On some integrals of theta-functions and incomplete elliptic integrals of the first kind: new possible mathematical connections with ϕ, $\zeta(2)$, and various parameters of Particle Physics

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Abstract

In this paper we have described some Ramanujan's integrals of theta-functions and incomplete elliptic integrals of the first kind. Furthermore, we describe new possible mathematical connections with ϕ, $\zeta(2)$, and various parameters of Particle Physics

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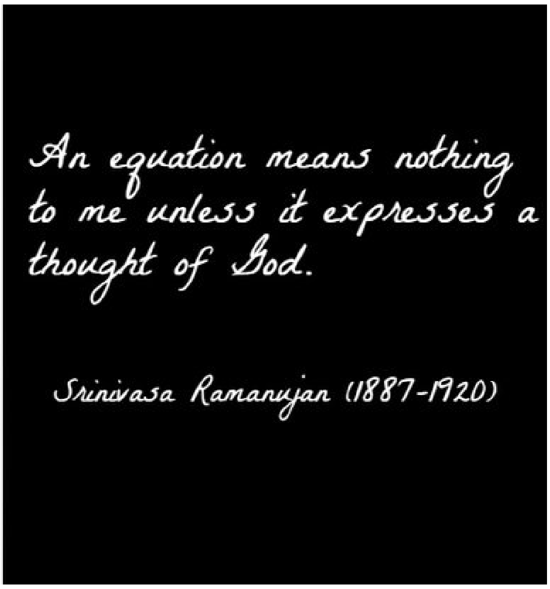


https://mobygeek.com/features/indian-mathematician-srinivasa-ramanujan-quotes-11012

We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation

From

INCOMPLETE ELLIPTIC INTEGRALS IN RAMANUJAN'S LOST NOTEBOOK
BRUCE C. BERNDT, HENG HUAT CHAN, AND SEN–SHAN HUANG

We have that:

Proof. By cubing both sides of (10.1), we deduce that

$$(PQ)^3 + \frac{9^3}{(PQ)^3} + 27\left(PQ + \frac{9}{PQ}\right) = \left(R^3 + \frac{1}{R^3}\right)^3.$$

For PQ = 1;

1+9^3+27(1+9)

Input:

$$1 + 9^3 + 27 (1 + 9)$$

Result:

1000

1000

Input:

$$1000 = \left(x^3 + \frac{1}{x^3}\right)^3$$

Alternate form assuming x is real:

$$x^5 + \frac{1}{x} = 10 x^2$$

Alternate form:

$$1000 = \frac{(x^2 + 1)^3 (x^4 - x^2 + 1)^3}{x^9}$$

Alternate form assuming x is positive:

$$x^6 + 1 = 10 x^3$$

Expanded form:

$$1000 = x^9 + \frac{1}{x^9} + 3 x^3 + \frac{3}{x^3}$$

Real solutions:

$$x = \sqrt[3]{5 - 2\sqrt{6}}$$

$$x = \sqrt[3]{5 + 2\sqrt{6}}$$

Indeed:

(((((5 + 2 sqrt(6))^(1/3))^3+1/((5 + 2 sqrt(6))^(1/3))^3)))^3

Input:

$$\left(\sqrt[3]{5+2\sqrt{6}}^{\,3}+\frac{1}{\sqrt[3]{5+2\sqrt{6}}^{\,3}}\right)^3$$

Result:

1000

1000

R = (5 + 2 sqrt(6))^(1/3) = 2.14715538606750

$$Q^6+\frac{9^3}{Q^6}+P^6+\frac{9^3}{P^6}=\left(R^3+\frac{1}{R^3}\right)^4-27\left(R^3+\frac{1}{R^3}\right)^2 \quad (10.7)$$

(((((5 + 2 sqrt(6))^(1/3))^3+1/((5 + 2 sqrt(6))^(1/3))^3)))^4 - 27(((((5 + 2 sqrt(6))^(1/3))^3+1/((5 + 2 sqrt(6))^(1/3))^3)))^2

Input:

$$\left(\sqrt[3]{5+2\sqrt{6}}^{\,3}+\frac{1}{\sqrt[3]{5+2\sqrt{6}}^{\,3}}\right)^4-27\left(\sqrt[3]{5+2\sqrt{6}}^{\,3}+\frac{1}{\sqrt[3]{5+2\sqrt{6}}^{\,3}}\right)^2$$

Result:

7300

7300

x^6+9^3/x^6+x^6+9^3/x^6 = 7300

Input:

$$x^6 + \frac{9^3}{x^6} + x^6 + \frac{9^3}{x^6} = 7300$$

Result:

$$2\,x^6 + \frac{1458}{x^6} = 7300$$

Alternate forms:

$$x^{12} - 3650\,x^6 = -729 \quad (\text{for } x \neq 0)$$

$$\frac{2\,(x^{12} + 729)}{x^6} = 7300$$

$$\frac{2\,(x^4 + 9)\,(x^8 - 9\,x^4 + 81)}{x^6} = 7300$$

Alternate form assuming x is positive:

$$x^{12} + 729 = 3650\,x^6 \quad (\text{for } x \neq 0)$$

Real solutions:

$$x = -\frac{3}{\sqrt[6]{1825 + 2\sqrt{832474}}}$$

$$x = \frac{3}{\sqrt[6]{1825 + 2\sqrt{832474}}}$$

$$x = -\sqrt[6]{1825 + 2\sqrt{832474}}$$

$$x = \sqrt[6]{1825 + 2\sqrt{832474}}$$

Real solutions:

$x \approx -0.76456$

$x \approx 0.76456$

$x \approx -3.9238$

$x \approx 3.9238$

Q = P =

$$x = \sqrt[6]{1825 + 2\sqrt{832474}}$$

= 3.9238

3.9238^6+9^3/3.9238^6+3.9238^6+9^3/3.9238^6

Input interpretation:

$$3.9238^6 + \frac{9^3}{3.9238^6} + 3.9238^6 + \frac{9^3}{3.9238^6}$$

Result:

7299.530745088849365915298680576856903148201746378856021336...

7299.530745… ≈ 7300

From (10.8), we have that:

Q = 2, P = 3

$$Q^6 + \frac{9^3}{Q^6} - P^6 - \frac{9^3}{P^6} = \left(R^3 - \frac{1}{R^3}\right)\left\{\left(S^3 - \frac{8}{S^3}\right)^3 + 27\left(S^3 - \frac{8}{S^3}\right)\right\}$$

2^6+9^3/2^6 – 3^6 – 9^3/3^6

Input:

$$2^6 + \frac{9^3}{2^6} - 3^6 - \frac{9^3}{3^6}$$

Exact result:

$$-\frac{41895}{64}$$

Decimal form:

-654.609375

-654.609375

(x^3-1/x^3) [(x^3-8/x^3)^3+27(x^3-8/x^3)] = 654.609375

Input interpretation:

$$\left(x^3-\frac{1}{x^3}\right)\left(\left(x^3-\frac{8}{x^3}\right)^3+27\left(x^3-\frac{8}{x^3}\right)\right)=654.609375$$

Result:

$$\left(x^3-\frac{1}{x^3}\right)\left(\left(x^3-\frac{8}{x^3}\right)^3+27\left(x^3-\frac{8}{x^3}\right)\right)=654.609$$

Alternate form assuming x is real:

$$x^{11}+\frac{14.7794}{x}=26.0561\,x^5$$

Alternate forms:

$$\frac{(x^6-8)(x^6-1)(x^{12}+11\,x^6+64)}{x^{12}}=654.609$$

$$\frac{1}{x^{12}}(x-1)(x+1)(x^2-2)(x^2-x+1)(x^2+x+1)(x^4-x^2+4)(x^4+2\,x^2+4)(x^4-3\,x^3+5\,x^2-6\,x+4)(x^4+3\,x^3+5\,x^2+6\,x+4)=654.609$$

Alternate form assuming x>0:

$$\left(x^3-\frac{8}{x^3}\right)^3x^3+27\left(x^3-\frac{8}{x^3}\right)x^3-\frac{\left(x^3-\frac{8}{x^3}\right)^3}{x^3}-\frac{27\left(x^3-\frac{8}{x^3}\right)}{x^3}=654.609$$

Alternate form assuming x is positive:

$(x-2.62861)\,x+1.56655=0$ (for $x\neq 0$)

Expanded form:

$$x^{12}+\frac{512}{x^{12}}+2\,x^6-\frac{488}{x^6}-27=654.609$$

Real solutions:

$x\approx-1.71536$

$x\approx-0.913246$

$x\approx0.913246$

$x\approx1.71536$

and for R = S = 1.71536, we obtain:

-(1.71536^3-1/1.71536^3) [(1.71536^3-8/1.71536^3)^3+27(1.71536^3-8/1.71536^3)]

Input interpretation:

$$-\left(1.71536^3-\frac{1}{1.71536^3}\right)\left(\left(1.71536^3-\frac{8}{1.71536^3}\right)^3+27\left(1.71536^3-\frac{8}{1.71536^3}\right)\right)$$

Result:

-654.6135250217214954301367403907709807410846267605488641 20...

