On some Ramanujan equations: various mathematical connections with ϕ, $\zeta(2)$, and some Physical parameters

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Abstract

In this paper we have described several Ramanujan equations and obtained various mathematical connections with ϕ, $\zeta(2)$, and some Physical parameters

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From:

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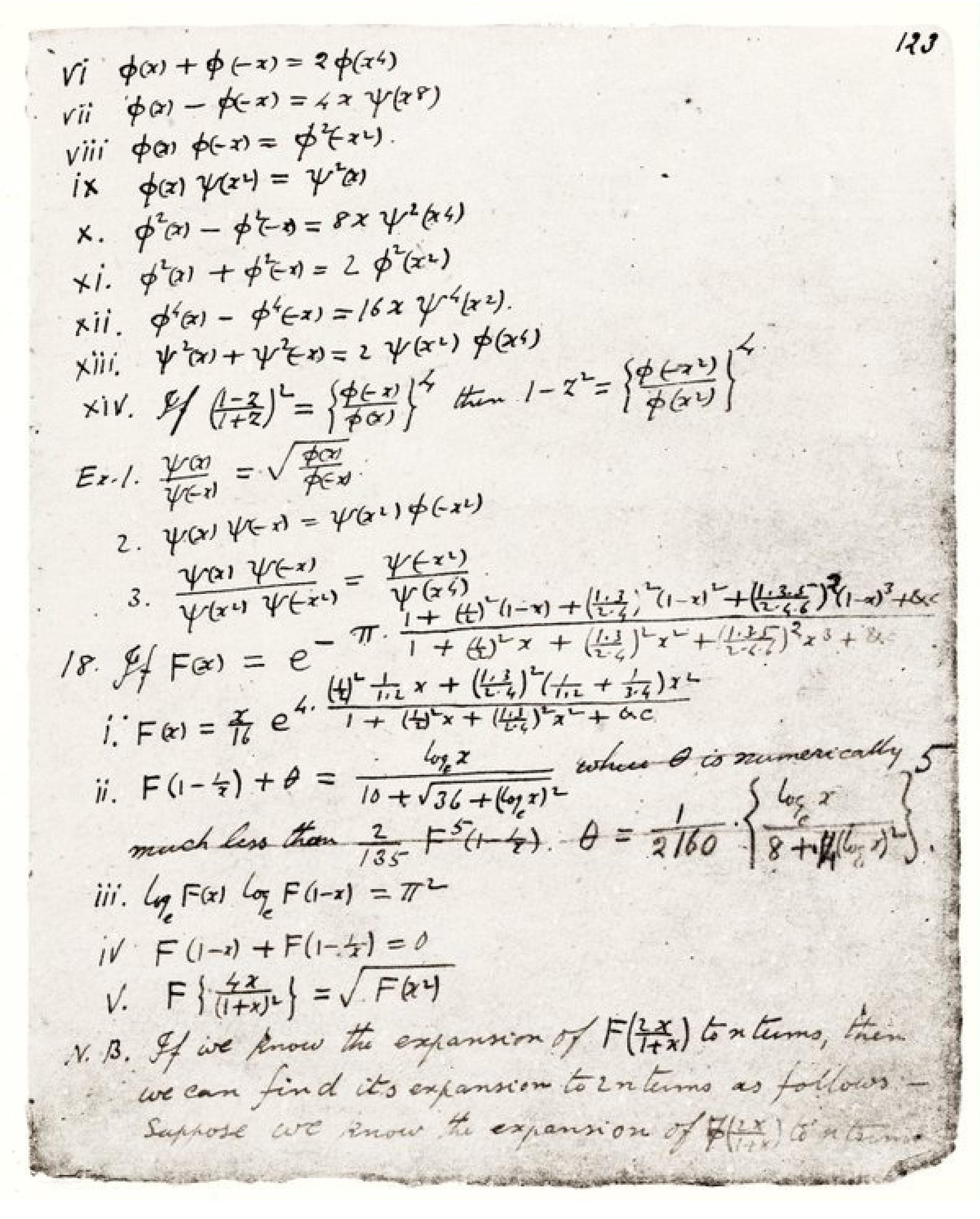


123

vi $\phi(x) + \phi(-x) = 2\phi(x^4)$

vii $\phi(x) - \phi(-x) = 4x\,\psi(x^8)$

viii $\phi(x)\,\phi(-x) = \phi^2(-x^2)$.

ix $\phi(x)\,\psi(x^2) = \psi^2(x)$

x. $\phi^2(x) - \phi^2(-x) = 8x\,\psi^2(x^4)$

xi. $\phi^2(x) + \phi^2(-x) = 2\,\phi^2(x^2)$

xii. $\phi^4(x) - \phi^4(-x) = 16x\,\psi^4(x^2)$.

xiii. $\psi^2(x) + \psi^2(-x) = 2\,\psi(x^2)\,\phi(x^4)$

xiv. If $\left(\frac{1-z}{1+z}\right)^2 = \left\{\frac{\phi(-x)}{\phi(x)}\right\}^4$ then $1 - z^2 = \left\{\frac{\phi(-x^2)}{\phi(x^2)}\right\}^4$

Ex. 1. $\frac{\psi(x)}{\psi(-x)} = \sqrt{\frac{\phi(x)}{\phi(-x)}}$.

2. $\psi(x)\,\psi(-x) = \psi(x^2)\,\phi(-x^2)$

3. $\frac{\psi(x)\,\psi(-x)}{\psi(x^2)\,\psi(-x^2)} = \frac{\psi(-x^2)}{\psi(x^4)}$

18. If $F(x) = e^{-\pi\cdot\frac{1 + (\frac{1}{2})^2(1-x) + (\frac{1.3}{2.4})^2(1-x)^2 + (\frac{1.3.5}{2.4.6})^2(1-x)^3 + \&c}{1 + (\frac{1}{2})^2 x + (\frac{1.3}{2.4})^2 x^2 + (\frac{1.3.5}{2.4.6})^2 x^3 + \&c}}$

i. $F(x) = \frac{x}{16}\,e^{4\cdot\frac{(\frac{1}{2})^2\frac{1}{1.2}x + (\frac{1.3}{2.4})^2(\frac{1}{1.2} + \frac{1}{3.4})x^2}{1 + (\frac{1}{2})^2 x + (\frac{1.3}{2.4})^2 x^2 + \&c}}$

ii. $F(1 - \frac{1}{x}) + \theta = \frac{\log_e x}{10 + \sqrt{36 + (\log_e x)^2}}$ ~~where θ is numerically much less than $\frac{2}{135}F^5(1-\frac{1}{x})$~~. $\theta = \frac{1}{2160}\cdot\left\{\frac{\log_e x}{8 + (\log_e x)^2}\right\}^5$.

iii. $\log_e F(x)\,\log_e F(1-x) = \pi^2$

iv $F(1-x) + F(1-\frac{1}{x}) = 0$

v. $F\left\{\frac{4x}{(1+x)^2}\right\} = \sqrt{F(x^2)}$

N. B. If we know the expansion of $F(\frac{2x}{1+x})$ to n terms, then we can find its expansion to $2n$ terms as follows — Suppose we know the expansion of $F(\frac{2x}{1+x})$ to n terms

From (18), we have the following equations:

exp(((((-Pi* ((1+1/4(1-x)+(3/8)^2(1-x)^2+(15/48)^2(1-x)^3)) / ((1+1/4 x+(3/8)^2 x^2+(15/48)^2 x^3)))))))

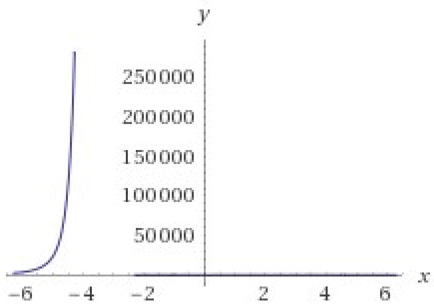
Input:

$$\exp\left(-\pi\times\frac{1+\frac{1}{4}(1-x)+\left(\frac{3}{8}\right)^2(1-x)^2+\left(\frac{15}{48}\right)^2(1-x)^3}{1+\frac{1}{4}x+\left(\frac{3}{8}\right)^2x^2+\left(\frac{15}{48}\right)^2x^3}\right)$$

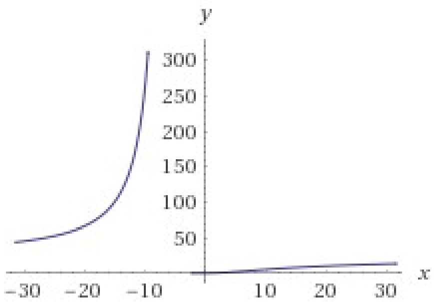
Exact result:

$$e^{-\frac{\pi\left(\frac{25}{256}(1-x)^3+\frac{9}{64}(1-x)^2+\frac{1-x}{4}+1\right)}{\frac{25x^3}{256}+\frac{9x^2}{64}+\frac{x}{4}+1}}$$

Plots:



(x from -6.3 to 6.3)



(x from -31.4 to 31.4)

Alternate forms:

$$e^{\frac{\pi(x(x(25x-111)+211)-381)}{x(x(25x+36)+64)+256}}$$

$$e^{\frac{\pi\left(25x^3-111x^2+211x-381\right)}{25x^3+36x^2+64x+256}}$$

Alternate form assuming x is real:

$$e^{-\frac{-\frac{25\pi x^3}{256}+\frac{111\pi x^2}{256}-\frac{211\pi x}{256}+\frac{381\pi}{256}}{\frac{25x^3}{256}+\frac{9x^2}{64}+\frac{x}{4}+1}}$$

Roots:

(no roots exist)

Properties as a real function:

Domain

$\{x \in \mathbb{R} : x \neq -2.28315\}$

Range

$\{y \in \mathbb{R} : y > 23.1407 \text{ or } 0 < y < 23.1407\}$

Injectivity

injective (one-to-one)

$\mathbb{R}$ is the set of real numbers

Series expansion at x = 0:

$$e^{-(381\pi)/256} + \frac{1225\, e^{-(381\pi)/256}\, \pi\, x}{1024} + \frac{8575\, e^{-(381\pi)/256}\, \pi\,(175\pi - 128)\, x^2}{2\,097\,152} + \frac{1225\, e^{-(381\pi)/256}\, \pi\,(1\,081\,344 - 3\,292\,800\,\pi + 1\,500\,625\,\pi^2)\, x^3}{6\,442\,450\,944} + \frac{1}{26\,388\,279\,066\,624} 1225\, e^{-(381\pi)/256}\, \pi\, (-2\,038\,431\,744 + 8\,248\,934\,400\,\pi - 8\,067\,360\,000\,\pi^2 + 1\,838\,265\,625\,\pi^3)\, x^4 + O(x^5)$$

(Taylor series)

Series expansion at x = ∞:

$$e^{\pi} - \frac{147\,(e^{\pi}\pi)}{25\,x} + \frac{147\, e^{\pi}\,\pi\,(122 + 147\,\pi)}{1250\,x^2} - \frac{49\,(e^{\pi}\,\pi\,(19\,826 + 53\,802\,\pi + 21\,609\,\pi^2))}{31\,250\,x^3} + O\left(\left(\frac{1}{x}\right)^4\right)$$

(Laurent series)

Derivative:

$$\frac{d}{dx}\left(\exp\left(-\frac{\pi\left(1 + \frac{1-x}{4} + \left(\frac{3}{8}\right)^2 (1-x)^2 + \left(\frac{15}{48}\right)^2 (1-x)^3\right)}{1 + \frac{x}{4} + \left(\frac{3}{8}\right)^2 x^2 + \left(\frac{15}{48}\right)^2 x^3}\right)\right) = \frac{1225\,\pi\,(3x^4 - 6x^3 + 27x^2 - 24x + 64)\exp\left(\frac{\pi(25x^3 - 111x^2 + 211x - 381)}{25x^3 + 36x^2 + 64x + 256}\right)}{(25x^3 + 36x^2 + 64x + 256)^2}$$

Limit:

$$\lim_{x \to \pm\infty} \exp\left(-\frac{\pi\left(1 + \frac{1-x}{4} + \frac{9}{64}(1-x)^2 + \frac{25}{256}(1-x)^3\right)}{1 + \frac{x}{4} + \frac{9x^2}{64} + \frac{25x^3}{256}}\right) = e^{\pi} \approx 23.1407$$

Now, for x = 2

exp(((((-Pi* ((1+1/4(1-2)+(3/8)^2(1-2)^2+(15/48)^2(1-2)^3)) / ((1+1/4 * 2+(3/8)^2 * 2^2+(15/48)^2 * 2^3)))))))

we obtain:

Input:

$$\exp\left(-\pi \times \frac{1+\frac{1}{4}(1-2)+\left(\frac{3}{8}\right)^2 (1-2)^2+\left(\frac{15}{48}\right)^2 (1-2)^3}{1+\frac{1}{4}\times 2+\left(\frac{3}{8}\right)^2 \times 2^2+\left(\frac{15}{48}\right)^2 \times 2^3}\right)$$

Exact result:

$$e^{-(29\pi)/104}$$

Decimal approximation:

0.4164366089307012351981966541617455222297165340221635985 79...

0.4164366089307……

Property:

$e^{-(29\pi)/104}$ is a transcendental number

Alternative representations:

$$\exp\left(-\frac{\pi\left(1+\frac{1-2}{4}+\left(\frac{3}{8}\right)^2 (1-2)^2+\left(\frac{15}{48}\right)^2 (1-2)^3\right)}{1+\frac{2}{4}+\left(\frac{3}{8}\right)^2 2^2+\left(\frac{15}{48}\right)^2 2^3}\right) = \exp\left(-\frac{180°\left(1-\frac{1}{4}+\left(\frac{3}{8}\right)^2-\left(\frac{15}{48}\right)^2\right)}{1+\frac{2}{4}+4\left(\frac{3}{8}\right)^2+8\left(\frac{15}{48}\right)^2}\right)$$

$$\exp\left(-\frac{\pi\left(1+\frac{1-2}{4}+\left(\frac{3}{8}\right)^2 (1-2)^2+\left(\frac{15}{48}\right)^2 (1-2)^3\right)}{1+\frac{2}{4}+\left(\frac{3}{8}\right)^2 2^2+\left(\frac{15}{48}\right)^2 2^3}\right) = \exp\left(\frac{i\log(-1)\left(1-\frac{1}{4}+\left(\frac{3}{8}\right)^2-\left(\frac{15}{48}\right)^2\right)}{1+\frac{2}{4}+4\left(\frac{3}{8}\right)^2+8\left(\frac{15}{48}\right)^2}\right)$$

$$\exp\left(-\frac{\pi\left(1+\frac{1-2}{4}+\left(\frac{3}{8}\right)^2 (1-2)^2+\left(\frac{15}{48}\right)^2 (1-2)^3\right)}{1+\frac{2}{4}+\left(\frac{3}{8}\right)^2 2^2+\left(\frac{15}{48}\right)^2 2^3}\right) = \exp\left(-\frac{2\,i\log\left(\frac{1-i}{1+i}\right)\left(1-\frac{1}{4}+\left(\frac{3}{8}\right)^2-\left(\frac{15}{48}\right)^2\right)}{1+\frac{2}{4}+4\left(\frac{3}{8}\right)^2+8\left(\frac{15}{48}\right)^2}\right)$$

Series representations:

$$\exp\left(-\frac{\pi\left(1+\frac{1-2}{4}+\left(\frac{3}{8}\right)^2(1-2)^2+\left(\frac{15}{48}\right)^2(1-2)^3\right)}{1+\frac{2}{4}+\left(\frac{3}{8}\right)^2 2^2+\left(\frac{15}{48}\right)^2 2^3}\right)=e^{-29/26\sum_{k=0}^{\infty}(-1)^k/(1+2k)}$$

$$\exp\left(-\frac{\pi\left(1+\frac{1-2}{4}+\left(\frac{3}{8}\right)^2(1-2)^2+\left(\frac{15}{48}\right)^2(1-2)^3\right)}{1+\frac{2}{4}+\left(\frac{3}{8}\right)^2 2^2+\left(\frac{15}{48}\right)^2 2^3}\right)=\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{-(29\pi)/104}$$

$$\exp\left(-\frac{\pi\left(1+\frac{1-2}{4}+\left(\frac{3}{8}\right)^2(1-2)^2+\left(\frac{15}{48}\right)^2(1-2)^3\right)}{1+\frac{2}{4}+\left(\frac{3}{8}\right)^2 2^2+\left(\frac{15}{48}\right)^2 2^3}\right)=\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{-(29\pi)/104}$$

Integral representations:

$$\exp\left(-\frac{\pi\left(1+\frac{1-2}{4}+\left(\frac{3}{8}\right)^2(1-2)^2+\left(\frac{15}{48}\right)^2(1-2)^3\right)}{1+\frac{2}{4}+\left(\frac{3}{8}\right)^2 2^2+\left(\frac{15}{48}\right)^2 2^3}\right)=e^{-29/26\int_0^1\sqrt{1-t^2}\,dt}$$

$$\exp\left(-\frac{\pi\left(1+\frac{1-2}{4}+\left(\frac{3}{8}\right)^2(1-2)^2+\left(\frac{15}{48}\right)^2(1-2)^3\right)}{1+\frac{2}{4}+\left(\frac{3}{8}\right)^2 2^2+\left(\frac{15}{48}\right)^2 2^3}\right)=e^{-29/52\int_0^1 1/\sqrt{1-t^2}\,dt}$$

$$\exp\left(-\frac{\pi\left(1+\frac{1-2}{4}+\left(\frac{3}{8}\right)^2(1-2)^2+\left(\frac{15}{48}\right)^2(1-2)^3\right)}{1+\frac{2}{4}+\left(\frac{3}{8}\right)^2 2^2+\left(\frac{15}{48}\right)^2 2^3}\right)=e^{-29/52\int_0^\infty 1/(1+t^2)\,dt}$$

From which:

4((exp(((((-Pi* ((1+1/4(1-2)+(3/8)^2(1-2)^2+(15/48)^2(1-2)^3)) / ((1+1/4 * 2+(3/8)^2 * 2^2+(15/48)^2 * 2^3)))))))))

Input:

$$4\exp\left(-\pi\times\frac{1+\frac{1}{4}(1-2)+\left(\frac{3}{8}\right)^2(1-2)^2+\left(\frac{15}{48}\right)^2(1-2)^3}{1+\frac{1}{4}\times 2+\left(\frac{3}{8}\right)^2\times 2^2+\left(\frac{15}{48}\right)^2\times 2^3}\right)$$

Exact result:

$$4\,e^{-(29\pi)/104}$$

Decimal approximation:

1.665746435722804940792786616646982088918866136088654394319...

