-19)+(9/64)*(2.706625789245e-19)^2))))))

Input interpretation:

$$\frac{\pi}{\frac{55}{2}+\frac{2}{\pi}}\left(\sqrt{210}\left(1+\frac{1}{4}\times 2.706625789245\times 10^{-19}+\frac{9}{64}\left(2.706625789245\times 10^{-19}\right)^2\right)\right)$$

Result:

1.6180338325835746004004566413281...

1.6180338325…

(((7 ln(((sqrt210 * (((1+1/4*(2.706625789245e-19)+(9/64)*(2.706625789245e-19)^2)))))))))

Input interpretation:

$$7\log\left(\sqrt{210}\left(1+\frac{1}{4}\times 2.706625789245\times 10^{-19}+\frac{9}{64}\left(2.706625789245\times 10^{-19}\right)^2\right)\right)$$

$\log(x)$ is the natural logarithm

Result:

18.714876357511140382288722535793 1...

18.714876….. result very near to the black hole entropy 18.7328

(((2Pi* ln(((sqrt210 * (((1+1/4*(2.706625789245e-19)+(9/64)*(2.706625789245e-19)^2))))))))

Input interpretation:

$$2\pi \log\left(\sqrt{210}\left(1+\frac{1}{4}\times 2.706625789245\times 10^{-19}+\frac{9}{64}\left(2.706625789245\times 10^{-19}\right)^2\right)\right)$$

log(x) is the natural logarithm

Result:

16.798433736456659292849804818869 2...

16.798433736….. result very near to the black hole entropy 16.8741, that is equal to ln(21296876)

64*2(((sqrt210 * (((1+1/4*(2.706625789245e-19)+(9/64)*(2.706625789245e-19)^2))))))-123-sqrt7-1/4

Input interpretation:

$$64\times 2\left(\sqrt{210}\left(1+\frac{1}{4}\times 2.706625789245\times 10^{-19}+\frac{9}{64}\left(2.706625789245\times 10^{-19}\right)^2\right)\right)-123-\sqrt{7}-\frac{1}{4}$$

Result:

1729.0004722011835469710001452374...

1729.0004722…

and also:

Pi^2(((sqrt210 * (((1+1/4*(2.706625789245e-19)+(9/64)*(2.706625789245e-19)^2))))))-5

Input interpretation:

$$\pi^2\left(\sqrt{210}\left(1+\frac{1}{4}\times 2.706625789245\times 10^{-19}+\frac{9}{64}\left(2.706625789245\times 10^{-19}\right)^2\right)\right)-5$$

Result:

138.02415571203527234762046087748...

138.024155712035… ≈ 138 (Ramanujan taxicab number)

Pi^2(((sqrt210 * (((1+1/4*(2.706625789245e-19)+(9/64)*(2.706625789245e-19)^2))))))-8

Input interpretation:

$$\pi^2\left(\sqrt{210}\left(1+\frac{1}{4}\times 2.706625789245\times 10^{-19}+\frac{9}{64}\left(2.706625789245\times 10^{-19}\right)^2\right)\right)-8$$

Result:

135.02415571203527234762046087748...

135.0241557…. ≈ 135 (Ramanujan taxicab number)

Pi^2(((sqrt210 * (((1+1/4*(2.706625789245e-19)+(9/64)*(2.706625789245e-19)^2))))))+29

Input interpretation:

$$\pi^2\left(\sqrt{210}\left(1+\frac{1}{4}\times 2.706625789245\times 10^{-19}+\frac{9}{64}\left(2.706625789245\times 10^{-19}\right)^2\right)\right)+29$$

Result:

172.02415571203527234762046087748...

172.0241557… ≈ 172 (Ramanujan taxicab number)

Now, we have that:

$$(1.5)\int_0^\infty \frac{1+\left(\frac{x}{b+1}\right)^2}{1+\left(\frac{x}{a}\right)^2}\cdot\frac{1+\left(\frac{x}{b+2}\right)^2}{1+\left(\frac{x}{a+1}\right)^2}\cdots dx=\tfrac{1}{2}\pi^{\frac{1}{2}}\frac{\Gamma(a+\frac{1}{2})\,\Gamma(b+1)\,\Gamma(b-a+\frac{1}{2})}{\Gamma(a)\,\Gamma(b+\frac{1}{2})\,\Gamma(b-a+1)}.$$

From the left-hand side, we obtain:

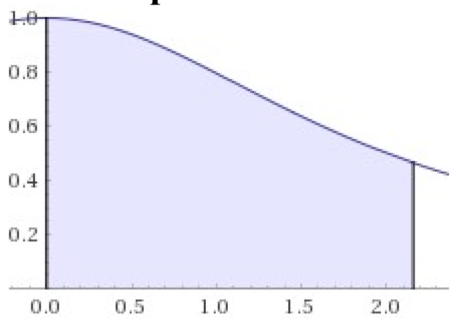
Integrate (1+(x/4)^2) / (1+(x/2)^2) * (1+(x/5)^2) / (1+(x/3)^2) , x = 0..2.16

Definite integral:

$$\int_0^{2.16} \frac{\left(1+\left(\frac{x}{4}\right)^2\right)\left(1+\left(\frac{x}{5}\right)^2\right)}{\left(1+\left(\frac{x}{2}\right)^2\right)\left(1+\left(\frac{x}{3}\right)^2\right)} dx = 1.64353$$

1.64353

Visual representation of the integral:



Indefinite integral:

$$\int \frac{\left(1+\left(\frac{x}{4}\right)^2\right)\left(1+\left(\frac{x}{5}\right)^2\right)}{\left(1+\left(\frac{x}{2}\right)^2\right)\left(1+\left(\frac{x}{3}\right)^2\right)} dx = \frac{9}{100}\left(x - \frac{112}{15}\tan^{-1}\left(\frac{x}{3}\right) + \frac{126}{5}\tan^{-1}\left(\frac{x}{2}\right)\right) + \text{constant}$$

$\tan^{-1}(x)$ is the inverse tangent function

Integrate (1+(x/4)^2) / (1+(x/2)^2) * (1+(x/5)^2) / (1+(x/3)^2) , x = 0..19183

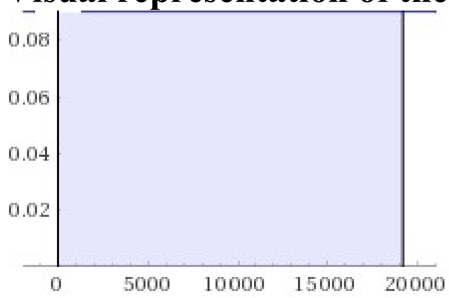
Definite integral:

$$\int_0^{19183} \frac{\left(1+\left(\frac{x}{4}\right)^2\right)\left(1+\left(\frac{x}{5}\right)^2\right)}{\left(1+\left(\frac{x}{2}\right)^2\right)\left(1+\left(\frac{x}{3}\right)^2\right)} dx = \frac{3}{500}\left(287\,745 + 133\pi - 378\tan^{-1}\left(\frac{2}{19\,183}\right) + 112\tan^{-1}\left(\frac{3}{19\,183}\right)\right) \approx 1729.0$$

1729

$\tan^{-1}(x)$ is the inverse tangent function

Visual representation of the integral:



Indefinite integral:

$$\int \frac{\left(1+\left(\frac{x}{4}\right)^2\right)\left(1+\left(\frac{x}{5}\right)^2\right)}{\left(1+\left(\frac{x}{2}\right)^2\right)\left(1+\left(\frac{x}{3}\right)^2\right)}\,dx = \frac{9}{100}\left(x-\frac{112}{15}\tan^{-1}\left(\frac{x}{3}\right)+\frac{126}{5}\tan^{-1}\left(\frac{x}{2}\right)\right)+\text{constant}$$

With regard 19183:

19 183 is an odd number.

19181 and 19183 form a twin prime pair.

(-11*2)+(1/(Pi))* Integrate (1+(x/4)^2) / (1+(x/2)^2) * (1+(x/5)^2) / (1+(x/3)^2) , x = 0..196884

Input:

$$-11\times 2+\frac{1}{\pi}\int_0^{196884}\frac{1+\left(\frac{x}{4}\right)^2}{1+\left(\frac{x}{2}\right)^2}\times\frac{1+\left(\frac{x}{5}\right)^2}{1+\left(\frac{x}{3}\right)^2}\,dx$$

Result:

$$\frac{3\left(2953260+133\pi-378\tan^{-1}\left(\frac{1}{98442}\right)+112\tan^{-1}\left(\frac{1}{65628}\right)\right)}{500\pi}-22\approx 5619.11$$

5619.11 result practically equal to the rest mass of bottom Lambda baryon 5619.4

$\tan^{-1}(x)$ is the inverse tangent function

Computation result:

$$-11\times 2+\frac{1}{\pi}\int_0^{196884}\frac{\left(1+\left(\frac{x}{4}\right)^2\right)\left(1+\left(\frac{x}{5}\right)^2\right)}{\left(1+\left(\frac{x}{2}\right)^2\right)\left(1+\left(\frac{x}{3}\right)^2\right)}\,dx = \frac{3\left(2\,953\,260+133\,\pi-378\tan^{-1}\left(\frac{1}{98\,442}\right)+112\tan^{-1}\left(\frac{1}{65\,628}\right)\right)}{500\,\pi}-22$$

Alternate forms:

$$\frac{8\,859\,780-10\,601\,\pi+336\cot^{-1}(65\,628)-1134\cot^{-1}(98\,442)}{500\,\pi}$$

$$-\frac{-8\,859\,780+10\,601\,\pi+1134\tan^{-1}\left(\frac{1}{98\,442}\right)-336\tan^{-1}\left(\frac{1}{65\,628}\right)}{500\,\pi}$$

$$\frac{8\,859\,780-10\,601\,\pi-1134\tan^{-1}\left(\frac{1}{98\,442}\right)+336\tan^{-1}\left(\frac{1}{65\,628}\right)}{500\,\pi}$$

$\cot^{-1}(x)$ is the inverse cotangent function

From:

Andrews GE. - 2019 **How Ramanujan may have discovered the mock theta functions. -** Phil. Trans. R. Soc. A **378**: 20180436. http://dx.doi.org/10.1098/rsta.2018.0436

We have that:

$$f_3(q) := \sum_{n=0}^{\infty}\frac{q^{n^2}}{(-q;q)_n^2} = \phi_3(-q)+2\psi_3(-q). \tag{5.1}$$

The proof is identical with that of (3.3) apart from some changes of sign.

Hence eliminating $\phi_3(-q)$ from (4.3) and (5.1), we find

$$f(q)-\frac{1}{(-q;q)_\infty}\sum_{n=-\infty}^{\infty}(-1)^n q^{n^2} = -4\psi(-q), \tag{5.2}$$

and by the Theorem from §4, as Ramanujan indicates in his letter, and as Folsom *et al.* [3,4] prove

$$\lim_{q\to 1^-}\left(f(q)-\frac{1}{(-q;q)_\infty}\sum_{n=-\infty}^{\infty}(-1)^n q^{n^2}\right)=4. \tag{5.3}$$

and again:

$$\sum_{n\geq 0}\frac{q^{n^2}}{(q;q)_n}=\sum_{n\geq 0}\frac{q^{n^2}}{(q^2;q^2)_n}(-q;q)_n=(-q;q)_\infty\sum_{n\geq 0}\frac{q^{n^2}}{(q^2;q^2)_n(-q^{n+1};q)_\infty}$$

As with the functions $f_3(q)$, $\phi_3(q)$ and $\psi_3(q)$, Ramanujan would likely have observed that the series $H(q^4)$ where

$$H(q):=\sum_{n\geq 0}\frac{q^{n^2+n}}{(q;q)_n}=\frac{1}{(q^2;q^5)_\infty(q^3;q^5)_\infty}$$

We have also:

$$\sum_{n\geq 0}\frac{q^{n^2}}{(q;q)_n}=\sum_{n\geq 0}\frac{q^{n^2}}{(q^2;q^2)_n}(-q;q)_n=(-q;q)_\infty\sum_{n\geq 0}\frac{q^{n^2}}{(q^2;q^2)_n(-q^{n+1};q)_\infty}$$

We have that

$$q=e^{2\pi i\tau}$$

For iτ = i(1+i), we obtain:

exp(2Pi*i*(1+i))

Input:

$\exp(2\pi i(1+i))$

i is the imaginary unit

Exact result:

$e^{-2\pi}$

Decimal approximation:

0.001867442731707988814430212934827030393422805002475317199...

