

On the Ramanujan Taxicab numbers: new mathematical connections with some equations of Einstein Dilaton Gauss-Bonnet Gravity

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Abstract

In this paper we have described new mathematical connections between Ramanujan Taxicab numbers and some equations of Einstein Dilaton Gauss-Bonnet Gravity

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ff

(i) $\frac{1+53x+9x^2}{1-82x-82x^2+x^3} = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$
 or $\frac{a_0}{x} + \frac{a_1}{x^2} + \frac{a_2}{x^3} + \dots$

(ii) $\frac{2-26x-49x^2}{1-82x-82x^2+x^3} = b_0 + b_1x + b_2x^2 + b_3x^3 + \dots$
 or $\frac{b_0}{x} + \frac{b_1}{x^2} + \frac{b_2}{x^3} + \dots$

(iii) $\frac{2+8x-10x^2}{1-82x-82x^2+x^3} = c_0 + c_1x + c_2x^2 + c_3x^3 + \dots$
 or $\frac{c_0}{x} + \frac{c_1}{x^2} + \frac{c_2}{x^3} + \dots$

then

$$\left. \begin{aligned} a_n^3 + b_n^3 &= c_n^3 + (-1)^n \\ \text{and } a_n^3 + b_n^3 &= c_n^3 + (-1)^n \end{aligned} \right\}$$

Examples

$$135^3 + 138^3 = 172^3 - 1$$

$$11161^3 + 11468^3 = 14258^3 + 1$$

$$791^3 + 812^3 = 1010^3 - 1$$

$$65601^3 + 67402^3 = 83802^3 + 1$$

$$9^3 + 10^3 = 12^3 + 1$$

$$6^3 + 8^3 = 9^3 - 1$$

<https://medium.com/cantors-paradise/the-hardy-ramanujan-number-1729-929766161647>

From:

Gravitational Collapse in Einstein Dilaton Gauss-Bonnet Gravity

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Department of Physics, Princeton University, Princeton, New Jersey 08544, USA.

E-mail: jripley@princeton.edu and fpretori@princeton.edu - February 2019

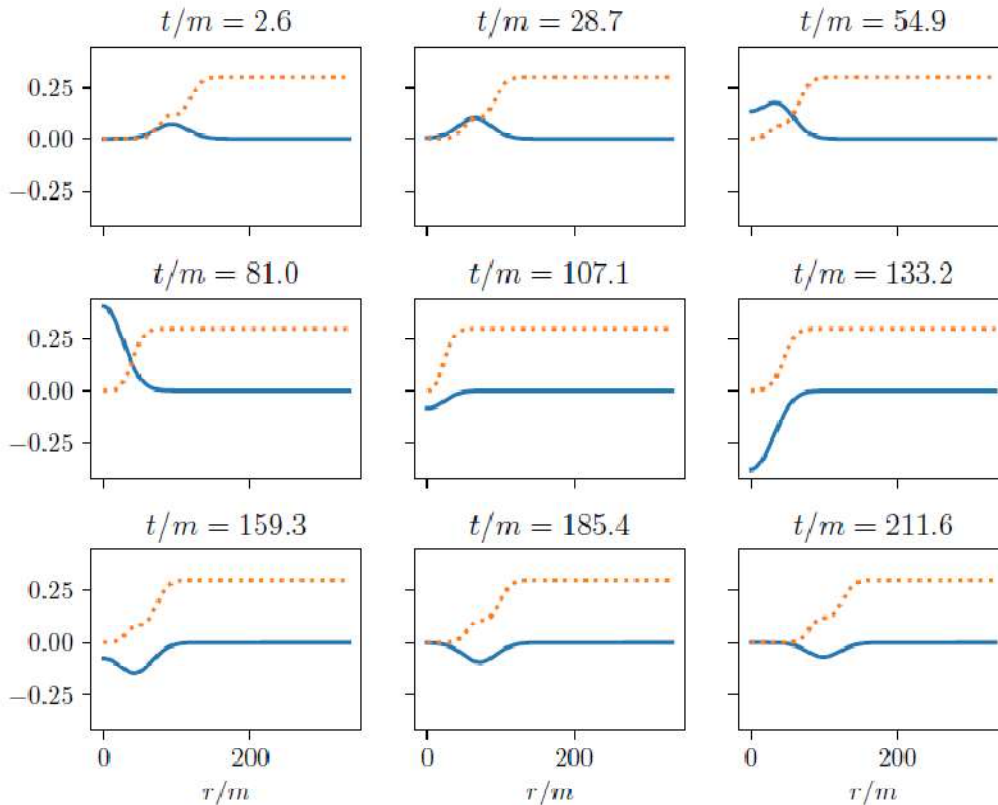


Figure 1: The scalar field profile ϕ (blue solid line) and Misner-Sharp mass m_{MS} (orange dashed line) from a weak field, weak coupling run with scalar field initial data parameters (10) $a_0 = 0.01$, $r_0 = 25$, $w_0 = 10$, and $\lambda = 0.1$, $r_{\max} = 100$ (discretized with $N_r = 2^{12} + 1$ points). Here, and in all figures, we normalize units with respect to $m \equiv m_{MS}(t = 0, r = r_{\max})$. The metric fields (not shown) remain smooth and close to their Minkowski spacetime values throughout.

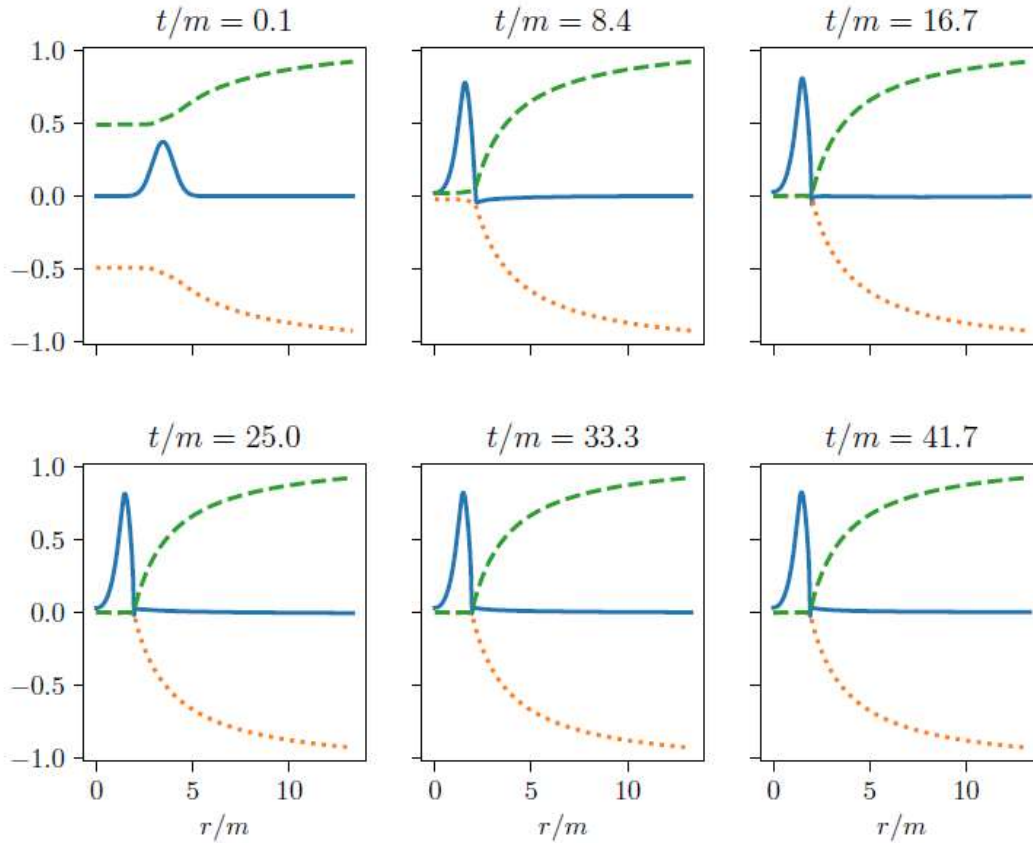


Figure 2: Run with strong field and weak coupling: $a_0 = 0.02$, $w_0 = 6$, $r_0 = 25$, $\lambda = 1$, $r_{max} = 100$, $N_r = 2^{12} + 1$, and $m \sim 7.5$. Shown is the scalar field ϕ (blue line), and corresponding ingoing (orange dots) and outgoing (green dashes) characteristic speeds. An apparent horizon begins to form soon after evolution begins. That both characteristic speeds go to zero inside the horizon $r \lesssim 2m$ is an artifact of the horizon-avoiding nature of the coordinates, as time flow “freezes” in this region as $A \rightarrow -\infty$ here. Outside the horizon the scalar field slowly grows, and appears to asymptote to the profile expected for a “hairy” black hole in EdGB gravity—see Figure 3 for a zoom-in of the late-time profile (though “late” is not particularly so in these coordinates, as we quickly loose convergence once A and B start to diverge).

