On some integral equations and incomplete elliptic integrals of the first kind: new possible mathematical connections with ϕ, $\zeta(2)$, and various parameters of Particle Physics. II

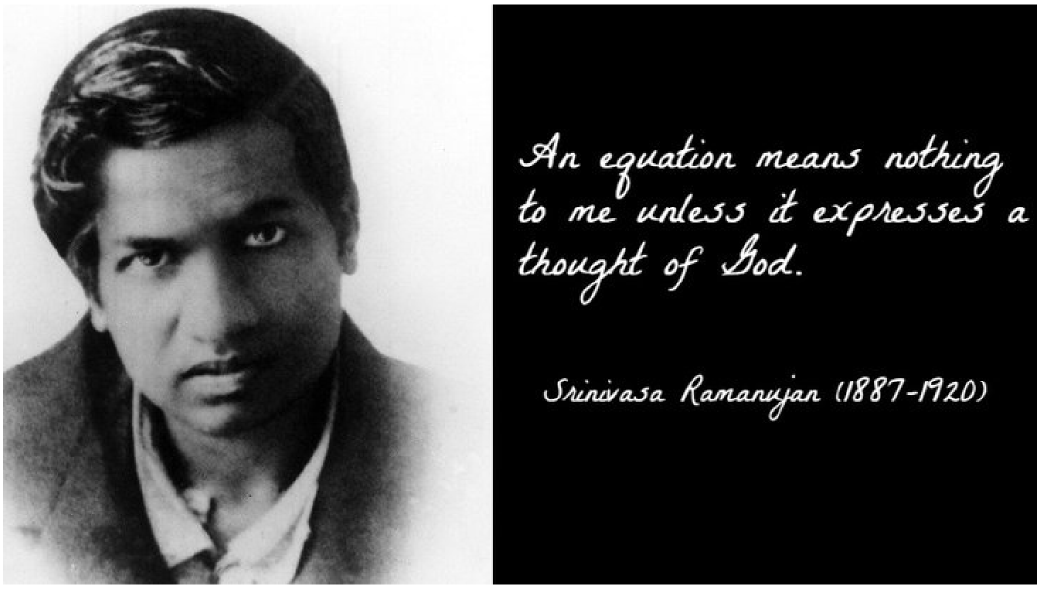
Michele Nardelli[1], **Antonio Nardelli**[2]

Abstract

In this paper we have described some Ramanujan's integral equations and incomplete elliptic integrals of the first kind. Furthermore, we describe new possible mathematical connections with ϕ, $\zeta(2)$, and various parameters of Particle Physics

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

[2] A. Nardelli studies at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**



https://mobygeek.com/features/indian-mathematician-srinivasa-ramanujan-quotes-11012

We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation

From

INCOMPLETE ELLIPTIC INTEGRALS IN RAMANUJAN'S LOST NOTEBOOK
BRUCE C. BERNDT, HENG HUAT CHAN, AND SEN–SHAN HUANG

From:

$$\frac{1}{\sqrt{v_2}} = \frac{1}{2}\left(\sqrt{\frac{1}{v} + 81v + 18} + \sqrt{\frac{1}{v} + 81v + 14}\right)$$
$$= \frac{1}{2}\left(\sqrt{\frac{1}{v_1} + 64v_1 + 20} + \sqrt{\frac{1}{v_1} + 64v_1 + 16}\right).$$

For v = 2, we obtain:

1/2(sqrt(1/2+81*2+18)+sqrt(1/2+81*2+14))

Input:

$$\frac{1}{2}\left(\sqrt{\frac{1}{2} + 81 \times 2 + 18} + \sqrt{\frac{1}{2} + 81 \times 2 + 14}\right)$$

Result:

$$\frac{1}{2}\left(\frac{19}{\sqrt{2}} + \sqrt{\frac{353}{2}}\right)$$

Decimal approximation:

13.36017954906541276590071237798891408231531394562172185317...

13.360179549…

Alternate forms:

$$\frac{19 + \sqrt{353}}{2\sqrt{2}}$$

$$\sqrt{\frac{357}{4} + \frac{19\sqrt{353}}{4}}$$

$$\frac{19}{2\sqrt{2}} + \frac{\sqrt{\frac{353}{2}}}{2}$$

Minimal polynomial:

$$2x^4 - 357x^2 + 2$$

1/(sqrtx) = 1/2(sqrt(1/2+81*2+18)+sqrt(1/2+81*2+14))

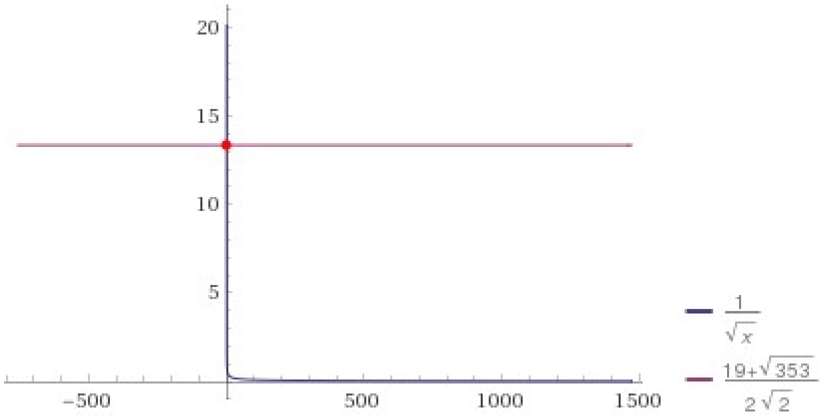
Input:

$$\frac{1}{\sqrt{x}} = \frac{1}{2}\left(\sqrt{\frac{1}{2} + 81\times 2 + 18} + \sqrt{\frac{1}{2} + 81\times 2 + 14}\right)$$

Exact result:

$$\frac{1}{\sqrt{x}} = \frac{1}{2}\left(\frac{19}{\sqrt{2}} + \sqrt{\frac{353}{2}}\right)$$

Plot:



Alternate form assuming x is real:

$$\frac{4}{\sqrt{x}} = 19\sqrt{2} + \sqrt{706}$$

Alternate forms:

$$\frac{1}{\sqrt{x}} = \frac{19 + \sqrt{353}}{2\sqrt{2}}$$

$$\left(19 + \sqrt{353}\right)\sqrt{x} = 2\sqrt{2}$$

$$\frac{1}{\sqrt{x}} = \sqrt{\frac{357}{4} + \frac{19\sqrt{353}}{4}}$$

Alternate form assuming x is positive:

$$\sqrt{2}\left(19 + \sqrt{353}\right)\sqrt{x} = 4$$

Expanded form:

$$\frac{1}{\sqrt{x}} = \frac{19}{2\sqrt{2}} + \frac{\sqrt{\frac{353}{2}}}{2}$$

Solution:

$$x = \frac{357}{4} - \frac{19\sqrt{353}}{4}$$

v_2 = (357/4 - (19 sqrt(353))/4) = x ≈ 0.0056024

indeed:

1/((sqrt(357/4 - (19 sqrt(353))/4)))

Input:

$$\frac{1}{\sqrt{\frac{357}{4} - \frac{1}{4}\left(19\sqrt{353}\right)}}$$

Result:

$$\frac{1}{\sqrt{\frac{357}{4} - \frac{19\sqrt{353}}{4}}}$$

Decimal approximation:

13.360179549065412765900712377988914082315313945621721853 17...

13.360179549…

Alternate forms:

$$\frac{2}{\sqrt{357 - 19\sqrt{353}}}$$

$$\sqrt{\frac{357}{4} + \frac{19\sqrt{353}}{4}}$$

root of $2x^4 - 357x^2 + 2$ near $x = 13.3602$

Minimal polynomial:

$$2x^4 - 357x^2 + 2$$

From:

$$\frac{1}{2}\left(\sqrt{\frac{1}{v_1}+64v_1+20}+\sqrt{\frac{1}{v_1}+64v_1+16}\right).$$

1/((sqrt(357/4 - (19 sqrt(353))/4))) = 1/2(sqrt(1/x+64x+20)+sqrt(1/x+64x+16))

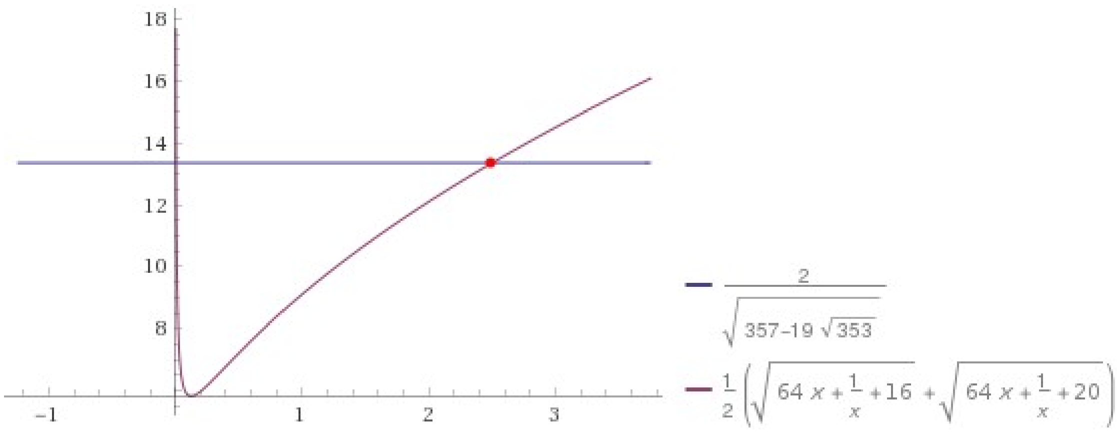
Input:

$$\frac{1}{\sqrt{\frac{357}{4}-\frac{1}{4}\left(19\sqrt{353}\right)}}=\frac{1}{2}\left(\sqrt{\frac{1}{x}+64x+20}+\sqrt{\frac{1}{x}+64x+16}\right)$$

Exact result:

$$\frac{1}{\sqrt{\frac{357}{4}-\frac{19\sqrt{353}}{4}}}=\frac{1}{2}\left(\sqrt{64x+\frac{1}{x}+16}+\sqrt{64x+\frac{1}{x}+20}\right)$$

Plot:



Alternate forms:

$$\frac{2}{\sqrt{357-19\sqrt{353}}}=\frac{1}{2}\left(\sqrt{64x+\frac{1}{x}+16}+\sqrt{64x+\frac{1}{x}+20}\right)$$

$$\sqrt{\frac{357}{4}+\frac{19\sqrt{353}}{4}}=\frac{1}{2}\left(\sqrt{64x+\frac{1}{x}+16}+\sqrt{64x+\frac{1}{x}+20}\right)$$

$$\frac{2}{\sqrt{357 - 19\sqrt{353}}} = \frac{1}{2}\left(\sqrt{\frac{(8x+1)^2}{x}} + \sqrt{64x + \frac{1}{x} + 20}\right)$$

Solutions:

$$x = \frac{321}{256} - \frac{17\sqrt{353}}{256}$$

$$x = \frac{321}{256} + \frac{17\sqrt{353}}{256}$$

$v_1 =$ 321/256 + (17 sqrt(353))/256 = x ≈ 2.5016

1/2(sqrt(1/((321/256 + (17 sqrt(353))/256))+64((321/256 + (17 sqrt(353))/256))+20)+(sqrt(1/((321/256 + (17 sqrt(353))/256))+64((321/256 + (17 sqrt(353))/256))+16)

Input:

$$\frac{1}{2}\left(\sqrt{\frac{1}{\frac{321}{256} + \frac{1}{256}\left(17\sqrt{353}\right)} + 64\left(\frac{321}{256} + \frac{1}{256}\left(17\sqrt{353}\right)\right) + 20} + \sqrt{\frac{1}{\frac{321}{256} + \frac{1}{256}\left(17\sqrt{353}\right)} + 64\left(\frac{321}{256} + \frac{1}{256}\left(17\sqrt{353}\right)\right) + 16}\right)$$

Result:

$$\frac{1}{2}\left(\sqrt{16 + \frac{1}{\frac{321}{256} + \frac{17\sqrt{353}}{256}} + 64\left(\frac{321}{256} + \frac{17\sqrt{353}}{256}\right)} + \sqrt{20 + \frac{1}{\frac{321}{256} + \frac{17\sqrt{353}}{256}} + 64\left(\frac{321}{256} + \frac{17\sqrt{353}}{256}\right)}\right)$$

Decimal approximation:

13.36017954906541276590071237798891408231531394562172185317...

13.3601979549…

Alternate forms:

$$\frac{1}{4}\left(19\sqrt{2} + \sqrt{706}\right)$$

$$\frac{19 + \sqrt{353}}{2\sqrt{2}}$$

$$\sqrt{\frac{357}{4} + \frac{19\sqrt{353}}{4}}$$

Minimal polynomial:

$2x^4 - 357x^2 + 2$

We have also:

1+9/((1/2(sqrt(1/((321/256 + (17 sqrt(353))/256))+64((321/256 + (17 sqrt(353))/256))+20)+ (sqrt(1/((321/256 + (17 sqrt(353))/256))+64((321/256 + (17 sqrt(353))/256))+16)))))-29/10^3

Input:

$$1 + 9\Bigg/\left(\frac{1}{2}\left(\sqrt{\frac{1}{\frac{321}{256} + \frac{1}{256}(17\sqrt{353})} + 64\left(\frac{321}{256} + \frac{1}{256}\left(17\sqrt{353}\right)\right) + 20} + \sqrt{\frac{1}{\frac{321}{256} + \frac{1}{256}(17\sqrt{353})} + 64\left(\frac{321}{256} + \frac{1}{256}\left(17\sqrt{353}\right)\right) + 16}\right)\right) - \frac{29}{10^3}$$

Result:

$$\frac{971}{1000} + \frac{18}{\sqrt{16 + \frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}} + 64\left(\frac{321}{256} + \frac{17\sqrt{353}}{256}\right)} + \sqrt{20 + \frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}} + 64\left(\frac{321}{256} + \frac{17\sqrt{353}}{256}\right)}}$$

Decimal approximation:

1.644643641310911779437974518028958976869119834133563578010...

