On some Ramanujan's Nested Radicals: mathematical connections with ϕ, $\zeta(2)$, and various parameters of Cosmology and Particle Physics.

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Abstract

In this paper we have described and analyzed some Ramanujan's Nested Radicals. Furthermore, we have obtained various mathematical connections with ϕ, $\zeta(2)$, and several parameters of Cosmology and Particle Physics.

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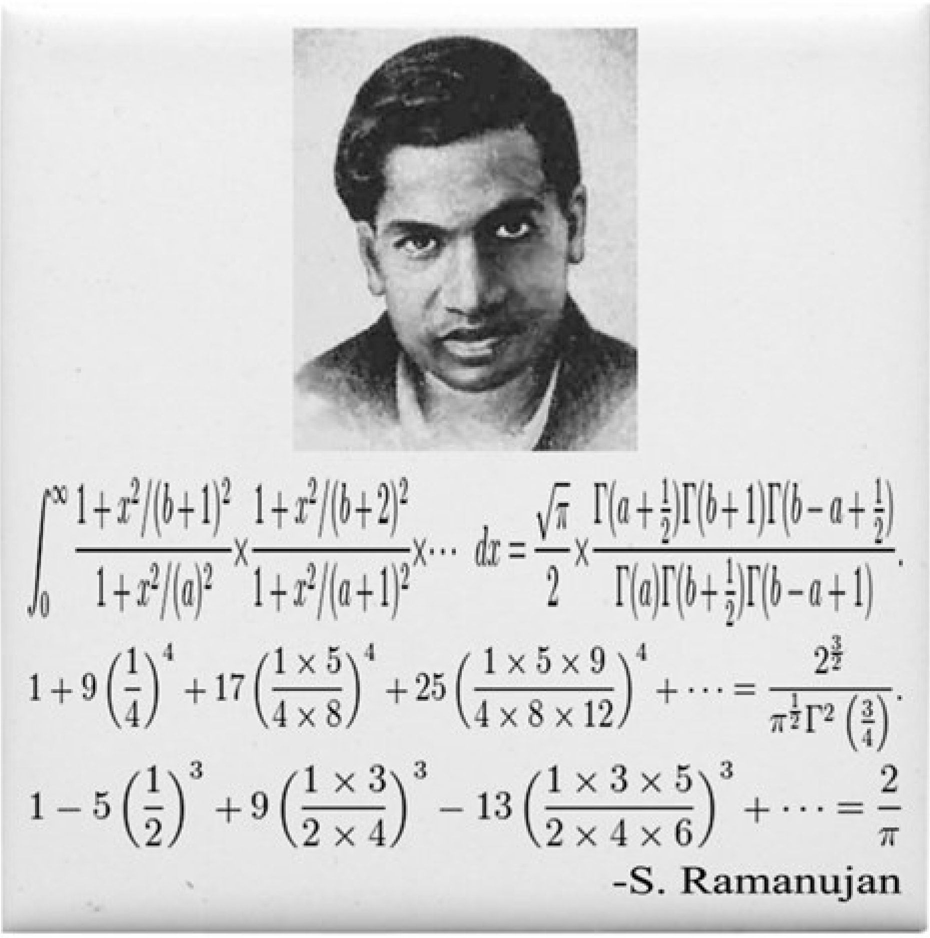
From:

https://www.cafepress.com/+ramanujan_and_his_equations_tile_coaster,1118423143



https://spiritoframanujan.com/elementor-popup/sor-logo-explanation/

$$3=\sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+\cdots}}}}$$

Sqrt((((1+2sqrt(((1+3sqrt((1+4sqrt(1+5sqrt(1+6(sqrt(1+7sqrt(1+8sqrt(1+9sqrt(1+14sqrt(1+19sqrt(1+26))))))))))))))))

Input:

$$\sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+5\sqrt{1+6\sqrt{1+7\sqrt{1+8\sqrt{1+9\sqrt{1+14\sqrt{1+19\sqrt{1+26}}}}}}}}}}}$$

Exact result:

$$\sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+5\sqrt{1+6\sqrt{1+7\sqrt{1+8\sqrt{1+9\sqrt{1+14\sqrt{1+57\sqrt{3}}}}}}}}}}}$$

Decimal approximation:

3.000652179457976056768846201107765783832522692312596263702...

3.0006521794…

Alternate form:

$$\sqrt{\left(1+2\sqrt{\left(1+3\sqrt{\left(1+4\sqrt{\left(1+5\sqrt{\left(1+6\sqrt{\left(1+7\sqrt{\left(1+8\sqrt{\left(1+\frac{1}{\sqrt[4]{2}}9\sqrt{\left(14\sqrt{1-i\sqrt{9746}}+\sqrt{2}\left(1+7\sqrt{2i\left(\sqrt{9746}+-i\right)}\right)\right)}\right)}\right)}\right)}\right)}\right)}\right)}\right)}\right)}$$

10^3+((([sqrt(1+2sqrt(1+3sqrt(1+4sqrt(1+5sqrt(1+6sqrt(1+7sqrt(1+8sqrt(1+9sqrt(1+14sqrt(1+19sqrt(1+26)])))))))))))))^6-1/golden ratio

Input:

$$10^3 + \left(\sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+5\sqrt{1+6\sqrt{1+7\sqrt{1+8\sqrt{1+9\sqrt{1+14\sqrt{1+19\sqrt{1+26}}}}}}}}}}}\right)^6 - \frac{1}{\phi}$$

ϕ is the golden ratio

Decimal approximation:

1729.333360596523026837844259532392509477883343763566915218...

1729.3333605965…

https://curiosamathematica.tumblr.com/post/116792561352/one-of-the-abundance-of-beautiful-formulas

$$\sqrt{1+1\cdot\sqrt{1+2\cdot\sqrt{1+3\cdot\sqrt{1+4\cdot\sqrt{1+5\cdot\sqrt{\cdots}}}}}} \longrightarrow 2$$

[[[[sqrt(1+1sqrt(1+2sqrt(1+3sqrt(1+4sqrt(1+5sqrt(1+6sqrt(1+7sqrt(1+8sqrt(1+9sqrt(1+14sqrt(1+19sqrt(1+26)]]]]

Input:

$$\sqrt{1+1\sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+5\sqrt{1+6\sqrt{1+7\sqrt{1+8\sqrt{1+9\sqrt{1+14\sqrt{1+19\sqrt{1+26}}}}}}}}}}}}$$

Exact result:

$$\sqrt{1+\sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+5\sqrt{1+6\sqrt{1+7\sqrt{1+8\sqrt{1+9\sqrt{1+14\sqrt{1+57\sqrt{3}}}}}}}}}}}}$$

Decimal approximation:

2.000163038219128790020373522793788581407738969817464091803...

2.000163038219...

Alternate form:

$$\sqrt{1+\sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+5\sqrt{1+6\sqrt{1+7\sqrt{1+8\sqrt{1+\frac{1}{\sqrt[4]{2}}9\sqrt{14\sqrt{1-i\sqrt{9746}}+\sqrt{2}\left(1+7\sqrt{2i\left(\sqrt{9746}+-i\right)}\right)}}}}}}}}}}$$

1/2+27[[[[sqrt(1+1sqrt(1+2sqrt(1+3sqrt(1+4sqrt(1+5sqrt(1+6sqrt(1+7sqrt(1+8sqrt(1+9sqrt(1+14sqrt(1+19sqrt(1+26))))))))))))]]]]^6

Input:

$$\frac{1}{2}+27\sqrt{\left(1+1\sqrt{\left(1+2\sqrt{\left(1+3\sqrt{\left(1+4\sqrt{\left(1+5\sqrt{\left(1+6\sqrt{\left(1+7\sqrt{\left(1+8\sqrt{1+9\sqrt{1+14\sqrt{1+19\sqrt{1+26}}}}\right)}\right)}\right)}\right)}\right)}\right)}\right)}\right)^6}$$

Exact result:

$$\frac{1}{2}+27\left(1+\sqrt{\left(1+2\sqrt{\left(1+3\sqrt{\left(1+4\sqrt{\left(1+5\sqrt{\left(1+6\sqrt{\left(1+7\sqrt{1+8\sqrt{1+9\sqrt{1+14\sqrt{1+57\sqrt{3}}}}}\right)}\right)}\right)}\right)}\right)}\right)}\right)^3$$

Decimal approximation:

1729.345362394553392967575467370089140613967694200606639172...

1729.345362394…

1/2-Pi+2[[[[sqrt(1+1sqrt(1+2sqrt(1+3sqrt(1+4sqrt(1+5sqrt(1+6sqrt(1+7sqrt(1+8sqrt(1+9sqrt(1+14sqrt(1+19sqrt(1+26)))))))))))]]]]^6

Input:

$$\frac{1}{2}-\pi+2\sqrt{\left(1+1\sqrt{\left(1+2\sqrt{\left(1+3\sqrt{\left(1+4\sqrt{\left(1+5\sqrt{\left(1+6\sqrt{\left(1+7\sqrt{\left(1+8\sqrt{1+9\sqrt{1+14\sqrt{1+19\sqrt{1+26}}}}\right)}\right)}\right)}\right)}\right)}\right)}\right)}\right)}^6$$

Exact result:

$$\frac{1}{2}+2\left(1+\sqrt{\left(1+2\sqrt{\left(1+3\sqrt{\left(1+4\sqrt{\left(1+5\sqrt{\left(1+6\sqrt{\left(1+7\sqrt{1+8\sqrt{1+9\sqrt{1+14\sqrt{1+57\sqrt{3}}}}}\right)}\right)}\right)}\right)}\right)}\right)}\right)^3-\pi$$

Decimal approximation:

125.421026783043791425802206051541914939059696837706867451 0...

125.421026783…

Property:

$$\frac{1}{2}+2\left(1+\sqrt{\left(1+2\sqrt{\left(1+3\sqrt{\left(1+4\sqrt{\left(1+5\sqrt{\left(1+6\sqrt{\left(1+7\sqrt{1+8\sqrt{1+9\sqrt{1+14\sqrt{1+57\sqrt{3}}}}}\right)}\right)}\right)}\right)}\right)}\right)}\right)^{3}-\pi$$

is a transcendental number

Alternate forms:

$$\frac{17}{2} + 12\sqrt{1+3\sqrt{1+4\sqrt{1+5\sqrt{1+6\sqrt{1+7\sqrt{1+8\sqrt{1+9\sqrt{1+14\sqrt{1+57\sqrt{3}}}}}}}}}} + 6\sqrt{\left(1+2\sqrt{\left(1+3\sqrt{\left(1+4\sqrt{\left(1+5\sqrt{\left(1+6\sqrt{\left(1+7\sqrt{1+8\sqrt{1+9\sqrt{1+14\sqrt{1+57\sqrt{3}}}}}\right)}\right)}\right)}\right)}\right)}\right)} \Bigg) + 2\left(1+2\sqrt{\left(1+3\sqrt{\left(1+4\sqrt{\left(1+5\sqrt{\left(1+6\sqrt{\left(1+7\sqrt{1+8\sqrt{1+9\sqrt{1+14\sqrt{1+57\sqrt{3}}}}}\right)}\right)}\right)}\right)}\right)}\right)^{3/2} - \pi$$

$$\frac{1}{2}\left(17+24\sqrt{1+3\sqrt{1+4\sqrt{1+5\sqrt{1+6\sqrt{1+7\sqrt{1+8\sqrt{1+9\sqrt{1+14\sqrt{1+57\sqrt{3}}}}}}}}}}\right.$$

$$+12\sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+5\sqrt{1+6\sqrt{1+7\sqrt{1+8\sqrt{1+9\sqrt{1+14\sqrt{1+57\sqrt{3}}}}}}}}}}}$$

$$\Bigg)+$$

$$4\left(1+2\sqrt{1+3\sqrt{1+4\sqrt{1+5\sqrt{1+6\sqrt{1+7\sqrt{1+8\sqrt{1+9\sqrt{1+14\sqrt{1+57\sqrt{3}}}}}}}}}}\right)^{3/2}-2$$

$$\left.\pi\right)$$

$$\frac{1}{2}\left(17+24\sqrt{1+3\sqrt{1+4\sqrt{1+5\sqrt{1+6\sqrt{1+7\sqrt{1+8\sqrt{1+\frac{1}{\sqrt[4]{2}}9\sqrt{14\sqrt{1-i\sqrt{9746}}+\sqrt{2}\left(1+7\sqrt{2i\left(\sqrt{9746}+-i\right)}\right)}}}}}}}}+\right.$$

$$12\sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+5\sqrt{1+6\sqrt{1+7\sqrt{1+8\sqrt{1+\frac{1}{\sqrt[4]{2}}9\sqrt{14\sqrt{1-i\sqrt{9746}}+\sqrt{2}\left(1+7\sqrt{2i\left(\sqrt{9746}+-i\right)}\right)}}}}}}}}}+$$

$$\left.4\left(1+2\sqrt{1+3\sqrt{1+4\sqrt{1+5\sqrt{1+6\sqrt{1+7\sqrt{1+8\sqrt{1+\frac{1}{\sqrt[4]{2}}9\sqrt{14\sqrt{1-i\sqrt{9746}}+\sqrt{2}\left(1+7\sqrt{2i\left(\sqrt{9746}+-i\right)}\right)}}}}}}}}\right)^{3/2}-2\pi\right)$$

11+1/2+2[[[[sqrt(1+1sqrt(1+2sqrt(1+3sqrt(1+4sqrt(1+5sqrt(1+6sqrt(1+7sqrt(1+8sqrt(1+9sqrt(1+14sqrt(1+19sqrt(1+26)))))))))))))]]]]^6

Input:

$$11+\frac{1}{2}+2\sqrt{1+1\sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+5\sqrt{1+6\sqrt{1+7\sqrt{1+8\sqrt{1+9\sqrt{1+14\sqrt{1+19\sqrt{1+26}}}}}}}}}}}^{6}$$

Exact result:

$$\frac{23}{2}+2\left(1+\sqrt{\left(1+2\sqrt{\left(1+3\sqrt{\left(1+4\sqrt{\left(1+5\sqrt{\left(1+6\sqrt{\left(1+7\sqrt{1+8\sqrt{1+9\sqrt{1+14\sqrt{1+57\sqrt{3}}}}}\right)}\right)}\right)}\right)}\right)}\right)}\right)^3$$

Decimal approximation:

139.5626194366335846642648494348214178232568662370819732720…

139.5626194366…

From Wikipedia:

Srinivasa Ramanujan demonstrated a number of curious identities involving nested radicals. Among them are the following:[2]

$$\sqrt[4]{\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}}=\frac{\sqrt[4]{5}+1}{\sqrt[4]{5}-1}=\frac{1}{2}\left(3+\sqrt[4]{5}+\sqrt{5}+\sqrt[4]{125}\right),$$

$$\sqrt{\sqrt[3]{28}-\sqrt[3]{27}}=\frac{1}{3}\left(\sqrt[3]{98}-\sqrt[3]{28}-1\right),$$

$$\sqrt[3]{\sqrt[5]{\frac{32}{5}}-\sqrt[5]{\frac{27}{5}}}=\sqrt[5]{\frac{1}{25}}+\sqrt[5]{\frac{3}{25}}-\sqrt[5]{\frac{9}{25}},$$

$$\sqrt[3]{\sqrt[3]{2}-1}=\sqrt[3]{\frac{1}{9}}-\sqrt[3]{\frac{2}{9}}+\sqrt[3]{\frac{4}{9}}.$$ [3]

Other odd-looking radicals inspired by Ramanujan include:

$$\sqrt[4]{49+20\sqrt{6}}+\sqrt[4]{49-20\sqrt{6}}=2\sqrt{3},$$

$$\sqrt[3]{\left(\sqrt{2}+\sqrt{3}\right)\left(5-\sqrt{6}\right)+3\left(2\sqrt{3}+3\sqrt{2}\right)}=\sqrt{10-\frac{13-5\sqrt{6}}{5+\sqrt{6}}}.$$

Ramanujan stated the following infinite radical denesting in his lost notebook:

$$\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\cdots}}}}}}}=\frac{2+\sqrt{5}+\sqrt{15-6\sqrt{5}}}{2}.$$

The repeating pattern of the signs is $(+,+,-)$.

We have that:

sqrt(5+sqrt(5+sqrt(5-sqrt(5+sqrt(5+sqrt(5-sqrt(5)))))))

Input:

$$\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5}}}}}}}$$

Decimal approximation:

2.7475967194394948322006630808045747333139757810443908805 85...

2.74759671943…

Alternate form:

$$\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\frac{\sqrt{5\sqrt{2}-\sqrt{5-2\sqrt{5}}+\sqrt{5+2\sqrt{5}}}}{\sqrt[4]{2}}}}}}$$

All 2nd roots of 5 + sqrt(5 + sqrt(5 - sqrt(5 + sqrt(5 + sqrt(5 - sqrt(5)))))):

$$\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5}}}}}}}\; e^{0} \approx 2.7476 \text{ (real, principal root)}$$

$$\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5+\sqrt{5+\sqrt{5-\sqrt{5}}}}}}}\; e^{i\pi} \approx -2.7476 \text{ (real root)}$$

1/2(2+sqrt5+(15-6sqrt5)^0.5)

Input:

$$\frac{1}{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)$$

Decimal approximation:

2.7472382749323043330574651861342028267581638787761679877 83...

2.7472382749…

Alternate forms:

$$\frac{1}{2}\left(2+\sqrt{5}+\sqrt{3\left(5-2\sqrt{5}\right)}\right)$$

$$1+\frac{\sqrt{5}}{2}+\frac{1}{2}\sqrt{15-6\sqrt{5}}$$

$$1+\frac{\sqrt{5}}{2}+\frac{1}{2}\sqrt{3\left(5-2\sqrt{5}\right)}$$

Minimal polynomial:

$$x^4-4x^3-4x^2+31x-29$$

From which:

((1/2(2+sqrt5+(15-6sqrt5)^0.5)))^1/2

Input:

$$\sqrt{\frac{1}{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)}$$

Decimal approximation:

1.6574794945737049247404830474067751903476230940183222056 69...

1.657479495…. result very near to the 14th root of the following Ramanujan's class invariant $Q=\left(G_{505}/G_{101/5}\right)^3$ = 1164.2696 i.e. 1.65578...

Alternate form:

$$\sqrt{\frac{1}{2}\left(2+\sqrt{5}+\sqrt{3\left(5-2\sqrt{5}\right)}\right)}$$

Minimal polynomial:

$$x^8-4x^6-4x^4+31x^2-29$$

All 2nd roots of 1/2 (2 + sqrt(5) + sqrt(15 - 6 sqrt(5))):

$$\sqrt{\frac{1}{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)}\, e^{0} \approx 1.6575 \quad \text{(real, principal root)}$$

$$\sqrt{\frac{1}{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)}\, e^{i\pi} \approx -1.6575 \quad \text{(real root)}$$

We have also:

(Pi/2)[6*((((1/2(2+sqrt5+(15-6sqrt5)^0.5)))^1/2))]^3+144+34+5

Input:

$$\frac{\pi}{2}\left(6\sqrt{\frac{1}{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)}\right)^3+144+34+5$$

Exact result:

$$183+27\sqrt{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)^{3/2}\pi$$

Decimal approximation:

1727.963134811810809209576308733239071871987614602680671529...