From

$$\phi(t, r) \Big|_{t=0} = a_0 \left(\frac{r}{w_0} \right)^2 \exp \left(- \left(\frac{r - r_0}{w_0} \right)^2 \right),$$

$$a_0 = 0.01, r_0 = 25, w_0 = 10, \text{ and } \lambda = 0.1, r_{max} = 100$$

we obtain:

$$0.01(100/10)^2 \exp(-((100-25)/10)^2)$$

Input:

$$0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100 - 25}{10} \right)^2 \right)$$

Result:

$$3.72336... \times 10^{-25}$$

$$3.72336... * 10^{-25}$$

For

$$a_0 = 0.02, w_0 = 6, r_0 = 25, \lambda = 1, r_{max} = 100,$$

$$\phi(t, r) \Big|_{t=0} = a_0 \left(\frac{r}{w_0} \right)^2 \exp \left(- \left(\frac{r - r_0}{w_0} \right)^2 \right),$$

we obtain:

$$0.02(100/6)^2 \exp(-((100-25)/6)^2)$$

Input:

$$0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100 - 25}{6} \right)^2 \right)$$

Result:

$$7.69511... \times 10^{-68}$$

$$7.69511... * 10^{-68}$$

For

$$a_0 = 0.02, r_0 = 20, w_0 = 8, r_{max} = 100,$$

we obtain:

$$0.02(100/8)^2 \exp(-((100-20)/8)^2)$$

Input:

$$0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100 - 20}{8} \right)^2 \right)$$

Result:

$$1.16252... \times 10^{-43}$$

$$1.16252... * 10^{-43}$$

$$\left(\left(\left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100 - 25}{10} \right)^2 \right) \right) \right) / \left(\left(\left(0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100 - 25}{6} \right)^2 \right) \right) \right) \right) \right) * \left(\left(\left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100 - 20}{8} \right)^2 \right) \right) \right) \right)$$

Input:

$$\frac{0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100 - 25}{10} \right)^2 \right)}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100 - 25}{6} \right)^2 \right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100 - 20}{8} \right)^2 \right) \right)$$

Result:

$$0.5625$$

0.5625

Rational form:

$$\frac{9}{16}$$

From which:

$$2 \left(\left(\left(\left(\left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100 - 25}{10} \right)^2 \right) \right) \right) / \left(\left(\left(0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100 - 25}{6} \right)^2 \right) \right) \right) \right) \right) * \left(\left(\left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100 - 20}{8} \right)^2 \right) \right) \right) \right) \right) \right)^{1/3}$$

Input:

$$2 \sqrt[3]{ \frac{0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100 - 25}{10} \right)^2 \right)}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100 - 25}{6} \right)^2 \right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100 - 20}{8} \right)^2 \right) \right) }$$

Result:

$$1.650963624447313341937304976204542478423442322159246668486...$$

1.65096362444... result very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5} \right)^3 = 1164.2696$ i.e. 1.65578...

$$8/[1/9((((0.01(100/10)^2 \exp(-((100-25)/10)^2))) 1 /(((0.02(100/6)^2 \exp(-((100-25)/6)^2)))) * (((0.02(100/8)^2 \exp(-((100-20)/8)^2))))))]]$$

Input:

$$\frac{8}{\frac{1}{9} \left(\left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100-25}{10} \right)^2 \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100-25}{6} \right)^2 \right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100-20}{8} \right)^2 \right) \right) \right)}$$

Result:

128

128

$$27*4/[1/9((((0.01(100/10)^2 \exp(-((100-25)/10)^2))) 1 /(((0.02(100/6)^2 \exp(-((100-25)/6)^2)))) * (((0.02(100/8)^2 \exp(-((100-20)/8)^2))))))] +1$$

Input:

$$27 \times 4 / \left(\frac{1}{9} \left(\left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100-25}{10} \right)^2 \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100-25}{6} \right)^2 \right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100-20}{8} \right)^2 \right) \right) \right) \right) + 1$$

Result:

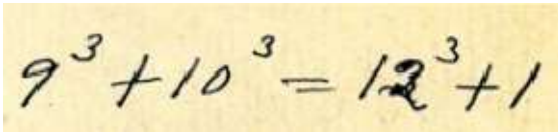
1729

1729

With regard 27 (From Wikipedia):

“The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group $\mathbf{Z}/3\mathbf{Z}$, and its outer automorphism group is the cyclic group $\mathbf{Z}/2\mathbf{Z}$. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories”.

From Ramanujan cubes:



$$12^3 + 1$$

Input:

$$12^3 + 1$$

Result:

$$1729$$

1729 (Ramanujan taxicab number)

and:

$$(47-2)/[1/9((((0.01(100/10)^2 \exp(-((100-25)/10)^2)))) 1 /(((0.02(100/6)^2 \exp(-((100-25)/6)^2)))) * (((0.02(100/8)^2 \exp(-((100-20)/8)^2)))))]+8$$

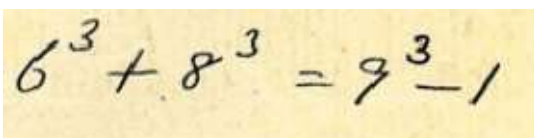
Input:

$$\frac{47 - 2}{\frac{1}{9} \left(\left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100 - 25}{10} \right)^2 \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100 - 25}{6} \right)^2 \right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100 - 20}{8} \right)^2 \right) \right) \right) + 8}$$

Result:

$$728$$

728 (Ramanujan taxicab number)



$$9^3 - 1$$

Input:

$$9^3 - 1$$

Result:

728

728 as above

$$8/[1/9((((0.01(100/10)^2 \exp(-((100-25)/10)^2))) 1 /(((0.02(100/6)^2 \exp(-((100-25)/6)^2)))) * (((0.02(100/8)^2 \exp(-((100-20)/8)^2))))))] + 7$$

Input:

$$\frac{8}{\frac{1}{9} \left(\left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100-25}{10} \right)^2 \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100-25}{6} \right)^2 \right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100-20}{8} \right)^2 \right) \right) \right) + 7}$$

Result:

135

135 (Ramanujan taxicab number)

$$8/[1/9((((0.01(100/10)^2 \exp(-((100-25)/10)^2))) 1 /(((0.02(100/6)^2 \exp(-((100-25)/6)^2)))) * (((0.02(100/8)^2 \exp(-((100-20)/8)^2))))))] + 7 + 3$$

Input:

$$\frac{8}{\frac{1}{9} \left(\left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100-25}{10} \right)^2 \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100-25}{6} \right)^2 \right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100-20}{8} \right)^2 \right) \right) \right) + 7 + 3}$$

Result:

138

138 (Ramanujan taxicab number)

Note that 135 and 138, are also very near the mass values of the two Pion mesons, that are 134.9766 and 139.57

$$8/[1/9((((0.01(100/10)^2 \exp(-((100-25)/10)^2))) 1 /(((0.02(100/6)^2 \exp(-((100-25)/6)^2)))) * (((0.02(100/8)^2 \exp(-((100-20)/8)^2)))))]+47-3$$

Input:

$$\frac{8}{\frac{1}{9} \left(\left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100-25}{10} \right)^2 \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100-25}{6} \right)^2 \right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100-20}{8} \right)^2 \right) \right) \right) + 47 - 3}$$

Result:

172

172 (Ramanujan taxicab number)

$$(((8/[1/9((((0.01(100/10)^2 \exp(-((100-25)/10)^2))) 1 /(((0.02(100/6)^2 \exp(-((100-25)/6)^2)))) * (((0.02(100/8)^2 \exp(-((100-20)/8)^2)))))]+47-3))))^3 - 1$$

Input:

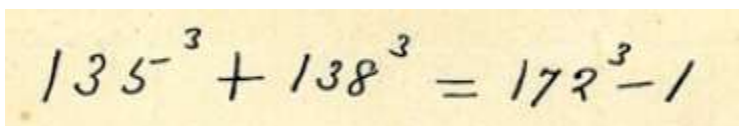
$$\left(8 / \left(\frac{1}{9} \left(\left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100-25}{10} \right)^2 \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100-25}{6} \right)^2 \right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100-20}{8} \right)^2 \right) \right) \right) \right) + 47 - 3 \right)^3 - 1$$

Result:

5088447

5088447

Indeed, from



$$135^3 + 138^3 = 172^3 - 1$$

we obtain:

$$172^3 - 1$$

Input:

$$172^3 - 1$$

Result:

5 088 447

5088447

Now, we have that:

$$(47+2)/[1/9((((((0.01(100/10)^2 \exp(-((100-25)/10)^2)))))) 1 /((((0.02(100/6)^2 \exp(-((100-25)/6)^2)))) * (((0.02(100/8)^2 \exp(-((100-20)/8)^2)))))))]+7$$

Input:

$$\frac{47 + 2}{\frac{1}{9} \left(\left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100-25}{10} \right)^2 \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100-25}{6} \right)^2 \right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100-20}{8} \right)^2 \right) \right) \right) + 7}$$

Result:

791

791 (Ramanujan taxicab number)

Note that 791 is also very near to the rest mass of Omega meson 782.65 (diff. about 9)

$$(47+4)/[1/9((((((0.01(100/10)^2 \exp(-((100-25)/10)^2)))))) 1 /((((0.02(100/6)^2 \exp(-((100-25)/6)^2)))) * (((0.02(100/8)^2 \exp(-((100-20)/8)^2)))))))]-4$$

Input:

$$\frac{47 + 4}{\frac{1}{9} \left(\left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100-25}{10} \right)^2 \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100-25}{6} \right)^2 \right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100-20}{8} \right)^2 \right) \right) \right) - 4}$$

Result:

812

812 (Ramanujan taxicab number)

$$(55+8)/[1/9((((((0.01(100/10)^2 \exp(-((100-25)/10)^2)))) 1 /(((0.02(100/6)^2 \exp(-((100-25)/6)^2)))) * (((0.02(100/8)^2 \exp(-((100-20)/8)^2)))))))]+2$$

Input:

$$\frac{55 + 8}{\frac{1}{9} \left(\left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100-25}{10} \right)^2 \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100-25}{6} \right)^2 \right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100-20}{8} \right)^2 \right) \right) \right) + 2}$$

Result:

1010

1010 (Ramanujan taxicab number)

Note that 1010 is also very near to the rest mass of Phi meson 1019.445 (diff. 9)

$$\left(\frac{(55+8)}{1/9((((((0.01(100/10)^2 \exp(-((100-25)/10)^2)))) 1 /(((0.02(100/6)^2 \exp(-((100-25)/6)^2)))) * (((0.02(100/8)^2 \exp(-((100-20)/8)^2)))))))]+2) \right)^3 - 1$$

Input:

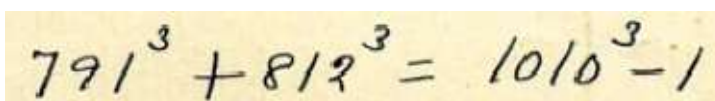
$$\left(\frac{(55 + 8)}{\frac{1}{9} \left(\left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100 - 25}{10} \right)^2 \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100-25}{6} \right)^2 \right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100 - 20}{8} \right)^2 \right) \right) \right) + 2 \right)^3 - 1$$

Result:

1030300999

1030300999

Indeed, from



$$791^3 + 812^3 = 1010^3 - 1$$

we obtain:

$$1010^3 - 1$$

Input:

$$1010^3 - 1$$

Result:

$$1030300999$$

$$1030300999$$

Now, we have that:

$$(521+123+47+7)/[1/9((((0.01(100/10)^2 \exp(-((100-25)/10)^2)))) 1 /(((0.02(100/6)^2 \exp(-((100-25)/6)^2)))) * (((0.02(100/8)^2 \exp(-((100-20)/8)^2)))))]-7$$