0.0018674427…

Property:

$e^{-2\pi}$ is a transcendental number

Alternative representations:

$$e^{2\pi i(1+i)} = e^{360^\circ i(1+i)}$$

$$e^{2\pi i(1+i)} = e^{-2i^2(1+i)\log(-1)}$$

$$e^{2\pi i(1+i)} = \exp^{2\pi i(1+i)}(z) \quad \text{for } z = 1$$

Series representations:

$$e^{2\pi i(1+i)} = e^{-8\sum_{k=0}^{\infty}(-1)^k/(1+2k)}$$

$$e^{2\pi i(1+i)} = \left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-2\pi}$$

$$e^{2\pi i(1+i)} = \left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{-2\pi}$$

and:

1/((exp(2Pi*i*(1+i))))

Input:

$$\frac{1}{\exp(2\pi i(1+i))}$$

i is the imaginary unit

Exact result:

$$e^{2\pi}$$

Decimal approximation:

535.491655524764736503049329589047181477805797603294915507 2...

535.491655524…

Property:

$e^{2\pi}$ is a transcendental number

From Wikipedia:

a)

$$\nu(q) = \sum_{n\geq 0} \frac{q^{n(n+1)}}{(-q;q^2)_{n+1}} = \frac{1}{\prod_{n>0}(1-q^n)} \sum_{n\geq 0}(-1)^n q^{3n(n+1)/2} \frac{1-q^{2n+1}}{1+q^{2n+1}}$$ (sequence A053254 in the OEIS).

from which:

sum [q^(n(n+1))/((1+q)(1+q^3)(1+q^(2n+1)))], n = 0..n

Input interpretation:

$$\sum_{n=0}^{n} \frac{q^{n(n+1)}}{(1+q)(1+q^3)(1+q^{2n+1})}$$

Result:

$$\sum_{n=0}^{n} \frac{q^{n(n+1)}}{(1+q)(1+q^3)(1+q^{2n+1})}$$

b)

$$\phi(q) = \sum_{n\geq 0} \frac{q^{n^2}}{(-q^2;q^2)_n} = \frac{1}{\prod_{n>0}(1-q^n)} \sum_{n\in\mathbf{Z}} \frac{(-1)^n(1+q^n)q^{n(3n+1)/2}}{1+q^{2n}}$$ (sequence A053250 in the OEIS).

From which:

1 + Sum x^(2*n+1), n = 0..n * Product (x^(2*k-1) - 1), k=1..n

Input interpretation:

$$1 + \left(\sum_{n=0}^{n} x^{2n+1}\right) \prod_{k=1}^{n} \left(x^{2k-1} - 1\right)$$

Result:

$$\frac{(-1)^n x^2 \left(x^{2n+2} - 1\right) \left(\frac{1}{x}; x^2\right)_{n+1}}{(x-1)\left(x^2-1\right)} + 1$$

$(a;q)_n$ gives the q-Pochhammer symbol

Values:

n	
0	$\frac{\left(1-\frac{1}{x}\right)x^2}{x-1}+1$
1	$1-\frac{x^2\left(x^4-1\right)\left(\frac{1}{x};x^2\right)_2}{(x-1)\left(x^2-1\right)}$
2	$\frac{\left(x^6-1\right)x^2\left(\frac{1}{x};x^2\right)_3}{(x-1)\left(x^2-1\right)}+1$
3	$1-\frac{x^2\left(x^8-1\right)\left(\frac{1}{x};x^2\right)_4}{(x-1)\left(x^2-1\right)}$

Alternate form:

$$\frac{(-1)^{n+1}x^2\left(\frac{1}{x};x^2\right)_{n+1}+(-1)^n x^{2n+4}\left(\frac{1}{x};x^2\right)_{n+1}+x^3-x^2-x+1}{(x-1)^2(x+1)}$$

Expanded form:

$$-\frac{(-1)^n x^2\left(\frac{1}{x};x^2\right)_{n+1}}{(x-1)\left(x^2-1\right)}+\frac{(-1)^n x^{2n+4}\left(\frac{1}{x};x^2\right)_{n+1}}{(x-1)\left(x^2-1\right)}+1$$

Alternate forms assuming n and x are positive:

$$1+\frac{e^{i\pi n}x^2\left(x^{2n+2}-1\right)\left(\frac{1}{x};x^2\right)_{n+1}}{(x-1)^2(x+1)}$$

$$1+\frac{e^{i\pi n}x^2\left(x^{2n+2}-1\right)\left(\frac{1}{x};x^2\right)_{n+1}}{(x-1)\left(x^2-1\right)}$$

Now, from (5.1)

$$f_3(q):=\sum_{n=0}^{\infty}\frac{q^{n^2}}{(-q;q)_n^2}$$
$$=\phi_3(-q)+2\psi_3(-q).$$

we have that:

$$f(q) = \sum_{n\ge 0} \frac{q^{n^2}}{(-q;q)_n^2} = \frac{2}{\prod_{n>0}(1-q^n)} \sum_{n\in\mathbf{Z}} \frac{(-1)^n q^{n(3n+1)/2}}{1+q^n},$$ (sequence A000025 in the OEIS).

that is (see sequence A000025 OEIS):

(1 + 4 * Sum_{n>0} (-1)^n * q^(n*(3*n+1)/2) / (1 + q^n)) / Product_{i>0} (1 - q^i).

We obtain, for n = 1..1 and n = 1..2

sum (1 + 4 *(-1)^n * q^(n*(3*n+1)/2) / (1 + q^n)), n=1..1 / Product(1 - q^n), n=1..1

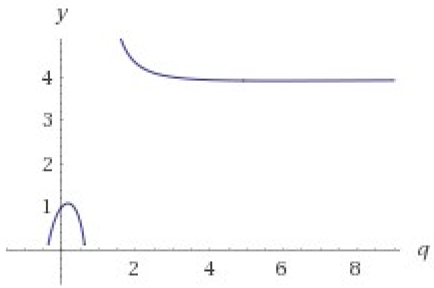
Input interpretation:

$$\frac{\sum_{n=1}^{1}\left(1+4(-1)^n \times \frac{q^{n(1/2(3n+1))}}{1+q^n}\right)}{\prod_{n=1}^{1}(1-q^n)}$$

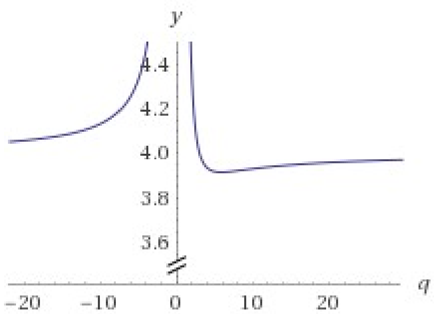
Result:

$$\frac{1-\frac{4q^2}{q+1}}{1-q}$$

Plots:



(q from -1.3 to 9)



(q from -21.9 to 29.6)

Alternate forms:

$$-\frac{2}{q+1}+\frac{1}{q-1}+4$$

$$\frac{-4q^2+q+1}{1-q^2}$$

$$\frac{4q^2-q-1}{(q-1)(q+1)}$$

Expanded form:

$$\frac{4q^2-q-1}{q^2-1}$$

(((1 - (4 *0.0018674427^2)/(1 + 0.0018674427))/(1 - 0.0018674427)))

Input interpretation:

$$\frac{1-\frac{4\times 0.0018674427^2}{1+0.0018674427}}{1-0.0018674427}$$

Result:

1.0018569871492363706827772427629639417993200285805395209 52...

1.0018569871…. result very near to the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}-\varphi}}=1+\frac{e^{-2\pi}}{1+\frac{e^{-4\pi}}{1+\frac{e^{-6\pi}}{1+\frac{e^{-8\pi}}{1+\ldots}}}}\approx 1.0018674362$$

Or:

(1 - (4 *535.491655524^2)/(1 + 535.491655524))/(1 - 535.491655524)

Input interpretation:

$$\frac{1-\frac{4\times 535.491655524^2}{1+535.491655524}}{1-535.491655524}$$

Result:

3.998143012819407951321451294098299066230010396817551637809...

3.9981430128…

Further:

sum (1 + 4 *(-1)^n * q^(n*(3*n+1)/2) / (1 + q^n)), n=1..2 / Product(1 - q^n), n=1..2

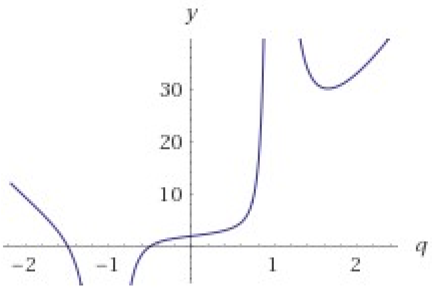
Input interpretation:

$$\frac{\sum_{n=1}^{2}\left(1+4(-1)^n \times \frac{q^{n(1/2(3n+1))}}{1+q^n}\right)}{\prod_{n=1}^{2}(1-q^n)}$$

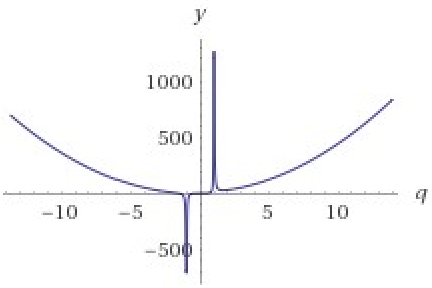
Result:

$$\frac{-\frac{4q^2}{q+1}+\frac{4q^7}{q^2+1}+2}{(1-q)(1-q^2)}$$

Plots:



(q from −2.2 to 2.4)



(q from −13.7 to 13.9)

Alternate forms:

$$\frac{2(2q^8+2q^7-2q^4+q^3-q^2+q+1)}{(q-1)^2(q+1)^2(q^2+1)}$$

$$4q^2 + \frac{1-q}{q^2+1} + 4q + \frac{4}{q-1} + \frac{1}{q+1} + \frac{1}{(q-1)^2} - \frac{1}{(q+1)^2} + 4$$

Expanded form:

$$-\frac{4q^2}{(1-q)(q+1)(1-q^2)} + \frac{2}{(1-q)(1-q^2)} + \frac{4q^7}{(1-q)(1-q^2)(q^2+1)}$$

(2 - (4 *0.0018674427^2)/(1 + 0.0018674427) + (4 *0.0018674427^7)/(1 + 0.0018674427^2))/((1 - 0.0018674427) (1 - 0.0018674427^2))

Input interpretation:

$$\frac{2 - \frac{4\times 0.0018674427^2}{1+0.0018674427} + \frac{4\times 0.0018674427^7}{1+0.0018674427^2}}{(1-0.0018674427)(1-0.0018674427^2)}$$

Result:

2.003734911425460202990043272560801864533795970630145205474...

2.003734911… ≈ 2

Or:

(2 - (4 *535.491655524^2)/(1 + 535.491655524) + (4 *535.491655524^7)/(1 + 535.491655524^2))/((1 - 535.491655524) (1 - 535.491655524^2))

Input interpretation:

$$\frac{2 - \frac{4\times 535.491655524^2}{1+535.491655524} + \frac{4\times 535.491655524^7}{1+535.491655524^2}}{(1-535.491655524)(1-535.491655524^2)}$$

Result:

$1.14915122664921856384298037106110381055676277469113138... \times 10^6$

$1.1491512266…*10^6$

1149151.2266… result very near to 1149851 (Lucas number)

From:

We recall, as Ramanujan would have, that [12, p. 17]

$$\frac{(q;q)_\infty}{(-q;q)_\infty} = \sum_{n=-\infty}^{\infty} (-1)^n q^{n^2}. \tag{4.1}$$

Also [12, p. 6], we need Euler's theorem

$$\frac{1}{(-q;q)_\infty} = (q;q^2)_\infty. \tag{4.2}$$

Hence

$$(q;q^2)_\infty \sum_{n=-\infty}^{\infty} (-1)^n q^{n^2} = \sum_{n\geq 0} \frac{(-1)^n q^{n^2}}{(-q^2;q^2)_n} + 2\sum_{n\geq 1} \frac{(-1)^n q^{n^2}}{(-q;q^2)_n}$$

$$:= \phi_3(-q) + 2\psi_3(-q), \tag{4.3}$$

Now, we have:

From Wikipedia:

$$\phi(q) = \sum_{n\geq 0} \frac{q^{n^2}}{(-q^2;q^2)_n} = \frac{1}{\prod_{n>0}(1-q^n)} \sum_{n\in\mathbf{Z}} \frac{(-1)^n(1+q^n)q^{n(3n+1)/2}}{1+q^{2n}}$$ (sequence A053250 in the OEIS).