-654.613525...

From which:

-(-(1.71536^3-1/1.71536^3) [(1.71536^3-8/1.71536^3)^3+27(1.71536^3-8/1.71536^3)])+123+4+1/golden ratio

Input interpretation:

$$-\left(-\left(1.71536^3-\frac{1}{1.71536^3}\right)\left(\left(1.71536^3-\frac{8}{1.71536^3}\right)^3+27\left(1.71536^3-\frac{8}{1.71536^3}\right)\right)\right)+123+4+\frac{1}{\phi}$$

ϕ is the golden ratio

Result:

782.232...

782.232… result practically equal to the rest mass of Omega meson 782.65

Alternative representations:

$$-(-1)\left(\left(1.71536^3-\frac{1}{1.71536^3}\right)\left(\left(1.71536^3-\frac{8}{1.71536^3}\right)^3+27\left(1.71536^3-\frac{8}{1.71536^3}\right)\right)\right)+123+4+\frac{1}{\phi}=127+\left(1.71536^3-\frac{1}{1.71536^3}\right)\left(27\left(1.71536^3-\frac{8}{1.71536^3}\right)+\left(1.71536^3-\frac{8}{1.71536^3}\right)^3\right)+\frac{1}{2\sin(54^\circ)}$$

$$-(-1)\left(\left(1.71536^3-\frac{1}{1.71536^3}\right)\left(\left(1.71536^3-\frac{8}{1.71536^3}\right)^3+27\left(1.71536^3-\frac{8}{1.71536^3}\right)\right)\right)+123+4+\frac{1}{\phi}=127+-\frac{1}{2\cos(216°)}+\left(1.71536^3-\frac{1}{1.71536^3}\right)\left(27\left(1.71536^3-\frac{8}{1.71536^3}\right)+\left(1.71536^3-\frac{8}{1.71536^3}\right)^3\right)$$

$$-(-1)\left(\left(1.71536^3-\frac{1}{1.71536^3}\right)\left(\left(1.71536^3-\frac{8}{1.71536^3}\right)^3+27\left(1.71536^3-\frac{8}{1.71536^3}\right)\right)\right)+123+4+\frac{1}{\phi}=127+\left(1.71536^3-\frac{1}{1.71536^3}\right)\left(27\left(1.71536^3-\frac{8}{1.71536^3}\right)+\left(1.71536^3-\frac{8}{1.71536^3}\right)^3\right)+-\frac{1}{2\sin(666°)}$$

From (10.10)

$$R^3-\frac{1}{R^3}=S^3+\frac{8}{S^3}.$$

We obtain:

((5 + 2 sqrt(6))^(1/3))^3-1/((5 + 2 sqrt(6))^(1/3))^3 = x^3+8/x^3

Input:

$$\sqrt[3]{5+2\sqrt{6}}^{\,3}-\frac{1}{\sqrt[3]{5+2\sqrt{6}}^{\,3}}=x^3+\frac{8}{x^3}$$

Exact result:

$$5+2\sqrt{6}-\frac{1}{5+2\sqrt{6}}=x^3+\frac{8}{x^3}$$

Alternate forms:

$$4\sqrt{6}=x^3+\frac{8}{x^3}$$

$$4\sqrt{6}=\frac{x^6+8}{x^3}$$

$$\left(5+2\sqrt{6}\right)x^6+\left(-48-20\sqrt{6}\right)x^3=-40-16\sqrt{6}\quad(\text{for } x\neq 0)$$

Alternate form assuming x is positive:

$4\sqrt{6}\ x^3 = x^6 + 8$ (for $x \neq 0$)

Real solutions:

$x = \sqrt[3]{2\left(\sqrt{6} - 2\right)}$

$x = \sqrt[3]{2\left(2 + \sqrt{6}\right)}$

S = x = (2 (2 + sqrt(6)))^(1/3)

Real solutions:

$x \approx 0.96512$

$x \approx 2.0723$

S = 2.0723

We have that:

For P = 3.9238

$$P^6 + \frac{9^3}{P^6} = S^6 - 6 + \frac{768}{S^6} + \frac{4096}{S^{12}},$$

3.9238^6+9^3/3.9238^6

Input interpretation:

$3.9238^6 + \frac{9^3}{3.9238^6}$

Result:

3649.765372544424682957649340288428451574100873189428010668...

3649.7653725444246...

(x^6-6+768/x^6+4096/x^12) = 3649.7653725444246

Input interpretation:

$x^6 - 6 + \frac{768}{x^6} + \frac{4096}{x^{12}} = 3649.7653725444246$

Result:

$$\frac{4096}{x^{12}}+x^6+\frac{768}{x^6}-6=3649.7653725444246$$

Alternate form assuming x is real:

$$1.000000000000000\,x^{11}+\frac{4273.13835838529}{x}=3656.72391861911\,x^5$$

Alternate form:

$$\frac{x^{18}-6\,x^{12}+768\,x^6+4096}{x^{12}}=3649.7653725444246$$

Alternate form assuming x is positive:

$$x\,(1.0000000000000000\,x-4.95123065987505)+4.02832491538684=0\quad(\text{for } x\neq 0)$$

Real solutions:

$x\approx-3.9248725005027882$

$x\approx-1.02635815937225912$

$x\approx1.02635815937225912$

$x\approx3.9248725005027882$

For S = 3.9248725005027882

(3.9248725005027882^6-6+768/3.9248725005027882^6+4096/3.9248725005027882^12)

Input interpretation:

$$3.9248725005027882^6-6+\frac{768}{3.9248725005027882^6}+\frac{4096}{3.9248725005027882^{12}}$$

Result:

3649.76537254442442746056761610514815358621254820446743657 3...

3649.7653725444244…

Or, reducing the decimal digits, we obtain:

(3.9248725^6-6+768/3.9248725^6+4096/3.9248725^12)

Input interpretation:

$$3.9248725^6-6+\frac{768}{3.9248725^6}+\frac{4096}{3.9248725^{12}}$$

Result:

3649.765369734859583735855257404617178131290732802516307174...

3649.765369734…

From which:

(3.9248725^6-6+768/3.9248725^6+4096/3.9248725^12)/2 +47-sqrt5

Input interpretation:

$$\frac{1}{2}\left(3.9248725^6 - 6 + \frac{768}{3.9248725^6} + \frac{4096}{3.9248725^{12}}\right) + 47 - \sqrt{5}$$

Result:

1869.647...

1869.647… result practically equal to the rest mass of D meson 1869.62

From:

$$\frac{1}{\sqrt{v}} - \sqrt{v} = \sqrt{K^2 - 4},$$

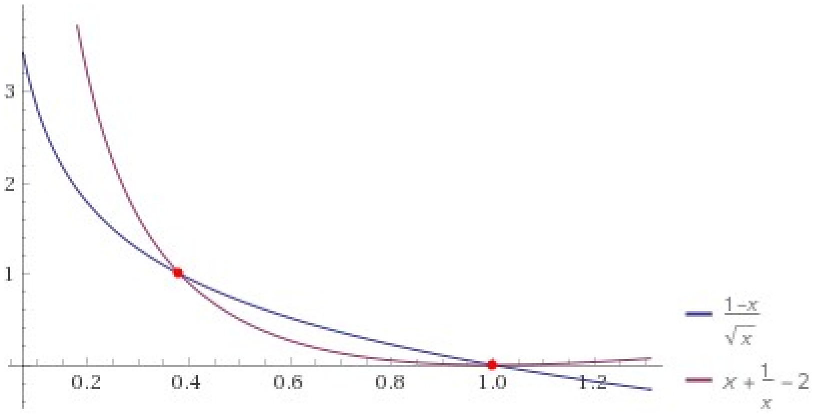
$$K = \frac{1}{\sqrt{v}} + \sqrt{v}.$$

(1/(sqrtx))-sqrt(x) = [(((1/(sqrtx))+sqrt(x)))^2 – 4]

Input:

$$\frac{1}{\sqrt{x}} - \sqrt{x} = \left(\frac{1}{\sqrt{x}} + \sqrt{x}\right)^2 - 4$$

Plot:



Alternate forms:

$$\frac{1-x}{\sqrt{x}} = x + \frac{1}{x} - 2$$

$$-\frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}} = \frac{(\sqrt{x}-1)^2(\sqrt{x}+1)^2}{x}$$

Alternate form assuming x is positive:

$$\sqrt{x}\,(x-2)+1=0$$

Expanded form:

$$\frac{1}{\sqrt{x}} - \sqrt{x} = x + \frac{1}{x} - 2$$

Solutions:

$$x = 1$$

$$x = \frac{3}{2} - \frac{\sqrt{5}}{2}$$

v = 0.381966011250 = (3/2 - sqrt(5)/2)

Now:

$$K = \frac{1}{\sqrt{v}} + \sqrt{v}.$$

1/(sqrt(3/2 - sqrt(5)/2)) + (sqrt(3/2 - sqrt(5)/2))

Input:

$$\frac{1}{\sqrt{\frac{3}{2} - \frac{\sqrt{5}}{2}}} + \sqrt{\frac{3}{2} - \frac{\sqrt{5}}{2}}$$

Decimal approximation:

2.2360679774997896964091736687312762354406183596115257242708...

