Four Color Theorem - a simpler proof

R. L. Hudson
3-29-2020

Abstract

The conjecture [1] stated that four colors are sufficient for any 2-dimensional plane map so that no two regions with a shared border are the same color. The conjecture is now a theorem [2], the result of a lengthy and complex proof, involving over 1000 classifications of graph objects and over 1000 hours of computer time. Instead of considering all maps, this paper examines which elements of a map are possible and the issue of causality and redundancy.

1. Method

To simplify the analysis, map configurations will be transformed to graph objects.

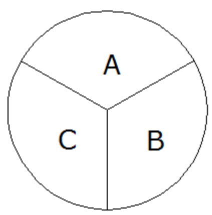


fig. 1

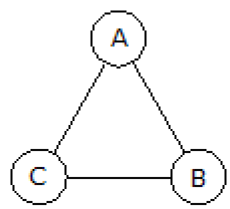


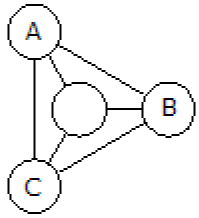
fig.2

The regions colored A, B, and C in fig.1, are represented by small circles (nodes) in fig.2. A line connecting the nodes represents any path crossing the border between any two regions in fig.1. This eliminates dealing with regions having highly irregular borders, and focuses on connectivity. Since borders cannot overlap, lines cannot cross.

1.1 The color restriction for maps can be restated for the graph as: no paired nodes are the same color.

2. Proposed cause

The color of a node is determined by the nodes directly linked to it. It and the linked nodes form a complete object, defined as having all possible links formed.



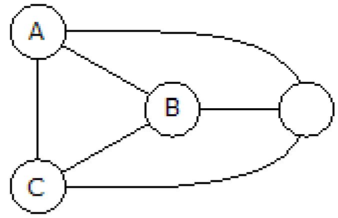
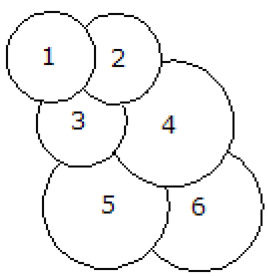


fig.3 fig.4

When 3 nodes form a closed boundary as ABC in fig.3, the surface is divided into 2 independent portions. The blank node in the interior of ABC (fig.3) is isolated from a blank node in the exterior of ABC (fig.4). The color of the first blank does not affect the color of the second blank. The color of both is determined by the adjacent A, B, & C nodes, requiring a 4th color D for both.

3. Coloring the map



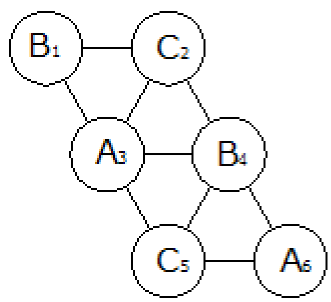


fig.5 fig.6

Fig.5 shows a formation of regions with a general geographic form and an id number. After numbering the nodes in any random order, they are sorted into a sequence by decreasing link count, as shown in fig.7.

links per node				
2	3	4	5	6
1	2	3		
6	5	4		

fig.7

node	A	B	C	D	plink
3	1				
4	0	1			3
2	0	0	1		3 4
5	0	0	1		3 4
1	0	1	0		2 3
6	1	0	0		4 5

fig.7a

A color assignment table can be formed satisfying 1.1 based on connectivity using this sequence (fig.7a). In general, the more links per node, the more color restrictions for that node, and the reason it is given priority. Colors are assigned using the set A to D. The process for each node is to list preceding colored links (plink) on the right and place a 0 in each column containing a 1 for each plink, which prohibits using its color. From left to right, place a 1 in the cell of the first available color. This method is based on the idea that nodes without color cannot affect the node being assigned a color.
Processing the nodes from fig.5 assigns a color to each id as shown in fig.6.

