Title: Prime numbers and composite numbers congruent to 1 (mod 9)
Author: Zeolla, Gabriel Martin
Comments: 7 pages, 1 graphic table
Subj-class: Number Theory, Prime numbers.
gabrielzvirgo@hotmail.com
ORCID: 0000-0003-0257-1897

Abstract: This paper develops a modified an old and well-known expression for calculating and obtaining all prime numbers greater than three and composite numbers divisible by numbers greater than three. This paper develops formulas to break down the prime numbers and the composite numbers in their reductions, these formulas based on equalities allow to regroup them according to congruence characteristics.

Keywords: Prime numbers, composite numbers, congruence.

Introduction

The study of the prime numbers is wonderful, I have discovered a brilliant expression that contains all the prime numbers greater than 3.

The Prime numbers and composite numbers expressed in the form $(6n \pm 1)$ have the characteristic of having 6 different reductions, the reductions are obtained by adding their digits. These 6 reductions are 1,4,7,2,5,8, this paper develops the correct formulas to obtain prime numbers whit reduction 1.

Methods

The way to solve the exercises will be looking for the pattern of the composite numbers that have the same congruences, then it will be very easy to find the prime numbers. Three reductions are within the sequence $(6n + 1)$ and the other three reductions within the sequence $(6n - 1)$. These reductions are repeated every 18 numbers. The data supplied are sufficient for obtaining a new formula.

Definition

There are 6 types of reductions for prime numbers greater than 3 and for composite numbers divisible by numbers greater than 3.

These 6 reductions are divided into two groups:

$A = 1,4,7$ This is associated with the expression $A = (6n + 1)$	$B = 2,5,8$ This is associated with the expression $B = (6n - 1)$.

The reductions are equivalent with the congruences with (mod 9)

A) A simple way to know which reduction has a number is as follows.

The reductions are obtained by adding the digits of a number. Also if we divide the numbers by 9 we obtain in their decimals the value of their reduction. The reductions are equivalent to the rest in the division.

Example

A) 13= 1+3=**4** 13/9= 1,4444....

B) 67=6+7=13=1+3=**4** 67/9= 7,4444.....

C) 29=2+9=11=1+1=2 29/9=3,22222.......

Chapter1

At point A we establish the original sequence $\beta = (6n \pm 1)$ on which we will try to calculate composite numbers and prime numbers >3.
In point B we will apply the subsequence $N_1 = (18n + 19)$ for the calculation of numbers with reduction 1.
At point C, I demonstrate and apply the expression $\beta * (\delta + 18z)$ by means of equality, which allows the calculation and obtaining only composite numbers with reduction 1.
At point D, I demonstrate how composite numbers are distributed.
At point E, I demonstrate and apply this same expression $\beta * (\delta + 18z)$ to cancel the composite numbers by inequality and thus obtain only prime numbers >3 with reduction 1.

A) Sequence β

This known expression shows in a disordered way, without following any possible pattern, prime numbers greater than 3 and composite numbers divisible by numbers greater than 3.

$$\beta = (6n \pm 1)$$

β_a=$(6n + 1) =$ 7,13,19,25,31,37,43,49,55,61,67,73,79,85,........
β_b= $(6n - 1) =$ 5,11,17,23,29,35,41,47,53,59,65,71,77,83, ...

B) Formula for numbers with reduction 1

Numbers with reduction 1 that are within the sequence β_a=$(6n + 1)$

$$Numbers\ \mathbf{in}\ \beta_a \equiv\ \mathbf{1\ (mod\ 9)}.$$

$$N_1 = (18n_{\geq 0} + 19)$$

$N_1 =$ 19, 37, 55, 73, 91, 109, 127, 145, 163, 181, 199, 217, 235, 253, 271, 289, 307, …

C) Formula for Composite number with reduction 1

My contribution will be in equating the values of the previous formula with the expression $\beta * (\delta + 18\, z)$. The discovery of this expression allows obtaining only the numbers composed with reduction 1. In this way the prime numbers with reduction are discarded 1.

C_1= $Composite$ numbers $\boldsymbol{in}$ $\beta_{\boldsymbol{a}} \equiv \mathbf{1}\ (\mathbf{mod}\ \mathbf{9})$.

$C_1 > 1$

$$C_1 = \beta * (\delta + 18\, z_{\geq 0})$$

$\beta = (6 * n \pm 1)$ = 5,7,11,13,17,19,23,25,29,31,35,37,.........