Input:

$$\frac{521 + 123 + 47 + 7}{\frac{1}{9} \left(\left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100-25}{10} \right)^2 \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100-25}{6} \right)^2 \right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100-20}{8} \right)^2 \right) \right) \right) - 7}$$

Result:

$$11161$$

$$11161 \text{ (Ramanujan taxicab number)}$$

$$(521+123+47+18+7)/[1/9((((0.01(100/10)^2 \exp(-((100-25)/10)^2)))) 1 /(((0.02(100/6)^2 \exp(-((100-25)/6)^2)))) * (((0.02(100/8)^2 \exp(-((100-20)/8)^2)))))]+11+1$$

Input:

$$\frac{521 + 123 + 47 + 18 + 7}{\frac{1}{9} \left(\left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100-25}{10} \right)^2 \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100-25}{6} \right)^2 \right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100-20}{8} \right)^2 \right) \right) \right) + 11 + 1}$$

Result:

11468

11468 (Ramanujan taxicab number)

$$(843+47)/[1/9((((0.01(100/10)^2 \exp(-((100-25)/10)^2)))) 1 /(((0.02(100/6)^2 \exp(-((100-25)/6)^2)))) * (((0.02(100/8)^2 \exp(-((100-20)/8)^2)))))]+18$$

Input:

$$\frac{843 + 47}{\frac{1}{9} \left(\left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100-25}{10} \right)^2 \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100-25}{6} \right)^2 \right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100-20}{8} \right)^2 \right) \right) \right) + 18}$$

Result:

14258

14258 (Ramanujan taxicab number)

$$(((843+47)/[1/9((((0.01(100/10)^2 \exp(-((100-25)/10)^2)))) 1 /(((0.02(100/6)^2 \exp(-((100-25)/6)^2)))) * (((0.02(100/8)^2 \exp(-((100-20)/8)^2)))))]+18)))^3 + 1$$

Input:

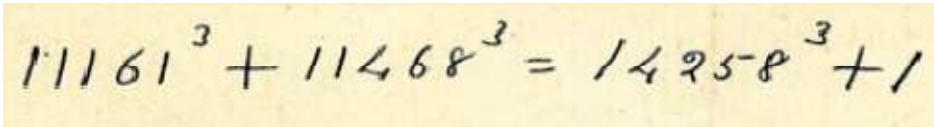
$$\left((843 + 47) / \left[\frac{1}{9} \left(\left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100 - 25}{10} \right)^2 \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100-25}{6} \right)^2 \right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100 - 20}{8} \right)^2 \right) \right) \right] + 18 \right)^3 + 1$$

Result:

2898516861513

2898516861513

Indeed, from



$$11161^3 + 11468^3 = 14258^3 + 1$$

we obtain:

$$14258^3 + 1$$

Input:

$$14258^3 + 1$$

Result:

2898516861513

[2898516861513](#)

From this last expression, we obtain:

$$5 \ln \left(\frac{((843+47) / [1/9 \left(\frac{1}{0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100-25}{10} \right)^2 \right) \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100-25}{6} \right)^2 \right) \right) * \left(\left(\frac{0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100-20}{8} \right)^2 \right) \right) \right) \right) + 18 \right)^3 + 1 \right) - 2\pi$$

Input:

$$5 \log \left(\left((843 + 47) / \left(\frac{1}{9} \left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100 - 25}{10} \right)^2 \right) \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100 - 25}{6} \right)^2 \right) \right) * \left(\left(\frac{0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100 - 20}{8} \right)^2 \right) \right) \right) \right) + 18 \right)^3 + 1 \right) - 2\pi$$

$\log(x)$ is the natural logarithm

Result:

137.193...

[137.193...](#)

This result is very near to the inverse of fine-structure constant [137,035](#)

Alternative representations:

$$5 \log \left(\left(\frac{843 + 47}{\left(0.01 \left(\frac{100}{10}\right)^2 \exp\left(-\left(\frac{100-25}{10}\right)^2\right)\right)\left(0.02 \left(\frac{100}{8}\right)^2 \exp\left(-\left(\frac{100-20}{8}\right)^2\right)\right)}{\left(0.02 \left(\frac{100}{6}\right)^2 \exp\left(-\left(\frac{100-25}{6}\right)^2\right)\right)\varphi} + 18 \right)^3 + 1 \right) - 2\pi =$$

$$-2\pi + 5 \log_e \left(1 + \left(18 + \frac{890}{\frac{0.0002 \exp\left(-\left(\frac{80}{8}\right)^2\right) \exp\left(-\left(\frac{75}{10}\right)^2\right) \left(\frac{100}{8}\right)^2 \left(\frac{100}{10}\right)^2}{\varphi \left(0.02 \exp\left(-\left(\frac{75}{6}\right)^2\right) \left(\frac{100}{6}\right)^2\right)}} \right)^3 \right)$$

$$5 \log \left(\left(\frac{843 + 47}{\left(0.01 \left(\frac{100}{10}\right)^2 \exp\left(-\left(\frac{100-25}{10}\right)^2\right)\right)\left(0.02 \left(\frac{100}{8}\right)^2 \exp\left(-\left(\frac{100-20}{8}\right)^2\right)\right)}{\left(0.02 \left(\frac{100}{6}\right)^2 \exp\left(-\left(\frac{100-25}{6}\right)^2\right)\right)\varphi} + 18 \right)^3 + 1 \right) - 2\pi =$$

$$-2\pi + 5 \log(a) \log_a \left(1 + \left(18 + \frac{890}{\frac{0.0002 \exp\left(-\left(\frac{80}{8}\right)^2\right) \exp\left(-\left(\frac{75}{10}\right)^2\right) \left(\frac{100}{8}\right)^2 \left(\frac{100}{10}\right)^2}{\varphi \left(0.02 \exp\left(-\left(\frac{75}{6}\right)^2\right) \left(\frac{100}{6}\right)^2\right)}} \right)^3 \right)$$

Series representations:

$$5 \log \left(\left(\frac{843 + 47}{\left(0.01 \left(\frac{100}{10}\right)^2 \exp\left(-\left(\frac{100-25}{10}\right)^2\right)\right)\left(0.02 \left(\frac{100}{8}\right)^2 \exp\left(-\left(\frac{100-20}{8}\right)^2\right)\right)}{\left(0.02 \left(\frac{100}{6}\right)^2 \exp\left(-\left(\frac{100-25}{6}\right)^2\right)\right)\varphi} + 18 \right)^3 + 1 \right) - 2\pi =$$

$$-2\pi + 5 \log \left(\left(18 + \frac{14240 \exp\left(-\frac{625}{4}\right)}{\exp(-100) \exp\left(-\frac{225}{4}\right)} \right)^3 \right) - 5 \sum_{k=1}^{\infty} \frac{(-1)^k \left(\left(18 + \frac{14240 \exp\left(-\frac{625}{4}\right)}{\exp(-100) \exp\left(-\frac{225}{4}\right)} \right)^3 \right)^{-k}}{k}$$

$$\begin{aligned}
& 5 \log \left(\left(\frac{843 + 47}{\frac{(0.01 \left(\frac{100}{10}\right)^2 \exp(-\frac{(100-25)^2}{10})) (0.02 \left(\frac{100}{8}\right)^2 \exp(-\frac{(100-20)^2}{8}))}{(0.02 \left(\frac{100}{6}\right)^2 \exp(-\frac{(100-25)^2}{6})) \varphi}} + 18 \right)^3 + 1 \right) - 2\pi = \\
& -2\pi + 10i\pi \left[\frac{\arg \left(1 - x + \left(18 + \frac{14240 \exp(-\frac{625}{4})}{\exp(-100) \exp(-\frac{225}{4})} \right)^3 \right)}{2\pi} \right] + 5 \log(x) - \\
& 5 \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} \left(1 - x + \left(18 + \frac{14240 \exp(-\frac{625}{4})}{\exp(-100) \exp(-\frac{225}{4})} \right)^3 \right)^k}{k} \quad \text{for } x < 0
\end{aligned}$$

$$\begin{aligned}
& 5 \log \left(\left(\frac{843 + 47}{\frac{(0.01 \left(\frac{100}{10}\right)^2 \exp(-\frac{(100-25)^2}{10})) (0.02 \left(\frac{100}{8}\right)^2 \exp(-\frac{(100-20)^2}{8}))}{(0.02 \left(\frac{100}{6}\right)^2 \exp(-\frac{(100-25)^2}{6})) \varphi}} + 18 \right)^3 + 1 \right) - 2\pi = \\
& -2\pi + 5 \left[\frac{\arg \left(1 + \left(18 + \frac{14240 \exp(-\frac{625}{4})}{\exp(-100) \exp(-\frac{225}{4})} \right)^3 - z_0 \right)}{2\pi} \right] \log \left(\frac{1}{z_0} \right) + \\
& 5 \log(z_0) + 5 \left[\frac{\arg \left(1 + \left(18 + \frac{14240 \exp(-\frac{625}{4})}{\exp(-100) \exp(-\frac{225}{4})} \right)^3 - z_0 \right)}{2\pi} \right] \log(z_0) - \\
& 5 \sum_{k=1}^{\infty} \frac{(-1)^k \left(1 + \left(18 + \frac{14240 \exp(-\frac{625}{4})}{\exp(-100) \exp(-\frac{225}{4})} \right)^3 - z_0 \right)^k z_0^{-k}}{k}
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& 5 \log \left(\left(\frac{843 + 47}{\frac{(0.01 \left(\frac{100}{10}\right)^2 \exp(-\frac{(100-25)^2}{10})) (0.02 \left(\frac{100}{8}\right)^2 \exp(-\frac{(100-20)^2}{8}))}{(0.02 \left(\frac{100}{6}\right)^2 \exp(-\frac{(100-25)^2}{6})) \varphi}} + 18 \right)^3 + 1 \right) - 2\pi = \\
& -2\pi + 5 \int_1^{1 + \left(18 + \frac{14240 \exp(-\frac{625}{4})}{\exp(-100) \exp(-\frac{225}{4})} \right)^3} \frac{1}{t} dt
\end{aligned}$$

$$5 \log \left(\left(\frac{843 + 47}{\frac{(0.01 \left(\frac{100}{10}\right)^2 \exp\left(-\left(\frac{100-25}{10}\right)^2\right)) (0.02 \left(\frac{100}{8}\right)^2 \exp\left(-\left(\frac{100-20}{8}\right)^2\right))}{(0.02 \left(\frac{100}{6}\right)^2 \exp\left(-\left(\frac{100-25}{6}\right)^2\right))^\varphi} + 18 \right) + 1 \right)^3 - 2\pi =$$

$$-2\pi + \frac{5}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(\left(18 + \frac{14240 \exp\left(-\frac{625}{4}\right)}{\exp(-100)\exp\left(-\frac{225}{4}\right)} \right)^3 \right)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

and again:

$$\left(\frac{843+47}{\frac{1}{9} \left(\frac{0.01 \left(\frac{100}{10}\right)^2 \exp\left(-\left(\frac{100-25}{10}\right)^2\right)}{0.02 \left(\frac{100}{6}\right)^2 \exp\left(-\left(\frac{100-25}{6}\right)^2\right)} \right)} \left(\frac{0.02 \left(\frac{100}{8}\right)^2 \exp\left(-\left(\frac{100-20}{8}\right)^2\right)}{0.02 \left(\frac{100}{6}\right)^2 \exp\left(-\left(\frac{100-25}{6}\right)^2\right)} \right) + 18 \right)^3 + 1 \right)^{3/13} + 24$$

