On the Ramanujan's elliptic integrals and BH-Wormholes equations: further mathematical connections with ϕ, $\zeta(2)$, and several parameters of High Energy Physics. IV

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Abstract

In this paper we have described some Ramanujan incomplete elliptic integrals and Black Holes-Wormholes formulas. Furthermore, we describe new possible mathematical connections with ϕ, $\zeta(2)$, and various parameters of High Energy Physics

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https://www.cse.iitk.ac.in/users/amit/books/hardy-1999-ramanujan-twelve-lectures.html

From:

On a New Approach for Constructing Wormholes in Einstein-Born-Infeld Gravity - *Jin Young Kim and Mu-In Park* - arXiv:1608.00445v3 [hep-th] 10 Oct 2016

We have that:

$$f = 1 - \frac{2C}{r} - \frac{\Lambda}{3}r^2 + \frac{2\beta^2}{r}\int_0^r r^2\left(1 - \sqrt{1 + \frac{Q^2}{\beta^2 r^4}}\right) dr, \qquad (13)$$

where C is an integration constant.

With a straightforward integration, one can express the solution in a compact form in terms of the incomplete elliptic integral of the first kind,

We note that the solution is in terms of the "incomplete elliptic integral of the first kind" (see Ramanujan incomplete elliptic integrals in the next pages)

$$f = 1 - \frac{2C}{r} - \frac{\Lambda}{3}r^2 + \frac{2\beta^2}{3}r^2\left(1 - \sqrt{1 + \frac{Q^2}{\beta^2 r^4}}\right) - \frac{4}{3r}\sqrt{-i\beta Q^3}\ \text{EllipticF}\left(r\sqrt{\frac{i\beta}{Q}}, i\right)$$

We have that:

$$M \equiv C + M_0,$$

$C = M - M_0 = 4.37902 * 10^{31} - 2.7638913425379 =$
$4.3790199999999999999999999999997236108657 4621 \times 10^{31}$; for $Q = 1$; $\beta = 5$ and $r = 1.94973 * 10^{13}$:

1-2*(4.3790199999e+31)/(1.94973e+13)- 1/3(1.1056e-52)(1.94973e+13)^2+50/3*(1.94973e+13)^2*(1-(1+1/(25*(1.94973e+13)^4))^0.5)-4/(3*1.94973e+13)*sqrt(-5i)

Input interpretation:

$$1 - 2\times\frac{4.3790199999\times10^{31}}{1.94973\times10^{13}} - \frac{1}{3}\times1.1056\times10^{-52}\,(1.94973\times10^{13})^2 + \frac{50}{3}(1.94973\times10^{13})^2\left(1 - \sqrt{1 + \frac{1}{25\,(1.94973\times10^{13})^4}}\right) - \frac{4}{3\times1.94973\times10^{13}}\sqrt{-5\,i}$$

i is the imaginary unit

Result:

$-4.49192... \times 10^{18} + 1.08127... \times 10^{-13}\, i$

Polar coordinates:

$r = 4.49192\times10^{18}$ (radius), $\theta = 180°$ (angle)

Alternate form:

-4.49192×10^{18}

***f* = 4.49192 * 10^18^**

From which:

(((1-2*(4.3790199999e+31)/(1.94973e+13)- 1/3(1.1056e-52)(1.94973e+13)^2+50/3*(1.94973e+13)^2*(1-(1+1/(25*(1.94973e+13)^4))^0.5)-4/(3*1.94973e+13)*sqrt(-5i)))^1/89-2/10^3

Input interpretation:

$$\left(1-2\times\frac{4.3790199999\times 10^{31}}{1.94973\times 10^{13}}-\frac{1}{3}\times 1.1056\times 10^{-52}\,(1.94973\times 10^{13})^2+\frac{50}{3}\,(1.94973\times 10^{13})^2\left(1-\sqrt{1+\frac{1}{25\,(1.94973\times 10^{13})^4}}\right)-\frac{4}{3\times 1.94973\times 10^{13}}\sqrt{-5\,i}\right)\hat{}\,(1/89)-\frac{2}{10^3}$$

i is the imaginary unit

Result:

1.617225... +
0.05718045... *i*

Polar coordinates:

$r = 1.61824$ (radius), $\theta = 2.02497°$ (angle)

1.61824

We have also, for r = 0.6160254037844… β = Q = 1

1-2*(4.3790199999e+31)/(0.6160254)- 1/3(1.1056e-52)(0.6160254)^2+2/3*(0.6160254)^2*((1-(1+1/(0.6160254)^4))^0.5)-4/(3*0.6160254)*sqrt(-i)

Input interpretation:

$$1 - 2\times\frac{4.3790199999\times 10^{31}}{0.6160254} - \frac{1}{3}\times 1.1056\times 10^{-52}\times 0.6160254^2 + \frac{2}{3}\times 0.6160254^2\sqrt{1-\left(1+\frac{1}{0.6160254^4}\right)} - \frac{4}{3\times 0.6160254}\sqrt{-i}$$

i is the imaginary unit

Result:

$-1.421701... \times 10^{32} + 2.197138...\, i$

Alternate form:

-1.4217×10^{32}

$f = -1.4217 * 10^{32}$

From which:

(((1-2*(4.3790199999e+31)/(0.6160254)- 1/3(1.1056e-52)(0.6160254)^2+2/3*(0.6160254)^2*((1-(1+1/(0.6160254)^4))^0.5)-4/(3*0.6160254)*sqrt(-i))))^1/154

Input interpretation:

$$\left(1 - 2\times\frac{4.3790199999\times 10^{31}}{0.6160254} - \frac{1}{3}\times 1.1056\times 10^{-52}\times 0.6160254^2 + \frac{2}{3}\times 0.6160254^2\sqrt{1-\left(1+\frac{1}{0.6160254^4}\right)} - \frac{4}{3\times 0.6160254}\sqrt{-i}\right)^{\wedge}(1/154)$$

i is the imaginary unit

Result:

$1.61694071... + 0.0329900899...\, i$

Polar coordinates:

$r = 1.61728$ (radius), $\theta = 1.16883°$ (angle)

1.61728

Now, we have that (see our previous paper "*On Some Incomplete Elliptic Integrals and Black Holes-Wormholes Formulas: New Possible Mathematical Connections with ϕ, $\zeta(2)$, and Various Parameters of Particle Physics. III*")

The solution (14) or (16) has a curvature singularity at the origin, $r = 0$,

$$R = \frac{4\beta Q}{r^2} + 4(\Lambda - 2\beta^2) + \frac{6\beta^3}{Q}r^2 + \mathcal{O}(r^4),$$

$$R^{\mu\nu}R_{\mu\nu} = \frac{8\beta^2 Q^2}{r^4} + \frac{8\beta Q(\Lambda - 2\beta^2)}{r^2} + 4(\Lambda^2 - 4\Lambda\beta^2 + 6\beta^4) + \mathcal{O}(r^2),$$

$$R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta} = \frac{48C^2}{r^6} + \frac{32\beta QC}{r^5} + \frac{16\beta^2 Q^2}{r^4} + \mathcal{O}(r^{-2}). \quad (20)$$

$C = M - M_0 = 4.37902 * 10^{31} - 2.7638913425379 = 4.379019999 \times 10^{31}$; $Q = 1$; $\beta = 5$; $r = 1.94973e+13$ and $\Lambda = 1.1056e-52$, we obtain:

a)

$$R = \frac{4\beta Q}{r^2} + 4(\Lambda - 2\beta^2) + \frac{6\beta^3}{Q}r^2 + \mathcal{O}(r^4),$$

(4*5)/(1.94973e+13)^2 + 4((1.1056e-52)-2*25) + 6*5^3*(1.94973e+13)^2+(1.94973e+13)^4

Input interpretation:

$$\frac{4\times 5}{(1.94973\times 10^{13})^2} + 4\,(1.1056\times 10^{-52} - 2\times 25) + 6\times 5^3\,(1.94973\times 10^{13})^2 + (1.94973\times 10^{13})^4$$

Result:

$1.445099984805997791441002851085304674999999999999980... \times 10^{53}$

1.4450999848....*10^53

b)

$$R^{\mu\nu}R_{\mu\nu} = \frac{8\beta^2 Q^2}{r^4} + \frac{8\beta Q(\Lambda - 2\beta^2)}{r^2} + 4(\Lambda^2 - 4\Lambda\beta^2 + 6\beta^4) + \mathcal{O}(r^2)$$

(8*25)/(1.94973e+13)^4+((8*5)(((1.1056e-52) – 2*25)))/(1.94973e+13)^2+4((((1.1056e-52)^2-4(1.1056e-52)*25+6*5^4)))+(1.94973e+13)^2

Input interpretation:

$$\frac{8\times 25}{(1.94973\times 10^{13})^4}+\frac{(8\times 5)(1.1056\times 10^{-52}-2\times 25)}{(1.94973\times 10^{13})^2}+4\left((1.1056\times 10^{-52})^2-4\times 1.1056\times 10^{-52}\times 25+6\times 5^4\right)+(1.94973\times 10^{13})^2$$

Result:

$3.80144707290000000000014999999999999999999999999999473\ldots\times 10^{26}$

3.8014470729....*10^26

c)

$$R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}=\frac{48C^2}{r^6}+\frac{32\beta QC}{r^5}+\frac{16\beta^2Q^2}{r^4}+\mathcal{O}(r^{-2}).$$

(48*(4.379019999e+31)^2)/(1.94973e+13)^6 + (32*5*4.379019999e+31)/(1.94973e+13)^5+(16*25)/(1.94973e+13)^4+(1.94973e+13)^-2

Input interpretation:

$$\frac{48\,(4.379019999\times 10^{31})^2}{(1.94973\times 10^{13})^6}+\frac{32\times 5\times 4.379019999\times 10^{31}}{(1.94973\times 10^{13})^5}+\frac{16\times 25}{(1.94973\times 10^{13})^4}+\frac{1}{(1.94973\times 10^{13})^2}$$

Result:

$1.675514728855212128034094315066321385687013290432012\ldots\times 10^{-15}$

1.67551472885....*10^-15

We have from the ratio between the first and the second equation:

1.4450999848059977914 41 × 10^53 / ((((8*25)/(1.94973e+13)^4+((8*5)(((1.1056e-52) – 2*25)))/(1.94973e+13)^2+4((((1.1056e-52)^2-4(1.1056e-52)*25+6*5^4)))+(1.94973e+13)^2)))

Input interpretation:

$$(1.4450999848059977914441 \times 10^{53}) / \left(\frac{8 \times 25}{(1.94973 \times 10^{13})^4} + \frac{(8 \times 5)(1.1056 \times 10^{-52} - 2 \times 25)}{(1.94973 \times 10^{13})^2} + 4((1.1056 \times 10^{-52})^2 - 4 \times 1.1056 \times 10^{-52} \times 25 + 6 \times 5^4) + (1.94973 \times 10^{13})^2\right)$$

Result:

$3.801447072899999999999985000000000000000000000059188513... \times 10^{26}$

$3.80144707289....*10^{26}$

Multiplying the result by the third equation, we obtain:

3.8014470728999999999985 × 10^26 ((((48*(4.379019999e+31)^2)/(1.94973e+13)^6 + (32*5*4.379019999e+31)/(1.94973e+13)^5+(16*25)/(1.94973e+13)^4+(1.94973e+13)^-2)))

Input interpretation:

$$3.8014470728999999999985 \times 10^{26} \left(\frac{48(4.379019999 \times 10^{31})^2}{(1.94973 \times 10^{13})^6} + \frac{32 \times 5 \times 4.379019999 \times 10^{31}}{(1.94973 \times 10^{13})^5} + \frac{16 \times 25}{(1.94973 \times 10^{13})^4} + \frac{1}{(1.94973 \times 10^{13})^2}\right)$$

Result:

$6.369380561607483312023536538202069519171363514308 9131... \times 10^{11}$

$6.369380561607483....*10^{11}$

From which:

8+1/5((((3.8014470728999999999985 × 10^26 ((((48*(4.379019999e+31)^2)/(1.94973e+13)^6 + (32*5*4.379019999e+31)/(1.94973e+13)^5+(16*25)/(1.94973e+13)^4+(1.94973e+13)^-2)))))))^1/3

Input interpretation:

$$8+\frac{1}{5}\left(3.801447072899999999999985\times 10^{26}\left(\frac{48\,(4.379019999\times 10^{31})^2}{(1.94973\times 10^{13})^6}+\frac{32\times 5\times 4.379019999\times 10^{31}}{(1.94973\times 10^{13})^5}+\frac{16\times 25}{(1.94973\times 10^{13})^4}+\frac{1}{(1.94973\times 10^{13})^2}\right)\right)\hat{}\,(1/3)$$

Result:

1728.79...

1728.79...

5ln((((3.801447072899999999999985 × 10^26 ((((48*(4.379019999e+31)^2)/(1.94973e+13)^6 + (32*5*4.379019999e+31)/(1.94973e+13)^5+(16*25)/(1.94973e+13)^4+(1.94973e+13)^-2)))))))+Pi+1/golden ratio

Input interpretation:

$$5\log\left(3.801447072899999999999985\times 10^{26}\left(\frac{48\,(4.379019999\times 10^{31})^2}{(1.94973\times 10^{13})^6}+\frac{32\times 5\times 4.379019999\times 10^{31}}{(1.94973\times 10^{13})^5}+\frac{16\times 25}{(1.94973\times 10^{13})^4}+\frac{1}{(1.94973\times 10^{13})^2}\right)\right)+\pi+\frac{1}{\phi}$$

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

Result:

139.659...

139.659…

5ln((((3.8014470728999999999999985 × 10^26 ((((48*(4.379019999e+31)^2)/(1.94973e+13)^6 + (32*5*4.379019999e+31)/(1.94973e+13)^5+(16*25)/(1.94973e+13)^4+(1.94973e+13)^-2)))))))-11+1/golden ratio

Input interpretation:

$$5 \log\left(3.8014470728999999999999985 \times 10^{26} \left(\frac{48\,(4.379019999 \times 10^{31})^2}{(1.94973 \times 10^{13})^6} + \frac{32 \times 5 \times 4.379019999 \times 10^{31}}{(1.94973 \times 10^{13})^5} + \frac{16 \times 25}{(1.94973 \times 10^{13})^4} + \frac{1}{(1.94973 \times 10^{13})^2}\right)\right) - 11 + \frac{1}{\phi}$$

log(x) is the natural logarithm

ϕ is the golden ratio

Result:

125.518...

125.518…

((((3.8014470728999999999999985 × 10^26 ((((48*(4.379019999e+31)^2)/(1.94973e+13)^6 + (32*5*4.379019999e+31)/(1.94973e+13)^5+(16*25)/(1.94973e+13)^4+(1.94973e+13)^-2)))))))^1/57+7/10^3

Input interpretation:

$$\left(3.8014470728999999999999985 \times 10^{26} \left(\frac{48\,(4.379019999 \times 10^{31})^2}{(1.94973 \times 10^{13})^6} + \frac{32 \times 5 \times 4.379019999 \times 10^{31}}{(1.94973 \times 10^{13})^5} + \frac{16 \times 25}{(1.94973 \times 10^{13})^4} + \frac{1}{(1.94973 \times 10^{13})^2}\right)\right)\text{^}(1/57) + \frac{7}{10^3}$$

Result:

1.617977313481417230268499452065026250232924119939848466412...

1.61797731348….

Now, multiplying the results of the three equations, we obtain:

1.44509998480599779144 1 × 10^53 ((((8*25)/(1.94973e+13)^4+((8*5)(((1.1056e-52) – 2*25)))/(1.94973e+13)^2+4((((1.1056e-52)^2-4(1.1056e-52)*25+6*5^4)))+(1.94973e+13)^2)))

Input interpretation:

$$1.445099984805997791441\times 10^{53}\left(\frac{8\times 25}{(1.94973\times 10^{13})^4}+\frac{(8\times 5)(1.1056\times 10^{-52}-2\times 25)}{(1.94973\times 10^{13})^2}+4\left((1.1056\times 10^{-52})^2-4\times 1.1056\times 10^{-52}\times 25+6\times 5^4\right)+(1.94973\times 10^{13})^2\right)$$

Result:

$5.493471107288594778637470888046620899668716149999 9239... \times 10^{79}$

5.49347110728…*10^{79}

5.493471107288594778 × 10^79 ((((48*(4.379019999e+31)^2)/(1.94973e+13)^6 + (32*5*4.379019999e+31)/(1.94973e+13)^5+(16*25)/(1.94973e+13)^4+(1.94973e+13)^-2)))

Input interpretation:

$$5.493471107288594778\times 10^{79}\left(\frac{48(4.379019999\times 10^{31})^2}{(1.94973\times 10^{13})^6}+\frac{32\times 5\times 4.379019999\times 10^{31}}{(1.94973\times 10^{13})^5}+\frac{16\times 25}{(1.94973\times 10^{13})^4}+\frac{1}{(1.94973\times 10^{13})^2}\right)$$

Result:

$9.204391752802591812920981968962698346335437931692 2584... \times 10^{64}$

9.2043917528...*10^{64} (final result)

Now, we have that:

12ln(((5.493471107288594778 × 10^79 ((((48*(4.379019999e+31)^2)/(1.94973e+13)^6 + (32*5*4.379019999e+31)/(1.94973e+13)^5+(16*25)/(1.94973e+13)^4+(1.94973e+13)^-2))))))-64-2

Input interpretation:

$$12 \log\left(5.493471107288594778 \times 10^{79} \left(\frac{48 (4.379019999 \times 10^{31})^2}{(1.94973 \times 10^{13})^6} + \frac{32 \times 5 \times 4.379019999 \times 10^{31}}{(1.94973 \times 10^{13})^5} + \frac{16 \times 25}{(1.94973 \times 10^{13})^4} + \frac{1}{(1.94973 \times 10^{13})^2}\right)\right) - 64 - 2$$

log(x) is the natural logarithm

Result:

1729.022...

1729.022…

ln(((5.493471107288594778 × 10^79 ((((48*(4.379019999e+31)^2)/(1.94973e+13)^6 + (32*5*4.379019999e+31)/(1.94973e+13)^5+(16*25)/(1.94973e+13)^4+(1.94973e+13)^-2))))))-8-2

Input interpretation:

$$\log\left(5.493471107288594778 \times 10^{79} \left(\frac{48 (4.379019999 \times 10^{31})^2}{(1.94973 \times 10^{13})^6} + \frac{32 \times 5 \times 4.379019999 \times 10^{31}}{(1.94973 \times 10^{13})^5} + \frac{16 \times 25}{(1.94973 \times 10^{13})^4} + \frac{1}{(1.94973 \times 10^{13})^2}\right)\right) - 8 - 2$$

log(x) is the natural logarithm

Result:

139.5851...

139.5851…

ln(((5.493471107288594778 × 10^79 ((((48*(4.379019999e+31)^2)/(1.94973e+13)^6 + (32*5*4.379019999e+31)/(1.94973e+13)^5+(16*25)/(1.94973e+13)^4+(1.94973e+13)^-2))))))-24

Input interpretation:

$$\log\left(5.493471107288594778\times10^{79}\left(\frac{48\,(4.379019999\times10^{31})^2}{(1.94973\times10^{13})^6}+\frac{32\times5\times4.379019999\times10^{31}}{(1.94973\times10^{13})^5}+\frac{16\times25}{(1.94973\times10^{13})^4}+\frac{1}{(1.94973\times10^{13})^2}\right)\right)-24$$

log(x) is the natural logarithm

Result:

125.5851...

125.5851…

(((5.493471107288594778 × 10^79 ((((48*(4.379019999e+31)^2)/(1.94973e+13)^6 +(32*5*4.379019999e+31)/(1.94973e+13)^5+(16*25)/(1.94973e+13)^4+(1.94973e+13)^-2))))))^1/311

Input interpretation:

$$\left(5.493471107288594778\times10^{79}\left(\frac{48\,(4.379019999\times10^{31})^2}{(1.94973\times10^{13})^6}+\frac{32\times5\times4.379019999\times10^{31}}{(1.94973\times10^{13})^5}+\frac{16\times25}{(1.94973\times10^{13})^4}+\frac{1}{(1.94973\times10^{13})^2}\right)\right)^{\wedge}(1/311)$$

Result:

1.617660734680411365521440032929994720082533837598092348981...

1.61766073468….

From

George E. Andrews Bruce C. Berndt
Ramanujan's Lost Notebook Part I - 2005 Springer Science+Business Media, Inc.