δ has 6 variants

These 6 variables are always ordered in the same way and are repeated simultaneously until infinity as we add more values of β.

$$\delta = 11,13,23,7,17,19$$

11	**13**	**23**	**7**	**17**	**19**

Demonstration: Composite number with reduction 1

$C_1 = \beta * (\delta + 18\, Z_{\geq 0})$

$=\beta_1 * (11+18*z)$
$=\beta_2 * (13+18*z)$
$=\beta_3 * (23+18*z)$
$=\beta_4 * (7+18*z)$
$=\beta_5 * (17+18*z)$
$=\beta_6 * (19+18*z)$
$=\beta_7 * (11+18*z)$
$=\beta_8 * (13+18*z)$
$=\beta_9 * (23+18*z)$
$=\beta_{10} * (7+18*z)$
$=\beta_{11} * (17+18*z)$
$=\beta_{12} * (19+18*z)$
continue infinitely

$C_1 = \beta * (\delta + 18\, Z_{\geq 0})$

$=5*(11+18*z)$
$=7*(13+18*z)$
$=11*(23+18*z)$
$=13*(7+18*z)$
$=17*(17+18*z)$
$=19*(19+18*z)$
$=23*(11+18*z)$
$=25*(13+18*z)$
$=29*(23+18*z)$
$=31*(7+18*z)$
$=35*(17+18*z)$
$=37*(19+18*z)$
continue infinitely

We can add more β numbers and expand the formula infinitely.

$$We\ solve\ the\ previous\ example\ when\ Z = 0, Z = 1, Z = 2, \ldots\ldots\ldots\ldots$$

Therefore it is
$C_1 > 1$

C_1

$=55,145,235,....$
$=91,217,343,....$
$=253,451,649,....$
$=91,325,559,....$
$=289,595,901,....$
$=361,703,1045,...$
$=253,667,1081,.....$
$=325,775,1225....$
$=667,1189,1711,...$
$=217,775,1333,...$
$=595,1225,1855,......$
$=703,1369,2035,...$
continue infinetely

Reference: A247681 (Composite number with reduction 1)

55, 91, 145, 217, 235, 253, 289, 325, 343, 361, 415, 451, 469, 505, 559, 595, 649, 667, 685, 703, 721, 775, 793, 847, 865, 901, 955, 973, 1027, 1045, 1081, 1099, 1135, 1189, 1207, 1225, 1243, 1261, 1315, 1333, 1351, 1369, 1387, 1405,.....

D) Distances between composite numbers with reduction 1.

The distance between composite numbers with reduction 1 when we use the same value for β is equal to:

Distance between composite number $\boldsymbol{D_1}$= 18* $\boldsymbol{\beta}$

$\boldsymbol{D_1}$= Distance between composite number (Reduction 1).

Example

A. $\beta = 5;\quad D_1 = 18 * 5 = \mathbf{90}$
B. $\beta = 7;\quad D_1 = 18 * 7 = \mathbf{126}$
C. $\beta = 11;\quad D_1 = 18 * 11 = \mathbf{198}$
D. $\beta = 13;\quad D_1 = 18 * 13 = \mathbf{234}$

E) Formula for Prime numbers with reduction 1

To calculate the prime numbers we will use the same formula as for the calculation of the composite numbers although we will use the inequality symbol. By using this symbol we are canceling all the numbers composed of the expression $(18n + 19)$, so only the prime numbers will remain.

$$P_1 = \ Prime\ numbers\ in\ \boldsymbol{\beta_a} \equiv \ 1\ (mod\ 9).$$

Formula Prime numbers

$$P_1 = N_1 \wedge (\neg C_1)$$

$$P_1 = (18n_{\geq 0} + 19) \wedge \neq \beta * (\delta + 18z_{\geq 0})$$

Demonstration

$$P_1 = (18n + 19) \wedge \neq \beta * (\delta + 18z)$$

$(18n + 19) =$	$\neq \beta_n * (\delta + 18z)$
19, 37, ~~55~~, 73, ~~91~~, 109, 127, ~~145~~, 163 181, 199, ~~217~~, ~~235~~, ~~253~~, 271, ~~289~~, 307 ~~325~~, ~~343~~, 361, 379, 397, ~~415~~, 433, ~~451~~ ~~469~~, 487, ~~505~~, … … … ..	$\neq$ 55,145,235,325,415,505 … . $\neq$ 91,217,343, 469, … . $\neq$ 253,451,649, … . $\neq$ 91,325,559, … . $\neq$ 289,595,901, … . $\neq$ 361,703,1045, … $\neq$ 253,667,1081, … … $\neq$ 325,775,1225, … . $\neq$ 667,1189,1711, … $\neq$ 217,775,1333 …