1 + Sum_{k>0} x^k^2 / ((1 + x^2) (1 + x^4) ... (1 + x^(2*k)))

1 + Sum ((q^n^2 / ((1 + q^2) (1 + q^4) (1 + q^(2*n))))), n=1..1

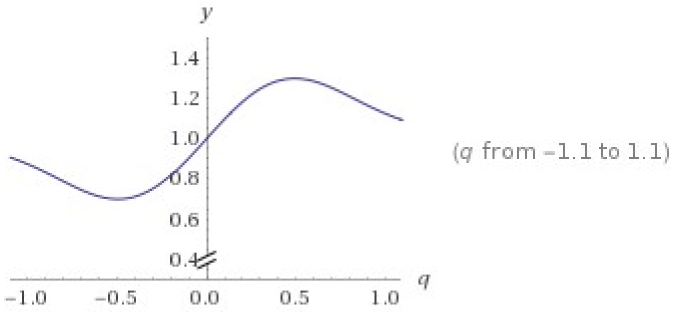
Input interpretation:

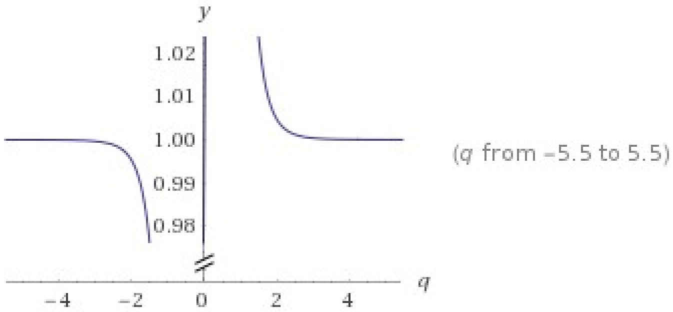
$$1 + \sum_{n=1}^{1} \frac{q^{n^2}}{(1+q^2)(1+q^4)(1+q^{2n})}$$

Result:

$$\frac{q}{(q^2+1)^2 (q^4+1)} + 1$$

Plots:





Alternate forms:

$$\frac{q}{2\left(q^2+1\right)}+\frac{q}{2\left(q^2+1\right)^2}-\frac{q^3}{2\left(q^4+1\right)}+1$$

$$\frac{\left(q^2+q+1\right)\left(q^6-q^5+2q^4-q^3+q^2+1\right)}{\left(q^2+1\right)^2\left(q^4+1\right)}$$

From the result

Input:

$$1+\frac{q}{\left(1+q^2\right)^2\left(1+q^4\right)}$$

We obtain:

1 + (0.0018674427)/((1 + 0.0018674427^2)^2 (1 + 0.0018674427^4))

Input interpretation:

$$1+\frac{0.0018674427}{\left(1+0.0018674427^2\right)^2\left(1+0.0018674427^4\right)}$$

Result:

1.0018674296752218131589708204114074360949406239550046864 33...

1.001867429675… result very near to the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}-\varphi}} = 1+\frac{e^{-2\pi}}{1+\frac{e^{-4\pi}}{1+\frac{e^{-6\pi}}{1+\frac{e^{-8\pi}}{1+\ldots}}}} \approx 1.0018674362$$

Or:

1 + (535.491655524)/((1 + 535.491655524^2)^2 (1 + 535.491655524^4))

Input interpretation:

$$1+\frac{535.491655524}{(1+535.491655524^2)^2\,(1+535.491655524^4)}$$

Result:

1.00000000000000000007920051710821057117589270745994761511 0...

1.000000000000000000079200517108210571175892707459947615110 ≈ 1

We have also:

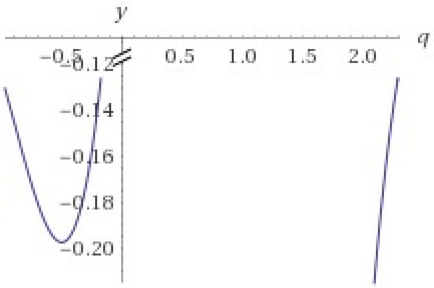
$$\psi(q)=\sum_{n>0}\frac{q^{n^2}}{(q;q^2)_n}=\frac{q}{\prod_{n>0}(1-q^{4n})}\sum_{n\in\mathbb{Z}}\frac{(-1)^n q^{6n(n+1)}}{1-q^{4n+1}}$$ (sequence A053251 in the OEIS).

sum(n>=1, q^(n^2) / ((1-q)*(1-q^3)*...*(1-q^(2*n-1)))) sum(q^(n^2) / ((1-q)*(1-q^3)*(1-q^(2*n-1)))), n=1..1

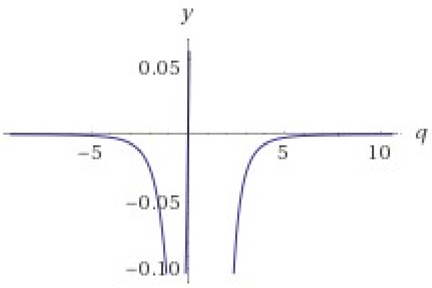
Sum:

$$\sum_{n=1}^{1} \frac{q^{n^2}}{(1-q)(1-q^3)(1-q^{2n-1})} = -\frac{q}{(q-1)^3(q^2+q+1)}$$

Plots:



(q from −1 to 2.3)



(q from −9.2 to 10.5)

Alternate form:

$$-\frac{q}{q(q(q((q-2)q+1)-1)+2)-1}$$

Partial fraction expansion:

$$\frac{-q-2}{9(q^2+q+1)} + \frac{1}{9(q-1)} - \frac{1}{3(q-1)^3}$$

Expanded form:

$$-\frac{q}{q^5-2q^4+q^3-q^2+2q-1}$$

From the result

Input:

$$-\frac{q}{(-1+q)^3(1+q+q^2)}$$

we obtain:

-0.0018674427/((-1 + 0.0018674427)^3 (1 + 0.0018674427 + 0.0018674427^2))

Input interpretation:

$$-\frac{0.0018674427}{(-1+0.0018674427)^3\,(1+0.0018674427+0.0018674427^2)}$$

Result:

0.001874436982678118764833981382269624475786427837754181739...

0.0018744369826…

Or:

-535.491655524/((-1 + 535.491655524)^3 (1 + 535.491655524 + 535.491655524^2))

Input interpretation:

$$-\frac{535.491655524}{(-1+535.491655524)^3\,(1+535.491655524+535.491655524^2)}$$

Result:

$-1.220710636242424172881301329795696468662660317960769... \times 10^{-11}$

$-1.220710636...*10^{-11}$

From:

$$(q;q^2)_\infty \sum_{n=-\infty}^{\infty} (-1)^n q^{n^2} = \sum_{n\geq 0} \frac{(-1)^n q^{n^2}}{(-q^2;q^2)_n} + 2\sum_{n\geq 1} \frac{(-1)^n q^{n^2}}{(-q;q^2)_n}$$
$$:= \phi_3(-q) + 2\psi_3(-q),$$

We obtain:

((1 + (0.0018674427)/((1 + 0.0018674427^2)^2 (1 + 0.0018674427^4)))) + 2* ((-0.0018674427/((-1 + 0.0018674427)^3 (1 + 0.0018674427 + 0.0018674427^2))))

Input interpretation:

$$\left(1+\frac{0.0018674427}{(1+0.0018674427^2)^2\,(1+0.0018674427^4)}\right)+ 2\left(-\frac{0.0018674427}{(-1+0.0018674427)^3\,(1+0.0018674427+0.0018674427^2)}\right)$$

Result:

1.005616303640578050688638783175946685046513479630513049912...

1.0056163036…

1/(((((1 + (0.0018674427)/((1 + 0.0018674427^2)^2 (1 + 0.0018674427^4)))) + 2* ((-0.0018674427/((-1 + 0.0018674427)^3 (1 + 0.0018674427 + 0.0018674427^2)))))))

Input interpretation:

$$1 \Big/ \left(\left(1 + \frac{0.0018674427}{(1 + 0.0018674427^2)^2 (1 + 0.0018674427^4)}\right) + 2\left(-\frac{0.0018674427}{(-1 + 0.0018674427)^3 (1 + 0.0018674427 + 0.0018674427^2)}\right)\right)$$

Result:

0.994415063061084379942586072028609172272801205533655281373...

0.99441506306… result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\dfrac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}}-1}}-\varphi+1} = 1 - \cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$$

((((((1 + (0.0018674427)/((1 + 0.0018674427^2)^2 (1 + 0.0018674427^4)))) + 2* ((-0.0018674427/((-1 + 0.0018674427)^3 (1 + 0.0018674427 + 0.0018674427^2))))))))^86

Input interpretation:

$$\left(\left(1 + \frac{0.0018674427}{(1 + 0.0018674427^2)^2 (1 + 0.0018674427^4)}\right) + 2\left(-\frac{0.0018674427}{(-1 + 0.0018674427)^3 (1 + 0.0018674427 + 0.0018674427^2)}\right)\right)^{86}$$

Result:

1.6187444646757909492693563918382935352523085840972920133 28...

1.61874446467579…

The first Jacobsthal numbers are:

0, 1, 1, 3, 5, 11, 21, 43, 85, 171, 341, 683, 1365, 2731, 5461, 10923, 21845, 43691, 87381, 174763, 349525,

The first Jacobsthal-Lucas numbers are:

2, 1, 5, 7, 17, 31, 65, 127, 257, 511, 1025, 2047

We note that the sum of the seventh number of each sequence, i.e. 21 and 65, is equal to 86, i.e. to the exponent of the expression

21 + 65 = 86

Now, from (5.3):

$$\lim_{q\to 1^-}\left(f(q)-\frac{1}{(-q;q)_\infty}\sum_{n=-\infty}^{\infty}(-1)^n q^{n^2}\right)=4. \qquad (5.3)$$

Multiplied by

$$\sum_{n\geq 0}\frac{(-1)^n q^{n^2}}{(-q^2;q^2)_n}+2\sum_{n\geq 1}\frac{(-1)^n q^{n^2}}{(-q;q^2)_n}$$

We obtain:

4*((((((1 + (0.0018674427)/((1 + 0.0018674427^2)^2 (1 + 0.0018674427^4)))) + 2*((-0.0018674427/((-1 + 0.0018674427)^3 (1 + 0.0018674427 + 0.0018674427^2)))))))

Input interpretation:

$$4\left(\left(1+\frac{0.0018674427}{(1+0.0018674427^2)^2\,(1+0.0018674427^4)}\right)+2\left(-\frac{0.0018674427}{(-1+0.0018674427)^3\,(1+0.0018674427+0.0018674427^2)}\right)\right)$$

Result:

4.022465214562312202754555132703786740186053918522052199649...

4.0224652145623…

From which:

5* 1/((4*((((1+(0.0018674427)/((1+0.0018674427^2)^2 (1+0.0018674427^4))))+2*((-0.0018674427/((-1+0.0018674427)^3 (1+0.0018674427+0.0018674427^2))))))))^1/3

Input interpretation:

$$5\times 1\Big/\left(\left(4\left(\left(1+\frac{0.0018674427}{(1+0.0018674427^2)^2\,(1+0.0018674427^4)}\right)+2\left(-\frac{0.0018674427}{(-1+0.0018674427)^3\,(1+0.0018674427+0.0018674427^2)}\right)\right)\right)^\wedge(1/3)\right)$$

Result:

3.1439278580...

3.143927858… ≈ π

1/6(((5* 1/((4*((((1+(0.0018674427)/((1+0.0018674427^2)^2 (1+0.0018674427^4))))+2*((-0.0018674427/((-1+0.0018674427)^3 (1+0.0018674427+0.0018674427^2))))))))^1/3)))^2

Input interpretation:

$$\frac{1}{6}\left(5\times 1\Big/\left(\left(4\left(\left(1+\frac{0.0018674427}{(1+0.0018674427^2)^2\,(1+0.0018674427^4)}\right)+2\left(-(0.0018674427/((-1+0.0018674427)^3\,(1+0.0018674427+0.0018674427^2)))\right)\right)\right)^\wedge(1/3)\right)\right)^2$$

Result:

1.647380396...

1.647380396…

1/6(((5* 1/((4*((((1+(0.0018674427)/((1+0.0018674427^2)^2 (1+0.0018674427^4))))+2*((-0.0018674427/((-1+0.0018674427)^3 (1+0.0018674427+0.0018674427^2))))))))^1/3)))^2-29/10^3

Input interpretation:

$$\frac{1}{6}\left(5\times 1/\left(\left(4\left(\left(1+\frac{0.0018674427}{(1+0.0018674427^2)^2\,(1+0.0018674427^4)}\right)+2\left(-(0.0018674427/((-1+0.0018674427)^3\,(1+0.0018674427+0.0018674427^2))))\right)\right)\right)\wedge(1/3)\right)\right)^2-\frac{29}{10^3}$$

Result:

1.618380396...