We have that:

Entry 15.4.1 (p. 52). *With* $f(-q)$, $\psi(q)$, *and* $u(q)$ *defined by* (15.2.4), (15.2.3), *and* (15.2.11), *respectively, and with* $\epsilon = (\sqrt{5}+1)/2$,

$$5^{3/4}\int_0^q \frac{f^2(-t)f^2(-t^5)}{\sqrt{t}}dt = 2\int_{\cos^{-1}\left((\epsilon u)^{5/2}\right)}^{\pi/2} \frac{d\varphi}{\sqrt{1-\epsilon^{-5}5^{-3/2}\sin^2\varphi}} \tag{15.4.1}$$

$$= \int_0^{2\tan^{-1}\left(5^{3/4}\sqrt{q}f^3(-q^5)/f^3(-q)\right)} \frac{d\varphi}{\sqrt{1-\epsilon^{-5}5^{-3/2}\sin^2\varphi}} \tag{15.4.2}$$

$$= \sqrt{5}\int_0^{2\tan^{-1}\left(5^{1/4}\sqrt{q}\psi(q^5)/\psi(q)\right)} \frac{d\varphi}{\sqrt{1-\epsilon 5^{-1/2}\sin^2\varphi}}. \tag{15.4.3}$$

Proof of (15.4.1). Let

$$\cos^2\varphi = \epsilon^5 u^5(t). \tag{15.4.4}$$

If $t=0$, then $\varphi=\pi/2$; if $t=q$, then $\varphi=\cos^{-1}\left((\epsilon u)^{5/2}\right)$. Upon differentiation and the use of Lemma 15.4.1,

$$\begin{aligned}2\cos\varphi(-\sin\varphi)\frac{d\varphi}{dt} &= 5\epsilon^5 u^4(t)u'(t)\\ &= \epsilon^5\frac{u^5(t)}{t}\frac{f^5(-t)}{f(-t^5)} = \cos^2\varphi\frac{f^5(-t)}{tf(-t^5)}.\end{aligned} \tag{15.4.5}$$

Hence, by (15.4.5), (15.2.13), and (15.4.4),

$$\begin{aligned}&5^{3/4}\int_0^q \frac{f^2(-t)f^2(-t^5)}{\sqrt{t}}dt\\ &= 5^{3/4}\int_{\pi/2}^{\cos^{-1}\left((\epsilon u)^{5/2}\right)} \frac{f^2(-t)f^2(-t^5)}{\sqrt{t}}\frac{-2tf(-t^5)}{f^5(-t)}\frac{\sin\varphi}{\cos\varphi}d\varphi\\ &= 2\cdot 5^{3/4}\int_{\cos^{-1}\left((\epsilon u)^{5/2}\right)}^{\pi/2}\sqrt{t}\frac{f^3(-t^5)}{f^3(-t)}\frac{\sin\varphi}{\cos\varphi}d\varphi\\ &= 2\cdot 5^{3/4}\int_{\cos^{-1}\left((\epsilon u)^{5/2}\right)}^{\pi/2}\frac{1}{\sqrt{1/u^5(t)-11-u^5(t)}}\frac{\sin\varphi}{\cos\varphi}d\varphi\\ &= 2\cdot 5^{3/4}\int_{\cos^{-1}\left((\epsilon u)^{5/2}\right)}^{\pi/2}\frac{\sin\varphi}{\sqrt{\epsilon^5-11\cos^2\varphi-\epsilon^{-5}\cos^4\varphi}}d\varphi.\end{aligned} \tag{15.4.6}$$

Since $\epsilon^{\pm 5}=(5\sqrt{5}\pm 11)/2$,

$$\begin{aligned}\epsilon^5-11\cos^2\varphi-\epsilon^{-5}\cos^4\varphi &= \epsilon^5-11(1-\sin^2\varphi)-\epsilon^{-5}\cos^4\varphi\\ &= \epsilon^{-5}+11\sin^2\varphi-\epsilon^{-5}\cos^4\varphi\\ &= \epsilon^{-5}(1-\cos^2\varphi)(1+\cos^2\varphi)+11\sin^2\varphi\\ &= \epsilon^{-5}\sin^2\varphi(2-\sin^2\varphi)+11\sin^2\varphi\\ &= \sin^2\varphi(2\epsilon^{-5}+11-\epsilon^{-5}\sin^2\varphi)\\ &= \sin^2\varphi(5\sqrt{5}-\epsilon^{-5}\sin^2\varphi)\\ &= 5\sqrt{5}\sin^2\varphi(1-\epsilon^{-5}5^{-3/2}\sin^2\varphi).\end{aligned}$$

Thus, from (15.4.6),

$$5^{3/4}\int_0^q \frac{f^2(-t)f^2(-t^5)}{\sqrt{t}}dt = 2\int_{\cos^{-1}\left((\epsilon u)^{5/2}\right)}^{\pi/2}\frac{d\varphi}{\sqrt{1-\epsilon^{-5}5^{-3/2}\sin^2\varphi}},$$

which is (15.4.1). ∎

We observe that:

$$5^{3/4}\int_0^q \frac{f^2(-t)f^2(-t^5)}{\sqrt{t}}dt$$
$$= 2\cdot 5^{3/4}\int_{\cos^{-1}\left((\epsilon u)^{5/2}\right)}^{\pi/2} \frac{\sin\varphi}{\sqrt{\epsilon^5 - 11\cos^2\varphi - \epsilon^{-5}\cos^4\varphi}}d\varphi.$$

and:

$$\epsilon^5 - 11\cos^2\varphi - \epsilon^{-5}\cos^4\varphi \qquad = 5\sqrt{5}\sin^2\varphi(1-\epsilon^{-5}5^{-3/2}\sin^2\varphi).$$

From which:

((5sqrt5)) *((sin^2(Pi/2)))*[1-((sqrt5+1)/2)^(-5)*5^(-3/2) sin^2(Pi/2)]

Input:

$$\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\sin^2\left(\frac{\pi}{2}\right)\right)$$

Exact result:

$$5\sqrt{5}\left(1-\frac{32}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)$$

Decimal approximation:

11.090169943749474241022934171828190588601545899028814310677...

11.0901699437…

Alternate forms:

$$\frac{1}{2}\left(11+5\sqrt{5}\right)$$

$$\frac{5\sqrt{5}}{2}+\frac{11}{2}$$

$$5\sqrt{5}-\frac{32}{\left(1+\sqrt{5}\right)^5}$$

Minimal polynomial:

$$x^2-11x-1$$

Alternative representations:

$$\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5} = 5\cos^2(0)\left(1-\frac{\cos^2(0)}{5^{3/2}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^5}\right)\sqrt{5}$$

$$\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5} = 5\cosh^2(0)\left(1-\frac{\cosh^2(0)}{5^{3/2}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^5}\right)\sqrt{5}$$

$$\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5} = 5\left(\frac{1}{\sec(0)}\right)^2\left(1-\frac{\left(\frac{1}{\sec(0)}\right)^2}{5^{3/2}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^5}\right)\sqrt{5}$$

Series representations:

$$\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5} = \left(4\sqrt{4}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\left(25-128\sqrt{5}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2+125\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+250\sqrt{4}^2\left(\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^2+250\sqrt{4}^3\left(\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^3+125\sqrt{4}^4\left(\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^4+25\sqrt{4}^5\left(\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^5\right)\right)\Big/\left(5\left(1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^5\right)$$

$$\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5} = \left(4\sqrt{4}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\left(25-128\sqrt{5}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2+125\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+250\sqrt{4}^2\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2+250\sqrt{4}^3\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^3+125\sqrt{4}^4\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^4+25\sqrt{4}^5\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^5\right)\right)\Big/\left(5\left(1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^5\right)$$

$$\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5} =$$

$$\left(4\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right.$$
$$\left(25-128\sqrt{5}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2+\right.$$
$$125\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+$$
$$250\exp^2\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2+$$
$$250\exp^3\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3+$$
$$125\exp^4\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4+$$
$$\left.\left.25\exp^5\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}^5\left(\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^5\right)\right)\Big/$$
$$\left(5\left(1+\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^5\right)$$

for $(x \in \mathbb{R}$ and $x < 0)$

Half-argument formulas:

$$\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5} =$$

$$\frac{5\sqrt{10}\sqrt{\frac{1}{2}(1-\cos(\pi))}^2\left(1-\frac{32\sqrt{2}^5\sqrt{\frac{1}{2}(1-\cos(\pi))}^2}{5\sqrt{5}\left(\sqrt{2}+\sqrt{10}\right)^5}\right)}{\sqrt{2}}$$

$$\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5} = \frac{1}{\sqrt{2}}\,5\,(-1)^{2\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}$$

$$\sqrt{10}\,\sqrt{\frac{1}{2}\,(1-\cos(\pi))}^{\,2}\left(1-\left(1+(-1)^{\lfloor -\mathrm{Re}(\pi)/(2\pi)\rfloor+\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(\pi))\right)^2$$

$$\left(1-\frac{1}{5\sqrt{5}\left(1+\frac{\sqrt{10}}{\sqrt{2}}\right)^5}\,32\,(-1)^{2\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\sqrt{\frac{1}{2}\,(1-\cos(\pi))}^{\,2}\right.$$

$$\left.\left(1-\left(1+(-1)^{\lfloor -\mathrm{Re}(\pi)/(2\pi)\rfloor+\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(\pi))\right)^2\right)$$

Multiple-argument formulas:

$$\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5} =$$

$$20\cos^2\left(\frac{\pi}{4}\right)\sin^2\left(\frac{\pi}{4}\right)\sqrt{5} - \frac{512\cos^4\left(\frac{\pi}{4}\right)\sin^4\left(\frac{\pi}{4}\right)\sqrt{5}}{\sqrt{5}\,(1+\sqrt{5})^5}$$

$$\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5} =$$

$$5\,U_{-\frac{1}{2}}(\cos(\pi))^2\sin^2(\pi)\sqrt{5} - \frac{32\,U_{-\frac{1}{2}}(\cos(\pi))^4\sin^4(\pi)\sqrt{5}}{\sqrt{5}\,(1+\sqrt{5})^5}$$

$$\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5} =$$

$$5\left(3\sin\left(\frac{\pi}{6}\right)-4\sin^3\left(\frac{\pi}{6}\right)\right)^2\sqrt{5}\left(1-\frac{32\left(-3\sin\left(\frac{\pi}{6}\right)+4\sin^3\left(\frac{\pi}{6}\right)\right)^2}{5\sqrt{5}\,(1+\sqrt{5})^5}\right)$$

We have:

$$2\cdot 5^{3/4}\int_{\cos^{-1}\left((\epsilon u)^{5/2}\right)}^{\pi/2}\frac{\sin\varphi}{\sqrt{\epsilon^5-11\cos^2\varphi-\epsilon^{-5}\cos^4\varphi}}\,d\varphi.$$

We observe that:

$$5^{3/4}\int_0^q \frac{f^2(-t)f^2(-t^5)}{\sqrt{t}}dt$$
$$= 2\cdot 5^{3/4}\int_{\cos^{-1}((\epsilon u)^{5/2})}^{\pi/2} \frac{\sin\varphi}{\sqrt{\epsilon^5 - 11\cos^2\varphi - \epsilon^{-5}\cos^4\varphi}}d\varphi.$$

and:

$$\epsilon^5 - 11\cos^2\varphi - \epsilon^{-5}\cos^4\varphi \qquad = 5\sqrt{5}\sin^2\varphi(1-\epsilon^{-5}5^{-3/2}\sin^2\varphi).$$

$$\frac{\sin\varphi}{\sqrt{\epsilon^5 - 11\cos^2\varphi - \epsilon^{-5}\cos^4\varphi}}$$

For φ = π / 2, we obtain:

((([sin(Pi/2) / sqrt(((((5sqrt5)) *((sin^2(Pi/2)))*[1-((sqrt5+1)/2)^(-5)*5^(-3/2) sin^2(Pi/2)])))])))

Input:

$$\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\sin^2\left(\frac{\pi}{2}\right)\right)}}$$

Exact result:

$$\frac{1}{5^{3/4}\sqrt{1-\frac{32}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}}}$$

Decimal approximation:

0.300283106000777607886694709948426367330636073835350226713...