Input:

$$\left(\frac{843 + 47}{\frac{1}{9} \left(\left(0.01 \left(\frac{100}{10}\right)^2 \exp\left(-\left(\frac{100 - 25}{10}\right)^2\right) \right) \times \frac{1}{0.02 \left(\frac{100}{6}\right)^2 \exp\left(-\left(\frac{100-25}{6}\right)^2\right)} \right)} \left(\frac{0.02 \left(\frac{100}{8}\right)^2 \exp\left(-\left(\frac{100 - 20}{8}\right)^2\right)}{0.02 \left(\frac{100}{6}\right)^2 \exp\left(-\left(\frac{100-25}{6}\right)^2\right)} \right) + 18 \right)^3 + 1 \right)^{3/13} + 24$$

Result:

775.427...

775.427... result practically equal to the rest mass of Neutral rho meson 775.26

$$\left(\frac{843+47}{\frac{1}{9} \left(\frac{0.01 \left(\frac{100}{10}\right)^2 \exp\left(-\left(\frac{100-25}{10}\right)^2\right)}{0.02 \left(\frac{100}{6}\right)^2 \exp\left(-\left(\frac{100-25}{6}\right)^2\right)} \right)} \left(\frac{0.02 \left(\frac{100}{8}\right)^2 \exp\left(-\left(\frac{100-20}{8}\right)^2\right)}{0.02 \left(\frac{100}{6}\right)^2 \exp\left(-\left(\frac{100-25}{6}\right)^2\right)} \right) + 18 \right)^3 + 1 \right)^{(29 \cdot 3^2)/10^3} - 4$$

Input:

$$\left(\frac{843 + 47}{\frac{1}{9} \left(\left(0.01 \left(\frac{100}{10}\right)^2 \exp\left(-\left(\frac{100 - 25}{10}\right)^2\right) \right) \times \frac{1}{0.02 \left(\frac{100}{6}\right)^2 \exp\left(-\left(\frac{100-25}{6}\right)^2\right)} \right)} \left(\frac{0.02 \left(\frac{100}{8}\right)^2 \exp\left(-\left(\frac{100 - 20}{8}\right)^2\right)}{0.02 \left(\frac{100}{6}\right)^2 \exp\left(-\left(\frac{100-25}{6}\right)^2\right)} \right) + 18 \right)^3 + 1 \right)^{(29 \cdot 3^2)/10^3} - 4$$

Result:

1785.07...

1785.07 as below

Baryon modes

B^0 DECAY MODES	Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
$D^*(2010)^- p \bar{n}$	$(1.4 \pm 0.4) \times 10^{-3}$		1785

 $D^*(2010)^\pm$ MASS

The fit includes D^\pm , D^0 , D_s^\pm , $D^{*\pm}$, D^{*0} , $D_s^{*\pm}$, $D_1(2420)^0$, $D_2^*(2460)^0$, and $D_{s1}(2536)^\pm$ mass and mass difference measurements.

<u>VALUE (MeV)</u>	<u>DOCUMENT ID</u>	<u>TECN</u>	<u>CHG</u>	<u>COMMENT</u>
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2010.26±0.05 OUR FIT

• • • We do not use the following data for averages, fits, limits, etc. • • •

2008 ±3	¹ GOLDHABER 77	MRK1 ±	$e^+ e^-$
2008.6 ±1.0	² PERUZZI 77	LGW ±	$e^+ e^-$

¹ From simultaneous fit to $D^*(2010)^+$, $D^*(2007)^0$, D^+ , and D^0 ; not independent of FELDMAN 77B mass difference below.

² PERUZZI 77 mass not independent of FELDMAN 77B mass difference below and PERUZZI 77 D^0 mass value.

We have also the following results:

From:

Input:

$$\left((843 + 47) / \left(\frac{1}{9} \left(\left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100 - 25}{10} \right)^2 \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100 - 25}{6} \right)^2 \right)} \right. \right. \right. \\ \left. \left. \left. \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100 - 20}{8} \right)^2 \right) \right) \right) + 18 \right)^3 + 1 \right)$$

Result:

2898516861513

2898516861513

$$\left(\frac{(((((843+47)/[1/9((((0.01(100/10)^2 \exp(-((100-25)/10)^2)))) 1 /(((0.02(100/6)^2 \exp(-((100-25)/6)^2)))) * (((0.02(100/8)^2 \exp(-((100-20)/8)^2)))))))]+18)))^3 + 1)))^{1/4} - 233 - 55 + 3 \right)$$

Input:

$$\left(\left((843 + 47) / \left(\frac{1}{9} \left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100 - 25}{10} \right)^2 \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100 - 25}{6} \right)^2 \right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100 - 20}{8} \right)^2 \right) \right) \right) + 18 \right)^3 + 1 \right)^{(1/4) - 233 - 55 + 3}$$

Result:

1019.80...

1019.80... result practically equal to the rest mass of Phi meson 1019.445

From

$$\frac{521 + 123 + 47 + 18 + 7}{\frac{1}{9} \left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100 - 25}{10} \right)^2 \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100 - 25}{6} \right)^2 \right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100 - 20}{8} \right)^2 \right) \right) + 11 + 1}$$

= 11468 , we obtain:

$$\left(\frac{521+123+47+18+7}{\left(\frac{1}{9} \left(0.01 \left(\frac{100}{10} \right)^2 \exp\left(-\left(\frac{100-25}{10}\right)^2\right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp\left(-\left(\frac{100-25}{6}\right)^2\right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp\left(-\left(\frac{100-20}{8}\right)^2\right) \right) \right) + 11 + 1 \right)^{3/4} - 89$$

Input:

$$\left(\frac{521 + 123 + 47 + 18 + 7}{\left(\frac{1}{9} \left(0.01 \left(\frac{100}{10} \right)^2 \exp\left(-\left(\frac{100 - 25}{10}\right)^2\right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp\left(-\left(\frac{100-25}{6}\right)^2\right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp\left(-\left(\frac{100 - 20}{8}\right)^2\right) \right) \right) + 11 + 1 \right)^{3/4} - 89$$

Result:

1019.19...

1019.19... result practically equal to the rest mass of Phi meson 1019.445

From:

Input:

$$\frac{521 + 123 + 47 + 7}{\left(\frac{1}{9} \left(0.01 \left(\frac{100}{10} \right)^2 \exp\left(-\left(\frac{100-25}{10}\right)^2\right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp\left(-\left(\frac{100-25}{6}\right)^2\right)} \left(0.02 \left(\frac{100}{8} \right)^2 \exp\left(-\left(\frac{100-20}{8}\right)^2\right) \right) \right) - 7$$

Result:

11161

11161

$$\left(\frac{(521+123+47+7) / \left(\frac{1}{9} \left(\left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100-25}{10} \right)^2 \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100-25}{6} \right)^2 \right)} \right. \right. \right. \\ \left. \left. \left. \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100-20}{8} \right)^2 \right) \right) \right) - 7 \right)^3 \right)^{(1/4) - 89 + 21 + 2}$$

Input:

$$\left((521 + 123 + 47 + 7) / \left(\frac{1}{9} \left(\left(0.01 \left(\frac{100}{10} \right)^2 \exp \left(- \left(\frac{100 - 25}{10} \right)^2 \right) \right) \times \frac{1}{0.02 \left(\frac{100}{6} \right)^2 \exp \left(- \left(\frac{100 - 25}{6} \right)^2 \right)} \right. \right. \right. \\ \left. \left. \left. \left(0.02 \left(\frac{100}{8} \right)^2 \exp \left(- \left(\frac{100 - 20}{8} \right)^2 \right) \right) \right) - 7 \right)^3 \right)^{(1/4) - 89 + 21 + 2}$$

Result:

1019.87...

1019.87... result practically equal to the rest mass of Phi meson 1019.445

Now, we have that:

With this, (D.1) reduces to

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \left(1 - \frac{2M}{r} \right) \frac{d\phi}{dr} \right) + \lambda \frac{48M^2}{r^6} = 0. \quad (D.3)$$

Imposing regularity of $\partial_r \phi$ at the geometric horizon $r = 2M$, setting $\lim_{r \rightarrow \infty} \phi = 0$, and changing variables to $x \equiv r/M$, we obtain

$$\phi(x) = \frac{2\lambda}{M^2} \left(\frac{1}{x} + \frac{1}{x^2} + \frac{4}{3x^3} \right), \quad (D.4)$$

From

$$\phi(x) = \frac{2\lambda}{M^2} \left(\frac{1}{x} + \frac{1}{x^2} + \frac{4}{3x^3} \right),$$

For $x = r/M$, where $M = 13.1266\text{e}+39$ and $r = 1.9501008226000\text{e}+13$,

we obtain:

$$(19501008226000) / 13.1266e+39 = 1.48561e-27$$

Indeed:

Input interpretation:

$$1.48561 \times 10^{-27} = \frac{19501008226000}{13.1266 \times 10^{39}}$$

Result:

True

Thence, we have:

$$\left(\frac{2 \times 50}{(13.12666e+39)^2} \right) \left(\frac{1}{1.48561e-27} + \frac{1}{(1.48561e-27)^2} + \frac{4}{3(1.48561e-27)^3} \right)$$

Input interpretation:

$$\frac{2 \times 50}{(13.12666 \times 10^{39})^2} \left(\frac{1}{1.48561 \times 10^{-27}} + \frac{1}{(1.48561 \times 10^{-27})^2} + \frac{4}{3(1.48561 \times 10^{-27})^3} \right)$$

Result:

236.0020749430157184085465925864200989116751681427078960454...