1.618380396…

Furthermore, we have also:

27[4((1+(0.0018674427)/((1.0018674427^2)^2 (1.0018674427^4))+2((-0.0018674427/((-1+0.0018674427)^3(1.0018674427+0.0018674427^2))))))]^3-29

Input interpretation:

$$27\left(4\left(1+\frac{0.0018674427}{(1.0018674427^2)^2\times 1.0018674427^4}+2\left(-\frac{0.0018674427}{(-1+0.0018674427)^3\,(1.0018674427+0.0018674427^2)}\right)\right)\right)^3-29$$

Result:

1728.1337799758222658068533159270986007347818382923064747 87...

1728.1337799758…

Furthermore, we have:

$$\sum_{n\geq 0}\frac{q^{n^2}}{(q;q)_n}=\sum_{n\geq 0}\frac{q^{n^2}}{(q^2;q^2)_n}(-q;q)_n=(-q;q)_\infty\sum_{n\geq 0}\frac{q^{n^2}}{(q^2;q^2)_n(-q^{n+1};q)_\infty}$$

From

$$f_0(q)=\sum_{n\geq 0}\frac{q^{n^2}}{(-q;q)_n}$$

That is:

1 + Sum_{k>0} q^k^2 / ((1 + q) * (1 + q^2) * ... * (1 + q^k)).

We obtain:

(((1 + Sum (((0.0018674427^k^2 / ((1 + 0.0018674427) * (1 + 0.0018674427^2) * (1 + 0.0018674427^k))))), k = 1..infinity)))

Input interpretation:

$$1+\sum_{k=1}^{\infty}\frac{0.0018674427^{k^2}}{(1+0.0018674427)(1+0.0018674427^2)(1+0.0018674427^k)}$$

Result:

1.00186

1.00186 as above

And

Input interpretation:

$$1+\sum_{k=1}^{\infty}-\frac{0.0018674427^{k^2}}{(1+0.0018674427)(1+0.0018674427^2)(1+0.0018674427^k)}$$

Result:

0.99814

0.99814 as above

(((1 + Sum (((0.0018674427^k^2 / ((1 + 0.0018674427) * (1 + 0.0018674427^2) * (1 + 0.0018674427^k))))), k = 1..infinity)))^259

Input interpretation:

$$\left(1+\sum_{k=1}^{\infty} \frac{0.0018674427^{k^2}}{(1+0.0018674427)\left(1+0.0018674427^2\right)\left(1+0.0018674427^k\right)}\right)^{259}$$

Result:

1.61837

1.61837

(((1 + Sum (((0.0018674427^k^2 / ((1 + 0.0018674427) * (1 + 0.0018674427^2) * (1 + 0.0018674427^k))))), k = 1..infinity)))^4096-256-47+7

Input interpretation:

$$\left(1+\sum_{k=1}^{\infty} \frac{0.0018674427^{k^2}}{(1+0.0018674427)\left(1+0.0018674427^2\right)\left(1+0.0018674427^k\right)}\right)^{4096} - 256 - 47 + 7$$

Result:

1729.25

1729.25

With regard:

$$\frac{(-q;q)_\infty}{(q;q)_\infty}\left\{\sum_{n\geq 0} q^{2n^2}(q^2;q^4)_n - q(-q^2;q^2)_\infty(q^4;q^4)_\infty \sum_{n\geq 0}\frac{q^{4n^2+4n}}{(q^4;q^4)_n} + \sum_{n\geq 1}\frac{q^{4n^2}}{(q^2;q^4)_n}\right\}.$$

We have that from:

$$F_0(q) = \sum_{n\geq 0} \frac{q^{2n^2}}{(q;q^2)_n} \text{ (sequence A053264 in the OEIS)}$$

$$f_1(q) = \sum_{n\geq 0} \frac{q^{n^2+n}}{(-q;q)_n} \text{ (sequence A053257 in the OEIS)}$$

We obtain:

sum q^(2n^2)/((1-q)(1-q^3)...(1-q^(2n-1))), n = 1..1

from which:

sum (0.0018674427^2)^(2n^2)/((1-0.0018674427^2)(1-((0.0018674427)^2)^3)(1-0.0018674427^2)^(2n-1))), n = 1..1

Sum:

$$\sum_{n=1}^{1} \frac{(0.00186744^2)^{2n^2}}{(1-0.00186744^2)(1-(0.00186744^2)^3)(1-0.00186744^2)^{2n-1}} =$$

78 373 577 212 774 187 115 481 792 865 537 642 234 583 744 615 020 060 700 054 406 547 817 718 788 698 330 551 609 / 6 444 325 983 779 305 343 660 272 653 648 025 630 470 445 938 013 103 394 135 020 625 428 197 812 260 556 950 675 213 194 944 000

Decimal approximation:

$1.2161640706886096007866888594041266557992453754364719... \times 10^{-11}$

1.216164070688…*10^{-11}

and:

sum q^(n^2+n)/((1+q)(1+q^2)...(1+q^n)), n = 1..1

sum 0.0018674427^(4(n^2+n))/((1+0.0018674427^4)(1+(0.0018674427^4)^2)...(1+(0.0018674427^4)^n)), n = 1..1

Sum:

$$\sum_{n=1}^{1} \frac{0.00186744^{4(n^2+n)}}{(1+0.00186744^4)((0.00186744^4)^2+1)((0.00186744^4)^n+1)} =$$

11 605 587 426 663 353 577 950 844 174 494 313 988 124 989 748 917 249 835 830 337 851 035 738 430 928 114 778 351 579 492 144 220 081 493 609 009 000 852 234 486 079 784 012 321 / 78 467 325 092 609 665 874 599 622 287 400 336 634 705 543 440 668 732 580 580 583 657 569 273 048 043 933 777 506 089 587 761 587 892 317 674 130 464 684 822 977 812 645 566 151 523 365 909 947 663 292 168

Decimal approximation:

$1.4790344150212926563681458523537073277474801938378060... \times 10^{-22}$

$1.47903441502...*10^{-22}$

From the ratio of the two results, we obtain:

(1.216164070688609 × 10^-11 / 1.47903441502129 × 10^-22)

Input interpretation:

$$\frac{1.216164070688609 \times 10^{-11}}{1.47903441502129 \times 10^{-22}}$$

Result:

$8.2226894677843104423171059415024286737137305175800352... \times 10^{10}$

$8.2226894677...*10^{10}$

From which:

(1.216164070688609 × 10^-11 / 1.47903441502129 × 10^-22)^1/52 - 3/10^3

Input interpretation:

$$\sqrt[52]{\frac{1.216164070688609 \times 10^{-11}}{1.47903441502129 \times 10^{-22}}} - \frac{3}{10^3}$$

Result:

1.6184520616712169...

1.6184520616712169…

We have also:

$$\phi_0(q) = \sum_{n\geq 0} q^{n^2} (-q; q^2)_n \text{ (sequence A053258 in the OEIS)}$$

From which:

sum for n >= 0 of q^n^2 (1+q)(1+q^3)...(1+q^(2n-1))

sum (0.0018674427^2)^n^2 (1+0.0018674427^2)(1+((0.0018674427)^2)^3)(1+((0.0018674427^2))^(2n-1)), n = 1..1

Sum:

$$\sum_{n=1}^{1}(1+0.00186744^2)\left((0.00186744^2)^3+1\right)(0.00186744^2)^{n^2}\left((0.00186744^2)^{2n-1}+1\right) =$$

517 028 204 409 612 377 993 520 312 801 316 757 199 367 623 096 526 569 336 396 680 161 458 710 583 620 611 756 291 026 697 702 038 530 165 508 008 / 148 257 487 526 813 259 225 846 222 831 340 464 079 214 669 691 942 390 464 900 212 585 350 985 602 322 527 038 818 908 072 390 999 978 323 420 636 168 881

Decimal approximation:

$3.4873665609374684322743303167495821788245183683170335\ldots \times 10^{-6}$

$3.4873665609374684322743303167495821788245183683170335 \times 10^{-6}$

Thence, we obtain:

$$\sum_{n\geq 0}\frac{q^{n^2}}{(q;q)_n}=\sum_{n\geq 0}\frac{q^{n^2}}{(q^2;q^2)_n}(-q;q)_n=(-q;q)_\infty\sum_{n\geq 0}\frac{q^{n^2}}{(q^2;q^2)_n(-q^{n+1};q)_\infty}$$

$$=\frac{(-q;q)_\infty}{(q;q)_\infty}\left\{\sum_{n\geq 0}q^{2n^2}(q^2;q^4)_n-q(-q^2;q^2)_\infty(q^4;q^4)_\infty\sum_{n\geq 0}\frac{q^{4n^2+4n}}{(q^4;q^4)_n}+\sum_{n\geq 1}\frac{q^{4n^2}}{(q^2;q^4)_n}\right\}.$$

(((((3.48736656093e-6)-y*(1.479034e-22+1.216164e-11))))) = 0.99814*1/x

Input interpretation:

$$3.48736656093\times 10^{-6}-y\left(1.479034\times 10^{-22}+1.216164\times 10^{-11}\right)=0.99814\times\frac{1}{x}$$

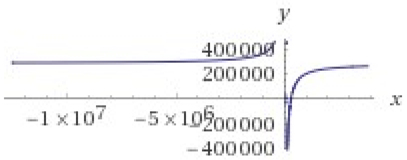
Result:

$$3.48736656093\times10^{-6} - 1.21616\times10^{-11}\, y = \frac{0.99814}{x}$$

Geometric figure:

line

Implicit plot:



Alternate forms:

$$y = 286\,751. - \frac{8.20728\times10^{10}}{x}$$

$$-1.21616\times10^{-11}\,(y - 286\,751.) = \frac{0.99814}{x}$$

Alternate form assuming x and y are positive:

$$x\, y + 8.20728\times10^{10} = 286\,751.\, x$$

Alternate forms assuming x and y are real:

$$3.48737\times10^{-6} - 1.21616\times10^{-11}\, y = \frac{0.99814}{x} + 0$$

$$\frac{8.20728\times10^{10}}{x} + y = 286\,751.$$

Solution:

$$x \neq 0\,, \quad y \approx \frac{2.61364\times10^{-8}\,(1.09713\times10^{13}\, x - 3.14017\times10^{18})}{x}$$

Solution for the variable y:

$$y \approx -8.22258\times10^{10}\left(\frac{0.99814}{x} - 3.48737\times10^{-6}\right)$$

Implicit derivatives:

$$\frac{\partial x(y)}{\partial y} = \frac{6\,100\,000\, x^2}{500\,644\,156\,538\,587\,919}$$

$$\frac{\partial y(x)}{\partial x} = \frac{500\,644\,156\,538\,587\,919}{6\,100\,000\, x^2}$$

(((((3.48736656093e-6)-(286751- (8.20728×10^10)/x)*(1.479034e-22+1.216164e-11))))) = 0.99814*1/x

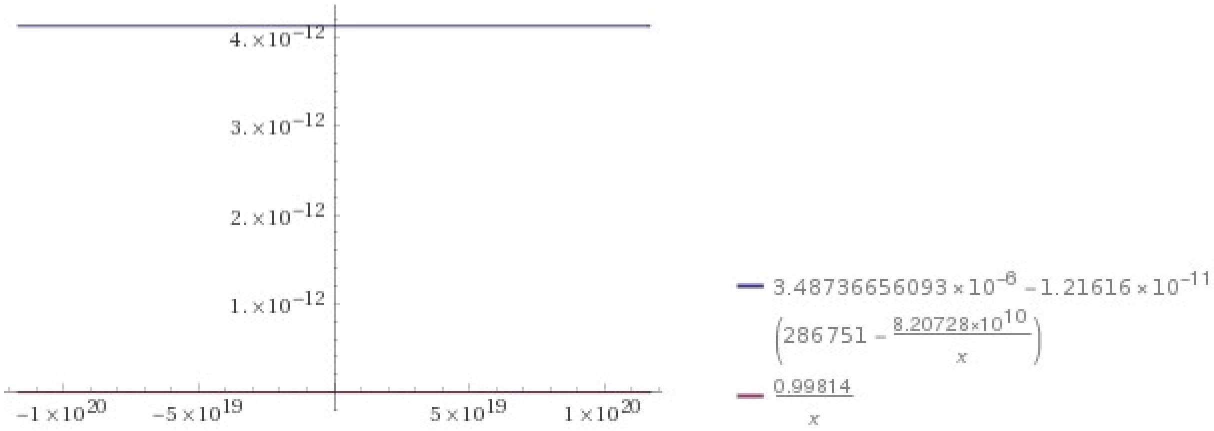
Input interpretation:

$$3.48736656093\times10^{-6} - \left(286\,751 - \frac{8.20728\times10^{10}}{x}\right)\left(1.479034\times10^{-22} + 1.216164\times10^{-11}\right) = 0.99814\times\frac{1}{x}$$