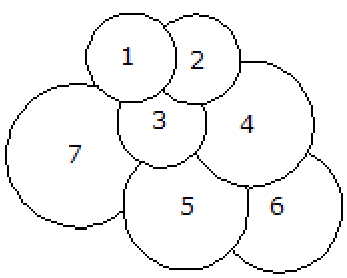


fig.8

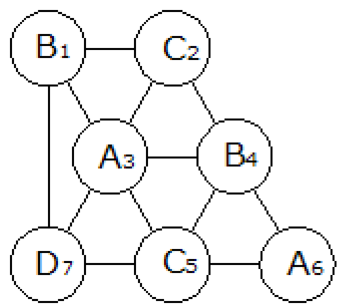


fig.9

node	A	B	C	D	plink
3	1				
4	0	1			3
5	0	0	1		3 4
1	0	1			3
2	0	0	1		1 3 4
7	0	0	0	1	1 3 5
6	1	0	0		4 5

fig.10

Addition of a node adds links to some existing nodes, thus a new sequence is formed that modifies the previous table. Fig.10 is the revised table used to assign colors for fig.9.

4. Map reduction and redundancy

Inspection of fig.10 reveals nodes (1, 2, & 6) reuse (A, B, & C), and therefore do not force the need of a 4th color.

node	A	B	C	D	plink
3	1				
4	0	1			3
5	0	0	1		3 4
7	0	0	0	1	3 4 5

fig.11

Removal of the redundant nodes and a different sequence modifies fig.10 to fig.11.

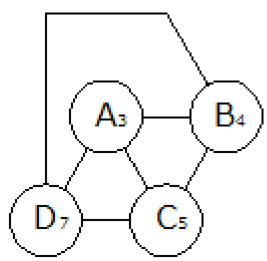


fig.12

The map of fig.10 is reduced to the complete 4c object in fig.12 which supports section 2.

5. The causality issue and redundancy

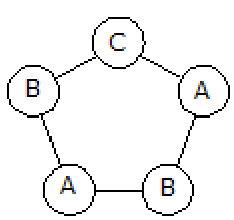


fig. 13

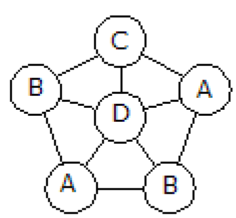


fig. 14

Some authors use fig. 13 as an argument against completeness as the cause of color number, reasoning that there are not 3 mutually connected nodes in fig.13, yet it still requires 3 colors. Fig. 14 is the 4 color counterpart for the same argument.

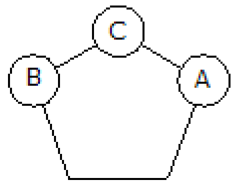


fig.15

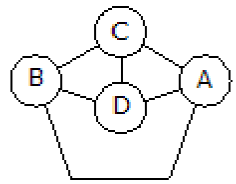


fig.16

Using the coloring table to remove redundant nodes, the revised maps are fig.15, a complete 3c object and fig.16, a complete 4c object.

The color number is determined by the complete object that remains after removal of the redundant parts of the configuration. In each case redundancy obscured completeness.

6. Conclusion

With a plane surface tiled with 3c objects as in fig. 2, the 4th color is required wherever a node is inserted within an arbitrarily selected boundary. This is independent of n. Sections 4, 5, & 6 show any map can be reduced to a 4c object, with the color being determined locally by adjacent contact supporting the proposed cause (section 2). The notion of causality spreading through the links, and therefore a larger map providing a possible configuration for the 5c object, has been shown to be a false assumption.

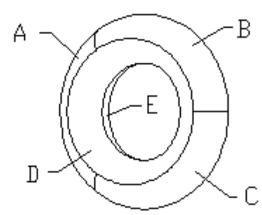


fig.17

The only solution to defeat isolation is by adding another dimension, forming the torus (fig.17).

references

[1] http://en.wikipedia.org/wiki/Four color theorem
[2] Appel, Kenneth; Haken, Wolfgang; Koch, John (1977), "Every Planar Map is Four Colorable", *Illinois Journal of Mathematics* **21**: 439–567