1727.96313481…

Property:

$183+27\sqrt{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)^{3/2}\pi$ is a transcendental number

Alternate forms:

$$183+27\sqrt{2}\left(2+\sqrt{5}+\sqrt{3\left(5-2\sqrt{5}\right)}\right)^{3/2}\pi$$

$$183+54\sqrt{19+13\sqrt{5}+3\sqrt{6\left(65-19\sqrt{5}\right)}}\,\pi$$

$$3\left(61+9\sqrt{2}\left(2+\sqrt{5}+\sqrt{3\left(5-2\sqrt{5}\right)}\right)^{3/2}\pi\right)$$

Series representations:

$$\frac{1}{2}\left(6\sqrt{\frac{1}{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)}\right)^3\pi+144+34+5=$$

$$3\left(61+9\sqrt{2}\,\pi\left(2+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+\sqrt{3}\sqrt{5-2\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}}\right)^{3/2}\right)$$

$$\frac{1}{2}\left(6\sqrt{\frac{1}{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)}\right)^3\pi+144+34+5=$$

$$3\left(61+9\sqrt{2}\,\pi\left(2+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\sqrt{3}\sqrt{5-2\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)^{3/2}\right)$$

$$\frac{1}{2}\left(6\sqrt{\frac{1}{2}\left(2+\sqrt{5}+\sqrt{15-6\sqrt{5}}\right)}\right)^3\pi+144+34+5=$$

$$3\left(61+9\sqrt{2}\,\pi\left(2+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\sqrt{3}\sqrt{5-2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}\right)^{3/2}\right)$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0$))

Now, we have that:

$$\sqrt{\sqrt[3]{28}-\sqrt[3]{27}}=\frac{1}{3}\left(\sqrt[3]{98}-\sqrt[3]{28}-1\right)$$

((28)^(1/3)-(27)^(1/3))^0.5

Input:

$$\sqrt{\sqrt[3]{28}-\sqrt[3]{27}}$$

Exact result:

$$\sqrt{2^{2/3}\sqrt[3]{7}-3}$$

Decimal approximation:

0.191282440060928016751295506478335098972307207254571910553...

0.19128244…

Alternate form:

$\frac{1}{3}\left(-1-2^{2/3}\sqrt[3]{7}+\sqrt[3]{2}\ 7^{2/3}\right)$

Minimal polynomial:

x^3+x^2+5x-1

All 2nd roots of 2^(2/3) 7^(1/3) - 3:

$\sqrt{2^{2/3}\sqrt[3]{7}-3}\ e^0 \approx 0.191$ (real, principal root)

$\sqrt{2^{2/3}\sqrt[3]{7}-3}\ e^{i\pi} \approx -0.191$ (real root)

1/3((98)^(1/3)-(28)^(1/3)-1)

Input:

$\frac{1}{3}\left(\sqrt[3]{98}-\sqrt[3]{28}-1\right)$

Result:

$\frac{1}{3}\left(-1-2^{2/3}\sqrt[3]{7}+\sqrt[3]{2}\ 7^{2/3}\right)$

Decimal approximation:

0.191282440060928016751295506478335098972307207254571910553...

0.19128244…

Alternate forms:

root of x^3+x^2+5x-1 near $x=0.191282$

$-\frac{1}{3}-\frac{1}{3}\times 2^{2/3}\sqrt[3]{7}+\frac{1}{3}\sqrt[3]{2}\ 7^{2/3}$

Minimal polynomial:

x^3+x^2+5x-1

From

$$\sqrt{\sqrt[3]{28}-\sqrt[3]{27}}$$

we obtain:

27(((12/(((((28)^(1/3)-(27)^(1/3))^0.5)))+golden ratio)))-8-1/2

Input:

$$27\left(\frac{12}{\sqrt{\sqrt[3]{28}-\sqrt[3]{27}}}+\phi\right)-8-\frac{1}{2}$$

ϕ is the golden ratio

Exact result:

$$27\left(\phi+\frac{12}{\sqrt{2^{2/3}\sqrt[3]{7}-3}}\right)-\frac{17}{2}$$

Decimal approximation:

1729.017255163702494621285892062689763173866971633624900227...

1729.0172551637…

With regard 27 (From Wikipedia):

*"The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group **Z**/3**Z**, and its outer automorphism group is the cyclic group **Z**/2**Z**. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories".*

Alternate forms:

$$27\phi-\frac{17}{2}+\frac{324}{\sqrt{2^{2/3}\sqrt[3]{7}-3}}$$

$$5+\frac{27\sqrt{5}}{2}+\frac{324}{\sqrt{2^{2/3}\sqrt[3]{7}-3}}$$

$$27\left(\frac{1}{2}\left(1+\sqrt{5}\right)+\frac{12}{\sqrt{2^{2/3}\sqrt[3]{7}-3}}\right)-\frac{17}{2}$$

Minimal polynomial:

$$64x^6-209\,280x^5+159\,557\,712x^4+14\,653\,178\,048x^3+7\,208\,631\,334\,284x^2+351\,845\,703\,912\,552x+78\,030\,570\,956\,222\,359$$

24*1/(((((28)^(1/3)-(27)^(1/3))^0.5)))

Input:

$$24\times\frac{1}{\sqrt{\sqrt[3]{28}-\sqrt[3]{27}}}$$

Exact result:

$$\frac{24}{\sqrt{2^{2/3}\sqrt[3]{7}-3}}$$

Decimal approximation:

125.4689138864781728681305220396161136292902684280643928111...

125.46891388…

Alternate form:

$$8\left(5+2\times2^{2/3}\sqrt[3]{7}+\sqrt[3]{2}\,7^{2/3}\right)$$

Minimal polynomial:

$$x^3-120x^2-576x-13\,824$$

24*1/(((((28)^(1/3)-(27)^(1/3))^0.5)))+13+1/golden ratio+1/2

Input:

$$24\times\frac{1}{\sqrt{\sqrt[3]{28}-\sqrt[3]{27}}}+13+\frac{1}{\phi}+\frac{1}{2}$$

ϕ is the golden ratio

Exact result:

$$\frac{1}{\phi}+\frac{27}{2}+\frac{24}{\sqrt{2^{2/3}\sqrt[3]{7}-3}}$$

Decimal approximation:

139.58694787522806771633510887398175174701057760787015567 32...

139.5869478…

Minimal polynomial:

$64 x^6 - 20352 x^5 + 2008272 x^4 - 65731136 x^3 + 1178107980 x^2 - 11188527864 x + 53122326431$

Alternate forms:

$$\frac{27}{2} + \frac{2}{1+\sqrt{5}} + \frac{24}{\sqrt{2^{2/3}\sqrt[3]{7} - 3}}$$

root of $64 x^6 - 20352 x^5 + 2008272 x^4 - 65731136 x^3 + 1178107980 x^2 - 11188527864 x + 53122326431$ near $x = 139.587$

$$\frac{1}{\frac{1}{2} + \frac{\sqrt{5}}{2}} + \frac{3\left(16 + 9\sqrt{2^{2/3}\sqrt[3]{7} - 3}\right)}{2\sqrt{2^{2/3}\sqrt[3]{7} - 3}}$$

Now, we have that:

$$\sqrt[4]{\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}} = \frac{\sqrt[4]{5}+1}{\sqrt[4]{5}-1} = \frac{1}{2}\left(3+\sqrt[4]{5}+\sqrt{5}+\sqrt[4]{125}\right)$$

[((3+2(5)^(1/4))) / ((3-2(5)^(1/4)))]^1/4

Input:

$$\sqrt[4]{\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}}$$

Decimal approximation:

5.0375591418015601791686190145827146563721270377443099468 18...

5.0375591418

Alternate forms:

$$\sqrt[4]{\frac{2\sqrt[4]{5}}{3-2\sqrt[4]{5}}+\frac{3}{3-2\sqrt[4]{5}}}$$

$$\sqrt[4]{\frac{6}{3-2\sqrt[4]{5}}-1}$$

$$\frac{3}{2}+\frac{\sqrt{5}}{2}+\sqrt{\frac{1}{2}\left(5+3\sqrt{5}\right)}$$

Minimal polynomial:

$$x^4-6x^3+6x^2-6x+1$$

All 4th roots of (3 + 2 5^(1/4))/(3 - 2 5^(1/4)):

$$\sqrt[4]{\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}}\; e^0 \approx 5.04 \text{ (real, principal root)}$$

$$\sqrt[4]{\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}}\; e^{(i\pi)/2} \approx 5.04\, i$$

$$\sqrt[4]{\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}}\; e^{i\pi} \approx -5.04 \text{ (real root)}$$

$$\sqrt[4]{\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}}\; e^{-(i\pi)/2} \approx -5.04\, i$$

From which:

((([((3+2(5)^(1/4))) / ((3-2(5)^(1/4)))]^1/4)))^3-e

Input:

$$\sqrt[4]{\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}}^{\,3}-e$$

Result:

$$\left(\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}\right)^{3/4}-e$$

Decimal approximation:

125.1198671279241931538321300790662578948777755128760341821...

125.1198671...

Property:

$\left(\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}\right)^{3/4} - e$ is a transcendental number

Alternate forms:

$$\sqrt[4]{\frac{2\sqrt[4]{5}\left(\frac{4\sqrt{5}}{\left(3-2\sqrt[4]{5}\right)^2}+\frac{12\sqrt[4]{5}}{\left(3-2\sqrt[4]{5}\right)^2}+\frac{9}{\left(3-2\sqrt[4]{5}\right)^2}\right)}{3-2\sqrt[4]{5}}+\frac{3\left(\frac{4\sqrt{5}}{\left(3-2\sqrt[4]{5}\right)^2}+\frac{12\sqrt[4]{5}}{\left(3-2\sqrt[4]{5}\right)^2}+\frac{9}{\left(3-2\sqrt[4]{5}\right)^2}\right)}{3-2\sqrt[4]{5}}} - e$$

$$\left(\frac{6}{3-2\sqrt[4]{5}}-1\right)^{3/4} - e$$

$$\frac{63}{2}+\frac{29\sqrt{5}}{2}+\sqrt{\frac{1}{2}\left(4085+1827\sqrt{5}\right)} - e$$

Alternative representation:

$$\sqrt[4]{\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}}^{3} - e = \sqrt[4]{\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}}^{3} - \exp(z) \text{ for } z = 1$$

Series representations:

$$\sqrt[4]{\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}}^{3} - e = \left(\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}\right)^{3/4} - \sum_{k=0}^{\infty}\frac{1}{k!}$$

$$\sqrt[4]{\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}}^{3} - e = \left(\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}\right)^{3/4} - \frac{\sum_{k=0}^{\infty}\frac{-1+k+z}{k!}}{z}$$

$$\sqrt[4]{\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}}^{3} - e = -3+\left(\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}\right)^{3/4} + \sum_{k=0}^{\infty}\frac{1+k}{(3+k)!}$$

1/2(3+(5)^(1/4)+sqrt5+(125)^(1/4))

Input:

$$\frac{1}{2}\left(3+\sqrt[4]{5}+\sqrt{5}+\sqrt[4]{125}\right)$$

Result:

$$\frac{1}{2}\left(3+\sqrt[4]{5}+\sqrt{5}+5^{3/4}\right)$$

Decimal approximation:

5.037559141801560179168619014582714656372127037744309946818...

5.03755914…

Alternate forms:

$$\frac{3}{2}+\frac{\sqrt[4]{5}}{2}+\frac{\sqrt{5}}{2}+\frac{5^{3/4}}{2}$$

$$\frac{3}{2}+\frac{\sqrt{5}}{2}+\sqrt{\frac{1}{2}\left(5+3\sqrt{5}\right)}$$

Minimal polynomial:

$$x^4-6x^3+6x^2-6x+1$$

Now, we have that:

$$\sqrt[3]{\sqrt[5]{\frac{32}{5}}-\sqrt[5]{\frac{27}{5}}}=\sqrt[5]{\frac{1}{25}}+\sqrt[5]{\frac{3}{25}}-\sqrt[5]{\frac{9}{25}}$$

((((32/5)^(1/5)-(27/5)^(1/5))))^1/3

Input:

$$\sqrt[3]{\sqrt[5]{\frac{32}{5}}-\sqrt[5]{\frac{27}{5}}}$$

Exact result:

$$\sqrt[3]{\frac{2}{\sqrt[5]{5}} - \frac{3^{3/5}}{\sqrt[5]{5}}}$$

Decimal approximation:

0.3645018412160680351762606744652073763731953651713141449 36...

0.3645018412…

Alternate forms:

$$\frac{\sqrt[3]{2 - 3^{3/5}}}{\sqrt[15]{5}}$$

$$\sqrt[5]{\text{root of } x^5 - 5x^4 - 5x^3 + 65x^2 + 155x - 1 \text{ near } x = 0.00643426}$$

Minimal polynomial:

$$x^{25} - 5x^{20} - 5x^{15} + 65x^{10} + 155x^5 - 1$$

All 3rd roots of 2/5^(1/5) - 3^(3/5)/5^(1/5):

$$\sqrt[3]{\frac{2}{\sqrt[5]{5}} - \frac{3^{3/5}}{\sqrt[5]{5}}}\ e^0 \approx 0.365 \text{ (real, principal root)}$$

$$\sqrt[3]{\frac{2}{\sqrt[5]{5}} - \frac{3^{3/5}}{\sqrt[5]{5}}}\ e^{(2i\pi)/3} \approx -0.1823 + 0.3157i$$

$$\sqrt[3]{\frac{2}{\sqrt[5]{5}} - \frac{3^{3/5}}{\sqrt[5]{5}}}\ e^{-(2i\pi)/3} \approx -0.1823 - 0.3157i$$

(1/25)^(1/5)+(3/25)^(1/5)-(9/25)^(1/5)

Input:

$$\sqrt[5]{\frac{1}{25}} + \sqrt[5]{\frac{3}{25}} - \sqrt[5]{\frac{9}{25}}$$

Result:

$$-\left(\frac{3}{5}\right)^{2/5} + \frac{1}{5^{2/5}} + \frac{\sqrt[5]{3}}{5^{2/5}}$$

Decimal approximation:

0.3645018412160680351762606744652073763731953651713141449 36...

0.3645018412…

Alternate forms:

$$\frac{1}{5}\left(1+\sqrt[5]{3}-3^{2/5}\right)5^{3/5}$$

$$\frac{1+\sqrt[5]{3}-3^{2/5}}{5^{2/5}}$$

$$-\frac{-1-\sqrt[5]{3}+3^{2/5}}{5^{2/5}}$$

Minimal polynomial:

$$x^{25}-5x^{20}-5x^{15}+65x^{10}+155x^5-1$$

From which:

sqrt((1/((((((32/5)^(1/5)-(27/5)^(1/5))))^1/3))))

Input:

$$\sqrt{\frac{1}{\sqrt[3]{\sqrt[5]{\frac{32}{5}}-\sqrt[5]{\frac{27}{5}}}}}$$

Exact result:

$$\frac{1}{\sqrt[6]{\frac{2}{\sqrt[5]{5}}-\frac{3^{3/5}}{\sqrt[5]{5}}}}$$

Decimal approximation:

1.6563424666246556401533050282211039614995419145821853615 37...

1.6563424666…. result very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164.2696$ i.e. 1.65578...

Alternate form:

$$\frac{\sqrt[30]{5}}{\sqrt[6]{2-3^{3/5}}}$$

All 2nd roots of 1/(2/5^(1/5) - 3^(3/5)/5^(1/5))^(1/3):

$$\frac{e^0}{\sqrt[6]{\frac{2}{\sqrt[5]{5}}-\frac{3^{3/5}}{\sqrt[5]{5}}}} \approx 1.656 \text{ (real, principal root)}$$

$$\frac{e^{i\pi}}{\sqrt[6]{\frac{2}{\sqrt[5]{5}}-\frac{3^{3/5}}{\sqrt[5]{5}}}} \approx -1.656 \text{ (real root)}$$

3*13*16/((((((32/5)^(1/5)-(27/5)^(1/5))))^1/3))+13+5-1/golden ratio

Input:

$$3\times 13\times \frac{16}{\sqrt[3]{\sqrt[5]{\frac{32}{5}}-\sqrt[5]{\frac{27}{5}}}}+13+5-\frac{1}{\phi}$$

ϕ is the golden ratio

Exact result:

$$-\frac{1}{\phi}+18+\frac{624}{\sqrt[3]{\frac{2}{\sqrt[5]{5}}-\frac{3^{3/5}}{\sqrt[5]{5}}}}$$

Decimal approximation:

1729.3074748596611586778695543618843649272329106067568596 94...

1729.307474859…

Alternate forms:

$$-\frac{1}{\phi}+18+\frac{624\sqrt[15]{5}}{\sqrt[3]{2-3^{3/5}}}$$

$$18+\frac{624}{\sqrt[3]{\frac{2}{\sqrt[5]{5}}-\frac{3^{3/5}}{\sqrt[5]{5}}}}-\frac{2}{1+\sqrt{5}}$$

$$\frac{\sqrt[15]{5}\left(6\left(104+\frac{3\sqrt[3]{2-3^{3/5}}}{\sqrt[15]{5}}\right)\phi-\frac{\sqrt[3]{2-3^{3/5}}}{\sqrt[15]{5}}\right)}{\sqrt[3]{2-3^{3/5}}\,\phi}$$

Alternative representations:

$$\frac{(3\times 13)\,16}{\sqrt[3]{\sqrt[5]{\frac{32}{5}}-\sqrt[5]{\frac{27}{5}}}}+13+5-\frac{1}{\phi}=18+\frac{624}{\sqrt[3]{-\sqrt[5]{\frac{27}{5}}+\sqrt[5]{\frac{32}{5}}}}-\frac{1}{2\sin(54^\circ)}$$

$$\frac{(3\times 13)\,16}{\sqrt[3]{\sqrt[5]{\frac{32}{5}}-\sqrt[5]{\frac{27}{5}}}}+13+5-\frac{1}{\phi}=18--\frac{1}{2\cos(216^\circ)}+\frac{624}{\sqrt[3]{-\sqrt[5]{\frac{27}{5}}+\sqrt[5]{\frac{32}{5}}}}$$

$$\frac{(3\times 13)\,16}{\sqrt[3]{\sqrt[5]{\frac{32}{5}}-\sqrt[5]{\frac{27}{5}}}}+13+5-\frac{1}{\phi}=18+\frac{624}{\sqrt[3]{-\sqrt[5]{\frac{27}{5}}+\sqrt[5]{\frac{32}{5}}}}--\frac{1}{2\sin(666^\circ)}$$

Now, we have that:

$$\pi\approx\sqrt{6+\sqrt{5+\sqrt{94+\sqrt{6+\sqrt{542+\sqrt{691+\sqrt{207046+\sqrt{278255738}}}}}}}}$$

sqrt(6+sqrt(5+sqrt(94+sqrt(6+sqrt(542+sqrt(691+sqrt(207046+sqrt(278255738))))))))

Input:

$$\sqrt{6+\sqrt{5+\sqrt{94+\sqrt{6+\sqrt{542+\sqrt{691+\sqrt{207\,046+\sqrt{278\,255\,738}}}}}}}}$$

Decimal approximation:

3.141592653589793238450109266756044852957448370402056408754...