Result:

$$3.48736656093\times10^{-6} - 1.21616\times10^{-11}\left(286\,751 - \frac{8.20728\times10^{10}}{x}\right) = \frac{0.99814}{x}$$

Plot:



Alternate forms:

$$\frac{0.99814}{x} + 4.12925\times10^{-12} = \frac{0.99814}{x}$$

$$\frac{4.12925\times10^{-12}\left(x + 2.41724\times10^{11}\right)}{x} = \frac{0.99814}{x}$$

Alternate form assuming x is positive:

$x = 36\,954.9$ (for $x \neq 0$)

Alternate forms assuming x is real:

$$\frac{0.99814}{x} + 4.12925\times10^{-12} = \frac{0.99814}{x} + 0$$

$$\frac{36\,954.9}{x} = 1$$

Solution:
$x \approx 36954.9$

36954.9

286751- (8.20728×10^10)/(36954.9)

Input interpretation:
$$286751-\frac{8.20728\times 10^{10}}{36954.9}$$

Result:
$-1.934139869681693090767394851562309734297752124887362 7... \times 10^6$

$-1.9341398696... * 10^6$

Repeating decimal:
$-1.934139869681693090767394851562309734297752124887362 7... \times 10^6$
(period 4562)

-1934139.86968

Thence:

(((((3.48736656093e-6)-(-1934139.86968)*(1.479034e-22+1.216164e-11))))) = 0.99814*1/(36954.9)

(((((3.48736656093e-6)+(1934139.86968)*(1.479034e-22+1.216164e-11)))))

Input interpretation:
$$3.48736656093\times 10^{-6}+1.93413986968\times 10^{6}\,(1.479034\times 10^{-22}+1.216164\times 10^{-11})$$

Result:
0.0000270096793659111410628012289 12

0.000027009679365911141062801228912 (final result)

0.99814*1/(36954.9)

Input interpretation:

$$0.99814\times\frac{1}{36\,954.9}$$

Result:

0.000027009679365929822567507962408232737742491523451558521...

0.00002700967936…

Thence:

$$\frac{(-q;q)_\infty}{(q;q)_\infty}\left\{\sum_{n\geq 0} q^{2n^2}(q^2;q^4)_n - q(-q^2;q^2)_\infty(q^4;q^4)_\infty \sum_{n\geq 0}\frac{q^{4n^2+4n}}{(q^4;q^4)_n} + \sum_{n\geq 1}\frac{q^{4n^2}}{(q^2;q^4)_n}\right\}$$

$= 0.000027009679365929822567507962408232737742491523451558521...$

From which:

[1/(((((3.48736656093e-6)+(1934139.86968)*(1.479034e-22+1.216164e-11)))))]^1/22+5/10^3

Input interpretation:

$$\left(1/\left(3.48736656093\times 10^{-6}+1.93413986968\times 10^{6}\left(1.479034\times 10^{-22}+1.216164\times 10^{-11}\right)\right)\right)\hat{}\,(1/22)+\frac{5}{10^3}$$

Result:

1.618088547333836553158998837425388957547707110665450003520...

1.618088547333…

From:

Congruence properties of the partition function
Tony Forbes - Talks for LSBU Mathematics Study Group, 24 Sep, 8 Oct & 19 Nov 2008

Ramanujan's tau function $\tau(n)$ is $p_{24}(n-1)$.

n	1	2	3	4	5	6	7	8	9	10
$\tau(n)$	1	−24	252	−1472	4830	−6048	−16744	84480	−113643	−115920

Now:

Lectures on Black Holes, Topological Strings and Quantum Attractors (2.0)
Boris Pioline - arXiv:hep-th/0607227v5 6 Feb 2007

$$p_{24}(N) = \frac{16}{2\pi i}\int_C d\beta \left(\frac{\beta}{2\pi}\right)^{12} e^{\beta(N-1)+4\frac{\pi^2}{\beta}} \qquad (6.5)$$

This integral may be evaluated by steepest descent: the saddle point occurs at $\beta = 2\pi/\sqrt{N-1}$, leading to the characteristic Hagedorn growth

$$p_{24}(N) \sim \exp\left(4\pi\sqrt{nw}\right) \qquad (6.6)$$

for the spectrum of DH states.

Using the standard asymptotic expansion of $\hat{I}_\nu(z)$ at large z

$$\hat{I}_\nu(z) \sim 2^\nu \left(\frac{z}{2\pi}\right)^{-\nu-\frac{1}{2}} \left[1 - \frac{(\mu-1)}{8z} + \frac{(\mu-1)(\mu-3^2)}{2!(8z)^2} - \frac{(\mu-1)(\mu-3^2)(\mu-5^2)}{3!(8z)^3} + \dots\right], \qquad (6.9)$$

where $\mu = 4\nu^2$, we can compute the subleading corrections to the microscopic entropy of DH states to arbitrary high order,

$$\log\Omega(n,w) \sim 4\pi\sqrt{|nw|} - \frac{27}{4}\log|nw| + \frac{15}{2}\log 2 - \frac{675}{32\pi\sqrt{|nw|}} - \frac{675}{2^8\pi^2|nw|} - \dots \qquad (6.10)$$

For N = (n – 1) = 8, from OEIS sequence, we obtain:

exp(4Pi*sqrt(x)) = 84480

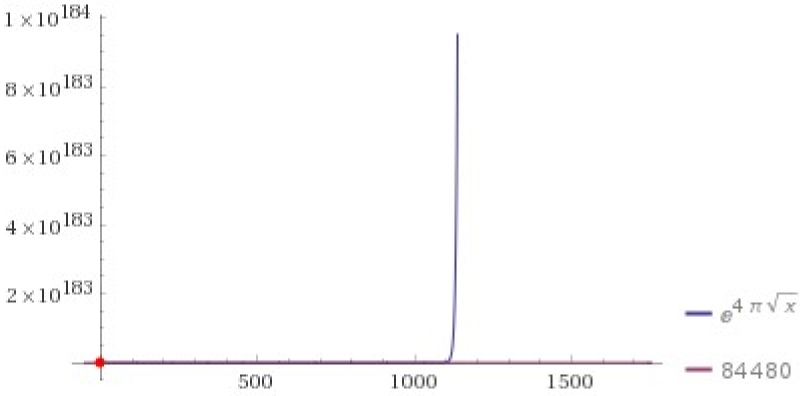
Input:

$\exp\left(4\pi\sqrt{x}\right) = 84480$

Exact result:

$e^{4\pi\sqrt{x}} = 84480$

Plot:



Numerical solutions:

$x \approx -5.43505 - 4.51374\,i \ldots$

$x \approx -5.43505 + 4.51374\,i \ldots$

Input interpretation:

$-5.43505 + 4.51374\,i$

i is the imaginary unit

Result:

$-5.43505\ldots +$
$4.51374\ldots\,i$

(-5.43505 + 4.51374 i)

Polar coordinates:

$r = 7.06496$ (radius), $\theta = 140.291°$ (angle)

7.06496

Indeed:

exp(4Pi*sqrt(-5.43505 + 4.51374 i))

Input interpretation:

$$\exp\left(4\pi\sqrt{-5.43505 + 4.51374\,i}\right)$$

i is the imaginary unit

Result:

84479.4... +
0.742205... i

Polar coordinates:

$r = 84479.4$ (radius), $\theta = 0.00050338°$ (angle)

84479.4

Thence nw = (-5.43505 + 4.51374 i)

Now, from (6.10)

$$\log \Omega(n,w) \sim 4\pi\sqrt{|nw|} - \frac{27}{4}\log|nw| + \frac{15}{2}\log 2 - \frac{675}{32\pi\sqrt{|nw|}} - \frac{675}{2^8\pi^2|nw|} - \ldots$$

4Pi*sqrt(-5.43505 + 4.51374 i)-27/4*ln(-5.43505 + 4.51374 i)+15/2 ln(2) – 675/(32Pi*sqrt(-5.43505 + 4.51374 i)) – 675/(2^8*Pi^2*(-5.43505 + 4.51374 i))

Input interpretation:

$$4\pi\sqrt{-5.43505 + 4.51374\,i} - \frac{27}{4}\log(-5.43505 + 4.51374\,i) + \frac{15}{2}\log(2) - \frac{675}{32\pi\sqrt{-5.43505 + 4.51374\,i}} - \frac{675}{2^8\pi^2 \times (-5.43505 + 4.51374\,i)}$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Result:

2.51677... +
17.2884... i

Polar coordinates:

$r = 17.4706$ (radius), $\theta = 81.7173°$ (angle)

17.4706 result very near to the black hole entropy 17.5764, that is equal to ln(42987519)

Alternative representations:

$$4\pi\sqrt{-5.43505+4.51374\,i} - \frac{1}{4}\log(-5.43505+4.51374\,i)\,27 + \frac{1}{2}\log(2)\,15 - \frac{675}{32\pi\sqrt{-5.43505+4.51374\,i}} - \frac{675}{2^8\pi^2(-5.43505+4.51374\,i)} = \frac{15}{2}\log(a)\log_a(2) - \frac{27}{4}\log(a)\log_a(-5.43505+4.51374\,i) - \frac{675}{(-5.43505+4.51374\,i)\,2^8\pi^2} - \frac{675}{32\pi\sqrt{-5.43505+4.51374\,i}} + 4\pi\sqrt{-5.43505+4.51374\,i}$$

$$4\pi\sqrt{-5.43505+4.51374\,i} - \frac{1}{4}\log(-5.43505+4.51374\,i)\,27 + \frac{1}{2}\log(2)\,15 - \frac{675}{32\pi\sqrt{-5.43505+4.51374\,i}} - \frac{675}{2^8\pi^2(-5.43505+4.51374\,i)} = \frac{15\log_e(2)}{2} - \frac{27}{4}\log_e(-5.43505+4.51374\,i) - \frac{675}{(-5.43505+4.51374\,i)\,2^8\pi^2} - \frac{675}{32\pi\sqrt{-5.43505+4.51374\,i}} + 4\pi\sqrt{-5.43505+4.51374\,i}$$

$$4\pi\sqrt{-5.43505+4.51374\,i} - \frac{1}{4}\log(-5.43505+4.51374\,i)\,27 + \frac{1}{2}\log(2)\,15 - \frac{675}{32\pi\sqrt{-5.43505+4.51374\,i}} - \frac{675}{2^8\pi^2(-5.43505+4.51374\,i)} = -\frac{15\,\text{Li}_1(-1)}{2} + \frac{27}{4}\text{Li}_1(6.43505-4.51374\,i) - \frac{675}{(-5.43505+4.51374\,i)\,2^8\pi^2} - \frac{675}{32\pi\sqrt{-5.43505+4.51374\,i}} + 4\pi\sqrt{-5.43505+4.51374\,i}$$