1.64464364131…

Alternate forms:

$$\frac{971 + 42750\sqrt{2} - 2250\sqrt{706}}{1000}$$

$$\frac{971}{1000} + \frac{18}{\sqrt{357 + 19\sqrt{353}}}$$

$$\frac{971}{1000} + \frac{18\sqrt{2}}{19 + \sqrt{353}}$$

Minimal polynomial:

$$1\,000\,000\,000\,000\,x^4 - 3\,884\,000\,000\,000\,x^3 - 14\,452\,842\,954\,000\,000\,x^2 + 28\,074\,745\,005\,556\,000\,x - 7\,070\,177\,649\,348\,719$$

1+9/((1/2(sqrt(1/((321/256 + (17 sqrt(353))/256))+64((321/256 + (17 sqrt(353))/256))+20)+ (sqrt(1/((321/256 + (17 sqrt(353))/256))+64((321/256 + (17 sqrt(353))/256))+16)))))-55/10^3

Input:

$$1+9\Big/\left(\frac{1}{2}\left(\sqrt{\frac{1}{\frac{321}{256}+\frac{1}{256}\left(17\sqrt{353}\right)}+64\left(\frac{321}{256}+\frac{1}{256}\left(17\sqrt{353}\right)\right)+20}+\sqrt{\frac{1}{\frac{321}{256}+\frac{1}{256}\left(17\sqrt{353}\right)}+64\left(\frac{321}{256}+\frac{1}{256}\left(17\sqrt{353}\right)\right)+16}\right)\right)-\frac{55}{10^3}$$

Result:

$$\frac{189}{200}+\frac{18}{\sqrt{16+\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)}+\sqrt{20+\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)}}$$

Decimal approximation:

1.618643641310911779437974518028958976869119834133563578010...

1.6186436413…

Alternate forms:

$$\frac{1}{200}\left(189+8550\sqrt{2}-450\sqrt{706}\right)$$

$$\frac{189}{200}+\frac{18}{\sqrt{357+19\sqrt{353}}}$$

$$\frac{189}{200}+\frac{18\sqrt{2}}{19+\sqrt{353}}$$

Minimal polynomial:

$$1\,600\,000\,000\,x^4 - 6\,048\,000\,000\,x^3 - 23\,125\,026\,960\,000\,x^2 + 43\,717\,102\,984\,800\,x - 10\,160\,007\,150\,159$$

and:

((((1+9/((1/2(sqrt(1/((321/256 + (17 sqrt(353))/256))+64((321/256 + (17 sqrt(353))/256))+20)+ (sqrt(1/((321/256 + (17 sqrt(353))/256))+64((321/256 + (17 sqrt(353))/256))+16)))))-29/10^3))))^15-13

Input:

$$\left(1+9\Big/\left(\frac{1}{2}\left(\sqrt{\frac{1}{\frac{321}{256}+\frac{1}{256}\left(17\sqrt{353}\right)}+64\left(\frac{321}{256}+\frac{1}{256}\left(17\sqrt{353}\right)\right)+20}+\sqrt{\frac{1}{\frac{321}{256}+\frac{1}{256}\left(17\sqrt{353}\right)}+64\left(\frac{321}{256}+\frac{1}{256}\left(17\sqrt{353}\right)\right)+16}\right)\right)-\frac{29}{10^3}\right)^{15}-13$$

Result:

$$\left(\frac{971}{1000}+18\Big/\left(\sqrt{16+\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)}+\sqrt{20+\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)}\right)\right)^{15}-13$$

Decimal approximation:

1729.116338997694503938317729719056450959749597083838794791...

1729.1163389976…

Alternate forms:

$$\left(963\,621\,211\,761\,867\,577\,772\,448\,653\,611\,833\,358\,637\,947\,168\,594\,486\,194\,379\,256\,833\,563\,206\,524\,051+5\,652\,533\,890\,021\,160\,724\,842\,454\,203\,383\,848\,959\,347\,495\,307\,528\,707\,051\,640\,068\,130\,104\,834\,691\,250\sqrt{2}-51\,288\,382\,014\,101\,312\,853\,097\,702\,760\,547\,132\,573\,347\,169\,220\,679\,487\,709\,894\,676\,725\,886\,250\,000\sqrt{353}-300\,854\,022\,265\,651\,957\,435\,390\,906\,929\,861\,955\,269\,021\,897\,512\,661\,072\,129\,360\,498\,831\,964\,983\,750\sqrt{706}\right)\Big/1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000$$

$$
\Big(11\,979\,157\,714\,156\,995\,954\,944\,444\,829\,627\,408\,609\,879\,846\,607\,136\,438\,808\,828 \ddots
$$
$$
106\,680\,275 +
$$
$$
637\,585\,750\,026\,260\,313\,445\,704\,375\,947\,081\,862\,613\,598\,686\,071\,187\,897\,898 \ddots
$$
$$
555\,533\,501\sqrt{353} + 3\,915\,000\sqrt{2}
$$
$$
\Big(2\,196\,398\,418\,905\,287\,599\,705\,601\,006\,625\,071\,440\,157\,682\,688\,004\,064 \ddots
$$
$$
712\,852\,477 +
$$
$$
116\,901\,770\,690\,694\,287\,727\,679\,077\,367\,194\,362\,812\,183\,489\,169\,248 \ddots
$$
$$
718\,590\,579\sqrt{353}\Big)\Big)\Big/
$$
$$
\left(61\,035\,156\,250\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(19+\sqrt{353}\right)^{15}\right)
$$
$$
\Big(963\,621\,211\,761\,867\,577\,772\,448\,653\,611\,833\,358\,637\,947\,168\,594\,486\,194\,379\,256 \ddots
$$
$$
833\,563\,206\,524\,051 +
$$
$$
5\,652\,533\,890\,021\,160\,724\,842\,454\,203\,383\,848\,959\,347\,495\,307\,528\,707\,051 \ddots
$$
$$
640\,068\,130\,104\,834\,691\,250\sqrt{2} -
$$
$$
500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000
$$
$$
\sqrt{}\Big(\Big(20\,745\,870\,270\,538\,534\,750\,286\,890\,954\,461\,038\,728\,892\,152\,106\,450\,145 \ddots
$$
$$
984\,150\,081\,524\,202\,208\,820\,027\,031\,531\,554\,298\,371\,008\,767\,976 \ddots
$$
$$
293\,026\,280\,799\,951\,327\,731\,792\,476\,888\,281\,935\,328\,743\,657 /
$$
$$
80\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \ddots
$$
$$
000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 +
$$
$$
3\,486\,016\,996\,241\,416\,970\,473\,870\,010\,392\,605\,739\,598\,293\,018\,742 \ddots
$$
$$
959\,628\,999\,177\,570\,903\,447\,523\,368\,660\,817\,287\,985\,477\,967\,370 \ddots
$$
$$
211\,592\,798\,319\,333\,420\,850\,917\,022\,715\,148\,181\,047\,370\,399 \Big/
$$
$$
\Big(40\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000 \ddots
$$
$$
000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000
$$
$$
\sqrt{2}\Big)\Big)\Big)\Big/
$$
$$
1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000
$$

Minimal polynomial:

1 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 ⋱
000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 ⋱
000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000
x^4 −
3 854 484 847 047 470 311 089 794 614 447 333 434 551 788 674 377 944 777 517 ⋱
027 334 252 826 096 204 000 000 000 000 000 000 000 000 000 000 000 000 000 ⋱
000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 ⋱
000 000 000 000 000 000 000 000 000 000 000 000 000 x^3 −
251 894 851 663 672 404 079 307 020 848 975 994 453 921 296 632 637 239 084 224 ⋱
590 848 256 674 810 836 158 192 964 917 039 927 678 846 679 602 217 883 976 ⋱
331 982 645 538 626 385 627 564 921 313 796 394 000 000 000 000 000 000 000 ⋱
000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 ⋱
000 000 000 000 x^2 +
432 280 870 652 593 832 133 648 264 757 691 848 056 727 382 926 801 867 145 109 ⋱
018 456 655 555 269 985 633 876 537 046 931 873 203 363 680 161 362 122 102 ⋱
356 748 256 024 520 250 860 332 297 413 571 522 638 869 582 981 048 753 907 ⋱
512 449 372 864 607 030 035 381 396 000 000 000 000 000 000 000 000 000 000 ⋱
000 000 000 000 000 x +
5 662 221 542 901 031 130 527 838 477 815 979 255 584 613 190 511 022 857 640 ⋱
063 755 891 785 042 032 042 649 052 165 488 875 760 542 545 759 233 591 542 ⋱
974 743 023 833 960 655 542 528 776 309 370 313 916 974 699 787 667 580 146 ⋱
752 979 036 187 948 117 180 473 171 262 370 150 546 438 043 848 640 102 455 ⋱
719 924 439 860 761 201

2Pi+10*1/2(sqrt(1/((321/256 + (17 sqrt(353))/256))+64((321/256 + (17 sqrt(353))/256))+20)+ (sqrt(1/((321/256 + (17 sqrt(353))/256))+64((321/256 + (17 sqrt(353))/256))+16)

Input:

$$2\pi + 10\times\frac{1}{2}\left(\sqrt{\frac{1}{\frac{321}{256}+\frac{1}{256}\left(17\sqrt{353}\right)}+64\left(\frac{321}{256}+\frac{1}{256}\left(17\sqrt{353}\right)\right)+20}+\sqrt{\frac{1}{\frac{321}{256}+\frac{1}{256}\left(17\sqrt{353}\right)}+64\left(\frac{321}{256}+\frac{1}{256}\left(17\sqrt{353}\right)\right)+16}\right)$$

Result:

$$5\left(\sqrt{16+\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)}+\sqrt{20+\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)}\right)+2\pi$$

Decimal approximation:

139.88498079783371413593241054644814659154747825496743017 37...

139.884980797…

Property:

$$5\left(\sqrt{16+\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)}+\sqrt{20+\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)}\right)+$$

2π is a transcendental number

Alternate forms:

$$\frac{1}{2}\left(4\pi+95\sqrt{2}+5\sqrt{706}\right)$$

$$\frac{5\left(19+\sqrt{353}\right)}{\sqrt{2}}+2\pi$$

$$\frac{95}{\sqrt{2}}+5\sqrt{\frac{353}{2}}+2\pi$$

Series representations:

$$2\pi+\frac{10}{2}\left(\sqrt{\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)+20}+\sqrt{\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)+16}\right)=$$

$$2\pi+\sum_{k=0}^{\infty}5\binom{\frac{1}{2}}{k}\left(\left(\frac{381}{4}+\frac{17\sqrt{353}}{4}+\frac{256}{321+17\sqrt{353}}\right)^{-k}\sqrt{\frac{123325+11934\sqrt{353}+289\sqrt{353}^{2}}{1284+68\sqrt{353}}}+\left(\frac{397}{4}+\frac{17\sqrt{353}}{4}+\frac{256}{321+17\sqrt{353}}\right)^{-k}\sqrt{\frac{128461+12206\sqrt{353}+289\sqrt{353}^{2}}{1284+68\sqrt{353}}}\right)$$

$$2\pi + \frac{10}{2}\left(\sqrt{\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}} + 64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right) + 20} + \sqrt{\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}} + 64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right) + 16}\right) =$$

$$2\pi + \sum_{k=0}^{\infty} \frac{1}{k!} 5(-1)^k \left(-\frac{1}{2}\right)_k \left(\left(\frac{381}{4}+\frac{17\sqrt{353}}{4}+\frac{256}{321+17\sqrt{353}}\right)^{-k} \sqrt{\frac{123\,325+11\,934\sqrt{353}+289\sqrt{353}^2}{1284+68\sqrt{353}}} + \left(\frac{397}{4}+\frac{17\sqrt{353}}{4}+\frac{256}{321+17\sqrt{353}}\right)^{-k} \sqrt{\frac{128\,461+12\,206\sqrt{353}+289\sqrt{353}^2}{1284+68\sqrt{353}}}\right)$$

$$2\pi + \frac{10}{2}\left(\sqrt{\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}} + 64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right) + 20} + \sqrt{\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}} + 64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right) + 16}\right) =$$

$$2\pi + 5\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\left(\frac{401}{4}+\frac{17\sqrt{353}}{4}+\frac{256}{321+17\sqrt{353}}-z_0\right)^k + \left(\frac{(353+17\sqrt{353})^2}{1284+68\sqrt{353}}-z_0\right)^k\right) z_0^{-k}}{k!} \quad \text{for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

10*1/2(sqrt(1/((321/256 + (17 sqrt(353))/256))+64((321/256 + (17 sqrt(353))/256))+20)+ (sqrt(1/((321/256 + (17 sqrt(353))/256))+64((321/256 + (17 sqrt(353))/256))+16)))-2e-Pi

Input:

$$10 \times \frac{1}{2}\left(\sqrt{\frac{1}{\frac{321}{256}+\frac{1}{256}(17\sqrt{353})} + 64\left(\frac{321}{256}+\frac{1}{256}\left(17\sqrt{353}\right)\right) + 20} + \sqrt{\frac{1}{\frac{321}{256}+\frac{1}{256}(17\sqrt{353})} + 64\left(\frac{321}{256}+\frac{1}{256}\left(17\sqrt{353}\right)\right) + 16}\right) - 2e - \pi$$

Result:

$$5\left(\sqrt{16+\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)}+\sqrt{20+\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)}\right)-2e-\pi$$

Decimal approximation:

125.0236391801462439498239054539043129434414758694421935608...