3.141592653589….. = π

All 2nd roots of 6 + sqrt(5 + sqrt(94 + sqrt(6 + sqrt(542 + sqrt(691 + sqrt(207046 + sqrt(278255738))))))):

$$\sqrt{6+\sqrt{5+\sqrt{94+\sqrt{6+\sqrt{542+\sqrt{691+\sqrt{207046+\sqrt{278255738}}}}}}}}\; e^{0} \approx 3.1416$$

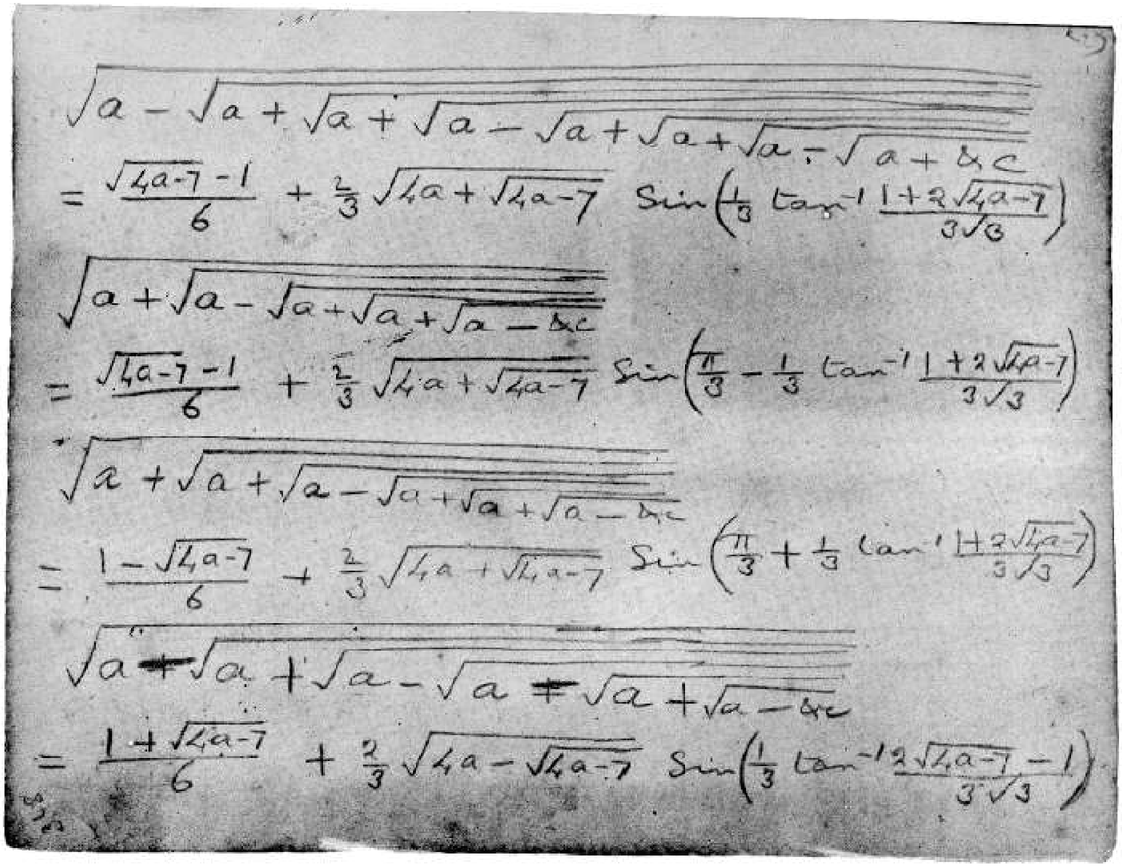
(real, principal root)

$$\sqrt{6+\sqrt{5+\sqrt{94+\sqrt{6+\sqrt{542+\sqrt{691+\sqrt{207046+\sqrt{278255738}}}}}}}}\; e^{i\pi}$$

≈ -3.1416 (real root)

Now, from:

Rediscovering Ramanujan - *a thesis presented by Momin Meraj Malik*
to The Department of the History of Science in partial fulfillment
for an honors degree in History and Science - Harvard University
Cambridge, Massachusetts - 8 December 2008



$$\sqrt{a-\sqrt{a+\sqrt{a+\sqrt{a-\sqrt{a+\sqrt{a+\sqrt{a-\sqrt{a+\&c}}}}}}}}$$
$$=\frac{\sqrt{4a-7}-1}{6}+\frac{2}{3}\sqrt{4a+\sqrt{4a-7}}\,\sin\left(\frac{1}{3}\tan^{-1}\frac{1+2\sqrt{4a-7}}{3\sqrt{3}}\right)$$

$$\sqrt{a+\sqrt{a-\sqrt{a+\sqrt{a+\sqrt{a-\&c}}}}}$$
$$=\frac{\sqrt{4a-7}-1}{6}+\frac{2}{3}\sqrt{4a+\sqrt{4a-7}}\,\sin\left(\frac{\pi}{3}-\frac{1}{3}\tan^{-1}\frac{1+2\sqrt{4a-7}}{3\sqrt{3}}\right)$$

$$\sqrt{a+\sqrt{a+\sqrt{a-\sqrt{a+\sqrt{a+\sqrt{a-\&c}}}}}}$$
$$=\frac{1-\sqrt{4a-7}}{6}+\frac{2}{3}\sqrt{4a+\sqrt{4a-7}}\,\sin\left(\frac{\pi}{3}+\frac{1}{3}\tan^{-1}\frac{1+2\sqrt{4a-7}}{3\sqrt{3}}\right)$$

$$\sqrt{a\mp\sqrt{a+\sqrt{a-\sqrt{a\mp\sqrt{a+\sqrt{a-\&c}}}}}}$$
$$=\frac{1+\sqrt{4a-7}}{6}+\frac{2}{3}\sqrt{4a-\sqrt{4a-7}}\,\sin\left(\frac{1}{3}\tan^{-1}\frac{2\sqrt{4a-7}-1}{3\sqrt{3}}\right)$$

For a = 3 , we obtain:

1/6(1+sqrt(12-7)) + 2/3 (12-sqrt(12-7))^0.5 * sin(((1/3*tan^-1 (((2sqrt(12-7)-1)/(3sqrt3))))))

Input:

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{2}{3}\sqrt{12-\sqrt{12-7}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)$$

Exact Result:

$$\frac{1}{6}\left(1+\sqrt{5}\right)+\frac{2}{3}\sqrt{12-\sqrt{5}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{5}-1}{3\sqrt{3}}\right)\right)$$

(result in radians)

Decimal approximation:

0.945763722196398446155536122455865440817511522937106097926...

(result in radians)

0.945763722196…. result very near to the spectral index n_s , to the mesonic Regge slope, to the inflaton value at the end of the inflation 0.9402 and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1}=1-\cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\ldots}}}}\approx 0.9568666373$$

Alternate forms:

$$\frac{1}{6}\left(1+\sqrt{5}+4\sqrt{12-\sqrt{5}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{5}-1}{3\sqrt{3}}\right)\right)\right)$$

$$\frac{1}{6}+\frac{\sqrt{5}}{6}+\frac{2}{3}\sqrt{12-\sqrt{5}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{5}-1}{3\sqrt{3}}\right)\right)$$

$$\frac{2}{3}\sqrt{12-\sqrt{5}}\ \left(\text{root of } 569344x^{12}-1708032x^{10}+1921536x^8-990464x^6+224688x^4-16704x^2+361 \text{ near } x=0.195098\right)+\frac{1}{6}\left(1+\sqrt{5}\right)$$

Alternative representations:

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2=$$

$$\frac{2}{3}\cos\left(\frac{\pi}{2}-\frac{1}{3}\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\sqrt{12-\sqrt{5}}+\frac{1}{6}\left(1+\sqrt{5}\right)$$

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2 = -\frac{2}{3}\cos\left(\frac{\pi}{2}+\frac{1}{3}\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\sqrt{12-\sqrt{5}}+\frac{1}{6}\left(1+\sqrt{5}\right)$$

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2 = \frac{2\left(-e^{-\frac{1}{3}i\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)}+e^{1/3\,i\tan^{-1}\left(\left(-1+2\sqrt{5}\right)/\left(3\sqrt{3}\right)\right)}\right)\sqrt{12-\sqrt{5}}}{3(2i)}+\frac{1}{6}\left(1+\sqrt{5}\right)$$

Series representations:

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2 = \frac{1}{6}+\frac{\sqrt{5}}{6}+\frac{2}{3}\sqrt{12-\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-1-2k}\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)^{1+2k}}{(1+2k)!}$$

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2 = \frac{1}{6}+\frac{\sqrt{5}}{6}+\frac{2}{3}\sqrt{12-\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{\pi}{2}+\frac{1}{3}\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\right)^{2k}}{(2k)!}$$

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2 = \frac{1}{6}+\frac{\sqrt{5}}{6}+\frac{1}{9}\sqrt{\left(12-\sqrt{5}\right)\pi}\ \tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{36^s\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}$$

Integral representations:

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2 = \frac{1}{6}+\frac{\sqrt{5}}{6}+\frac{2}{9}\sqrt{12-\sqrt{5}}\ \tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\int_0^1\cos\left(\frac{1}{3}t\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\right)dt$$

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2=\frac{1}{6}+\frac{\sqrt{5}}{6}-\frac{1}{18}\,i\sqrt{\frac{12-\sqrt{5}}{\pi}}\,\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{s-\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)^2/(36s)}}{s^{3/2}}\,ds \quad \text{for } \gamma>0$$

$$\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2=\frac{1}{6}+\frac{\sqrt{5}}{6}-\frac{1}{3}\,i\sqrt{\frac{12-\sqrt{5}}{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\left(\frac{1}{6}\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\right)^{1-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds \quad \text{for } 0<\gamma<1$$

and:

(((1/(((1/6(1+sqrt(12-7)) + 2/3 (12-sqrt(12-7))^0.5 * sin(((1/3*tan^-1 (((2sqrt(12-7)-1)/(3sqrt3)))))))))))^9

Input:

$$\left(\frac{1}{\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{2}{3}\sqrt{12-\sqrt{12-7}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)}\right)^9$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\frac{1}{\left(\frac{1}{6}\left(1+\sqrt{5}\right)+\frac{2}{3}\sqrt{12-\sqrt{5}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{5}-1}{3\sqrt{3}}\right)\right)\right)^9}$$

(result in radians)

Decimal approximation:

1.651794968339342584809560014494074500933210200121297367702...

(result in radians)

1.651794968…… result very near to the 14th root of the following Ramanujan's class invariant $Q=(G_{505}/G_{101/5})^3=1164.2696$ i.e. 1.65578...

Alternate forms:

$$\frac{10077696}{\left(1+\sqrt{5}+4\sqrt{12-\sqrt{5}}\,\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{5}-1}{3\sqrt{3}}\right)\right)\right)^9}$$

$$1\Big/\left(\frac{2}{3}\sqrt{12-\sqrt{5}}\ \boxed{\text{root of } 569344x^{12}-1708032x^{10}+1921536x^{8}-990464x^{6}+224688x^{4}-16704x^{2}+361 \text{ near } x=0.195098}+\frac{1}{6}\left(1+\sqrt{5}\right)\right)^{9}$$

$$1\Big/\left(\frac{1}{6}\left(1+\sqrt{5}\right)+\frac{1}{3}\,i\sqrt{12-\sqrt{5}}\left(\exp\left(\frac{1}{6}\left(\log\left(1-\frac{i(2\sqrt{5}-1)}{3\sqrt{3}}\right)-\log\left(1+\frac{i(2\sqrt{5}-1)}{3\sqrt{3}}\right)\right)\right)-\exp\left(\frac{1}{6}\left(\log\left(1+\frac{i(2\sqrt{5}-1)}{3\sqrt{3}}\right)-\log\left(1-\frac{i(2\sqrt{5}-1)}{3\sqrt{3}}\right)\right)\right)\right)\right)^{9}$$

$\log(x)$ is the natural logarithm

Alternative representations:

$$\left(\frac{1}{\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2}\right)^{9}=\left(\frac{1}{\frac{2}{3}\cos\left(\frac{\pi}{2}-\frac{1}{3}\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\sqrt{12-\sqrt{5}}+\frac{1}{6}\left(1+\sqrt{5}\right)}\right)^{9}$$

$$\left(\frac{1}{\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2}\right)^{9}=\left(\frac{1}{-\frac{2}{3}\cos\left(\frac{\pi}{2}+\frac{1}{3}\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\right)\sqrt{12-\sqrt{5}}+\frac{1}{6}\left(1+\sqrt{5}\right)}\right)^{9}$$

$$\left(\frac{1}{\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2}\right)^{9}=\left(\frac{1}{\frac{2\left(-e^{-\frac{1}{3}i\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)}+e^{1/3\,i\tan^{-1}\left(\left(-1+2\sqrt{5}\right)/\left(3\sqrt{3}\right)\right)}\right)\sqrt{12-\sqrt{5}}}{3(2i)}+\frac{1}{6}\left(1+\sqrt{5}\right)}\right)^{9}$$

Series representations:

$$\left(\frac{1}{\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2}\right)^{9}=$$

$$\frac{1}{\left(\frac{1}{6}\left(1+\sqrt{5}\right)+\frac{2}{3}\sqrt{12-\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\,3^{-1-2k}\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)^{1+2k}}{(1+2k)!}\right)^{9}}$$

$$\left(\frac{1}{\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2}\right)^{9}=$$

$$\frac{1}{\left(\frac{1}{6}\left(1+\sqrt{5}\right)+\frac{2}{3}\sqrt{12-\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{\pi}{2}+\frac{1}{3}\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\right)^{2k}}{(2k)!}\right)^{9}}$$

$$\left(\frac{1}{\frac{1}{6}\left(1+\sqrt{12-7}\right)+\frac{1}{3}\left(\sqrt{12-\sqrt{12-7}}\ \sin\left(\frac{1}{3}\tan^{-1}\left(\frac{2\sqrt{12-7}-1}{3\sqrt{3}}\right)\right)\right)2}\right)^{9}=$$

$$1\Bigg/\left(\frac{1}{6}\left(1+\sqrt{5}\right)+\frac{1}{9}\sqrt{\left(12-\sqrt{5}\right)\pi}\ \tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{36^s\tan^{-1}\left(\frac{-1+2\sqrt{5}}{3\sqrt{3}}\right)^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)^{9}$$

From:

Gravitational Wave Anisotropies from Primordial Black Holes
N. Bartolo, D. Bertacca, V. De Luca, G. Franciolini, S. Matarrese, M. Peloso, A. Ricciardone, A. Riotto, G. Tasinato - arXiv:1909.12619v2 [astro-ph.CO] 9 Mar 2020

The contraction forced $k_1 = k_2$ in Eq. (2.2) using Eq. (2.1). In this case the time average procedure in (2.2) became straightforward, namely

$$\left\langle \sin^2 (k_1 \eta) \right\rangle_T = \left\langle \cos^2 (k_1 \eta) \right\rangle_T = 1/2 \,, \quad \left\langle \sin (k_1 \eta) \cos (k_1 \eta) \right\rangle_T = 0. \tag{2.5}$$

Following the standard convention, in the first line of (2.4) we defined the fractional energy density in the GW for log interval. The quantity $\rho_c = 3H^2 M_p^2$ denotes the critical energy density of a spatially flat universe with Hubble rate H. The notation in the second line of (2.4) exploits the fact that the integral over the two angles $d\Omega_k$ can be made trivial by exploiting that the only angular dependence of the integrand is on the angle between $\vec{k}_1$ and $\vec{p}_1$ (this is a consequence of the statistical isotropy of the background). By introducing the rescaled magnitudes $x \equiv p_1/k_1$ and $y \equiv |\vec{k}_1 - \vec{p}_1|/k_1$, the expression (2.4) reduces to [49]

$$\Omega_{\rm GW}(k,\eta) = \frac{1}{972 a^2 H^2 \eta^2} \iint_{\mathcal{S}} dx dy \frac{x^2}{y^2} \left[1 - \frac{\left(1 + x^2 - y^2\right)^2}{4x^2} \right]^2 \mathcal{P}_\zeta (kx) \mathcal{P}_\zeta (ky) \mathcal{I}^2 (x,y) \,, \tag{2.6}$$

where the integration region $\mathcal{S}$ extends to $x > 0$ and to $|1-x| \leq y \leq 1+x$ and where we defined $\mathcal{I}^2 \equiv \mathcal{I}_c^2 + \mathcal{I}_s^2$. For a Dirac delta power spectrum of the scalar curvature perturbation on small scales, $\mathcal{P}_{\zeta_s}(k) = A_s \, k_* \, \delta(k - k_*)$, this expression then becomes [5]

$$\Omega_{\rm GW}(k,\eta) = \frac{1}{a^2 H^2 \eta^2} \frac{A_s^2}{15552} \frac{k^2}{k_*^2} \left[\frac{4k_*^2}{k^2} - 1 \right]^2 \theta\left(2k_* - k\right) \, \mathcal{I}^2 \left(\frac{k_*}{k}, \frac{k_*}{k} \right) \tag{2.7}$$

where θ is the Heaviside step function, and

$$\begin{aligned} \mathcal{I}^2 \left(\frac{k_*}{k}, \frac{k_*}{k} \right) &\equiv \mathcal{I}_c^2 \left(\frac{k_*}{k}, \frac{k_*}{k} \right) + \mathcal{I}_s^2 \left(\frac{k_*}{k}, \frac{k_*}{k} \right) \\ &= \frac{729}{16} \left(\frac{k}{k_*} \right)^{12} \left(3 - \frac{2k_*^2}{k^2} \right)^4 \left\{ \left[4 \left(2 - 3\frac{k^2}{k_*^2} \right)^{-1} - \log \left(\left| 1 - \frac{4k_*^2}{3k^2} \right| \right) \right]^2 + \pi^2 \theta \left(\frac{2k_*}{\sqrt{3}k} - 1 \right) \right\}. \end{aligned} \tag{2.8}$$

We note that the result (2.7) for the one-point expectation value of the GW energy density is independent of position. This follows from statistical homogeneity of the FLRW background universe (at the technical level, it is due to the fact that the contraction of the four ζ operators in Eq. (2.2) forces $\vec{k}_1 + \vec{k}_2 = 0$). However, as explained in the introduction, one does not expect that the sourced GWs are perfectly homogeneous across the universe. As a consequence, the SGWB reaching us from different directions will present some angular anisotropies.