0.300283106…

Alternate forms:

$$\sqrt{\frac{5\sqrt{5}}{2}-\frac{11}{2}}$$

root of $x^4 + 11x^2 - 1$ near $x = 0.300283$

$$\sqrt[4]{5}\sqrt{\frac{2}{25+11\sqrt{5}}}$$

Minimal polynomial:

x^4+11x^2-1

Alternative representations:

$$\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}=\frac{\cos(0)}{\sqrt{5\cos^2(0)\left(1-\frac{\cos^2(0)}{5^{3/2}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^5}\right)\sqrt{5}}}$$

$$\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}=\frac{\cosh(0)}{\sqrt{5\cosh^2(0)\left(1-\frac{\cosh^2(0)}{5^{3/2}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^5}\right)\sqrt{5}}}$$

$$\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}=\frac{1}{\sec(0)\sqrt{5\left(\frac{1}{\sec(0)}\right)^2\left(1-\frac{\left(\frac{1}{\sec(0)}\right)^2}{5^{3/2}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^5}\right)\sqrt{5}}}$$

Series representations:

$$\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}=$$

$$\frac{2\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)}{\sqrt{-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)^{-k}}$$

$$\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}} =$$

$$\frac{2\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)}{\sqrt{-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)^{-k}}{k!}}$$

$$\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}} =$$

$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\, 2^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)\Bigg/\left(\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)\right)^k}{k!}\right) \quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

Half-argument formulas:

$$\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}} = \frac{\sqrt{2}\sqrt{\frac{1}{2}(1-\cos(\pi))}}{\sqrt{10\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{64\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}}}$$

$$\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}} =$$

$$\frac{(-1)^{\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\sqrt{2}\sqrt{\frac{1}{2}(1-\cos(\pi))}\left(1-\left(1+(-1)^{\lfloor -\mathrm{Re}(\pi)/(2\pi)\rfloor+\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(\pi))\right)}{\sqrt{10\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}}$$

Multiple-argument formulas:

$$\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}} = \frac{3\sin\left(\frac{\pi}{6}\right)-4\sin^3\left(\frac{\pi}{6}\right)}{\sqrt{5}\sqrt{\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}}$$

$$\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}} = \frac{2\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)}{\sqrt{5}\sqrt{\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}}$$

$$\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}} = \frac{U_{-\frac{1}{2}}(\cos(\pi))\sin(\pi)}{\sqrt{5}\sqrt{\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}}$$

From which:

1/2*1/((([sin(Pi/2) 1/sqrt(((((5sqrt5)) *((sin^2(Pi/2)))*[1-((sqrt5+1)/2)^(-5)*5^(-3/2) sin^2(Pi/2)])))])))

Input:

$$\frac{1}{2}\times\frac{1}{\sin\left(\frac{\pi}{2}\right)\times\frac{1}{\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\sin^2\left(\frac{\pi}{2}\right)\right)}}}$$

Exact result:

$$\frac{1}{2}\times 5^{3/4}\sqrt{1-\frac{32}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}}$$

Decimal approximation:

1.665095338392780607287201754658970956477169070758957867449...

1.66509533839278… result very near to the 14th root of the following Ramanujan’s class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164{,}2696$ i.e. 1,65578...

Alternate forms:

$$\sqrt{\frac{11}{8} + \frac{5\sqrt{5}}{8}}$$

$$\frac{\sqrt{25 + 11\sqrt{5}}}{2\sqrt{2}\sqrt[4]{5}}$$

$$\frac{2\sqrt{275 + 123\sqrt{5}}}{\sqrt[4]{5}\,(1 + \sqrt{5})^{5/2}}$$

Minimal polynomial:

$16x^4 - 44x^2 - 1$

Alternative representations:

$$\frac{1}{\frac{\sin\left(\frac{\pi}{2}\right)2}{\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1 - \frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}}} = \frac{1}{\frac{2\cos(0)}{\sqrt{5\cos^2(0)\left(1 - \frac{\cos^2(0)}{5^{3/2}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^5}\right)\sqrt{5}}}}$$

$$\frac{1}{\frac{\sin\left(\frac{\pi}{2}\right)2}{\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1 - \frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}}} = \frac{1}{\frac{2\cosh(0)}{\sqrt{5\cosh^2(0)\left(1 - \frac{\cosh^2(0)}{5^{3/2}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^5}\right)\sqrt{5}}}}$$

$$\frac{1}{\frac{\sin\left(\frac{\pi}{2}\right)2}{\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1 - \frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}}} = \frac{1}{\frac{2}{\sec(0)\sqrt{5\left(\frac{1}{\sec(0)}\right)^2\left(1 - \frac{\left(\frac{1}{\sec(0)}\right)^2}{5^{3/2}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^5}\right)\sqrt{5}}}}$$

Series representations:

$$\frac{1}{\frac{\frac{\sin\left(\frac{\pi}{2}\right)2}{\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}}}{}} = \frac{\sqrt{-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)^{-k}}{4\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)}$$

$$\frac{1}{\frac{\frac{\sin\left(\frac{\pi}{2}\right)2}{\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}}}{}} = \frac{\sqrt{-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)^{-k}}{k!}}{4\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)}$$

$$\frac{1}{\frac{\frac{\sin\left(\frac{\pi}{2}\right)2}{\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}}}{}} = \left(\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)\right)^k}{k!}\right) \Big/ \left(4\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

Half-argument formulas:

$$\frac{1}{\frac{\sin\left(\frac{\pi}{2}\right)2}{\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}}} = \frac{\sqrt{10\sin^2\left(\frac{\pi}{2}\right)\sqrt{5} - \frac{64\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}}}{2\sqrt{2}\sqrt{\frac{1}{2}(1-\cos(\pi))}}$$

$$\frac{1}{\frac{\sin\left(\frac{\pi}{2}\right)2}{\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}}} =$$

$$\frac{(-1)^{-\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\sqrt{10\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}}{2\sqrt{2}\sqrt{\frac{1}{2}(1-\cos(\pi))}\left(1-\left(1+(-1)^{\lfloor -\mathrm{Re}(\pi)/(2\pi)\rfloor+\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(\pi))\right)}$$

Multiple-argument formulas:

$$\frac{1}{\frac{\sin\left(\frac{\pi}{2}\right)2}{\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}}} = \frac{\sqrt{5}\sqrt{\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}}{4\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)}$$

$$\frac{1}{\frac{\sin\left(\frac{\pi}{2}\right)2}{\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}}} = -\frac{\sqrt{5}\sqrt{\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}}{-6\sin\left(\frac{\pi}{6}\right)+8\sin^3\left(\frac{\pi}{6}\right)}$$

$$\frac{1}{\frac{\sin\left(\frac{\pi}{2}\right)2}{\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}}} = \frac{\sqrt{20\cos^2\left(\frac{\pi}{4}\right)\sin^2\left(\frac{\pi}{4}\right)\sqrt{5} - \frac{512\cos^4\left(\frac{\pi}{4}\right)\sin^4\left(\frac{\pi}{4}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}}}{4\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)}$$

and:

1/((([sin(Pi/2) / sqrt(((((5sqrt5)) *((sin^2(Pi/2)))*[1-((sqrt5+1)/2)^(-5)*5^(-3/2) sin^2(Pi/2)])))])))^1/15

Input:

$$\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\sin^2\left(\frac{\pi}{2}\right)\right)}}}}$$

Exact result:

$$\sqrt[20]{5}\ \sqrt[30]{1-\frac{32}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}}$$

Decimal approximation:

1.083505882173847965292569060944839635460216042424984787953...

1.08350588217… result very near to 1+1/12 = 1.083333...

Alternate forms:

$$\sqrt[6]{\frac{1}{2}\left(1+\sqrt{5}\right)}$$

$$\frac{\sqrt[30]{25+11\sqrt{5}}}{\sqrt[30]{2}\ \sqrt[60]{5}}$$

$$\frac{\sqrt[30]{4400+1968\sqrt{5}}}{\sqrt[60]{5}\ \sqrt[6]{1+\sqrt{5}}}$$

Minimal polynomial:

$$x^{12}-x^6-1$$

Alternative representations:

$$\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}}=\frac{1}{\sqrt[15]{\frac{\cos(0)}{\sqrt{5\cos^2(0)\left(1-\frac{\cos^2(0)}{5^{3/2}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^5}\right)\sqrt{5}}}}}$$

$$\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}} = \frac{1}{\sqrt[15]{\frac{\cosh(0)}{\sqrt{5\cosh^2(0)\left(1-\frac{\cosh^2(0)}{5^{3/2}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^5}\right)\sqrt{5}}}}}$$

$$\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}} = \frac{1}{\sqrt[15]{\frac{1}{\sec(0)\sqrt{5\left(\frac{1}{\sec(0)}\right)^2\left(1-\frac{\left(\frac{1}{\sec(0)}\right)^2}{5^{3/2}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^5}\right)\sqrt{5}}}}}$$

Series representations:

$$\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}} = \left(\sqrt{-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}\right.$$

$$\left.\left(\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)\Big/\left(\sqrt{-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}\right.\right.$$

$$\left.\left.\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)^{-k}\right)\right)^{14/15}$$

$$\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)^{-k}\Bigg)\Big/\left(\sqrt[15]{2}\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)$$

$$\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}} = \left(\sqrt{-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\,(1+\sqrt{5})^5}}\right.$$

$$\left(\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)\Bigg/\left(\sqrt{-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\,(1+\sqrt{5})^5}}\right.\right.$$

$$\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\,\left(1+\sqrt{5}\right)^5}\right)^{-k}}{k!}\right)\right)^{14/15}$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\,\left(1+\sqrt{5}\right)^5}\right)^{-k}}{k!}\right)\Bigg/$$

$$\left(\sqrt[15]{2}\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)$$

$$\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}} =$$

$$\left(\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}{2\pi}\right\rfloor\right)\sqrt{x}\left(\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)\Bigg/\right.\right.$$

$$\left(\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}{2\pi}\right\rfloor\right)\sqrt{x}\right.$$

$$\left.\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\,x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)^k}{k!}\right)\right)\right)^{14/15}$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\,x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)^k}{k!}\right)\Bigg/$$

$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

Half-argument formulas:

$$\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}} = \frac{1}{\sqrt[15]{\frac{\sqrt{2}\sqrt{\frac{1}{2}\left(1-\cos(\pi)\right)}}{\sqrt{10\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{64\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}}}}}$$

$$\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}} =$$

$$\frac{1}{\sqrt[15]{\frac{(-1)^{\lfloor \operatorname{Re}(\pi)/(2\pi)\rfloor}\sqrt{2}\sqrt{\frac{1}{2}(1-\cos(\pi))}\left(1-\left(1+(-1)^{\lfloor -\operatorname{Re}(\pi)/(2\pi)\rfloor+\lfloor \operatorname{Re}(\pi)/(2\pi)\rfloor}\right)\theta(-\operatorname{Im}(\pi))\right)}{\sqrt{10\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}}}}$$

Multiple-argument formulas:

$$\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}} = \frac{1}{\sqrt[15]{2}\sqrt[15]{\frac{\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)}{\sqrt{5}\sqrt{\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}}}}$$

$$\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}} = \frac{1}{\sqrt[15]{\frac{3\sin\left(\frac{\pi}{6}\right)-4\sin^3\left(\frac{\pi}{6}\right)}{\sqrt{5}\sqrt{\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}}}}$$

$$\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}} = \frac{1}{\sqrt[15]{\frac{U_{-\frac{1}{2}}(\cos(\pi))\sin(\pi)}{\sqrt{5}\sqrt{\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}}}}$$

(((1/((([sin(Pi/2) / sqrt(((((5sqrt5)) *((sin^2(Pi/2)))*[1-((sqrt5+1)/2)^(-5)*5^(-3/2) sin^2(Pi/2)])))])))^1/15)))^6

Input:

$$\left(\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\sin^2\left(\frac{\pi}{2}\right)\right)}}}}\right)^6$$

Exact result:

$$5^{3/10}\sqrt[5]{1-\frac{32}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}}$$

Decimal approximation:

1.6180339887498948482045868343656381177203091798057628621354...