K = 2.23606797749...

Alternate forms:

$\sqrt{5}$

$$\frac{5 - \sqrt{5}}{\sqrt{6 - 2\sqrt{5}}}$$

$$\frac{5 - \sqrt{5}}{\sqrt{2(3 - \sqrt{5})}}$$

Now, we have that:

$$1 - \frac{16\sqrt{2} - 13}{32\sqrt{2}} \sin^2 \varphi = \frac{32\sqrt{2}(1-v)^2 - (16\sqrt{2} - 13)\left\{(1-v)^2 - c^2(1+v)^2\right\}}{32\sqrt{2}(1-v)^2}$$

$$(8.19) \qquad = \frac{(16\sqrt{2} + 13)(1-v)^2 + 7(1+v)^2}{32\sqrt{2}(1-v)^2}.$$

((((16sqrt2+13)(1-(3/2 - sqrt(5)/2)^2+7(1+(3/2 - sqrt(5)/2)^2))) / (((32sqrt2(1-((3/2 - sqrt(5)/2)^2)))

Input:

$$\frac{(16\sqrt{2}+13)\left(1-\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+7\left(1+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\right)\right)}{32\sqrt{2}\left(1-\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2\right)}$$

Decimal approximation:

8.180823011437741911858663781534938015714258633106972203745...

8.18082301143…

Alternate forms:

$$\frac{1}{320}\left(32+13\sqrt{2}\right)\left(5+21\sqrt{5}\right)$$

$$\frac{1}{2}\left(1+\frac{21}{\sqrt{5}}+\sqrt{\frac{37687}{1280}+\frac{3549}{256\sqrt{5}}}\right)$$

$$\frac{13\sqrt{2}}{64}+\frac{21\sqrt{5}}{10}+\frac{273\sqrt{10}}{320}+\frac{1}{2}$$

Minimal polynomial:

$$6553600x^4-13107200x^3-375662080x^2+191421440x+1397787769$$

We have that:

$$(7.20)\qquad \frac{d\varphi/dt}{dv/dt}=-\frac{5\sqrt{1-\frac{9}{25}\sin^2\varphi}}{\sqrt{1-10v-13v^2+10v^3+v^4}}.$$

For (3/2 - sqrt(5)/2) = v , and φ = π / 6, we obtain:

(5sqrt(1-9/25sin^2(Pi/6))) / [1-10((3/2 - sqrt(5)/2))-13((3/2 - sqrt(5)/2))^2+10((3/2 - sqrt(5)/2))^3+((3/2 - sqrt(5)/2))^4]^(1/2)

Input:

$$-\frac{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}$$

Exact result:

$$-\frac{1}{2\sqrt{\frac{1}{91}\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}}$$

Decimal approximation:

2.3448100286494452130170095369073747646916877797734239875 49... i

Polar coordinates:

$r \approx 2.34481$ (radius), $\theta = 90°$ (angle)

2.34481

Alternate forms:

$$\frac{1}{4} i \sqrt{\frac{91}{58}\left(27+13\sqrt{5}\right)}$$

$$\frac{1}{2} i \sqrt{\frac{91}{26\sqrt{5}-54}}$$

$$\frac{1}{2} i \sqrt{\frac{91}{2\left(13\sqrt{5}-27\right)}}$$

Minimal polynomial:

$7424 x^4 + 39\,312 x^2 - 8281$

Alternative representations:

$$-\frac{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}} =$$

$$-\frac{5\sqrt{1-\frac{9}{25}\left(\frac{1}{\csc\left(\frac{\pi}{6}\right)}\right)^2}}{\sqrt{1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}}$$

$$-\frac{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}} = -\frac{5\sqrt{1-\frac{9}{25}\cos^2\left(\frac{\pi}{2}-\frac{\pi}{6}\right)}}{\sqrt{1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}}$$

$$-\frac{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}} = -\frac{5\sqrt{1-\frac{9}{25}\left(-\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)\right)^2}}{\sqrt{1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}}$$

Series representations:

$$-\frac{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}} = -\frac{20\sqrt{1-\frac{36}{25}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2}}{\sqrt{-71-256\sqrt{5}+182\sqrt{5}^2-32\sqrt{5}^3+\sqrt{5}^4}}$$

$$-\frac{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}} = -\frac{20\sqrt{1-\frac{9}{25}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{9}\right)^k(-\pi)^{2k}}{(2k)!}\right)^2}}{\sqrt{-71-256\sqrt{5}+182\sqrt{5}^2-32\sqrt{5}^3+\sqrt{5}^4}}$$

$$-\frac{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}} =$$

$$-\frac{20\sqrt{1-\frac{9}{25}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,6^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2}}{\sqrt{-71-256\sqrt{5}+182\sqrt{5}^2-32\sqrt{5}^3+\sqrt{5}^4}}$$

and from:

$$\int_0^q f(-t)f(-t^3)f(-t^5)f(-t^{15})dt$$
$$=\frac{1}{5}\int_{2\tan^{-1}\left(\frac{1}{\sqrt{5}}\sqrt{\frac{1-11v-v^2}{1+v-v^2}}\right)}^{2\tan^{-1}(1/\sqrt{5})} f(-t)f(-t^3)f(-t^5)f(-t^{15})$$
$$\times\frac{\sqrt{1-10v-13v^2+10v^3+v^4}}{\frac{dv}{dt}\sqrt{1-\frac{9}{25}\sin^2\varphi}}d\varphi.$$

from the last term, we obtain:

sqrt [1-10((3/2 - sqrt(5)/2))-13((3/2 - sqrt(5)/2))^2+10((3/2 - sqrt(5)/2))^3+((3/2 - sqrt(5)/2))^4] / (5sqrt(1-9/25sin^2(Pi/6)))

Input:

$$\frac{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}$$

Exact result:

$$2\sqrt{\frac{1}{91}\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}$$

Decimal approximation:

0.4264737815779371291894470793798453588704922234186870533 43... i

Polar coordinates:

$r \approx 0.426474$ (radius), $\theta = 90°$ (angle)

0.426474

Alternate forms:

$$2\,i\sqrt{\frac{2\sqrt{5}}{7}-\frac{54}{91}}$$

$$2\,i\sqrt{\frac{2}{91}\left(13\sqrt{5}-27\right)}$$

$$\frac{2\,i}{\sqrt{\frac{91}{-\frac{61}{2}+\frac{31\sqrt{5}}{2}-\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}}$$

Minimal polynomial:

$8281\,x^4 - 39\,312\,x^2 - 7424$

Alternative representations:

$$\frac{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{5\sqrt{1-\frac{1}{25}\sin^2\left(\frac{\pi}{6}\right)9}} = \frac{\sqrt{1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}}{5\sqrt{1-\frac{9}{25}\left(\frac{1}{\csc\left(\frac{\pi}{6}\right)}\right)^2}}$$

$$\frac{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{5\sqrt{1-\frac{1}{25}\sin^2\left(\frac{\pi}{6}\right)9}} = \frac{\sqrt{1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}}{5\sqrt{1-\frac{9}{25}\cos^2\left(\frac{\pi}{2}-\frac{\pi}{6}\right)}}$$

$$\frac{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{5\sqrt{1-\frac{1}{25}\sin^2\left(\frac{\pi}{6}\right)9}} = \frac{\sqrt{1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}}{5\sqrt{1-\frac{9}{25}\left(-\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)\right)^2}}$$