From (2.8)

$$\mathcal{I}^2\left(\frac{k_*}{k},\frac{k_*}{k}\right) \equiv \mathcal{I}_c^2\left(\frac{k_*}{k},\frac{k_*}{k}\right)+\mathcal{I}_s^2\left(\frac{k_*}{k},\frac{k_*}{k}\right)$$
$$=\frac{729}{16}\left(\frac{k}{k_*}\right)^{12}\left(3-\frac{2k_*^2}{k^2}\right)^4\left\{\left[4\left(2-3\frac{k^2}{k_*^2}\right)^{-1}-\log\left(\left|1-\frac{4k_*^2}{3k^2}\right|\right)\right]^2+\pi^2\theta\left(\frac{2k_*}{\sqrt{3}k}-1\right)\right\}.$$

where θ = 1/2 and k / k* = 10 k* / k = 1/10, we obtain:

729/16 (10)^12 (3-(2*1/100))^4 [((((((4(2-3*100)^-1) – ln(1-(4/(3*100)))))))^2+Pi^2*1/2(((2/sqrt3)*1/10-1))]

Input:

$$\frac{729}{16}\times 10^{12}\left(3-2\times\frac{1}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\pi^2\times\frac{1}{2}\left(\frac{2}{\sqrt{3}}\times\frac{1}{10}-1\right)\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$3593127283290000\left(\frac{1}{2}\left(\frac{1}{5\sqrt{3}}-1\right)\pi^2+\left(\log\left(\frac{75}{74}\right)-\frac{2}{149}\right)^2\right)$$

Decimal approximation:

$-1.5683929895948783131527018419074606125869956387074 35\ldots\times 10^{16}$

-1.5683929…*10^16

Alternate forms:

$$119770909443000\left(\sqrt{3}-15\right)\pi^2+161845290000\left(149\log\left(\frac{75}{74}\right)-2\right)^2$$

$$3593127283290000\left(\frac{1}{30}\left(\sqrt{3}-15\right)\pi^2+\left(\log\left(\frac{75}{74}\right)-\frac{2}{149}\right)^2\right)$$

$$647381160000-1796563641645000\,\pi^2+119770909443000\sqrt{3}\,\pi^2+3593127283290000\log^2\left(\frac{75}{74}\right)-96459792840000\log\left(\frac{75}{74}\right)$$

Alternative representations:

$$\frac{1}{16}\times 10^{12}\times 729\left(3-\frac{2}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\frac{1}{2}\pi^2\left(\frac{2}{\sqrt{3}\ 10}-1\right)\right)=$$
$$\frac{729}{16}\times 10^{12}\left(3-\frac{2}{100}\right)^4\left(\left(\text{Li}_1\left(\frac{4}{300}\right)+-\frac{4}{298}\right)^2+\frac{1}{2}\pi^2\left(-1+\frac{2}{10\sqrt{3}}\right)\right)$$

$$\frac{1}{16}\times 10^{12}\times 729\left(3-\frac{2}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\frac{1}{2}\pi^2\left(\frac{2}{\sqrt{3}\ 10}-1\right)\right)=$$
$$\frac{729}{16}\times 10^{12}\left(3-\frac{2}{100}\right)^4\left(\left(-\log_e\left(1-\frac{4}{300}\right)+-\frac{4}{298}\right)^2+\frac{1}{2}\pi^2\left(-1+\frac{2}{10\sqrt{3}}\right)\right)$$

$$\frac{1}{16}\times 10^{12}\times 729\left(3-\frac{2}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\frac{1}{2}\pi^2\left(\frac{2}{\sqrt{3}\ 10}-1\right)\right)=$$
$$\frac{729}{16}\times 10^{12}\left(3-\frac{2}{100}\right)^4\left(\left(-\log(a)\log_a\left(1-\frac{4}{300}\right)+-\frac{4}{298}\right)^2+\frac{1}{2}\pi^2\left(-1+\frac{2}{10\sqrt{3}}\right)\right)$$

Series representations:

$$\frac{1}{16}\times 10^{12}\times 729\left(3-\frac{2}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\frac{1}{2}\pi^2\left(\frac{2}{\sqrt{3}\ 10}-1\right)\right)=$$
$$5\,394\,843\,000\left(120-333\,015\,\pi^2+22\,201\sqrt{3}\,\pi^2+17\,880\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{74}\right)^k}{k}+666\,030\left(\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{74}\right)^k}{k}\right)^2\right)$$

$$\frac{1}{16}\times 10^{12}\times 729\left(3-\frac{2}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\frac{1}{2}\pi^2\left(\frac{2}{\sqrt{3}\ 10}-1\right)\right)=$$
$$3\,593\,127\,283\,290\,000\left(\frac{1}{30}\left(-15+\sqrt{3}\right)\pi^2+\left(-\frac{2}{149}+2\,i\,\pi\left\lfloor\frac{\arg\left(\frac{75}{74}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{75}{74}-x\right)^k x^{-k}}{k}\right)^2\right)\quad\text{for } x<0$$

$$\frac{1}{16}\times 10^{12}\times 729\left(3-\frac{2}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\frac{1}{2}\pi^2\left(\frac{2}{\sqrt{3}\ 10}-1\right)\right)=$$
$$3\,593\,127\,283\,290\,000\left(\frac{1}{2}\left(-1+\frac{1}{5\sqrt{3}}\right)\pi^2+\left(-\frac{2}{149}+2\,i\,\pi\left\lfloor\frac{\arg\left(\frac{75}{74}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{75}{74}-x\right)^k x^{-k}}{k}\right)^2\right)\quad\text{for } x<0$$

Integral representations:

$$\frac{1}{16}\times 10^{12}\times 729\left(3-\frac{2}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\frac{1}{2}\pi^2\left(\frac{2}{\sqrt{3}\ 10}-1\right)\right)=$$
$$5\,394\,843\,000\left(120-333\,015\,\pi^2+22\,201\sqrt{3}\,\pi^2-17\,880\int_1^{\frac{75}{74}}\frac{1}{t}\,dt+666\,030\left(\int_1^{\frac{75}{74}}\frac{1}{t}\,dt\right)^2\right)$$

$$\frac{1}{16}\times 10^{12}\times 729\left(3-\frac{2}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\frac{1}{2}\pi^2\left(\frac{2}{\sqrt{3}\ 10}-1\right)\right)=$$
$$3\,593\,127\,283\,290\,000\left(\frac{1}{30}\left(-15+\sqrt{3}\right)\pi^2+\left(\frac{2}{149}+\frac{i}{2\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{74^s\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2\right)\quad \text{for } -1<\gamma<0$$

From which:

(x+1)/16 (10)^12 (3-(2*1/100))^4 [(((((((4(2-3*100)^-1) – ln(1-(4/(3*100)))))))))^2+Pi^2*1/2(((2/sqrt3)*1/10-1))] = -1.568392989e+16

Input interpretation:

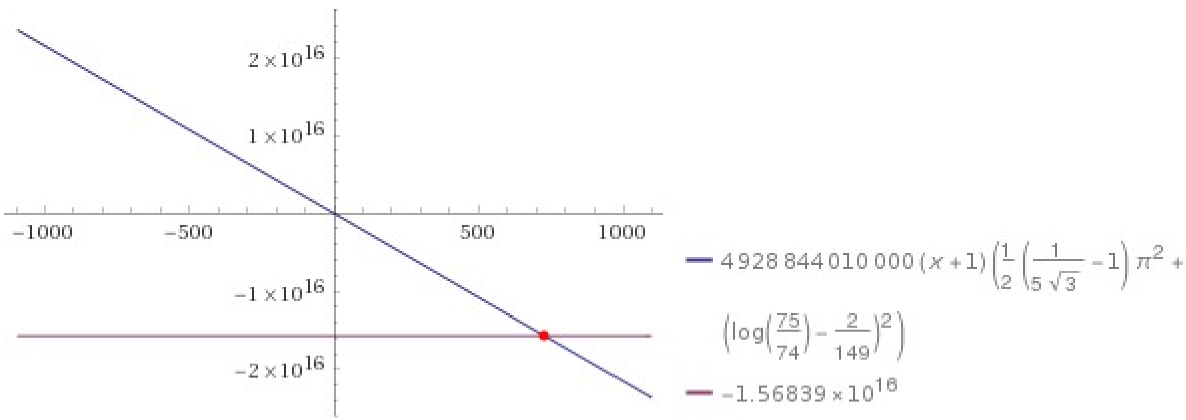
$$\frac{x+1}{16}\times 10^{12}\left(3-2\times\frac{1}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\pi^2\times\frac{1}{2}\left(\frac{2}{\sqrt{3}}\times\frac{1}{10}-1\right)\right)=$$
$$-1.568392989\times 10^{16}$$

log(x) is the natural logarithm

Result:

$$4\,928\,844\,010\,000\,(x+1)\left(\frac{1}{2}\left(\frac{1}{5\sqrt{3}}-1\right)\pi^2+\left(\log\left(\frac{75}{74}\right)-\frac{2}{149}\right)^2\right)=-1.56839\times 10^{16}$$

Plot:



Alternate forms:

$$4\,928\,844\,010\,000\,(x+1)\left(\frac{1}{30}\left(\sqrt{3}-15\right)\pi^2+\left(\log\left(\frac{75}{74}\right)-\frac{2}{149}\right)^2\right)=-1.56839\times 10^{16}$$

$$\frac{22\,201\,000}{3}(x+1)\left(22\,201\left(\sqrt{3}-15\right)\pi^2+30\left(2-149\log\left(\frac{75}{74}\right)\right)^2\right)=-1.56839\times10^{16}$$

$$\frac{492\,884\,401\,000\,\pi^2\,x}{\sqrt{3}}-2\,464\,422\,005\,000\,\pi^2\,x+888\,040\,000\,x+4\,928\,844\,010\,000\,x\log^2\left(\frac{75}{74}\right)-132\,317\,960\,000\,x\log\left(\frac{75}{74}\right)+1.56624\times10^{16}=0$$

Alternate form assuming x>0:

$$2\,464\,422\,005\,000\left(\frac{1}{5\sqrt{3}}-1\right)\pi^2\,x+4\,928\,844\,010\,000\,x\left(-\frac{2}{149}-\log(2)+\log(3)+2\log(5)-\log(37)\right)^2+2\,464\,422\,005\,000\left(\frac{1}{5\sqrt{3}}-1\right)\pi^2+4\,928\,844\,010\,000\left(-\frac{2}{149}-\log(2)+\log(3)+2\log(5)-\log(37)\right)^2=-1.56839\times10^{16}$$

Expanded form:

$$\frac{492\,884\,401\,000\,\pi^2\,x}{\sqrt{3}}-2\,464\,422\,005\,000\,\pi^2\,x+888\,040\,000\,x+4\,928\,844\,010\,000\,x\log^2\left(\frac{75}{74}\right)-132\,317\,960\,000\,x\log\left(\frac{75}{74}\right)+\frac{492\,884\,401\,000\,\pi^2}{\sqrt{3}}-2\,464\,422\,005\,000\,\pi^2+888\,040\,000+4\,928\,844\,010\,000\log^2\left(\frac{75}{74}\right)-132\,317\,960\,000\log\left(\frac{75}{74}\right)=-15\,683\,929\,890\,000\,000$$

Solution:

$x \approx 728.$

728 (Ramanujan taxicab number)

and:

729/16 (10)^12 (3-(2*1/100))^4 [((((((4(2-3*100)^-1) – ln(1-(4/(3*100))))))))^2+x^2*1/2(((2/sqrt3)*1/10-1))] = -1.568392989e+16

Input interpretation:

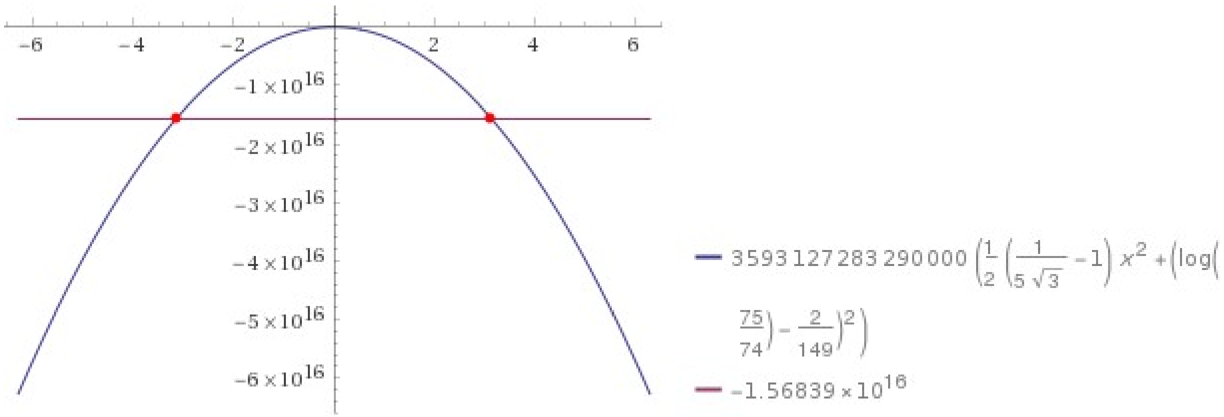
$$\frac{729}{16}\times10^{12}\left(3-2\times\frac{1}{100}\right)^4\left(\left(\frac{4}{2-3\times100}-\log\left(1-\frac{4}{3\times100}\right)\right)^2+x^2\times\frac{1}{2}\left(\frac{2}{\sqrt{3}}\times\frac{1}{10}-1\right)\right)=-1.568392989\times10^{16}$$

log(x) is the natural logarithm

Result:

$$3593127283290000\left(\frac{1}{2}\left(\frac{1}{5\sqrt{3}}-1\right)x^2+\left(\log\left(\frac{75}{74}\right)-\frac{2}{149}\right)^2\right)=-1.56839\times10^{16}$$

Plot:



Alternate forms:

$$119770909443000\sqrt{3}\,x^2-1796563641645000\,x^2+1.56839\times10^{16}=0$$

$$119770909443000\left(\sqrt{3}-15\right)x^2+161845290000\left(149\log\left(\frac{75}{74}\right)-2\right)^2=$$
$$-1.56839\times10^{16}$$

$$3593127283290000\left(\frac{1}{30}\left(\sqrt{3}-15\right)x^2+\left(\log\left(\frac{75}{74}\right)-\frac{2}{149}\right)^2\right)=-1.56839\times10^{16}$$

Expanded form:

$$119770909443000\sqrt{3}\,x^2-1796563641645000\,x^2+$$
$$647381160000+3593127283290000\log^2\left(\frac{75}{74}\right)-$$
$$96459792840000\log\left(\frac{75}{74}\right)=-15683929890000000$$

Alternate forms assuming x>0:

$$1796563641645000\left(\frac{1}{15}\left(\sqrt{3}-15\right)x^2+2\left(\log\left(\frac{75}{74}\right)-\frac{2}{149}\right)^2\right)=-1.56839\times10^{16}$$

$$1796563641645000\left(\frac{1}{5\sqrt{3}}-1\right)x^2+3593127283290000\left(-\frac{2}{149}-\log(2)+\log(3)+2\log(5)-\log(37)\right)^2=-1.56839\times10^{16}$$

Solutions:

$x \approx -3.14159$

$x \approx 3.14159$

3.14159 ≈ π

We have also:

((((((((((-729/16 (10)^12 (3-(2*1/100))^4 [((((((4(2-3*100)^-1) – ln(1-(4/(3*100))))))))^2+Pi^2*1/2(((2/sqrt3)*1/10-1))]))))))))^1/5

Input:

$$\left(-\frac{729}{16}\times10^{12}\left(3-2\times\frac{1}{100}\right)^4\left(\left(\frac{4}{2-3\times100}-\log\left(1-\frac{4}{3\times100}\right)\right)^2+\pi^2\times\frac{1}{2}\left(\frac{2}{\sqrt{3}}\times\frac{1}{10}-1\right)\right)\right)\text{^}(1/5)$$

log(x) is the natural logarithm

Exact result:

$$3\times1490^{4/5}\sqrt[5]{3\left(-\frac{1}{2}\left(\frac{1}{5\sqrt{3}}-1\right)\pi^2-\left(\log\left(\frac{75}{74}\right)-\frac{2}{149}\right)^2\right)}$$

Decimal approximation:

1734.167242948033764195427187860357602266057441302800682852...

1734.167242948…..

Alternate forms:

$$3\times1490^{4/5}\sqrt[5]{-\frac{1}{10}\left(\sqrt{3}-15\right)\pi^2-3\left(\log\left(\frac{75}{74}\right)-\frac{2}{149}\right)^2}$$

$$3\times1490^{4/5}\sqrt[5]{3\left(\frac{1}{30}\left(15-\sqrt{3}\right)\pi^2-\left(\log\left(\frac{75}{74}\right)-\frac{2}{149}\right)^2\right)}$$

$$3\times1490^{4/5}\sqrt[5]{3\left(\left(\frac{1}{2}-\frac{1}{10\sqrt{3}}\right)\pi^2-\left(\log\left(\frac{75}{74}\right)-\frac{2}{149}\right)^2\right)}$$

All 5th roots of -3593127283290000 (1/2 (1/(5 sqrt(3)) - 1) π^2 + (log(75/74) - 2/149)^2):

$$3\times 1490^{4/5}\, e^{0}\, \sqrt[5]{3\left(-\frac{1}{2}\left(\frac{1}{5\sqrt{3}}-1\right)\pi^2-\left(\log\left(\frac{75}{74}\right)-\frac{2}{149}\right)^2\right)} \approx 1734.2 \text{ (real, principal root)}$$

$$3\times 1490^{4/5}\, e^{(2 i \pi)/5}\, \sqrt[5]{3\left(-\frac{1}{2}\left(\frac{1}{5\sqrt{3}}-1\right)\pi^2-\left(\log\left(\frac{75}{74}\right)-\frac{2}{149}\right)^2\right)} \approx 535.9 + 1649.3\, i$$

$$3\times 1490^{4/5}\, e^{(4 i \pi)/5}\, \sqrt[5]{3\left(-\frac{1}{2}\left(\frac{1}{5\sqrt{3}}-1\right)\pi^2-\left(\log\left(\frac{75}{74}\right)-\frac{2}{149}\right)^2\right)} \approx -1403.0 + 1019.3\, i$$

$$3\times 1490^{4/5}\, e^{-(4 i \pi)/5}\, \sqrt[5]{3\left(-\frac{1}{2}\left(\frac{1}{5\sqrt{3}}-1\right)\pi^2-\left(\log\left(\frac{75}{74}\right)-\frac{2}{149}\right)^2\right)} \approx -1403.0 - 1019.3\, i$$

$$3\times 1490^{4/5}\, e^{-(2 i \pi)/5}\, \sqrt[5]{3\left(-\frac{1}{2}\left(\frac{1}{5\sqrt{3}}-1\right)\pi^2-\left(\log\left(\frac{75}{74}\right)-\frac{2}{149}\right)^2\right)} \approx 535.9 - 1649.3\, i$$

Alternative representations:

$$\sqrt[5]{\frac{1}{16}\times 10^{12}\,(-729)\left(3-\frac{2}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\frac{1}{2}\,\pi^2\left(\frac{2}{\sqrt{3}\;10}-1\right)\right)}$$
$$=\sqrt[5]{-\frac{729}{16}\times 10^{12}\left(3-\frac{2}{100}\right)^4\left(\left(\text{Li}_1\left(\frac{4}{300}\right)+-\frac{4}{298}\right)^2+\frac{1}{2}\,\pi^2\left(-1+\frac{2}{10\sqrt{3}}\right)\right)}$$

$$\sqrt[5]{\frac{1}{16}\times 10^{12}\,(-729)\left(3-\frac{2}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\frac{1}{2}\,\pi^2\left(\frac{2}{\sqrt{3}\;10}-1\right)\right)}$$
$$=\sqrt[5]{-\frac{729}{16}\times 10^{12}\left(3-\frac{2}{100}\right)^4\left(\left(-\log_e\left(1-\frac{4}{300}\right)+-\frac{4}{298}\right)^2+\frac{1}{2}\,\pi^2\left(-1+\frac{2}{10\sqrt{3}}\right)\right)}$$

$$\sqrt[5]{\frac{1}{16}\times 10^{12}\,(-729)\left(3-\frac{2}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\frac{1}{2}\,\pi^2\left(\frac{2}{\sqrt{3}\;10}-1\right)\right)}$$
$$=\sqrt[5]{-\frac{729}{16}\times 10^{12}\left(3-\frac{2}{100}\right)^4\left(\left(S_{0,1}\left(\frac{4}{300}\right)+-\frac{4}{298}\right)^2+\frac{1}{2}\,\pi^2\left(-1+\frac{2}{10\sqrt{3}}\right)\right)}$$