We get the following prime numbers

P_1=19, 37, 73, 109, 127, 163, 181, 199, 271, 307, 379, 397, 433, 487, 523, 541, 577, 613, 631, 739, 757, 811, 829, 883, 919, 937, 991, 1009, 1063, 1117, 1153, 1171, 1279, 1297, 1423, 1459, 1531, 1549, 1567, 1621, 1657, 1693, 1747, 1783, 1801, 1873, 1999,……..

All the Prime numbers are reduced to 1.

Reference A061237 **(The On-line Enciclopedia of integers sequences)**

Graphics tables 1

In the graph we can see how the numbers with reduction 1 are systematically ordered every 18 numbers.

Reduction 1

			1						
			7						
			13	1					
			19	2	1				
			25	3	2	1			
		1	31	4	3	2	1		
		2	37	5	4	3	2	1	
		3	43	6	5	4	3	2	1
		4	49	7	6	5	4	3	2
		5	55	1	7	6	5	4	3
	1	1	61	2	8	7	6	5	4
	2	2	67	3	9	8	7	6	5
	3	3	73	4	10	9	8	7	6
	4	4	79	5	11	10	9	8	7
	5	5	85	6	12	11	10	9	8
1	6	1	91	7	13	12	11	10	9
2	7	2	97	1	1	13	12	11	10
3	8	3	103	2	2	14	13	12	11
4	9	4	109	3	3	15	14	13	12
5	10	5	115	4	4	16	15	14	13
6	11	1	121	5	5	17	16	15	14
7	1	2	127	6	6	18	17	16	15
8	2	3	133	7	7	19	18	17	16
9	3	4	139	1	8	1	19	18	17
10	4	5	145	2	9	2	20	19	18
11	5	1	151	3	10	3	21	20	19

Conclusion

The order of the prime numbers and composite numbers is done by combining the β numbers.
These formulas are simple and easy although extensive, and infinity.
These formulas allow to obtain in a simple way the prime numbers greater than three congruent to (1) Mod 9. Also the composite numbers (1) Mod 9.
The prime numbers 7,11,13,17,19 and 23 are the key to the formula to understand how these numbers are distributed.

References

1. Burton, J. (1964). The Theory of Numbers. United States of America. Library of Congress Catalog Card Number: 55-6187. Página 41
2. Burton, D. M. "The Theory of Congruences." Ch. 4 in *Elementary Number Theory, 4th ed.* Boston, MA: Allyn and Bacon, pp. 80-105, 1989.
3. Dickson L. E., History of the Theory of Numbers.
4. Euclides, Elementorum Libri XV (Lipsiae 1769, books 7,8,9)
5. Guy, Unsolved problems in Number Theory (1981).
6. Hardy, G. H. and Wright, E. M. An Introduction to the Theory of Numbers, 5th ed. Oxford, England: Clarendon Press, pp. 13 and 219, 1979.
7. H. Riesel, Prime Numbers and Computer Methods for Factorization, Birkhaeuser Boston, Cambridge MA 1994.
8. Hua Loo Keng, Introduction to Number theory (1982).
9. M. Dietzfelbinger, Primality Testing in Polynomial Time, Springer NY 2004.
10. Nagell, T. "Theory of Congruences." Ch. 3 in *Introduction to Number Theory.* New York: Wiley, pp. 68-131, 1951.
11. Shanks, Solved and unsolved problems in Number theory (3 ed. 1985).
12. T. M. Apostol, Introduction to Analytic Number Theory, Springer-Verlag, 1976, page 2.
13. W. & F. Ellison, Prime Numbers, Hermann Paris 1985
14. **Zeolla Gabriel M., Patrón Dorado II, ISBN 978-987-42-6105-2, 10/2017 (This book contains what was written in this paper.)**
15. Prime numbers and twin prime numbers algorithm
https://www.academia.edu/39938028/Prime_numbers_and_twin_prime_numbers_algorithm

Professor Zeolla Gabriel Martin
Buenos Aires, Argentina
November 2017