Series representations:

$$\frac{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{5\sqrt{1-\frac{1}{25}\sin^2\left(\frac{\pi}{6}\right)9}} = \frac{\sqrt{\frac{1}{16}\left(-71-256\sqrt{5}+182\sqrt{5}^2-32\sqrt{5}^3+\sqrt{5}^4\right)}}{5\sqrt{1-\frac{36}{25}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2}}$$

$$\frac{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{5\sqrt{1-\frac{1}{25}\sin^2\left(\frac{\pi}{6}\right)9}} = \frac{\sqrt{\frac{1}{16}\left(-71-256\sqrt{5}+182\sqrt{5}^2-32\sqrt{5}^3+\sqrt{5}^4\right)}}{5\sqrt{1-\frac{9}{25}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{9}\right)^k(-\pi)^{2k}}{(2k)!}\right)^2}}$$

$$\frac{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{5\sqrt{1-\frac{1}{25}\sin^2\left(\frac{\pi}{6}\right)9}} = \frac{\sqrt{\frac{1}{16}\left(-71-256\sqrt{5}+182\sqrt{5}^2-32\sqrt{5}^3+\sqrt{5}^4\right)}}{5\sqrt{1-\frac{9}{25}\left(\sum_{k=0}^{\infty}\frac{(-1)^k 6^{-1-2k}\pi^{1+2k}}{(1+2k)!}\right)^2}}$$

y = 1/5x * sqrt [1-10((3/2 - sqrt(5)/2))-13((3/2 - sqrt(5)/2))^2+10((3/2 - sqrt(5)/2))^3+((3/2 - sqrt(5)/2))^4] / (5sqrt(1-9/25sin^2(Pi/6)))

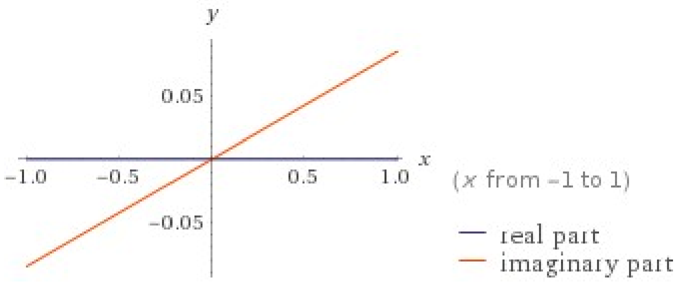
Input:

$$y = \frac{1}{5}\,x \times \frac{\sqrt{1 - 10\left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right) - 13\left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)^2 + 10\left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)^3 + \left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)^4}}{5\sqrt{1 - \frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}$$

Exact result:

$$y = \frac{2}{5}\sqrt{\frac{1}{91}\left(1 - 10\left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right) - 13\left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)^2 + 10\left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)^3 + \left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)^4\right)}\;x$$

Plot:



Alternate forms:

$$y = \frac{2}{5}\,i\sqrt{\frac{2\sqrt{5}}{7} - \frac{54}{91}}\;x$$

$$y = \frac{2}{5}\,i\sqrt{\frac{2}{91}\left(13\sqrt{5} - 27\right)}\;x$$

Alternate form assuming x and y are real:

$$y = \frac{2\,i\,x}{5\sqrt{\frac{91}{-\frac{61}{2} + \frac{31\sqrt{5}}{2} - \left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)^4}}}$$

Properties as a function:

Domain

$\mathbb{R}$ (all real numbers)

Range

$\{y \in \mathbb{R} : (-i)\,\infty < y < i\,\infty\}$

Parity

odd

$\mathbb{R}$ is the set of real numbers

Partial derivatives:

$$\frac{\partial}{\partial x}\left(\frac{2}{5}\sqrt{\frac{1}{91}\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\,x\right)=$$

$$\frac{2}{5}\sqrt{\frac{1}{91}\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}$$

$$\frac{\partial}{\partial y}\left(\frac{2}{5}\sqrt{\frac{1}{91}\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\,x\right)=0$$

From

$$y=\frac{2}{5}\,i\sqrt{\frac{2\sqrt{5}}{7}-\frac{54}{91}}\;x$$

we obtain:

y = 1/5 ((y / (((2/5 i sqrt(-54/91 + (2 sqrt(5))/7)))))) * sqrt [1-10((3/2 - sqrt(5)/2))-13((3/2 - sqrt(5)/2))^2+10((3/2 - sqrt(5)/2))^3+((3/2 - sqrt(5)/2))^4] / (5sqrt(1-9/25sin^2(Pi/6)))

Input:

$$y=\frac{1}{5}\times\frac{y}{\frac{2}{5}\,i\sqrt{-\frac{54}{91}+\frac{1}{7}\left(2\sqrt{5}\right)}}\times\frac{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}$$

i is the imaginary unit

Alternate form:

True

Alternate form assuming y is real:

$$y=\frac{y}{\sqrt{\frac{91\left(\frac{2\sqrt{5}}{7}-\frac{54}{91}\right)}{-\frac{61}{2}+\frac{31\sqrt{5}}{2}-\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}}$$

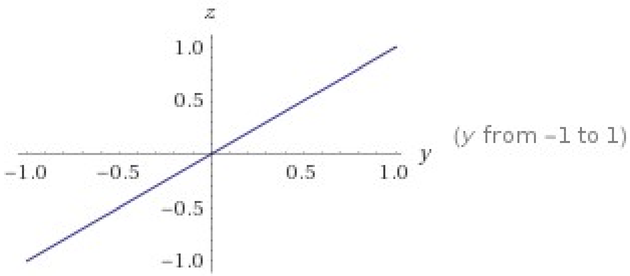
Input:

$$\frac{y}{\sqrt{\frac{91\left(-\frac{54}{91}+\frac{1}{7}\left(2\sqrt{5}\right)\right)}{-\frac{61}{2}+\frac{1}{2}\left(31\sqrt{5}\right)-\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}}$$

Result:

$$\frac{y}{\sqrt{\frac{91\left(\frac{2\sqrt{5}}{7}-\frac{54}{91}\right)}{-\frac{61}{2}+\frac{31\sqrt{5}}{2}-\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}}$$

Plot:



Geometric figure:

line

Alternate forms:

$\frac{1}{1}y$

y

Alternate form assuming y is real:

$$\sqrt{\frac{26\sqrt{5}-54}{91\left(\frac{2\sqrt{5}}{7}-\frac{54}{91}\right)}}\ y$$

Properties as a real function:

Domain

ℝ (all real numbers)

Range

ℝ (all real numbers)

Bijectivity

bijective from its domain to ℝ

Parity

odd

ℝ is the set of real numbers

Derivative:

$$\frac{d}{dy}\left(\frac{y}{\sqrt{\frac{91\left(-\frac{54}{91}+\frac{2\sqrt{5}}{7}\right)}{-\frac{61}{2}+\frac{31\sqrt{5}}{2}-\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}}\right)=\frac{1}{\sqrt{\frac{91\left(\frac{2\sqrt{5}}{7}-\frac{54}{91}\right)}{-\frac{61}{2}+\frac{31\sqrt{5}}{2}-\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}}$$

Indefinite integral:

$$\int\frac{y}{\sqrt{\frac{91\left(-\frac{54}{91}+\frac{2\sqrt{5}}{7}\right)}{-\frac{61}{2}+\frac{31\sqrt{5}}{2}-\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}}\,dy=\frac{y^2}{2\sqrt{\frac{91\left(\frac{2\sqrt{5}}{7}-\frac{54}{91}\right)}{-\frac{61}{2}+\frac{31\sqrt{5}}{2}-\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}}+\text{constant}$$

If y = 2:

Input:

$$\frac{2}{\sqrt{\frac{91\left(-\frac{54}{91}+\frac{1}{7}\left(2\sqrt{5}\right)\right)}{-\frac{61}{2}+\frac{1}{2}\left(31\sqrt{5}\right)-\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}}$$

Result:

2

2

Thence, for y = 2 we obtain 2, for y = 3, we obtain 3, and so on.