125.02363918…

Alternate forms:

$$\frac{1}{2}\left(95\sqrt{2}+5\sqrt{706}-4e-2\pi\right)$$

$$\frac{5\left(19+\sqrt{353}\right)}{\sqrt{2}}-2e-\pi$$

$$\frac{95}{\sqrt{2}}+5\sqrt{\frac{353}{2}}-2e-\pi$$

Series representations:

$$\frac{10}{2}\left(\sqrt{\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)+20}+\sqrt{\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)+16}\right)-2e-\pi=$$

$$-2e-\pi+\sum_{k=0}^{\infty}\left(5\binom{\frac{1}{2}}{k}\left(15+\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)\right)^{-k}\sqrt{15+\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)}+5\binom{\frac{1}{2}}{k}\left(19+\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)\right)^{-k}\sqrt{19+\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)}\right)$$

$$\frac{10}{2}\left(\sqrt{\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)+20}+\sqrt{\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)+16}\right)-2e-\pi=$$

$$-2e-\pi+\sum_{k=0}^{\infty}\frac{1}{k!}5(-1)^k\left(-\frac{1}{2}\right)_k\left(\left(\frac{381}{4}+\frac{17\sqrt{353}}{4}+\frac{256}{321+17\sqrt{353}}\right)^{-k}\sqrt{\frac{123\,325+11\,934\sqrt{353}+289\sqrt{353}^2}{1284+68\sqrt{353}}}+\left(\frac{397}{4}+\frac{17\sqrt{353}}{4}+\frac{256}{321+17\sqrt{353}}\right)^{-k}\sqrt{\frac{128\,461+12\,206\sqrt{353}+289\sqrt{353}^2}{1284+68\sqrt{353}}}\right)$$

$$\frac{10}{2}\left(\sqrt{\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)+20}+\sqrt{\frac{1}{\frac{321}{256}+\frac{17\sqrt{353}}{256}}+64\left(\frac{321}{256}+\frac{17\sqrt{353}}{256}\right)+16}\right)-2e-\pi=$$

$$-2e-\pi+5\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\left(\frac{401}{4}+\frac{17\sqrt{353}}{4}+\frac{256}{321+17\sqrt{353}}-z_0\right)^k+\left(\frac{\left(353+17\sqrt{353}\right)^2}{1284+68\sqrt{353}}-z_0\right)^k\right)z_0^{-k}}{k!}\quad \text{for (not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\leq 0))$$

Now, we have that;

Theorem 6.3 (p. 52). *We have*

(i) $$G(e^{-2\pi/\sqrt{5}})=4\left(\frac{\sqrt{5}+1}{2}\right)^{5/2},$$

(ii) $$G(e^{-2\pi})=G(e^{-2\pi/5})=6\cdot 5^{1/4}(3+\sqrt{5}).$$

we obtain:

4((sqrt5+1)/2)^(5/2)

Input:

$$4\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^{5/2}$$

Result:

$$\frac{\left(1+\sqrt{5}\right)^{5/2}}{\sqrt{2}}$$

Decimal approximation:

13.32076270714224485829761403727176765181735256607166293959...

13.320762707142…

Alternate forms:

$$2\sqrt{\left(11+5\sqrt{5}\right)2}$$

$$2\sqrt{22+10\sqrt{5}}$$

Minimal polynomial:

$$x^4-176x^2-256$$

6*5^(1/4) (3+sqrt5)

Input:

$$6\sqrt[4]{5}\left(3+\sqrt{5}\right)$$

Decimal approximation:

46.97848721127463047451117409229587920818333138663904116239...

46.97848721127…

Alternate forms:

$$6\left(3\sqrt[4]{5}+5^{3/4}\right)$$

$$\sqrt[4]{5}\left(18+6\sqrt{5}\right)$$

$$18\sqrt[4]{5}+6\times5^{3/4}$$

Minimal polynomial:

$$x^4 - 2160 x^2 - 103680$$

(Pi+e) *((4((sqrt5+1)/2)^(5/2))) / ((6*5^(1/4) (3+sqrt5))) -(47-4)1/10^3

Input:

$$(\pi + e) \times \frac{4\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^{5/2}}{6\sqrt[4]{5}\left(3+\sqrt{5}\right)} - (47-4) \times \frac{1}{10^3}$$

Result:

$$\frac{\left(1+\sqrt{5}\right)^{5/2}(e+\pi)}{6\sqrt{2}\sqrt[4]{5}\left(3+\sqrt{5}\right)} - \frac{43}{1000}$$

Decimal approximation:

1.618568988332111919490591941207530357391054290795324157644...

1.618568988332...

Alternate forms:

$$\frac{1000\,\pi\sqrt{2\left(11+5\sqrt{5}\right)} + 1000\,e\sqrt{2\left(11+5\sqrt{5}\right)} - 387\sqrt[4]{5} - 129 \times 5^{3/4}}{3000\left(3\sqrt[4]{5}+5^{3/4}\right)}$$

$$\frac{1}{6}\sqrt{2+\frac{2}{\sqrt{5}}}\,(e+\pi) - \frac{43}{1000}$$

$$\frac{1}{3}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\,(e+\pi) - \frac{43}{1000}$$

Series representations:

$$\frac{(\pi+e)\left(4\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^{5/2}\right)}{6\sqrt[4]{5}\left(3+\sqrt{5}\right)} - \frac{47-4}{10^3} =$$

$$\left(-387 - 129\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 50\sqrt{2}\,5^{3/4}\,e\left(1+\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)^{5/2} + 50\sqrt{2}\,5^{3/4}\,\pi\left(1+\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)^{5/2}\right) \Big/ \left(3000\left(3+\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$\frac{(\pi+e)\left(4\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^{5/2}\right)}{6\sqrt[4]{5}\left(3+\sqrt{5}\right)}-\frac{47-4}{10^3}=$$

$$\left(-387-129\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+50\sqrt{2}\,5^{3/4}\,e\left(1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{5/2}+50\sqrt{2}\,5^{3/4}\,\pi\left(1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{5/2}\right)\Bigg/\left(3000\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$\frac{(\pi+e)\left(4\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^{5/2}\right)}{6\sqrt[4]{5}\left(3+\sqrt{5}\right)}-\frac{47-4}{10^3}=\left(-387-129\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+50\sqrt{2}\,5^{3/4}\,e\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)^{5/2}+50\sqrt{2}\,5^{3/4}\,\pi\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)^{5/2}\right)\Bigg/\left(3000\left(3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

e *((4((sqrt5+1)/2)^(5/2))) * ((6*5^(1/4) (3+sqrt5)))+(21+5+2)

Input:

$$e\left(4\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^{5/2}\right)\left(6\sqrt[4]{5}\left(3+\sqrt{5}\right)\right)+(21+5+2)$$

Result:

$$28+3\sqrt{2}\sqrt[4]{5}\left(1+\sqrt{5}\right)^{5/2}\left(3+\sqrt{5}\right)e$$

Decimal approximation:

1729.071629578425684411301619694005250852871070371597469351...

1729.071629578…

Property:

$28+3\sqrt{2}\sqrt[4]{5}\left(1+\sqrt{5}\right)^{5/2}\left(3+\sqrt{5}\right)e$ is a transcendental number

Alternate forms:

$$36\sqrt[4]{5}\,e\sqrt{2\left(11+5\sqrt{5}\right)}+12\times 5^{3/4}\,e\sqrt{2\left(11+5\sqrt{5}\right)}+28$$

$$28+48\sqrt{85+38\sqrt{5}}\,e$$

$$28+84\sqrt[4]{5}\sqrt{2\left(1+\sqrt{5}\right)}\,e+36\times 5^{3/4}\sqrt{2\left(1+\sqrt{5}\right)}\,e$$

Series representations:

$$\left(e\left(6\sqrt[4]{5}\left(3+\sqrt{5}\right)\right)\right)4\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^{5/2}+(21+5+2)=
28+9\sqrt{2}\sqrt[4]{5}\,e\left(1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^{5/2}+
3\sqrt{2}\sqrt[4]{5}\,e\sqrt{4}\left(\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\left(1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^{5/2}$$

$$\left(e\left(6\sqrt[4]{5}\left(3+\sqrt{5}\right)\right)\right)4\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^{5/2}+(21+5+2)=
28+9\sqrt{2}\sqrt[4]{5}\,e\left(1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{5/2}+
3\sqrt{2}\sqrt[4]{5}\,e\sqrt{4}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\left(1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{5/2}$$

$$\left(e\left(6\sqrt[4]{5}\left(3+\sqrt{5}\right)\right)\right)4\left(\frac{1}{2}\left(\sqrt{5}+1\right)\right)^{5/2}+(21+5+2)=
28+9\sqrt{2}\sqrt[4]{5}\,e\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)^{5/2}+3\sqrt{2}\sqrt[4]{5}\,e\sqrt{z_0}
\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\left(1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)^{5/2}$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\le 0$))

From:

$$(9.16)\quad \left(\frac{1}{qf(-q)f(-q^5)f(-q^7)f(-q^{35})}\frac{dv}{dq}\right)^2$$
$$= \frac{v^2}{36}\left\{\frac{49}{R^2}\left(Q^6+\frac{125}{Q^6}+22\right)+R^2\left(P^6+\frac{125}{P^6}+22\right) - 14\sqrt{\left(Q^6+\frac{125}{Q^6}+22\right)\left(P^6+\frac{125}{P^6}+22\right)}\right\}$$
$$= \frac{v^2}{36}\left\{\frac{(V_1-V_2)}{2}\left(\frac{U_1+U_2}{2}+22\right)+\frac{(V_1+V_2)}{2}\left(\frac{U_1-U_2}{2}+22\right) - 14\sqrt{\left(\frac{U_1}{2}+22\right)^2-\frac{U_2^2}{4}}\right\}$$
$$= \frac{v^2}{36}\left\{\frac{1}{2}(U_1V_1-U_2V_2)+22V_1-14\sqrt{\left(\frac{U_1}{2}+22\right)^2-\frac{U_2^2}{4}}\right\}$$
$$= v^2(K^4-4K^3-2K^2-16K-19),$$

we analyze:

$$\frac{v^2}{36}\left\{\frac{49}{R^2}\left(Q^6+\frac{125}{Q^6}+22\right)+R^2\left(P^6+\frac{125}{P^6}+22\right) - 14\sqrt{\left(Q^6+\frac{125}{Q^6}+22\right)\left(P^6+\frac{125}{P^6}+22\right)}\right\}$$

For (3/2 - sqrt(5)/2) = v , Q = 2, P = 3 and for R = S = 1.71536, we obtain

((((3/2 - sqrt(5)/2))^2))/36 * [49/(1.71536^2)*(2^6+125/2^6+22)+1.71536^2(3^6+125/3^6+22)-14 sqrt((2^6+125/2^6+22)(3^6+125/3^6+22))]

Input interpretation:

$$\left(\frac{1}{36}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\right)\left(\frac{49}{1.71536^2}\left(2^6+\frac{125}{2^6}+22\right)+1.71536^2\left(3^6+\frac{125}{3^6}+22\right)-14\sqrt{\left(2^6+\frac{125}{2^6}+22\right)\left(3^6+\frac{125}{3^6}+22\right)}\right)$$

Result:

0.309780894089221985017072436061415250224695400612356878021...

0.30978089408922…

1/2 * 1/(((((((3/2 - sqrt(5)/2))^2))/36 * [49/(1.71536^2)*(2^6+125/2^6+22)+1.71536^2(3^6+125/3^6+22)-14 sqrt((2^6+125/2^6+22)(3^6+125/3^6+22))])))+4/10^3

Input interpretation:

$$\frac{1}{2}\times 1\Big/\left(\left(\frac{1}{36}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\right)\left(\frac{49}{1.71536^2}\left(2^6+\frac{125}{2^6}+22\right)+1.71536^2\left(3^6+\frac{125}{3^6}+22\right)-14\sqrt{\left(2^6+\frac{125}{2^6}+22\right)\left(3^6+\frac{125}{3^6}+22\right)}\right)\right)+\frac{4}{10^3}$$

Result:

1.618044021242936268754416453844496617446768832041739267106...

1.6180440212429…

1/2 * 1/(((((((3/2 - sqrt(5)/2))^2))/36 * [49/(1.71536^2)*(2^6+125/2^6+22)+1.71536^2(3^6+125/3^6+22)-14 sqrt((2^6+125/2^6+22)(3^6+125/3^6+22))])))+4/10^3+26/10^3

Input interpretation:

$$\frac{1}{2}\times 1\Big/\left(\left(\frac{1}{36}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\right)\left(\frac{49}{1.71536^2}\left(2^6+\frac{125}{2^6}+22\right)+1.71536^2\left(3^6+\frac{125}{3^6}+22\right)-14\sqrt{\left(2^6+\frac{125}{2^6}+22\right)\left(3^6+\frac{125}{3^6}+22\right)}\right)\right)+\frac{4}{10^3}+\frac{26}{10^3}$$

Result:

1.644044021242936268754416453844496617446768832041739267106...