236.00207494... \approx 236

From which:

$$\left(\frac{2 \times 50}{(13.12666e+39)^2} \right) \left(\frac{1}{1.48561e-27} + \frac{1}{(1.48561e-27)^2} + \frac{4}{3(1.48561e-27)^3} \right) - 64$$

Input interpretation:

$$\frac{2 \times 50}{(13.12666 \times 10^{39})^2} \left(\frac{1}{1.48561 \times 10^{-27}} + \frac{1}{(1.48561 \times 10^{-27})^2} + \frac{4}{3(1.48561 \times 10^{-27})^3} \right) - 64$$

Result:

172.0020749430157184085465925864200989116751681427078960454...

172.00207494... \approx 172 (Ramanujan taxicab number)

$$\left(\frac{2 \times 50}{(13.12666 \times 10^{39})^2} \right) \left(\frac{1}{1.48561 \times 10^{-27}} + \frac{1}{(1.48561 \times 10^{-27})^2} + \frac{4}{3(1.48561 \times 10^{-27})^3} \right) - 8^2 - 34$$

Input interpretation:

$$\frac{2 \times 50}{(13.12666 \times 10^{39})^2} \left(\frac{1}{1.48561 \times 10^{-27}} + \frac{1}{(1.48561 \times 10^{-27})^2} + \frac{4}{3(1.48561 \times 10^{-27})^3} \right) - 8^2 - 34$$

Result:

138.0020749430157184085465925864200989116751681427078960454...

138.002074943.... \approx 138 (Ramanujan taxicab number)

$$\left(\frac{2 \times 50}{(13.12666 \times 10^{39})^2} \right) \left(\frac{1}{1.48561 \times 10^{-27}} + \frac{1}{(1.48561 \times 10^{-27})^2} + \frac{4}{3(1.48561 \times 10^{-27})^3} \right) - 8^2 - 34 - 3$$

Input interpretation:

$$\frac{2 \times 50}{(13.12666 \times 10^{39})^2} \left(\frac{1}{1.48561 \times 10^{-27}} + \frac{1}{(1.48561 \times 10^{-27})^2} + \frac{4}{3(1.48561 \times 10^{-27})^3} \right) - 8^2 - 34 - 3$$

Result:

135.0020749430157184085465925864200989116751681427078960454...

135.002074943.... \approx 135 (Ramanujan taxicab number)

Results can be applicate to the following Ramanujan cube (Taxicab):

$$135^3 + 138^3 = 172^3 - 1$$

From:

<https://twitter.com/Adityadhar004/status/997520422815059969>

123

vi. $\phi(x) + \phi(-x) = 2\phi(x^2)$
vii. $\phi(x) - \phi(-x) = 4x\psi(x^2)$
viii. $\phi(x)\phi(-x) = \phi^2(x^2)$
ix. $\phi(x)\psi(x^2) = \psi^2(x)$
x. $\phi^2(x) - \phi^2(-x) = 8x\psi^2(x^2)$
xi. $\phi^2(x) + \phi^2(-x) = 2\phi^2(x^2)$
xii. $\phi^4(x) - \phi^4(-x) = 16x\psi^4(x^2)$
xiii. $\psi^2(x) + \psi^2(-x) = 2\psi(x^2)\phi(x^2)$
xiv. If $\left(\frac{1-z}{1+z}\right)^2 = \left\{\frac{\phi(-x)}{\phi(x)}\right\}^4$ then $1-z^2 = \left\{\frac{\phi(-x^2)}{\phi(x^2)}\right\}^2$

Ex-1. $\frac{\psi(x)}{\psi(-x)} = \sqrt{\frac{\phi(x)}{\phi(-x)}}$

2. $\psi(x)\psi(-x) = \psi(x^2)\phi(-x^2)$

3. $\frac{\psi(x)\psi(-x)}{\psi(x^2)\psi(-x^2)} = \frac{\psi(-x^2)}{\psi(x^2)}$

18. $\int_0^1 F(x) = e^{-\pi \cdot \frac{1 + (4)^{-1}x + \left(\frac{1.3}{2.4}\right)^2 x^2 + \left(\frac{1.3.5}{2.4.6}\right)^2 x^3 + \dots}{1 + (4)^{-1}x + \left(\frac{1.3}{2.4}\right)^2 x^2 + \left(\frac{1.3.5}{2.4.6}\right)^2 x^3 + \dots}}$

i. $F(x) = \frac{x}{16} e^{4 \cdot \frac{(4)^{-1} \frac{1}{1.2} x + \left(\frac{1.3}{2.4}\right)^2 \left(\frac{1}{1.2} + \frac{1}{3.4}\right) x^2}{1 + (4)^{-1}x + \left(\frac{1.3}{2.4}\right)^2 x^2 + \dots}}$

ii. $F(1-\frac{1}{2}) + \theta = \frac{\log_e x}{10 + \sqrt{36 + (\log_e x)^2}}$ where θ is numerically 5
much less than $\frac{2}{135} F^5(1-\frac{1}{2})$. $\theta = \frac{1}{2160} \cdot \left\{ \frac{\log_e x}{8 + \sqrt{16 + (\log_e x)^2}} \right\}$

iii. $\log_e F(x) \log_e F(1-x) = \pi^2$
iv. $F(1-x) + F(1-\frac{1}{2}) = 0$
v. $F\left\{\frac{4x}{(1+x)^2}\right\} = \sqrt{F(x^2)}$

x. B. If we know the expansion of $F\left(\frac{2x}{1+x}\right)$ to n terms, then we can find its expansion to $2n$ terms as follows -
Suppose we know the expansion of $F\left(\frac{2x}{1+x}\right)$ to n terms.

We have the following equation:

$$\exp\left(\frac{-\pi \left(\left(1 + \frac{1}{4}(1-x) + \left(\frac{3}{8}\right)^2(1-x)^2 + \left(\frac{15}{48}\right)^2(1-x)^3 \right) \right)}{\left(1 + \frac{1}{4}x + \left(\frac{3}{8}\right)^2x^2 + \left(\frac{15}{48}\right)^2x^3 \right)}\right)$$

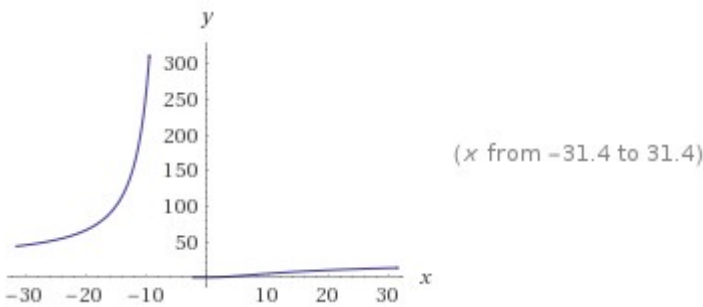
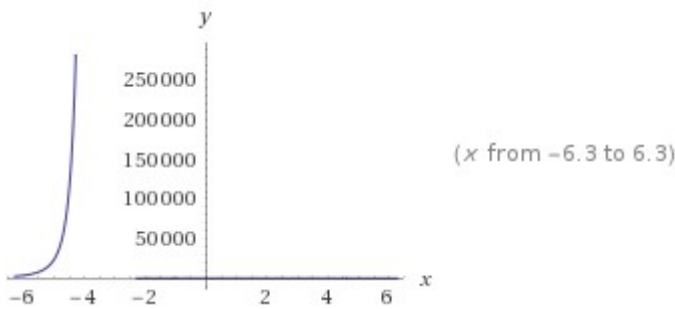
Input:

$$\exp\left(-\pi \times \frac{1 + \frac{1}{4}(1-x) + \left(\frac{3}{8}\right)^2(1-x)^2 + \left(\frac{15}{48}\right)^2(1-x)^3}{1 + \frac{1}{4}x + \left(\frac{3}{8}\right)^2x^2 + \left(\frac{15}{48}\right)^2x^3}\right)$$

Exact result:

$$e^{-\frac{\pi \left(\frac{25}{256}(1-x)^3 + \frac{9}{64}(1-x)^2 + \frac{1-x}{4} + 1 \right)}{\frac{25x^3}{256} + \frac{9x^2}{64} + \frac{x}{4} + 1}}$$

Plots:



Alternate forms:

$$e^{-\frac{\pi(x(25x-111)+211)-381}{x(x(25x+36)+64)+256}}$$

$$e^{-\frac{\pi(25x^3-111x^2+211x-381)}{25x^3+36x^2+64x+256}}$$

Alternate form assuming x is real:

$$e^{-\frac{\frac{25\pi x^3}{256} + \frac{111\pi x^2}{256} - \frac{211\pi x}{256} + \frac{381\pi}{256}}{\frac{25x^3}{256} + \frac{9x^2}{64} + \frac{x}{4} + 1}}$$

Roots:

(no roots exist)

Properties as a real function:

Domain

$\{x \in \mathbb{R} : x \neq -2.28315\}$

Range

$\{y \in \mathbb{R} : y > 23.1407 \text{ or } 0 < y < 23.1407\}$

Injectivity

injective (one-to-one)

\mathbb{R} is the set of real numbers

Series expansion at $x = 0$:

$$e^{-(381\pi)/256} + \frac{1225 e^{-(381\pi)/256} \pi x}{1024} + \frac{8575 e^{-(381\pi)/256} \pi (175\pi - 128) x^2}{2097152} +$$

$$\frac{1225 e^{-(381\pi)/256} \pi (1081344 - 3292800\pi + 1500625\pi^2) x^3}{6442450944} +$$

$$\frac{1}{26388279066624} 1225 e^{-(381\pi)/256} \pi$$

$$(-2038431744 + 8248934400\pi - 8067360000\pi^2 + 1838265625\pi^3) x^4 + O(x^5)$$