Now, for y = 2, we obtain from the previous expression:

2 = 1/5x * sqrt [1-10((3/2 - sqrt(5)/2))-13((3/2 - sqrt(5)/2))^2+10((3/2 - sqrt(5)/2))^3+((3/2 - sqrt(5)/2))^4] / (5sqrt(1-9/25sin^2(Pi/6)))

Input:

$$2 = \frac{1}{5}x \times \frac{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}$$

Exact result:

$$2 = \frac{2}{5}\sqrt{\frac{1}{91}\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\,x$$

Alternate forms:

$$2 = \frac{2}{5}i\sqrt{\frac{2\sqrt{5}}{7}-\frac{54}{91}}\,x$$

$$2 = \frac{2}{5}i\sqrt{\frac{2}{91}\left(13\sqrt{5}-27\right)}\,x$$

$$\frac{2}{5}i\,x\,\left(\text{root of } 8281x^4+9828x^2-464 \text{ near } x = 0.213237\right) = 2$$

Alternate form assuming x is real:

$$2 = \frac{2ix}{5\sqrt{\frac{91}{-\frac{61}{2}+\frac{31\sqrt{5}}{2}-\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}}$$

Complex solution:

$$x = -5i\sqrt{\frac{91}{26\sqrt{5}-54}}$$

Complex solution:

$x \approx -23.448\,i$

-23.448*i*

and:

1/5(((-5 i sqrt(91/(-54 + 26 sqrt(5))))))

Input:

$$\frac{1}{5}\left(-5\,i\sqrt{\frac{91}{-54+26\sqrt{5}}}\right)$$

i is the imaginary unit

Result:

$$-i\sqrt{\frac{91}{26\sqrt{5}-54}}$$

Decimal approximation:

-4.68962005729889042603401907381474952938337555954684797509... i

Polar coordinates:

$r \approx 4.68962$ (radius), $\theta = -90°$ (angle)

4.68962

Alternate forms:

$$-\frac{1}{116}\,i\sqrt{5278\left(27+13\sqrt{5}\right)}$$

root of $464x^4 + 9828x^2 - 8281$ near $x = -4.68962\,i$

$$-\frac{1}{2}\,i\sqrt{\frac{91}{58}\left(27+13\sqrt{5}\right)}$$

Minimal polynomial:

$$464x^4 + 9828x^2 - 8281$$

Thence, from

$$\int_0^q f(-t)f(-t^3)f(-t^5)f(-t^{15})dt$$
$$=\frac{1}{5}\int_{2\tan^{-1}\left(\frac{1}{\sqrt{5}}\sqrt{\frac{1-11v-v^2}{1+v-v^2}}\right)}^{2\tan^{-1}(1/\sqrt{5})} f(-t)f(-t^3)f(-t^5)f(-t^{15})$$
$$\times\frac{\sqrt{1-10v-13v^2+10v^3+v^4}}{\frac{dv}{dt}\sqrt{1-\frac{9}{25}\sin^2\varphi}}d\varphi.$$

we obtain:

$$2 = \frac{1}{5} x \times \frac{\sqrt{1 - 10\left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right) - 13\left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)^2 + 10\left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)^3 + \left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)^4}}{5\sqrt{1 - \frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}$$

$$x = -5\, i \sqrt{\frac{91}{26\sqrt{5} - 54}}$$

$$x \approx -23.448\, i$$

and

$$\frac{1}{5}\int_{2\tan^{-1}\left(\frac{1}{\sqrt{5}}\sqrt{\frac{1-11v-v^2}{1+v-v^2}}\right)}^{2\tan^{-1}(1/\sqrt{5})} f(-t)f(-t^3)f(-t^5)f(-t^{15})$$

$$= -4.68962005729889042603401907381474952938337555954684797509\ldots\, i$$

then:

-4.68962005729889 i * sqrt [1-10((3/2 - sqrt(5)/2))-13((3/2 - sqrt(5)/2))^2+10((3/2 - sqrt(5)/2))^3+((3/2 - sqrt(5)/2))^4] / (5sqrt(1-9/25sin^2(Pi/6)))

Input interpretation:

$$-4.68962005729889\, i \times \frac{\sqrt{1 - 10\left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right) - 13\left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)^2 + 10\left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)^3 + \left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)^4}}{5\sqrt{1 - \frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}$$

i is the imaginary unit

Result:

2.00000000000000...

2 result equal to the graviton spin

Alternative representations:

$$\frac{1}{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}\left(i\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}\right)(-1)$$

$$4.689620057298890000 = -\frac{1}{5\sqrt{1-\frac{9}{25}\left(\frac{1}{\csc\left(\frac{\pi}{6}\right)}\right)^2}} 4.689620057298890000$$

$$i\sqrt{1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}$$

$$\frac{1}{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}\left(i\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}\right)$$

$$(-1)\,4.689620057298890000 = -\left(\left(4.689620057298890000\, i \sqrt{1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}\right)\Big/ \left(5\sqrt{1-\frac{9}{25}\cos^2\left(\frac{\pi}{2}-\frac{\pi}{6}\right)}\right)\right)$$

$$\frac{1}{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}\left(i\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}\right)$$

$$(-1)\,4.689620057298890000 = -\left(\left(4.689620057298890000\, i \sqrt{1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}\right)\Big/ \left(5\sqrt{1-\frac{9}{25}\left(-\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)\right)^2}\right)\right)$$

Series representations:

$$\frac{1}{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}\left(i\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}\right)$$
$$(-1)\,4.689620057298890000 = -\frac{0.9379240114597780000\, i\sqrt{\frac{1}{16}\left(-71-256\sqrt{5}+182\sqrt{5}^{\,2}-32\sqrt{5}^{\,3}+\sqrt{5}^{\,4}\right)}}{\sqrt{1-\frac{36}{25}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2}}$$

$$\frac{1}{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}\left(i\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}\right)$$
$$(-1)\,4.689620057298890000 = -\frac{0.9379240114597780000\, i\sqrt{\frac{1}{16}\left(-71-256\sqrt{5}+182\sqrt{5}^{\,2}-32\sqrt{5}^{\,3}+\sqrt{5}^{\,4}\right)}}{\sqrt{1-\frac{9}{25}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{9}\right)^k(-\pi)^{2k}}{(2k)!}\right)^2}}$$

$$\frac{1}{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}\left(i\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}\right)$$
$$(-1)\,4.689620057298890000 = -\frac{0.9379240114597780000\, i\sqrt{\frac{1}{16}\left(-71-256\sqrt{5}+182\sqrt{5}^{\,2}-32\sqrt{5}^{\,3}+\sqrt{5}^{\,4}\right)}}{\sqrt{1-\frac{9}{25}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\, 6^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2}}$$

Thence:

$$\int_0^q f(-t)f(-t^3)f(-t^5)f(-t^{15})dt$$
$$=\frac{1}{5}\int_{2\tan^{-1}\left(\frac{1}{\sqrt{5}}\sqrt{\frac{1-11v-v^2}{1+v-v^2}}\right)}^{2\tan^{-1}(1/\sqrt{5})} f(-t)f(-t^3)f(-t^5)f(-t^{15})$$
$$\times \frac{\sqrt{1-10v-13v^2+10v^3+v^4}}{\frac{dv}{dt}\sqrt{1-\frac{9}{25}\sin^2\varphi}}d\varphi.$$

$$-4.68962005729889\, i \times \frac{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}$$

$= 2.00000000000000...$

from:

2 = 1/5x * sqrt [1-10((3/2 - sqrt(5)/2))-13((3/2 - sqrt(5)/2))^2+10((3/2 - sqrt(5)/2))^3+((3/2 - sqrt(5)/2))^4] / (5sqrt(1-9/25sin^2(Pi/6)))

for x = -23.448 i , we obtain:

((((1/5*(-23.448i) * sqrt [1-10((3/2 - sqrt(5)/2))-13((3/2 - sqrt(5)/2))^2+10((3/2 - sqrt(5)/2))^3+((3/2 - sqrt(5)/2))^4] / (5sqrt(1-9/25sin^2(Pi/6)))))))

Input interpretation:

$$\frac{1}{5}(-23.448\, i) \times \frac{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}$$

i is the imaginary unit

Result:

1.99999...

1.99999… ≈ 2 as above

Alternative representations:

$$-\frac{(23.448\,i)\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5} = -\frac{23.448\,i\sqrt{1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}}{5\left(5\sqrt{1-\frac{9}{25}\left(\frac{1}{\csc\left(\frac{\pi}{6}\right)}\right)^2}\right)}$$

$$-\frac{(23.448\,i)\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5} = -\frac{23.448\,i\sqrt{1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}}{5\left(5\sqrt{1-\frac{9}{25}\cos^2\left(\frac{\pi}{2}-\frac{\pi}{6}\right)}\right)}$$

$$-\frac{(23.448\,i)\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5} = -\frac{23.448\,i\sqrt{1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)}}{5\left(5\sqrt{1-\frac{9}{25}\left(-\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)\right)^2}\right)}$$

Series representations:

$$-\frac{(23.448\,i)\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5} = -\frac{0.93792\,i\sqrt{\frac{1}{16}\left(-71-256\sqrt{5}+182\sqrt{5}^2-32\sqrt{5}^3+\sqrt{5}^4\right)}}{\sqrt{1-\frac{36}{25}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2}}$$

$$-\frac{(23.448\,i)\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5}=$$

$$-\frac{0.93792\,i\sqrt{\frac{1}{16}\left(-71-256\sqrt{5}+182\sqrt{5}^2-32\sqrt{5}^3+\sqrt{5}^4\right)}}{\sqrt{1-\frac{9}{25}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{9}\right)^k(-\pi)^{2k}}{(2k)!}\right)^2}}$$

$$-\frac{(23.448\,i)\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5}=$$

$$-\frac{0.93792\,i\sqrt{\frac{1}{16}\left(-71-256\sqrt{5}+182\sqrt{5}^2-32\sqrt{5}^3+\sqrt{5}^4\right)}}{\sqrt{1-\frac{9}{25}\left(\sum_{k=0}^{\infty}\frac{(-1)^k 6^{-1-2k}\pi^{1+2k}}{(1+2k)!}\right)^2}}$$

From which:

((((1/5*(-23.448i) * sqrt [1-10((3/2 - sqrt(5)/2))-13((3/2 - sqrt(5)/2))^2+10((3/2 - sqrt(5)/2))^3+((3/2 - sqrt(5)/2))^4] / (5sqrt(1-9/25sin^2(Pi/6)))))))^6

Input interpretation:

$$\left(\frac{1}{5}(-23.448\,i)\times\frac{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}\right)^6$$

i is the imaginary unit

Result:

63.9984...