1.6657464357… result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

Property:

$4 e^{-(29\pi)/104}$ is a transcendental number

Series representations:

$$4 \exp\left(-\frac{\pi\left(1+\frac{1-2}{4}+\left(\frac{3}{8}\right)^2 (1-2)^2+\left(\frac{15}{48}\right)^2 (1-2)^3\right)}{1+\frac{2}{4}+\left(\frac{3}{8}\right)^2 2^2+\left(\frac{15}{48}\right)^2 2^3}\right) = 4 e^{-29/26 \sum_{k=0}^{\infty} (-1)^k/(1+2k)}$$

$$4 \exp\left(-\frac{\pi\left(1+\frac{1-2}{4}+\left(\frac{3}{8}\right)^2 (1-2)^2+\left(\frac{15}{48}\right)^2 (1-2)^3\right)}{1+\frac{2}{4}+\left(\frac{3}{8}\right)^2 2^2+\left(\frac{15}{48}\right)^2 2^3}\right) = 4 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{-(29\pi)/104}$$

$$4 \exp\left(-\frac{\pi\left(1+\frac{1-2}{4}+\left(\frac{3}{8}\right)^2 (1-2)^2+\left(\frac{15}{48}\right)^2 (1-2)^3\right)}{1+\frac{2}{4}+\left(\frac{3}{8}\right)^2 2^2+\left(\frac{15}{48}\right)^2 2^3}\right) = 4 \left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}\right)^{-(29\pi)/104}$$

Integral representations:

$$4 \exp\left(-\frac{\pi\left(1+\frac{1-2}{4}+\left(\frac{3}{8}\right)^2 (1-2)^2+\left(\frac{15}{48}\right)^2 (1-2)^3\right)}{1+\frac{2}{4}+\left(\frac{3}{8}\right)^2 2^2+\left(\frac{15}{48}\right)^2 2^3}\right) = 4 e^{-29/26 \int_0^1 \sqrt{1-t^2}\, dt}$$

$$4 \exp\left(-\frac{\pi\left(1+\frac{1-2}{4}+\left(\frac{3}{8}\right)^2 (1-2)^2+\left(\frac{15}{48}\right)^2 (1-2)^3\right)}{1+\frac{2}{4}+\left(\frac{3}{8}\right)^2 2^2+\left(\frac{15}{48}\right)^2 2^3}\right) = 4 e^{-29/52 \int_0^1 1/\sqrt{1-t^2}\, dt}$$

$$4 \exp\left(-\frac{\pi\left(1+\frac{1-2}{4}+\left(\frac{3}{8}\right)^2 (1-2)^2+\left(\frac{15}{48}\right)^2 (1-2)^3\right)}{1+\frac{2}{4}+\left(\frac{3}{8}\right)^2 2^2+\left(\frac{15}{48}\right)^2 2^3}\right) = 4 e^{-29/52 \int_0^{\infty} 1/(1+t^2)\, dt}$$

Thence, we have the following equations:

x/16 * exp(4*((((1/4)*(1/2) x + (3/8)^2(1/2+1/12) x^2))) / (((1+1/4 x + (3/8)^2 x^2)))

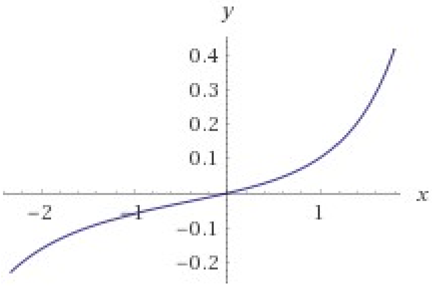
Input:

$$\frac{x}{16} \times \frac{\exp\left(4\left(\frac{1}{4} \times \frac{1}{2}\, x + \left(\frac{3}{8}\right)^2 \left(\frac{1}{2} + \frac{1}{12}\right) x^2\right)\right)}{1 + \frac{1}{4}\, x + \left(\frac{3}{8}\right)^2 x^2}$$

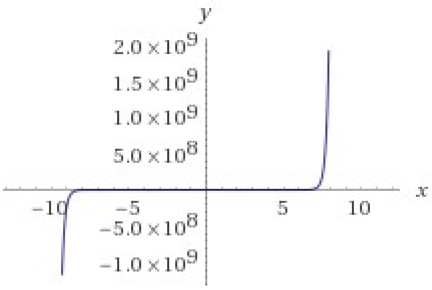
Exact result:

$$\frac{e^{4\left(\left(21 x^2\right)/256 + x/8\right)}\, x}{16\left(\frac{9 x^2}{64} + \frac{x}{4} + 1\right)}$$

Plots:



(x from −2.3 to 1.8)



(x from −12.6 to 12.1)

Alternate forms:

$$\frac{4\, e^{1/64\, x (21 x + 32)}\, x}{x (9 x + 16) + 64}$$

$$\frac{4\, e^{1/64\, x (21 x + 32)}\, x}{9 x^2 + 16 x + 64}$$

$$\frac{4\, e^{\left(21 x^2\right)/64 + x/2}\, x}{9 x^2 + 16 x + 64}$$

Alternate form assuming x is real:

$$\frac{e^{(21 x^2)/64+x/2}\, x}{16\left(\frac{9x^2}{64}+\frac{x}{4}+1\right)}$$

Root:

$x = 0$

Properties as a real function:

Domain

$\mathbb{R}$ (all real numbers)

Range

$\mathbb{R}$ (all real numbers)

Bijectivity

bijective from its domain to $\mathbb{R}$

$\mathbb{R}$ is the set of real numbers

Series expansion at x = 0:

$$\frac{x}{16}+\frac{x^2}{64}+\frac{x^3}{64}+\frac{67 x^4}{12288}+\frac{995 x^5}{393216}+O\left(x^6\right)$$

(Taylor series)

Series expansion at x = ∞:

$$e^{1/64\, x (21 x+32)}\left(\frac{4}{9 x}-\frac{64}{81 x^2}-\frac{1280}{729 x^3}+\frac{57344}{6561 x^4}-\frac{180224}{59049 x^5}+O\left(\left(\frac{1}{x}\right)^6\right)\right)$$

Derivative:

$$\frac{d}{dx}\left(\frac{x \exp\left(4\left(\frac{x}{4\times 2}+\left(\frac{3}{8}\right)^2\left(\frac{1}{2}+\frac{1}{12}\right)x^2\right)\right)}{16\left(1+\frac{x}{4}+\left(\frac{3}{8}\right)^2 x^2\right)}\right)=\frac{e^{1/64\, x (21 x+32)}\left(189 x^4+480 x^3+1312 x^2+1024 x+2048\right)}{8\left(9 x^2+16 x+64\right)^2}$$

Definite integral:

$$\int_{-\frac{32}{21}}^{0} \frac{e^{4\left(x/8+(21 x^2)/256\right)}\, x}{16\left(1+\frac{x}{4}+\frac{9x^2}{64}\right)}\, dx \approx -0.0703575\ldots$$

For x = 2, we obtain:

2/16 * exp(4*((((1/4)*(1/2)*2 + (3/8)^2(1/2+1/12) 2^2))))/ (((1+1/4 *2 + (3/8)^2 2^2)))

Input:

$$\frac{2}{16} \times \frac{\exp\left(4\left(\frac{1}{4} \times \frac{1}{2} \times 2 + \left(\frac{3}{8}\right)^2 \left(\frac{1}{2} + \frac{1}{12}\right) \times 2^2\right)\right)}{1 + \frac{1}{4} \times 2 + \left(\frac{3}{8}\right)^2 \times 2^2}$$

Exact result:

$$\frac{2 e^{37/16}}{33}$$

Decimal approximation:

0.61209952881697292928390906360874910048191519348044947807 4...

0.6120995288169…..

Property:

$\frac{2 e^{37/16}}{33}$ is a transcendental number

Series representations:

$$\frac{\exp\left(4\left(\frac{2}{4 \times 2} + \left(\frac{3}{8}\right)^2 \left(\frac{1}{2} + \frac{1}{12}\right) 2^2\right)\right) 2}{\left(1 + \frac{2}{4} + \left(\frac{3}{8}\right)^2 2^2\right) 16} = \frac{2}{33} \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{37/16}$$

$$\frac{\exp\left(4\left(\frac{2}{4 \times 2} + \left(\frac{3}{8}\right)^2 \left(\frac{1}{2} + \frac{1}{12}\right) 2^2\right)\right) 2}{\left(1 + \frac{2}{4} + \left(\frac{3}{8}\right)^2 2^2\right) 16} = \frac{2}{33} \left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{37/16}$$

$$\frac{\exp\left(4\left(\frac{2}{4 \times 2} + \left(\frac{3}{8}\right)^2 \left(\frac{1}{2} + \frac{1}{12}\right) 2^2\right)\right) 2}{\left(1 + \frac{2}{4} + \left(\frac{3}{8}\right)^2 2^2\right) 16} = \frac{\left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{37/16}}{66 \times 2^{5/16}}$$

and:

1 + 2/16 * exp(4*((((1/4)*(1/2)*2 + (3/8)^2(1/2+1/12) 2^2))))/ (((1+1/4 *2 + (3/8)^2 2^2)))

Input:

$$1 + \frac{2}{16} \times \frac{\exp\left(4\left(\frac{1}{4} \times \frac{1}{2} \times 2 + \left(\frac{3}{8}\right)^2 \left(\frac{1}{2} + \frac{1}{12}\right) \times 2^2\right)\right)}{1 + \frac{1}{4} \times 2 + \left(\frac{3}{8}\right)^2 \times 2^2}$$

Exact result:

$$1 + \frac{2 e^{37/16}}{33}$$

Decimal approximation:

1.612099528816972929283909063608749100481915193480449478074...

1.6120995288…

Property:

$1 + \frac{2 e^{37/16}}{33}$ is a transcendental number

Alternate form:

$$\frac{1}{33} \left(33 + 2 e^{37/16}\right)$$

Series representations:

$$1 + \frac{\exp\left(4\left(\frac{2}{4 \times 2} + \left(\frac{3}{8}\right)^2 \left(\frac{1}{2} + \frac{1}{12}\right) 2^2\right)\right) 2}{\left(1 + \frac{2}{4} + \left(\frac{3}{8}\right)^2 2^2\right) 16} = 1 + \frac{2}{33} \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{37/16}$$

$$1 + \frac{\exp\left(4\left(\frac{2}{4 \times 2} + \left(\frac{3}{8}\right)^2 \left(\frac{1}{2} + \frac{1}{12}\right) 2^2\right)\right) 2}{\left(1 + \frac{2}{4} + \left(\frac{3}{8}\right)^2 2^2\right) 16} = 1 + \frac{2}{33} \left(\sum_{k=0}^{\infty} \frac{(-1 + k)^2}{k!}\right)^{37/16}$$

$$1 + \frac{\exp\left(4\left(\frac{2}{4 \times 2} + \left(\frac{3}{8}\right)^2 \left(\frac{1}{2} + \frac{1}{12}\right) 2^2\right)\right) 2}{\left(1 + \frac{2}{4} + \left(\frac{3}{8}\right)^2 2^2\right) 16} = 1 + \frac{\left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{37/16}}{66 \times 2^{5/16}}$$

We have that:

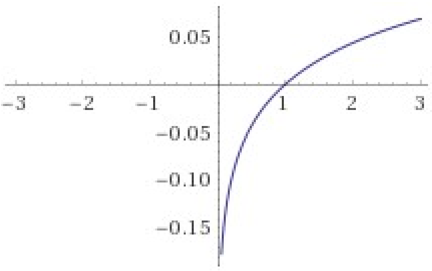
ln (x) / ((10+sqrt(36+ln(x)^2)))

Input:

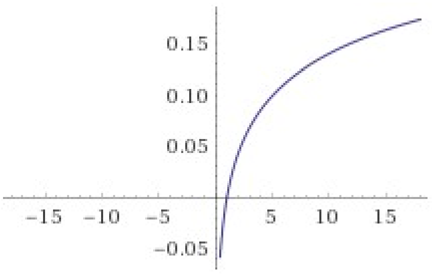
$$\frac{\log(x)}{10+\sqrt{36+\log^2(x)}}$$

log(x) is the natural logarithm

Plots:



(x from −3 to 3)



(x from −18 to 18)

Alternate form:

$$\frac{\sqrt{\log^2(x)+36}}{2(\log(x)-8)}+\frac{\sqrt{\log^2(x)+36}}{2(\log(x)+8)}-\frac{5}{\log(x)-8}-\frac{5}{\log(x)+8}$$

Root:

$x = 1$

Properties as a real function:

Domain

$\{x \in \mathbb{R} : x > 0\}$ (all positive real numbers)

Range

$\{y \in \mathbb{R} : -1 < y < 1\}$

Injectivity

injective (one-to-one)

$\mathbb{R}$ is the set of real numbers

Derivative:

$$\frac{d}{dx}\left(\frac{\log(x)}{10+\sqrt{36+\log^2(x)}}\right) = \frac{\frac{36}{\sqrt{36+\log^2(x)}}+10}{x\left(\log^2(x)+20\sqrt{36+\log^2(x)}+136\right)}$$

Limit:

$$\lim_{x\to\pm\infty} \frac{\log(x)}{10+\sqrt{36+\log^2(x)}} = 1$$

where for x = 2, we obtain:

ln (2) / ((10+sqrt(36+ln(2)^2)))

Input:

$$\frac{\log(2)}{10+\sqrt{36+\log^2(2)}}$$

$\log(x)$ is the natural logarithm

Decimal approximation:

0.043213920431325497378424857306636332317331255774345672994...

0.04321392043…..

Alternative representations:

$$\frac{\log(2)}{10+\sqrt{36+\log^2(2)}} = \frac{\log_e(2)}{10+\sqrt{36+\log_e^2(2)}}$$

$$\frac{\log(2)}{10+\sqrt{36+\log^2(2)}} = \frac{\log(a)\log_a(2)}{10+\sqrt{36+(\log(a)\log_a(2))^2}}$$

$$\frac{\log(2)}{10+\sqrt{36+\log^2(2)}} = \frac{2\coth^{-1}(3)}{10+\sqrt{36+(2\coth^{-1}(3))^2}}$$

Series representations:

$$\frac{\log(2)}{10+\sqrt{36+\log^2(2)}} = \frac{2 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}}{10+\sqrt{36+\left(2 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}\right)^2}} \quad \text{for } x < 0$$

$$\frac{\log(2)}{10+\sqrt{36+\log^2(2)}} = \frac{\left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}{10+\sqrt{36+\left(\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}\right)^2}}$$

$$\frac{\log(2)}{10+\sqrt{36+\log^2(2)}} = \frac{2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}{10+\sqrt{36+\left(2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}\right)^2}}$$

Integral representations:

$$\frac{\log(2)}{10+\sqrt{36+\log^2(2)}} = \frac{1}{10+\sqrt{36+\left(\int_1^2 \frac{1}{t}\,dt\right)^2}} \int_1^2 \frac{1}{t}\,dt$$

$$\frac{\log(2)}{10+\sqrt{36+\log^2(2)}} = -\frac{i}{20\pi+\sqrt{144\pi^2 - \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1 < \gamma < 0$$

From which:

1+1/(((1/(((ln (2) 1/((10+sqrt(36+ln(2)^2))))))^1/7)))

Input:

$$1+\frac{1}{\frac{1}{\sqrt[7]{\log(2)\times\frac{1}{10+\sqrt{36+\log^2(2)}}}}}$$

log(x) is the natural logarithm

Exact result:

$$1+\sqrt[7]{\frac{\log(2)}{10+\sqrt{36+\log^2(2)}}}$$

Decimal approximation:

1.638394439240531095111690161363660706530772593802614154266...

1.638394439… ≈ ζ(2) = 1.644934…

Alternative representations:

$$1+\frac{1}{\frac{1}{\sqrt[7]{\frac{\log(2)}{10+\sqrt{36+\log^2(2)}}}}}=1+\frac{1}{\frac{1}{\sqrt[7]{\frac{\log_e(2)}{10+\sqrt{36+\log_e^2(2)}}}}}$$

$$1+\frac{1}{\frac{1}{\sqrt[7]{\frac{\log(2)}{10+\sqrt{36+\log^2(2)}}}}}=1+\frac{1}{\frac{1}{\sqrt[7]{\frac{2\coth^{-1}(3)}{10+\sqrt{36+(2\coth^{-1}(3))^2}}}}}$$

$$1+\frac{1}{\frac{1}{\sqrt[7]{\frac{\log(2)}{10+\sqrt{36+\log^2(2)}}}}}=1+\frac{1}{\frac{1}{\sqrt[7]{-\frac{\text{Li}_1(-1)}{10+\sqrt{36+(-\text{Li}_1(-1))^2}}}}}$$

Series representations:

$$1+\frac{1}{\frac{1}{\sqrt[7]{\frac{\log(2)}{10+\sqrt{36+\log^2(2)}}}}}=\frac{\sqrt[7]{10+\sqrt{36+\log^2(2)}}-\sum_{k=0}^{\infty}\binom{-\frac{1}{7}+k}{k}\sum_{j=0}^{k}\frac{(-1)^j\binom{k}{j}p_{j,k}}{-1+7j}}{\sqrt[7]{10+\sqrt{36+\log^2(2)}}}$$

$$\text{for }\left(c_k=\frac{(-1)^k}{(1+k)\left(10+\sqrt{36+\log^2(2)}\right)}\text{ and }p_{j,0}=1\text{ and }p_{j,k}=\frac{\left(10+\sqrt{36+\log^2(2)}\right)\sum_{m=1}^{k}(-k+m+j\,m)\,c_m\,p_{j,k-m}}{k}\text{ and }k\in\mathbb{Z}\text{ and }k>0\right)$$

$$1+\frac{1}{\frac{1}{\sqrt[7]{\frac{\log(2)}{10+\sqrt{36+\log^2(2)}}}}}=\frac{7\sqrt[7]{10+\sqrt{36+\log^2(2)}}+e^{2/7\,i\pi\left\lfloor 1/2-\arg\left(\frac{1}{e}\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\binom{-\frac{1}{7}+k}{k}\sum_{j=0}^{k}-\frac{7(-1)^j\binom{k}{j}p_{j,k}}{-1+7j}}{7\sqrt[7]{10+\sqrt{36+\log^2(2)}}}$$

$$\text{for }\left(c_k=\frac{(-1)^k}{(1+k)\left(10+\sqrt{36+\log^2(2)}\right)}\text{ and }p_{j,0}=1\text{ and }p_{j,k}=\frac{\left(10+\sqrt{36+\log^2(2)}\right)\sum_{m=1}^{k}(-k+m+j\,m)\,c_m\,p_{j,k-m}}{k}\text{ and }k\in\mathbb{Z}\text{ and }k>0\right)$$

$$1+\frac{1}{\frac{1}{\sqrt[7]{\frac{\log(2)}{10+\sqrt{36+\log^2(2)}}}}}=1+\sqrt[7]{\frac{2i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}}{10+\sqrt{36+\left(2i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}\right)^2}}}\quad\text{for }x<0$$

Integral representations:

$$1+\frac{1}{\frac{1}{\sqrt[7]{\frac{\log(2)}{10+\sqrt{36+\log^2(2)}}}}} = 1+\sqrt[7]{\frac{1}{10+\sqrt{36+\left(\int_1^2 \frac{1}{t}\,dt\right)^2}}\int_1^2 \frac{1}{t}\,dt}$$

$$1+\frac{1}{\frac{1}{\sqrt[7]{\frac{\log(2)}{10+\sqrt{36+\log^2(2)}}}}} = \frac{\sqrt[7]{\pi}+\sqrt[7]{\pi}\sqrt[7]{-\frac{i}{20\pi+\sqrt{144\pi^2-\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}}{\sqrt[7]{\pi}} \text{ for } -1<\gamma<0$$

We have:

ln F(x) ln F(1 – x) = π^2 = 9.8696044010893586188344

From the sum of the above results:

(0.04321392043+0.6120995288169+0.4164366089307+9.869604401089358618834
4)

Input interpretation:

0.04321392043 + 0.6120995288169 +
0.4164366089307 + 9.8696044010893586188344

Result:

10.9413544592669586188344

10.9413544592669586188344

From which:

(0.043213920+0.612099528+0.416436608+9.869604401)^1/5

Input interpretation:

$\sqrt[5]{0.043213920+0.612099528+0.416436608+9.869604401}$

Result:

1.613668114336964199325764339840498200924370396883789577812...