1.6440440212429…

((((1/2 /((((((((3/2 - sqrt(5)/2))^2))/36 *
[49/(1.71536^2)*(2^6+125/2^6+22)+1.71536^2(3^6+125/3^6+22)-14
sqrt((2^6+125/2^6+22)(3^6+125/3^6+22))]))))+4/10^3+26/10^3))))^15-Pi-1/2

Input interpretation:

$$\left(\frac{1}{2}\Big/\left(\left(\frac{1}{36}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\right)\left(\frac{49}{1.71536^2}\left(2^6+\frac{125}{2^6}+22\right)+1.71536^2\left(3^6+\frac{125}{3^6}+22\right)-14\sqrt{\left(2^6+\frac{125}{2^6}+22\right)\left(3^6+\frac{125}{3^6}+22\right)}\right)\right)+\frac{4}{10^3}+\frac{26}{10^3}\right)^{15}-\pi-\frac{1}{2}$$

Result:

1728.97…

1728.97… ≈ 1729

We have also:

((((3/2 - sqrt(5)/2))^2))/36 *
[49/(1.71536^2)*(2^6+x/2^6+22)+1.71536^2(3^6+x/3^6+22)-14
sqrt((2^6+x/2^6+22)(3^6+x/3^6+22))] = 0.309781

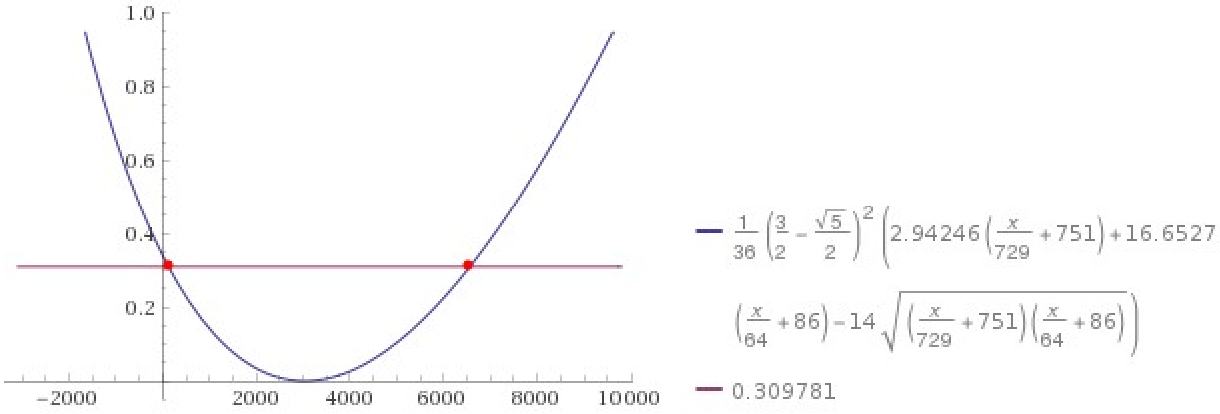
Input interpretation:

$$\left(\frac{1}{36}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\right)\left(\frac{49}{1.71536^2}\left(2^6+\frac{x}{2^6}+22\right)+1.71536^2\left(3^6+\frac{x}{3^6}+22\right)-14\sqrt{\left(2^6+\frac{x}{2^6}+22\right)\left(3^6+\frac{x}{3^6}+22\right)}\right)=0.309781$$

Result:

$$\frac{1}{36}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\left(2.94246\left(\frac{x}{729}+751\right)+16.6527\left(\frac{x}{64}+86\right)-14\sqrt{\left(\frac{x}{729}+751\right)\left(\frac{x}{64}+86\right)}\right)=0.309781$$

Plot:



Alternate forms:

$$\sqrt{x^2+552\,983\,x+3\,013\,324\,416}-4.07677\,x=55\,010.3$$

$$-0.000262677\left(-4.07677\,x+\sqrt{(x+5504)(x+547\,479)}-56\,189.7\right)=0.309781$$

$$-0.000262677\left(\sqrt{x^2+552\,983\,x+3\,013\,324\,416}-4.07677\,x-56\,189.7\right)=0.309781$$

Alternate form assuming x is positive:

$$\sqrt{(x+5504)(x+547\,479)}=4.07677\,x+55\,010.3$$

Expanded form:

$$0.00107087\,x+\frac{7}{12}\sqrt{5}\sqrt{\left(\frac{x}{729}+751\right)\left(\frac{x}{64}+86\right)}-\frac{49}{36}\sqrt{\left(\frac{x}{729}+751\right)\left(\frac{x}{64}+86\right)}+14.7597=0.309781$$

Alternate forms assuming x>0:

$$-0.000262677\sqrt{x^2+552\,983\,x+3\,013\,324\,416}+0.00107087\,x+14.7597=0.309781$$

$$0.011925\left(\frac{x}{729}+751\right)+0.0674889\left(\frac{x}{64}+86\right)-\frac{7}{18}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\sqrt{\left(\frac{x}{729}+751\right)\left(\frac{x}{64}+86\right)}=0.309781$$

Solutions:

$x \approx 125.$

125

$x \approx 6562.15$

((((3/2 - sqrt(5)/2))^2))/36 * [49/(1.71536^2)*(2^6+(x-14)/2^6+22)+1.71536^2(3^6+(x-14)/3^6+22)-14 sqrt((2^6+(x-14)/2^6+22)(3^6+(x-14)/3^6+22))] = 0.309781

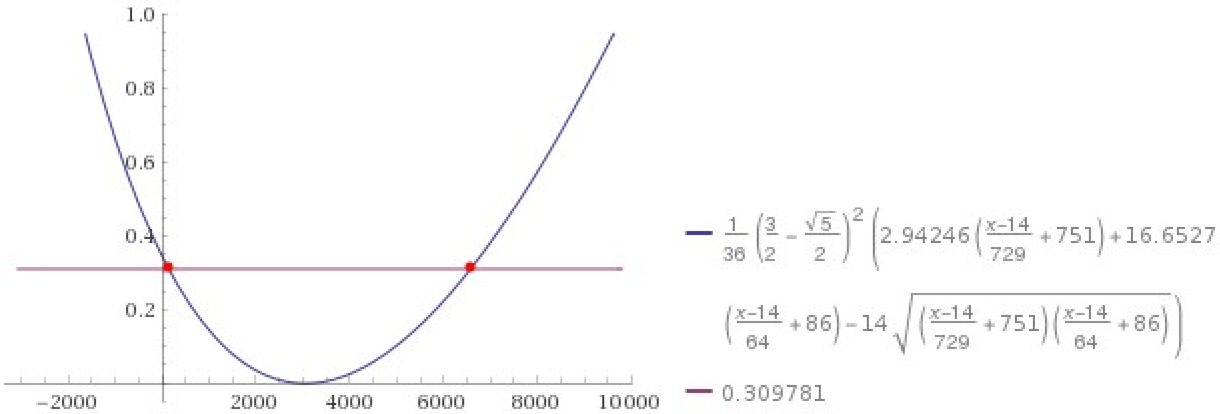
Input interpretation:

$$\left(\frac{1}{36}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\right)\left(\frac{49}{1.71536^2}\left(2^6+\frac{x-14}{2^6}+22\right)+1.71536^2\left(3^6+\frac{x-14}{3^6}+22\right)-14\sqrt{\left(2^6+\frac{x-14}{2^6}+22\right)\left(3^6+\frac{x-14}{3^6}+22\right)}\right)=0.309781$$

Result:

$$\frac{1}{36}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\left(2.94246\left(\frac{x-14}{729}+751\right)+16.6527\left(\frac{x-14}{64}+86\right)-14\sqrt{\left(\frac{x-14}{729}+751\right)\left(\frac{x-14}{64}+86\right)}\right)=0.309781$$

Plot:



Alternate forms:

$$\sqrt{x^2+552\,955\,x+3\,005\,582\,850}-4.07677\,x=54\,953.3$$

$$-0.000262677\left(-4.07677\,x+\sqrt{(x+5490)(x+547\,465)}-56\,132.6\right)=0.309781$$

$$-0.000262677\left(\sqrt{x^2+552\,955\,x+3\,005\,582\,850}-4.07677\,x-56\,132.6\right)=0.309781$$

Alternate form assuming x is positive:

$$\sqrt{(x+5490)(x+547\,465)}=4.07677\,x+54\,953.3$$

Expanded form:

$$0.00107087\,x+\frac{7}{12}\sqrt{5}\sqrt{\left(\frac{x-14}{729}+751\right)\left(\frac{x-14}{64}+86\right)}-\frac{49}{36}\sqrt{\left(\frac{x-14}{729}+751\right)\left(\frac{x-14}{64}+86\right)}+14.7447=0.309781$$

Alternate forms assuming x>0:

$$-0.000262677\sqrt{x^2+552\,955\,x+3\,005\,582\,850}+0.00107087\,x+14.7447=0.309781$$

$$0.011925\left(\frac{x-14}{729}+751\right)+0.0674889\left(\frac{x-14}{64}+86\right)-\frac{7}{18}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\sqrt{\left(\frac{x-14}{729}+751\right)\left(\frac{x-14}{64}+86\right)}=0.309781$$

Solutions:

$x\approx 139.$

139

$x\approx 6576.15$

and:

((((3/2 - sqrt(5)/2))^2))/36 * [49/(1.71536^2)*(2^6+(x-13)/2^6+22)+1.71536^2(3^6+(x-13)/3^6+22)-14 sqrt((2^6+(x-13)/2^6+22)(3^6+(x-13)/3^6+22))] = 0.309781

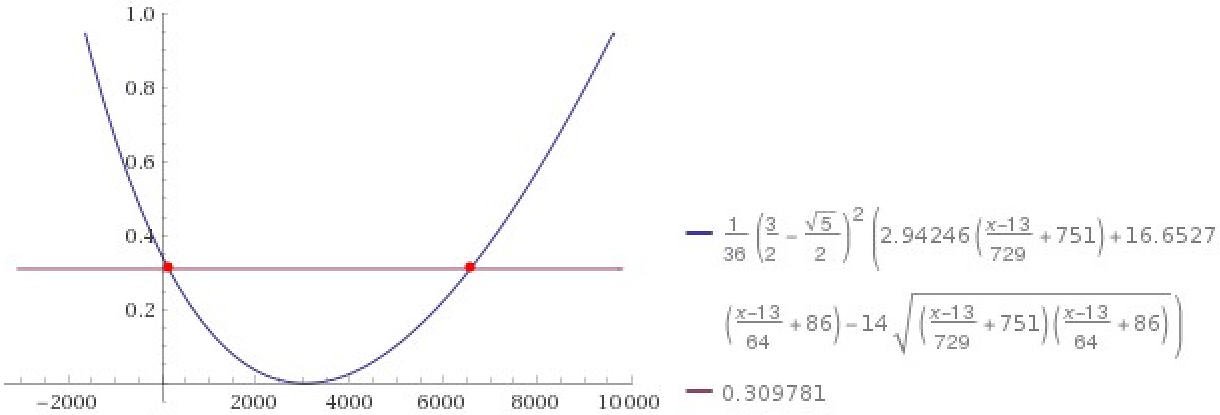
Input interpretation:

$$\left(\frac{1}{36}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\right)\left(\frac{49}{1.71536^2}\left(2^6+\frac{x-13}{2^6}+22\right)+1.71536^2\left(3^6+\frac{x-13}{3^6}+22\right)-14\sqrt{\left(2^6+\frac{x-13}{2^6}+22\right)\left(3^6+\frac{x-13}{3^6}+22\right)}\right)=0.309781$$

Result:

$$\frac{1}{36}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\left(2.94246\left(\frac{x-13}{729}+751\right)+16.6527\left(\frac{x-13}{64}+86\right)-14\sqrt{\left(\frac{x-13}{729}+751\right)\left(\frac{x-13}{64}+86\right)}\right)=0.309781$$

Plot:



Alternate forms:

$$\sqrt{x^2+552957x+3006135806}-4.07677x=54957.3$$

$$-0.000262677\left(-4.07677x+\sqrt{(x+5491)(x+547466)}-56136.7\right)=0.309781$$

$$-0.000262677\left(\sqrt{x^2+552957x+3006135806}-4.07677x-56136.7\right)=0.309781$$

Alternate form assuming x is positive:

$$\sqrt{(x+5491)(x+547466)}=4.07677x+54957.3$$

Expanded form:

$$0.00107087x+\frac{7}{12}\sqrt{5}\sqrt{\left(\frac{x-13}{729}+751\right)\left(\frac{x-13}{64}+86\right)}-\frac{49}{36}\sqrt{\left(\frac{x-13}{729}+751\right)\left(\frac{x-13}{64}+86\right)}+14.7458=0.309781$$

Alternate forms assuming x>0:

$$-0.000262677\sqrt{x^2+552957x+3006135806}+0.00107087x+14.7458=0.309781$$

$$0.011925\left(\frac{x-13}{729}+751\right)+0.0674889\left(\frac{x-13}{64}+86\right)-\frac{7}{18}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\sqrt{\left(\frac{x-13}{729}+751\right)\left(\frac{x-13}{64}+86\right)}=0.309781$$

Solutions:

$x \approx 138.$

138

$x \approx 6575.15$

((((3/2 - sqrt(5)/2))^2))/36 * [49/(1.71536^2)*(2^6+(x-10)/2^6+22)+1.71536^2(3^6+(x-10)/3^6+22)-14 sqrt((2^6+(x-10)/2^6+22)(3^6+(x-10)/3^6+22))] = 0.309781

Input interpretation:

$$\left(\frac{1}{36}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\right)\left(\frac{49}{1.71536^2}\left(2^6+\frac{x-10}{2^6}+22\right)+1.71536^2\left(3^6+\frac{x-10}{3^6}+22\right)-14\sqrt{\left(2^6+\frac{x-10}{2^6}+22\right)\left(3^6+\frac{x-10}{3^6}+22\right)}\right)=0.309781$$