Series representations:

$$\sqrt[5]{\frac{1}{16} \times 10^{12} (-729) \left(3 - \frac{2}{100}\right)^4 \left(\left(\frac{4}{2 - 3 \times 100} - \log\left(1 - \frac{4}{3 \times 100}\right)\right)^2 + \frac{1}{2} \pi^2 \left(\frac{2}{\sqrt{3}\ 10} - 1\right)\right)}$$
$$= 3 \times 1490^{4/5} \sqrt[5]{-\frac{1}{10} \left(-15 + \sqrt{3}\right) \pi^2 - 3 \left(-\frac{2}{149} - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{74}\right)^k}{k}\right)^2}$$

$$\sqrt[5]{\frac{1}{16} \times 10^{12} (-729) \left(3 - \frac{2}{100}\right)^4 \left(\left(\frac{4}{2 - 3 \times 100} - \log\left(1 - \frac{4}{3 \times 100}\right)\right)^2 + \frac{1}{2} \pi^2 \left(\frac{2}{\sqrt{3}\ 10} - 1\right)\right)}$$
$$= 3 \sqrt[5]{3}\ 1490^{4/5} \sqrt[5]{-\frac{1}{2} \left(-1 + \frac{1}{5 \sqrt{3}}\right) \pi^2 - \left(-\frac{2}{149} - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{74}\right)^k}{k}\right)^2}$$

$$\sqrt[5]{\frac{1}{16} \times 10^{12} (-729) \left(3 - \frac{2}{100}\right)^4 \left(\left(\frac{4}{2 - 3 \times 100} - \log\left(1 - \frac{4}{3 \times 100}\right)\right)^2 + \frac{1}{2} \pi^2 \left(\frac{2}{\sqrt{3}\ 10} - 1\right)\right)}$$
$$= 3 \sqrt[5]{3}\ 1490^{4/5} \left(-\frac{1}{2} \left(-1 + \frac{1}{5 \sqrt{3}}\right) \pi^2 - \left(-\frac{2}{149} + 2 i \pi \left\lfloor \frac{\arg\left(\frac{75}{74} - x\right)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{75}{74} - x\right)^k x^{-k}}{k}\right)^2\right) \hat{}\ (1/5) \quad \text{for } x < 0$$

Integral representations:

$$\sqrt[5]{\frac{1}{16} \times 10^{12} (-729) \left(3 - \frac{2}{100}\right)^4 \left(\left(\frac{4}{2 - 3 \times 100} - \log\left(1 - \frac{4}{3 \times 100}\right)\right)^2 + \frac{1}{2} \pi^2 \left(\frac{2}{\sqrt{3}\ 10} - 1\right)\right)}$$
$$= 3 \sqrt[5]{3}\ 1490^{4/5} \sqrt[5]{-\frac{1}{2} \left(-1 + \frac{1}{5 \sqrt{3}}\right) \pi^2 - \left(-\frac{2}{149} + \int_1^{\frac{75}{74}} \frac{1}{t}\, dt\right)^2}$$

$$\sqrt[5]{\frac{1}{16} \times 10^{12} (-729) \left(3 - \frac{2}{100}\right)^4 \left(\left(\frac{4}{2 - 3 \times 100} - \log\left(1 - \frac{4}{3 \times 100}\right)\right)^2 + \frac{1}{2} \pi^2 \left(\frac{2}{\sqrt{3}\ 10} - 1\right)\right)}$$
$$= 3 \sqrt[5]{3}\ 1490^{4/5} \sqrt[5]{-\frac{1}{2} \left(-1 + \frac{1}{5 \sqrt{3}}\right) \pi^2 - \left(-\frac{2}{149} - \frac{i}{2 \pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{74^s\, \Gamma(-s)^2\, \Gamma(1 + s)}{\Gamma(1 - s)}\, ds\right)^2} \quad \text{for } -1 < \gamma < 0$$

From the formula of coefficients of the '5th order' mock theta function $\psi_1(q)$:
(A053261 OEIS Sequence)

sqrt(golden ratio) * exp(Pi*sqrt(n/15)) / (2*5^(1/4)*sqrt(n)) for n = 181 , we obtain:

sqrt(golden ratio) * exp(Pi*sqrt(181/15)) / (2*5^(1/4)*sqrt(181))

Input:

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{181}{15}}\right)}{2 \sqrt[4]{5} \sqrt{181}}$$

ϕ is the golden ratio

Exact result:

$$\frac{e^{\sqrt{181/15}\,\pi} \sqrt{\frac{\phi}{181}}}{2 \sqrt[4]{5}}$$

Decimal approximation:

1735.125533153011008619695955154105542578248297442381526157...

1735.125533153...

Property:

$$\frac{e^{\sqrt{181/15}\,\pi} \sqrt{\frac{\phi}{181}}}{2 \sqrt[4]{5}}$$ is a transcendental number

Alternate forms:

$$\frac{1}{2} \sqrt{\frac{5+\sqrt{5}}{1810}}\, e^{\sqrt{181/15}\,\pi}$$

$$\frac{\sqrt{\frac{1}{362}\left(1+\sqrt{5}\right)}\, e^{\sqrt{181/15}\,\pi}}{2 \sqrt[4]{5}}$$

Series representations:

$$\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{181}{15}}\right)}{2\sqrt[4]{5}\sqrt{181}} = \frac{\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{181}{15}-z_0\right)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}}{2\sqrt[4]{5}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(181-z_0)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{181}{15}}\right)}{2\sqrt[4]{5}\sqrt{181}} = \left(\exp\left(i\pi\left\lfloor\frac{\arg(\phi-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{181}{15}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{181}{15}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(\phi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right) \Big/ \left(2\sqrt[4]{5}\exp\left(i\pi\left\lfloor\frac{\arg(181-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(181-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{181}{15}}\right)}{2\sqrt[4]{5}\sqrt{181}} = \left(\exp\left(\pi\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{181}{15}-z_0\right)/(2\pi)\right\rfloor} z_0^{1/2\left(1+\left\lfloor\arg\left(\frac{181}{15}-z_0\right)/(2\pi)\right\rfloor\right)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{181}{15}-z_0\right)^k z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(181-z_0)/(2\pi)\rfloor+1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor} z_0^{-1/2\lfloor\arg(181-z_0)/(2\pi)\rfloor+1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right) \Big/ \left(2\sqrt[4]{5}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(181-z_0)^k z_0^{-k}}{k!}\right)$$

Furthermore, we obtain also:

((((((((((-729/16 (10)^12 (3-(2*1/100))^4 [(((((((4(2-3*100)^-1) – ln(1-(4/(3*100))))))))^2+Pi^2*1/2(((2/sqrt3)*1/10-1))])))))))))^1/5 -5

Input:

$$\left(-\frac{729}{16}\times 10^{12}\left(3-2\times\frac{1}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\pi^2\times\frac{1}{2}\left(\frac{2}{\sqrt{3}}\times\frac{1}{10}-1\right)\right)\right)\text{^}(1/5)-5$$

$\log(x)$ is the natural logarithm

Exact result:

$$3\times 1490^{4/5}\sqrt[5]{3\left(-\frac{1}{2}\left(\frac{1}{5\sqrt{3}}-1\right)\pi^2-\left(\log\left(\frac{75}{74}\right)-\frac{2}{149}\right)^2\right)}-5$$

Decimal approximation:

1729.167242948033764195427187860357602266057441302800682852...

1729.1672429….

Alternate forms:

$$3\times 1490^{4/5}\sqrt[5]{-\frac{1}{10}\left(\sqrt{3}-15\right)\pi^2-3\left(\log\left(\frac{75}{74}\right)-\frac{2}{149}\right)^2}-5$$

$$3\times 1490^{4/5}\sqrt[5]{3\left(\frac{1}{30}\left(15-\sqrt{3}\right)\pi^2-\left(\log\left(\frac{75}{74}\right)-\frac{2}{149}\right)^2\right)}-5$$

$$3\times 1490^{4/5}\sqrt[5]{3\left(\left(\frac{1}{2}-\frac{1}{10\sqrt{3}}\right)\pi^2-\left(\log\left(\frac{75}{74}\right)-\frac{2}{149}\right)^2\right)}-5$$

Alternative representations:

$$\sqrt[5]{\frac{1}{16}\times 10^{12}(-729)\left(3-\frac{2}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\frac{1}{2}\pi^2\left(\frac{2}{\sqrt{3}\ 10}-1\right)\right)}-5=$$
$$-5+\sqrt[5]{-\frac{729}{16}\times 10^{12}\left(3-\frac{2}{100}\right)^4\left(\left(\text{Li}_1\left(\frac{4}{300}\right)+-\frac{4}{298}\right)^2+\frac{1}{2}\pi^2\left(-1+\frac{2}{10\sqrt{3}}\right)\right)}$$

$$\sqrt[5]{\frac{1}{16}\times 10^{12}(-729)\left(3-\frac{2}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\frac{1}{2}\pi^2\left(\frac{2}{\sqrt{3}\ 10}-1\right)\right)}-5=$$
$$-5+\sqrt[5]{-\frac{729}{16}\times 10^{12}\left(3-\frac{2}{100}\right)^4\left(\left(-\log_e\left(1-\frac{4}{300}\right)+-\frac{4}{298}\right)^2+\frac{1}{2}\pi^2\left(-1+\frac{2}{10\sqrt{3}}\right)\right)}$$

$$\sqrt[5]{\frac{1}{16}\times 10^{12}(-729)\left(3-\frac{2}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\frac{1}{2}\pi^2\left(\frac{2}{\sqrt{3}\ 10}-1\right)\right)}-5=$$
$$-5+\sqrt[5]{-\frac{729}{16}\times 10^{12}\left(3-\frac{2}{100}\right)^4\left(\left(S_{0,1}\left(\frac{4}{300}\right)+-\frac{4}{298}\right)^2+\frac{1}{2}\pi^2\left(-1+\frac{2}{10\sqrt{3}}\right)\right)}$$

Series representations:

$$\sqrt[5]{\frac{1}{16}\times 10^{12}(-729)\left(3-\frac{2}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\frac{1}{2}\pi^2\left(\frac{2}{\sqrt{3}\ 10}-1\right)\right)}-5=-5+3\times 1490^{4/5}\sqrt[5]{-\frac{1}{10}\left(-15+\sqrt{3}\right)\pi^2-3\left(-\frac{2}{149}-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{74}\right)^k}{k}\right)^2}$$

$$\sqrt[5]{\frac{1}{16}\times 10^{12}(-729)\left(3-\frac{2}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\frac{1}{2}\pi^2\left(\frac{2}{\sqrt{3}\ 10}-1\right)\right)}-5=-5+3\sqrt[5]{3}\ 1490^{4/5}\sqrt[5]{-\frac{1}{2}\left(-1+\frac{1}{5\sqrt{3}}\right)\pi^2-\left(-\frac{2}{149}-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{74}\right)^k}{k}\right)^2}$$

$$\sqrt[5]{\frac{1}{16}\times 10^{12}(-729)\left(3-\frac{2}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\frac{1}{2}\pi^2\left(\frac{2}{\sqrt{3}\ 10}-1\right)\right)}-5=$$
$$-5+3\sqrt[5]{3}\ 1490^{4/5}\left(-\frac{1}{2}\left(-1+\frac{1}{5\sqrt{3}}\right)\pi^2-\left(-\frac{2}{149}+2i\pi\left\lfloor\frac{\arg\left(\frac{75}{74}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{75}{74}-x\right)^k x^{-k}}{k}\right)^2\right)\hat{}\,(1/5)\quad \text{for } x<0$$

Integral representations:

$$\sqrt[5]{\frac{1}{16}\times 10^{12}(-729)\left(3-\frac{2}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\frac{1}{2}\pi^2\left(\frac{2}{\sqrt{3}\ 10}-1\right)\right)}-5=-5+3\sqrt[5]{3}\ 1490^{4/5}\sqrt[5]{-\frac{1}{2}\left(-1+\frac{1}{5\sqrt{3}}\right)\pi^2-\left(-\frac{2}{149}+\int_1^{\frac{75}{74}}\frac{1}{t}\,dt\right)^2}$$

$$\sqrt[5]{\frac{1}{16}\times 10^{12}(-729)\left(3-\frac{2}{100}\right)^4\left(\left(\frac{4}{2-3\times 100}-\log\left(1-\frac{4}{3\times 100}\right)\right)^2+\frac{1}{2}\pi^2\left(\frac{2}{\sqrt{3}\ 10}-1\right)\right)}-5=-5+3\sqrt[5]{3}\ 1490^{4/5}$$
$$\sqrt[5]{-\frac{1}{2}\left(-1+\frac{1}{5\sqrt{3}}\right)\pi^2-\left(-\frac{2}{149}-\frac{i}{2\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{74^s\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2}$$
for $-1<\gamma<0$

Now, for:

$$-\ \sigma = 0.5$$

$$\mathcal{P}_{\zeta_s} = \mathcal{P}_0 \exp\left[-\frac{\log^2(k/k_*)}{2\sigma^2}\right]$$

$f_{NL} = 3.5$

$u_* = 0.238095$ $\zeta_c = 0.28$ or 0.272 $\zeta_L = 0.6$ or $0.61803398...$

from:

$$\delta_\beta \equiv \frac{\beta - \bar{\beta}}{\bar{\beta}} = \left(\frac{25 + 30\zeta_c f_{\rm NL} + 36 f_{\rm NL}^2 \sigma_s^2 - 5\sqrt{25 + 60\zeta_c f_{\rm NL} + 36 f_{\rm NL}^2 \sigma_s^2}}{3 f_{\rm NL}\sigma_s^2 \sqrt{25 + 60\zeta_c f_{\rm NL} + 36 f_{\rm NL}^2 \sigma_s^2}}\right) \zeta_L \equiv b\,\zeta_L. \tag{4.4}$$

we obtain:

(((25+30*0.28*3.5+36*3.5^2*0.5^2-
5*sqrt(25+60*0.28*3.5+36*3.5^2*0.5^2))))/(((3*3.5*0.5^2*
sqrt(25+60*0.28*3.5+36*3.5^2*0.5^2))))0.6

Input:

$$\frac{25 + 30 \times 0.28 \times 3.5 + 36 \times 3.5^2 \times 0.5^2 - 5\sqrt{25 + 60 \times 0.28 \times 3.5 + 36 \times 3.5^2 \times 0.5^2}}{3 \times 3.5 \times 0.5^2 \sqrt{25 + 60 \times 0.28 \times 3.5 + 36 \times 3.5^2 \times 0.5^2}} \times 0.6$$

Result:

1.55878...

1.558778977819865287107442319063385563915034662116612179682...

1.55877897781…

1+1/(((((25+30*0.28*3.5+36*3.5^2*0.5^2-
5*sqrt(25+60*0.28*3.5+36*3.5^2*0.5^2))))/(((3*3.5*0.5^2*
sqrt(25+60*0.28*3.5+36*3.5^2*0.5^2))))0.6))

Input:

$$1+\frac{1}{\frac{25+30\times0.28\times3.5+36\times3.5^2\times0.5^2-5\sqrt{25+60\times0.28\times3.5+36\times3.5^2\times0.5^2}}{3\times3.5\times0.5^2\sqrt{25+60\times0.28\times3.5+36\times3.5^2\times0.5^2}}\times0.6}$$

Result:

1.641527768996870203160293248899554686971019123902890029778...

1.641527768…

1+1/(((((25+30*0.28*3.5+36*3.5^2*0.5^2-
5*sqrt(25+60*0.28*3.5+36*3.5^2*0.5^2))))/(((3*3.5*0.5^2*
sqrt(25+60*0.28*3.5+36*3.5^2*0.5^2))))0.6)) - (21+2)1/10^3

Input:

$$1+\frac{1}{\frac{25+30\times0.28\times3.5+36\times3.5^2\times0.5^2-5\sqrt{25+60\times0.28\times3.5+36\times3.5^2\times0.5^2}}{3\times3.5\times0.5^2\sqrt{25+60\times0.28\times3.5+36\times3.5^2\times0.5^2}}\times0.6}-(21+2)\times\frac{1}{10^3}$$

Result:

1.618527768996870203160293248899554686971019123902890029778...

1.61852776899...

(((((25+30*0.28*3.5+36*3.5^2*0.5^2-
5*sqrt(25+60*0.28*3.5+36*3.5^2*0.5^2))))/(((3*3.5*0.5^2*
sqrt(25+60*0.28*3.5+36*3.5^2*0.5^2))))0.6))^17-123-47+7-golden ratio

Input:

$$\left(\left(25+30\times0.28\times3.5+36\times3.5^2\times0.5^2-5\sqrt{25+60\times0.28\times3.5+36\times3.5^2\times0.5^2}\right)/\left(3\times3.5\times0.5^2\sqrt{25+60\times0.28\times3.5+36\times3.5^2\times0.5^2}\right)\times 0.6\right)^{17}-123-47+7-\phi$$

ϕ is the golden ratio

Result:

1729.20…

1729.2…

Series representations:

$$\left(\left(0.6\left(25+30\times 0.28\times 3.5+36\times 3.5^2\times 0.5^2-5\sqrt{25+60\times 0.28\times 3.5+36\times 3.5^2\times 0.5^2}\right)\right)/\left(3\times 3.5\times 0.5^2\sqrt{25+60\times 0.28\times 3.5+36\times 3.5^2\times 0.5^2}\right)\right)^{17}-123-47+7-\phi=-163-\phi-\frac{9.67973\left(-32.93+\sqrt{193.05}\sum_{k=0}^{\infty}e^{-5.26295k}\binom{\frac{1}{2}}{k}\right)^{17}}{\sqrt{193.05}^{17}\left(\sum_{k=0}^{\infty}e^{-5.26295k}\binom{\frac{1}{2}}{k}\right)^{17}}$$

$$\left(\left(0.6\left(25+30\times 0.28\times 3.5+36\times 3.5^2\times 0.5^2-5\sqrt{25+60\times 0.28\times 3.5+36\times 3.5^2\times 0.5^2}\right)\right)/\left(3\times 3.5\times 0.5^2\sqrt{25+60\times 0.28\times 3.5+36\times 3.5^2\times 0.5^2}\right)\right)^{17}-123-47+7-\phi=-163-\phi-\frac{9.67973\left(-32.93+\sqrt{193.05}\sum_{k=0}^{\infty}\frac{(-0.00518001)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{17}}{\sqrt{193.05}^{17}\left(\sum_{k=0}^{\infty}\frac{(-0.00518001)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{17}}$$

$$\left(\left(0.6\left(25+30\times 0.28\times 3.5+36\times 3.5^2\times 0.5^2-5\sqrt{25+60\times 0.28\times 3.5+36\times 3.5^2\times 0.5^2}\right)\right)/\left(3\times 3.5\times 0.5^2\sqrt{25+60\times 0.28\times 3.5+36\times 3.5^2\times 0.5^2}\right)\right)^{17}-123-47+7-\phi=-163-\phi+\frac{7.9879\times 10^{31}\left(\sqrt{\pi}-0.0151837\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}e^{-5.26295s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^{17}}{\left(\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}e^{-5.26295s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^{17}}$$

Pi(((((25+30*0.272*3.5+36*3.5^2*0.5^2-5*sqrt(25+60*0.272*3.5+36*3.5^2*0.5^2))))/(((3*3.5*0.5^2* sqrt(25+60*0.272*3.5+36*3.5^2*0.5^2))))1/golden ratio))

Input:

$$\pi\left(\frac{25+30\times0.272\times3.5+36\times3.5^2\times0.5^2-5\sqrt{25+60\times0.272\times3.5+36\times3.5^2\times0.5^2}}{3\times3.5\times0.5^2\sqrt{25+60\times0.272\times3.5+36\times3.5^2\times0.5^2}}\times\frac{1}{\phi}\right)$$

ϕ is the golden ratio

Result:

5.037533085615440475927679384730873132513519921270768323747...