63.9984… ≈ 64

and:

2* ((((1/5*(-23.448i) * sqrt [1-10((3/2 - sqrt(5)/2))-13((3/2 - sqrt(5)/2))^2+10((3/2 - sqrt(5)/2))^3+((3/2 - sqrt(5)/2))^4] / (5sqrt(1-9/25sin^2(Pi/6)))))))^6-golden ratio^2

Input interpretation:

$$2\left(\frac{1}{5}(-23.448\, i)\times\frac{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}\right)^6-\phi^2$$

i is the imaginary unit

ϕ is the golden ratio

Result:

125.379...

125.379…

Alternative representations:

$$2\left(-\left(\left((23.448\, i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6-\phi^2 = -\phi^2+2\left(-\left(\left(23.448\, i\sqrt{\left(1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)\right)}\right)\Big/\left(5\left(5\sqrt{1-\frac{9}{25}\left(\frac{1}{\csc\left(\frac{\pi}{6}\right)}\right)^2}\right)\right)\right)\right)^6$$

$$2\left(-\left(\left((23.448\, i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6-\phi^2 = -\phi^2+2\left(-\left(\left(23.448\, i\sqrt{\left(1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)\right)}\right)\Big/\left(5\left(5\sqrt{1-\frac{9}{25}\cos^2\left(\frac{\pi}{2}-\frac{\pi}{6}\right)}\right)\right)\right)\right)^6$$

$$2\left(-\left(\left((23.448\,i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6-\phi^2 =$$

$$-\phi^2+2\left(-\left(\left(23.448\,i\sqrt{\left(1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)\right)}\right)\Big/\left(5\left(5\sqrt{1-\frac{9}{25}\left(-\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)\right)^2}\right)\right)\right)\right)^6$$

Series representations:

$$2\left(-\left(\left((23.448\,i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6-\phi^2 =$$

$$-\phi^2+\frac{1.36152\,i^6\sqrt{\frac{1}{16}\left(-71-256\sqrt{5}+182\sqrt{5}^2-32\sqrt{5}^3+\sqrt{5}^4\right)}^6}{\sqrt{1-\frac{36}{25}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2}^6}$$

$$2\left(-\left(\left((23.448\,i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6-\phi^2 =$$

$$-\phi^2+\frac{1.36152\,i^6\sqrt{\frac{1}{16}\left(-71-256\sqrt{5}+182\sqrt{5}^2-32\sqrt{5}^3+\sqrt{5}^4\right)}^6}{\sqrt{1-\frac{9}{25}\left(\sum_{k=0}^{\infty}\frac{(-1)^k 6^{-1-2k}\pi^{1+2k}}{(1+2k)!}\right)^2}^6}$$

$$2\left(-\left(\left((23.448\,i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6-\phi^2=$$

$$-\left(\left(-1.36152\,i^6\left(\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{1}{16}\right)^k\left(-\frac{1}{2}\right)_k\left(-71-256\sqrt{5}+182\sqrt{5}^2-32\sqrt{5}^3+\sqrt{5}^4-16\,z_0\right)^k z_0^{-k}\right)^6+\phi^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)-z_0\right)^k z_0^{-k}}{k!}\right)^6\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)-z_0\right)^k z_0^{-k}}{k!}\right)^6\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

2* ((((1/5*(-23.448i) * sqrt [1-10((3/2 - sqrt(5)/2))-13((3/2 - sqrt(5)/2))^2+10((3/2 - sqrt(5)/2))^3+((3/2 - sqrt(5)/2))^4] / (5sqrt(1-9/25sin^2(Pi/6)))))))^6+11+1/golden ratio

Input interpretation:

$$2\left(\frac{1}{5}(-23.448\,i)\times\frac{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}\right)^6+11+\frac{1}{\phi}$$

i is the imaginary unit

ϕ is the golden ratio

Result:

139.615...

139.615…

Alternative representations:

$$2\left(-\left(\left((23.448\,i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6+11+\frac{1}{\phi}=$$

$$11+\frac{1}{\phi}+2\left(-\left(\left(23.448\,i\sqrt{\left(1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)\right)}\right)\Big/\left(5\left(5\sqrt{1-\frac{9}{25}\left(\frac{1}{\csc\left(\frac{\pi}{6}\right)}\right)^2}\right)\right)\right)\right)^6$$

$$2\left(-\left(\left((23.448\,i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6+11+\frac{1}{\phi}=$$

$$11+\frac{1}{\phi}+2\left(-\left(\left(23.448\,i\sqrt{\left(1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)\right)}\right)\Big/\left(5\left(5\sqrt{1-\frac{9}{25}\cos^2\left(\frac{\pi}{2}-\frac{\pi}{6}\right)}\right)\right)\right)\right)^6$$

$$2\left(-\left(\left((23.448\,i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6+11+\frac{1}{\phi}=$$

$$11+\frac{1}{\phi}+2\left(-\left(\left(23.448\,i\sqrt{\left(1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)\right)}\right)\Big/\left(5\left(5\sqrt{1-\frac{9}{25}\left(-\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)\right)^2}\right)\right)\right)\right)^6$$

Series representations:

$$2\left(-\left(\left((23.448\,i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6+11+\frac{1}{\phi}=$$

$$11+\frac{1}{\phi}+\frac{1.36152\,i^6\sqrt{\frac{1}{16}\left(-71-256\sqrt{5}+182\sqrt{5}^2-32\sqrt{5}^3+\sqrt{5}^4\right)}^6}{\sqrt{1-\frac{36}{25}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2}^6}$$

$$2\left(-\left(\left((23.448\,i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6+11+\frac{1}{\phi}=$$

$$11+\frac{1}{\phi}+\frac{1.36152\,i^6\sqrt{\frac{1}{16}\left(-71-256\sqrt{5}+182\sqrt{5}^2-32\sqrt{5}^3+\sqrt{5}^4\right)}^6}{\sqrt{1-\frac{9}{25}\left(\sum_{k=0}^{\infty}\frac{(-1)^k 6^{-1-2k}\pi^{1+2k}}{(1+2k)!}\right)^2}^6}$$

$$2\left(-\left(\left((23.448\,i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6+11+\frac{1}{\phi}=$$

$$\left(1.36152\left(\phi\,i^6\left(\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{1}{16}\right)^k\left(-\frac{1}{2}\right)_k\left(-71-256\sqrt{5}+182\sqrt{5}^2-32\sqrt{5}^3+\sqrt{5}^4-16z_0\right)^k z_0^{-k}\right)^6+0.734472\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)-z_0\right)^k z_0^{-k}}{k!}\right)^6+8.07919\,\phi\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)-z_0\right)^k z_0^{-k}}{k!}\right)^6\right)\right)\Big/\left(\phi\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)-z_0\right)^k z_0^{-k}}{k!}\right)^6\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

27* ((((1/5*(-23.448i) * sqrt [1-10((3/2 - sqrt(5)/2))-13((3/2 - sqrt(5)/2))^2+10((3/2 - sqrt(5)/2))^3+((3/2 - sqrt(5)/2))^4] / (5sqrt(1-9/25sin^2(Pi/6)))))))^6+1

Input interpretation:

$$27\left(\frac{1}{5}(-23.448\,i)\times\frac{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}\right)^6+1$$

i is the imaginary unit

Result:

1728.96...