1.613668114336…

We have that:

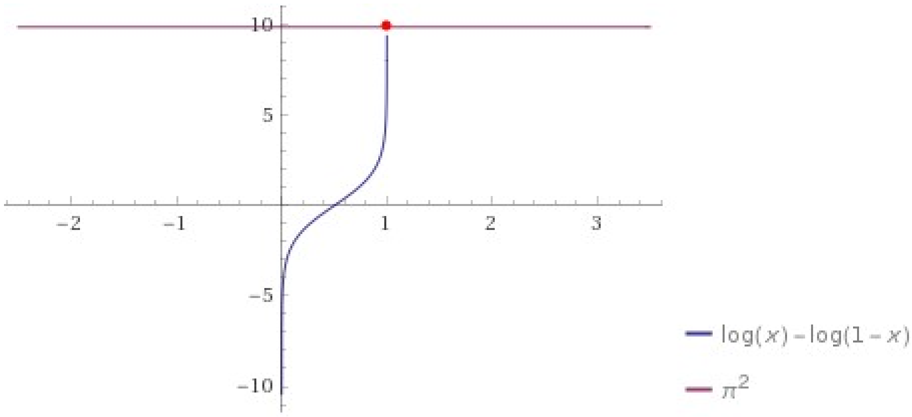
ln(x) - ln(1-x) = Pi^2

Input:

$\log(x) - \log(1 - x) = \pi^2$

$\log(x)$ is the natural logarithm

Plot:



Alternate form assuming x is real:

$\log(1 - x) + \pi^2 = \log(x)$

Alternate form:

$-2 \tanh^{-1}(1 - 2 x) = \pi^2$

$\tanh^{-1}(x)$ is the inverse hyperbolic tangent function

Alternate form assuming x is positive:

$\log\left(\frac{1}{x} - 1\right) + \pi^2 = 0$

Solution:

$$x = \frac{e^{\pi^2}}{1 + e^{\pi^2}}$$

Solution:

$x \approx 0.99995$

0.99995

Thence:

ln(0.99995) - ln(1-0.99995)

Input:

$\log(0.99995) - \log(1 - 0.99995)$

$\log(x)$ is the natural logarithm

Result:

9.90344...

9.90344…

Alternative representations:

$$\log(0.99995) - \log(1 - 0.99995) = \log\left(\frac{0.99995}{0.00005}\right)$$

$$\log(0.99995) - \log(1 - 0.99995) = -\log_e(0.00005) + \log_e(0.99995)$$

$$\log(0.99995) - \log(1 - 0.99995) = -\log(a)\log_a(0.00005) + \log(a)\log_a(0.99995)$$

Series representations:

$$\log(0.99995) - \log(1 - 0.99995) = \sum_{k=1}^{\infty} \frac{(-1)^k \left((-0.99995)^k - (-0.00005)^k\right)}{k}$$

$$\log(0.99995) - \log(1 - 0.99995) = -2 i \pi \left\lfloor \frac{\arg(0.00005 - x)}{2 \pi} \right\rfloor + 2 i \pi \left\lfloor \frac{\arg(0.99995 - x)}{2 \pi} \right\rfloor + \sum_{k=1}^{\infty} \frac{(-1)^k \left((0.00005 - x)^k - (0.99995 - x)^k\right) x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.99995) - \log(1 - 0.99995) = -\left\lfloor \frac{\arg(0.00005 - z_0)}{2 \pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \left\lfloor \frac{\arg(0.99995 - z_0)}{2 \pi} \right\rfloor \log\left(\frac{1}{z_0}\right) - \left\lfloor \frac{\arg(0.00005 - z_0)}{2 \pi} \right\rfloor \log(z_0) + \left\lfloor \frac{\arg(0.99995 - z_0)}{2 \pi} \right\rfloor \log(z_0) + \sum_{k=1}^{\infty} \frac{(-1)^k \left((0.00005 - z_0)^k - (0.99995 - z_0)^k\right) z_0^{-k}}{k}$$

Integral representation:

$$\log(0.99995) - \log(1 - 0.99995) = \int_1^{0.00005} \left(-\frac{1}{t} + \frac{1}{19998. + t}\right) dt$$

Indeed:

sqrt[ln(0.99995) - ln(1-0.99995)]

Input:

$$\sqrt{\log(0.99995) - \log(1 - 0.99995)}$$

$\log(x)$ is the natural logarithm

Result:

3.14697…

3.14697… ≈ π

All 2nd roots of 9.90344:

$3.14697 e^0 \approx 3.1470$ (real, principal root)

$3.14697 e^{i \pi} \approx -3.1470$ (real root)

Alternative representations:

$$\sqrt{\log(0.99995) - \log(1 - 0.99995)} = \sqrt{\log\left(\frac{0.99995}{0.00005}\right)}$$

$$\sqrt{\log(0.99995) - \log(1 - 0.99995)} = \sqrt{-\log_e(0.00005) + \log_e(0.99995)}$$

$$\sqrt{\log(0.99995) - \log(1 - 0.99995)} = \sqrt{-\log(a)\log_a(0.00005) + \log(a)\log_a(0.99995)}$$

Series representations:

$$\sqrt{\log(0.99995) - \log(1 - 0.99995)} = \sqrt{\sum_{k=1}^{\infty} \frac{(-1)^k \left((-0.99995)^k - (-0.00005)^k\right)}{k}}$$

$$\sqrt{\log(0.99995) - \log(1 - 0.99995)} = \sqrt{-1 - \log(0.00005) + \log(0.99995)} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} (-1 - \log(0.00005) + \log(0.99995))^{-k}$$

$$\sqrt{\log(0.99995) - \log(1 - 0.99995)} = \sqrt{-1 - \log(0.00005) + \log(0.99995)} \sum_{k=0}^{\infty} \frac{(-1)^k (-1 - \log(0.00005) + \log(0.99995))^{-k} \left(-\frac{1}{2}\right)_k}{k!}$$

Integral representation:

$$\sqrt{\log(0.99995) - \log(1 - 0.99995)} = \sqrt{\int_1^{0.00005} \left(-\frac{1}{t} + \frac{1}{19998. + t}\right) dt}$$

We have that:

(4x)/(1+x)^2 = sqrt(x^2)

Input:

$$\frac{4x}{(1+x)^2} = \sqrt{x^2}$$

Alternate form assuming x is real:

$$\frac{4x}{(x+1)^2} = |x|$$

$|z|$ is the absolute value of z

Alternate form assuming x>0:

$$\frac{4x}{(x+1)^2} = x$$

Real solutions:

$x = 0$

$x = 1$

x = 1

For x = 1, we obtain:

(4x)/(1+x)^2 = sqrt(x^2)

Input:

$$\frac{4}{(1+1)^2} = \sqrt{1^2}$$

Result:

True

Left hand side:

$$\frac{4}{(1+1)^2} = 1$$

Right hand side:

$$\sqrt{1^2} = 1$$

We have that:

1/2160(((ln x)/(8+4(ln x)^2)))^5

Input:

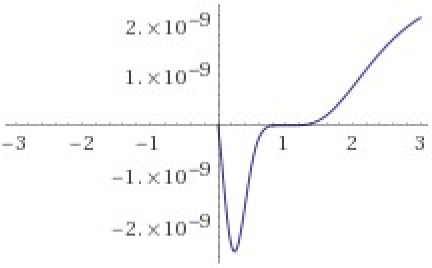
$$\frac{1}{2160}\left(\frac{\log(x)}{8+4\log^2(x)}\right)^5$$

$\log(x)$ is the natural logarithm

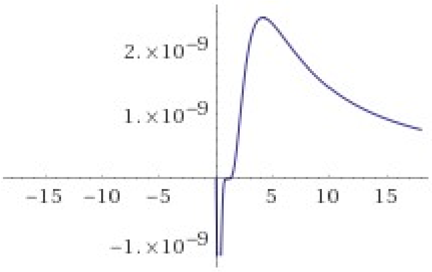
Result:

$$\frac{\log^5(x)}{2160\left(4\log^2(x)+8\right)^5}$$

Plots:



(x from −3 to 3)



(x from −18 to 18)

Alternate forms:

$$\frac{\log^5(x)}{2\,211\,840\,(\log^2(x)+2)^5}$$

$$\frac{\log(x)}{2\,211\,840\,(\log^2(x)+2)^3} - \frac{\log(x)}{552\,960\,(\log^2(x)+2)^4} + \frac{\log(x)}{552\,960\,(\log^2(x)+2)^5}$$

Root:

$x = 1$

Properties as a real function:

Domain

$\{x \in \mathbb{R} : x > 0\}$ (all positive real numbers)

Range

$$\left\{y \in \mathbb{R} : -\frac{1}{283\,115\,520\,\sqrt{2}} \le y \le \frac{1}{283\,115\,520\,\sqrt{2}}\right\}$$

$\mathbb{R}$ is the set of real numbers

Series expansion at x = 0:

$$\frac{\log^5(x)}{2\,211\,840\,(\log^2(x)+2)^5} + O(x^2)$$

(generalized Puiseux series)

Series expansion at x = ∞:

$$\frac{\log^5(x)}{2\,211\,840\,(\log^2(x)+2)^5} + O\left(\left(\frac{1}{x}\right)^2\right)$$

(generalized Puiseux series)

Derivative:

$$\frac{d}{dx}\left(\frac{\left(\frac{\log(x)}{8+4\log^2(x)}\right)^5}{2160}\right) = -\frac{\log^4(x)\,(\log^2(x)-2)}{442\,368\,x\,(\log^2(x)+2)^6}$$

Indefinite integral:

$$\int \frac{\left(\frac{\log(x)}{8+4\log^2(x)}\right)^5}{2160}\,dx = \frac{1}{6\,794\,772\,480}\Big(-i\,(35\sqrt{2} + -26\,i)\,e^{i\sqrt{2}}\,\mathrm{Ei}(\log(x) - i\sqrt{2}) +$$

$$i\,(35\sqrt{2} + 26\,i)\,e^{-i\sqrt{2}}\,\mathrm{Ei}(\log(x) + i\sqrt{2}) +$$

$$\frac{4\,x\,(13\log^7(x) - 22\log^6(x) + 34\log^5(x) - 316\log^4(x) + 12\log^3(x) - 488\log^2(x) - 8\log(x) - 272)}{(\log^2(x)+2)^4}\Big) +$$

constant

(assuming a complex-valued logarithm)

Ei(x) is the exponential integral Ei

Global maximum:

$$\max\left\{\frac{\left(\frac{\log(x)}{8+4\log^2(x)}\right)^5}{2160}\right\} = \frac{1}{283\,115\,520\,\sqrt{2}} \text{ at } x = e^{\sqrt{2}}$$

Global minimum:

$$\min\left\{\frac{\left(\frac{\log(x)}{8+4\log^2(x)}\right)^5}{2160}\right\} = -\frac{1}{283\,115\,520\,\sqrt{2}} \text{ at } x = e^{-\sqrt{2}}$$

Limit:

$$\lim_{x\to\pm\infty} \frac{\log^5(x)}{2160\,(8+4\log^2(x))^5} = 0$$

Definite integrals:

integral_0^∞ (log^5(x))/(2160 (8 + 4 log^2(x))^5) dx ≈

$3.5179658656039185 83\ldots \times 10^{27919}$

$$\int_0^1 \frac{\log^5(x)}{2160\,(8+4\log^2(x))^5}\,dx \approx -8.93936\times 10^{-10}\ldots$$

Alternative representations:

$$\frac{\left(\frac{\log(x)}{8+4\log^2(x)}\right)^5}{2160} = \frac{\left(\frac{\log_e(x)}{8+4\log_e^2(x)}\right)^5}{2160}$$

$$\frac{\left(\frac{\log(x)}{8+4\log^2(x)}\right)^5}{2160} = \frac{\left(\frac{\log(a)\log_a(x)}{8+4(\log(a)\log_a(x))^2}\right)^5}{2160}$$

$$\frac{\left(\frac{\log(x)}{8+4\log^2(x)}\right)^5}{2160} = \frac{\left(-\frac{\text{Li}_1(1-x)}{8+4(-\text{Li}_1(1-x))^2}\right)^5}{2160}$$

Series representations:

$$\frac{\left(\frac{\log(x)}{8+4\log^2(x)}\right)^5}{2160} = -\frac{\left(\sum_{k=1}^{\infty}\frac{(-1)^k(-1+x)^k}{k}\right)^5}{2211840\left(2+\left(\sum_{k=1}^{\infty}\frac{(-1)^k(-1+x)^k}{k}\right)^2\right)^5} \quad \text{for } |-1+x|<1$$

$$\frac{\left(\frac{\log(x)}{8+4\log^2(x)}\right)^5}{2160} = \left(\log(-1+x)-\sum_{k=1}^{\infty}\frac{(-1)^k(-1+x)^{-k}}{k}\right)^5 \Big/ \left(2211840\left(2+\log^2(-1+x)-2\log(-1+x)\sum_{k=1}^{\infty}\frac{(-1)^k(-1+x)^{-k}}{k}+\left(\sum_{k=1}^{\infty}\frac{(-1)^k(-1+x)^{-k}}{k}\right)^2\right)^5\right) \quad \text{for } |-1+x|>1$$

$$\frac{\left(\frac{\log(x)}{8+4\log^2(x)}\right)^5}{2160} = \frac{\left(\sum_{j=1}^{\infty} \text{Res}_{s=-j} \frac{(-1+x)^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\right)^5}{2\,211\,840\left(2+\left(\sum_{j=1}^{\infty} \text{Res}_{s=-j} \frac{(-1+x)^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\right)^2\right)^5} \quad \text{for } |-1+x|<1$$

Integral representations:

$$\frac{\left(\frac{\log(x)}{8+4\log^2(x)}\right)^5}{2160} = \frac{\left(\int_1^x \frac{1}{t}\,dt\right)^5}{2\,211\,840\left(2+\left(\int_1^x \frac{1}{t}\,dt\right)^2\right)^5}$$

$$\frac{\left(\frac{\log(x)}{8+4\log^2(x)}\right)^5}{2160} = -\frac{i\,\pi^5\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{(-1+x)^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^5}{69\,120\left(8\,\pi^2-\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{(-1+x)^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2\right)^5}$$

for $(-1<\gamma<0$ and $|\arg(-1+x)|<\pi)$

We note that from:

integral_0^∞ (log^5(x))/(2160 (8 + 4 log^2(x))^5) dx ≈

$3.5179658656039185835\ldots \times 10^{27919}$

we obtain:

[ln(3.5179658656039185835 ×10^27919)]^1/23

Input interpretation:

$$\sqrt[23]{\log(3.5179658656039185835 \times 10^{27919})}$$

log(x) is the natural logarithm

Result:

1.6182621318189089555520447...

1.618262131...

For x = 2 , we obtain:

1/2160(((ln 2)/(8+4(ln 2)^2)))^5

Input:

$$\frac{1}{2160}\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{\log^5(2)}{2160\,(8+4\log^2(2))^5}$$

Decimal approximation:

$7.7040407871603987256963481272524300872643243728415654... \times 10^{-10}$

7.70404078716…*10^-10

Property:

$$\frac{\log^5(2)}{2160\,(8+4\log^2(2))^5}$$ is a transcendental number

Alternate forms:

$$\frac{\log^5(2)}{2\,211\,840\,(2+\log^2(2))^5}$$

$$\frac{\log(2)}{552\,960\,(2+\log^2(2))^5}-\frac{\log(2)}{552\,960\,(2+\log^2(2))^4}+\frac{\log(2)}{2\,211\,840\,(2+\log^2(2))^3}$$

Alternative representations:

$$\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}=\frac{\left(\frac{\log_e(2)}{8+4\log_e^2(2)}\right)^5}{2160}$$

$$\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}=\frac{\left(\frac{2\coth^{-1}(3)}{8+4(2\coth^{-1}(3))^2}\right)^5}{2160}$$

$$\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}=\frac{\left(\frac{\log(a)\log_a(2)}{8+4(\log(a)\log_a(2))^2}\right)^5}{2160}$$

Series representations:

$$\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160} = \left(2\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor - i\log(x) + i\sum_{k=1}^{\infty}\frac{(-1)^k (2-x)^k x^{-k}}{k}\right)^5 \Bigg/ \left(2211840\left(-2i+4i\pi^2\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor^2 + 4\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\log(x) - i\log^2(x) - 4\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\sum_{k=1}^{\infty}\frac{(-1)^k (2-x)^k x^{-k}}{k} + 2i\log(x)\sum_{k=1}^{\infty}\frac{(-1)^k (2-x)^k x^{-k}}{k} - i\left(\sum_{k=1}^{\infty}\frac{(-1)^k (2-x)^k x^{-k}}{k}\right)^2\right)^5\right) \quad \text{for } x < 0$$

$$\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160} = \left(2\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor - i\log(z_0) + i\sum_{k=1}^{\infty}\frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}\right)^5 \Bigg/ \left(2211840\left(-2i+4i\pi^2\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor^2 + 4\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor\log(z_0) - i\log^2(z_0) - 4\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor\sum_{k=1}^{\infty}\frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} + 2i\log(z_0)\sum_{k=1}^{\infty}\frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} - i\left(\sum_{k=1}^{\infty}\frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}\right)^2\right)^5\right)$$

$$\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160} = \left(\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^5 \Big/$$

$$\Bigg(2\,211\,840\Bigg(2+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor^2\log^2\left(\frac{1}{z_0}\right)+2\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)\log(z_0)+2\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor^2 \log\left(\frac{1}{z_0}\right)\log(z_0)+\log^2(z_0)+2\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log^2(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor^2 \log^2(z_0)-2\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}-2\log(z_0)\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}-2\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log(z_0)\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}+\left(\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^2\Bigg)^5\Bigg)$$

Integral representations:

$$\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160} = \frac{\left(\int_1^2 \frac{1}{t}\,dt\right)^5}{2\,211\,840\left(2+\left(\int_1^2\frac{1}{t}\,dt\right)^2\right)^5}$$

$$\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160} = -\frac{i\,\pi^5\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^5}{69\,120\left(8\,\pi^2-\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2\right)^5} \quad \text{for } -1<\gamma<0$$

We know the following Ramanujan expression, for to obtain a highly precise value of ϕ:

((((1/(((1/32(-1+sqrt(5))^5+5*(e^((-sqrt(5)*Pi))^5)))-(-1.6382898797095665677239458827012056245798314722584 × 10^-7429)))^1/5

$$\sqrt[5]{\frac{1}{\left(\frac{1}{32}\left(-1+\sqrt{5}\right)^5+5\,e^{\left(-\sqrt{5}\,\pi\right)^5}\right)+\frac{1.6382898797095665677239458827012056245798314722584}{10^{7429}}}}$$

((((1/(((1/32(-1+sqrt(5))^5+5*(e^((-sqrt(5)*Pi))^5)))-(-1.63828979709566567723945882701205624579831472258 4 × 10^-7429)))^1/5

If we put 1/(3.517965865603918583 ×10^27919) =

$$\frac{1}{3.517965865603918583\times 10^{27919}}$$

$2.842551742122526302509527761722104330350466636218 08\ldots \times 10^{-27920}$

$2.842551742122526302509527761722104330350466636218 08 \times 10^{-27920}$

we obtain:

((((1/(((1/32(-1+sqrt(5))^5+5*(e^((-sqrt(5)*Pi))^5)))+(2.842551742122526302509527761722104330350466636218 × 10^-27920)))^1/5

Input interpretation:

$$\sqrt[5]{\frac{1}{\left(\frac{1}{32}\left(-1+\sqrt{5}\right)^5+5\,e^{\left(-\sqrt{5}\,\pi\right)^5}\right)+\frac{2.842551742122526302509527761722104330350466636218}{10^{27920}}}}$$

Result:

1.6180339887498948482045868343656381177203091798057628 62135...