5.0375330856... result practically equal to the solution of the above analyzed formula:

$$\sqrt[4]{\frac{3+2\sqrt[4]{5}}{3-2\sqrt[4]{5}}}=\frac{\sqrt[4]{5}+1}{\sqrt[4]{5}-1}=\frac{1}{2}\left(3+\sqrt[4]{5}+\sqrt{5}+\sqrt[4]{125}\right)$$

= **5.0375591418**

Series representations:

$$\frac{\pi\left(25+30\times0.272\times3.5+36\times3.5^2\times0.5^2-5\sqrt{25+60\times0.272\times3.5+36\times3.5^2\times0.5^2}\right)}{\left(3\times3.5\times0.5^2\sqrt{25+60\times0.272\times3.5+36\times3.5^2\times0.5^2}\right)\phi}=-\frac{1.90476\,\pi}{\phi}+\frac{62.4038\,\pi}{\phi\sqrt{191.37}\sum_{k=0}^{\infty}e^{-5.25421k}\binom{\frac{1}{2}}{k}}$$

$$\frac{\pi\left(25+30\times0.272\times3.5+36\times3.5^2\times0.5^2-5\sqrt{25+60\times0.272\times3.5+36\times3.5^2\times0.5^2}\right)}{\left(3\times3.5\times0.5^2\sqrt{25+60\times0.272\times3.5+36\times3.5^2\times0.5^2}\right)\phi}=-\frac{1.90476\,\pi}{\phi}+\frac{62.4038\,\pi}{\phi\sqrt{191.37}\sum_{k=0}^{\infty}\frac{(-0.00522548)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

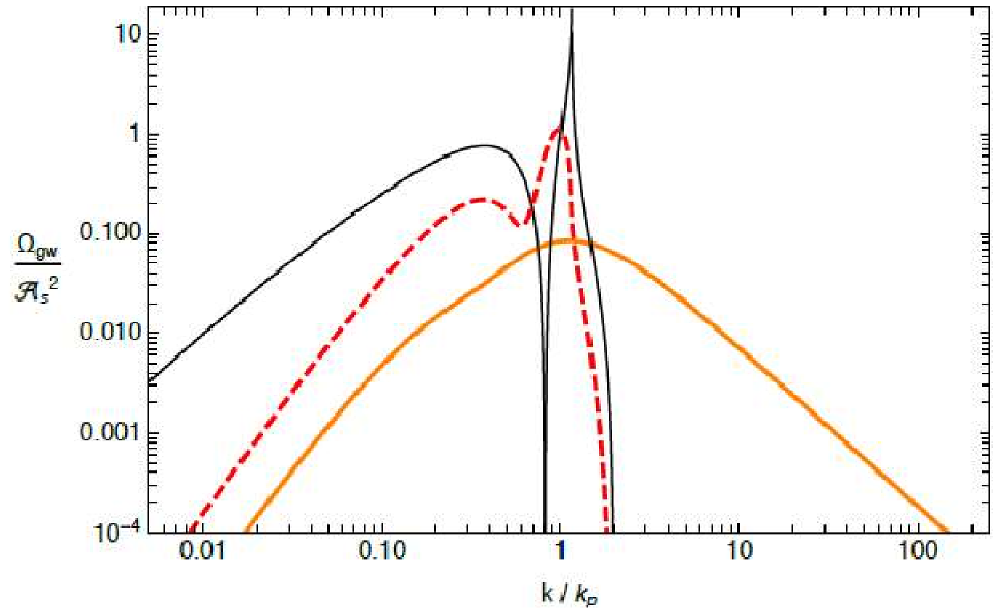
$$\frac{\pi\left(25+30\times0.272\times3.5+36\times3.5^2\times0.5^2-5\sqrt{25+60\times0.272\times3.5+36\times3.5^2\times0.5^2}\right)}{\left(3\times3.5\times0.5^2\sqrt{25+60\times0.272\times3.5+36\times3.5^2\times0.5^2}\right)\phi}$$

$$=-\frac{1.90476\,\pi}{\phi}+\frac{124.808\,\pi\sqrt{\pi}}{\phi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}e^{-5.25421\,s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}$$

From

On the slope of the curvature power spectrum in non-attractor inflation

Ogan Ozsoy, Gianmassimo Tasinato - arXiv:1912.01061v3 [astro-ph.CO] 31 Mar 2020



We have:

$$\mathcal{P}_{\mathcal{R}}(k)=\frac{2.674\,\mathcal{A}_s}{\left[c_4(k/k_p)^{-4}+c_3\,(k/k_p)^{-3}+c_{3/2}\,(k/k_p)^{-3/2}+c_{-0.4}\,(k/k_p)^{0.4}\right]^2} \tag{4.5}$$

with an aim to compare our results with the results of [39]. In the last choice of power spectrum in eq. (4.5), we have the following parameter choices $c_4 = 2.42\times10^{-5}$, $c_3 = 2.94\times10^{-4}$, $c_{3/2} = 5.7\times10^{-1}$, $c_{-0.4} - 2.15$. For the power law profile in (4.4), we make the hypothesis that

For:

$$\frac{\Omega_{gw}}{\mathcal{A}_s^{2}}$$

= 0.0035 from which: $\Omega_{gw} = k^{2.3} = 4^{2.3} = 24.251465$;

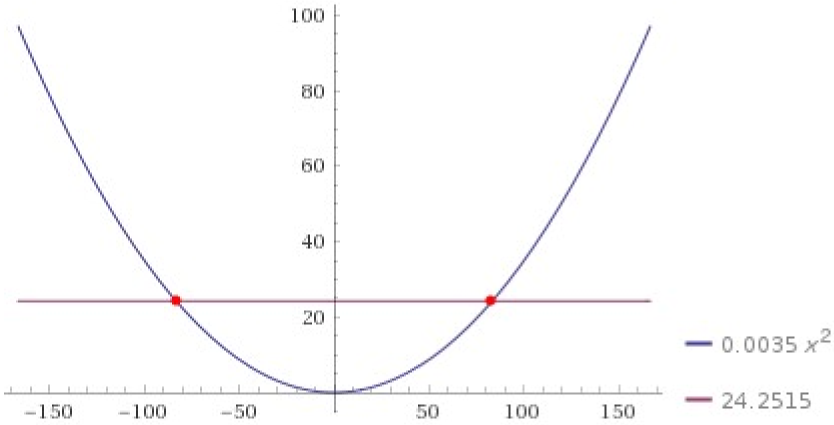
Thence:

0.0035x^2=24.251465

Input interpretation:

$0.0035\,x^2 = 24.251465$

Plot:



Alternate form:

$0.0035\,x^2 - 24.2515 = 0$

Alternate form assuming x is real:

$0.0035\,x^2 + 0 = 24.2515$

Solutions:

$x \approx -83.2406$

$x \approx 83.2406$

83.2406

For $k / k_p = 4 / 5 = 0.80$; $\mathcal{A}_s = 83.2406$; $\Omega_{gw} = 24.251465$;

From

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{2.674\ \mathcal{A}_s}{\left[c_4(k/k_p)^{-4} + c_3\,(k/k_p)^{-3} + c_{3/2}\,(k/k_p)^{-3/2} + c_{-0.4}\,(k/k_p)^{0.4}\right]^2} \qquad (4.5)$$

with an aim to compare our results with the results of [39]. In the last choice of power spectrum in eq. (4.5), we have the following parameter choices $c_4 = 2.42 \times 10^{-5}$, $c_3 = 2.94 \times 10^{-4}$, $c_{3/2} = 5.7 \times 10^{-1}$, $c_{-0.4} = 2.15$. For the power law profile in (4.4), we make the hypothesis that

we obtain:

(2.674*83.2406)/[(2.42e-5)*(0.80)^-4 + (2.94e-4)*(0.80)^-3 + (5.7e-1)*(0.80)^-1.5 + (2.15)*(0.80)^0.4]^2

Input interpretation:

$$\frac{2.674 \times 83.2406}{\left(\frac{2.42 \times 10^{-5}}{0.8^4} + \frac{2.94 \times 10^{-4}}{0.8^3} + \frac{5.7 \times 10^{-1}}{0.8^{1.5}} + 2.15 \times 0.8^{0.4}\right)^2}$$

Result:

29.142870883186583411894057658170484746707520584663904065 45...

29.14287088318… ≈ 29 (Lucas number)

5 /10^3+47*1/((((2.674*83.2406) 1/[(2.42e-5)*(0.80)^-4 + (2.94e-4)*(0.80)^-3 + (5.7e-1)*(0.80)^-1.5 + (2.15)*(0.80)^0.4]^2)))

Input interpretation:

$$\frac{5}{10^3} + 47 \times \frac{1}{(2.674 \times 83.2406) \times \frac{1}{\left(\frac{2.42 \times 10^{-5}}{0.8^4} + \frac{2.94 \times 10^{-4}}{0.8^3} + \frac{5.7 \times 10^{-1}}{0.8^{1.5}} + 2.15 \times 0.8^{0.4}\right)^2}}$$

Result:

1.617744337659463138176189379286239523839495054601458069787...

1.6177443376…

and also:

((((2.674*83.2406)/[(2.42e-5)*(0.80)^-4 + (2.94e-4)*(0.80)^-3 + (5.7e-1)*(0.80)^-1.5 + (2.15)*(0.80)^0.4]^2)))^1/7

Input interpretation:

$$\sqrt[7]{\frac{2.674\times 83.2406}{\left(\frac{2.42\times 10^{-5}}{0.8^4}+\frac{2.94\times 10^{-4}}{0.8^3}+\frac{5.7\times 10^{-1}}{0.8^{1.5}}+2.15\times 0.8^{0.4}\right)^2}}$$

Result:

1.618895830739667306067153533897225516207796095747679133677...

1.61889583073…

and again:

(55+5)*((((2.674*83.2406)/[(2.42e-5)*(0.80)^-4 + (2.94e-4)*(0.80)^-3 + (5.7e-1)*(0.80)^-1.5 + (2.15)*(0.80)^0.4]^2)))-21+golden ratio

Input interpretation:

$$(55+5)\times\frac{2.674\times 83.2406}{\left(\frac{2.42\times 10^{-5}}{0.8^4}+\frac{2.94\times 10^{-4}}{0.8^3}+\frac{5.7\times 10^{-1}}{0.8^{1.5}}+2.15\times 0.8^{0.4}\right)^2}-21+\phi$$

ϕ is the golden ratio

Result:

1729.190286979944899561848046324594722920171544259640006789...

1729.19028697…

(((((55+5)*((((2.674*83.2406)/[(2.42e-5)*(0.80)^-4 + (2.94e-4)*(0.80)^-3 + (5.7e-1)*(0.80)^-1.5 + (2.15)*(0.80)^0.4]^2)))-21+golden ratio))))^1/15

Input interpretation:

$$\sqrt[15]{(55+5)\times\frac{2.674\times 83.2406}{\left(\frac{2.42\times 10^{-5}}{0.8^4}+\frac{2.94\times 10^{-4}}{0.8^3}+\frac{5.7\times 10^{-1}}{0.8^{1.5}}+2.15\times 0.8^{0.4}\right)^2}-21+\phi}$$

ϕ is the golden ratio

Result:

1.6438272889211408475977364792599622008514591408155954983 30...

1.6438272889…

From which:

sqrt(((6(((((55+5)*((((2.674*83.2406)/[(2.42e-5)*(0.80)^-4 + (2.94e-4)*(0.80)^-3 + (5.7e-1)*(0.80)^-1.5 + (2.15)*(0.80)^0.4]^2)))-21+golden ratio))))^1/15)))

Input interpretation:

$$\sqrt{6\sqrt[15]{(55+5)\times\frac{2.674\times 83.2406}{\left(\frac{2.42\times 10^{-5}}{0.8^4}+\frac{2.94\times 10^{-4}}{0.8^3}+\frac{5.7\times 10^{-1}}{0.8^{1.5}}+2.15\times 0.8^{0.4}\right)^2}-21+\phi}}$$

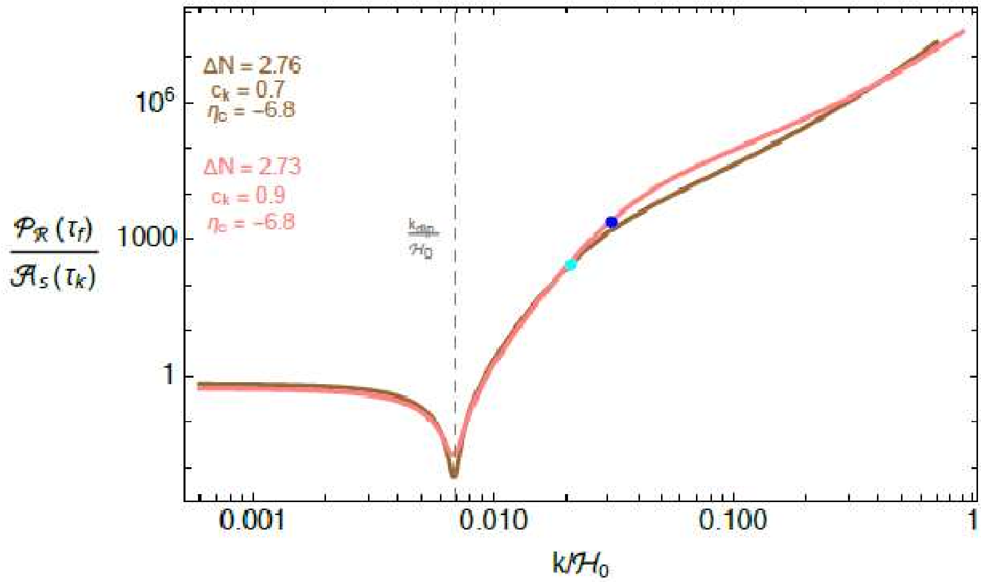
ϕ is the golden ratio

Result:

3.1405355806815571064349952033066490246769328654383075281 00...

3.1405355806815…… ≈ π

Now, we have that:



For modes that leave the horizon during the initial slow-roll era, *i.e.* $x_k > 1$, shape of the power spectrum in Model 1 can be determined solely through the k dependence of the functions $D^{(0)}(\tau_k)$ $F(\tau_k)$, $G_1(\tau_k)$ $G_2(\tau_k)$ in (C.4), (C.10), (C.22) and (C.23) as they appear inside the enhancement factor in (3.7) and (3.8).

In these expressions, it is important to realize that k dependent terms have coefficients that can be organized in a hierarchal way in powers (determined by η_c) of $a(\tau_f)/a(\tau_0) = e^{\Delta N}$ where ΔN is the duration of non-attractor era in number of e-folds. This result reflects the fact that modes that leave during the slow-roll era are enhanced due to the presence of non-attractor era that follows it. It should be also noted that for modes that leave the horizon during slow-roll phase, k dependence of the functions $D^{(0)}(\tau_k)$ $F(\tau_k)$, $G_1(\tau_k)$ $G_2(\tau_k)$ is fixed and do not depend on the properties of the background model (such as the value of η_c) during the non-attractor era that follows it.

We have that:

$$\frac{G_1(\tau_k)}{(\tau_0)^4} = \frac{\eta_c^2\ e^{-(2\eta_c+6)\Delta N}}{(\eta_c+3)^2(\eta_c+1)^2} + \frac{\eta_c^2(3\eta_c+1)\ e^{-(\eta_c+3)\Delta N}}{2(\eta_c+3)^2(\eta_c-1)(\eta_c+1)} + \frac{\eta_c\ (3\eta_c+11)\ e^{-(\eta_c+5)\Delta N}}{2(\eta_c+1)^2(\eta_c+5)(\eta_c+3)}$$

$$+ \frac{\eta_c\left(3\eta_c^2+19\eta_c+18\right)}{4(\eta_c+3)^2(\eta_c+5)} + \frac{\eta_c\ e^{-2\Delta N}}{4(\eta_c+3)(\eta_c+1)} + \frac{(\eta_c-3)\ e^{-4\Delta N}}{8(\eta_c+1)^2(\eta_c-1)}$$
$$+ x_k^{-2}\left[\frac{e^{-(2\eta_c+6)\Delta N}}{(\eta_c+3)^2} + \frac{2\eta_c\ e^{-(\eta_c+3)\Delta N}}{3(\eta_c+3)^2} + \frac{\eta_c^2}{9(\eta_c+3)^2}\right]$$

$$+ x_k^{-1}\left[-\frac{2\eta_c\ e^{-(2\eta_c+6)\Delta N}}{(\eta_c+3)^2(\eta_c+1)} - \frac{\eta_c(11\eta_c+9)\ e^{-(\eta_c+3)\Delta N}}{6(\eta_c+3)^2(\eta_c+1)} - \frac{(3\eta_c+11)\ e^{-(\eta_c+5)\Delta N}}{2(\eta_c+5)(\eta_c+3)(\eta_c+1)}\right.$$
$$\left. - \frac{\eta_c\left(22\eta_c^2+122\eta_c+48\right)}{60(\eta_c+3)^2(\eta_c+5)} - \frac{\eta_c\ e^{-2\Delta N}}{6(\eta_c+3)(\eta_c+1)}\right]$$

$$+ x_k\left[-\frac{e^{-(\eta_c+3)\Delta N}}{6(\eta_c+3)} - \frac{\eta_c}{18(\eta_c+3)}\right]$$

$$+ x_k^2\left[-\frac{\eta_c\ e^{-(\eta_c+3)\Delta N}}{6(\eta_c+3)(\eta_c+1)} - \frac{\eta_c}{12(\eta_c+3)} - \frac{e^{-2\Delta N}}{12(\eta_c+1)}\right] + \frac{7x_k^4}{360}, \tag{C.22}$$
$$\equiv \mathcal{C}_0^{G_1} + \mathcal{C}_2^{G_1}\ x_k^{-2} + \mathcal{C}_1^{G_1}\ x_k^{-1} + \mathcal{C}_{-1}^{G_1}\ x_k + \mathcal{C}_{-2}^{G_1}\ x_k^2 + \frac{7x_k^4}{360}, \qquad x_k > 1.$$

$$\frac{G_2(\tau_k)}{(\tau_0)^2} = x_k^{-4}\left[-\frac{3\ e^{-(2\eta_c+6)\Delta N}}{(\eta_c+3)^2} - \frac{2\eta_c\ e^{-(\eta_c+3)\Delta N}}{(\eta_c+3)^2} - \frac{\eta_c^2}{3(\eta_c+3)^2}\right] \tag{C.23}$$
$$+ x_k^{-3}\left[\frac{3\eta_c\ e^{-(2\eta_c+6)\Delta N}}{(\eta_c+3)^2(\eta_c+1)} + \frac{3\eta_c\ e^{-(\eta_c+3)\Delta N}}{(\eta_c+3)^2} + \frac{3\ e^{-(\eta_c+5)\Delta N}}{(\eta_c+5)(\eta_c+1)}\right.$$
$$\left. + \frac{6\left(\eta_c^2+6\eta_c+4\right)\eta_c}{10(\eta_c+3)^2(\eta_c+5)}\right]$$
$$+ x_k^{-1}\left[-\frac{e^{-(\eta_c+3)\Delta N}}{(\eta_c+3)} - \frac{\eta_c}{3(\eta_c+3)}\right] + \frac{x_k^2}{15},$$
$$\equiv \mathcal{C}_4^{G_2}\ x_k^{-4} + \mathcal{C}_3^{G_2}\ x_k^{-3} + \mathcal{C}_1^{G_2}\ x_k^{-1} + \frac{x_k^2}{15}, \qquad x_k > 1.$$

From (C.22), we obtain:

$$\frac{\eta_c^2\ e^{-(2\eta_c+6)\Delta N}}{(\eta_c+3)^2(\eta_c+1)^2} + \frac{\eta_c^2(3\eta_c+1)\ e^{-(\eta_c+3)\Delta N}}{2(\eta_c+3)^2(\eta_c-1)(\eta_c+1)} + \frac{\eta_c\ (3\eta_c+11)\ e^{-(\eta_c+5)\Delta N}}{2(\eta_c+1)^2(\eta_c+5)(\eta_c+3)}$$

[(-6.8^2 e^-(((2*-6.8+6)*2.73)))/((-3.8)^2(-5.8)^2)] + [(-6.8^2(3*-6.8+1) e^-(((-3.8)*2.73)))/(2(-3.8)^2(-6.8)(-5.8))] + [(-6.8^2(3*-6.8+11)e^-(((-1.8)*2.73)))/(2(-3.8)^2(-1.8)(-3.8))]

Input:

$$-\frac{6.8^2\, e^{-((2\times(-6.8)+6)\times 2.73)}}{(-3.8)^2\,(-5.8)^2} - \frac{6.8^2\,(3\times(-6.8)+1)\,e^{-(-3.8\times 2.73)}}{2\,(-3.8)^2\times(-6.8)\times(-5.8)} - \frac{6.8^2\,(3\times(-6.8)+11)\,e^{-(-1.8\times 2.73)}}{2\,(-3.8)^2\times(-1.8)\times(-3.8)}$$

Result:

$-9.75490\ldots \times 10^7$

-9.7549…*10^7

$$+\frac{\eta_c\,(3\eta_c^2+19\eta_c+18)}{4(\eta_c+3)^2(\eta_c+5)} + \frac{\eta_c\; e^{\;\;2\Delta N}}{4(\eta_c+3)(\eta_c+1)} + \frac{(\eta_c-3)\; e^{\;\;4\Delta N}}{8(\eta_c+1)^2(\eta_c-1)}$$

[(-6.8(3*6.8^2+19*-6.8+18))/(4(-3.8)^2(1.8))]+[(-6.8*e^(-2*2.73))/(4(-3.8)(-5.8))]+[(-9.8*e^(-4*2.73))/(8(-5.8)^2(-7.8))]

Input:

$$\frac{-6.8\,(3\times 6.8^2+19\times(-6.8)+18)}{4\,(-3.8)^2\times 1.8} + \frac{-6.8\,e^{-2\times 2.73}}{4\times(-3.8)\times(-5.8)} + \frac{-9.8\,e^{-4\times 2.73}}{8\,(-5.8)^2\times(-7.8)}$$

Result:

$-1.80027\ldots$

-1.80027…

$$+\,x_k^{-2}\left[\frac{e^{-(2\eta_c+6)\Delta N}}{(\eta_c+3)^2} + \frac{2\eta_c\; e^{-(\eta_c+3)\Delta N}}{3(\eta_c+3)^2} + \frac{\eta_c^2}{9(\eta_c+3)^2}\right]$$

1/9 [(e^-(2*-6.8+6*2.73))/((-3.8)^2)+(2*-6.8*e^-(-3.8*2.73))/(3(-3.8)^2)+((-6.8)^2)/(9(-3.8)^2)]

Input:

$$\frac{1}{9}\left(\frac{e^{-(2\times(-6.8)+6\times 2.73)}}{(-3.8)^2}+\frac{2\times(-6.8)\,e^{-(-3.8\times 2.73)}}{3\,(-3.8)^2}+\frac{(-6.8)^2}{9\,(-3.8)^2}\right)$$

Result:

-1116.77...