1728.96… ≈ 1729

With regard 27 (From Wikipedia):

*"The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group **Z**/3**Z**, and its outer automorphism group is the cyclic group **Z**/2**Z**. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories".*

Alternative representations:

$$27\left(-\left(\left((23.448\,i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6+1=$$

$$1+27\left(-\left(\left(23.448\,i\sqrt{\left(1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)\right)}\right)\Big/\left(5\left(5\sqrt{1-\frac{9}{25}\left(\frac{1}{\csc\left(\frac{\pi}{6}\right)}\right)^2}\right)\right)\right)\right)^6$$

$$27\left(-\left(\left((23.448\,i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6+1=$$

$$1+27\left(-\left(\left(23.448\,i\sqrt{\left(1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)\right)}\right)\Big/\left(5\left(5\sqrt{1-\frac{9}{25}\cos^2\left(\frac{\pi}{2}-\frac{\pi}{6}\right)}\right)\right)\right)\right)^6$$

$$27\left(-\left(\left((23.448\,i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6+1=$$

$$1+27\left(-\left(\left(23.448\,i\sqrt{\left(1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)\right)}\right)\Big/\left(5\left(5\sqrt{1-\frac{9}{25}\left(-\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)\right)^2}\right)\right)\right)\right)^6$$

Series representations:

$$27\left(-\left(\left((23.448\,i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6+1=$$

$$1+\frac{18.3806\,i^6\sqrt{\frac{1}{16}\left(-71-256\sqrt{5}+182\sqrt{5}^2-32\sqrt{5}^3+\sqrt{5}^4\right)}^6}{\sqrt{1-\frac{36}{25}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2}^6}$$

$$27\left(-\left(\left((23.448\,i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6+1=$$

$$1+\frac{18.3806\,i^6\sqrt{\frac{1}{16}\left(-71-256\sqrt{5}+182\sqrt{5}^2-32\sqrt{5}^3+\sqrt{5}^4\right)}^6}{\sqrt{1-\frac{9}{25}\left(\sum_{k=0}^{\infty}\frac{(-1)^k 6^{-1-2k}\pi^{1+2k}}{(1+2k)!}\right)^2}^6}$$

$$27\left(-\left(\left((23.448\, i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6+1=$$

$$\left(18.3806\left(i^6\left(\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{1}{16}\right)^k\left(-\frac{1}{2}\right)_k\left(-71-256\sqrt{5}+182\sqrt{5}^2-32\sqrt{5}^3+\sqrt{5}^4-16 z_0\right)^k z_0^{-k}\right)^6+0.0544053\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)-z_0\right)^k z_0^{-k}}{k!}\right)^6\right)\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)-z_0\right)^k z_0^{-k}}{k!}\right)^6$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

((((27((((1/5*(-23.448i) sqrt [1-10((3/2 - sqrt(5)/2))-13((3/2 - sqrt(5)/2))^2+10((3/2 - sqrt(5)/2))^3+((3/2 - sqrt(5)/2))^4] / (5sqrt(1-9/25sin^2(Pi/6)))))))^6+1))))^1/15

Input interpretation:

$$\left(27\left(\frac{1}{5}(-23.448\, i)\times\frac{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}\right)^6+1\right)^{\wedge}(1/15)$$

i is the imaginary unit

Result:

1.643812418161439507326731187216329870204397741313524744923...

1.64381241816…

((((27(((((1/5*(-23.448i) sqrt [1-10((3/2 - sqrt(5)/2))-13((3/2 - sqrt(5)/2))^2+10((3/2 - sqrt(5)/2))^3+((3/2 - sqrt(5)/2))^4] / (5sqrt(1-9/25sin^2(Pi/6)))))))^6+1))))^1/15-26/10^3

Input interpretation:

$$\left(27\left(\frac{1}{5}(-23.448\,i)\times\frac{\sqrt{1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4}}{5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}}\right)^6+1\right)\text{^}(1/15)-\frac{26}{10^3}$$

i is the imaginary unit

Result:

1.617812418161439507326731187216329870204397741313524744923...

1.61781241816…

Alternative representations:

$$\left(27\left(-\left(\left((23.448\,i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6+1\right)\text{^}(1/15)-\frac{26}{10^3}=-\frac{26}{10^3}+\left(1+27\left(-\left(\left(23.448\,i\sqrt{\left(1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)\right)}\right)\Big/\left(5\left(5\sqrt{1-\frac{9}{25}\left(\frac{1}{\csc\left(\frac{\pi}{6}\right)}\right)^2}\right)\right)\right)\right)^6\right)\text{^}(1/15)$$

$$\left(27\left(-\left(\left((23.448\,i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6+1\right)\hat{}\,(1/15)-\frac{26}{10^3}=-\frac{26}{10^3}+\left(1+27\left(-\left(\left(23.448\,i\sqrt{\left(1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)\right)}\right)\Big/\left(5\left(5\sqrt{1-\frac{9}{25}\cos^2\left(\frac{\pi}{2}-\frac{\pi}{6}\right)}\right)\right)\right)\right)^6\right)\hat{}\,(1/15)$$

$$\left(27\left(-\left(\left((23.448\,i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6+1\right)\hat{}\,(1/15)-\frac{26}{10^3}=-\frac{26}{10^3}+\left(1+27\left(-\left(\left(23.448\,i\sqrt{\left(1-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)\right)}\right)\Big/\left(5\left(5\sqrt{1-\frac{9}{25}\left(-\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)\right)^2}\right)\right)\right)\right)^6\right)\hat{}\,(1/15)$$

Series representations:

$$\left(27\left(-\left(\left((23.448\,i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)\right)^6+1\right)\hat{}\,(1/15)-\frac{26}{10^3}=$$

$$-\frac{13}{500}+\sqrt[15]{1+\frac{18.3806\,i^6\sqrt{\frac{1}{16}\left(-71-256\sqrt{5}+182\sqrt{5}^2-32\sqrt{5}^3+\sqrt{5}^4\right)}^6}{\sqrt{1-\frac{36}{25}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2}^6}}$$

$$\left(27\left(-\left(\left((23.448\, i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)^6+1\right)\hat{}\,(1/15)-\frac{26}{10^3}=$$

$$-\frac{13}{500}+\sqrt[15]{1+\frac{18.3806\, i^6\sqrt{\frac{1}{16}\left(-71-256\sqrt{5}+182\sqrt{5}^2-32\sqrt{5}^3+\sqrt{5}^4\right)}^{\,6}}{\sqrt{1-\frac{9}{25}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\, 6^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}\right)^2}^{\,6}}}$$

$$\left(27\left(-\left(\left((23.448\, i)\sqrt{\left(1-10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)-13\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2+10\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^3+\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^4\right)}\right)\Big/\left(\left(5\sqrt{1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)}\right)5\right)\right)^6+1\right)\hat{}\,(1/15)-\frac{26}{10^3}=$$

$$\frac{1}{500}\left(-13+500\left(1+\left(18.3806\, i^6\left(\sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{1}{16}\right)^k\left(-\frac{1}{2}\right)_k\left(-71-256\sqrt{5}+182\sqrt{5}^2-32\sqrt{5}^3+\sqrt{5}^4-16z_0\right)^k z_0^{-k}\right)^6\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(1-\frac{9}{25}\sin^2\left(\frac{\pi}{6}\right)-z_0\right)^k z_0^{-k}}{k!}\right)^6\right)\hat{}\,(1/15)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

Now, we have that:

Lemma 7.7 (Second version of Landen's transformation). *If* $0 \le \alpha, \beta \le \pi/2$, $0 < x < 1$, *and* $\tan(\beta-\alpha) = \sqrt{1-x^2}\tan\alpha$, *then*

$$\int_0^{\alpha}\frac{d\varphi}{\sqrt{1-x^2\sin^2\varphi}} = \frac{1}{1+\sqrt{1-x^2}}\int_0^{\beta}\frac{d\varphi}{\sqrt{1-\left(\frac{1-\sqrt{1-x^2}}{1+\sqrt{1-x^2}}\right)^2\sin^2\varphi}}.$$

If $q = 0$, then $\alpha = 2\tan^{-1}(1/\sqrt{5})$. In comparing (7.11) and (7.12), we must prove that, with the agrument q deleted for brevity, 2 tan^-1(1/(sqrt5)) = α

$$\tan(\beta/2) = \frac{(1-v\epsilon^{-3})}{(1+v\epsilon^{3})}\sqrt{\frac{(1+v\epsilon)(1-v\epsilon^{5})}{(1-v\epsilon^{-1})(1+v\epsilon^{-5})}}, \tag{7.22}$$

for if $q=0$, then $\beta = \pi/2$.