1.6180339887…

Possible closed forms:

$\phi \approx 1.618033988749894848204586834365638117720309179805762862135$

$\Phi + 1 \approx 1.618033988749894848204586834365638117720309179805762862135$

$\frac{1}{\Phi} \approx 1.618033988749894848204586834365638117720309179805762862135$

ϕ is the golden ratio

Φ is the golden ratio conjugate

Now, from the previous expression, we have also that:

1/(((1/2160(((ln 2)/(8+4(ln 2)^2)))^5)))

Input:

$$\frac{1}{\frac{1}{2160}\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}$$

log(x) is the natural logarithm

Exact result:

$$\frac{2160\,(8+4\log^2(2))^5}{\log^5(2)}$$

Decimal approximation:

$1.298020126874984991654748918252200984528279450062919 40\ldots \times 10^9$

1.29802012687…*10^9

Property:

$$\frac{2160\,(8+4\log^2(2))^5}{\log^5(2)}$$ is a transcendental number

Alternate forms:

$$\frac{2\,211\,840\,(2+\log^2(2))^5}{\log^5(2)}$$

$$\frac{70\,778\,880}{\log^5(2)} + 2\,211\,840\log^5(2) + \frac{176\,947\,200}{\log^3(2)} + 22\,118\,400\log^3(2) + \frac{176\,947\,200}{\log(2)} + 88\,473\,600\log(2)$$

Alternative representations:

$$\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}} = \frac{1}{\frac{\left(\frac{\log_e(2)}{8+4\log_e^2(2)}\right)^5}{2160}}$$

$$\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}} = \frac{1}{\frac{\left(\frac{2\coth^{-1}(3)}{8+4\left(2\coth^{-1}(3)\right)^2}\right)^5}{2160}}$$

$$\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}} = \frac{1}{\frac{\left(-\frac{\mathrm{Li}_1(-1)}{8+4(-\mathrm{Li}_1(-1))^2}\right)^5}{2160}}$$

Series representations:

$$\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}} = \left(2\,211\,840\left(-2+4\pi^2\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor^2 - 4i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\log(x) - \log^2(x) + 4 i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k} + 2\log(x)\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k} - \left(\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}\right)^2\right)^5\right) \Big/ \left(-2i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor - \log(x) + \sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}\right)^5 \quad \text{for } x < 0$$

$$\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}} = \left(2\,211\,840\left(-2+4\pi^2\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor^2 - 4i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor\log(z_0) - \log^2(z_0) + 4i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k} + 2\log(z_0)\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k} - \left(\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^2\right)^5\right) \Big/ \left(-2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor - \log(z_0) + \sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^5$$

$$\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}} =$$

$$\left(2\,211\,840\left(2+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor^2\log^2\left(\frac{1}{z_0}\right)+2\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)\log(z_0)+ 2\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor^2\log\left(\frac{1}{z_0}\right)\log(z_0)+\log^2(z_0)+ 2\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log^2(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor^2\log^2(z_0)- 2\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}- 2\log(z_0)\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}-2\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log(z_0) \sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}+\left(\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^2\right)^5\right)\Big/ \left(\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^5$$

Integral representations:

$$\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}} = \frac{2\,211\,840\left(2+\left(\int_1^2\frac{1}{t}\,dt\right)^2\right)^5}{\left(\int_1^2\frac{1}{t}\,dt\right)^5}$$

$$\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}} = \frac{69\,120\,i\left(8\pi^2-\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2\right)^5}{\pi^5\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^5} \quad \text{for } -1<\gamma<0$$

From which, performing the ln and putting 34 as numerator, we obtain:

34/ ln(((1/(((1/2160(((ln 2)/(8+4(ln 2)^2)))^5))))))

Input:

$$\frac{34}{\log\left(\frac{1}{\frac{1}{2160}\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}\right)}$$

log(x) is the natural logarithm

Exact result:

$$\frac{34}{\log\left(\frac{2160\,(8+4\log^2(2))^5}{\log^5(2)}\right)}$$

Decimal approximation:

1.620273937946508825618602494169893250837918702809029179647...

1.62027393794…

Alternate forms:

$$\frac{34}{5\log(8+4\log^2(2))+\log(2160)-5\log(\log(2))}$$

$$\frac{34}{5\log(2+\log^2(2))+14\log(2)+3\log(3)+\log(5)-5\log(\log(2))}$$

$$\frac{34}{5\log(2+\log^2(2))+\log(2\,211\,840)-5\log(\log(2))}$$

Alternative representations:

$$\frac{34}{\log\left(\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}\right)}=\frac{34}{\log_e\left(\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}\right)}$$

$$\frac{34}{\log\left(\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}\right)}=\frac{34}{\log(a)\log_a\left(\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}\right)}$$

$$\frac{34}{\log\left(\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}\right)}=-\frac{34}{\text{Li}_1\left(1-\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}\right)}$$

Series representations:

$$\frac{34}{\log\left(\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}\right)} = \frac{34}{\log\left(-1+\frac{2160\left(8+4\log^2(2)\right)^5}{\log^5(2)}\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+\frac{2160\left(8+4\log^2(2)\right)^5}{\log^5(2)}}\right)^k}{k}}$$

$$\frac{34}{\log\left(\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}\right)} = \frac{34}{2 i \pi \left\lfloor \frac{\arg\left(-x+\frac{2160\left(8+4\log^2(2)\right)^5}{\log^5(2)}\right)}{2\pi}\right\rfloor + \log(x) - \sum_{k=1}^{\infty}\frac{(-1)^k x^{-k}\left(-x+\frac{2160\left(8+4\log^2(2)\right)^5}{\log^5(2)}\right)^k}{k}} \quad \text{for } x < 0$$

$$\frac{34}{\log\left(\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}\right)} = \frac{34}{2 i \pi \left\lfloor \frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor + \log(z_0) - \sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{2160\left(8+4\log^2(2)\right)^5}{\log^5(2)}-z_0\right)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{34}{\log\left(\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}\right)} = \frac{34}{\int_1^{\frac{2160\left(8+4\log^2(2)\right)^5}{\log^5(2)}}\frac{1}{t}\,dt}$$

$$\frac{34}{\log\left(\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}\right)} = \frac{68 i \pi}{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)\left(-1+\frac{2160\left(8+4\log^2(2)\right)^5}{\log^5(2)}\right)^{-s}}{\Gamma(1-s)}\,ds} \quad \text{for } -1 < \gamma < 0$$

Now, from the various obtained results, we have the following expression:

34/ln((((1/(7.70404078716*10^-10)
+0.04321392043+0.6120995288169+0.4164366089307+9.869604401089))))-2/10^3

Input interpretation:

$$34 / \log\left(\frac{1}{7.70404078716 \times 10^{-10}} + 0.04321392043 + 0.6120995288169 + 0.4164366089307 + 9.869604401089\right) - \frac{2}{10^3}$$

log(x) is the natural logarithm

Result:

1.618273937295644749691874443206729629993271662139453806395...

1.6182739372956....

From the same above expression, we have also:

(((1/(((1/2160(((ln 2)/(8+4(ln 2)^2))^5))))))^(1/e)-233-144*2-2

Input:

$$\sqrt[e]{\frac{1}{\frac{1}{2160}\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}} - 233 - 144 \times 2 - 2$$

log(x) is the natural logarithm

Exact result:

$$\sqrt[e]{2160}\ \log^{-5/e}(2)\,(8 + 4\log^2(2))^{5/e} - 523$$

Decimal approximation:

1729.106263622115568932970520538426888939162016401381284425...

1729.106263622…

Alternate forms:

$$\sqrt[e]{2160}\ \left(\frac{8}{\log(2)} + \log(16)\right)^{5/e} - 523$$

$$4^{7/e}\sqrt[e]{135}\left(\frac{2+\log^2(2)}{\log(2)}\right)^{5/e}-523$$

$$\log^{-5/e}(2)\left(\sqrt[e]{2160}\left(8+4\log^2(2)\right)^{5/e}-523\log^{5/e}(2)\right)$$

Alternative representations:

$$\sqrt[e]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}}-233-144\times 2-2=-523+\sqrt[e]{\frac{1}{\frac{\left(\frac{\log_e(2)}{8+4\log_e^2(2)}\right)^5}{2160}}}$$

$$\sqrt[e]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}}-233-144\times 2-2=-523+\sqrt[e]{\frac{1}{\frac{\left(\frac{2\coth^{-1}(3)}{8+4\left(2\coth^{-1}(3)\right)^2}\right)^5}{2160}}}$$

$$\sqrt[e]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}}-233-144\times 2-2=-523+\sqrt[e]{\frac{1}{\frac{\left(-\frac{\text{Li}_1(-1)}{8+4\left(-\text{Li}_1(-1)\right)^2}\right)^5}{2160}}}$$

Series representations:

$$\sqrt[e]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}}-233-144\times 2-2=$$

$$\left(\sqrt[e]{2160}\left(8+4\left(2i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}\right)^2\right)^{5/e}-523\left(2i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}\right)^{5/e}\right)\left(2i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}\right)^{-5/e}\quad\text{for } x<0$$

$$\sqrt[e]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}} - 233 - 144 \times 2 - 2 =$$

$$\left(\sqrt[e]{2160}\left(8+4\left(\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^2\right)^{5/e} - 523\left(\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^{5/e}\right) \left(\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^{-5/e}$$

$$\sqrt[e]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}} - 233 - 144 \times 2 - 2 =$$

$$\left(\sqrt[e]{2160}\left(8+4\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^2\right)^{5/e} - 523\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^{5/e}\right) \left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^{-5/e}$$

Integral representations:

$$\sqrt[e]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}} - 233 - 144 \times 2 - 2 =$$

$$\left(\int_1^2 \frac{1}{t}\,dt\right)^{-5/e}\left(-523\left(\int_1^2\frac{1}{t}\,dt\right)^{5/e}+\sqrt[e]{2160}\left(8+4\left(\int_1^2\frac{1}{t}\,dt\right)^2\right)^{5/e}\right)$$

$$\sqrt[e]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}} - 233 - 144\times 2 - 2 =$$

$$\left(-i\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,d\,s\right)^{-5/e}\left(-523\left(-i\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,d\,s\right)^{5/e}+\right.$$

$$\left.2^{9/e}\sqrt[e]{135}\,\pi^{5/e}\left(8-\frac{\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,d\,s\right)^2}{\pi^2}\right)^{5/e}\right)\quad \text{for } -1<\gamma<0$$

and again:

(((1/(((1/2160(((ln 2)/(8+4(ln 2)^2))^5))))))^1/4-55

Input:

$$\sqrt[4]{\frac{1}{\frac{1}{2160}\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}} - 55$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{2\times 3^{3/4}\sqrt[4]{5}\,\left(8+4\log^2(2)\right)^{5/4}}{\log^{5/4}(2)} - 55$$

Decimal approximation:

134.8105539550056929248112219716756776208226193057392542243...

134.81055395…..

Alternate forms:

$$2\times 3^{3/4}\sqrt[4]{5}\left(\frac{8}{\log(2)}+\log(16)\right)^{5/4} - 55$$

$$\frac{8\sqrt{2}\;3^{3/4}\sqrt[4]{5}\,\left(2+\log^2(2)\right)^{5/4}}{\log^{5/4}(2)} - 55$$

$$\frac{8\sqrt{2}\;3^{3/4}\sqrt[4]{5}\,\left(2+\log^2(2)\right)^{5/4} - 55\log^{5/4}(2)}{\log^{5/4}(2)}$$

Alternative representations:

$$\sqrt[4]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}}-55=-55+\sqrt[4]{\frac{1}{\frac{\left(\frac{\log_e(2)}{8+4\log_e^2(2)}\right)^5}{2160}}}$$

$$\sqrt[4]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}}-55=-55+\sqrt[4]{\frac{1}{\frac{\left(\frac{2\coth^{-1}(3)}{8+4\left(2\coth^{-1}(3)\right)^2}\right)^5}{2160}}}$$

$$\sqrt[4]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}}-55=-55+\sqrt[4]{\frac{1}{\frac{\left(-\frac{\text{Li}_1(-1)}{8+4(-\text{Li}_1(-1))^2}\right)^5}{2160}}}$$

Series representations:

$$\sqrt[4]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}}-55=$$

$$-55+\left(2\times 3^{3/4}\sqrt[4]{5}\left(8+4\left(\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^2\right)^{5/4}\right)\Big/$$

$$\left(\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^{5/4}$$

$$\sqrt[4]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}} - 55 =$$

$$\left(-55\left(2 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}\right)^{5/4} + 8\sqrt{2}\, 3^{3/4} \sqrt[4]{5} \left(2 - 4\pi^2 \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor^2 + 4 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor \log(x) + \log^2(x) - 4 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} - 2\log(x) \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} + \left(\sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}\right)^2\right)^{5/4}\right) \Big/ \left(2 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}\right)^{5/4} \quad \text{for } x < 0$$

$$\sqrt[4]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}} - 55 =$$

$$\left(-55\left(2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}\right)^{5/4} + 8\sqrt{2}\, 3^{3/4} \sqrt[4]{5} \left(2 - 4\pi^2 \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor^2 + 4 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor \log(z_0) + \log^2(z_0) - 4 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} - 2\log(z_0) \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} + \left(\sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}\right)^2\right)^{5/4}\right) \Big/ \left(2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}\right)^{5/4}$$

Integral representations:

$$\sqrt[4]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}} - 55 = \frac{1}{\left(\int_1^2 \frac{1}{t}\,dt\right)^{5/4}} \left(-55\left(\int_1^2 \frac{1}{t}\,dt\right)^{5/4} + 16\sqrt{2}\;3^{3/4}\sqrt[4]{5}\sqrt[4]{2+\left(\int_1^2 \frac{1}{t}\,dt\right)^2} + 8\sqrt{2}\;3^{3/4}\sqrt[4]{5}\left(\int_1^2 \frac{1}{t}\,dt\right)^2\sqrt[4]{2+\left(\int_1^2 \frac{1}{t}\,dt\right)^2}\right)$$

$$\sqrt[4]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}} - 55 = \left(-55\,\pi^{5/4}\left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2 - 32\times 3^{3/4}\sqrt[4]{10}\,\pi^2 \left(-i\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^{3/4}\sqrt[4]{8\pi^2-\left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2} + 4\times 3^{3/4}\sqrt[4]{10}\left(-i\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^{3/4} \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2\sqrt[4]{8\pi^2-\left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2}\right) \Big/ \left(\pi^{5/4}\left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2\right) \quad \text{for } -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

(((1/(((1/2160(((ln 2)/(8+4(ln 2)^2)))^5))))))^1/4-34-21-8-golden ratio

Input:

$$\sqrt[4]{\frac{1}{\frac{1}{2160}\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}} - 34 - 21 - 8 - \phi$$

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

Exact result:

$$-\phi - 63 + \frac{2\times 3^{3/4}\sqrt[4]{5}\,(8+4\log^2(2))^{5/4}}{\log^{5/4}(2)}$$

Decimal approximation:

125.1925199662557980766066351373100395031023101259334913621...

125.192519….