(Taylor series)

Series expansion at $x = \infty$:

$$e^\pi - \frac{147(e^\pi \pi)}{25x} + \frac{147e^\pi \pi (122 + 147\pi)}{1250x^2} -$$

$$\frac{49(e^\pi \pi (19826 + 53802\pi + 21609\pi^2))}{31250x^3} + O\left(\left(\frac{1}{x}\right)^4\right)$$

(Laurent series)

Derivative:

$$\frac{d}{dx} \left(\exp \left(- \frac{\pi \left(1 + \frac{1-x}{4} + \left(\frac{3}{8}\right)^2 (1-x)^2 + \left(\frac{15}{48}\right)^2 (1-x)^3 \right)}{1 + \frac{x}{4} + \left(\frac{3}{8}\right)^2 x^2 + \left(\frac{15}{48}\right)^2 x^3} \right) \right) =$$

$$\frac{1225 \pi (3x^4 - 6x^3 + 27x^2 - 24x + 64) \exp \left(\frac{\pi (25x^3 - 111x^2 + 211x - 381)}{25x^3 + 36x^2 + 64x + 256} \right)}{(25x^3 + 36x^2 + 64x + 256)^2}$$

Limit:

$$\lim_{x \rightarrow \pm\infty} \exp\left(-\frac{\pi\left(1 + \frac{1-x}{4} + \frac{9}{64}(1-x)^2 + \frac{25}{256}(1-x)^3\right)}{1 + \frac{x}{4} + \frac{9x^2}{64} + \frac{25x^3}{256}}\right) = e^\pi \approx 23.1407$$

Now, for $x = \sqrt[4]{3^2/2}$

$$\exp\left(\frac{\pi\left(\left(1 + \frac{1}{4}\left(1 - \sqrt[4]{\frac{3^2}{2}}\right) + \left(\frac{3}{8}\right)^2\left(1 - \sqrt[4]{\frac{3^2}{2}}\right)^2 + \left(\frac{15}{48}\right)^2\left(1 - \sqrt[4]{\frac{3^2}{2}}\right)^3\right)\right)}{\left(1 + \frac{1}{4}\sqrt[4]{\frac{3^2}{2}} + \left(\frac{3}{8}\right)^2\sqrt[4]{\frac{3^2}{2}}^2 + \left(\frac{15}{48}\right)^2\sqrt[4]{\frac{3^2}{2}}^3\right)}\right)$$

Input:

$$\exp\left(-\pi \times \frac{1 + \frac{1}{4}\left(1 - \sqrt[4]{\frac{3^2}{2}}\right) + \left(\frac{3}{8}\right)^2\left(1 - \sqrt[4]{\frac{3^2}{2}}\right)^2 + \left(\frac{15}{48}\right)^2\left(1 - \sqrt[4]{\frac{3^2}{2}}\right)^3}{\left(1 + \frac{1}{4}\sqrt[4]{\frac{3^2}{2}} + \left(\frac{3}{8}\right)^2\sqrt[4]{\frac{3^2}{2}}^2 + \left(\frac{15}{48}\right)^2\sqrt[4]{\frac{3^2}{2}}^3\right)}\right)$$

Exact result:

$$e^{-\frac{\left(1 + \frac{1}{4}\left(1 - \frac{\sqrt{3}}{\sqrt[4]{2}}\right) + \frac{9}{64}\left(1 - \frac{\sqrt{3}}{\sqrt[4]{2}}\right)^2 + \frac{25}{256}\left(1 - \frac{\sqrt{3}}{\sqrt[4]{2}}\right)^3\right)\pi}{\left(1 + \frac{27}{64\sqrt{2}} + \frac{89\sqrt{3}}{256\sqrt[4]{2}}\right)^3}}$$

Decimal approximation:

0.616180280493643121288779290775434429457405890246816407783...

0.6161802804936.....

Property:

$$\exp\left(-\frac{\left(1 + \frac{1}{4}\left(1 - \frac{\sqrt{3}}{\sqrt[4]{2}}\right) + \frac{9}{64}\left(1 - \frac{\sqrt{3}}{\sqrt[4]{2}}\right)^2 + \frac{25}{256}\left(1 - \frac{\sqrt{3}}{\sqrt[4]{2}}\right)^3\right)\pi}{\left(1 + \frac{27}{64\sqrt{2}} + \frac{89\sqrt{3}}{256\sqrt[4]{2}}\right)^3}\right)$$

is a transcendental number

Alternate forms:

$$e^{\frac{262144 \left(-762 - 333\sqrt{2} + 75\sqrt[4]{2}\sqrt{3} + 211 \times 2^{3/4}\sqrt{3} \right) \pi}{\left(512 + 108\sqrt{2} + 89 \times 2^{3/4}\sqrt{3} \right)^3}}$$

$$e^{-\frac{65536 \times 2^{3/4} \left(333\sqrt[4]{2} + 381 \times 2^{3/4} - 75\sqrt{3} - 211\sqrt{6} \right) \pi}{\left(108 + 256\sqrt{2} + 89\sqrt[4]{2}\sqrt{3} \right)^3}}$$

Series representations:

$$\exp \left[-\frac{\pi \left(1 + \frac{1}{4} \left(1 - \sqrt{\sqrt{\frac{3^2}{2}}} \right) + \left(\frac{3}{8} \right)^2 \left(1 - \sqrt{\sqrt{\frac{3^2}{2}}} \right)^2 + \left(\frac{15}{48} \right)^2 \left(1 - \sqrt{\sqrt{\frac{3^2}{2}}} \right)^3 \right)}{\left(1 + \frac{1}{4} \sqrt{\sqrt{\frac{3^2}{2}}} + \left(\frac{3}{8} \right)^2 \sqrt{\sqrt{\frac{3^2}{2}}} + \left(\frac{15}{48} \right)^2 \sqrt{\sqrt{\frac{3^2}{2}}} \right)^3} \right] =$$

$$\exp \left[-\left(\left(\pi \left(1 + \frac{1}{4} \left(1 - \sqrt{\sqrt{\frac{7}{2}} \sum_{k=0}^{\infty} \left(\frac{7}{2} \right)^{-k} \binom{\frac{1}{2}}{k} \right)} + \frac{9}{64} \left(1 - \sqrt{\sqrt{\frac{7}{2}} \sum_{k=0}^{\infty} \left(\frac{7}{2} \right)^{-k} \binom{\frac{1}{2}}{k} \right)} \right)^2 + \frac{25}{256} \left(1 - \sqrt{\sqrt{\frac{7}{2}} \sum_{k=0}^{\infty} \left(\frac{7}{2} \right)^{-k} \binom{\frac{1}{2}}{k} \right)} \right)^3 \right) \right] /$$

$$\left(1 + \frac{9}{64} \sqrt{\frac{7}{2}} \sum_{k=0}^{\infty} \left(\frac{7}{2} \right)^{-k} \binom{\frac{1}{2}}{k} + \frac{89}{256} \sqrt{\sqrt{\frac{7}{2}} \sum_{k=0}^{\infty} \left(\frac{7}{2} \right)^{-k} \binom{\frac{1}{2}}{k}} \right)^3 \right)$$

$$\exp \left[- \frac{\pi \left(1 + \frac{1}{4} \left(1 - \sqrt{\sqrt{\frac{3^2}{2}}} \right) + \left(\frac{3}{8} \right)^2 \left(1 - \sqrt{\sqrt{\frac{3^2}{2}}} \right)^2 + \left(\frac{15}{48} \right)^2 \left(1 - \sqrt{\sqrt{\frac{3^2}{2}}} \right)^3 \right)}{\left(1 + \frac{1}{4} \sqrt{\sqrt{\frac{3^2}{2}}} + \left(\frac{3}{8} \right)^2 \sqrt{\sqrt{\frac{3^2}{2}}}^2 + \left(\frac{15}{48} \right)^2 \sqrt{\sqrt{\frac{3^2}{2}}}^3 \right)} \right] =$$

$$\exp \left[- \left(\left(\left(\pi \left(1 + \frac{1}{4} \left(1 - \sqrt{\sqrt{\frac{7}{2}} \sum_{k=0}^{\infty} \frac{\left(-\frac{2}{7} \right)^k \left(-\frac{1}{2} \right)_k}{k!} \right)} + \frac{9}{64} \left(1 - \sqrt{\sqrt{\frac{7}{2}} \sum_{k=0}^{\infty} \frac{\left(-\frac{2}{7} \right)^k \left(-\frac{1}{2} \right)_k}{k!} \right)} \right)^2 + \right. \right.$$

$$\left. \left. \frac{25}{256} \left(1 - \sqrt{\sqrt{\frac{7}{2}} \sum_{k=0}^{\infty} \frac{\left(-\frac{2}{7} \right)^k \left(-\frac{1}{2} \right)_k}{k!} \right)} \right)^3 \right) \right] /$$

$$\left(1 + \frac{9}{64} \sqrt{\sqrt{\frac{7}{2}} \sum_{k=0}^{\infty} \frac{\left(-\frac{2}{7} \right)^k \left(-\frac{1}{2} \right)_k}{k!}} + \frac{89}{256} \sqrt{\sqrt{\frac{7}{2}} \sum_{k=0}^{\infty} \frac{\left(-\frac{2}{7} \right)^k \left(-\frac{1}{2} \right)_k}{k!}} \right)^3 \right)$$

$$\exp \left[- \frac{\pi \left(1 + \frac{1}{4} \left(1 - \sqrt{\sqrt{\frac{3^2}{2}}} \right) + \left(\frac{3}{8} \right)^2 \left(1 - \sqrt{\sqrt{\frac{3^2}{2}}} \right)^2 + \left(\frac{15}{48} \right)^2 \left(1 - \sqrt{\sqrt{\frac{3^2}{2}}} \right)^3 \right)}{\left(1 + \frac{1}{4} \sqrt{\sqrt{\frac{3^2}{2}}} + \left(\frac{3}{8} \right)^2 \sqrt{\sqrt{\frac{3^2}{2}}}^2 + \left(\frac{15}{48} \right)^2 \sqrt{\sqrt{\frac{3^2}{2}}}^3 \right)} \right] =$$

$$\exp \left[- \left(\left(\left(\pi \left(1 + \frac{1}{4} \left(1 - \sqrt{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2} \right)_k \left(\frac{9}{2} - z_0 \right)^k z_0^{-k}}{k!} \right)} + \right. \right. \right.$$

$$\left. \left. \frac{9}{64} \left(1 - \sqrt{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2} \right)_k \left(\frac{9}{2} - z_0 \right)^k z_0^{-k}}{k!} \right)} \right)^2 + \right.$$

$$\left. \left. \frac{25}{256} \left(1 - \sqrt{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2} \right)_k \left(\frac{9}{2} - z_0 \right)^k z_0^{-k}}{k!} \right)} \right)^3 \right) \right] /$$

$$\left(1 + \frac{9}{64} \sqrt{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2} \right)_k \left(\frac{9}{2} - z_0 \right)^k z_0^{-k}}{k!}} + \right.$$

$$\left. \frac{89}{256} \sqrt{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2} \right)_k \left(\frac{9}{2} - z_0 \right)^k z_0^{-k}}{k!}} \right)^3 \right)$$

for (not $(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)$)