1.6180339887… = golden ratio

Alternate forms:

$$\frac{1}{2}\left(1+\sqrt{5}\right)$$

$$\frac{\sqrt[5]{25+11\sqrt{5}}}{\sqrt[5]{2}\sqrt[10]{5}}$$

$$\frac{\sqrt[5]{4400+1968\sqrt{5}}}{\sqrt[10]{5}\left(1+\sqrt{5}\right)}$$

Minimal polynomial:

$$x^2-x-1$$

Alternative representations:

$$\left(\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}}\right)^6=\left(\frac{1}{\sqrt[15]{\frac{\cos(0)}{\sqrt{5\cos^2(0)\left(1-\frac{\cos^2(0)}{5^{3/2}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^5}\right)\sqrt{5}}}}}\right)^6$$

$$\left(\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}}\right)^6 = \left(\frac{1}{\sqrt[15]{\frac{\cosh(0)}{\sqrt{5\cosh^2(0)\left(1-\frac{\cosh^2(0)}{5^{3/2}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^5}\right)\sqrt{5}}}}}\right)^6$$

$$\left(\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}}\right)^6 = \left(\frac{1}{\sqrt[15]{\frac{1}{\sec(0)\sqrt{5\left(\frac{1}{\sec(0)}\right)^2\left(1-\frac{\left(\frac{1}{\sec(0)}\right)^2}{5^{3/2}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^5}\right)\sqrt{5}}}}}\right)^6$$

Series representations:

$$\left(\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}}\right)^6 = \left(\sqrt{-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}\right.$$

$$\left.\left(\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)\Big/\left(\sqrt{-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}\right.\right.\right.$$

$$\left.\left.\left.\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)^{-k}\right)\right)^{3/5}\right)$$

$$\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)^{-k}\Bigg/\left(2^{2/5}\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)$$

$$\left(\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}}\right)^6 = \left(\sqrt{-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\,(1+\sqrt{5})^5}}\right.$$

$$\left(\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)\Big/\left(\sqrt{-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\,(1+\sqrt{5})^5}}\right.\right.$$

$$\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)^{-k}}{k!}\right)\right)^{3/5}$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)^{-k}}{k!}\right)\Big/$$

$$\left(2^{2/5}\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)$$

$$\left(\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}}\right)^6 =$$

$$\left(\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}{2\pi}\right\rfloor\right)\sqrt{x}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)\Big/\right.$$

$$\left(\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}{2\pi}\right\rfloor\right)\sqrt{x}\right.$$

$$\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\,x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)^k}{k!}\right)\right)^{3/5}$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\,x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+5\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{32\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)^k}{k!}\right)\Big/$$

$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

Half-argument formulas:

$$\left(\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}}\right)^6 = \frac{1}{\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}(1-\cos(\pi))}}{\sqrt{10\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}-\frac{64\sin^4\left(\frac{\pi}{2}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}}}\right)^{2/5}}$$

$$\left(\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}}\right)^6 = \frac{1}{\left(\frac{(-1)^{\lfloor \operatorname{Re}(\pi)/(2\pi)\rfloor}\sqrt{2}\sqrt{\frac{1}{2}(1-\cos(\pi))}\left(1-\left(1+(-1)^{\lfloor -\operatorname{Re}(\pi)/(2\pi)\rfloor+\lfloor \operatorname{Re}(\pi)/(2\pi)\rfloor}\right)\theta(-\operatorname{Im}(\pi))\right)}{\sqrt{10\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}}\right)^{2/5}}$$

Multiple-argument formulas:

$$\left(\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}}\right)^6 = \frac{1}{2^{2/5}\left(\frac{\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)}{\sqrt{5}\sqrt{\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}}\right)^{2/5}}$$

$$\left(\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}}\right)^6 = \frac{1}{\left(\frac{3\sin\left(\frac{\pi}{6}\right)-4\sin^3\left(\frac{\pi}{6}\right)}{\sqrt{5}\sqrt{\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}}\right)^{2/5}}$$

$$\left(\frac{1}{\sqrt[15]{\frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}}}}}\right)^6 = \frac{1}{\left(\frac{U_{-\frac{1}{2}}(\cos(\pi))\sin(\pi)}{\sqrt{5}\sqrt{\sin^2\left(\frac{\pi}{2}\right)\sqrt{5}\left(1-\frac{32\sin^2\left(\frac{\pi}{2}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}}\right)^{2/5}}$$

We note that from:

$$\epsilon^5 - 11\cos^2\varphi - \epsilon^{-5}\cos^4\varphi \qquad = 5\sqrt{5}\sin^2\varphi(1-\epsilon^{-5}5^{-3/2}\sin^2\varphi).$$

we obtain:

(((5sqrt5)) *((sin^2(Pi/2)))*[1-((sqrt5+1)/2)^(-5)*5^(-3/2) sin^2(Pi/2)])^1/5

Input:

$$\sqrt[5]{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\sin^2\left(\frac{\pi}{2}\right)\right)}$$

Exact result:

$$5^{3/10}\sqrt[5]{1-\frac{32}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}}$$

Decimal approximation:

1.6180339887498948482045868343656381177203091798057628621354...

1.6180339887… = golden ratio

Alternate forms:

$$\frac{1}{2}\left(1+\sqrt{5}\right)$$

$$\frac{\sqrt[5]{25+11\sqrt{5}}}{\sqrt[5]{2}\sqrt[10]{5}}$$

$$\frac{\sqrt[5]{4400+1968\sqrt{5}}}{\sqrt[10]{5}\left(1+\sqrt{5}\right)}$$

Minimal polynomial:

$x^2 - x - 1$

All 5th roots of 5 sqrt(5) (1 - 32/(5 sqrt(5) (1 + sqrt(5))^5)):

$$5^{3/10} \sqrt[5]{1 - \frac{32}{5\sqrt{5}\,(1+\sqrt{5})^5}}\; e^0 \approx 1.61803 \text{ (real, principal root)}$$

$$5^{3/10} \sqrt[5]{1 - \frac{32}{5\sqrt{5}\,(1+\sqrt{5})^5}}\; e^{(2 i \pi)/5} \approx 0.5000 + 1.5388\, i$$

$$5^{3/10} \sqrt[5]{1 - \frac{32}{5\sqrt{5}\,(1+\sqrt{5})^5}}\; e^{(4 i \pi)/5} \approx -1.3090 + 0.9511\, i$$

$$5^{3/10} \sqrt[5]{1 - \frac{32}{5\sqrt{5}\,(1+\sqrt{5})^5}}\; e^{-(4 i \pi)/5} \approx -1.3090 - 0.9511\, i$$

$$5^{3/10} \sqrt[5]{1 - \frac{32}{5\sqrt{5}\,(1+\sqrt{5})^5}}\; e^{-(2 i \pi)/5} \approx 0.5000 - 1.5388\, i$$

Alternative representations:

$$\sqrt[5]{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1 - \frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right) 5\sqrt{5}} = \sqrt[5]{5\cos^2(0)\left(1 - \frac{\cos^2(0)}{5^{3/2}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^5}\right)\sqrt{5}}$$

$$\sqrt[5]{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1 - \frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right) 5\sqrt{5}} = \sqrt[5]{5\cosh^2(0)\left(1 - \frac{\cosh^2(0)}{5^{3/2}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^5}\right)\sqrt{5}}$$

$$\sqrt[5]{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1 - \frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right) 5\sqrt{5}} = \sqrt[5]{5\left(\frac{1}{\sec(0)}\right)^2\left(1 - \frac{\left(\frac{1}{\sec(0)}\right)^2}{5^{3/2}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^5}\right)\sqrt{5}}$$

Series representations:

$$\sqrt[5]{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}} =$$

$$\frac{1}{\sqrt[5]{5}}\,2^{2/5}\left(\left(\sqrt{4}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right.\right.$$
$$\left(25-128\sqrt{5}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2+125\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+\right.$$
$$250\sqrt{4}^2\left(\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^2+250\sqrt{4}^3\left(\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^3+$$
$$\left.\left.125\sqrt{4}^4\left(\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^4+25\sqrt{4}^5\left(\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^5\right)\right)\Big/$$
$$\left.\left(1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^5\right)\wedge(1/5)$$

$$\sqrt[5]{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}} =$$

$$2^{2/5}\sqrt[5]{5}\left(\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2\right.$$
$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\left(1-\left(128\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2\right)\Big/\right.$$
$$\left(5\sqrt{5}\left(1+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\right.\right.$$
$$\left.\left.\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)^5\right)\right)\right)\wedge(1/5)$$

$$\sqrt[5]{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}} =$$

$$\sqrt[5]{5}\left(\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\, 2^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\right.$$
$$\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\, z_0^{-k}}{k!}\right)\left(1-\left(32\left(\sum_{k=0}^{\infty}\frac{(-1)^k\, 2^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2\right)\right/$$
$$\left(5\sqrt{5}\left(1+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\right.\right.$$
$$\left.\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\, z_0^{-k}}{k!}\right)^5\right)\right)\right)\hat{}\,(1/5)$$

Half-argument formulas:

$$\sqrt[5]{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}} =$$

$$\sqrt[5]{5}\sqrt[5]{\frac{\sqrt{10}\sqrt{\frac{1}{2}(1-\cos(\pi))}^{\,2}\left(1-\frac{32\sqrt{\frac{1}{2}(1-\cos(\pi))}^{\,2}}{5\sqrt{5}\left(1+\frac{\sqrt{10}}{\sqrt{2}}\right)^5}\right)}{\sqrt{2}}}$$

$$\sqrt[5]{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}} =$$

$$\sqrt[5]{5}\left(\frac{1}{\sqrt{2}}(-1)^{2\lfloor\mathrm{Re}(\pi)/(2\pi)\rfloor}\sqrt{10}\sqrt{\frac{1}{2}(1-\cos(\pi))}^{\,2}\right.$$
$$\left(1-\left(1+(-1)^{\lfloor-\mathrm{Re}(\pi)/(2\pi)\rfloor+\lfloor\mathrm{Re}(\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(\pi))\right)^2$$
$$\left(1-\frac{1}{5\sqrt{5}\left(1+\frac{\sqrt{10}}{\sqrt{2}}\right)^5}\,32\,(-1)^{2\lfloor\mathrm{Re}(\pi)/(2\pi)\rfloor}\sqrt{\frac{1}{2}(1-\cos(\pi))}^{\,2}\right.$$
$$\left.\left.\left(1-\left(1+(-1)^{\lfloor-\mathrm{Re}(\pi)/(2\pi)\rfloor+\lfloor\mathrm{Re}(\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(\pi))\right)^2\right)\right)\hat{}\,(1/5)$$