Alternate forms:

$$-\phi - 63 + 2\times 3^{3/4}\sqrt[4]{5}\left(\frac{8}{\log(2)} + \log(16)\right)^{5/4}$$

$$-\phi - 63 + \frac{8\sqrt{2}\,3^{3/4}\sqrt[4]{5}\,(2+\log^2(2))^{5/4}}{\log^{5/4}(2)}$$

$$-\frac{127}{2} - \frac{\sqrt{5}}{2} + \frac{2\times 3^{3/4}\sqrt[4]{5}\,(8+4\log^2(2))^{5/4}}{\log^{5/4}(2)}$$

Alternative representations:

$$\sqrt[4]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}} - 34 - 21 - 8 - \phi = -63 - \phi + \sqrt[4]{\frac{1}{\frac{\left(\frac{\log_e(2)}{8+4\log_e^2(2)}\right)^5}{2160}}}$$

$$\sqrt[4]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}} - 34 - 21 - 8 - \phi = -63 - \phi + \sqrt[4]{\frac{1}{\frac{\left(\frac{2\coth^{-1}(3)}{8+4\left(2\coth^{-1}(3)\right)^2}\right)^5}{2160}}}$$

$$\sqrt[4]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}} - 34 - 21 - 8 - \phi = -63 - \phi + \sqrt[4]{\frac{1}{\frac{\left(-\frac{\mathrm{Li}_1(-1)}{8+4(-\mathrm{Li}_1(-1))^2}\right)^5}{2160}}}$$

Series representations:

$$\sqrt[4]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}} - 34 - 21 - 8 - \phi =$$

$$-63 - \phi + \left(2 \times 3^{3/4} \sqrt[4]{5} \left(8 + 4\left(\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}\right)^2\right)^{5/4}\right) \Big/ \left(\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}\right)^{5/4}$$

$$\sqrt[4]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}} - 34 - 21 - 8 - \phi =$$

$$-\left(\left(127\left(2 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}\right)^{5/4} + \sqrt{5}\left(2 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}\right)^{5/4} - 16\sqrt{2}\, 3^{3/4} \sqrt[4]{5} \left(2 - 4\pi^2 \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor^2 + 4 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor \log(x) + \log^2(x) - 4 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} - 2\log(x) \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} + \left(\sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}\right)^2\right)^{5/4}\right) \Big/ \left(2\left(2 i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}\right)^{5/4}\right)\right) \quad \text{for } x < 0$$

$$\sqrt[4]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}} - 34 - 21 - 8 - \phi =$$

$$-\left(\left(127\left(2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}\right)^{5/4} + \sqrt{5}\left(2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}\right)^{5/4} - 16\sqrt{2}\, 3^{3/4} \sqrt[4]{5} \left(2 - 4\pi^2 \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor^2 + 4 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor \log(z_0) + \log^2(z_0) - 4 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} - 2\log(z_0) \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} + \left(\sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}\right)^2\right)^{5/4}\right) \Big/ \left(2\left(2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}\right)^{5/4}\right)\right)$$

Integral representations:

$$\sqrt[4]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}} - 34 - 21 - 8 - \phi = -\frac{1}{2\left(\int_1^2 \frac{1}{t}\,dt\right)^{5/4}} \left(127\left(\int_1^2 \frac{1}{t}\,dt\right)^{5/4} + \sqrt{5}\left(\int_1^2 \frac{1}{t}\,dt\right)^{5/4} - 32\sqrt{2}\, 3^{3/4} \sqrt[4]{5} \sqrt[4]{2 + \left(\int_1^2 \frac{1}{t}\,dt\right)^2} - 16\sqrt{2}\, 3^{3/4} \sqrt[4]{5} \left(\int_1^2 \frac{1}{t}\,dt\right)^2 \sqrt[4]{2 + \left(\int_1^2 \frac{1}{t}\,dt\right)^2}\right)$$

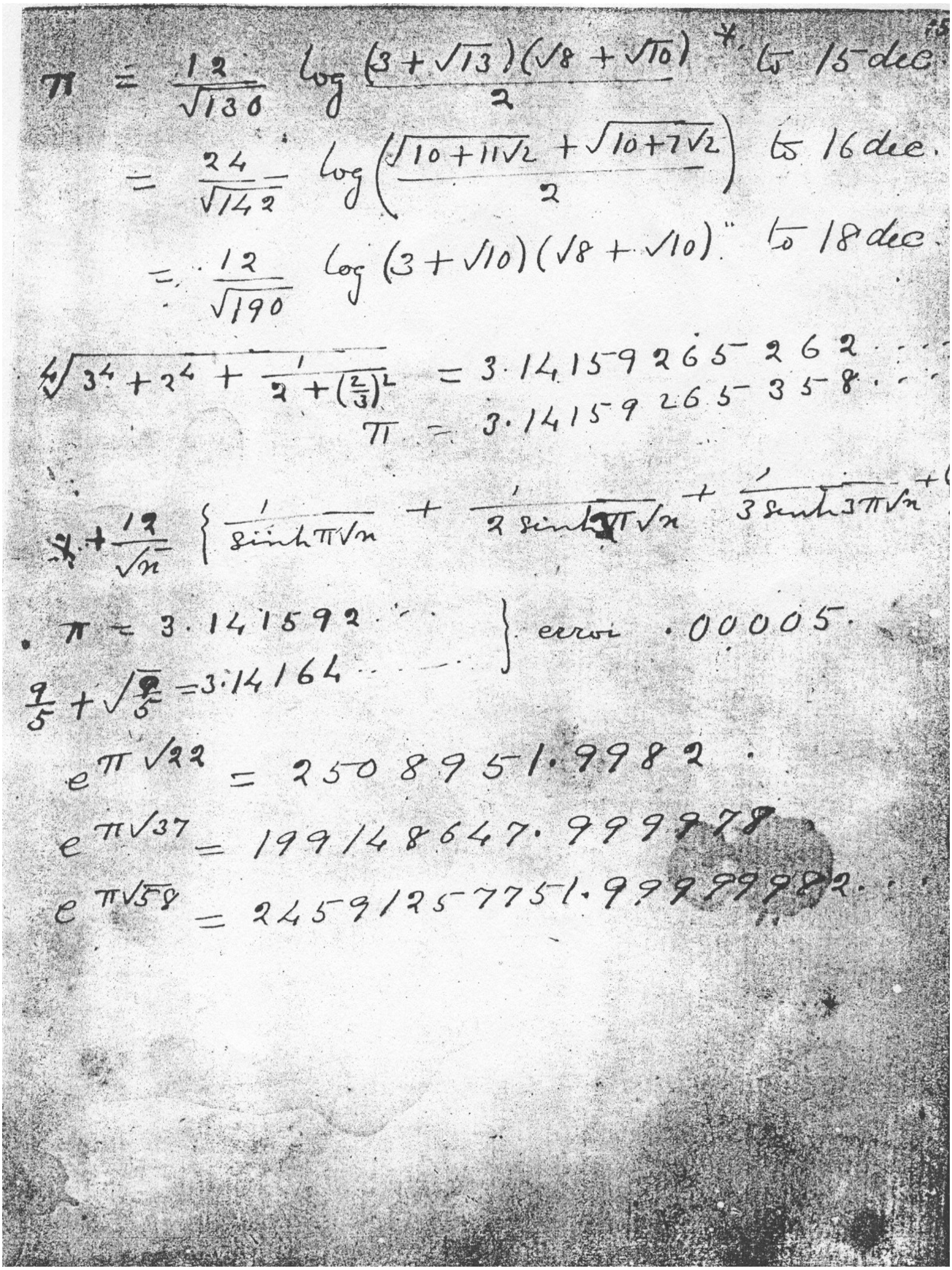
$$\sqrt[4]{\frac{1}{\frac{\left(\frac{\log(2)}{8+4\log^2(2)}\right)^5}{2160}}} - 34 - 21 - 8 - \phi =$$

$$-\left(\left(127\pi^{5/4}\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2 + \sqrt{5}\,\pi^{5/4}\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2 + 64\times 3^{3/4}\sqrt[4]{10}\,\pi^2\left(-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^{3/4}\sqrt[4]{8\pi^2-\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2} - 8\times 3^{3/4}\sqrt[4]{10}\left(-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^{3/4}\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2\sqrt[4]{8\pi^2-\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2}\right)\middle/\left(2\pi^{5/4}\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2\right)\right) \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

From:

https://webpages.ciencias.ulisboa.pt/~ommartins/seminario/Ramanujan/biografia.htm



$$\pi = \frac{12}{\sqrt{130}} \log \frac{(3+\sqrt{13})(\sqrt{8}+\sqrt{10})}{2} \text{ to 15 dec}$$

$$= \frac{24}{\sqrt{142}} \log \left(\frac{\sqrt{10+11\sqrt{2}} + \sqrt{10+7\sqrt{2}}}{2} \right) \text{ to 16 dec.}$$

$$= \frac{12}{\sqrt{190}} \log (3+\sqrt{10})(\sqrt{8}+\sqrt{10}) \text{ to 18 dec.}$$

$$\sqrt[4]{3^4+2^4+\frac{1}{2+\left(\frac{2}{3}\right)^2}} = 3.14159265262\ldots$$

$$\pi = 3.14159265358\ldots$$

$$[illegible] + \frac{12}{\sqrt{n}} \left\{ \frac{1}{\sinh \pi\sqrt{n}} + \frac{1}{2\sinh 2\pi\sqrt{n}} + \frac{1}{3\sinh 3\pi\sqrt{n}} + \ldots \right.$$

$$\left. \begin{array}{l} \pi = 3.141592\ldots \\ \frac{9}{5}+\sqrt{\frac{9}{5}} = 3.14164\ldots \end{array} \right\} \text{ error } .00005$$

$$e^{\pi\sqrt{22}} = 2508951.9982$$

$$e^{\pi\sqrt{37}} = 199148647.999978$$

$$e^{\pi\sqrt{58}} = 24591257751.99999982$$

From the second expression, we have:

24/(sqrt142) ln((((((10+11sqrt2)^0.5)+((10+7sqrt2)^0.5))/2)))

Input:

$$\frac{24}{\sqrt{142}} \log\left(\frac{1}{2}\left(\sqrt{10+11\sqrt{2}} + \sqrt{10+7\sqrt{2}}\right)\right)$$

log(x) is the natural logarithm

Exact result:

$$12\sqrt{\frac{2}{71}} \log\left(\frac{1}{2}\left(\sqrt{10+7\sqrt{2}} + \sqrt{10+11\sqrt{2}}\right)\right)$$

Decimal approximation:

3.141592653589793127379949506290255350331758331654956045001...

3.1415926535……. ≈ π

Property:

$12\sqrt{\frac{2}{71}} \log\left(\frac{1}{2}\left(\sqrt{10+7\sqrt{2}} + \sqrt{10+11\sqrt{2}}\right)\right)$ is a transcendental number

Alternate forms:

$$6\sqrt{\frac{2}{71}} \log\left(5 + \frac{9}{\sqrt{2}} + \sqrt{\frac{127}{2} + 45\sqrt{2}}\right)$$

$$6\sqrt{\frac{2}{71}} \log\left(5 + \frac{9}{\sqrt{2}} + \sqrt{\frac{1}{2}\left(127 + 90\sqrt{2}\right)}\right)$$

$$6\sqrt{\frac{2}{71}} \left(-5\log(2) + 2\log\left(\sqrt{2\left(10-7\sqrt{2}\right)} + 2\sqrt{10-\sqrt{2}} + 2^{3/4}\sqrt{7+5\sqrt{2}} + 2\sqrt{10-i\sqrt{142}} + 2\sqrt{10+i\sqrt{142}}\right)\right)$$

Alternative representations:

$$\frac{\log\left(\frac{1}{2}\left(\sqrt{10+11\sqrt{2}}+\sqrt{10+7\sqrt{2}}\right)\right)24}{\sqrt{142}} = \frac{24\log_e\left(\frac{1}{2}\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)\right)}{\sqrt{142}}$$

$$\frac{\log\left(\frac{1}{2}\left(\sqrt{10+11\sqrt{2}}+\sqrt{10+7\sqrt{2}}\right)\right)24}{\sqrt{142}} = \frac{24\log(a)\log_a\left(\frac{1}{2}\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)\right)}{\sqrt{142}}$$

$$\frac{\log\left(\frac{1}{2}\left(\sqrt{10+11\sqrt{2}}+\sqrt{10+7\sqrt{2}}\right)\right)24}{\sqrt{142}} = -\frac{24\,\mathrm{Li}_1\left(1+\frac{1}{2}\left(-\sqrt{10+7\sqrt{2}}-\sqrt{10+11\sqrt{2}}\right)\right)}{\sqrt{142}}$$

Series representations:

$$\frac{\log\left(\frac{1}{2}\left(\sqrt{10+11\sqrt{2}}+\sqrt{10+7\sqrt{2}}\right)\right)24}{\sqrt{142}} = 12\sqrt{\frac{2}{71}}\log\left(\frac{1}{2}\left(-2+\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)\right) - 12\sqrt{\frac{2}{71}}\sum_{k=1}^{\infty}\frac{\left(-\frac{2}{-2+\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}}\right)^k}{k}$$

$$\frac{\log\left(\frac{1}{2}\left(\sqrt{10+11\sqrt{2}}+\sqrt{10+7\sqrt{2}}\right)\right)24}{\sqrt{142}} = 12\sqrt{\frac{2}{71}}\log\left(-1+\frac{1}{2}\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)\right) - 12\sqrt{\frac{2}{71}}\sum_{k=1}^{\infty}\frac{\left(-\frac{2}{-2+\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}}\right)^k}{k}$$

$$\frac{\log\left(\frac{1}{2}\left(\sqrt{10+11\sqrt{2}}+\sqrt{10+7\sqrt{2}}\right)\right)24}{\sqrt{142}} =$$

$$24\,i\sqrt{\frac{2}{71}}\,\pi\left\lfloor\frac{\arg\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}-2x\right)}{2\pi}\right\rfloor+12\sqrt{\frac{2}{71}}\log(x)-$$

$$12\sqrt{\frac{2}{71}}\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}-2x\right)^k x^{-k}}{k} \quad \text{for } x<0$$

Integral representations:

$$\frac{\log\left(\frac{1}{2}\left(\sqrt{10+11\sqrt{2}}+\sqrt{10+7\sqrt{2}}\right)\right)24}{\sqrt{142}} = 12\sqrt{\frac{2}{71}}\int_1^{\frac{1}{2}\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)}\frac{1}{t}\,dt$$

$$\frac{\log\left(\frac{1}{2}\left(\sqrt{10+11\sqrt{2}}+\sqrt{10+7\sqrt{2}}\right)\right)24}{\sqrt{142}} =$$

$$-\frac{6\,i\sqrt{\frac{2}{71}}}{\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\left(-1+\frac{1}{2}\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

From this expression, we can also to obtain:

24/(sqrt(x+3)) ln((((((10+11sqrt2)^0.5)+((10+7sqrt2)^0.5))/2))) = 3.1415926535897931273

Input interpretation:

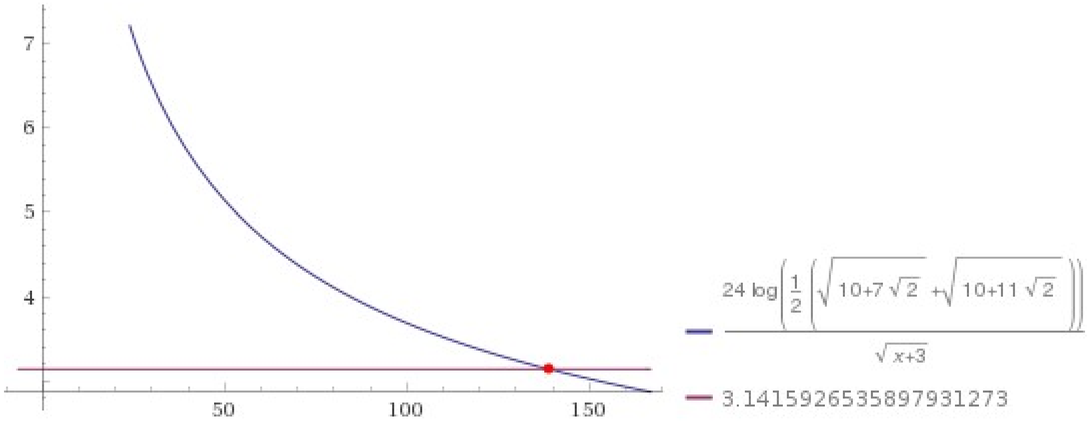
$$\frac{24}{\sqrt{x+3}}\log\left(\frac{1}{2}\left(\sqrt{10+11\sqrt{2}}+\sqrt{10+7\sqrt{2}}\right)\right) = 3.1415926535897931273$$

$\log(x)$ is the natural logarithm

Result:

$$\frac{24\log\left(\frac{1}{2}\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)\right)}{\sqrt{x+3}} = 3.1415926535897931273$$

Plot:



Alternate form assuming x is real:

$$\frac{1.00000000000000}{\sqrt{x+3}} = 0.0839181358296689$$

Alternate forms:

$$\frac{12\log\left(5+\frac{9}{\sqrt{2}}+\sqrt{\frac{127}{2}+45\sqrt{2}}\right)}{\sqrt{x+3}} = 3.1415926535897931273$$

$$\frac{12\log\left(5+\frac{9}{\sqrt{2}}+\sqrt{\frac{1}{2}\left(127+90\sqrt{2}\right)}\right)}{\sqrt{x+3}} = 3.1415926535897931273$$

Alternate form assuming x is positive:

$$1.00000000000000000\sqrt{x+3} = 0.083918135829668908\,x + 0.2517544074890067 24$$

Alternate forms assuming x>0:

$$-\frac{24\log\left(\frac{2}{\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}}\right)}{\sqrt{x+3}} = 3.1415926535897931273$$

$$\frac{24\log\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)}{\sqrt{x+3}} - \frac{24\log(2)}{\sqrt{x+3}} = 3.1415926535897931273$$

Solution:

$x \approx 139.00000000000000001$

139

24/(sqrt(x+18-golden ratio)) ln((((((10+11sqrt2)^0.5)+((10+7sqrt2)^0.5))/2))) = 3.14159265358979312 73

Input interpretation:

$$\frac{24}{\sqrt{x+18-\phi}} \log\left(\frac{1}{2}\left(\sqrt{10+11\sqrt{2}}+\sqrt{10+7\sqrt{2}}\right)\right) = 3.1415926535897931273$$

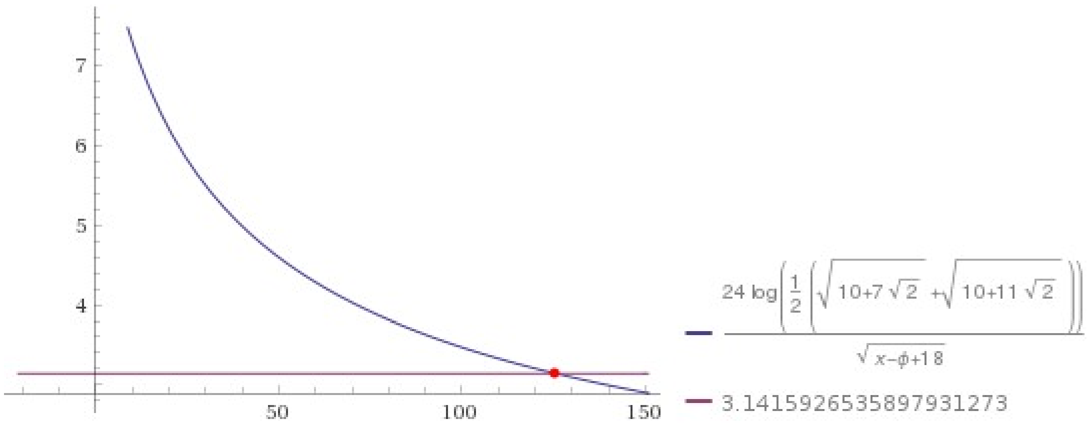
log(x) is the natural logarithm

ϕ is the golden ratio

Result:

$$\frac{24 \log\left(\frac{1}{2}\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)\right)}{\sqrt{x-\phi+18}} = 3.1415926535897931273$$

Plot:



Alternate form assuming x is real:

$$\frac{1.0000000000\sqrt{2x-\sqrt{5}+35}}{1.0000000000\,x+16.381966011} = 0.118678165819$$

Alternate forms:

$$\frac{12 \log\left(5+\frac{9}{\sqrt{2}}+\sqrt{\frac{127}{2}+45\sqrt{2}}\right)}{\sqrt{x-\phi+18}} = 3.1415926535897931273$$

$$\frac{12\log\left(5+\frac{9}{\sqrt{2}}+\sqrt{\frac{1}{2}(127+90\sqrt{2})}\right)}{\sqrt{x+\frac{1}{2}(35-\sqrt{5})}} = 3.1415926535897931273$$

$$\frac{24\sqrt{2}\log\left(\frac{1}{2}\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)\right)}{\sqrt{2x-\sqrt{5}+35}} = 3.1415926535897931273$$

Alternate form assuming x is positive:

$$1.0000000000000000\sqrt{2x-\sqrt{5}+35} = 0.11867816581938533x+1.9441816787306745$$

Expanded form:

$$\frac{24\log\left(\frac{1}{2}\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)\right)}{\sqrt{x+\frac{1}{2}(-1-\sqrt{5})+18}} = 3.1415926535897931273$$

Alternate forms assuming x>0:

$$\frac{24\left(\log\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)-\log(2)\right)}{\sqrt{x-\frac{\sqrt{5}}{2}+\frac{35}{2}}} = 3.1415926535897931273$$

$$\frac{24\log\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)}{\sqrt{x-\phi+18}} - \frac{24\log(2)}{\sqrt{x-\phi+18}} = 3.1415926535897931273$$

Solution:

$x \approx 125.61803398874989486$

125.61803398874989486

24/(sqrt((x-24)/12)) ln((((((10+11sqrt2)^0.5)+((10+7sqrt2)^0.5))/2))) = 3.1415926535897931273