Series representations:

$$4\pi\sqrt{-5.43505+4.51374\,i} - \frac{1}{4}\log(-5.43505+4.51374\,i)\,27 + \frac{1}{2}\log(2)\,15 - \frac{675}{32\pi\sqrt{-5.43505+4.51374\,i}} - \frac{675}{2^8\pi^2(-5.43505+4.51374\,i)} = -\frac{0.584154}{(-1.20411+i)\pi^2} + \frac{15\log(2)}{2} - \frac{27}{4}\log(-5.43505+4.51374\,i) - \frac{675}{32\pi\sqrt{-6.43505+4.51374\,i}\sum_{k=0}^{\infty}(-6.43505+4.51374\,i)^{-k}\binom{\frac{1}{2}}{k}} + 4\pi\sqrt{-6.43505+4.51374\,i}\sum_{k=0}^{\infty}(-6.43505+4.51374\,i)^{-k}\binom{\frac{1}{2}}{k}$$

$$4\pi\sqrt{-5.43505+4.51374 i} - \frac{1}{4}\log(-5.43505+4.51374 i)\, 27 + \frac{1}{2}\log(2)\, 15 - \frac{675}{32\pi\sqrt{-5.43505+4.51374 i}} - \frac{675}{2^8\pi^2(-5.43505+4.51374 i)} = -\frac{0.584154}{(-1.20411+i)\pi^2} + \frac{15\log(2)}{2} - \frac{27}{4}\log(-5.43505+4.51374 i) - \frac{675}{32\pi\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(-5.43505+4.51374 i-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(-5.43505+4.51374 i-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}} + 4\pi\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(-5.43505+4.51374 i-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(-5.43505+4.51374 i-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!} \quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$4\pi\sqrt{-5.43505+4.51374 i} - \frac{1}{4}\log(-5.43505+4.51374 i)\, 27 + \frac{1}{2}\log(2)\, 15 - \frac{675}{32\pi\sqrt{-5.43505+4.51374 i}} - \frac{675}{2^8\pi^2(-5.43505+4.51374 i)} = -\frac{675}{256(-5.43505+4.51374 i)\pi^2} + \frac{15\log(2)}{2} - \frac{27}{4}\log(-5.43505+4.51374 i) - \frac{675}{32\pi\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(-5.43505+4.51374 i-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(-5.43505+4.51374 i-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}} + 4\pi\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(-5.43505+4.51374 i-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(-5.43505+4.51374 i-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!} \quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

Integral representations:

$$4\pi\sqrt{-5.43505+4.51374 i} - \frac{1}{4}\log(-5.43505+4.51374 i)\, 27 + \frac{1}{2}\log(2)\, 15 - \frac{675}{32\pi\sqrt{-5.43505+4.51374 i}} - \frac{675}{2^8\pi^2(-5.43505+4.51374 i)} = -\frac{0.584154}{(-1.20411+i)\pi^2} + \int_1^2 \frac{12.354 + i(-7.5+0.75 t) - 1.06924 t}{t(1.6472 - i - 1.42566 t + i t)}\, dt - \frac{675}{32\pi\sqrt{-5.43505+4.51374 i}} + 4\pi\sqrt{-5.43505+4.51374 i}$$

$$4\pi\sqrt{-5.43505+4.51374\,i} - \frac{1}{4}\log(-5.43505+4.51374\,i)\,27 + \frac{1}{2}\log(2)\,15 - \frac{675}{32\pi\sqrt{-5.43505+4.51374\,i}} - \frac{675}{2^8\pi^2(-5.43505+4.51374\,i)} =$$

$$-\frac{675}{256(-5.43505+4.51374\,i)\pi^2} + \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{3(-6.43505+4.51374\,i)^{-s}(-9+10(-6.43505+4.51374\,i)^s)\Gamma(-s)^2\Gamma(1+s)}{8\pi\mathcal{A}\Gamma(1-s)}\,ds - \frac{675}{32\pi\sqrt{-5.43505+4.51374\,i}} + 4\pi\sqrt{-5.43505+4.51374\,i}$$

for $-1 < \gamma < 0$

exp(4Pi*sqrt(x)) = 987136; for n = 16

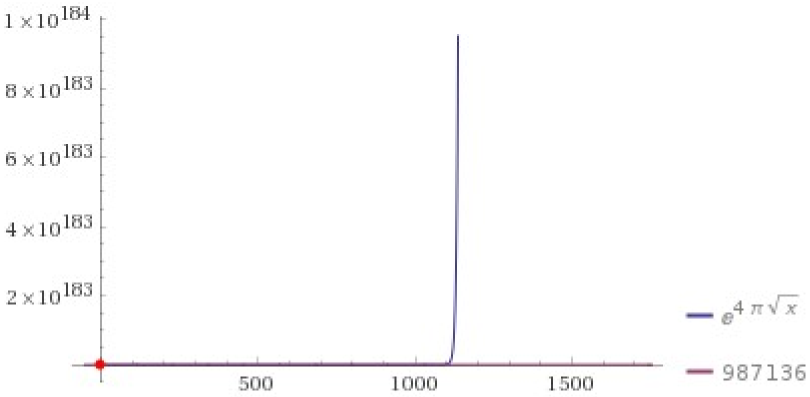
Input:

$$\exp\left(4\pi\sqrt{x}\right) = 987136$$

Exact result:

$$e^{4\pi\sqrt{x}} = 987136$$

Plot:



Numerical solutions:

$x \approx -19.0436 - 9.88536\,i\ldots$

$x \approx -19.0436 + 9.88536\,i\ldots$

4Pi*sqrt(-19.0436 - 9.88536 i)-27/4*ln(-19.0436 - 9.88536 i)+15/2 ln(2) – 675/(32Pi*sqrt(-19.0436 - 9.88536 i)) – 675/(2^8*Pi^2*(-19.0436 - 9.88536 i))

Input interpretation:

$$4\pi\sqrt{-19.0436+i\times(-9.88536)}-\frac{27}{4}\log(-19.0436+i\times(-9.88536))+\frac{15}{2}\log(2)-\frac{675}{32\pi\sqrt{-19.0436+i\times(-9.88536)}}-\frac{675}{2^8\pi^2(-19.0436+i\times(-9.88536))}$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Result:

$-2.02717\ldots-39.9888\ldots i$

Polar coordinates:

$r = 40.0401$ (radius), $\theta = -92.902°$ (angle)

40.0401

Alternative representations:

$$4\pi\sqrt{-19.0436-i\,9.88536}-\frac{1}{4}\log(-19.0436-i\,9.88536)\,27+\frac{1}{2}\log(2)\,15-\frac{675}{32\pi\sqrt{-19.0436-i\,9.88536}}-\frac{675}{2^8\pi^2(-19.0436-i\,9.88536)}=\frac{15}{2}\log(a)\log_a(2)-\frac{27}{4}\log(a)\log_a(-19.0436-9.88536\,i)-\frac{675}{(-19.0436-9.88536\,i)\,2^8\pi^2}-\frac{675}{32\pi\sqrt{-19.0436-9.88536\,i}}+4\pi\sqrt{-19.0436-9.88536\,i}$$

$$4\pi\sqrt{-19.0436-i\,9.88536}-\frac{1}{4}\log(-19.0436-i\,9.88536)\,27+\frac{1}{2}\log(2)\,15-\frac{675}{32\pi\sqrt{-19.0436-i\,9.88536}}-\frac{675}{2^8\pi^2(-19.0436-i\,9.88536)}=\frac{15\log_e(2)}{2}-\frac{27}{4}\log_e(-19.0436-9.88536\,i)-\frac{675}{(-19.0436-9.88536\,i)\,2^8\pi^2}-\frac{675}{32\pi\sqrt{-19.0436-9.88536\,i}}+4\pi\sqrt{-19.0436-9.88536\,i}$$

$$4\pi\sqrt{-19.0436 - i\,9.88536} - \frac{1}{4}\log(-19.0436 - i\,9.88536)\,27 + \frac{1}{2}\log(2)\,15 - \frac{675}{32\pi\sqrt{-19.0436 - i\,9.88536}} - \frac{675}{2^8\,\pi^2\,(-19.0436 - i\,9.88536)} = -\frac{15\,\text{Li}_1(-1)}{2} + \frac{27}{4}\,\text{Li}_1(20.0436 + 9.88536\,i) - \frac{675}{(-19.0436 - 9.88536\,i)\,2^8\,\pi^2} - \frac{675}{32\pi\sqrt{-19.0436 - 9.88536\,i}} + 4\pi\sqrt{-19.0436 - 9.88536\,i}$$

Integral representations:

$$4\pi\sqrt{-19.0436 - i\,9.88536} - \frac{1}{4}\log(-19.0436 - i\,9.88536)\,27 + \frac{1}{2}\log(2)\,15 - \frac{675}{32\pi\sqrt{-19.0436 - i\,9.88536}} - \frac{675}{2^8\,\pi^2\,(-19.0436 - i\,9.88536)} = \frac{0.26673}{(1.92644 + i)\,\pi^2} + \int_1^2 \frac{-15.9657 + i\,(-7.5 + 0.75\,t) + 1.5207\,t}{t\,(-2.12876 - i + 2.0276\,t + i\,t)}\,dt - \frac{675}{32\pi\sqrt{-19.0436 - 9.88536\,i}} + 4\pi\sqrt{-19.0436 - 9.88536\,i}$$

$$4\pi\sqrt{-19.0436 - i\,9.88536} - \frac{1}{4}\log(-19.0436 - i\,9.88536)\,27 + \frac{1}{2}\log(2)\,15 - \frac{675}{32\pi\sqrt{-19.0436 - i\,9.88536}} - \frac{675}{2^8\,\pi^2\,(-19.0436 - i\,9.88536)} = -\frac{675}{256\,(-19.0436 - 9.88536\,i)\,\pi^2} + \int_{-\mathcal{A}\,\infty+\gamma}^{\mathcal{A}\,\infty+\gamma} \frac{3\,(-9 + 10\,(-20.0436 - 9.88536\,i)^s)\,(-20.0436 - 9.88536\,i)^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{8\pi\,\mathcal{A}\,\Gamma(1-s)}\,ds - \frac{675}{32\pi\sqrt{-19.0436 - 9.88536\,i}} + 4\pi\sqrt{-19.0436 - 9.88536\,i}$$

for $-1 < \gamma < 0$

exp(4Pi*sqrt(x)) = 2699296768;   for n = 64

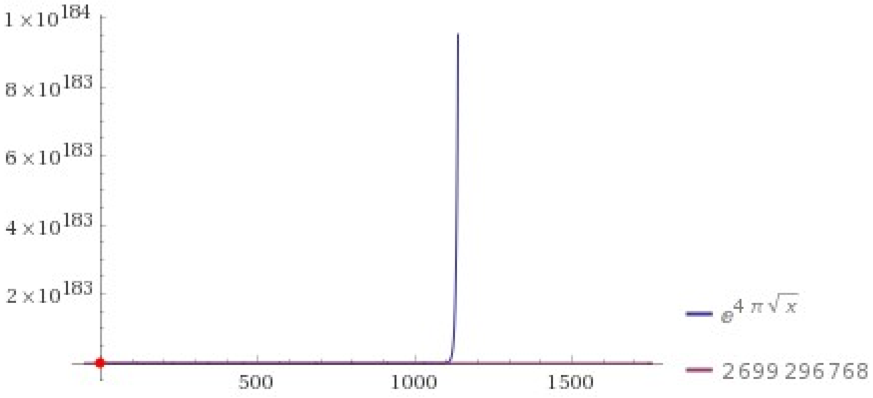
Input:

$$\exp\left(4\pi\sqrt{x}\right) = 2\,699\,296\,768$$

Exact result:

$$e^{4\pi\sqrt{x}} = 2\,699\,296\,768$$

Plot:



Numerical solution:

$x \approx 2.98641543853585...$

2.98641543853585…

4Pi*sqrt(2.98641543853585)-27/4*ln(2.98641543853585)+15/2 ln(2) – 675/(32Pi*sqrt(2.98641543853585)) – 675/(2^8*Pi^2*(2.98641543853585))

Input interpretation:

$$4\pi\sqrt{2.98641543853585} - \frac{27}{4}\log(2.98641543853585) + \frac{15}{2}\log(2) - \frac{675}{32\pi\sqrt{2.98641543853585}} - \frac{675}{2^8\pi^2 \times 2.98641543853585}$$

log(x) is the natural logarithm

Result:

15.5550677559998...