Result:

$$\frac{1}{36}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\left(2.94246\left(\frac{x-10}{729}+751\right)+16.6527\left(\frac{x-10}{64}+86\right)-14\sqrt{\left(\frac{x-10}{729}+751\right)\left(\frac{x-10}{64}+86\right)}\right)=0.309781$$

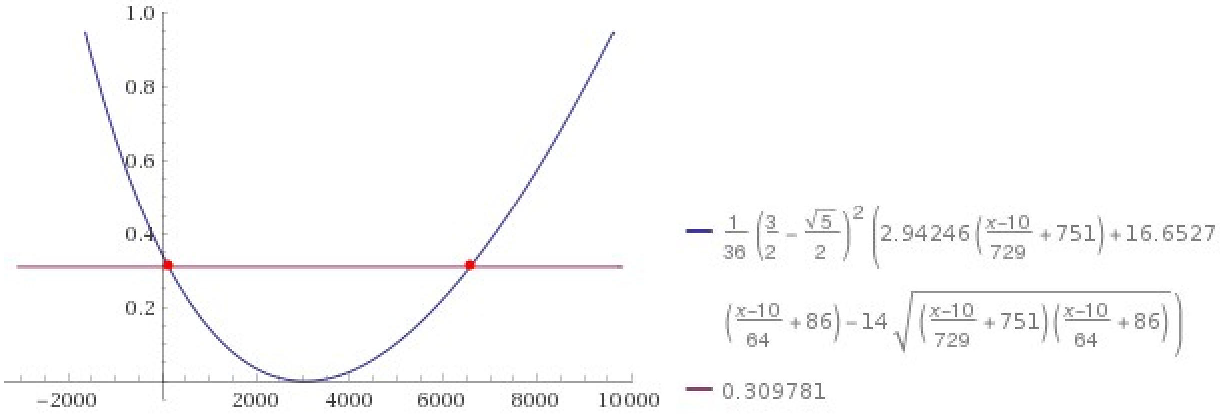
Alternate forms:

$$\sqrt{x^2+552\,963\,x+3\,007\,794\,686}-4.07677\,x=54\,969.6$$

$$-0.000262677\left(-4.07677\,x+\sqrt{(x+5494)(x+547\,469)}-56\,148.9\right)=0.309781$$

$$-0.000262677\left(\sqrt{x^2+552\,963\,x+3\,007\,794\,686}-4.07677\,x-56\,148.9\right)=0.309781$$

Plot:



Alternate form assuming x is positive:

$$\sqrt{(x+5494)(x+547469)} = 4.07677\,x + 54969.6$$

Expanded form:

$$0.00107087\,x + \frac{7}{12}\sqrt{5}\sqrt{\left(\frac{x-10}{729}+751\right)\left(\frac{x-10}{64}+86\right)} - \frac{49}{36}\sqrt{\left(\frac{x-10}{729}+751\right)\left(\frac{x-10}{64}+86\right)} + 14.749 = 0.309781$$

Alternate forms assuming x>0:

$$-0.000262677\sqrt{x^2+552963\,x+3007794686} + 0.00107087\,x + 14.749 = 0.309781$$

$$0.011925\left(\frac{x-10}{729}+751\right) + 0.0674889\left(\frac{x-10}{64}+86\right) - \frac{7}{18}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\sqrt{\left(\frac{x-10}{729}+751\right)\left(\frac{x-10}{64}+86\right)} = 0.309781$$

Solutions:

$x \approx 135.$

135

$x \approx 6572.15$

((((3/2 - sqrt(5)/2))^2))/36 * [49/(1.71536^2)*(2^6+(x-47)/2^6+22)+1.71536^2(3^6+(x-47)/3^6+22)-14 sqrt((2^6+(x-47)/2^6+22)(3^6+(x-47)/3^6+22))] = 0.309781

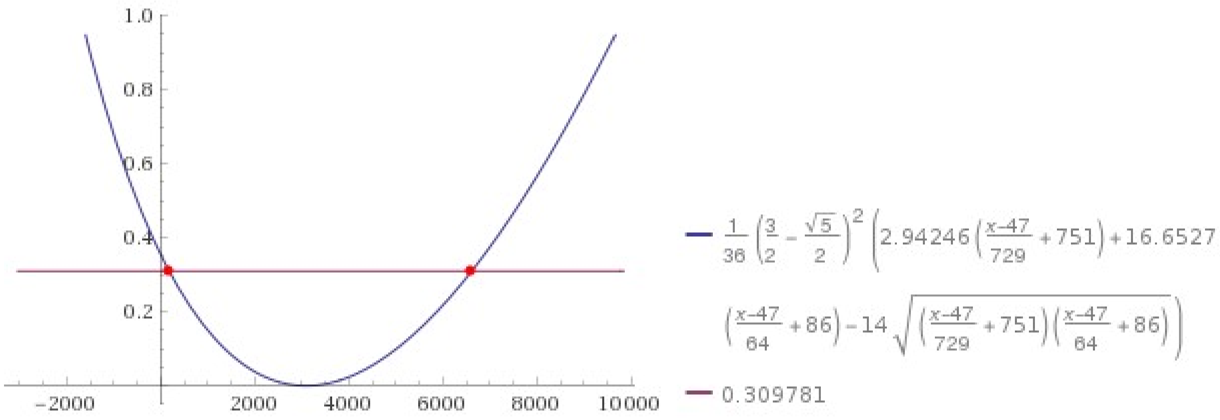
Input interpretation:

$$\left(\frac{1}{36}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\right)\left(\frac{49}{1.71536^2}\left(2^6+\frac{x-47}{2^6}+22\right)+1.71536^2\left(3^6+\frac{x-47}{3^6}+22\right)-14\sqrt{\left(2^6+\frac{x-47}{2^6}+22\right)\left(3^6+\frac{x-47}{3^6}+22\right)}\right)=0.309781$$

Result:

$$\frac{1}{36}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\left(2.94246\left(\frac{x-47}{729}+751\right)+16.6527\left(\frac{x-47}{64}+86\right)-14\sqrt{\left(\frac{x-47}{729}+751\right)\left(\frac{x-47}{64}+86\right)}\right)=0.309781$$

Plot:



Alternate forms:

$$\sqrt{x^2+552\,889\,x+2\,987\,336\,424}-4.07677\,x=54\,818.7$$

$$-0.000262677\left(-4.07677\,x+\sqrt{(x+5457)(x+547\,432)}-55\,998.1\right)=0.309781$$

$$-0.000262677\left(\sqrt{x^2+552\,889\,x+2\,987\,336\,424}-4.07677\,x-55\,998.1\right)=0.309781$$

Alternate form assuming x is positive:

$$\sqrt{(x+5457)(x+547\,432)}=4.07677\,x+54\,818.7$$

Expanded form:

$$0.00107087\,x + \frac{7}{12}\sqrt{5}\sqrt{\left(\frac{x-47}{729}+751\right)\left(\frac{x-47}{64}+86\right)} - \frac{49}{36}\sqrt{\left(\frac{x-47}{729}+751\right)\left(\frac{x-47}{64}+86\right)} + 14.7094 = 0.309781$$

Alternate forms assuming x>0:

$$-0.000262677\sqrt{x^2+552\,889\,x+2\,987\,336\,424} + 0.00107087\,x + 14.7094 = 0.309781$$

$$0.011925\left(\frac{x-47}{729}+751\right) + 0.0674889\left(\frac{x-47}{64}+86\right) - \frac{7}{18}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\sqrt{\left(\frac{x-47}{729}+751\right)\left(\frac{x-47}{64}+86\right)} = 0.309781$$

Solutions:

$x \approx 172.$

172

$x \approx 6609.15$

((((3/2 - sqrt(5)/2))^2))/36 [49/(1.71536^2)*(2^6+(x^(1/3)-46.9999)/2^6+22)+1.71536^2(3^6+(x^(1/3)-46.9999)/3^6+22)-14 sqrt((2^6+(x^(1/3)-46.9999)/2^6+22)(3^6+(x^(1/3)-46.9999)/3^6+22))] = 0.309781

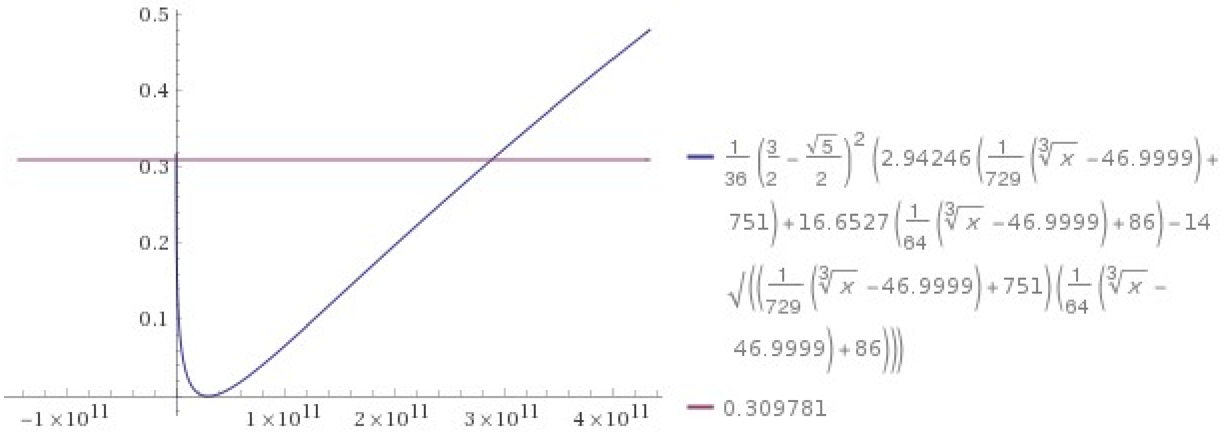
Input interpretation:

$$\left(\frac{1}{36}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\right)\left(\frac{49}{1.71536^2}\left(2^6+\frac{\sqrt[3]{x}-46.9999}{2^6}+22\right)+1.71536^2\left(3^6+\frac{\sqrt[3]{x}-46.9999}{3^6}+22\right)-14\sqrt{\left(2^6+\frac{\sqrt[3]{x}-46.9999}{2^6}+22\right)\left(3^6+\frac{\sqrt[3]{x}-46.9999}{3^6}+22\right)}\right) = 0.309781$$

Result:

$$\frac{1}{36}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2 \left(2.94246\left(\frac{1}{729}\left(\sqrt[3]{x}-46.9999\right)+751\right)+16.6527\left(\frac{1}{64}\left(\sqrt[3]{x}-46.9999\right)+86\right)-14\sqrt{\left(\frac{1}{729}\left(\sqrt[3]{x}-46.9999\right)+751\right)\left(\frac{1}{64}\left(\sqrt[3]{x}-46.9999\right)+86\right)}\right)=0.309781$$

Plot:



Alternate form:

$$\sqrt{\left(\sqrt[3]{x}+5457.\right)\left(\sqrt[3]{x}+547432.\right)}-4.07677\sqrt[3]{x}=54818.7$$

Alternate form assuming x is positive:

$$\sqrt{x^{2/3}+552889.\sqrt[3]{x}+2.98734\times10^{9}}=4.07677\sqrt[3]{x}+54818.7$$

Expanded form:

$$\frac{7}{12}\sqrt{5}\sqrt{\left(\frac{1}{729}\left(\sqrt[3]{x}-46.9999\right)+751\right)\left(\frac{1}{64}\left(\sqrt[3]{x}-46.9999\right)+86\right)}-\frac{49}{36}\sqrt{\left(\frac{1}{729}\left(\sqrt[3]{x}-46.9999\right)+751\right)\left(\frac{1}{64}\left(\sqrt[3]{x}-46.9999\right)+86\right)}+0.00107087\sqrt[3]{x}+14.7094=0.309781$$

Alternate forms assuming x>0:

$$-0.000262677\sqrt{x^{2/3}+552889.\sqrt[3]{x}+2.98734\times10^{9}}+0.00107087\sqrt[3]{x}+14.7094=0.309781$$

$$0.011925\left(\frac{1}{729}\left(\sqrt[3]{x}-46.9999\right)+751\right)+$$
$$0.0674889\left(\frac{1}{64}\left(\sqrt[3]{x}-46.9999\right)+86\right)-\frac{7}{18}\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2$$
$$\sqrt{\left(\frac{1}{729}\left(\sqrt[3]{x}-46.9999\right)+751\right)\left(\frac{1}{64}\left(\sqrt[3]{x}-46.9999\right)+86\right)} = 0.309781$$

Solutions:

$x = 5.0884\times 10^{6}$

$172^3 = 5088448$

$x = 2.88693\times 10^{11}$

Solutions that are equal to the numbers of the following Ramanujan cubes (taxicab):

$$135^3 + 138^3 = 172^3 - 1$$

From:

INTEGRALS ASSOCIATED WITH RAMANUJAN AND ELLIPTIC FUNCTIONS

BRUCE C. BERNDT

Now, we have that:

Theorem 2.1. *We have*

$$\int_{-\infty}^{\infty}\frac{dx}{\cos\sqrt{x}+\cosh\sqrt{x}} = \frac{\pi}{4}\frac{\Gamma^2(\frac{1}{4})}{\Gamma^2(\frac{3}{4})}. \qquad (2.1)$$

((Pi(gamma^2 (1/4)))) / ((4(gamma^2 (3/4))))

Input:

$$\frac{\pi\,\Gamma\left(\frac{1}{4}\right)^2}{4\,\Gamma\left(\frac{3}{4}\right)^2}$$

$\Gamma(x)$ is the gamma function

Decimal approximation:

6.875185818020372827490095779810557197900856451819160896274...