Input interpretation:

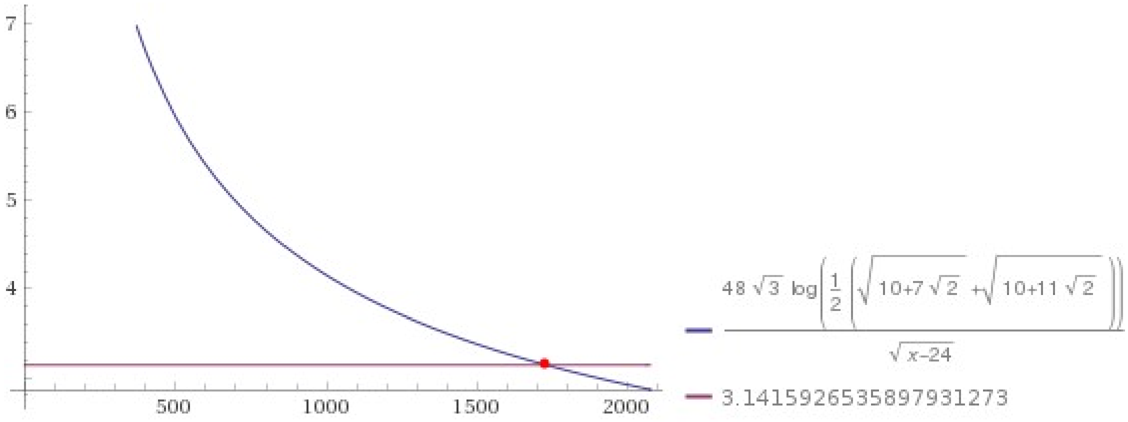
$$\frac{24}{\sqrt{\frac{x-24}{12}}}\log\left(\frac{1}{2}\left(\sqrt{10+11\sqrt{2}}+\sqrt{10+7\sqrt{2}}\right)\right) = 3.1415926535897931273$$

log(x) is the natural logarithm

Result:

$$\frac{48\sqrt{3}\,\log\left(\frac{1}{2}\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)\right)}{\sqrt{x-24}} = 3.1415926535897931273$$

Plot:



Alternate forms:

$$\frac{24\log\left(5+\frac{9}{\sqrt{2}}+\sqrt{\frac{127}{2}+45\sqrt{2}}\right)}{\sqrt{\frac{x}{3}-8}} = 3.1415926535897931273$$

$$\frac{24\sqrt{3}\,\log\left(5+\frac{9}{\sqrt{2}}+\sqrt{\frac{1}{2}\left(127+90\sqrt{2}\right)}\right)}{\sqrt{x-24}} = 3.1415926535897931273$$

$$\frac{48\log\left(\frac{1}{2}\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)\right)}{\sqrt{\frac{x}{3}-8}} = 3.1415926535897931273$$

Alternate form assuming x>0:

$$\frac{48\sqrt{3}\,\log\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)}{\sqrt{x-24}} - \frac{48\sqrt{3}\,\log(2)}{\sqrt{x-24}} = 3.1415926535897931273$$

Alternate form assuming x is positive:

$$\frac{1.00000000000000}{\sqrt{x-24}} = 0.02422507915558$$

Solution:

$x \approx 1728.0000000000000001$

1728

x/(sqrt(142)) ln((((((10+11sqrt2)^0.5)+((10+7sqrt2)^0.5))/2))) = 3.14159265358979312 73

Input interpretation:

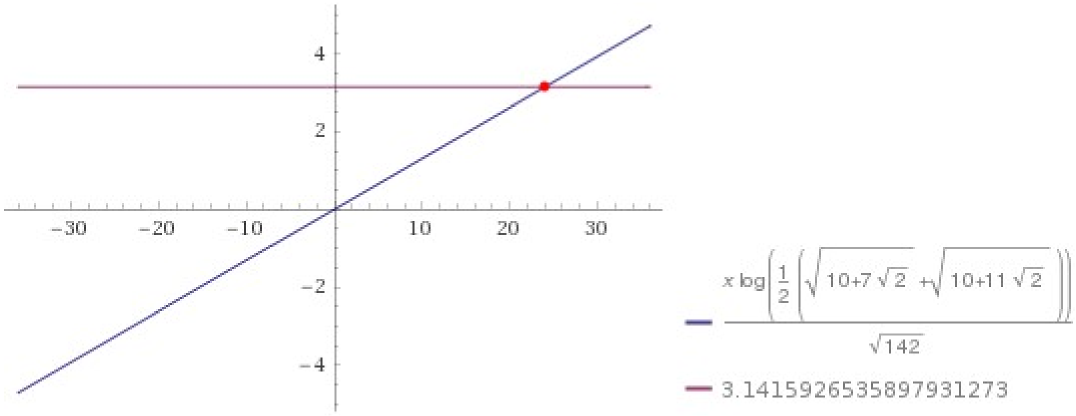
$$\frac{x}{\sqrt{142}} \log\left(\frac{1}{2}\left(\sqrt{10+11\sqrt{2}}+\sqrt{10+7\sqrt{2}}\right)\right) = 3.1415926535897931273$$

log(x) is the natural logarithm

Result:

$$\frac{x \log\left(\frac{1}{2}\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)\right)}{\sqrt{142}} = 3.1415926535897931273$$

Plot:



Alternate forms:

$$\frac{x \log\left(5+\frac{9}{\sqrt{2}}+\sqrt{\frac{127}{2}+45\sqrt{2}}\right)}{2\sqrt{142}} = 3.1415926535897931273$$

$$\frac{x \log\left(5+\frac{9}{\sqrt{2}}+\sqrt{\frac{1}{2}\left(127+90\sqrt{2}\right)}\right)}{2\sqrt{142}} = 3.1415926535897931273$$

$$\frac{x\left(\log\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)-\log(2)\right)}{\sqrt{142}} = 3.1415926535897931273$$

Alternate form assuming x>0:

$$\frac{x\log\left(\sqrt{10+7\sqrt{2}}+\sqrt{10+11\sqrt{2}}\right)}{\sqrt{142}} - \frac{x\log(2)}{\sqrt{142}} = 3.1415926535897931273$$

Solution:

$x \approx 24.000000000000000000$

Integer solution:

$x = 24$

24

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

12/(sqrt130) ln(1/2*(3+sqrt13)(sqrt8+sqrt10))

Input:

$$\frac{12}{\sqrt{130}}\log\left(\frac{1}{2}\left(3+\sqrt{13}\right)\left(\sqrt{8}+\sqrt{10}\right)\right)$$

log(x) is the natural logarithm

Exact result:

$$6\sqrt{\frac{2}{65}}\log\left(\frac{1}{2}\left(2\sqrt{2}+\sqrt{10}\right)\left(3+\sqrt{13}\right)\right)$$

Decimal approximation:

3.1415926535897926537322435251197540183892557971769513889 65...

3.141592653589… ≈ π

Property:

$6\sqrt{\frac{2}{65}}\log\left(\frac{1}{2}\left(2\sqrt{2}+\sqrt{10}\right)\left(3+\sqrt{13}\right)\right)$ is a transcendental number

Alternate forms:

$$6\sqrt{\frac{2}{65}}\left(-\frac{\log(2)}{2}+\log\left(3+\sqrt{13}\right)+\sinh^{-1}(2)\right)$$

$$6\sqrt{\frac{2}{65}}\log\left(\frac{(2+\sqrt{5})(3+\sqrt{13})}{\sqrt{2}}\right)$$

$$3\sqrt{\frac{2}{65}}\log\left(99+44\sqrt{5}+3\sqrt{13\left(161+72\sqrt{5}\right)}\right)$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternative representations:

$$\frac{\log\left(\frac{1}{2}(3+\sqrt{13})(\sqrt{8}+\sqrt{10})\right)12}{\sqrt{130}}=\frac{12\log_e\left(\frac{1}{2}(\sqrt{8}+\sqrt{10})(3+\sqrt{13})\right)}{\sqrt{130}}$$

$$\frac{\log\left(\frac{1}{2}(3+\sqrt{13})(\sqrt{8}+\sqrt{10})\right)12}{\sqrt{130}}=\frac{12\log(a)\log_a\left(\frac{1}{2}(\sqrt{8}+\sqrt{10})(3+\sqrt{13})\right)}{\sqrt{130}}$$

$$\frac{\log\left(\frac{1}{2}(3+\sqrt{13})(\sqrt{8}+\sqrt{10})\right)12}{\sqrt{130}}=-\frac{12\operatorname{Li}_1\left(1-\frac{1}{2}(\sqrt{8}+\sqrt{10})(3+\sqrt{13})\right)}{\sqrt{130}}$$

Series representations:

$$\frac{\log\left(\frac{1}{2}(3+\sqrt{13})(\sqrt{8}+\sqrt{10})\right)12}{\sqrt{130}}=$$

$$6\sqrt{\frac{2}{65}}\log\left(-1+\frac{(2+\sqrt{5})(3+\sqrt{13})}{\sqrt{2}}\right)-6\sqrt{\frac{2}{65}}\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+\frac{(2+\sqrt{5})(3+\sqrt{13})}{\sqrt{2}}}\right)^k}{k}$$

$$\frac{\log\left(\frac{1}{2}(3+\sqrt{13})(\sqrt{8}+\sqrt{10})\right)12}{\sqrt{130}} =$$

$$6\sqrt{\frac{2}{65}}\log\left(-1+\frac{1}{2}\left(2\sqrt{2}+\sqrt{10}\right)\left(3+\sqrt{13}\right)\right)-6\sqrt{\frac{2}{65}}\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+\frac{(2+\sqrt{5})(3+\sqrt{13})}{\sqrt{2}}}\right)^k}{k}$$

$$\frac{\log\left(\frac{1}{2}(3+\sqrt{13})(\sqrt{8}+\sqrt{10})\right)12}{\sqrt{130}} = 12\,i\sqrt{\frac{2}{65}}\,\pi\left\lfloor\frac{\arg\left(\frac{(2+\sqrt{5})(3+\sqrt{13})}{\sqrt{2}}-x\right)}{2\pi}\right\rfloor + 6\sqrt{\frac{2}{65}}\log(x)-6\sqrt{\frac{2}{65}}\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{(2+\sqrt{5})(3+\sqrt{13})}{\sqrt{2}}-x\right)^k x^{-k}}{k} \quad \text{for } x<0$$

Integral representations:

$$\frac{\log\left(\frac{1}{2}(3+\sqrt{13})(\sqrt{8}+\sqrt{10})\right)12}{\sqrt{130}} = 6\sqrt{\frac{2}{65}}\int_1^{\frac{(2+\sqrt{5})(3+\sqrt{13})}{\sqrt{2}}}\frac{1}{t}\,dt$$

$$\frac{\log\left(\frac{1}{2}(3+\sqrt{13})(\sqrt{8}+\sqrt{10})\right)12}{\sqrt{130}} = -\frac{3\,i\sqrt{\frac{2}{65}}}{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\left(-1+\frac{(2+\sqrt{5})(3+\sqrt{13})}{\sqrt{2}}\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1<\gamma<0$$

From which, we obtain:

12/(sqrt(x-5)) ln(1/2*(3+sqrt13)(sqrt8+sqrt10)) = 3.14159265358979265

Input interpretation:

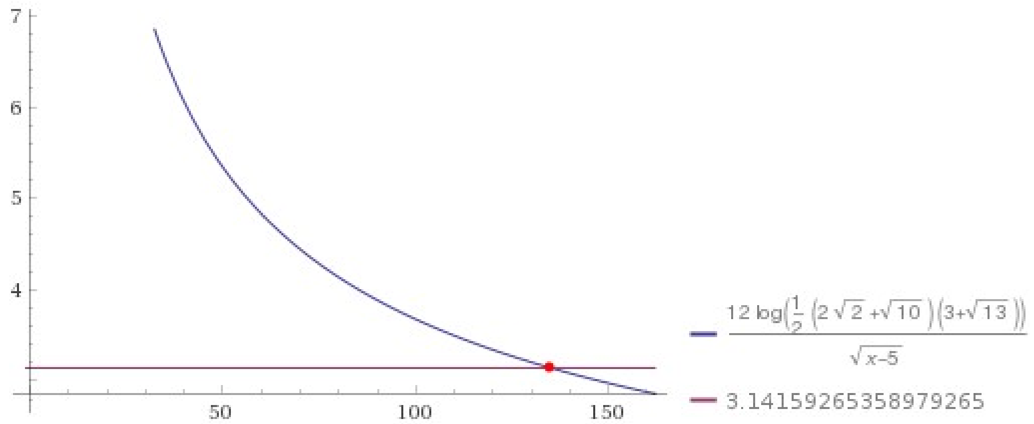
$$\frac{12}{\sqrt{x-5}}\log\left(\frac{1}{2}\left(3+\sqrt{13}\right)\left(\sqrt{8}+\sqrt{10}\right)\right) = 3.14159265358979265$$

log(x) is the natural logarithm

Result:

$$\frac{12\log\left(\frac{1}{2}\left(2\sqrt{2}+\sqrt{10}\right)\left(3+\sqrt{13}\right)\right)}{\sqrt{x-5}} = 3.14159265358979265$$

Plot:



Alternate forms:

$$\frac{6\left(\log\left(11+3\sqrt{13}\right)+2\sinh^{-1}(2)\right)}{\sqrt{x-5}} = 3.14159265358979265$$

$$\frac{6\log\left(99+44\sqrt{5}+3\sqrt{13\left(161+72\sqrt{5}\right)}\right)}{\sqrt{x-5}} = 3.14159265358979265$$

$$\frac{12\left(-\log(2)+\log\left(2\sqrt{2}+\sqrt{10}\right)+\log\left(3+\sqrt{13}\right)\right)}{\sqrt{x-5}} = 3.14159265358979265$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternate form assuming x>0:

$$\frac{12\log\left(3+\sqrt{13}\right)}{\sqrt{x-5}}+\frac{12\log\left(2\sqrt{2}+\sqrt{10}\right)}{\sqrt{x-5}}-\frac{12\log(2)}{\sqrt{x-5}} = 3.14159265358979265$$

Alternate form assuming x is positive:

$$\frac{1.00000000000000}{\sqrt{x-5}} = 0.0877058019307$$

Solution:

$x \approx 135.000000000000000$

135

12/(sqrt(x-8)) ln(1/2*(3+sqrt13)(sqrt8+sqrt10)) = 3.14159265358979265

Input interpretation:

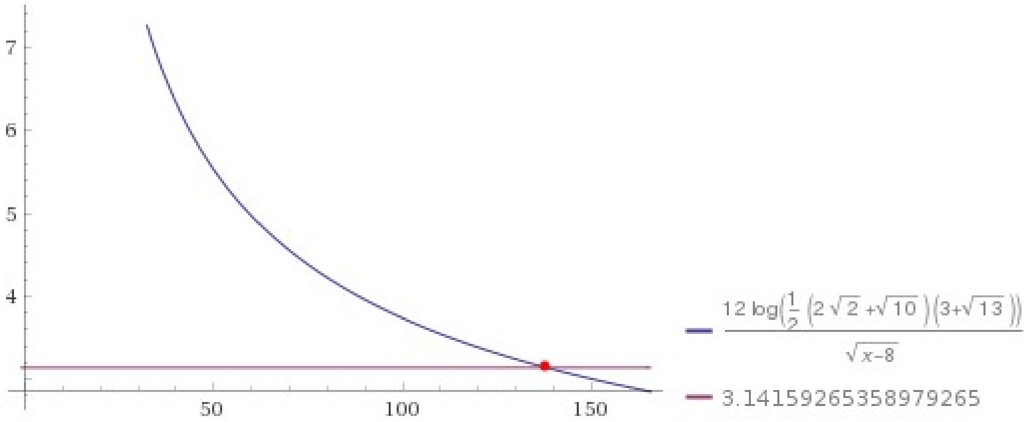
$$\frac{12}{\sqrt{x-8}} \log\left(\frac{1}{2}\left(3+\sqrt{13}\right)\left(\sqrt{8}+\sqrt{10}\right)\right) = 3.14159265358979265$$

log(x) is the natural logarithm

Result:

$$\frac{12 \log\left(\frac{1}{2}\left(2\sqrt{2}+\sqrt{10}\right)\left(3+\sqrt{13}\right)\right)}{\sqrt{x-8}} = 3.14159265358979265$$

Plot:



Alternate forms:

$$\frac{6\left(\log\left(11+3\sqrt{13}\right)+2\sinh^{-1}(2)\right)}{\sqrt{x-8}} = 3.14159265358979265$$

$$\frac{6 \log\left(99+44\sqrt{5}+3\sqrt{13\left(161+72\sqrt{5}\right)}\right)}{\sqrt{x-8}} = 3.14159265358979265$$

$$\frac{12\left(-\log(2)+\log\left(2\sqrt{2}+\sqrt{10}\right)+\log\left(3+\sqrt{13}\right)\right)}{\sqrt{x-8}} = 3.14159265358979265$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternate form assuming x>0:

$$\frac{12\log\left(3+\sqrt{13}\right)}{\sqrt{x-8}} + \frac{12\log\left(2\sqrt{2}+\sqrt{10}\right)}{\sqrt{x-8}} - \frac{12\log(2)}{\sqrt{x-8}} = 3.14159265358979265$$

Alternate form assuming x is positive:

$$\frac{1.00000000000000}{\sqrt{x-8}} = 0.0877058019307$$

Solution:

$x \approx 138.000000000000000$

138

12/(sqrt(x-42)) ln(1/2*(3+sqrt13)(sqrt8+sqrt10)) = 3.14159265358979265

Input interpretation:

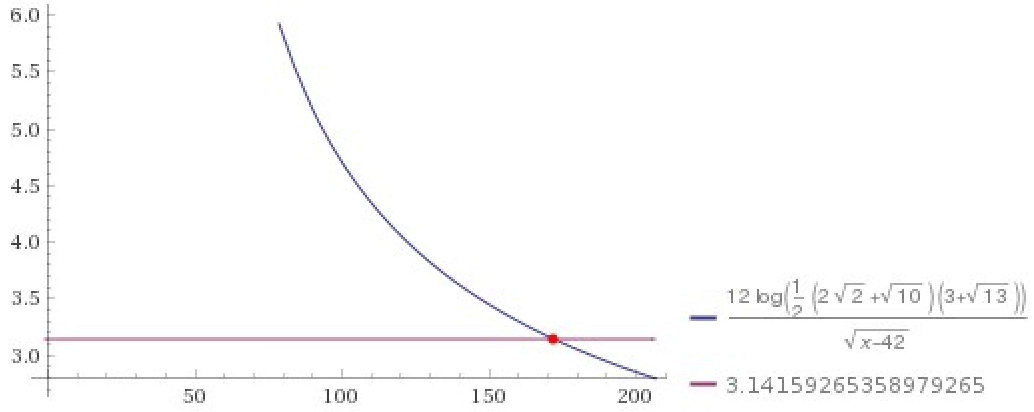
$$\frac{12}{\sqrt{x-42}}\log\left(\frac{1}{2}\left(3+\sqrt{13}\right)\left(\sqrt{8}+\sqrt{10}\right)\right) = 3.14159265358979265$$