15.5550677559998… result very near to the black hole entropy 15.6730

Alternative representations:

$$4\pi\sqrt{2.98641543853585000} - \frac{1}{4}\log(2.98641543853585000)\,27 + \frac{1}{2}\log(2)\,15 - \frac{675}{32\pi\sqrt{2.98641543853585000}} - \frac{675}{2^8\pi^2\,2.98641543853585000} = \frac{15}{2}\log(a)\log_a(2) - \frac{27}{4}\log(a)\log_a(2.98641543853585000) - \frac{675}{2.98641543853585000\times 2^8\pi^2} - \frac{675}{32\pi\sqrt{2.98641543853585000}} + 4\pi\sqrt{2.98641543853585000}$$

$$4\pi\sqrt{2.986415438535850000}-\frac{1}{4}\log(2.986415438535850000)\,27+\frac{1}{2}\log(2)\,15-\frac{675}{32\pi\sqrt{2.986415438535850000}}-\frac{675}{2^8\pi^2\,2.986415438535850000}=$$

$$\frac{15\log_e(2)}{2}-\frac{27\log_e(2.986415438535850000)}{4}-\frac{675}{2.986415438535850000\times 2^8\pi^2}-\frac{675}{32\pi\sqrt{2.986415438535850000}}+4\pi\sqrt{2.986415438535850000}$$

$$4\pi\sqrt{2.986415438535850000}-\frac{1}{4}\log(2.986415438535850000)\,27+\frac{1}{2}\log(2)\,15-\frac{675}{32\pi\sqrt{2.986415438535850000}}-\frac{675}{2^8\pi^2\,2.986415438535850000}=$$

$$15\coth^{-1}(3)-\frac{54}{4}\coth^{-1}\left(\frac{3.986415438535850000}{1.986415438535850000}\right)-\frac{675}{2.986415438535850000\times 2^8\pi^2}-\frac{675}{32\pi\sqrt{2.986415438535850000}}+4\pi\sqrt{2.986415438535850000}$$

Series representations:

$$4\pi\sqrt{2.986415438535850000}-\frac{1}{4}\log(2.986415438535850000)\,27+\frac{1}{2}\log(2)\,15-\frac{675}{32\pi\sqrt{2.986415438535850000}}-\frac{675}{2^8\pi^2\,2.986415438535850000}=$$

$$\left(4.0000000000000000\left(-5.2734375000000000\pi-0.2207260513705273531\sqrt{1.986415438535850000}\sum_{k=0}^{\infty}e^{-0.686331727299946687k}\binom{\frac{1}{2}}{k}+1.8750000000000000\pi^2\log(2)\sqrt{1.986415438535850000}\sum_{k=0}^{\infty}e^{-0.686331727299946687k}\binom{\frac{1}{2}}{k}-1.6875000000000000\pi^2\log(2.986415438535850000)\sqrt{1.986415438535850000}\sum_{k=0}^{\infty}e^{-0.686331727299946687k}\binom{\frac{1}{2}}{k}+1.0000000000000000\pi^3\sqrt{1.986415438535850000}^2\left(\sum_{k=0}^{\infty}e^{-0.686331727299946687k}\binom{\frac{1}{2}}{k}\right)^2\right)\right)\Big/\left(\pi^2\sqrt{1.986415438535850000}\sum_{k=0}^{\infty}e^{-0.686331727299946687k}\binom{\frac{1}{2}}{k}\right)$$

$$4\pi\sqrt{2.986415438535850000} - \frac{1}{4}\log(2.986415438535850000)\,27 + \frac{1}{2}\log(2)\,15 - \frac{675}{32\pi\sqrt{2.986415438535850000}} - \frac{675}{2^8\pi^2\,2.986415438535850000} =$$

$$\Bigg(4.0000000000000000\Bigg(-5.2734375000000000\,\pi - 0.2207260513705273531\exp\left(i\pi\left\lfloor\frac{\arg(2.986415438535850000-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2.986415438535850000-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!} + 1.8750000000000000\,\pi^2\exp\left(i\pi\left\lfloor\frac{\arg(2.986415438535850000-x)}{2\pi}\right\rfloor\right)\log(2)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2.986415438535850000-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!} - 1.6875000000000000\,\pi^2\exp\left(i\pi\left\lfloor\frac{\arg(2.986415438535850000-x)}{2\pi}\right\rfloor\right)\log(2.986415438535850000)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2.986415438535850000-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!} + 1.0000000000000000\,\pi^3\exp^2\left(i\pi\left\lfloor\frac{\arg(2.986415438535850000-x)}{2\pi}\right\rfloor\right)\sqrt{x}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2.986415438535850000-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2\Bigg)\Bigg)\Bigg/\left(\pi^2\exp\left(i\pi\left\lfloor\frac{\arg(2.986415438535850000-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2.986415438535850000-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$4\pi\sqrt{2.986415438535850000} - \frac{1}{4}\log(2.986415438535850000)\,27 + \frac{1}{2}\log(2)\,15 - \frac{675}{32\pi\sqrt{2.986415438535850000}} - \frac{675}{2^8\pi^2\, 2.986415438535850000} =$$

$$\Bigg(4.0000000000000000\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2.986415438535850000-z_0)/(2\pi)\rfloor} z_0^{-1-1/2\lfloor\arg(2.986415438535850000-z_0)/(2\pi)\rfloor}\Bigg(-5.2734375000000000\,\pi\sqrt{z_0} - 0.220726051370527353\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2.986415438535850000-z_0)/(2\pi)\rfloor} z_0^{1+1/2\lfloor\arg(2.986415438535850000-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2.986415438535850000-z_0)^k z_0^{-k}}{k!} + 1.87500000000000000\,\pi^2\log(2)\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2.986415438535850000-z_0)/(2\pi)\rfloor} z_0^{1+1/2\lfloor\arg(2.986415438535850000-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2.986415438535850000-z_0)^k z_0^{-k}}{k!} - 1.68750000000000000\,\pi^2\log(2.986415438535850000)\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2.986415438535850000-z_0)/(2\pi)\rfloor} z_0^{1+1/2\lfloor\arg(2.986415438535850000-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2.986415438535850000-z_0)^k z_0^{-k}}{k!} + 1.000000000000000000\,\pi^3\left(\frac{1}{z_0}\right)^{\lfloor\arg(2.986415438535850000-z_0)/(2\pi)\rfloor} z_0^{3/2+\lfloor\arg(2.986415438535850000-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2.986415438535850000-z_0)^k z_0^{-k}}{k!}\right)^2\Bigg)\Bigg) \Big/ \left(\pi^2\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2.986415438535850000-z_0)^k z_0^{-k}}{k!}\right)$$

Integral representations:

$$4\pi\sqrt{2.986415438535850000} - \frac{1}{4}\log(2.986415438535850000)\,27 + \frac{1}{2}\log(2)\,15 - \frac{675}{32\pi\sqrt{2.986415438535850000}} - \frac{675}{2^8\pi^2\,2.986415438535850000} = -\frac{0.8829042054821094122}{\pi^2} + \int_1^2 \frac{3.72435475756868 - 0.750000000000000\,t}{0.49658063434249108\,t - 1.0000000000000000\,t^2}\,dt - \frac{675}{32\pi\sqrt{2.986415438535850000}} + 4\pi\sqrt{2.986415438535850000}$$

$$4\pi\sqrt{2.986415438535850000} - \frac{1}{4}\log(2.986415438535850000)\,27 + \frac{1}{2}\log(2)\,15 - \frac{675}{32\pi\sqrt{2.986415438535850000}} - \frac{675}{2^8\pi^2\,2.986415438535850000} = -\frac{0.882904205482109412}{\pi^2} + \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1}{i\pi\,\Gamma(1-s)}\,3.7500000000000\,e^{-0.686331727299946687\,s}\left(-0.90000000000000 + 1.00000000000000\,e^{0.686331727299946687\,s}\right)\Gamma(-s)^2\,\Gamma(1+s)\,ds - \frac{21.093750000000000000}{\pi\sqrt{2.986415438535850000}} + 4.000000000000000000\,\pi\sqrt{2.986415438535850000} \quad \text{for } -1 < \gamma < 0$$

Or, approximating 2.986415… to 3, we obtain:

Input:

$$4\pi\sqrt{3} - \frac{27}{4}\log(3) + \frac{15}{2}\log(2) - \frac{675}{32\pi\sqrt{3}} - \frac{675}{2^8\pi^2\times 3}$$

log(x) is the natural logarithm

Exact result:

$$-\frac{225}{256\pi^2} - \frac{225\sqrt{3}}{32\pi} + 4\sqrt{3}\,\pi + \frac{15\log(2)}{2} - \frac{27\log(3)}{4}$$

Decimal approximation:

15.58298015900533983078066302769040347414832695013152123021…

15.582980159… as above

Alternate forms:

$$\frac{8\sqrt{3}\,\pi\left(128\,\pi^2-225\right)-225}{256\,\pi^2}+\frac{3}{4}\left(\log(1024)-9\log(3)\right)$$

$$\frac{-225-1800\sqrt{3}\,\pi+1024\sqrt{3}\,\pi^3+1920\,\pi^2\log(2)-1728\,\pi^2\log(3)}{256\,\pi^2}$$

$$-\frac{225}{256\,\pi^2}-\frac{225\sqrt{3}}{32\,\pi}+4\sqrt{3}\,\pi+\frac{3}{4}\left(10\log(2)-9\log(3)\right)$$

Alternative representations:

$$4\pi\sqrt{3}-\frac{1}{4}\log(3)\,27+\frac{1}{2}\log(2)\,15-\frac{675}{32\pi\sqrt{3}}-\frac{675}{2^8\,\pi^2\,3}=\frac{15}{2}\log(a)\log_a(2)-\frac{27}{4}\log(a)\log_a(3)-\frac{675}{3\times 2^8\,\pi^2}-\frac{675}{32\pi\sqrt{3}}+4\pi\sqrt{3}$$

$$4\pi\sqrt{3}-\frac{1}{4}\log(3)\,27+\frac{1}{2}\log(2)\,15-\frac{675}{32\pi\sqrt{3}}-\frac{675}{2^8\,\pi^2\,3}=\frac{15\log_e(2)}{2}-\frac{27\log_e(3)}{4}-\frac{675}{3\times 2^8\,\pi^2}-\frac{675}{32\pi\sqrt{3}}+4\pi\sqrt{3}$$

$$4\pi\sqrt{3}-\frac{1}{4}\log(3)\,27+\frac{1}{2}\log(2)\,15-\frac{675}{32\pi\sqrt{3}}-\frac{675}{2^8\,\pi^2\,3}=15\coth^{-1}(3)-\frac{54}{4}\coth^{-1}(2)-\frac{675}{3\times 2^8\,\pi^2}-\frac{675}{32\pi\sqrt{3}}+4\pi\sqrt{3}$$

Series representations:

$$4\pi\sqrt{3}-\frac{1}{4}\log(3)\,27+\frac{1}{2}\log(2)\,15-\frac{675}{32\pi\sqrt{3}}-\frac{675}{2^8\,\pi^2\,3}=-\frac{225}{256\,\pi^2}-\frac{225\sqrt{3}}{32\,\pi}+4\sqrt{3}\,\pi+\frac{3}{2}\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\frac{3\log(z_0)}{4}+\sum_{k=1}^{\infty}-\frac{3(-1)^k\left(10(2-z_0)^k-9(3-z_0)^k\right)z_0^{-k}}{4k}$$

$$4\pi\sqrt{3}-\frac{1}{4}\log(3)\,27+\frac{1}{2}\log(2)\,15-\frac{675}{32\pi\sqrt{3}}-\frac{675}{2^8\,\pi^2\,3}=-\frac{225}{256\,\pi^2}-\frac{225\sqrt{3}}{32\,\pi}+4\sqrt{3}\,\pi+15\,i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor-\frac{27}{2}\,i\,\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor+\frac{3\log(x)}{4}+\sum_{k=1}^{\infty}-\frac{3(-1)^k\left(10(2-x)^k-9(3-x)^k\right)x^{-k}}{4k}\quad\text{for } x<0$$

$$4\pi\sqrt{3} - \frac{1}{4}\log(3)\,27 + \frac{1}{2}\log(2)\,15 - \frac{675}{32\pi\sqrt{3}} - \frac{675}{2^8\pi^2\,3} = -\frac{225}{256\pi^2} - \frac{225\sqrt{3}}{32\pi} + 4\sqrt{3}\,\pi + \frac{15}{2}\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right) - \frac{27}{4}\left\lfloor\frac{\arg(3-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right) + \frac{3\log(z_0)}{4} + \frac{15}{2}\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log(z_0) - \frac{27}{4}\left\lfloor\frac{\arg(3-z_0)}{2\pi}\right\rfloor\log(z_0) + \sum_{k=1}^{\infty} -\frac{3(-1)^k\left(10(2-z_0)^k - 9(3-z_0)^k\right)z_0^{-k}}{4k}$$

Integral representations:

$$4\pi\sqrt{3} - \frac{1}{4}\log(3)\,27 + \frac{1}{2}\log(2)\,15 - \frac{675}{32\pi\sqrt{3}} - \frac{675}{2^8\pi^2\,3} = -\frac{225}{256\pi^2} - \frac{225\sqrt{3}}{32\pi} + 4\sqrt{3}\,\pi + \int_1^2 \frac{15-3t}{2t-4t^2}\,dt$$

$$4\pi\sqrt{3} - \frac{1}{4}\log(3)\,27 + \frac{1}{2}\log(2)\,15 - \frac{675}{32\pi\sqrt{3}} - \frac{675}{2^8\pi^2\,3} = -\frac{225}{256\pi^2} - \frac{225\sqrt{3}}{32\pi} + 4\sqrt{3}\,\pi + \int_{-i\infty+\gamma}^{i\infty+\gamma} -\frac{3\,i\,2^{-3-s}\left(-9+5\times 2^{1+s}\right)\Gamma(-s)^2\,\Gamma(1+s)}{\pi\,\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

[exp(((4Pi*sqrt(3)-27/4*ln(3)+15/2 ln(2) – 675/(32Pi*sqrt(3)) – 675/(2^8*Pi^2*(3)))))]^1/32-(Pi^2)/1024

Input:

$$\sqrt[32]{\exp\left(4\pi\sqrt{3} - \frac{27}{4}\log(3) + \frac{15}{2}\log(2) - \frac{675}{32\pi\sqrt{3}} - \frac{675}{2^8\pi^2\times 3}\right)} - \frac{\pi^2}{1024}$$

log(x) is the natural logarithm

Exact result:

$$\frac{2^{15/64}\,e^{-\frac{225}{8192\pi^2} - \frac{225\sqrt{3}}{1024\pi} + \frac{\sqrt{3}\,\pi}{8}}}{3^{27/128}} - \frac{\pi^2}{1024}$$

Decimal approximation:

1.617736458520499083675384422892781611719341332788717026252...