6.875185818…

Alternate forms:

$$\frac{\Gamma\left(\frac{1}{4}\right)^4}{8\,\pi}$$

$$\frac{4\,\pi\,\Gamma\left(\frac{5}{4}\right)^2}{\Gamma\left(\frac{3}{4}\right)^2}$$

$$\frac{9\,\pi\left(\frac{1}{4}\,!\right)^2}{4\left(\frac{3}{4}\,!\right)^2}$$

$n!$ is the factorial function

Alternative representations:

$$\frac{\pi\,\Gamma\left(\frac{1}{4}\right)^2}{4\,\Gamma\left(\frac{3}{4}\right)^2} = \frac{\pi\left(\left(-1+\frac{1}{4}\right)!\right)^2}{4\left(\left(-1+\frac{3}{4}\right)!\right)^2}$$

$$\frac{\pi\,\Gamma\left(\frac{1}{4}\right)^2}{4\,\Gamma\left(\frac{3}{4}\right)^2} = \frac{\pi\,\Gamma\left(\frac{1}{4},\,0\right)^2}{4\,\Gamma\left(\frac{3}{4},\,0\right)^2}$$

$$\frac{\pi\,\Gamma\left(\frac{1}{4}\right)^2}{4\,\Gamma\left(\frac{3}{4}\right)^2} = \frac{\pi\left(\frac{G\left(1+\frac{1}{4}\right)}{G\left(\frac{1}{4}\right)}\right)^2}{4\left(\frac{G\left(1+\frac{3}{4}\right)}{G\left(\frac{3}{4}\right)}\right)^2}$$

Series representations:

$$\frac{\pi\,\Gamma\left(\frac{1}{4}\right)^2}{4\,\Gamma\left(\frac{3}{4}\right)^2} = \frac{\pi\left(\sum_{k=1}^{\infty}\left(\frac{3}{4}\right)^k c_k\right)^2}{4\left(\sum_{k=1}^{\infty} 4^{-k} c_k\right)^2}$$

$$\text{for}\left(c_1 = 1 \text{ and } c_2 = 1 \text{ and } c_k = \frac{\gamma\, c_{-1+k} + \sum_{j=1}^{-2+k}(-1)^{1+j+k} c_j\, \zeta(-j+k)}{-1+k}\right)$$

$$\frac{\pi\,\Gamma\left(\frac{1}{4}\right)^2}{4\,\Gamma\left(\frac{3}{4}\right)^2} = \frac{9\,\pi\left(\sum_{k=0}^{\infty}\frac{4^{-k}\,\Gamma^{(k)}(1)}{k!}\right)^2}{4\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}\right)^k\Gamma^{(k)}(1)}{k!}\right)^2}$$

$$\frac{\pi\,\Gamma\left(\frac{1}{4}\right)^2}{4\,\Gamma\left(\frac{3}{4}\right)^2} = \frac{\pi\left(\sum_{k=0}^{\infty}\frac{\left(\frac{1}{4}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)^2}{4\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{4}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)^2} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$\frac{\pi\,\Gamma\left(\frac{1}{4}\right)^2}{4\,\Gamma\left(\frac{3}{4}\right)^2} = \frac{\pi\left(\sum_{k=0}^{\infty}\left(\frac{3}{4}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}(-j+k)\pi+\pi z_0\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)^2}{4\left(\sum_{k=0}^{\infty}\left(\frac{1}{4}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}(-j+k)\pi+\pi z_0\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)^2}$$

Integral representations:

$$\frac{\pi\,\Gamma\left(\frac{1}{4}\right)^2}{4\,\Gamma\left(\frac{3}{4}\right)^2} = \frac{1}{4}\exp\left(\gamma + \int_0^1 \frac{2\sqrt[4]{x} - 2\,x^{3/4} + \log(x)}{(-1+x)\log(x)}\,dx\right)\pi$$

$$\frac{\pi\,\Gamma\left(\frac{1}{4}\right)^2}{4\,\Gamma\left(\frac{3}{4}\right)^2} = \frac{1}{4}\,e^{\int_0^1 \frac{\left(-1+\sqrt[4]{x}\right)^2}{\left(1+\sqrt{x}\right)\log(x)}\,dx}\,\pi$$

$$\frac{\pi\,\Gamma\left(\frac{1}{4}\right)^2}{4\,\Gamma\left(\frac{3}{4}\right)^2} = \frac{\pi\left(\int_0^1 \frac{1}{\log^{3/4}\left(\frac{1}{t}\right)}\,dt\right)^2}{4\left(\int_0^1 \frac{1}{\sqrt[4]{\log\left(\frac{1}{t}\right)}}\,dt\right)^2}$$

We have that:

$$\int_0^\infty \frac{x^5 dx}{\cos x + \cosh x} = -\frac{\pi^6}{16}\left(\frac{\sqrt{\pi}}{\Gamma^2(\frac{3}{4})}\right)^6 \left\{1 - \frac{16}{4}\right\}\frac{1}{2}$$
$$= \frac{3\pi^9}{32\Gamma^{12}(\frac{3}{4})} = \frac{3\pi^3}{256}\frac{\Gamma^6(\frac{1}{4})}{\Gamma^6(\frac{3}{4})},$$

(3Pi^3 (gamma^6(1/4)))/(256(gamma^6(3/4)))

Input:

$$\frac{3\pi^3\,\Gamma\left(\frac{1}{4}\right)^6}{256\,\Gamma\left(\frac{3}{4}\right)^6}$$

$\Gamma(x)$ is the gamma function

Decimal approximation:

243.7331407513206852001947251977716653431983226563734391776...

243.73314075132…..

Alternate forms:

$$\frac{3\,\Gamma\left(\frac{1}{4}\right)^{12}}{2048\,\pi^3}$$

$$\frac{48\,\pi^3\,\Gamma\left(\frac{5}{4}\right)^6}{\Gamma\left(\frac{3}{4}\right)^6}$$

$$\frac{2187\,\pi^3\left(\frac{1}{4}!\right)^6}{256\left(\frac{3}{4}!\right)^6}$$

Now, we have that:

$$\int_0^\infty \frac{x^9 dx}{\cos x + \cosh x} = \frac{\pi^{10}}{2^6}\left(\frac{\sqrt{\pi}}{\Gamma^2(\frac{3}{4})}\right)^{10} \left\{1 - \frac{1232}{4} + \frac{7936}{16}\right\}\frac{1}{2}$$
$$= \frac{189\pi^{15}}{2^7\Gamma^{20}(\frac{3}{4})} = \frac{189\pi^{15}}{2^7\Gamma^{10}(\frac{3}{4})}\cdot\frac{\Gamma^{10}(\frac{1}{4})}{(\pi\sqrt{2})^{10}} = \frac{3^3\cdot 7\pi^5}{2^{12}}\frac{\Gamma^{10}(\frac{1}{4})}{\Gamma^{10}(\frac{3}{4})}.$$

[3^3 * (7Pi^5) * (gamma^10(1/4))] / [2^12 * (gamma^10(3/4))]

Input:

$$\frac{3^3 (7\pi^5) \Gamma\left(\frac{1}{4}\right)^{10}}{2^{12} \Gamma\left(\frac{3}{4}\right)^{10}}$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{189 \pi^5 \Gamma\left(\frac{1}{4}\right)^{10}}{4096 \Gamma\left(\frac{3}{4}\right)^{10}}$$

Decimal approximation:

725811.7845430244874980537425854957142684872912626410861573...

725811.78454302…

Alternate forms:

$$\frac{189 \Gamma\left(\frac{1}{4}\right)^{20}}{131072 \pi^5}$$

$$\frac{48384 \pi^5 \Gamma\left(\frac{5}{4}\right)^{10}}{\Gamma\left(\frac{3}{4}\right)^{10}}$$

$$\frac{11160261 \pi^5 \left(\frac{1}{4}!\right)^{10}}{4096 \left(\frac{3}{4}!\right)^{10}}$$

$n!$ is the factorial function

Alternative representations:

$$\frac{3^3 \left((7\pi^5) \Gamma\left(\frac{1}{4}\right)^{10}\right)}{2^{12} \Gamma\left(\frac{3}{4}\right)^{10}} = \frac{189 \pi^5 \left(\left(-1+\frac{1}{4}\right)!\right)^{10}}{2^{12} \left(\left(-1+\frac{3}{4}\right)!\right)^{10}}$$

$$\frac{3^3 \left((7\pi^5) \Gamma\left(\frac{1}{4}\right)^{10}\right)}{2^{12} \Gamma\left(\frac{3}{4}\right)^{10}} = \frac{189 \pi^5 \Gamma\left(\frac{1}{4}, 0\right)^{10}}{2^{12} \Gamma\left(\frac{3}{4}, 0\right)^{10}}$$

$$\frac{3^3 \left((7 \pi^5) \Gamma\left(\frac{1}{4}\right)^{10}\right)}{2^{12} \Gamma\left(\frac{3}{4}\right)^{10}} = \frac{189 \pi^5 \left(\frac{G\left(1+\frac{1}{4}\right)}{G\left(\frac{1}{4}\right)}\right)^{10}}{2^{12} \left(\frac{G\left(1+\frac{3}{4}\right)}{G\left(\frac{3}{4}\right)}\right)^{10}}$$

Series representations:

$$\frac{3^3 \left((7 \pi^5) \Gamma\left(\frac{1}{4}\right)^{10}\right)}{2^{12} \Gamma\left(\frac{3}{4}\right)^{10}} = \frac{189 \pi^5 \left(\sum_{k=1}^{\infty} \left(\frac{3}{4}\right)^k c_k\right)^{10}}{4096 \left(\sum_{k=1}^{\infty} 4^{-k} c_k\right)^{10}}$$

$$\text{for } \left(c_1 = 1 \text{ and } c_2 = 1 \text{ and } c_k = \frac{\gamma c_{-1+k} + \sum_{j=1}^{-2+k} (-1)^{1+j+k} c_j \zeta(-j+k)}{-1+k}\right)$$

$$\frac{3^3 \left((7 \pi^5) \Gamma\left(\frac{1}{4}\right)^{10}\right)}{2^{12} \Gamma\left(\frac{3}{4}\right)^{10}} = \frac{11\,160\,261 \pi^5 \left(\sum_{k=0}^{\infty} \frac{4^{-k} \Gamma^{(k)}(1)}{k!}\right)^{10}}{4096 \left(\sum_{k=0}^{\infty} \frac{\left(\frac{3}{4}\right)^k \Gamma^{(k)}(1)}{k!}\right)^{10}}$$

$$\frac{3^3 \left((7 \pi^5) \Gamma\left(\frac{1}{4}\right)^{10}\right)}{2^{12} \Gamma\left(\frac{3}{4}\right)^{10}} = \frac{189 \pi^5 \left(\sum_{k=0}^{\infty} \frac{\left(\frac{1}{4}-z_0\right)^k \Gamma^{(k)}(z_0)}{k!}\right)^{10}}{4096 \left(\sum_{k=0}^{\infty} \frac{\left(\frac{3}{4}-z_0\right)^k \Gamma^{(k)}(z_0)}{k!}\right)^{10}} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$\frac{3^3 \left((7 \pi^5) \Gamma\left(\frac{1}{4}\right)^{10}\right)}{2^{12} \Gamma\left(\frac{3}{4}\right)^{10}} = \frac{189 \pi^5 \left(\sum_{k=0}^{\infty} \left(\frac{3}{4} - z_0\right)^k \sum_{j=0}^{k} \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2}(-j+k)\pi + \pi z_0\right) \Gamma^{(j)}(1-z_0)}{j!(-j+k)!}\right)^{10}}{4096 \left(\sum_{k=0}^{\infty} \left(\frac{1}{4} - z_0\right)^k \sum_{j=0}^{k} \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2}(-j+k)\pi + \pi z_0\right) \Gamma^{(j)}(1-z_0)}{j!(-j+k)!}\right)^{10}}$$

Integral representations:

$$\frac{3^3 \left((7 \pi^5) \Gamma\left(\frac{1}{4}\right)^{10}\right)}{2^{12} \Gamma\left(\frac{3}{4}\right)^{10}} = \frac{189 \exp\left(5 \gamma + \int_0^1 \frac{5\left(2 \sqrt[4]{x} - 2 x^{3/4} + \log(x)\right)}{(-1+x)\log(x)} \, dx\right) \pi^5}{4096}$$

$$\frac{3^3 \left((7 \pi^5) \Gamma\left(\frac{1}{4}\right)^{10}\right)}{2^{12} \Gamma\left(\frac{3}{4}\right)^{10}} = \frac{189 \exp\left(\int_0^1 \frac{5\left(-1+\sqrt[4]{x}\right)^2}{\left(1+\sqrt{x}\right)\log(x)} \, dx\right) \pi^5}{4096}$$

$$\frac{3^3\left((7\pi^5)\,\Gamma\left(\frac{1}{4}\right)^{10}\right)}{2^{12}\,\Gamma\left(\frac{3}{4}\right)^{10}} = \frac{189\,\pi^5\left(\int_0^1 \frac{1}{\log^{3/4}\left(\frac{1}{t}\right)}\,dt\right)^{10}}{4096\left(\int_0^1 \frac{1}{\sqrt[4]{\log\left(\frac{1}{t}\right)}}\,dt\right)^{10}}$$

Dividing the three results and adding 64, we obtain:

(725811.784543024487 / 243.73314075132 *1 / 6.8751858)+64

Input interpretation:

$$\frac{725\,811.784543024487}{243.73314075132} \times \frac{1}{6.8751858} + 64$$

Result:

497.1367076705681966073657257157125181510453958183848065286...