-1116.77

$$+\,x_k^{-1}\left[-\frac{2\eta_c\ e^{-(2\eta_c+6)\Delta N}}{(\eta_c+3)^2(\eta_c+1)}-\frac{\eta_c(11\eta_c+9)\ e^{-(\eta_c+3)\Delta N}}{6(\eta_c+3)^2(\eta_c+1)}-\frac{(3\eta_c+11)\ e^{-(\eta_c+5)\Delta N}}{2(\eta_c+5)(\eta_c+3)(\eta_c+1)}-\frac{\eta_c\,(22\eta_c^2+122\eta_c+48)}{60(\eta_c+3)^2(\eta_c+5)}-\frac{\eta_c\ e^{-2\Delta N}}{6(\eta_c+3)(\eta_c+1)}\right]$$

1/3 [-(2*-6.8*e^-(2*-6.8+6)*2.73)/((-3.8)^2(-5.8))-(-6.8*(11*-6.8+9)e^-(-3.8)*2.73)/(6(-3.8)^2(-5.8))-((3*-6.8+11)*e^-(-1.8)*2.73)/(2(-1.8)(-3.8)(-5.8))-(-6.8*(22*-6.8^2+122*6.8+48)/(60(-3.8)^2(-1.8))-(-6.8*e^-(2*2.73))/(6(-3.8)(-5.8))]

1/3 ((-(2*-6.8*e^-(2*-6.8+6*2.73)/((-3.8)^2(-5.8))-(-6.8*(11*-6.8+9)e^-(-3.8*2.73)/(6(-3.8)^2(-5.8))-((3*-6.8+11)*e^-(-1.8*2.73)/(2(-1.8)(-3.8)(-5.8))))))

Input:

$$\frac{1}{3}\left(-\left(2\times(-6.8)\times\frac{e^{-(2\times(-6.8)+6\times 2.73)}}{(-3.8)^2\times(-5.8)}-\left(-6.8\,(11\times(-6.8)+9)\times\frac{e^{-(-3.8\times 2.73)}}{6\,(-3.8)^2\times(-5.8)}-(3\times(-6.8)+11)\times\frac{e^{-(-1.8\times 2.73)}}{2\times(-1.8)\times(-3.8)\times(-5.8)}\right)\right)\right)$$

Result:

-9507.89...

-9507.89...

-9507.89+1/3*(-(-6.8*(22*-6.8^2+122*6.8+48)/(60(-3.8)^2(-1.8))-(-6.8*e^-(2*2.73))/(6(-3.8)(-5.8))))

Input interpretation:

$$-9507.89+\frac{1}{3}\left(-\left(-6.8\times\frac{22\times(-1)\times 6.8^2+122\times 6.8+48}{60\,(-3.8)^2\times(-1.8)}-\frac{-6.8\,e^{-(2\times 2.73)}}{6\times(-3.8)\times(-5.8)}\right)\right)$$

Result:

$-9507.69\ldots$

-9507.69...

$$+ x_k \left[- \frac{e^{-(\eta_c+3)\Delta N}}{6(\eta_c+3)} - \frac{\eta_c}{18(\eta_c+3)} \right]$$

3[-(e^-(-3.8*2.73))/(6(-3.8))+(6.8)/(18(-3.8))]

Input:

$$3\left(-\frac{e^{-(-3.8\times 2.73)}}{6\times(-3.8)} + \frac{6.8}{18\times(-3.8)}\right)$$

Result:

$4212.37\ldots$

4212.37...

$$+ x_k^2 \left[- \frac{\eta_c\, e^{-(\eta_c+3)\Delta N}}{6(\eta_c+3)(\eta_c+1)} - \frac{\eta_c}{12(\eta_c+3)} - \frac{e^{-2\Delta N}}{12(\eta_c+1)} \right] + \frac{7x_k^4}{360},$$

9 [-(-6.8*e^-(-3.8*2.7))/((6(-3.8))(-5.8))+(6.8)/(12(-3.8))-(e^(-2*2.73))/(12(-5.8))]+(7*3^4)/(360)

Input:

$$9\left(-\frac{-6.8\, e^{-(-3.8\times 2.7)}}{(6\times(-3.8))\times(-5.8)} + \frac{6.8}{12\times(-3.8)} - \frac{e^{-2\times 2.73}}{12\times(-5.8)}\right) + \frac{1}{360}\left(7\times 3^4\right)$$

Result:

13220.8...

13220.8....

Thence:

(4212.37 -9507.69-1116.77-1.80027+13220.8)

Input interpretation:

4212.37 − 9507.69 − 1116.77 − 1.80027 + 13 220.8

Result:

6806.90973

6806.90973

The final result is:

[(-6.8^2 e^-(((2*-6.8+6)*2.73)))/((-3.8)^2(-5.8)^2)] + [(-6.8^2(3*-6.8+1) e^-(((-3.8)*2.73)))/(2(-3.8)^2(-6.8)(-5.8))] + [(-6.8^2(3*-6.8+11)e^-(((-1.8)*2.73)))/(2(-3.8)^2(-1.8)(-3.8))]+6806.90973

Input interpretation:

$$-\frac{6.8^2\,e^{-((2\times(-6.8)+6)\times 2.73)}}{(-3.8)^2\,(-5.8)^2}-\frac{6.8^2\,(3\times(-6.8)+1)\,e^{-(-3.8\times 2.73)}}{2\,(-3.8)^2\times(-6.8)\times(-5.8)}-\frac{6.8^2\,(3\times(-6.8)+11)\,e^{-(-1.8\times 2.73)}}{2\,(-3.8)^2\times(-1.8)\times(-3.8)}-151.69607$$

Result:

$-9.75492\ldots\times 10^7$

-9.75492…*10^7

Alternative representation:

$$-\frac{6.8^2\, e^{-((2(-6.8)+6)\,2.73)}}{(-3.8)^2\,(-5.8)^2} + -\frac{6.8^2\,(3(-6.8)+1)\, e^{-(-3.8\times 2.73)}}{2\,(-3.8)^2\,(-6.8)\,(-5.8)} + -\frac{6.8^2\,(3(-6.8)+11)\, e^{-(-1.8\times 2.73)}}{2\,(-3.8)^2\,(-1.8)\,(-3.8)} - 151.696 = -\frac{6.8^2\,\exp^{-((2(-6.8)+6)\,2.73)}(z)}{(-3.8)^2\,(-5.8)^2} + -\frac{6.8^2\,(3(-6.8)+1)\,\exp^{-(-3.8\times 2.73)}(z)}{2\,(-3.8)^2\,(-6.8)\,(-5.8)} + -\frac{6.8^2\,(3(-6.8)+11)\,\exp^{-(-1.8\times 2.73)}(z)}{2\,(-3.8)^2\,(-1.8)\,(-3.8)} - 151.696 \text{ for } z = 1$$

Series representations:

$$-\frac{6.8^2\, e^{-((2(-6.8)+6)\,2.73)}}{(-3.8)^2\,(-5.8)^2} + -\frac{6.8^2\,(3(-6.8)+1)\, e^{-(-3.8\times 2.73)}}{2\,(-3.8)^2\,(-6.8)\,(-5.8)} + -\frac{6.8^2\,(3(-6.8)+11)\, e^{-(-1.8\times 2.73)}}{2\,(-3.8)^2\,(-1.8)\,(-3.8)} - 151.696 = 2.20035 \left(-68.9417 + \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{4.914} + 0.357926 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{10.374} - 0.0432616 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{20.748}\right)$$

$$-\frac{6.8^2\, e^{-((2(-6.8)+6)\,2.73)}}{(-3.8)^2\,(-5.8)^2} + -\frac{6.8^2\,(3(-6.8)+1)\, e^{-(-3.8\times 2.73)}}{2\,(-3.8)^2\,(-6.8)\,(-5.8)} + -\frac{6.8^2\,(3(-6.8)+11)\, e^{-(-1.8\times 2.73)}}{2\,(-3.8)^2\,(-1.8)\,(-3.8)} - 151.696 = 0.0729846 \left(-2078.47 + \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{4.914} + 0.00813147 \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{10.374} - 7.40616\times 10^{-7} \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{20.748}\right)$$

$$-\frac{6.8^2\, e^{-((2(-6.8)+6)\,2.73)}}{(-3.8)^2\,(-5.8)^2} + -\frac{6.8^2\,(3(-6.8)+1)\, e^{-(-3.8\times 2.73)}}{2\,(-3.8)^2\,(-6.8)\,(-5.8)} + -\frac{6.8^2\,(3(-6.8)+11)\, e^{-(-1.8\times 2.73)}}{2\,(-3.8)^2\,(-1.8)\,(-3.8)} - 151.696 = 2.20035 \left(-68.9417 + \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{4.914} + 0.357926 \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{10.374} - 0.0432616 \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{20.748}\right)$$

Note that, the final result is practically equal to the following part of the expression:

[(-6.8^2 e^-(((2*-6.8+6)*2.73)))/((-3.8)^2(-5.8)^2)] + [(-6.8^2(3*-6.8+1) e^-(((-3.8)*2.73)))/(2(-3.8)^2(-6.8)(-5.8))] + [(-6.8^2(3*-6.8+11)e^-(((-1.8)*2.73)))/(2(-3.8)^2(-1.8)(-3.8))]

Input:

$$-\frac{6.8^2 e^{-((2\times(-6.8)+6)\times 2.73)}}{(-3.8)^2 (-5.8)^2} - \frac{6.8^2 (3\times(-6.8)+1) e^{-(-3.8\times 2.73)}}{2(-3.8)^2\times(-6.8)\times(-5.8)} - \frac{6.8^2 (3\times(-6.8)+11) e^{-(-1.8\times 2.73)}}{2(-3.8)^2\times(-1.8)\times(-3.8)}$$

Result:

$-9.75490\ldots\times 10^7$

-9.75490…*10^7

Alternative representation:

$$-\frac{6.8^2 e^{-((2(-6.8)+6)2.73)}}{(-3.8)^2 (-5.8)^2} + -\frac{6.8^2 (3(-6.8)+1) e^{-(-3.8\times 2.73)}}{2(-3.8)^2 (-6.8)(-5.8)} + -\frac{6.8^2 (3(-6.8)+11) e^{-(-1.8\times 2.73)}}{2(-3.8)^2 (-1.8)(-3.8)} = -\frac{6.8^2 \exp^{-((2(-6.8)+6)2.73)}(z)}{(-3.8)^2(-5.8)^2} + -\frac{6.8^2 (3(-6.8)+1)\exp^{-(-3.8\times 2.73)}(z)}{2(-3.8)^2(-6.8)(-5.8)} + -\frac{6.8^2 (3(-6.8)+11)\exp^{-(-1.8\times 2.73)}(z)}{2(-3.8)^2(-1.8)(-3.8)} \quad \text{for } z=1$$

Series representations:

$$-\frac{6.8^2 e^{-((2(-6.8)+6)2.73)}}{(-3.8)^2 (-5.8)^2} + -\frac{6.8^2 (3(-6.8)+1) e^{-(-3.8\times 2.73)}}{2(-3.8)^2 (-6.8)(-5.8)} + -\frac{6.8^2 (3(-6.8)+11) e^{-(-1.8\times 2.73)}}{2(-3.8)^2 (-1.8)(-3.8)} = 2.20035\left(\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4.914} + 0.357926\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{10.374} - 0.0432616\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{20.748}\right)$$

$$-\frac{6.8^2 e^{-((2(-6.8)+6)2.73)}}{(-3.8)^2(-5.8)^2}+-\frac{6.8^2(3(-6.8)+1)e^{-(-3.8\times 2.73)}}{2(-3.8)^2(-6.8)(-5.8)}+-\frac{6.8^2(3(-6.8)+11)e^{-(-1.8\times 2.73)}}{2(-3.8)^2(-1.8)(-3.8)}=0.0729846\left(\left(\sum_{k=0}^{\infty}\frac{1+k}{k!}\right)^{4.914}+0.00813147\left(\sum_{k=0}^{\infty}\frac{1+k}{k!}\right)^{10.374}-7.40616\times 10^{-7}\left(\sum_{k=0}^{\infty}\frac{1+k}{k!}\right)^{20.748}\right)$$

$$-\frac{6.8^2 e^{-((2(-6.8)+6)2.73)}}{(-3.8)^2(-5.8)^2}+-\frac{6.8^2(3(-6.8)+1)e^{-(-3.8\times 2.73)}}{2(-3.8)^2(-6.8)(-5.8)}+-\frac{6.8^2(3(-6.8)+11)e^{-(-1.8\times 2.73)}}{2(-3.8)^2(-1.8)(-3.8)}=2.20035\left(\left(\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}\right)^{4.914}+0.357926\left(\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}\right)^{10.374}-0.0432616\left(\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}\right)^{20.748}\right)$$

From which:

((([(-6.8^2 e^-(((2*-6.8+6)*2.73)))/((-3.8)^2(-5.8)^2)] + [(-6.8^2(3*-6.8+1) e^-(((-3.8)*2.73)))/(2(-3.8)^2(-6.8)(-5.8))] + [(-6.8^2(3*-6.8+11)e^-(((-1.8)*2.73)))/(2(-3.8)^2(-1.8)(-3.8))])))^1/37

Input:

$$\left(-\frac{6.8^2 e^{-((2\times(-6.8)+6)\times 2.73)}}{(-3.8)^2(-5.8)^2}-\frac{6.8^2(3\times(-6.8)+1)e^{-(-3.8\times 2.73)}}{2(-3.8)^2\times(-6.8)\times(-5.8)}-\frac{6.8^2(3\times(-6.8)+11)e^{-(-1.8\times 2.73)}}{2(-3.8)^2\times(-1.8)\times(-3.8)}\right)^{\wedge}(1/37)$$

Result:

1.63816... +
0.139428... i

Polar coordinates:

$r = 1.64409$ (radius), $\theta = 4.86486°$ (angle)

1.64409

Now, from (C.23), we obtain:

$$x_k^{-4}\left[-\frac{3\ e^{-(2\eta_c+6)\Delta N}}{(\eta_c+3)^2}-\frac{2\eta_c\ e^{-(\eta_c+3)\Delta N}}{(\eta_c+3)^2}-\frac{\eta_c^2}{3(\eta_c+3)^2}\right]$$

1/81 [-3*(e^-(2*-6.8+6*2.73))/((-3.8)^2)-(2*-6.8*e^-(-3.8*2.73))/((-3.8)^2)-((-6.8)^2)/(3(-3.8)^2)]

Input:

$$\frac{1}{81}\left(-3\times\frac{e^{-(2\times(-6.8)+6\times2.73)}}{(-3.8)^2}-\frac{2\times(-6.8)\,e^{-(-3.8\times2.73)}}{(-3.8)^2}-\frac{(-6.8)^2}{3\,(-3.8)^2}\right)$$

Result:

372.256...