$\log(x)$ is the natural logarithm

Result:

$$\frac{12\log\left(\frac{1}{2}\left(2\sqrt{2}+\sqrt{10}\right)\left(3+\sqrt{13}\right)\right)}{\sqrt{x-42}} = 3.14159265358979265$$

Plot:



Alternate forms:

$$\frac{6\left(\log(11+3\sqrt{13})+2\sinh^{-1}(2)\right)}{\sqrt{x-42}} = 3.14159265358979265$$

$$\frac{6\log\left(99+44\sqrt{5}+3\sqrt{13\,(161+72\sqrt{5})}\right)}{\sqrt{x-42}} = 3.14159265358979265$$

$$\frac{12\left(-\log(2)+\log(2\sqrt{2}+\sqrt{10})+\log(3+\sqrt{13})\right)}{\sqrt{x-42}} = 3.14159265358979265$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternate form assuming x>0:

$$\frac{12\log(3+\sqrt{13})}{\sqrt{x-42}} + \frac{12\log(2\sqrt{2}+\sqrt{10})}{\sqrt{x-42}} - \frac{12\log(2)}{\sqrt{x-42}} = 3.14159265358979265$$

Alternate form assuming x is positive:

$$\frac{1.00000000000000}{\sqrt{x-42}} = 0.0877058019307$$

Solution:

$x \approx 172.000000000000000$

172

12/(sqrt((x-169)/12)) ln(1/2*(3+sqrt13)(sqrt8+sqrt10)) = 3.14159265358979265

Input interpretation:

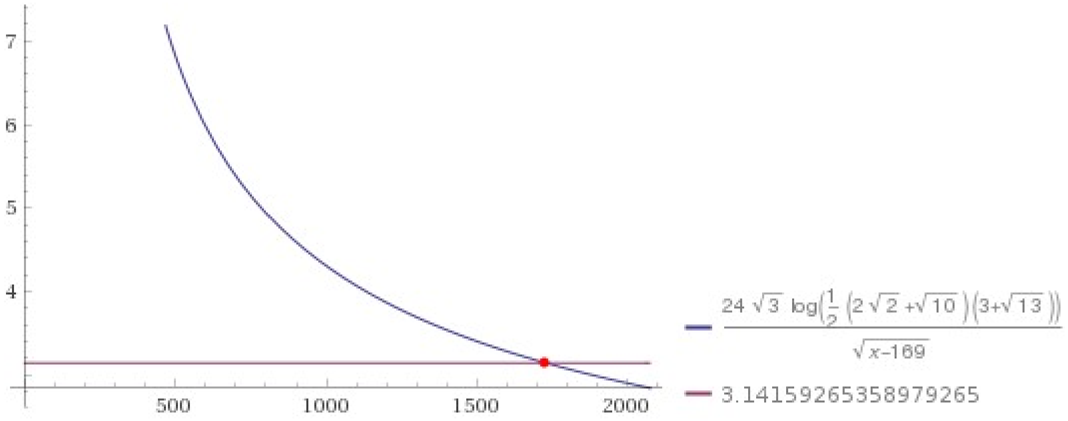
$$\frac{12}{\sqrt{\frac{x-169}{12}}} \log\left(\frac{1}{2}\left(3+\sqrt{13}\right)\left(\sqrt{8}+\sqrt{10}\right)\right) = 3.14159265358979265$$

log(x) is the natural logarithm

Result:

$$\frac{24\sqrt{3}\log\left(\frac{1}{2}\left(2\sqrt{2}+\sqrt{10}\right)\left(3+\sqrt{13}\right)\right)}{\sqrt{x-169}} = 3.14159265358979265$$

Plot:



Alternate forms:

$$\frac{12\sqrt{3}\left(\log\left(11+3\sqrt{13}\right)+2\sinh^{-1}(2)\right)}{\sqrt{x-169}} = 3.14159265358979265$$

$$\frac{24\sqrt{3}\left(-\log(2)+\log\left(2\sqrt{2}+\sqrt{10}\right)+\log\left(3+\sqrt{13}\right)\right)}{\sqrt{x-169}} = 3.14159265358979265$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternate form assuming x is positive:

$$\frac{1.00000000000000}{\sqrt{x-169}} = 0.02531848417709$$

Alternate forms assuming x>0:

$$\frac{24\sqrt{3}\left(-\frac{\log(2)}{2}+\log\left(2+\sqrt{5}\right)+\log\left(3+\sqrt{13}\right)\right)}{\sqrt{x-169}} = 3.14159265358979265$$

$$\frac{24\sqrt{3}\log(3+\sqrt{13})}{\sqrt{x-169}} + \frac{24\sqrt{3}\log(2\sqrt{2}+\sqrt{10})}{\sqrt{x-169}} - \frac{24\sqrt{3}\log(2)}{\sqrt{x-169}} = 3.14159265358979265$$

Solution:

$x \approx 1729.00000000000000$

1729

1/2*x / sqrt(130) ln(1/2*(3+sqrt13)(sqrt8+sqrt10)) = 3.14159265358979265

Input interpretation:

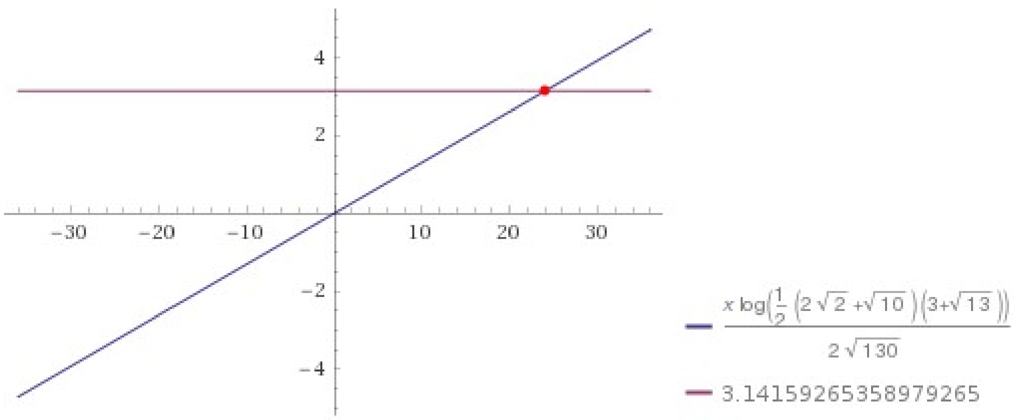
$$\frac{1}{2} \times \frac{x}{\sqrt{130}} \log\left(\frac{1}{2}\left(3+\sqrt{13}\right)\left(\sqrt{8}+\sqrt{10}\right)\right) = 3.14159265358979265$$

log(x) is the natural logarithm

Result:

$$\frac{x\log\left(\frac{1}{2}(2\sqrt{2}+\sqrt{10})(3+\sqrt{13})\right)}{2\sqrt{130}} = 3.14159265358979265$$

Plot:



Alternate forms:

$$\frac{x\left(\log(11+3\sqrt{13})+2\sinh^{-1}(2)\right)}{4\sqrt{130}} = 3.14159265358979265$$

$$\frac{x\left(-\log(2)+\log(2\sqrt{2}+\sqrt{10})+\log(3+\sqrt{13})\right)}{2\sqrt{130}} = 3.14159265358979265$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternate forms assuming x>0:

$$\frac{x\left(-\frac{\log(2)}{2}+\log(2+\sqrt{5})+\log(3+\sqrt{13})\right)}{2\sqrt{130}}=3.14159265358979265$$

$$\frac{x\log(3+\sqrt{13})}{2\sqrt{130}}+\frac{x\log(2\sqrt{2}+\sqrt{10})}{2\sqrt{130}}-\frac{x\log(2)}{2\sqrt{130}}=3.14159265358979265$$

Solution:

$x \approx 24.000000000000000$

Integer solution:

$x = 24$

24

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

Now, we have that:

https://writings.stephenwolfram.com/2016/04/who-was-ramanujan/

[xxvi]

VIII Theorems on approximate integration and summation of series.

(1). $1^2\log 1 + 2^2\log 2 + 3^2\log 3 + \cdots\cdots + x^2\log x$

$$= \frac{x(x+1)(2x+1)}{6}\log x - \frac{x^3}{9} + \frac{1}{4\pi^2}\left(\frac{1}{1^3}+\frac{1}{2^3}+\frac{1}{3^3}+\&c\right) + \frac{x}{12} - \frac{1}{360x} + \&c.$$

(2) $$1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \cdots\cdots + \frac{x^x}{\lfloor x}\theta = \frac{e^x}{2}.$$

where $\theta = \frac{1}{3} + \frac{4}{135(x+k)}$ where k lies between $\frac{8}{45}$ and $\frac{2}{21}$.

(3) $$1 + \left(\frac{x}{\lfloor 1}\right)^5 + \left(\frac{x^2}{\lfloor 2}\right)^5 + \left(\frac{x^3}{\lfloor 3}\right)^5 + \&c$$

$$= \frac{\sqrt{5}}{4\pi^2}\cdot\frac{e^{5x}}{5x^2 - x + \theta}$$ where θ vanishes when $x = \infty$.

(4) $$\frac{1^2}{e^x-1} + \frac{2^2}{e^{2x}-1} + \frac{3^2}{e^{3x}-1} + \frac{4^2}{e^{4x}-1} + \&c$$

$$= \frac{2}{x^3}\left(\frac{1}{1^3}+\frac{1}{2^3}+\frac{1}{3^3}+\&c\right) - \frac{1}{12x} + \frac{x}{1440} + \frac{x^3}{181440} + \frac{x^5}{7257600} + \frac{x^7}{159667200} + \&c.$$ when x is small.

(Note. x may be given values from 0 to 2.)

(5) $$\frac{1}{1001} + \frac{1}{1002^2} + \frac{3}{1003^3} + \frac{4^2}{1004^4} + \frac{5^3}{1005^5} + \&c$$

$$= \frac{1}{1000} - 10^{-440}\times 1.0125 \text{ nearly}.$$

(6) $$\int_0^a e^{-x^2}dx = \frac{\sqrt{\pi}}{2} - \cfrac{e^{-a^2}}{2a+\cfrac{1}{a+\cfrac{2}{2a+\cfrac{3}{a+\cfrac{4}{2a+\&c.}}}}}$$

(7) The coeff^t. of x^n in $\frac{1}{1-2x+2x^4-2x^9+2x^{16}-\&c}$

= The nearest integer to $\frac{1}{4n}\left\{\cosh(\pi\sqrt{n}) - \frac{\sinh(\pi\sqrt{n})}{\pi\sqrt{n}}\right\}$

From (4), we obtain:

2/8(1+1/8+1/27)-1/24+2/1440+8/181440+32/7257600+128/159667200

Input:

$$\frac{2}{8}\left(1+\frac{1}{8}+\frac{1}{27}\right)-\frac{1}{24}+\frac{2}{1440}+\frac{8}{181440}+\frac{32}{7257600}+\frac{128}{159667200}$$

Exact result:

$$\frac{416267}{1663200}$$

Decimal approximation:

0.25028078403078403078403078403078403078403078403078403078403078 4...

0.250280784...

From which:

1+1/(((1/(((2/8(1+1/8+1/27)-
1/24+2/1440+8/181440+32/7257600+128/159667200))))))^1/3

Input:

$$1+\frac{1}{\sqrt[3]{\frac{1}{\frac{2}{8}\left(1+\frac{1}{8}+\frac{1}{27}\right)-\frac{1}{24}+\frac{2}{1440}+\frac{8}{181440}+\frac{32}{7257600}+\frac{128}{159667200}}}}$$

Result:

$$1+\frac{\sqrt[3]{\frac{416267}{77}}}{6\times10^{2/3}}$$

Decimal approximation:

1.630196280514827893370094396031476109029933943405093001778...

1.63019628…

Alternate forms:

$$\frac{4620+77^{2/3}\sqrt[3]{4162670}}{4620}$$

$\left(\text{root of } 1663200x^3-416267 \text{ near } x=0.630196\right)+1$

$$\frac{6\times10^{2/3}+\sqrt[3]{\frac{416267}{77}}}{6\times10^{2/3}}$$

Minimal polynomial:

$1663200x^3-4989600x^2+4989600x-2079467$

From (7), we obtain:

1/8(((cosh(Pi*sqrt2)-sinh(Pi*sqrt2)/(Pi*sqrt2))))

Input:

$$\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)$$

cosh(x) is the hyperbolic cosine function

sinh(x) is the hyperbolic sine function

Decimal approximation:

4.118621867402802010655274119303959478741829296562199024633...

4.1186218674…

Alternate forms:

$$\frac{1}{8}\cosh\left(\sqrt{2}\,\pi\right)-\frac{\sinh\left(\sqrt{2}\,\pi\right)}{8\sqrt{2}\,\pi}$$

$$-\frac{\sqrt{2}\sinh\left(\sqrt{2}\,\pi\right)-2\pi\cosh\left(\sqrt{2}\,\pi\right)}{16\pi}$$

$$\frac{2\pi\cosh\left(\sqrt{2}\,\pi\right)-\sqrt{2}\sinh\left(\sqrt{2}\,\pi\right)}{16\pi}$$

Alternative representations:

$$\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh(\pi\sqrt{2})}{\pi\sqrt{2}}\right)=\frac{1}{8}\left(\cos\left(i\pi\sqrt{2}\right)-\frac{-e^{-\pi\sqrt{2}}+e^{\pi\sqrt{2}}}{2(\pi\sqrt{2})}\right)$$

$$\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh(\pi\sqrt{2})}{\pi\sqrt{2}}\right)=\frac{1}{8}\left(\cos\left(-i\pi\sqrt{2}\right)-\frac{-e^{-\pi\sqrt{2}}+e^{\pi\sqrt{2}}}{2(\pi\sqrt{2})}\right)$$

$$\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh(\pi\sqrt{2})}{\pi\sqrt{2}}\right)=\frac{1}{8}\left(\frac{1}{2}\left(e^{-\pi\sqrt{2}}+e^{\pi\sqrt{2}}\right)-\frac{-e^{-\pi\sqrt{2}}+e^{\pi\sqrt{2}}}{2(\pi\sqrt{2})}\right)$$

Series representations:

$$\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh(\pi\sqrt{2})}{\pi\sqrt{2}}\right)=\sum_{k=0}^{\infty}\frac{2^{-3+k}\pi^{2k}(-(2k)!+(1+2k)!)}{(2k)!(1+2k)!}$$

$$\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh(\pi\sqrt{2})}{\pi\sqrt{2}}\right)=\sum_{k=0}^{\infty}\left(-\frac{2^{-7/2+1/2(1+2k)}\pi^{2k}}{(1+2k)!}+\frac{i\left(-\frac{i\pi}{2}+\sqrt{2}\pi\right)^{1+2k}}{8(1+2k)!}\right)$$

$$\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh(\pi\sqrt{2})}{\pi\sqrt{2}}\right)=\frac{1}{16}\left(2\sum_{k=0}^{\infty}\frac{2^k\pi^{2k}}{(2k)!}-\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{\left(-\frac{1}{2}\right)^{-s}\pi^{-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)$$

Integral representations:

$$\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh(\pi\sqrt{2})}{\pi\sqrt{2}}\right)=\frac{1}{8}+\int_0^1\frac{1}{8}\left(-\cosh\left(\sqrt{2}\pi t\right)+\sqrt{2}\pi\sinh\left(\sqrt{2}\pi t\right)\right)dt$$

$$\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh(\pi\sqrt{2})}{\pi\sqrt{2}}\right)=\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i e^{\pi^2/(2s)+s}(-1+2s)}{32\sqrt{\pi}\,s^{3/2}}\,ds \quad \text{for } \gamma>0$$

$$\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh(\pi\sqrt{2})}{\pi\sqrt{2}}\right)=$$
$$\int_0^1\left(\frac{1}{8}i\left(-\frac{i\pi}{2}+\sqrt{2}\pi\right)\cos\left(\frac{\pi t}{2}+i\sqrt{2}\pi t\right)-\frac{1}{8}\cosh\left(\sqrt{2}\pi t\right)\right)dt$$

From which:

1/8(((cosh(Pi*sqrt2)-sinh(Pi*sqrt2)/(Pi*sqrt2))))-5/2

Input:

$$\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)-\frac{5}{2}$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

Decimal approximation:

1.618621867402802010655274119303959478741829296562199024633...

1.6186218674...

Alternate forms:

$$\frac{1}{8}\left(-20-\frac{\sinh\left(\sqrt{2}\ \pi\right)}{\sqrt{2}\ \pi}+\cosh\left(\sqrt{2}\ \pi\right)\right)$$

$$-\frac{5}{2}-\frac{\sinh\left(\sqrt{2}\ \pi\right)}{8\sqrt{2}\ \pi}+\frac{1}{8}\cosh\left(\sqrt{2}\ \pi\right)$$

$$-\frac{40\pi+\sqrt{2}\ \sinh\left(\sqrt{2}\ \pi\right)-2\pi\cosh\left(\sqrt{2}\ \pi\right)}{16\pi}$$

Alternative representations:

$$\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)-\frac{5}{2}=-\frac{5}{2}+\frac{1}{8}\left(\cos\left(i\pi\sqrt{2}\right)-\frac{-e^{-\pi\sqrt{2}}+e^{\pi\sqrt{2}}}{2\left(\pi\sqrt{2}\right)}\right)$$

$$\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)-\frac{5}{2}=-\frac{5}{2}+\frac{1}{8}\left(\cos\left(-i\pi\sqrt{2}\right)-\frac{-e^{-\pi\sqrt{2}}+e^{\pi\sqrt{2}}}{2\left(\pi\sqrt{2}\right)}\right)$$

$$\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)-\frac{5}{2}=-\frac{5}{2}+\frac{1}{8}\left(\frac{1}{2}\left(e^{-\pi\sqrt{2}}+e^{\pi\sqrt{2}}\right)-\frac{-e^{-\pi\sqrt{2}}+e^{\pi\sqrt{2}}}{2\left(\pi\sqrt{2}\right)}\right)$$

Series representations:

$$\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)-\frac{5}{2}=-\frac{5}{2}+\sum_{k=0}^{\infty}\frac{2^{-3+k}\,\pi^{2k}\,(-(2k)!+(1+2k)!)}{(2k)!\,(1+2k)!}$$

$$\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)-\frac{5}{2}=$$
$$-\frac{5}{2}+\sum_{k=0}^{\infty}\left(-\frac{2^{-7/2+1/2(1+2k)}\,\pi^{2k}}{(1+2k)!}+\frac{i\left(-\frac{i\pi}{2}+\sqrt{2}\,\pi\right)^{1+2k}}{8\,(1+2k)!}\right)$$

$$\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)-\frac{5}{2}=$$
$$\frac{1}{16}\left(-40+2\sum_{k=0}^{\infty}\frac{2^k\,\pi^{2k}}{(2k)!}-\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{\left(-\frac{1}{2}\right)^{-s}\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)$$

Integral representations:

$$\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)-\frac{5}{2}=$$
$$-\frac{19}{8}+\int_0^1\frac{1}{8}\left(-\cosh\left(\sqrt{2}\,\pi\,t\right)+\sqrt{2}\,\pi\sinh\left(\sqrt{2}\,\pi\,t\right)\right)dt$$

$$\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)-\frac{5}{2}=-\frac{5}{2}+\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i\,e^{\pi^2/(2s)+s}\,(-1+2s)}{32\sqrt{\pi}\,s^{3/2}}\,ds\quad\text{for }\gamma>0$$

$$\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)-\frac{5}{2}=$$
$$-\frac{5}{2}+\int_0^1\left(\frac{1}{8}\,i\left(-\frac{i\pi}{2}+\sqrt{2}\,\pi\right)\cos\left(\frac{\pi t}{2}+i\sqrt{2}\,\pi\,t\right)-\frac{1}{8}\cosh\left(\sqrt{2}\,\pi\,t\right)\right)dt$$

and again:

6(((1/8(((cosh(Pi*sqrt2)-sinh(Pi*sqrt2)/(Pi*sqrt2)))))))^4+golden ratio^2

Input:

$$6\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)\right)^4+\phi^2$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

ϕ is the golden ratio

Exact result:

$$\phi^2 + \frac{3\left(\cosh(\sqrt{2}\,\pi) - \frac{\sinh(\sqrt{2}\,\pi)}{\sqrt{2}\,\pi}\right)^4}{2048}$$

Decimal approximation:

1729.087629215324797949031458573506675038209871526632208639...