497.1367.... result practically equal to the rest mass of Kaon meson 497.614

From the above expression:

$$\frac{189\,\pi^5\,\Gamma\left(\frac{1}{4}\right)^{10}}{4096\,\Gamma\left(\frac{3}{4}\right)^{10}}$$

we obtain also:

(189 π^5 Γ(1/4)^10)/ (((x-1)/27)^2* Γ(3/4)^10) = 725811.784543024487498

Input interpretation:

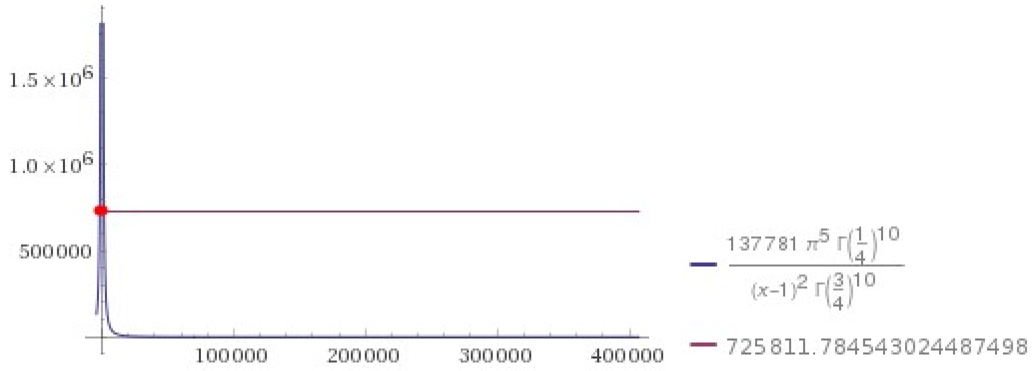
$$\frac{189\,\pi^5\,\Gamma\left(\frac{1}{4}\right)^{10}}{\left(\frac{x-1}{27}\right)^2\,\Gamma\left(\frac{3}{4}\right)^{10}} = 725\,811.784543024487498$$

$\Gamma(x)$ is the gamma function

Result:

$$\frac{137\,781\,\pi^5\,\Gamma\left(\frac{1}{4}\right)^{10}}{(x-1)^2\,\Gamma\left(\frac{3}{4}\right)^{10}} = 725\,811.784543024487498$$

Plot:



Alternate form assuming x is real:

$$1.0000000000000\, x = 1.000000000 - \frac{2.98598400000 \times 10^{6}}{1.00000000000000 - 1.0000000000000\, x}$$

Alternate forms:

$$\frac{-1.00000000000000000\, x^{2} + 2.000000000000\, x + 2.985983000000000 \times 10^{6}}{1.0000000000000000000 - 1.0000000000000000000\, x} = 0$$

$$\frac{4\,623\,163\,195\,392\, (2+\sqrt{2})^{10}\, K\!\left(\frac{(-2-2\sqrt{2})^{2}}{(4+2\sqrt{2})^{2}}\right)^{10}}{(4+2\sqrt{2})^{10}\,(x-1)^{2}} = 725\,811.784543024487498$$

$K(m)$ is the complete elliptic integral of the first kind with parameter $m = k^2$

Alternate form assuming x is positive:

$$-\frac{1728.000000000000}{1.00000000000000000 - 1.00000000000000000\, x} = 1.0000000000000000$$

Solutions:

$x \approx -1727.0000000000000000$

$x \approx 1729.0000000000000000$

Integer solutions:

$x = -1727$

$x = 1729$

1729

((x+55-5) π^5 Γ(1/4)^10)/(4096 Γ(3/4)^10) = 725811.784543024487498

Input interpretation:

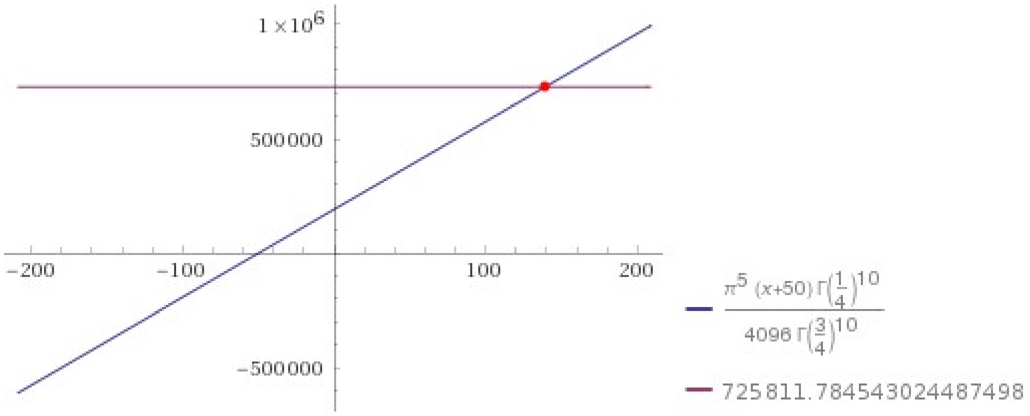
$$\frac{(x+55-5)\,\pi^5\,\Gamma\left(\frac{1}{4}\right)^{10}}{4096\,\Gamma\left(\frac{3}{4}\right)^{10}} = 725\,811.784543024487498$$

$\Gamma(x)$ is the gamma function

Result:

$$\frac{\pi^5\,(x+50)\,\Gamma\left(\frac{1}{4}\right)^{10}}{4096\,\Gamma\left(\frac{3}{4}\right)^{10}} = 725\,811.784543024487498$$

Plot:



Alternate forms:

$$1.000000000000000000\,x = 139.0000000000000000$$

$$\frac{256\,\pi^5\,(x+50)\,\Gamma\left(\frac{5}{4}\right)^{10}}{\Gamma\left(\frac{3}{4}\right)^{10}} = 725\,811.784543024487498$$

$$\frac{\pi^5\,x\,\Gamma\left(\frac{1}{4}\right)^{10}}{4096\,\Gamma\left(\frac{3}{4}\right)^{10}} - 533\,798.084928467744774 = 0$$

Expanded form:

$$\frac{\pi^5\,x\,\Gamma\left(\frac{1}{4}\right)^{10}}{4096\,\Gamma\left(\frac{3}{4}\right)^{10}} + \frac{25\,\pi^5\,\Gamma\left(\frac{1}{4}\right)^{10}}{2048\,\Gamma\left(\frac{3}{4}\right)^{10}} = 725\,811.784543024487498$$

Solution:

$x \approx 139.00000000000000000$

Integer solution:

$x = 139$

139

((x+64) π^5 Γ(1/4)^10)/(4096 Γ(3/4)^10) = 725811.784543024487498

Input interpretation:

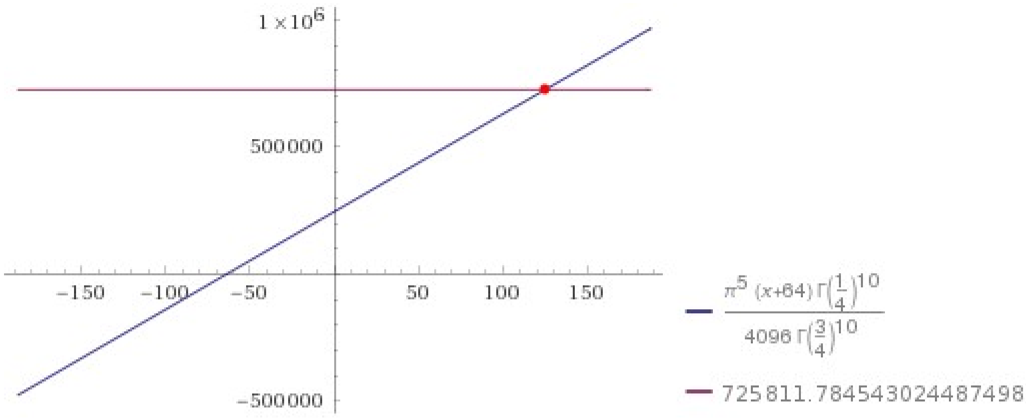
$$\frac{(x+64)\,\pi^5\,\Gamma\left(\frac{1}{4}\right)^{10}}{4096\,\Gamma\left(\frac{3}{4}\right)^{10}} = 725\,811.784543024487498$$

$\Gamma(x)$ is the gamma function

Result:

$$\frac{\pi^5\,(x+64)\,\Gamma\left(\frac{1}{4}\right)^{10}}{4096\,\Gamma\left(\frac{3}{4}\right)^{10}} = 725\,811.784543024487498$$

Plot:



Alternate forms:

$1.0000000000000000000\,x = 125.0000000000000000$

$$\frac{256\,\pi^5\,(x+64)\,\Gamma\left(\frac{5}{4}\right)^{10}}{\Gamma\left(\frac{3}{4}\right)^{10}} = 725\,811.784543024487498$$

$$\frac{\pi^5 x \Gamma\left(\frac{1}{4}\right)^{10}}{4096 \Gamma\left(\frac{3}{4}\right)^{10}} - 480034.249036391856811 = 0$$

Expanded form:

$$\frac{\pi^5 x \Gamma\left(\frac{1}{4}\right)^{10}}{4096 \Gamma\left(\frac{3}{4}\right)^{10}} + \frac{\pi^5 \Gamma\left(\frac{1}{4}\right)^{10}}{64 \Gamma\left(\frac{3}{4}\right)^{10}} = 725811.784543024487498$$

Solution:

$x \approx 125.00000000000000000$

Integer solution:

$x = 125$

125

From:

RAMANUJAN'S THEORIES OF ELLIPTIC FUNCTIONS TO ALTERNATIVE BASES
Bruce C. Berndt, S. Bhargava, and Frank G. Garvan

From

Theorem 2.12 (p. 257). *Let $q = q_3$ and $z = z(3)$. Then*

$$z = 1 + 6\sum_{n=1}^{\infty} \frac{q^n}{1 + q^n + q^{2n}}.$$

$$\beta = \frac{27p^2(1+p)^2}{4(1+p+p^2)^3}.$$

$0 < p < 1$

$|q| < 1.$

for n = 2, q = 0.5 , we obtain:

1+6 * (0.5^2) / (1+0.5^2+0.5^4)

Input:

$$1+6\times\frac{0.5^2}{1+0.5^2+0.5^4}$$

Result:

2.142857142857142857142857142857142857142857142857142857142...

2.142857142857…

Repeating decimal:

$2.\overline{142857}$ (period 6)

z(3) = z = 2.142857

from

$$\beta=\frac{27p^2(1+p)^2}{4(1+p+p^2)^3}.$$

for p = 1/5 = 0.2 , we obtain:

((27*0.2^2(1+0.2)^2)) / ((4(1+0.2+0.2^2)^3))

Input:

$$\frac{27\times0.2^2\,(1+0.2)^2}{4\,(1+0.2+0.2^2)^3}$$

Result:

0.203920647175321405793696082709543150615957839615991406800...

Repeating decimal:

0.203920647175321405793696082709543150615957839615991406800...

(period 14415)

β = 0.203920647….

From:

$$
\begin{aligned}
f(-q_3^{3}) - f(-q^2) - \sqrt{z}2^{-\frac{1}{8}}\{\alpha(1-\alpha)/q\}^{\frac{1}{12}} \\
&= \frac{(1+2p)^{\frac{1}{4}}}{(1+p+p^2)^{\frac{1}{2}}}\sqrt{z(3)}2^{\ \frac{1}{3}}\left(\frac{p^3(2+p)(1-p)(1+p)^3}{(1+2p)^2}\right)^{\frac{1}{12}}\frac{1}{q_3^{\frac{1}{8}}} \\
&= \frac{\sqrt{z(3)}}{q_3^{\frac{1}{8}}3^{\frac{3}{8}}}\left(\frac{27p^2(1+p)^2}{4(1+p+p^2)^3}\right)^{\frac{1}{8}}\left(\frac{(2+p)^2(1+2p)^2(1-p)^2}{4(1+p+p^2)^3}\right)^{\frac{1}{24}} \\
&= \frac{\sqrt{z(3)}}{q_3^{\frac{1}{8}}3^{\frac{3}{8}}}\beta^{\frac{1}{8}}(1-\beta)^{\frac{1}{24}},
\end{aligned}
$$

(((2.142857)^(1/2) *(0.203920647)^(1/8) (1-(0.203920647))^(1/24))) / (((0.5)^(1/8) * 3^(3/8)))

Input interpretation:

$$\frac{\sqrt{2.142857}\ \sqrt[8]{0.203920647}\ \sqrt[24]{1-0.203920647}}{\sqrt[8]{0.5}\times 3^{3/8}}$$

Result:

0.8585403...

0.8585403...

From which:

sqrt(((Pi [(((2.142857)^(1/2) *(0.203920647)^(1/8) (1-(0.203920647))^(1/24))) / (((0.5)^(1/8) * 3^(3/8)))])))

Input interpretation:

$$\sqrt{\pi\times\frac{\sqrt{2.142857}\ \sqrt[8]{0.203920647}\ \sqrt[24]{1-0.203920647}}{\sqrt[8]{0.5}\times 3^{3/8}}}$$

Result:

1.642311...