Multiple-argument formulas:

$$\sqrt[5]{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}} = \sqrt[5]{5}\sqrt[5]{U_{-\frac{1}{2}}(\cos(\pi))^2\sin^2(\pi)\sqrt{5}\left(1-\frac{32\,U_{-\frac{1}{2}}(\cos(\pi))^2\sin^2(\pi)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}$$

$$\sqrt[5]{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}} = \sqrt[5]{5\,U_{-\frac{1}{2}}(\cos(\pi))^2\sin^2(\pi)\sqrt{5}-\frac{32\,U_{-\frac{1}{2}}(\cos(\pi))^4\sin^4(\pi)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}}$$

$$\sqrt[5]{\left(\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)\right)5\sqrt{5}} = 2^{2/5}\sqrt[5]{5}\sqrt[5]{\cos^2\left(\frac{\pi}{4}\right)\sin^2\left(\frac{\pi}{4}\right)\sqrt{5}\left(1-\frac{128\cos^2\left(\frac{\pi}{4}\right)\sin^2\left(\frac{\pi}{4}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}$$

We obtain also:

1/2((((((5sqrt5)) *((sin^2(Pi/2)))*[1-((sqrt5+1)/2)^(-5)*5^(-3/2) sin^2(Pi/2)]))))^1/2-21/10^3

Input:

$$\frac{1}{2}\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\sin^2\left(\frac{\pi}{2}\right)\right)}-\frac{21}{10^3}$$

Exact result:

$$\frac{1}{2}\times 5^{3/4}\sqrt{1-\frac{32}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}}-\frac{21}{1000}$$

Decimal approximation:

1.644095338392780607287201754658970956477169070758957867449...

1.64409533839278…

Alternate forms:

$$\sqrt{\frac{11}{8}+\frac{5\sqrt{5}}{8}}-\frac{21}{1000}$$

$$\frac{250\sqrt{2\left(11+5\sqrt{5}\right)}-21}{1000}$$

$$\frac{50\times 5^{3/4}\sqrt{50+22\sqrt{5}}-21}{1000}$$

Minimal polynomial:

$1\,000\,000\,000\,000\,x^4+84\,000\,000\,000\,x^3-2\,747\,354\,000\,000\,x^2-115\,462\,956\,000\,x-63\,712\,555\,519$

Alternative representations:

$$\frac{1}{2}\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}-\frac{21}{10^3}=$$
$$-\frac{21}{10^3}+\frac{1}{2}\sqrt{5\cos^2(0)\left(1-\frac{\cos^2(0)}{5^{3/2}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^5}\right)\sqrt{5}}$$

$$\frac{1}{2}\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}-\frac{21}{10^3}=$$
$$-\frac{21}{10^3}+\frac{1}{2}\sqrt{5\cosh^2(0)\left(1-\frac{\cosh^2(0)}{5^{3/2}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^5}\right)\sqrt{5}}$$

$$\frac{1}{2}\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}-\frac{21}{10^3}=$$
$$-\frac{21}{10^3}+\frac{1}{2}\sqrt{5\left(\frac{1}{\sec(0)}\right)^2\left(1-\frac{\left(\frac{1}{\sec(0)}\right)^2}{5^{3/2}\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^5}\right)\sqrt{5}}$$

Series representations:

$$\frac{1}{2}\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}-\frac{21}{10^3}=$$

$$\frac{1}{1000}\left(-21+1000\sqrt{5}\sqrt{\left(\sqrt{4}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\left(1-\frac{128\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2}{5\sqrt{5}\left(1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^5}\right)\right)}\right)$$

$$\frac{1}{2}\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}-\frac{21}{10^3}=$$

$$\frac{1}{1000}\left(-21+1000\sqrt{5}\sqrt{\left(\sqrt{4}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\left(1-\frac{128\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2}{5\sqrt{5}\left(1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^5}\right)\right)}\right)$$

$$\frac{1}{2}\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}-\frac{21}{10^3}=$$

$$\frac{1}{1000}\left(-21+1000\sqrt{5}\sqrt{\left(\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\left(1-\frac{128\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2}{5\sqrt{5}\left(1+\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^5}\right)\right)}\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

Half-argument formulas:

$$\frac{1}{2}\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}-\frac{21}{10^3}=$$

$$-\frac{21}{1000}+\frac{1}{2}\sqrt{5}\sqrt{\frac{\sqrt{10}\sqrt{\frac{1}{2}(1-\cos(\pi))}^2\left(1-\frac{32\sqrt{\frac{1}{2}(1-\cos(\pi))}^2}{5\sqrt{5}\left(1+\frac{\sqrt{10}}{\sqrt{2}}\right)^5}\right)}{\sqrt{2}}}$$

$$\frac{1}{2}\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}-\frac{21}{10^3}=$$

$$-\frac{21}{1000}+\frac{1}{2}\sqrt{5}\sqrt{\left(\frac{1}{\sqrt{2}}(-1)^{2\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\sqrt{10}\sqrt{\frac{1}{2}(1-\cos(\pi))}^2\left(1-\left(1+(-1)^{\lfloor -\mathrm{Re}(\pi)/(2\pi)\rfloor+\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(\pi))\right)^2\left(1-\frac{1}{5\sqrt{5}\left(1+\frac{\sqrt{10}}{\sqrt{2}}\right)^5}32(-1)^{2\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\sqrt{\frac{1}{2}(1-\cos(\pi))}^2\left(1-\left(1+(-1)^{\lfloor -\mathrm{Re}(\pi)/(2\pi)\rfloor+\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(\pi))\right)^2\right)\right)}$$

Multiple-argument formulas:

$$\frac{1}{2}\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}-\frac{21}{10^3}=$$

$$-\frac{21}{1000}+\sqrt{5}\sqrt{\cos^2\left(\frac{\pi}{4}\right)\sin^2\left(\frac{\pi}{4}\right)\sqrt{5}\left(1-\frac{128\cos^2\left(\frac{\pi}{4}\right)\sin^2\left(\frac{\pi}{4}\right)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}$$

$$\frac{1}{2}\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}-\frac{21}{10^3}=$$

$$-\frac{21}{1000}+\sqrt{5\cos^2\left(\frac{\pi}{4}\right)\sin^2\left(\frac{\pi}{4}\right)\sqrt{5}-\frac{128\cos^4\left(\frac{\pi}{4}\right)\sin^4\left(\frac{\pi}{4}\right)\sqrt{5}}{\sqrt{5}\left(1+\sqrt{5}\right)^5}}$$

$$\frac{1}{2}\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1-\frac{5^{-3/2}\sin^2\left(\frac{\pi}{2}\right)}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\right)}-\frac{21}{10^3}=$$

$$-\frac{21}{1000}+\frac{1}{2}\sqrt{5}\sqrt{U_{-\frac{1}{2}}(\cos(\pi))^2\sin^2(\pi)\sqrt{5}\left(1-\frac{32\,U_{-\frac{1}{2}}(\cos(\pi))^2\sin^2(\pi)}{5\sqrt{5}\left(1+\sqrt{5}\right)^5}\right)}$$

From:

$$=2\cdot 5^{3/4}\int_{\cos^{-1}\left((\epsilon u)^{5/2}\right)}^{\pi/2}\frac{\sin\varphi}{\sqrt{\epsilon^5-11\cos^2\varphi-\epsilon^{-5}\cos^4\varphi}}d\varphi.$$

cos^-1(golden ratio)^2.5

Input:

$\cos^{-1}(\phi)^{2.5}$

$\cos^{-1}(x)$ is the inverse cosine function

ϕ is the golden ratio

Result:

$-0.820455\ldots-0.820455\ldots i$

(result in radians)

Polar coordinates:

$r = 1.1603$ (radius), $\theta = -135.°$ (angle)

1.1603

From

$$2\cdot 5^{3/4}\int_{\cos^{-1}\left((\epsilon u)^{5/2}\right)}^{\pi/2}\frac{\sin\varphi}{\sqrt{\epsilon^5-11\cos^2\varphi-\epsilon^{-5}\cos^4\varphi}}d\varphi.$$

we note that from:

$$\epsilon^5 - 11\cos^2\varphi - \epsilon^{-5}\cos^4\varphi \qquad = 5\sqrt{5}\sin^2\varphi(1 - \epsilon^{-5}5^{-3/2}\sin^2\varphi).$$

We obtain:

2 * 5^(3/4) integrate ((([sin(Pi/2) / sqrt(((((5sqrt5)) *((sin^2(Pi/2)))*[1-((sqrt5+1)/2)^(-5)*5^(-3/2) sin^2(Pi/2)]))))])))Pi/2 x =cos^-1(golden ratio)^2.5..Pi/2

Input:

$$2 \times 5^{3/4} \int_{\cos^{-1}(\phi)^{2.5}}^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1 - \frac{5^{-3/2}}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\sin^2\left(\frac{\pi}{2}\right)\right)}} \times \frac{\pi}{2}\, dx$$

$\cos^{-1}(x)$ is the inverse cosine function

ϕ is the golden ratio

Result:

$7.54282 + 2.58799\, i$

Input interpretation:

$7.54282 + 2.58799\, i$

i is the imaginary unit

Result:

$7.54282... +$
$2.58799... \, i$

Polar coordinates:

$r = 7.97445$ (radius), $\theta = 18.9374°$ (angle)

7.97445

From which:

10^18* ((((-Pi-0.73) + 2 * 5^(3/4) integrate ((([sin(Pi/2) / sqrt(((((5sqrt5)) *((sin^2(Pi/2)))*[1-((sqrt5+1)/2)^(-5)*5^(-3/2) sin^2(Pi/2)]))))])))Pi/2 x =cos^-1(golden ratio)^2.5..Pi/2)))

Input:

$$10^{18}\left((-\pi - 0.73) + 2 \times 5^{3/4} \int_{\cos^{-1}(\phi)^{2.5}}^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2}\right)}{\sqrt{\left(5\sqrt{5}\right)\sin^2\left(\frac{\pi}{2}\right)\left(1 - \frac{5^{-3/2}}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\sin^2\left(\frac{\pi}{2}\right)\right)}} \times \frac{\pi}{2}\, dx\right)$$

$\cos^{-1}(x)$ is the inverse cosine function

ϕ is the golden ratio

Result:

$3.67122\times10^{18}+2.58799\times10^{18}\,i$

Input interpretation:

$3.67122\times10^{18}+2.58799\times10^{18}\,i$

i is the imaginary unit

Result:

$3.67122...\times10^{18}+$
$2.58799...\times10^{18}\,i$

Polar coordinates:

$r=4.49172\times10^{18}$ (radius), $\theta=35.1815°$ (angle)

$4.49172*10^{18}$ value practically equal to the previous result ***f* = 4.49192 * 10^{18}**

or:

2 * integrate (Pi/2) / sqrt((((1-((sqrt5+1)/2)^(-5)*5^(-3/2) sin^2(Pi/2)))))dx x =cos^-1(golden ratio)^2.5..Pi/2

Input:

$$2\int_{\cos^{-1}(\phi)^{2.5}}^{\frac{\pi}{2}}\frac{\frac{\pi}{2}}{\sqrt{1-\frac{5^{-3/2}}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\sin^2\left(\frac{\pi}{2}\right)}}\,dx$$

$\cos^{-1}(x)$ is the inverse cosine function

ϕ is the golden ratio

Result:

$7.54282+2.58799\,i$

Input interpretation:

$7.54282+2.58799\,i$

i is the imaginary unit

Result:

7.54282... +
2.58799... i

Polar coordinates:

$r = 7.97445$ (radius), $\theta = 18.9374°$ (angle)

7.97445

Now, we have:

10^17((((5+2*5 integrate (Pi/2) / sqrt((((1-((sqrt5+1)/2)^(-5)*5^(-3/2) sin^2(Pi/2)))))dx x =cos^-1(golden ratio)^2.5..Pi/2))))

Input:

$$10^{17}\left(5+2\times 5\int_{\cos^{-1}(\phi)^{2.5}}^{\frac{\pi}{2}}\frac{\frac{\pi}{2}}{\sqrt{1-\frac{5^{-3/2}}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\sin^2\left(\frac{\pi}{2}\right)}}\,dx\right)$$

$\cos^{-1}(x)$ is the inverse cosine function

ϕ is the golden ratio

Result:

$4.27141\times 10^{18}+1.294\times 10^{18}\,i$

Input interpretation:

$4.27141\times 10^{18}+1.294\times 10^{18}\,i$

i is the imaginary unit

Result:

$4.27141...\times 10^{18}$ +
$1.29400...\times 10^{18}\,i$

Polar coordinates:

$r = 4.46311\times 10^{18}$ (radius), $\theta = 16.8539°$ (angle)

4.46311 * 10^{18} value very near to the previous result ***f* = 4.49192 * 10^{18}**

Furthermore, we have also:

((((10^17((((5+2*5 integrate (Pi/2) / sqrt((((1-((sqrt5+1)/2)^(-5)*5^(-3/2) sin^2(Pi/2)))))dx x =cos^-1(golden ratio)^2.5..Pi/2))))))))^1/89-2/10^3

Input:

$$\sqrt[89]{10^{17}\left(5+2\times 5\int_{\cos^{-1}(\phi)^{2.5}}^{\frac{\pi}{2}}\frac{\frac{\pi}{2}}{\sqrt{1-\frac{5^{-3/2}}{\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^5}\sin^2\left(\frac{\pi}{2}\right)}}\,dx\right)}-\frac{2}{10^3}$$

$\cos^{-1}(x)$ is the inverse cosine function

ϕ is the golden ratio

Result:

$1.61811 + 0.00535467\,i$

Input interpretation:

$1.61811 + 0.00535467\,i$

i is the imaginary unit

Result:

$1.61811\ldots +$
$0.00535467\ldots\,i$

Polar coordinates:

$r = 1.61812$ (radius), $\theta = 0.189603°$ (angle)

1.61812

Now, we have that:

Lemma 15.8.2. *Let*

$$P = \frac{1}{q}\left(\frac{f(-q)}{f(-q^7)}\right)^4 \qquad \textit{and} \qquad Q = \frac{1}{q^2}\left(\frac{f(-q^2)}{f(-q^{14})}\right)^4. \tag{15.8.4}$$

Then

$$P + \frac{49}{P} = R - 1 + \frac{48}{R} + \frac{64}{R^2} \tag{15.8.5}$$

and

$$Q + \frac{49}{Q} - R^2 + 6R - 1 + \frac{8}{R}. \tag{15.8.6}$$

For P = 7 and R = 8, from (15.8.5) we obtain:

7+49/7 = 8-1+48/8+64/8^2

Input:

$$7 + \frac{49}{7} = 8 - 1 + \frac{48}{8} + \frac{64}{8^2}$$

Result:

True

Left hand side:

$$7 + \frac{49}{7} = 14$$

Right hand side:

$$8 - 1 + \frac{48}{8} + \frac{64}{8^2} = 14$$

14

For Q = 7 and R = 1, from (15.8.6), we obtain:

7+49/7 = 1+6-1+8

Input:

$$7 + \frac{49}{7} = 1 + 6 - 1 + 8$$

Result:

True

Left hand side:

$7 + \frac{49}{7} = 14$

Right hand side:

$1 + 6 - 1 + 8 = 14$

14

From

$$\sqrt{PQ} + \frac{49}{\sqrt{PQ}} = v^{3/2} - 8v^{1/2} - 8v^{-1/2} + v^{-3/2}$$
$$= \left(\frac{1}{\sqrt{v}} + \sqrt{v}\right)^3 - 11\left(\frac{1}{\sqrt{v}} + \sqrt{v}\right)$$
$$- K(K^2 - 11), \qquad (15.8.7)$$

sqrt49+(49/sqrt49) = x(x^2-11)

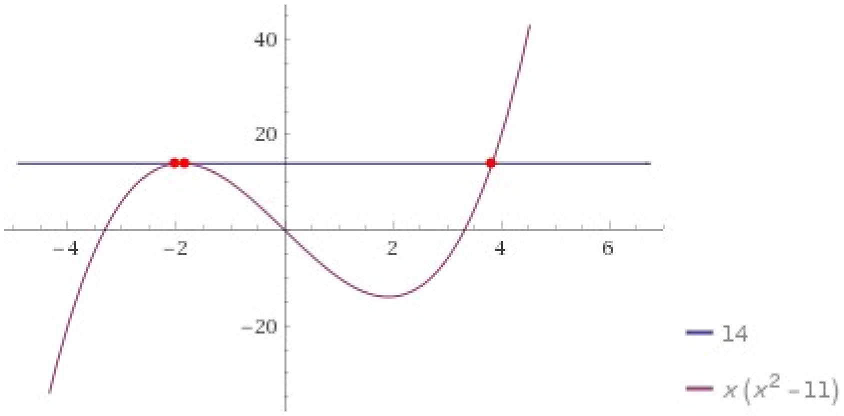
Input:

$\sqrt{49} + \frac{49}{\sqrt{49}} = x(x^2 - 11)$

Result:

$14 = x(x^2 - 11)$

Plot:



Alternate forms:

$-x^3 + 11\,x + 14 = 0$

$14 - x\,(x^2 - 11) = 0$

Expanded form:

$14 = x^3 - 11\,x$

Solutions:

$x = -2$

$x = 1 - 2\sqrt{2}$

$x = 1 + 2\sqrt{2}$

From

$$K\,(K^2 - 11)$$

for K = - 2 , we obtain:

-2((-2)^2-11)

Input:

$-2\left((-2)^2 - 11\right)$

Result:

14

14

or:

(1+2sqrt2)*((1+2sqrt2)^2-11)

Input:

$$\left(1+2\sqrt{2}\right)\left(\left(1+2\sqrt{2}\right)^2-11\right)$$

Result:

14

14

Now, from

$$K = \frac{1}{\sqrt{v}} + \sqrt{v}.$$

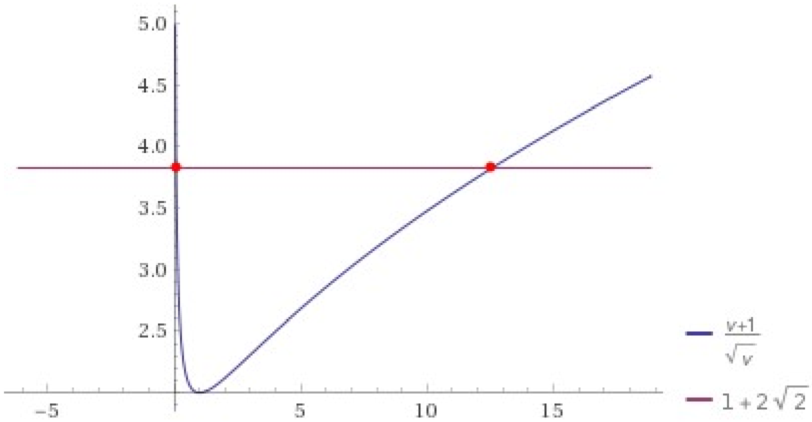
For $K = 1+2\sqrt{2}$, we have that:

1/sqrt(v) + sqrt(v) = 1+2sqrt2

Input:

$$\frac{1}{\sqrt{v}} + \sqrt{v} = 1+2\sqrt{2}$$

Plot:



Alternate form:

$$\frac{v+1}{\sqrt{v}} = 1+2\sqrt{2}$$

Alternate form assuming v is positive:

$$v+1=\left(1+2\sqrt{2}\right)\sqrt{v}$$

Solutions:

$$v=\frac{1}{2}\left(7+4\sqrt{2}-\sqrt{77+56\sqrt{2}}\right)$$

$$v=\frac{1}{2}\left(7+4\sqrt{2}+\sqrt{77+56\sqrt{2}}\right)$$

Solutions:

$$v\approx 0.079508$$

$$v\approx 12.577$$

Now:

$$\left(R+\frac{8}{R}+9\right)\left(R+\frac{8}{R}-2\right)$$

(R+8/R+9)(R+8/R-2) = 28

Input:

$$\left(R+\frac{8}{R}+9\right)\left(R+\frac{8}{R}-2\right)=28$$

Alternate form assuming R is real:

$$(R-3)\,R\,(R+10)+\frac{64}{R}+56=0$$

Alternate forms:

$$R^4+7R^3-30R^2+56R=-64 \quad (\text{for } R\neq 0)$$

$$\frac{(R+1)(R+8)((R-2)R+8)}{R^2}=28$$

$$\frac{(R+1)(R+8)\left(R^2-2R+8\right)}{R^2}=28$$

Expanded form:

$$R^2+\frac{64}{R^2}+7R+\frac{56}{R}-2=28$$

Real solutions:

$$R = \frac{1}{4}\left(-7 - \sqrt{233} - \sqrt{14\left(11 + \sqrt{233}\right)}\right)$$

$$R = \frac{1}{4}\left(-7 - \sqrt{233} + \sqrt{14\left(11 + \sqrt{233}\right)}\right)$$

Complex solutions:

$$R = \frac{1}{4}\left(-7 + \sqrt{233} - i\sqrt{14\left(\sqrt{233} - 11\right)}\right)$$

$$R = \frac{1}{4}\left(-7 + \sqrt{233} + i\sqrt{14\left(\sqrt{233} - 11\right)}\right)$$

$$\frac{(R+8)(R+1)(R^2-2R+8)}{R^2}$$

(((R+8)(R+1)(R^2-2R+8)))/R^2

Input:

$$\frac{(R+8)(R+1)\left(R^2-2R+8\right)}{R^2} = 28$$

Alternate form assuming R is real:

$$(R-3)R(R+10) + \frac{64}{R} + 56 = 0$$

Alternate forms:

$$R^4 + 7R^3 - 30R^2 + 56R = -64 \quad (\text{for } R \neq 0)$$

$$\frac{(R+1)(R+8)((R-2)R+8)}{R^2} = 28$$

Expanded form:

$$R^2 + \frac{64}{R^2} + 7R + \frac{56}{R} - 2 = 28$$

Real solutions:

$$R = \frac{1}{4}\left(-7 - \sqrt{233} - \sqrt{14\left(11 + \sqrt{233}\right)}\right)$$

$$R = \frac{1}{4}\left(-7 - \sqrt{233} + \sqrt{14\left(11 + \sqrt{233}\right)}\right)$$

Complex solutions:

$$R = \frac{1}{4}\left(-7 + \sqrt{233} - i\sqrt{14\left(\sqrt{233} - 11\right)}\right)$$

$$R = \frac{1}{4}\left(-7 + \sqrt{233} + i\sqrt{14\left(\sqrt{233} - 11\right)}\right)$$

Thence, we have:

$$R = \frac{1}{4}\left(-7 - \sqrt{233} + \sqrt{14\left(11 + \sqrt{233}\right)}\right)$$

$$R \approx -0.77220$$

-0.77220

for the two expressions.