Integral representation:

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-a-s)}{z^s} ds}{(2\pi i)\Gamma(-a)} \text{ for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

and:

$$1 + \exp\left(\frac{(-\pi\left(1 + \frac{1}{4}\left(1 - \sqrt{\frac{3^2}{2}}\right) + \left(\frac{3}{8}\right)^2\left(1 - \sqrt{\frac{3^2}{2}}\right)^2 + \left(\frac{15}{48}\right)^2\left(1 - \sqrt{\frac{3^2}{2}}\right)^3\right))}{\left(1 + \frac{1}{4}\sqrt{\frac{3^2}{2}} + \left(\frac{3}{8}\right)^2\sqrt{\frac{3^2}{2}}^2 + \left(\frac{15}{48}\right)^2\sqrt{\frac{3^2}{2}}^3\right)}\right)$$

Input:

$$1 + \exp\left(-\pi \times \frac{\left(1 + \frac{1}{4}\left(1 - \sqrt{\frac{3^2}{2}}\right) + \left(\frac{3}{8}\right)^2\left(1 - \sqrt{\frac{3^2}{2}}\right)^2 + \left(\frac{15}{48}\right)^2\left(1 - \sqrt{\frac{3^2}{2}}\right)^3\right)}{\left(1 + \frac{1}{4}\sqrt{\frac{3^2}{2}} + \left(\frac{3}{8}\right)^2\sqrt{\frac{3^2}{2}}^2 + \left(\frac{15}{48}\right)^2\sqrt{\frac{3^2}{2}}^3\right)}\right)$$

Exact result:

$$\exp\left(-\frac{\left(1 + \frac{1}{4}\left(1 - \frac{\sqrt{3}}{\sqrt[4]{2}}\right) + \frac{9}{64}\left(1 - \frac{\sqrt{3}}{\sqrt[4]{2}}\right)^2 + \frac{25}{256}\left(1 - \frac{\sqrt{3}}{\sqrt[4]{2}}\right)^3\right)\pi}{\left(1 + \frac{27}{64\sqrt{2}} + \frac{89\sqrt{3}}{256\sqrt[4]{2}}\right)^3}\right) + 1$$

Decimal approximation:

1.616180280493643121288779290775434429457405890246816407783...

[1.6161802804936.....](#)

Property:

$$1 + \exp\left(-\frac{\left(1 + \frac{1}{4}\left(1 - \frac{\sqrt{3}}{\sqrt[4]{2}}\right) + \frac{9}{64}\left(1 - \frac{\sqrt{3}}{\sqrt[4]{2}}\right)^2 + \frac{25}{256}\left(1 - \frac{\sqrt{3}}{\sqrt[4]{2}}\right)^3\right)\pi}{\left(1 + \frac{27}{64\sqrt{2}} + \frac{89\sqrt{3}}{256\sqrt[4]{2}}\right)^3}\right)$$

is a transcendental number

Alternate forms:

$$\exp\left(\left(\left(-269\,298\,588\,178\,482\,448\,500\,293\,763\,072 - \right.\right.\right. \\ \left.\left.\left.439\,373\,744\,880\,302\,651\,673\,667\,829\,760\sqrt{2} + \right.\right.\right. \\ \left.\left.\left.110\,967\,596\,607\,821\,716\,533\,638\,154\,497 \right.\right.\right. \\ \left.\left.\left.\sqrt{\left(362\,392\,128\,663\,198\,293\,294\,217\,559\,082\,619\,139\,777\,277\,248\right)} \right.\right.\right. \\ \left.\left.\left.\frac{880\,372\,361\,461\,760}{12\,313\,807\,496\,916\,245\,730\,109\,797\,366\,715\,732\,287\,648} \right.\right.\right. \\ \left.\left.\left.\frac{993\,638\,964\,041\,323\,009 + \left(262\,429\,531\,615\,646\,298\,197\,131\,784\,097\,209\,366\,376\,316\right)}{618\,792\,751\,245\,295\,616} \right.\right.\right. \\ \left.\left.\left.\sqrt{2}\right)\right)\right) / \\ \left.221\,935\,193\,215\,643\,433\,067\,276\,308\,994\right) + 1$$

$$\exp\left(\frac{262\,144\left(-762 - 333\sqrt{2} + 75\sqrt[4]{2}\sqrt{3} + 211 \times 2^{3/4}\sqrt{3}\right)\pi}{\left(512 + 108\sqrt{2} + 89 \times 2^{3/4}\sqrt{3}\right)^3}\right) + 1$$

$$\exp\left(-\frac{65\,536 \times 2^{3/4}\left(333\sqrt[4]{2} + 381 \times 2^{3/4} - 75\sqrt{3} - 211\sqrt{6}\right)\pi}{\left(108 + 256\sqrt{2} + 89\sqrt[4]{2}\sqrt{3}\right)^3}\right) + 1$$

Series representations:

$$1 + \exp\left(\frac{\pi\left(1 + \frac{1}{4}\left(1 - \sqrt{\sqrt{\frac{3^2}{2}}}\right) + \left(\frac{3}{8}\right)^2\left(1 - \sqrt{\sqrt{\frac{3^2}{2}}}\right)^2 + \left(\frac{15}{48}\right)^2\left(1 - \sqrt{\sqrt{\frac{3^2}{2}}}\right)^3\right)}{\left(1 + \frac{1}{4}\sqrt{\sqrt{\frac{3^2}{2}}} + \left(\frac{3}{8}\right)^2\sqrt{\sqrt{\frac{3^2}{2}}}^2 + \left(\frac{15}{48}\right)^2\sqrt{\sqrt{\frac{3^2}{2}}}^3\right)}\right) =$$

$$1 + \exp\left(\left(65\,536\pi\left(-381 - 111\sqrt{\frac{7}{2}}\sum_{k=0}^{\infty}\left(\frac{7}{2}\right)^{-k}\binom{\frac{1}{2}}{k} + \right.\right.\right. \\ \left.\left.\left.211\sqrt{\sqrt{\frac{7}{2}}}\sum_{k=0}^{\infty}\left(\frac{7}{2}\right)^{-k}\binom{\frac{1}{2}}{k} + 25\left(\sqrt{\frac{7}{2}}\sum_{k=0}^{\infty}\left(\frac{7}{2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)^{3/2}\right)\right) / \right. \\ \left.\left(256 + 36\sqrt{\frac{7}{2}}\sum_{k=0}^{\infty}\left(\frac{7}{2}\right)^{-k}\binom{\frac{1}{2}}{k} + 89\sqrt{\sqrt{\frac{7}{2}}}\sum_{k=0}^{\infty}\left(\frac{7}{2}\right)^{-k}\binom{\frac{1}{2}}{k}\right)^3\right)$$

$$1 + \exp \left[- \frac{\pi \left(1 + \frac{1}{4} \left(1 - \sqrt{\sqrt{\frac{3^2}{2}}} \right) + \left(\frac{3}{8} \right)^2 \left(1 - \sqrt{\sqrt{\frac{3^2}{2}}} \right)^2 + \left(\frac{15}{48} \right)^2 \left(1 - \sqrt{\sqrt{\frac{3^2}{2}}} \right)^3 \right)}{\left(1 + \frac{1}{4} \sqrt{\sqrt{\frac{3^2}{2}}} + \left(\frac{3}{8} \right)^2 \sqrt{\sqrt{\frac{3^2}{2}}}^2 + \left(\frac{15}{48} \right)^2 \sqrt{\sqrt{\frac{3^2}{2}}}^3 \right)} \right] =$$

$$1 + \exp \left[\left(65536 \pi \left(-381 - 111 \sqrt{\frac{7}{2}} \sum_{k=0}^{\infty} \frac{\left(-\frac{2}{7} \right)^k \binom{-1}{2}_k}{k!} + \right. \right. \right. \\ \left. \left. \left. 211 \sqrt{\sqrt{\frac{7}{2}}} \sum_{k=0}^{\infty} \frac{\left(-\frac{2}{7} \right)^k \binom{-1}{2}_k}{k!} + 25 \left(\sqrt{\frac{7}{2}} \sum_{k=0}^{\infty} \frac{\left(-\frac{2}{7} \right)^k \binom{-1}{2}_k}{k!} \right)^{3/2} \right) \right] / \\ \left(256 + 36 \sqrt{\frac{7}{2}} \sum_{k=0}^{\infty} \frac{\left(-\frac{2}{7} \right)^k \binom{-1}{2}_k}{k!} + 89 \sqrt{\sqrt{\frac{7}{2}}} \sum_{k=0}^{\infty} \frac{\left(-\frac{2}{7} \right)^k \binom{-1}{2}_k}{k!} \right)^3 \right)$$

$$1 + \exp \left[- \frac{\pi \left(1 + \frac{1}{4} \left(1 - \sqrt{\sqrt{\frac{3^2}{2}}} \right) + \left(\frac{3}{8} \right)^2 \left(1 - \sqrt{\sqrt{\frac{3^2}{2}}} \right)^2 + \left(\frac{15}{48} \right)^2 \left(1 - \sqrt{\sqrt{\frac{3^2}{2}}} \right)^3 \right)}{\left(1 + \frac{1}{4} \sqrt{\sqrt{\frac{3^2}{2}}} + \left(\frac{3}{8} \right)^2 \sqrt{\sqrt{\frac{3^2}{2}}}^2 + \left(\frac{15}{48} \right)^2 \sqrt{\sqrt{\frac{3^2}{2}}}^3 \right)} \right] = 1 +$$