1729.08762921…

Alternate forms:

$$\frac{1}{2}\left(3+\sqrt{5}\right) + \frac{3\left(\cosh(\sqrt{2}\,\pi) - \frac{\sinh(\sqrt{2}\,\pi)}{\sqrt{2}\,\pi}\right)^4}{2048}$$

$$\frac{1}{4}\left(1+\sqrt{5}\right)^2 + \frac{3\left(\cosh(\sqrt{2}\,\pi) - \frac{\sinh(\sqrt{2}\,\pi)}{\sqrt{2}\,\pi}\right)^4}{2048}$$

$$\phi^2 + \frac{3\left(\frac{1}{2}\left(e^{-\sqrt{2}\,\pi} + e^{\sqrt{2}\,\pi}\right) - \frac{e^{\sqrt{2}\,\pi} - e^{-\sqrt{2}\,\pi}}{2\sqrt{2}\,\pi}\right)^4}{2048}$$

Alternative representations:

$$6\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right) - \frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)\right)^4 + \phi^2 = \phi^2 + 6\left(\frac{1}{8}\left(\cos\left(i\pi\sqrt{2}\right) - \frac{-e^{-\pi\sqrt{2}} + e^{\pi\sqrt{2}}}{2\left(\pi\sqrt{2}\right)}\right)\right)^4$$

$$6\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right) - \frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)\right)^4 + \phi^2 = \phi^2 + 6\left(\frac{1}{8}\left(\cos\left(i\pi\sqrt{2}\right) + \frac{i\cos\left(\frac{\pi}{2} - i\left(\pi\sqrt{2}\right)\right)}{\pi\sqrt{2}}\right)\right)^4$$

$$6\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right) - \frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)\right)^4 + \phi^2 = \phi^2 + 6\left(\frac{1}{8}\left(\cos\left(-i\pi\sqrt{2}\right) - \frac{-e^{-\pi\sqrt{2}} + e^{\pi\sqrt{2}}}{2\left(\pi\sqrt{2}\right)}\right)\right)^4$$

Series representations:

$$6\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)\right)^4+\phi^2=\phi^2+\frac{3\left(\sum_{k=0}^{\infty}\frac{2^{1+k}\pi^{1+2k}\left((2k)!-(1+2k)!\right)}{(2k)!\,(1+2k)!}\right)^4}{32768\,\pi^4}$$

$$6\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)\right)^4+\phi^2=$$
$$\phi^2+\frac{3\left(\sum_{k=0}^{\infty}\left(-\frac{2^{-1/2+1/2(1+2k)}\pi^{2k}}{(1+2k)!}+\frac{i\left(-\frac{i\pi}{2}+\sqrt{2}\,\pi\right)^{1+2k}}{(1+2k)!}\right)\right)^4}{2048}$$

$$6\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)\right)^4+\phi^2=$$
$$\phi^2+\frac{3\left(\sum_{k=0}^{\infty}\frac{2^k\pi^{2k}}{(2k)!}-\frac{1}{2}\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{\left(-\frac{1}{2}\right)^{-s}\pi^{-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)^4}{2048}$$

1/2(((1/8(((cosh(Pi*sqrt2)-sinh(Pi*sqrt2)/(Pi*sqrt2)))))))^4-Pi-e

Input:

$$\frac{1}{2}\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)\right)^4-\pi-e$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

Exact result:

$$-e-\pi+\frac{\left(\cosh\left(\sqrt{2}\,\pi\right)-\frac{\sinh\left(\sqrt{2}\,\pi\right)}{\sqrt{2}\,\pi}\right)^4}{8192}$$

Decimal approximation:

138.0125917868324034512459751236295876947530470358271384188...

138.0125917… ≈ 138

Alternate forms:

$$-e+\frac{\left(\frac{1}{2}\left(e^{-\sqrt{2}\,\pi}+e^{\sqrt{2}\,\pi}\right)-\frac{e^{\sqrt{2}\,\pi}-e^{-\sqrt{2}\,\pi}}{2\sqrt{2}\,\pi}\right)^4}{8192}-\pi$$

$$-e-\pi+\frac{\sinh^4(\sqrt{2}\,\pi)}{32768\,\pi^4}+\frac{\cosh^4(\sqrt{2}\,\pi)}{8192}-\frac{\sinh(\sqrt{2}\,\pi)\cosh^3(\sqrt{2}\,\pi)}{2048\sqrt{2}\,\pi}+\frac{3\sinh^2(\sqrt{2}\,\pi)\cosh^2(\sqrt{2}\,\pi)}{8192\,\pi^2}-\frac{\sinh^3(\sqrt{2}\,\pi)\cosh(\sqrt{2}\,\pi)}{4096\sqrt{2}\,\pi^3}$$

$$-\frac{1}{32768\,\pi^4}\Big(32768\,e\,\pi^4+32768\,\pi^5-\sinh^4\left(\sqrt{2}\,\pi\right)-4\,\pi^4\cosh^4\left(\sqrt{2}\,\pi\right)+8\sqrt{2}\,\pi^3\sinh\left(\sqrt{2}\,\pi\right)\cosh^3\left(\sqrt{2}\,\pi\right)-12\,\pi^2\sinh^2\left(\sqrt{2}\,\pi\right)\cosh^2\left(\sqrt{2}\,\pi\right)+4\sqrt{2}\,\pi\sinh^3\left(\sqrt{2}\,\pi\right)\cosh\left(\sqrt{2}\,\pi\right)\Big)$$

Alternative representations:

$$\frac{1}{2}\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh(\pi\sqrt{2})}{\pi\sqrt{2}}\right)\right)^4-\pi-e=-e-\pi+\frac{1}{2}\left(\frac{1}{8}\left(\cos\left(i\,\pi\sqrt{2}\right)+\frac{i\cos\left(\frac{\pi}{2}-i\,(\pi\sqrt{2})\right)}{\pi\sqrt{2}}\right)\right)^4$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh(\pi\sqrt{2})}{\pi\sqrt{2}}\right)\right)^4-\pi-e=-e-\pi+\frac{1}{2}\left(\frac{1}{8}\left(\cos\left(i\,\pi\sqrt{2}\right)-\frac{-e^{-\pi\sqrt{2}}+e^{\pi\sqrt{2}}}{2\,(\pi\sqrt{2})}\right)\right)^4$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh(\pi\sqrt{2})}{\pi\sqrt{2}}\right)\right)^4-\pi-e=-e-\pi+\frac{1}{2}\left(\frac{1}{8}\left(\cos\left(-i\,\pi\sqrt{2}\right)+\frac{i\cos\left(\frac{\pi}{2}-i\,(\pi\sqrt{2})\right)}{\pi\sqrt{2}}\right)\right)^4$$

Series representations:

$$\frac{1}{2}\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh(\pi\sqrt{2})}{\pi\sqrt{2}}\right)\right)^4-\pi-e=-e-\pi+\frac{\left(\sum_{k=0}^{\infty}\frac{2^{1+k}\,\pi^{1+2k}\,((2k)!-(1+2k)!)}{(2k)!\,(1+2k)!}\right)^4}{131072\,\pi^4}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)\right)^4-\pi-e=$$
$$-e-\pi+\frac{\left(\sum_{k=0}^{\infty}\frac{\pi^{-1+2k}\left(-i\sqrt{2}\left(-\frac{i}{2}+\sqrt{2}\right)^{2k}+2^{1+k}\pi\right)}{2(2k)!}\right)^4}{8192}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)\right)^4-\pi-e=$$
$$-e-\pi+\frac{\left(\sum_{k=0}^{\infty}\left(-\frac{2^{-1/2+1/2(1+2k)}\pi^{2k}}{(1+2k)!}+\frac{i\left(-\frac{i\pi}{2}+\sqrt{2}\,\pi\right)^{1+2k}}{(1+2k)!}\right)\right)^4}{8192}$$

1/2(((1/8(((cosh(Pi*sqrt2)-sinh(Pi*sqrt2)/(Pi*sqrt2)))))))^4-Pi-3-e

Input:

$$\frac{1}{2}\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)\right)^4-\pi-3-e$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

Exact result:

$$-3-e-\pi+\frac{\left(\cosh\left(\sqrt{2}\,\pi\right)-\frac{\sinh\left(\sqrt{2}\,\pi\right)}{\sqrt{2}\,\pi}\right)^4}{8192}$$

Decimal approximation:

135.0125917868324034512459751236295876947530470358271384188...

135.012591786… ≈ 135

Alternate forms:

$$-3-e+\frac{\left(\frac{1}{2}\left(e^{-\sqrt{2}\,\pi}+e^{\sqrt{2}\,\pi}\right)-\frac{e^{\sqrt{2}\,\pi}-e^{-\sqrt{2}\,\pi}}{2\sqrt{2}\,\pi}\right)^4}{8192}-\pi$$

$$-3-e-\pi+\frac{\sinh^4(\sqrt{2}\,\pi)}{32\,768\,\pi^4}+\frac{\cosh^4(\sqrt{2}\,\pi)}{8192}-\frac{\sinh(\sqrt{2}\,\pi)\cosh^3(\sqrt{2}\,\pi)}{2048\sqrt{2}\,\pi}+\frac{3\sinh^2(\sqrt{2}\,\pi)\cosh^2(\sqrt{2}\,\pi)}{8192\,\pi^2}-\frac{\sinh^3(\sqrt{2}\,\pi)\cosh(\sqrt{2}\,\pi)}{4096\sqrt{2}\,\pi^3}$$

$$-\frac{1}{32\,768\,\pi^4}\left(98\,304\,\pi^4+32\,768\,e\,\pi^4+32\,768\,\pi^5-\sinh^4\left(\sqrt{2}\,\pi\right)-4\,\pi^4\cosh^4\left(\sqrt{2}\,\pi\right)+8\sqrt{2}\,\pi^3\sinh\left(\sqrt{2}\,\pi\right)\cosh^3\left(\sqrt{2}\,\pi\right)-12\,\pi^2\sinh^2\left(\sqrt{2}\,\pi\right)\cosh^2\left(\sqrt{2}\,\pi\right)+4\sqrt{2}\,\pi\sinh^3\left(\sqrt{2}\,\pi\right)\cosh\left(\sqrt{2}\,\pi\right)\right)$$

Alternative representations:

$$\frac{1}{2}\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh(\pi\sqrt{2})}{\pi\sqrt{2}}\right)\right)^4-\pi-3-e=-3-e-\pi+\frac{1}{2}\left(\frac{1}{8}\left(\cos\left(i\,\pi\sqrt{2}\right)+\frac{i\cos\left(\frac{\pi}{2}-i\left(\pi\sqrt{2}\right)\right)}{\pi\sqrt{2}}\right)\right)^4$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh(\pi\sqrt{2})}{\pi\sqrt{2}}\right)\right)^4-\pi-3-e=-3-e-\pi+\frac{1}{2}\left(\frac{1}{8}\left(\cos\left(i\,\pi\sqrt{2}\right)-\frac{-e^{-\pi\sqrt{2}}+e^{\pi\sqrt{2}}}{2\left(\pi\sqrt{2}\right)}\right)\right)^4$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh(\pi\sqrt{2})}{\pi\sqrt{2}}\right)\right)^4-\pi-3-e=-3-e-\pi+\frac{1}{2}\left(\frac{1}{8}\left(\cos\left(-i\,\pi\sqrt{2}\right)+\frac{i\cos\left(\frac{\pi}{2}-i\left(\pi\sqrt{2}\right)\right)}{\pi\sqrt{2}}\right)\right)^4$$

Series representations:

$$\frac{1}{2}\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh(\pi\sqrt{2})}{\pi\sqrt{2}}\right)\right)^4-\pi-3-e=-3-e-\pi+\frac{\left(\sum_{k=0}^{\infty}\frac{2^{1+k}\,\pi^{1+2k}\left((2k)!-(1+2k)!\right)}{(2k)!\,(1+2k)!}\right)^4}{131\,072\,\pi^4}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh(\pi\sqrt{2})}{\pi\sqrt{2}}\right)\right)^4-\pi-3-e=-3-e-\pi+\frac{\left(\sum_{k=0}^{\infty}\frac{\pi^{-1+2k}\left(-i\sqrt{2}\left(-\frac{i}{2}+\sqrt{2}\right)^{2k}+2^{1+k}\,\pi\right)}{2\,(2k)!}\right)^4}{8192}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)\right)^4-\pi-3-e=$$
$$-3-e-\pi+\frac{\left(\sum_{k=0}^{\infty}\left(-\frac{2^{-1/2+1/2(1+2k)}\pi^{2k}}{(1+2k)!}+\frac{i\left(-\frac{i\pi}{2}+\sqrt{2}\pi\right)^{1+2k}}{(1+2k)!}\right)\right)^4}{8192}$$

1/2(((1/8(((cosh(Pi*sqrt2)-sinh(Pi*sqrt2)/(Pi*sqrt2)))))))^4-Pi-e+34

Input:

$$\frac{1}{2}\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)\right)^4-\pi-e+34$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

Exact result:

$$34-e-\pi+\frac{\left(\cosh\left(\sqrt{2}\pi\right)-\frac{\sinh\left(\sqrt{2}\pi\right)}{\sqrt{2}\pi}\right)^4}{8192}$$

Decimal approximation:

172.01259178683240345124597512362958769475304703582713841 88...

172.0125917… ≈ 172

Alternate forms:

$$34-e+\frac{\left(\frac{1}{2}\left(e^{-\sqrt{2}\pi}+e^{\sqrt{2}\pi}\right)-\frac{e^{\sqrt{2}\pi}-e^{-\sqrt{2}\pi}}{2\sqrt{2}\pi}\right)^4}{8192}-\pi$$

$$34-e-\pi+\frac{\sinh^4\left(\sqrt{2}\pi\right)}{32768\pi^4}+\frac{\cosh^4\left(\sqrt{2}\pi\right)}{8192}-\frac{\sinh\left(\sqrt{2}\pi\right)\cosh^3\left(\sqrt{2}\pi\right)}{2048\sqrt{2}\pi}+$$
$$\frac{3\sinh^2\left(\sqrt{2}\pi\right)\cosh^2\left(\sqrt{2}\pi\right)}{8192\pi^2}-\frac{\sinh^3\left(\sqrt{2}\pi\right)\cosh\left(\sqrt{2}\pi\right)}{4096\sqrt{2}\pi^3}$$

$$-\frac{1}{32768\,\pi^4}\Big(-1114112\,\pi^4 + 32768\,e\,\pi^4 + 32768\,\pi^5 - \sinh^4\left(\sqrt{2}\,\pi\right) - 4\,\pi^4\cosh^4\left(\sqrt{2}\,\pi\right) + 8\sqrt{2}\,\pi^3\sinh\left(\sqrt{2}\,\pi\right)\cosh^3\left(\sqrt{2}\,\pi\right) - 12\,\pi^2\sinh^2\left(\sqrt{2}\,\pi\right)\cosh^2\left(\sqrt{2}\,\pi\right) + 4\sqrt{2}\,\pi\sinh^3\left(\sqrt{2}\,\pi\right)\cosh\left(\sqrt{2}\,\pi\right)\Big)$$

Alternative representations:

$$\frac{1}{2}\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right) - \frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)\right)^4 - \pi - e + 34 = 34 - e - \pi + \frac{1}{2}\left(\frac{1}{8}\left(\cos\left(i\,\pi\sqrt{2}\right) + \frac{i\cos\left(\frac{\pi}{2} - i\left(\pi\sqrt{2}\right)\right)}{\pi\sqrt{2}}\right)\right)^4$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right) - \frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)\right)^4 - \pi - e + 34 = 34 - e - \pi + \frac{1}{2}\left(\frac{1}{8}\left(\cos\left(i\,\pi\sqrt{2}\right) - \frac{-e^{-\pi\sqrt{2}} + e^{\pi\sqrt{2}}}{2\left(\pi\sqrt{2}\right)}\right)\right)^4$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right) - \frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)\right)^4 - \pi - e + 34 = 34 - e - \pi + \frac{1}{2}\left(\frac{1}{8}\left(\cos\left(-i\,\pi\sqrt{2}\right) + \frac{i\cos\left(\frac{\pi}{2} - i\left(\pi\sqrt{2}\right)\right)}{\pi\sqrt{2}}\right)\right)^4$$

Series representations:

$$\frac{1}{2}\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right) - \frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)\right)^4 - \pi - e + 34 = 34 - e - \pi + \frac{\left(\sum_{k=0}^{\infty}\frac{2^{1+k}\,\pi^{1+2k}\left((2k)! - (1+2k)!\right)}{(2k)!\,(1+2k)!}\right)^4}{131072\,\pi^4}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right) - \frac{\sinh\left(\pi\sqrt{2}\right)}{\pi\sqrt{2}}\right)\right)^4 - \pi - e + 34 = 34 - e - \pi + \frac{\left(\sum_{k=0}^{\infty}\frac{\pi^{-1+2k}\left(-i\sqrt{2}\left(-\frac{i}{2} + \sqrt{2}\right)^{2k} + 2^{1+k}\,\pi\right)}{2\,(2k)!}\right)^4}{8192}$$

$$\frac{1}{2}\left(\frac{1}{8}\left(\cosh\left(\pi\sqrt{2}\right)-\frac{\sinh(\pi\sqrt{2})}{\pi\sqrt{2}}\right)\right)^4-\pi-e+34=$$

$$34-e-\pi+\frac{\left(\sum_{k=0}^{\infty}\left(-\frac{2^{-1/2+1/2(1+2k)}\pi^{2k}}{(1+2k)!}+\frac{i\left(-\frac{i\pi}{2}+\sqrt{2}\pi\right)^{1+2k}}{(1+2k)!}\right)\right)^4}{8192}$$

Note that 135 and 138, are also very near the mass values of the two Pion mesons, that are 134.9766 and 139.57

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.

In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= \qquad\qquad 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} \quad = \quad 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

References

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