Now:

$$P + \frac{49}{P} + Q + \frac{49}{Q} = \frac{(R+8)(R+1)(R^2 - 2R + 8)}{R^2}$$

P+49/P+Q+49/Q = (((-0.77220+8)(-0.77220+1)(-0.77220^2-2*-0.77220+8)))/(-0.77220)^2

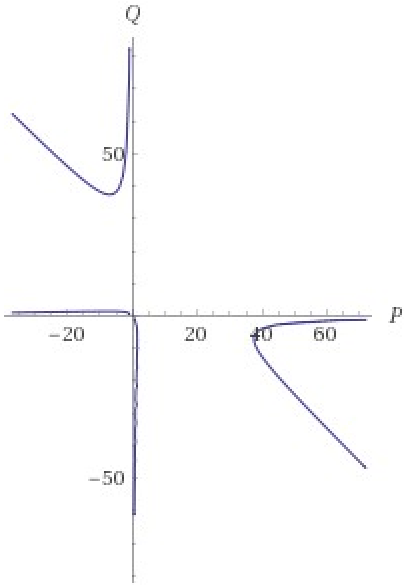
Input:

$$P + \frac{49}{P} + Q + \frac{49}{Q} = \frac{(-0.7722 + 8)(-0.7722 + 1)(-0.7722^2 - 2 \times (-0.7722) + 8)}{(-0.7722)^2}$$

Result:

$$P + \frac{49}{P} + Q + \frac{49}{Q} = 24.7076$$

Implicit plot:



Alternate form:

$$\frac{(P+Q)(PQ+49)}{PQ} = 24.7076$$

Solutions:

$$P \neq 0, \quad Q \approx \frac{1}{P} 2.64118 \times 10^{-8} \left(-1.89309 \times 10^{7} P^2 - \sqrt{\left(3.58379 \times 10^{14} P^4 - 1.77094 \times 10^{16} P^3 + 1.83658 \times 10^{17} P^2 - 8.67761 \times 10^{17} P + 8.60468 \times 10^{17}\right)} + 4.67738 \times 10^{8} P - 9.27614 \times 10^{8}\right)$$

$$P \neq 0, \quad Q \approx \frac{1}{P} 2.64118 \times 10^{-8} \left(-1.89309 \times 10^{7} P^2 + \sqrt{\left(3.58379 \times 10^{14} P^4 - 1.77094 \times 10^{16} P^3 + 1.83658 \times 10^{17} P^2 - 8.67761 \times 10^{17} P + 8.60468 \times 10^{17}\right)} + 4.67738 \times 10^{8} P - 9.27614 \times 10^{8}\right)$$

$P \approx 1.31025, \quad Q \approx -7$

$P \approx 37.3974, \quad Q \approx -7$

P = 1.31025 Q = -7 or P = 37.3974 Q = -7

We note that:

(((-0.77220+8)(-0.77220+1)(-0.77220^2-2*-0.77220+8)))/(-0.77220)^2

Input:

$$\frac{(-0.7722+8)(-0.7722+1)\left(-0.7722^2-2\times(-0.7722)+8\right)}{(-0.7722)^2}$$

Result:

24.707649299449469156798864128571458278787986117693447400 77...

24.707649299449…

and, with P = 1.31025 and Q = -7 , we obtain:

$$P+\frac{49}{P}+Q+\frac{49}{Q}$$

thus:

1.31025+49/(1.31025)+(-7)+49/(-7)

Input interpretation:

$$1.31025+\frac{49}{1.31025}-7+-\frac{49}{7}$$

Result:

24.707693236023659606945239458118679641289830185079183361 95...

24.707693236…

We have that:

$$\begin{aligned}
&-P-\frac{49}{P}+Q+\frac{49}{Q}\\
&=(K^2-4)\sqrt{K^4-18K^2+49}\\
&=\left(R+\frac{8}{R}+5\right)\sqrt{\left(R+\frac{8}{R}+9\right)^2-18\left(R+\frac{8}{R}+9\right)+49}\\
&=\frac{(R^2+5R+8)(R^2-8)}{R^2}. \qquad (15.8.11)
\end{aligned}$$

From:

$$\left(R+\frac{8}{R}+5\right)\sqrt{\left(R+\frac{8}{R}+9\right)^2-18\left(R+\frac{8}{R}+9\right)+49}$$

For R = -0.77220, we obtain:

(-0.77220+8/(-0.77220)+5) * sqrt(((((-0.77220+8/(-0.77220)+9))^2-18(-0.77220+8/(-0.77220)+9)+49)))

Input:

$$\left(-0.7722+-\frac{8}{0.7722}+5\right)\sqrt{\left(-0.7722+-\frac{8}{0.7722}+9\right)^2-18\left(-0.7722+-\frac{8}{0.7722}+9\right)+49}$$

Result:

−58.79447001947018917751888484859217829950800683771416742 14...

-58.794470019…

From

$$\frac{(R^2+5R+8)(R^2-8)}{R^2}.$$

we obtain:

((-0.77220)^2+5*(-0.77220)+8)((-0.7722)^2-8) / (-0.77220)^2

Input:

$$\left((-0.7722)^2+5\times(-0.7722)+8\right)\times\frac{(-0.7722)^2-8}{(-0.7722)^2}$$

Result:

−58.79447001947018917751888484859217829950800683771416742 14...

-58.794470019…

From:

$$-P-\frac{49}{P}+Q+\frac{49}{Q}$$

we obtain:

(((-P-49/(P)+(-7)+49/(-7)))) = ((-0.77220)^2+5*(-0.77220)+8)((-0.7722)^2-8) / (-0.77220)^2

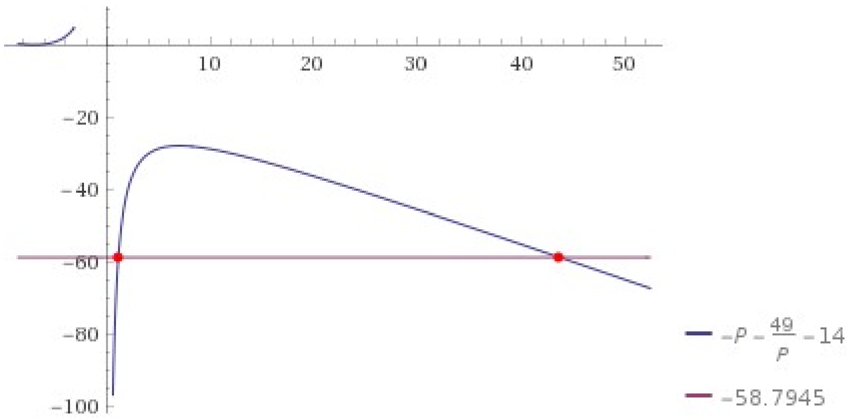
Input:

$$-P-\frac{49}{P}-7+-\frac{49}{7}=\left((-0.7722)^2+5\times(-0.7722)+8\right)\times\frac{(-0.7722)^2-8}{(-0.7722)^2}$$

Result:

$$-P-\frac{49}{P}-14=-58.7945$$

Plot:



Alternate form:

$$-\frac{(P+7)^2}{P}=-58.7945$$

Alternate form assuming P is positive:

$$\frac{(P+7)^2}{P}=58.7945$$

Solutions:

$P\approx 1.12199$

$P\approx 43.6725$

Thence, from

$$-P-\frac{49}{P}+Q+\frac{49}{Q}$$

we obtain:

(((-(43.6725)-49/(43.6725)+(-7)+49/(-7))))

Input interpretation:

$$-43.6725-\frac{49}{43.6725}-7+-\frac{49}{7}$$

Result:

-58.79448752075104470776804625336310034918999370313126109 10...

-58.79448752…

From:

$$(K^2-4)\sqrt{K^4-18K^2+49}$$

we obtain:

(K^2-4)(K^4-18K^2+49)^0.5 = ((-0.77220)^2+5*(-0.77220)+8)((-0.7722)^2-8) / (-0.77220)^2

Input:

$$(K^2-4)\sqrt{K^4-18K^2+49} = ((-0.7722)^2+5\times(-0.7722)+8)\times\frac{(-0.7722)^2-8}{(-0.7722)^2}$$

Result:

$$(K^2-4)\sqrt{K^4-18K^2+49} = -58.7945$$

Alternate form:

$$(K-2)(K+2)\sqrt{K^4-18K^2+49} = -58.7945$$

Expanded form:

$$K^2\sqrt{K^4-18K^2+49}-4\sqrt{K^4-18K^2+49} = -58.7945$$

Complex solutions:

$K = -1.46021\, i$

$K = 1.46021\, i$

Thence, we obtain:

((1.46021i)^2-4)((1.46021i)^4-18(1.46021i)^2+49)^0.5

Input interpretation:

$$((1.46021\, i)^2 - 4)\sqrt{(1.46021\, i)^4 - 18\,(1.46021\, i)^2 + 49}$$

i is the imaginary unit

Result:

-58.7945182062553436366836807719330725619138967182309555722...

-58.794518206…

Now, from the following equation

$$((-0.7722)^2 + 5\times(-0.7722) + 8)\times\frac{(-0.7722)^2 - 8}{(-0.7722)^2}$$

-58.7944700194701891775188848485921782995080068377141674214...

i.e.

((-0.77220)^2+5*(-0.77220)+8)((-0.7722)^2-8) / (-0.77220)^2

we obtain:

(((-((-0.77220)^2+5*(-0.77220)+8)((-0.7722)^2-8) / (-0.77220)^2)))^1/8-(29+11+4+2)1/10^3

Input:

$$\sqrt[8]{-((-0.7722)^2 + 5\times(-0.7722) + 8)\times\frac{(-0.7722)^2 - 8}{(-0.7722)^2}} - (29 + 11 + 4 + 2)\times\frac{1}{10^3}$$

Result:

1.618052638144921444271546835551804359180683755246596729317...

1.618052638144...

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.
In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the *golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.*

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the *second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.*

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:
2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

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