$$\exp \left[\left(\left(\left(\pi \left(1 + \frac{1}{4} \left(1 - \sqrt{\sqrt{\frac{7}{2}}} \sum_{k=0}^{\infty} \frac{\left(-\frac{2}{7} \right)^k \binom{-1}{2}_k}{k!} \right) + \frac{9}{64} \left(1 - \sqrt{\sqrt{\frac{7}{2}}} \sum_{k=0}^{\infty} \frac{\left(-\frac{2}{7} \right)^k \binom{-1}{2}_k}{k!} \right)^2 \right. \right. \right. \right. \\ \left. \left. \left. \frac{25}{256} \left(1 - \sqrt{\sqrt{\frac{7}{2}}} \sum_{k=0}^{\infty} \frac{\left(-\frac{2}{7} \right)^k \binom{-1}{2}_k}{k!} \right)^3 \right) \right) \right] / \\ \left(1 + \frac{9}{64} \sqrt{\frac{7}{2}} \sum_{k=0}^{\infty} \frac{\left(-\frac{2}{7} \right)^k \binom{-1}{2}_k}{k!} + \frac{89}{256} \sqrt{\sqrt{\frac{7}{2}}} \sum_{k=0}^{\infty} \frac{\left(-\frac{2}{7} \right)^k \binom{-1}{2}_k}{k!} \right)^3 \right)$$

$\binom{n}{m}$ is the binomial coefficient

While, for $x = 0.5$, we obtain:

$$\exp\left(\frac{-\pi \left(\left(1 + \frac{1}{4}(1-0.5) + \left(\frac{3}{8}\right)^2(1-0.5)^2 + \left(\frac{15}{48}\right)^2(1-0.5)^3 \right) \right)}{\left(1 + \frac{1}{4} \times 0.5 + \left(\frac{3}{8}\right)^2 \times 0.5^2 + \left(\frac{15}{48}\right)^2 \times 0.5^2\right)}\right)$$

Input:

$$\exp\left(-\pi \times \frac{1 + \frac{1}{4}(1-0.5) + \left(\frac{3}{8}\right)^2(1-0.5)^2 + \left(\frac{15}{48}\right)^2(1-0.5)^3}{1 + \frac{1}{4} \times 0.5 + \left(\frac{3}{8}\right)^2 \times 0.5^2 + \left(\frac{15}{48}\right)^2 \times 0.5^2}\right)$$

Result:

0.0446358...

0.0446358...

From which:

$$1 / \exp\left(\frac{-\pi \left(\left(1 + \frac{1}{4}(1-0.5) + \left(\frac{3}{8}\right)^2(1-0.5)^2 + \left(\frac{15}{48}\right)^2(1-0.5)^3 \right) \right)}{\left(1 + \frac{1}{4} \times 0.5 + \left(\frac{3}{8}\right)^2 \times 0.5^2 + \left(\frac{15}{48}\right)^2 \times 0.5^2\right)}\right) + \text{golden ratio}$$

Input:

$$\frac{1}{\exp\left(-\pi \times \frac{1 + \frac{1}{4}(1-0.5) + \left(\frac{3}{8}\right)^2(1-0.5)^2 + \left(\frac{15}{48}\right)^2(1-0.5)^3}{1 + \frac{1}{4} \times 0.5 + \left(\frac{3}{8}\right)^2 \times 0.5^2 + \left(\frac{15}{48}\right)^2 \times 0.5^2}\right)} + \phi$$

ϕ is the golden ratio

Result:

24.0216...

24.0216...

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

and:

$$6/\exp(((((-\pi * ((1+1/4(1-0.5)+(3/8)^2(1-0.5)^2+(15/48)^2(1-0.5)^3)) / ((1+1/4(0.5)+(3/8)^2 0.5^2+(15/48)^2 0.5^2)))))))+1/\text{golden ratio}$$

Input:

$$\frac{6}{\exp\left(-\pi \times \frac{1+\frac{1}{4}(1-0.5)+\left(\frac{3}{8}\right)^2(1-0.5)^2+\left(\frac{15}{48}\right)^2(1-0.5)^3}{1+\frac{1}{4} \times 0.5+\left(\frac{3}{8}\right)^2 \times 0.5^2+\left(\frac{15}{48}\right)^2 \times 0.5^2}\right)} + \frac{1}{\phi}$$

ϕ is the golden ratio

Result:

135.039...

135.039... \approx 138 (Ramanujan taxicab number)

$$6/\exp(((((-\pi * ((1+1/4(1-0.5)+(3/8)^2(1-0.5)^2+(15/48)^2(1-0.5)^3)) / ((1+1/4(0.5)+(3/8)^2 0.5^2+(15/48)^2 0.5^2)))))))+1/\text{golden ratio} + 3$$

Input:

$$\frac{6}{\exp\left(-\pi \times \frac{1+\frac{1}{4}(1-0.5)+\left(\frac{3}{8}\right)^2(1-0.5)^2+\left(\frac{15}{48}\right)^2(1-0.5)^3}{1+\frac{1}{4} \times 0.5+\left(\frac{3}{8}\right)^2 \times 0.5^2+\left(\frac{15}{48}\right)^2 \times 0.5^2}\right)} + \frac{1}{\phi} + 3$$

ϕ is the golden ratio

Result:

138.039...

138.039... \approx 138 (Ramanujan taxicab number)

$$6/\exp(((((-\pi * ((1+1/4(1-0.5)+(3/8)^2(1-0.5)^2+(15/48)^2(1-0.5)^3)) / ((1+1/4(0.5)+(3/8)^2 0.5^2+(15/48)^2 0.5^2)))))))+1/\text{golden ratio} + 3 + 34$$

Input:

$$\frac{6}{\exp\left(-\pi \times \frac{1+\frac{1}{4}(1-0.5)+\left(\frac{3}{8}\right)^2(1-0.5)^2+\left(\frac{15}{48}\right)^2(1-0.5)^3}{1+\frac{1}{4} \times 0.5+\left(\frac{3}{8}\right)^2 \times 0.5^2+\left(\frac{15}{48}\right)^2 \times 0.5^2}\right)} + \frac{1}{\phi} + 3 + 34$$

ϕ is the golden ratio

Result:

172.039...

172.039... \approx 172 (Ramanujan taxicab number)

$$27 \times \frac{1}{2} \left(\frac{6}{\exp\left(\frac{-\pi \left(\left(1 + \frac{1}{4}(1-0.5) + \left(\frac{3}{8}\right)^2(1-0.5)^2 + \left(\frac{15}{48}\right)^2(1-0.5)^3 \right)}{\left(1 + \frac{1}{4}(0.5) + \left(\frac{3}{8}\right)^2 \cdot 0.5^2 + \left(\frac{15}{48}\right)^2 \cdot 0.5^2 \right)}\right)}\right)} - 7 + \frac{1}{\phi} \right) + \frac{1}{2}$$

Input:

$$27 \times \frac{1}{2} \left(\frac{6}{\exp\left(-\pi \times \frac{1 + \frac{1}{4}(1-0.5) + \left(\frac{3}{8}\right)^2(1-0.5)^2 + \left(\frac{15}{48}\right)^2(1-0.5)^3}{1 + \frac{1}{4} \times 0.5 + \left(\frac{3}{8}\right)^2 \times 0.5^2 + \left(\frac{15}{48}\right)^2 \times 0.5^2}\right)} - 7 + \frac{1}{\phi} \right) + \frac{1}{2}$$

 ϕ is the golden ratio**Result:**

1729.03...

1729.03...

Jf

$$(i) \frac{1+53x+9x^2}{1-82x-82x^2+x^3} = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$$
$$\text{or } \frac{\alpha_0}{x} + \frac{\alpha_1}{x^2} + \frac{\alpha_2}{x^3} + \dots$$

$$(ii) \frac{2-26x-12x^2}{1-82x-82x^2+x^3} = b_0 + b_1x + b_2x^2 + b_3x^3 + \dots$$
$$\text{or } \frac{\beta_0}{x} + \frac{\beta_1}{x^2} + \frac{\beta_2}{x^3} + \dots$$

$$(iii) \frac{2+8x-10x^2}{1-82x-82x^2+x^3} = c_0 + c_1x + c_2x^2 + c_3x^3 + \dots$$
$$\text{or } \frac{\gamma_0}{x} + \frac{\gamma_1}{x^2} + \frac{\gamma_2}{x^3} + \dots$$

then

$$\left. \begin{aligned} a_n^3 + b_n^3 &= c_n^3 + (-1)^n \\ \text{and } d_n^3 + \beta_n^3 &= \gamma_n^3 + (-1)^n \end{aligned} \right\}$$

Examples

$$135^3 + 138^3 = 172^3 - 1$$

$$11161^3 + 11468^3 = 14258^3 + 1$$

$$791^3 + 812^3 = 1010^3 - 1$$

$$9^3 + 10^3 = 12^3 + 1$$

$$6^3 + 8^3 = 9^3 - 1$$

<https://plus.maths.org/content/ramanujan>

Ramanujan's manuscript. The representations of 1729 as the sum of two cubes appear in the bottom right corner. The equation expressing the near counter examples to Fermat's last theorem appears further up: $\alpha^3 + \beta^3 = \gamma^3 + (-1)^n$. Image courtesy Trinity College library.

Observations

From:

https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that $p(9) = 30$, $p(9 + 5) = 135$, $p(9 + 10) = 490$, $p(9 + 15) = 1,575$ and so on are all divisible by 5. Note that here the n 's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of $p(n)$ that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n 's separated by $5^3 = 125$ units, saying that the corresponding $p(n)$'s should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ . The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $T = 0$ and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \dots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and $4096 = 64^2$

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π , ϕ , $1/\phi$, the Fibonacci and Lucas numbers, linked to the

golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n , form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the n th Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.

Fibonacci numbers are also closely related to Lucas numbers, in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.^[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803.....

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the

second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ , the golden ratio.^[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies^[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **$f_0(1710)$ scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson π) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

References

Gravitational Collapse in Einstein Dilaton Gauss-Bonnet Gravity

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