1.6177364585…

Note that we have subtracted $\pi^2 / 1024$, where $1024 = 64^2 / 4$ and concerning the fundamental Ramanujan's paper "Modular equations and Approximations to π". (Antonio Nardelli)

Alternate forms:

$$\frac{2^{15/64}\, e^{-\left(225+8\sqrt{3}\,\pi\left(225-128\pi^2\right)\right)/\left(8192\pi^2\right)}}{3^{27/128}} - \frac{\pi^2}{1024}$$

$$\frac{2^{15/64}\, e^{-\left(225+1800\sqrt{3}\,\pi-1024\sqrt{3}\,\pi^3\right)/\left(8192\pi^2\right)}}{3^{27/128}} - \frac{\pi^2}{1024}$$

$$\frac{1024\times 2^{15/64}\, e^{-\frac{225}{8192\pi^2}-\frac{225\sqrt{3}}{1024\pi}+\frac{\sqrt{3}\,\pi}{8}} - 3^{27/128}\,\pi^2}{1024\times 3^{27/128}}$$

Alternative representations:

$$\sqrt[32]{\exp\left(4\pi\sqrt{3} - \frac{1}{4}\log(3)\,27 + \frac{1}{2}\log(2)\,15 - \frac{675}{32\pi\sqrt{3}} - \frac{675}{2^8\pi^2\,3}\right)} - \frac{\pi^2}{1024} = -\frac{\pi^2}{1024} + \sqrt[32]{\exp\left(\frac{15\log_e(2)}{2} - \frac{27\log_e(3)}{4} - \frac{675}{3\times 2^8\pi^2} - \frac{675}{32\pi\sqrt{3}} + 4\pi\sqrt{3}\right)}$$

$$\sqrt[32]{\exp\left(4\pi\sqrt{3} - \frac{1}{4}\log(3)\,27 + \frac{1}{2}\log(2)\,15 - \frac{675}{32\pi\sqrt{3}} - \frac{675}{2^8\pi^2\,3}\right)} - \frac{\pi^2}{1024} = -\frac{\pi^2}{1024} + \sqrt[32]{\exp\left(\frac{15}{2}\log(a)\log_a(2) - \frac{27}{4}\log(a)\log_a(3) - \frac{675}{3\times 2^8\pi^2} - \frac{675}{32\pi\sqrt{3}} + 4\pi\sqrt{3}\right)}$$

Integral representations:

$$\sqrt[32]{\exp\left(4\pi\sqrt{3} - \frac{1}{4}\log(3)\,27 + \frac{1}{2}\log(2)\,15 - \frac{675}{32\pi\sqrt{3}} - \frac{675}{2^8\pi^2\,3}\right)} - \frac{\pi^2}{1024} = -\frac{\pi^2}{1024} + \sqrt[32]{\exp\left(-\frac{225}{256\pi^2} + \int_1^2 \frac{15-3t}{2t-4t^2}\,dt - \frac{675}{32\pi\sqrt{3}} + 4\pi\sqrt{3}\right)}$$

$$\sqrt[32]{\exp\left(4\pi\sqrt{3} - \frac{1}{4}\log(3)\,27 + \frac{1}{2}\log(2)\,15 - \frac{675}{32\pi\sqrt{3}} - \frac{675}{2^8\pi^2\,3}\right)} - \frac{\pi^2}{1024} = -\frac{\pi^2}{1024} + \sqrt[32]{\exp\left(\frac{-225 + \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{3\times 2^{5-s}\left(-9+5\times 2^{1+s}\right)\pi\,\Gamma(-s)^2\,\Gamma(1+s)}{i\,\Gamma(1-s)}\,ds - \frac{5400\pi}{\sqrt{3}} + 1024\pi^3\sqrt{3}}{256\pi^2}\right)}$$

for $-1 < \gamma < 0$

The micro-states correspond to open strings attached to the D2 and D6 branes, in the background of the NS5-branes. In the limit where $Y \times S_1'$ is very small, they may be described by a two-dimensional field theory extending along the time and S_1 direction. In the absence of the NS5-branes, the open strings are described at low energy by $U(Q_2) \times U(Q_6)$ gauge bosons together with bi-fundamental matter, which is known to flow to a CFT with central charge $c = 6Q_2Q_6$ in the infrared (see [34] for a detailed analysis of this point). In the presence of the NS5-branes, localized at

Q_5 points along S_1', the D2-branes generally break at the points where they intersect the NS5-branes. This effectively leads to Q_5Q_2 independent D2-branes, hence a CFT with central charge $c_{\text{eff}} = 6Q_2Q_5Q_6$. The extremal micro-states correspond to the right-moving ground states of that field theory, with N units of left-moving momentum along S_1. By the Ramanujan-Hardy formula (Eq. (6.18) below), also known as the Cardy formula in the physics literature, the number of states carrying N units of momentum grows exponentially as

$$\Omega(Q_2, Q_5, Q_6, N) \sim \exp\left[2\pi\sqrt{\frac{c_{\text{eff}}}{6}N}\right] \sim \exp\left[2\pi\sqrt{Q_2\, Q_5\, Q_6\, N}\right] \tag{2.17}$$

central charge $c = 3\ell/2G = 24k$

N = 2.98641543853585+1

exp(2Pi*sqrt(24/6*3.98641543853585))

Input interpretation:

$$\exp\left(2\pi\sqrt{\frac{24}{6}\times 3.98641543853585}\right)$$

Result:

$7.8788097398245\ldots \times 10^{10}$

7.8788097398245…*10^10

From which:

ln((exp(2Pi*sqrt(24/6*3.98641543853585))))

Input interpretation:

$$\log\left(\exp\left(2\pi\sqrt{\frac{24}{6}\times 3.98641543853585}\right)\right)$$

$\log(x)$ is the natural logarithm

Result:

25.0900277741557...

25.0900277741557…

Alternative representations:

$$\log\left(\exp\left(2\pi\sqrt{\frac{3.98641543853585850000\times 24}{6}}\right)\right) = \log_e\left(\exp\left(2\pi\sqrt{\frac{95.67397052486040000}{6}}\right)\right)$$

$$\log\left(\exp\left(2\pi\sqrt{\frac{3.98641543853585850000\times 24}{6}}\right)\right) = \log(a)\log_a\left(\exp\left(2\pi\sqrt{\frac{95.67397052486040000}{6}}\right)\right)$$

Series representation:

$$\log\left(\exp\left(2\pi\sqrt{\frac{3.98641543853585850000\times 24}{6}}\right)\right) = \log\left(-1+\exp\left(2\pi\sqrt{15.94566175414340000}\right)\right) - \sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\exp\left(2\pi\sqrt{15.94566175414340000}\right)\right)^{-k}}{k}$$

Integral representations:

$$\log\left(\exp\left(2\pi\sqrt{\frac{3.98641543853585850000\times 24}{6}}\right)\right) = \int_1^{\exp\left(2\pi\sqrt{15.94566175414340000}\right)}\frac{1}{t}\,dt$$

$$\log\left(\exp\left(2\pi\sqrt{\frac{3.986415438535850000\times 24}{6}}\right)\right) = \frac{1}{2 i \pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\left(-1+\exp\left(2\pi\sqrt{15.94566175414340000}\right)\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \text{ for } -1<\gamma<0$$

((exp(2Pi*sqrt(24/6*3.98641543853585))))^1/53+(4Pi)/10^3

Input interpretation:

$$\sqrt[53]{\exp\left(2\pi\sqrt{\frac{24}{6}\times 3.98641543853585}\right)} + \frac{4\pi}{10^3}$$

Result:

1.618004585594071...

1.618004585594071…

Series representations:

$$\sqrt[53]{\exp\left(2\pi\sqrt{\frac{3.986415438535850000\times 24}{6}}\right)} + \frac{4\pi}{10^3} = \frac{\pi}{250} + \sqrt[53]{\exp\left(2\pi\sqrt{14.94566175414340000}\sum_{k=0}^{\infty} e^{-2.704421074055929769\,k}\binom{\frac{1}{2}}{k}\right)}$$

$$\sqrt[53]{\exp\left(2\pi\sqrt{\frac{3.986415438535850000\times 24}{6}}\right)} + \frac{4\pi}{10^3} = \frac{\pi}{250} + \sqrt[53]{\exp\left(2\pi\sqrt{14.94566175414340000}\sum_{k=0}^{\infty}\frac{(-0.0669090480200897795 9)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}$$

$$\sqrt[53]{\exp\left(2\pi\sqrt{\frac{3.986415438535850000\times 24}{6}}\right)} + \frac{4\pi}{10^3} = \frac{\pi}{250} + \sqrt[53]{\exp\left(\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} e^{-2.704421074055929769\,s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}\right)}$$

Or, for N = (-4.43505 +4.51374 i)

exp(2Pi*sqrt(24/6*(-4.43505 +4.51374 i)))

Input interpretation:

$$\exp\left(2\pi\sqrt{\frac{24}{6}\times(-4.43505+4.51374\,i)}\right)$$

i is the imaginary unit

Result:

$-1.30349\ldots\times10^5 -$
$1.56804\ldots\times10^5\,i$

Polar coordinates:

$r = 203907.$ (radius), $\theta = -129.736°$ (angle)

203907

(((exp(2Pi*sqrt(24/6*(-4.43505 +4.51374 i))))))^1/25-(4Pi)/10^3

Input interpretation:

$$\sqrt[25]{\exp\left(2\pi\sqrt{\frac{24}{6}\times(-4.43505+4.51374\,i)}\right)}-\frac{4\pi}{10^3}$$

i is the imaginary unit

Result:

$1.61146\ldots -$
$0.147497\ldots\,i$

Polar coordinates:

$r = 1.6182$ (radius), $\theta = -5.22969°$ (angle)

1.6182

Series representations:

$$\sqrt[25]{\exp\left(2\pi\sqrt{\frac{1}{6}(-4.43505+4.51374\,i)\,24}\right)} - \frac{4\pi}{10^3} = -\frac{\pi}{250} + \sqrt[25]{\exp\left(2\pi\sqrt{-18.7402+18.055\,i}\sum_{k=0}^{\infty}(-18.7402+18.055\,i)^{-k}\binom{\frac{1}{2}}{k}\right)}$$

$$\sqrt[25]{\exp\left(2\pi\sqrt{\frac{1}{6}(-4.43505+4.51374\,i)\,24}\right)} - \frac{4\pi}{10^3} = -\frac{\pi}{250} + \sqrt[25]{\exp\left(2\pi\sqrt{-18.7402+18.055\,i}\sum_{k=0}^{\infty}\frac{(-1)^k(-18.7402+18.055\,i)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}$$

$$\sqrt[25]{\exp\left(2\pi\sqrt{\frac{1}{6}(-4.43505+4.51374\,i)\,24}\right)} - \frac{4\pi}{10^3} = -\frac{\pi}{250} + \sqrt[25]{\exp\left(2\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(-17.7402+18.055\,i-z_0)^k z_0^{-k}}{k!}\right)}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.
In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64 g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276 e^{-\pi\sqrt{22}} - \cdots, \\ 64 g_{22}^{-24} &= \qquad\qquad 4096 e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372 e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64 g_{22}^{-24} = \qquad 4096 e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372 e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.

Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a *factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals*

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

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References

RAMANUJAN

TWELVE LECTURES ON
SUBJECTS SUGGESTED BY HIS LIFE AND WORK

BY

G. H. HARDY

Sadleirian Professor of Pure Mathematics in the University of Cambridge

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