372.256...

$$+\ x_k^{-3}\left[\frac{3\eta_c\ e^{-(2\eta_c+6)\Delta N}}{(\eta_c+3)^2(\eta_c+1)}+\frac{3\eta_c\ e^{-(\eta_c+3)\Delta N}}{(\eta_c+3)^2}+\frac{3\ e^{-(\eta_c+5)\Delta N}}{(\eta_c+5)(\eta_c+1)}+\frac{6\left(\eta_c^2+6\eta_c+4\right)\eta_c}{10(\eta_c+3)^2(\eta_c+5)}\right]$$

1/27 [3*-6.8(e^-((2*-6.8+6)*2.73))/((((-3.8)^2)(-5.8)))+(3*-6.8*e^-(-3.8*2.73))/((-3.8)^2)+3(e^-((-1.8*2.73)))/((-1.8)(-5.8))+((6(-6.8^2+6*-6.8+4)*-6.8))/((10(-3.8)^2(-1.8)))]

Input:

$$\frac{1}{27}\left(3\times(-6.8)\times\frac{e^{-((2\times(-6.8)+6)\times2.73)}}{(-3.8)^2\times(-5.8)}+\frac{3\times(-6.8)\,e^{-(-3.8\times2.73)}}{(-3.8)^2}+3\left(-\frac{e^{-(-1.8\times2.73)}}{1.8\times(-5.8)}\right)+\frac{6\left(-6.8^2+6\times(-6.8)+4\right)\times(-6.8)}{10\,(-3.8)^2\times(-1.8)}\right)$$

Result:

$9.24559\ldots\times10^6$

$9.24559\ldots*10^6$

Alternative representation:

$$\frac{1}{27}\left(\frac{3(-6.8)\,e^{-((2(-6.8)+6)\,2.73)}}{(-3.8)^2(-5.8)}+\frac{3(-6.8)\,e^{-(-3.8\times 2.73)}}{(-3.8)^2}+-\frac{3\,e^{-(-1.8\times 2.73)}}{1.8(-5.8)}+\frac{6\left(-6.8^2+6(-6.8)+4\right)(-6.8)}{10(-3.8)^2(-1.8)}\right)=$$

$$\frac{1}{27}\left(\frac{3(-6.8)\exp^{-((2(-6.8)+6)\,2.73)}(z)}{(-3.8)^2(-5.8)}+\frac{3(-6.8)\exp^{-(-3.8\times 2.73)}(z)}{(-3.8)^2}+-\frac{3\exp^{-(-1.8\times 2.73)}(z)}{1.8(-5.8)}+\frac{6\left(-6.8^2+6(-6.8)+4\right)(-6.8)}{10(-3.8)^2(-1.8)}\right)\text{ for } z=1$$

Series representations:

$$\frac{1}{27}\left(\frac{3(-6.8)\,e^{-((2(-6.8)+6)\,2.73)}}{(-3.8)^2(-5.8)}+\frac{3(-6.8)\,e^{-(-3.8\times 2.73)}}{(-3.8)^2}+-\frac{3\,e^{-(-1.8\times 2.73)}}{1.8(-5.8)}+\frac{6\left(-6.8^2+6(-6.8)+4\right)(-6.8)}{10(-3.8)^2(-1.8)}\right)=0.0106428\left(-45.3615+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4.914}-4.91634\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{10.374}+0.847645\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{20.748}\right)$$

$$\frac{1}{27}\left(\frac{3(-6.8)\,e^{-((2(-6.8)+6)\,2.73)}}{(-3.8)^2(-5.8)}+\frac{3(-6.8)\,e^{-(-3.8\times 2.73)}}{(-3.8)^2}+-\frac{3\,e^{-(-1.8\times 2.73)}}{1.8(-5.8)}+\frac{6\left(-6.8^2+6(-6.8)+4\right)(-6.8)}{10(-3.8)^2(-1.8)}\right)=0.000353017\left(-1367.57+\left(\sum_{k=0}^{\infty}\frac{1+k}{k!}\right)^{4.914}-0.111691\left(\sum_{k=0}^{\infty}\frac{1+k}{k!}\right)^{10.374}+0.0000145113\left(\sum_{k=0}^{\infty}\frac{1+k}{k!}\right)^{20.748}\right)$$

$$\frac{1}{27}\left(\frac{3(-6.8)\,e^{-((2(-6.8)+6)\,2.73)}}{(-3.8)^2(-5.8)}+\frac{3(-6.8)\,e^{-(-3.8\times 2.73)}}{(-3.8)^2}+-\frac{3\,e^{-(-1.8\times 2.73)}}{1.8(-5.8)}+\frac{6\left(-6.8^2+6(-6.8)+4\right)(-6.8)}{10(-3.8)^2(-1.8)}\right)=0.0106428\left(-45.3615+\left(\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}\right)^{4.914}-4.91634\left(\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}\right)^{10.374}+0.847645\left(\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}\right)^{20.748}\right)$$

$$+x_k^{-1}\left[-\frac{e^{-(\eta_c+3)\Delta N}}{(\eta_c+3)}-\frac{\eta_c}{3(\eta_c+3)}\right]+\frac{x_k^2}{15},$$

1/3 ((((-(e^-(-3.8*2.7)))/(-3.8)+(6.8)/(3(-3.8)))))+9/15

Input:

$$\frac{1}{3}\left(\frac{-e^{-(-3.8\times 2.7)}}{-3.8}+\frac{6.8}{3\times(-3.8)}\right)+\frac{9}{15}$$

Result:

2506.26...

2506.26...

We can write also:

1/27 [3*-6.8(e^-((2*-6.8+6)*2.73))/((((-3.8)^2)(-5.8)))+(3*-6.8*e^-(-3.8*2.73))/((-3.8)^2)+3(e^-((-1.8*2.73)))/((-1.8)(-5.8))+((6(-6.8^2+6*-6.8+4)*-6.8))/((10(-3.8)^2(-1.8)))]+372.256+2506.26

Input interpretation:

$$\frac{1}{27}\left(3\times(-6.8)\times\frac{e^{-((2\times(-6.8)+6)\times 2.73)}}{(-3.8)^2\times(-5.8)}+\frac{3\times(-6.8)\,e^{-(-3.8\times 2.73)}}{(-3.8)^2}+3\left(-\frac{e^{-(-1.8\times 2.73)}}{1.8\times(-5.8)}\right)+\frac{6\left(-6.8^2+6\times(-6.8)+4\right)\times(-6.8)}{10\,(-3.8)^2\times(-1.8)}\right)+372.256+2506.26$$

Result:

$9.24847\ldots\times 10^6$

9.24847…*10^6 (final result)

Alternative representation:

$$\frac{1}{27}\left(\frac{3\,(-6.8)\,e^{-((2\,(-6.8)+6)\,2.73)}}{(-3.8)^2\,(-5.8)}+\frac{3\,(-6.8)\,e^{-(-3.8\times 2.73)}}{(-3.8)^2}+-\frac{3\,e^{-(-1.8\times 2.73)}}{1.8\,(-5.8)}+\frac{6\left(-6.8^2+6\,(-6.8)+4\right)(-6.8)}{10\,(-3.8)^2\,(-1.8)}\right)+372.256+2506.26=$$

$$\frac{1}{27}\left(\frac{3\,(-6.8)\exp^{-((2\,(-6.8)+6)\,2.73)}(z)}{(-3.8)^2\,(-5.8)}+\frac{3\,(-6.8)\exp^{-(-3.8\times 2.73)}(z)}{(-3.8)^2}-\frac{3\exp^{-(-1.8\times 2.73)}(z)}{1.8\,(-5.8)}+\frac{6\left(-6.8^2+6\,(-6.8)+4\right)(-6.8)}{10\,(-3.8)^2\,(-1.8)}\right)+372.256+2506.26\quad\text{for } z=1$$

Series representations:

$$\frac{1}{27}\left(\frac{3(-6.8)\,e^{-((2(-6.8)+6)2.73)}}{(-3.8)^2(-5.8)}+\frac{3(-6.8)\,e^{-(-3.8\times 2.73)}}{(-3.8)^2}+-\frac{3\,e^{-(-1.8\times 2.73)}}{1.8(-5.8)}+\frac{6(-6.8^2+6(-6.8)+4)(-6.8)}{10(-3.8)^2(-1.8)}\right)+372.256+2506.26=0.0106428\left(270\,420.+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{4.914}-4.91634\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{10.374}+0.847645\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{20.748}\right)$$

$$\frac{1}{27}\left(\frac{3(-6.8)\,e^{-((2(-6.8)+6)2.73)}}{(-3.8)^2(-5.8)}+\frac{3(-6.8)\,e^{-(-3.8\times 2.73)}}{(-3.8)^2}+-\frac{3\,e^{-(-1.8\times 2.73)}}{1.8(-5.8)}+\frac{6(-6.8^2+6(-6.8)+4)(-6.8)}{10(-3.8)^2(-1.8)}\right)+372.256+2506.26=0.000353017\left(8.15268\times 10^6+\left(\sum_{k=0}^{\infty}\frac{1+k}{k!}\right)^{4.914}-0.111691\left(\sum_{k=0}^{\infty}\frac{1+k}{k!}\right)^{10.374}+0.0000145113\left(\sum_{k=0}^{\infty}\frac{1+k}{k!}\right)^{20.748}\right)$$

$$\frac{1}{27}\left(\frac{3(-6.8)\,e^{-((2(-6.8)+6)2.73)}}{(-3.8)^2(-5.8)}+\frac{3(-6.8)\,e^{-(-3.8\times 2.73)}}{(-3.8)^2}+-\frac{3\,e^{-(-1.8\times 2.73)}}{1.8(-5.8)}+\frac{6(-6.8^2+6(-6.8)+4)(-6.8)}{10(-3.8)^2(-1.8)}\right)+372.256+2506.26=0.0106428\left(270\,420.+\left(\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}\right)^{4.914}-4.91634\left(\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}\right)^{10.374}+0.847645\left(\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}\right)^{20.748}\right)$$

From the two results, we obtain:

(-9.754222825727129181817553294054189340959600888 79 × 10^7 / 9.248465512268988327179474246578765591388916539 75 × 10^6)

Input interpretation:

$$-\frac{9.75422282572712918181755329405418934095960088879\times 10^7}{9.24846551226898832717947424657876559138891653975\times 10^6}$$

Result:

$-10.5468553813464564749672079095730310208290773715267896714\ldots$

-10.54685538…

-(-9.754222825727129 × 10^7 / 9.24846551226898 × 10^6)

Input interpretation:

$$-\left(-\frac{9.754222825727129\times 10^{7}}{9.24846551226898\times 10^{6}}\right)$$

Result:

10.54685538134646577460529407183095824259574844505368098552...

10.54685538134....

From which:

1/(10^19)*(-(-9.754222825727129 × 10^7 / 9.24846551226898 × 10^6))^1/5

Input interpretation:

$$\frac{1}{10^{19}}\sqrt[5]{-\left(-\frac{9.754222825727129\times 10^{7}}{9.24846551226898\times 10^{6}}\right)}$$

Result:

$1.601860152831229... \times 10^{-19}$

$1.601860152831229...*10^{-19}$ result very near to the electric charge of Positron

(-(-9.754222825727129 × 10^7 + 9.24846551226898 × 10^6))^1/2+golden ratio

Input interpretation:

$$\sqrt{-\left(-9.754222825727129\times 10^{7}+9.24846551226898\times 10^{6}\right)}+\phi$$

ϕ is the golden ratio

Result:

9398.094115212088...

9398.094115212088... result practically equal to the rest mass of Bottom eta meson 9398

(-(-9.754222825727129 × 10^7 + 9.24846551226898 × 10^6))^1/38

Input interpretation:

$$\sqrt[38]{-\left(-9.754222825727129\times 10^{7}+9.24846551226898\times 10^{6}\right)}$$

Result:

1.6184654087395343...

1.6184654087395343…

Note that:

(-(-9.754222825727129 × 10^7 + 9.24846551226898 × 10^6)

Input interpretation:

$$-\left(-9.754222825727129\times 10^{7}+9.24846551226898\times 10^{6}\right)$$

Result:

$8.829376274500231\times 10^{7}$

$8.8293762…*10^7$

If we perform the following exponentiation:

(golden ratio)^38

Input:

ϕ^{38}

ϕ is the golden ratio

Decimal approximation:

$8.7403802999999988558850236756859070538293339952499045\ldots\times 10^{7}$

$8.74038029…*10^7$

Alternate forms:

$$\frac{87403803}{2}+\frac{39088169\sqrt{5}}{2}$$

$$\frac{(1+\sqrt{5})^{38}}{274877906944}$$

$$\frac{1}{2}\left(87403803+39088169\sqrt{5}\right)$$

Alternative representations:

$$\phi^{38}=(2\sin(54°))^{38}$$

$$\phi^{38}=(-2\cos(216°))^{38}$$

$$\phi^{38}=(-2\sin(666°))^{38}$$

Integral representation:

$$(1+z)^a=\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\Gamma(-a)} \quad \text{for } (0<\gamma<-\text{Re}(a) \text{ and } |\arg(z)|<\pi)$$

From the formula of coefficients of the '5th order' mock theta function ψ_1(q):
(A053261 OEIS Sequence)

sqrt(golden ratio) * exp(Pi*sqrt(n/15)) / (2*5^(1/4)*sqrt(n))

(((sqrt(golden ratio) * exp(Pi*sqrt(767/15)) / (2*5^(1/4)*sqrt(767)))))

Input:

$$\sqrt{\phi}\times\frac{\exp\left(\pi\sqrt{\frac{767}{15}}\right)}{2\sqrt[4]{5}\sqrt{767}}$$

ϕ is the golden ratio

Exact result:

$$\frac{e^{\sqrt{767/15}\,\pi}\sqrt{\frac{\phi}{767}}}{2\sqrt[4]{5}}$$

Decimal approximation:

$8.7629056376747427261454674697059005802807969704003840 2\ldots \times 10^7$

8.7629056…*10^7

Property:

$\frac{e^{\sqrt{767/15}\,\pi}\sqrt{\frac{\phi}{767}}}{2\sqrt[4]{5}}$ is a transcendental number

Alternate forms:

$$\frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{7670}}\,e^{\sqrt{767/15}\,\pi}$$

$$\frac{\sqrt{\frac{1+\sqrt{5}}{1534}}\,e^{\sqrt{767/15}\,\pi}}{2\sqrt[4]{5}}$$

Series representations:

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{767}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{767}} = \frac{\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{767}{15}-z_0\right)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}}{2\sqrt[4]{5}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(767-z_0)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{767}{15}}\right)}{2\sqrt[4]{5}\sqrt{767}} = \left(\exp\left(i\pi\left\lfloor\frac{\arg(\phi-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{767}{15}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{767}{15}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(\phi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right) \Big/ \left(2\sqrt[4]{5}\exp\left(i\pi\left\lfloor\frac{\arg(767-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(767-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{767}{15}}\right)}{2\sqrt[4]{5}\sqrt{767}} = \left(\exp\left(\pi\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{767}{15}-z_0\right)/(2\pi)\right\rfloor} z_0^{1/2\left(1+\left\lfloor\arg\left(\frac{767}{15}-z_0\right)/(2\pi)\right\rfloor\right)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{767}{15}-z_0\right)^k z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(767-z_0)/(2\pi)\rfloor+1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor} z_0^{-1/2\lfloor\arg(767-z_0)/(2\pi)\rfloor+1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right) \Big/ \left(2\sqrt[4]{5}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(767-z_0)^k z_0^{-k}}{k!}\right)$$

From which, we obtain:

(((sqrt(golden ratio) * exp(Pi*sqrt(767/15)) / (2*5^(1/4)*sqrt(767)))))^1/38

Input:

$$\sqrt[38]{\sqrt{\phi}\times\frac{\exp\left(\pi\sqrt{\frac{767}{15}}\right)}{2\sqrt[4]{5}\sqrt{767}}}$$

ϕ is the golden ratio

Exact result:

$$\frac{e^{1/38\sqrt{767/15}\,\pi}\sqrt[76]{\frac{\phi}{767}}}{\sqrt[38]{2}\sqrt[152]{5}}$$

Decimal approximation:

1.618143586265540551846619270313146005139478682197079957383...

1.618143586…

Property:

$$\frac{e^{1/38\sqrt{767/15}\,\pi}\sqrt[76]{\frac{\phi}{767}}}{\sqrt[38]{2}\sqrt[152]{5}}$$ is a transcendental number

Alternate form:

$$\frac{\sqrt[76]{\frac{1}{767}\left(1+\sqrt{5}\right)}\,e^{1/38\sqrt{767/15}\,\pi}}{2^{3/76}\sqrt[152]{5}}$$

All 38th roots of (e^(sqrt(767/15) π) sqrt(ϕ/767))/(2 5^(1/4)):

$$\frac{e^{1/38\sqrt{767/15}\,\pi}\,e^{0}\sqrt[76]{\frac{\phi}{767}}}{\sqrt[38]{2}\sqrt[152]{5}} \approx 1.6181$$ (real, principal root)

$$\frac{e^{1/38\sqrt{767/15}\,\pi}\,e^{(i\pi)/19}\sqrt[76]{\frac{\phi}{767}}}{\sqrt[38]{2}\sqrt[152]{5}} \approx 1.5961+0.26634\,i$$

$$\frac{e^{1/38\sqrt{767/15}\,\pi}\,e^{(2i\pi)/19}\sqrt[76]{\frac{\phi}{767}}}{\sqrt[38]{2}\sqrt[152]{5}} \approx 1.5305+0.5254\,i$$

$$\frac{e^{1/38\sqrt{767/15}\,\pi}\,e^{(3i\pi)/19}\sqrt[76]{\frac{\phi}{767}}}{\sqrt[38]{2}\sqrt[152]{5}} \approx 1.4231+0.7702\,i$$

$$\frac{e^{1/38\sqrt{767/15}\,\pi}\,e^{(4i\pi)/19}\sqrt[76]{\frac{\phi}{767}}}{\sqrt[38]{2}\sqrt[152]{5}} \approx 1.2769+0.9939\,i$$

Series representations:

$$\sqrt[38]{\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{767}{15}}\right)}{2\sqrt[4]{5}\sqrt{767}}} = \frac{\sqrt[38]{\frac{\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{767}{15}-z_0\right)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}}{\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(767-z_0)^k z_0^{-k}}{k!}}}}{\sqrt[38]{2}\sqrt[152]{5}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\sqrt[38]{\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{767}{15}}\right)}{2\sqrt[4]{5}\sqrt{767}}} = \frac{1}{\sqrt[38]{2}\sqrt[152]{5}}\left(\left(\left(\exp\left(i\pi\left\lfloor\frac{\arg(\phi-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{767}{15}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{767}{15}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(\phi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\middle/\left(\exp\left(i\pi\left\lfloor\frac{\arg(767-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(767-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\hat{}\,(1/38)\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\sqrt[38]{\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{767}{15}}\right)}{2\sqrt[4]{5}\sqrt{767}}} = \frac{1}{\sqrt[38]{2}\sqrt[152]{5}}\left(\left(\left(\exp\left(\pi\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{767}{15}-z_0\right)/(2\pi)\right\rfloor} z_0^{1/2\left(1+\left\lfloor\arg\left(\frac{767}{15}-z_0\right)/(2\pi)\right\rfloor\right)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{767}{15}-z_0\right)^k z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(767-z_0)/(2\pi)\rfloor+1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor} z_0^{-1/2\lfloor\arg(767-z_0)/(2\pi)\rfloor+1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right)\middle/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(767-z_0)^k z_0^{-k}}{k!}\right)\right)\hat{}\,(1/38)\right)$$

Integral representation:

$$(1+z)^a = \frac{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,d\,s}{(2\,\pi\,i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

The three values (there is also the result of the formula of coefficients of the '5th order' mock theta function $\psi_1(q)$)

$8.829376274500231*10^7$
$8.740380299999998*10^7$
$8.7629056*10^7$

are all very near and this means that the value or result of equations of many physical and / or cosmological parameters could very well correspond to powers, of positive or negative sign, of the golden ratio.

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.
In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the *golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.*

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the *second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.*

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:
2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

Appendix

The OEIS Foundation is supported by donations from users of the OEIS and by a grant from the Simons Foundation.

THE ON-LINE ENCYCLOPEDIA OF INTEGER SEQUENCES ®

founded in 1964 by N. J. A. Sloane

A053261 Coefficients of the '5th order' mock theta function psi_1(q). 17

1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6, 6, 7, 7, 7, 8, 8, 9, 10, 10, 10, 11, 11, 12, 13, 13, 14, 15, 16, 16, 17, 18, 19, 20, 20, 21, 23, 24, 25, 26, 27, 28, 30, 31, 32, 34, 35, 37, 39, 40, 41, 44, 45, 47, 50, 51, 53, 56, 58, 60, 63, 65 (list; graph; refs; listen; history; text; internal format)

OFFSET	0,7
COMMENTS	Number of partitions of n such that each part occurs at most twice and if k occurs as a part then all smaller positive integers occur. Strictly unimodal compositions with rising range 1, 2, 3, .., m where m is the largest part and distinct parts in the falling range (this follows trivially from the comment above). [Joerg Arndt, Mar 26 2014]
REFERENCES	Srinivasa Ramanujan, Collected Papers, Chelsea, New York, 1962, pp. 354-355 Srinivasa Ramanujan, The Lost Notebook and Other Unpublished Papers, Narosa Publishing House, New Delhi, 1988, pp. 19, 21, 22
LINKS	Vaclav Kotesovec, Table of n, a(n) for n = 0..10000 (terms 0..1000 from Alois P. Heinz)

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