1.642311…

((((sqrt(((Pi [(((2.142857)^(1/2) *(0.203920647)^(1/8) (1-(0.203920647))^(1/24))) / (((0.5)^(1/8) * 3^(3/8)))])))))))^15+24-2/5

Input interpretation:

$$\sqrt{\pi \times \frac{\sqrt{2.142857}\ \sqrt[8]{0.203920647}\ \sqrt[24]{1-0.203920647}}{\sqrt[8]{0.5} \times 3^{3/8}}}^{15} + 24 - \frac{2}{5}$$

Result:

1729.01…

1729.01…

Series representations:

$$\sqrt{\frac{\pi\left(\sqrt{2.14286}\ \sqrt[8]{0.203921}\ \sqrt[24]{1-0.203921}\right)}{\sqrt[8]{0.5}\ 3^{3/8}}}^{15} + 24 - \frac{2}{5} = \frac{118}{5} + \sqrt{-1+0.85854\pi}^{15} \left(\sum_{k=0}^{\infty} (-1+0.85854\pi)^{-k} \binom{\frac{1}{2}}{k}\right)^{15}$$

$$\sqrt{\frac{\pi\left(\sqrt{2.14286}\ \sqrt[8]{0.203921}\ \sqrt[24]{1-0.203921}\right)}{\sqrt[8]{0.5}\ 3^{3/8}}}^{15} + 24 - \frac{2}{5} = \frac{118}{5} + \sqrt{-1+0.85854\pi}^{15} \left(\sum_{k=0}^{\infty} \frac{(-1)^k (-1+0.85854\pi)^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)^{15}$$

$$\sqrt{\frac{\pi\left(\sqrt{2.14286}\ \sqrt[8]{0.203921}\ \sqrt[24]{1-0.203921}\right)}{\sqrt[8]{0.5}\ 3^{3/8}}}^{15} + 24 - \frac{2}{5} = \frac{118}{5} + \sqrt{z_0}^{15} \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (0.85854\pi - z_0)^k z_0^{-k}}{k!}\right)^{15}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

48Pi * 1/ ((((((2.142857)^(1/2) *(0.203920647)^(1/8) (1-(0.203920647))^(1/24))) 1 / (((0.5)^(1/8) * 3^(3/8))))))-29-7

Input interpretation:

$$48\pi\times\frac{1}{\left(\sqrt{2.142857}\,\sqrt[8]{0.203920647}\,\sqrt[24]{1-0.203920647}\right)\times\frac{1}{\sqrt[8]{0.5}\times 3^{3/8}}}-29-7$$

Result:

139.6428...

139.6428…

Series representations:

$$\frac{48\pi}{\frac{\sqrt{2.14286}\,\sqrt[8]{0.203921}\,\sqrt[24]{1-0.203921}}{\sqrt[8]{0.5}\,3^{3/8}}}-29-7=-36+223.635\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\frac{48\pi}{\frac{\sqrt{2.14286}\,\sqrt[8]{0.203921}\,\sqrt[24]{1-0.203921}}{\sqrt[8]{0.5}\,3^{3/8}}}-29-7=-147.818+111.818\sum_{k=1}^{\infty}\frac{2^k}{\binom{2k}{k}}$$

$$\frac{48\pi}{\frac{\sqrt{2.14286}\,\sqrt[8]{0.203921}\,\sqrt[24]{1-0.203921}}{\sqrt[8]{0.5}\,3^{3/8}}}-29-7=-36+55.9088\sum_{k=0}^{\infty}\frac{2^{-k}(-6+50k)}{\binom{3k}{k}}$$

Integral representations:

$$\frac{48\pi}{\frac{\sqrt{2.14286}\,\sqrt[8]{0.203921}\,\sqrt[24]{1-0.203921}}{\sqrt[8]{0.5}\,3^{3/8}}}-29-7=-36+111.818\int_0^{\infty}\frac{1}{1+t^2}\,dt$$

$$\frac{48\pi}{\frac{\sqrt{2.14286}\,\sqrt[8]{0.203921}\,\sqrt[24]{1-0.203921}}{\sqrt[8]{0.5}\,3^{3/8}}}-29-7=-36+223.635\int_0^{1}\sqrt{1-t^2}\,dt$$

$$\frac{48\pi}{\frac{\sqrt{2.14286}\,\sqrt[8]{0.203921}\,\sqrt[24]{1-0.203921}}{\sqrt[8]{0.5}\,3^{3/8}}}-29-7=-36+111.818\int_0^{\infty}\frac{\sin(t)}{t}\,dt$$

48Pi * 1/ ((((((2.142857)^(1/2) *(0.203920647)^(1/8) (1-(0.203920647))^(1/24))) 1 / (((0.5)^(1/8) * 3^(3/8))))))-55+5

Input interpretation:

$$48\,\pi\times\frac{1}{\left(\sqrt{2.142857}\,\sqrt[8]{0.203920647}\,\sqrt[24]{1-0.203920647}\right)\times\frac{1}{\sqrt[8]{0.5}\times 3^{3/8}}}-55+5$$

Result:

125.6428...

125.6428…

Series representations:

$$\frac{48\,\pi}{\frac{\sqrt{2.14286}\,\sqrt[8]{0.203921}\,\sqrt[24]{1-0.203921}}{\sqrt[8]{0.5}\,3^{3/8}}}-55+5=-50+223.635\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2\,k}$$

$$\frac{48\,\pi}{\frac{\sqrt{2.14286}\,\sqrt[8]{0.203921}\,\sqrt[24]{1-0.203921}}{\sqrt[8]{0.5}\,3^{3/8}}}-55+5=-161.818+111.818\sum_{k=1}^{\infty}\frac{2^k}{\binom{2\,k}{k}}$$

$$\frac{48\,\pi}{\frac{\sqrt{2.14286}\,\sqrt[8]{0.203921}\,\sqrt[24]{1-0.203921}}{\sqrt[8]{0.5}\,3^{3/8}}}-55+5=-50+55.9088\sum_{k=0}^{\infty}\frac{2^{-k}\,(-6+50\,k)}{\binom{3\,k}{k}}$$

Integral representations:

$$\frac{48\,\pi}{\frac{\sqrt{2.14286}\,\sqrt[8]{0.203921}\,\sqrt[24]{1-0.203921}}{\sqrt[8]{0.5}\,3^{3/8}}}-55+5=-50+111.818\int_0^{\infty}\frac{1}{1+t^2}\,dt$$

$$\frac{48\,\pi}{\frac{\sqrt{2.14286}\,\sqrt[8]{0.203921}\,\sqrt[24]{1-0.203921}}{\sqrt[8]{0.5}\,3^{3/8}}}-55+5=-50+223.635\int_0^{1}\sqrt{1-t^2}\,dt$$

$$\frac{48\,\pi}{\frac{\sqrt{2.14286}\,\sqrt[8]{0.203921}\,\sqrt[24]{1-0.203921}}{\sqrt[8]{0.5}\,3^{3/8}}}-55+5=-50+111.818\int_0^{\infty}\frac{\sin(t)}{t}\,dt$$

We have that:

Second proof of Theorem 4.5

$$N(q_3^3) = N(q^2) = z^6(1+\alpha)(1-\frac{1}{2}\alpha)(1-2\alpha)$$
$$= \frac{z^6(3)}{2(1+p+p^2)^6}\left(1+2p+p^3(2+p)\right)\left(2(1+2p)-p^3(2+p)\right)\left(1+2p-2p^3(2+p)\right)$$
$$= \frac{z^6(3)}{2(1+p+p^2)^6}\{2(1+p+p^2)^6 - 18p^2(1+p)^2(1+p+p^2)^3 + 27p^4(1+p)^4\}$$
$$= z^6(3)\left(1-\frac{4}{3}\beta+\frac{8}{27}\beta^2\right).$$

2.142857^6 (1-4/3*0.203920647+8/27*0.203920647^2)

Input interpretation:

$$2.142857^6\left(1+\frac{4}{3}\times(-0.203920647)+\frac{8}{27}\times0.203920647^2\right)$$

Result:

71.68715035647667828518408945516330508507917774183871853066...

71.6871503564766…

2.142857^6 (1-4/3*0.203920647+8/27*0.203920647^2) - 7 - 1/golden ratio

Input interpretation:

$$2.142857^6\left(1+\frac{4}{3}\times(-0.203920647)+\frac{8}{27}\times0.203920647^2\right)-7-\frac{1}{\phi}$$

ϕ is the golden ratio

Result:

64.0691...

64.0691…

Alternative representations:

$$2.14286^6 \left(1 - \frac{0.203921 \times 4}{3} + \frac{0.203921^2 \times 8}{27}\right) - 7 - \frac{1}{\phi} = -7 + 2.14286^6 \left(0.728106 + \frac{8 \times 0.203921^2}{27}\right) - \frac{1}{2 \sin(54°)}$$

$$2.14286^6 \left(1 - \frac{0.203921 \times 4}{3} + \frac{0.203921^2 \times 8}{27}\right) - 7 - \frac{1}{\phi} = -7 + 2.14286^6 \left(0.728106 + \frac{8 \times 0.203921^2}{27}\right) - -\frac{1}{2 \cos(216°)}$$

$$2.14286^6 \left(1 - \frac{0.203921 \times 4}{3} + \frac{0.203921^2 \times 8}{27}\right) - 7 - \frac{1}{\phi} = -7 + 2.14286^6 \left(0.728106 + \frac{8 \times 0.203921^2}{27}\right) - -\frac{1}{2 \sin(666°)}$$

2 * ((2.142857^6 (1-4/3*0.203920647+8/27*0.203920647^2) - 7 - 1/golden ratio))-3

Input interpretation:

$$2\left(2.142857^6 \left(1 + \frac{4}{3} \times (-0.203920647) + \frac{8}{27} \times 0.203920647^2\right) - 7 - \frac{1}{\phi}\right) - 3$$

ϕ is the golden ratio

Result:

125.138...

125.138…

Alternative representations:

$$2\left(2.14286^6 \left(1 - \frac{0.203921 \times 4}{3} + \frac{0.203921^2 \times 8}{27}\right) - 7 - \frac{1}{\phi}\right) - 3 = -3 + 2\left(-7 + 2.14286^6 \left(0.728106 + \frac{8 \times 0.203921^2}{27}\right) - \frac{1}{2 \sin(54°)}\right)$$

$$2\left(2.14286^6 \left(1 - \frac{0.203921 \times 4}{3} + \frac{0.203921^2 \times 8}{27}\right) - 7 - \frac{1}{\phi}\right) - 3 = -3 + 2\left(-7 + 2.14286^6 \left(0.728106 + \frac{8 \times 0.203921^2}{27}\right) - -\frac{1}{2 \cos(216°)}\right)$$

$$2\left(2.14286^6\left(1-\frac{0.203921\times 4}{3}+\frac{0.203921^2\times 8}{27}\right)-7-\frac{1}{\phi}\right)-3 = -3+2\left(-7+2.14286^6\left(0.728106+\frac{8\times 0.203921^2}{27}\right)--\frac{1}{2\sin(666^\circ)}\right)$$

27((2.142857^6 (1-4/3*0.203920647+8/27*0.203920647^2)-Pi-4-1/2))

Input interpretation:

$$27\left(2.142857^6\left(1+\frac{4}{3}\times(-0.203920647)+\frac{8}{27}\times 0.203920647^2\right)-\pi-4-\frac{1}{2}\right)$$

Result:

1729.23...

1729.23…

With regard 27 (From Wikipedia):

*"The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group **Z**/3**Z**, and its outer automorphism group is the cyclic group **Z**/2**Z**. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories".*

Alternative representations:

$$27\left(2.14286^6\left(1-\frac{0.203921\times 4}{3}+\frac{0.203921^2\times 8}{27}\right)-\pi-4-\frac{1}{2}\right) = 27\left(-\frac{9}{2}-180^\circ+2.14286^6\left(0.728106+\frac{8\times 0.203921^2}{27}\right)\right)$$

$$27\left(2.14286^6\left(1-\frac{0.203921\times 4}{3}+\frac{0.203921^2\times 8}{27}\right)-\pi-4-\frac{1}{2}\right) = 27\left(-\frac{9}{2}+i\log(-1)+2.14286^6\left(0.728106+\frac{8\times 0.203921^2}{27}\right)\right)$$

$$27\left(2.14286^6\left(1-\frac{0.203921\times 4}{3}+\frac{0.203921^2\times 8}{27}\right)-\pi-4-\frac{1}{2}\right)=$$
$$27\left(-\frac{9}{2}-\cos^{-1}(-1)+2.14286^6\left(0.728106+\frac{8\times 0.203921^2}{27}\right)\right)$$

Series representations:

$$27\left(2.14286^6\left(1-\frac{0.203921\times 4}{3}+\frac{0.203921^2\times 8}{27}\right)-\pi-4-\frac{1}{2}\right)=$$
$$1814.05-108\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$27\left(2.14286^6\left(1-\frac{0.203921\times 4}{3}+\frac{0.203921^2\times 8}{27}\right)-\pi-4-\frac{1}{2}\right)=$$
$$1868.05-54\sum_{k=1}^{\infty}\frac{2^k}{\binom{2k}{k}}$$

$$27\left(2.14286^6\left(1-\frac{0.203921\times 4}{3}+\frac{0.203921^2\times 8}{27}\right)-\pi-4-\frac{1}{2}\right)=$$
$$1814.05-27\sum_{k=0}^{\infty}\frac{2^{-k}(-6+50k)}{\binom{3k}{k}}$$

Integral representations:

$$27\left(2.14286^6\left(1-\frac{0.203921\times 4}{3}+\frac{0.203921^2\times 8}{27}\right)-\pi-4-\frac{1}{2}\right)=$$
$$1814.05-54\int_0^{\infty}\frac{1}{1+t^2}\,dt$$

$$27\left(2.14286^6\left(1-\frac{0.203921\times 4}{3}+\frac{0.203921^2\times 8}{27}\right)-\pi-4-\frac{1}{2}\right)=$$
$$1814.05-108\int_0^{1}\sqrt{1-t^2}\,dt$$

$$27\left(2.14286^6\left(1-\frac{0.203921\times 4}{3}+\frac{0.203921^2\times 8}{27}\right)-\pi-4-\frac{1}{2}\right)=$$
$$1814.05-54\int_0^{\infty}\frac{\sin(t)}{t}\,dt$$

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson:

125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] *The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.*

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

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