Set $t_1 = \tan(\alpha/2)$ and $t_2 = \tan\left((\beta-\alpha)/2\right)$. Then

$$\frac{2t_2}{1-t_2^2} = \tan(\beta-\alpha) = \tfrac{4}{5}\tan\alpha = \frac{8t_1}{5(1-t_1^2)}.$$

If we consider the extremal equality as a quadratic equation in t_2, a routine calculation gives

$$t_2 = \frac{5(1-t_1^2)}{8t_1} + \frac{1}{2}\sqrt{\frac{25(1-t_1^2)^2}{16t_1^2} + 4}, \tag{7.23}$$

since $t_2 > 0$. Using (7.21) and the definition of t_1, we find that

$$1 - t_1^2 = \frac{4(1+4v-v^2)}{5(1+v-v^2)} \tag{7.24}$$

and

$$\frac{25(1-t_1^2)^2}{16t_1^2} + 4 = \frac{9(1+v^2)^2}{(1+v-v^2)(1-11v-v^2)}, \tag{7.25}$$

after a lengthy calculation. Employing (7.21), (7.24), and (7.25) in (7.23), we conclude that

$$\begin{aligned} t_2 &= -\frac{\sqrt{5}(1+4v-v^2)+3(1+v^2)}{2\sqrt{(1+v-v^2)(1-11v-v^2)}} \\ &= \frac{\epsilon^2(\epsilon-v)(\epsilon^{-5}-v)}{\sqrt{(1-v\epsilon^{-1})(1+v\epsilon)(1-v\epsilon^{5})(1+v\epsilon^{-5})}} \\ &= \epsilon^{-2}\sqrt{\frac{(1-v\epsilon^{-1})(1-v\epsilon^{5})}{(1+v\epsilon)(1+v\epsilon^{-5})}}. \end{aligned} \tag{7.26}$$

We analyze (7.23) and (7.25):

$$t_2 = -\frac{5(1-t_1^2)}{8t_1} + \frac{1}{2}\sqrt{\frac{25(1-t_1^2)^2}{16t_1^2} + 4},$$

2 tan^-1(1/(sqrt5)) = α t_1 = tan (α / 2) = tan (((1/2 * 2 tan^-1(1/(sqrt5)))))

-5(1- [tan(((1/2 * 2 tan^-1(1/(sqrt5)))))]^2) / ((8*(tan (((1/2 * 2 tan^-1(1/(sqrt5))))))))+1/2*sqrt[(25(1- [tan(((1/2 * 2 tan^-1(1/(sqrt5)))))]^2)^2)/(16(tan (((1/2 * 2 tan^-1(1/(sqrt5)))))^2))+4]

Input:

$$-5\times\frac{1-\tan^2\left(\frac{1}{2}\times 2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}{8\tan\left(\frac{1}{2}\times 2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{1}{2}\times 2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{1}{2}\times 2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\frac{3}{2}-\frac{\sqrt{5}}{2}$$

(result in radians)

Decimal approximation:

0.3819660112501051517954131656343618822796908201942371378645...

(result in radians)

0.38196601125…

Alternate form:

$$\frac{1}{2}\left(3-\sqrt{5}\right)$$

Alternative representations:

$$-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}=$$
$$\frac{-5\left(1-\left(-\cot\left(-\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\right)}{-8\cot\left(-\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{4+\frac{25\left(1-\left(-\cot\left(-\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\right)^2}{16\left(-\cot\left(-\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}}$$

$$-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4} =$$

$$-\frac{5\left(1-\left(-i+\frac{2i}{1+e^{2i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2\right)}{8\left(-i+\frac{2i}{1+e^{2i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)}+\frac{1}{2}\sqrt{4+\frac{25\left(1-\left(-i+\frac{2i}{1+e^{2i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2\right)^2}{16\left(-i+\frac{2i}{1+e^{2i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2}}$$

$$-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4} =$$

$$-\frac{5\left(1-\left(\frac{i\left(e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}-e^{i\tan^{-1}\left(1/\sqrt{5}\right)}\right)}{e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}+e^{i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2\right)}{\frac{8i\left(e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}-e^{i\tan^{-1}\left(1/\sqrt{5}\right)}\right)}{e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}+e^{i\tan^{-1}\left(1/\sqrt{5}\right)}}}+$$

$$\frac{1}{2}\sqrt{4+\frac{25\left(1-\left(\frac{i\left(e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}-e^{i\tan^{-1}\left(1/\sqrt{5}\right)}\right)}{e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}+e^{i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2\right)^2}{16\left(\frac{i\left(e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}-e^{i\tan^{-1}\left(1/\sqrt{5}\right)}\right)}{e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}+e^{i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2}}$$

Series representations:

$$-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4} =$$

$$\left(-5+5i^2\left(\sum_{k=-\infty}^{\infty}(-1)^k e^{2ik\tan^{-1}\left(1/\sqrt{5}\right)}\operatorname{sgn}(k)\right)^2 + 4i\exp\left(i\pi\left\lfloor\frac{\arg\left(4-x+\frac{25\left(-1+\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k_1=-\infty}^{\infty}\sum_{k_2=0}^{\infty}\frac{1}{k_2!}(-1)^{k_1+k_2}e^{2i\tan^{-1}\left(1/\sqrt{5}\right)k_1}x^{-k_2}\left(-\frac{1}{2}\right)_{k_2}\operatorname{sgn}(k_1)\left(4-x+\frac{25\left(-1+\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}\right)^{k_2}\right) \Bigg/ \left(8i\sum_{k=-\infty}^{\infty}(-1)^k e^{2ik\tan^{-1}\left(1/\sqrt{5}\right)}\operatorname{sgn}(k)\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4} =$$

$$\left(-5+5\,i^2\left(\sum_{k=-\infty}^{\infty}(-1)^k e^{2ik\tan^{-1}\left(1/\sqrt{5}\right)}\operatorname{sgn}(k)\right)^2+\right.$$

$$4\,i\exp\left(i\pi\left\lfloor\frac{\arg\left(4-x+\frac{25\left(1-\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}\right)}{2\pi}\right\rfloor\right)\sqrt{x}$$

$$\sum_{k_1=-\infty}^{\infty}\sum_{k_2=0}^{\infty}\frac{1}{k_2!}(-1)^{k_1+k_2}e^{2i\tan^{-1}\left(1/\sqrt{5}\right)k_1}x^{-k_2}\left(-\frac{1}{2}\right)_{k_2}$$

$$\left.\operatorname{sgn}(k_1)\left(4-x+\frac{25\left(1-\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}\right)^{k_2}\right)\Big/$$

$$\left(8\,i\sum_{k=-\infty}^{\infty}(-1)^k e^{2ik\tan^{-1}\left(1/\sqrt{5}\right)}\operatorname{sgn}(k)\right)\quad\text{for }(x\in\mathbb{R}\text{ and }x<0)$$

$$-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4} =$$

$$\left(-5+320\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2\left(\sum_{k=1}^{\infty}\frac{1}{(1-2k)^2\pi^2-4\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}\right)^2+\right.$$

$$32\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\exp\left(i\pi\left\lfloor\frac{\arg\left(4-x+\frac{25\left(-1+\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}\right)}{2\pi}\right\rfloor\right)\sqrt{x}$$

$$\left.\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_2}\,x^{-k_2}\left(-\frac{1}{2}\right)_{k_2}\left(4-x+\frac{25\left(-1+\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}\right)^{k_2}}{k_2!\left(-4\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2+\pi^2(1-2k_1)^2\right)}\right)\Bigg/$$

$$\left(64\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\sum_{k=1}^{\infty}\frac{1}{(1-2k)^2\pi^2-4\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}\right)\quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

Continued fraction representations:

$$-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}=$$

$$\frac{1}{8}\left(\frac{5}{\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}}-5\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)+4\sqrt{4+\frac{25\left(-1+\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)^2\right)^2}{16\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)^2}}\right)=$$

$$\frac{1}{8}\left(\frac{5}{-\cfrac{1}{-\cfrac{1}{-\cfrac{1}{-\cfrac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\dots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}}-\right.$$

$$5\left(-\cfrac{1}{-\cfrac{1}{-\cfrac{1}{-\cfrac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\dots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)+$$

$$4\sqrt{\left(4+\left(25\left(-1+\left(-1\Big/\left(-1\Big/\left(-\cfrac{1}{-\cfrac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\dots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)\right)^2\right)^2\right)\Big/}$$

$$\left.\left.\left(16\left(-1\Big/\left(-1\Big/\left(-\cfrac{1}{-\cfrac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\dots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)\right)^2\right)\right)\right)$$

$$-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}=$$

$$\frac{1}{8}\left(4+\frac{5}{\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}}-5\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)+16\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{1}{16}\left(3+\frac{25\left(-1+\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}\right)}{\frac{1}{2}}\right)\right)=$$

$$\frac{1}{8}\left(4+\frac{5}{-\frac{1}{-\frac{1}{-\frac{1}{-\frac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\dots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}}-\right.$$

$$5\left(-\frac{1}{-\frac{1}{-\frac{1}{-\frac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\dots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)+$$

$$\left.16\,\frac{1}{2\left(\frac{1}{2}+\frac{1}{2\left(\frac{1}{2}+\frac{1}{2\left(\frac{1}{2}+\frac{1}{2\left(\frac{1}{2}+\dots\right)}\right)}\right)}\right)}\right)$$

$$-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}=$$

$$\frac{1}{8}\left(4+\frac{5\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}{\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{-1+2k}}-\frac{5\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{-1+2k}\right)}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+16\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{1}{16}\left(3+\frac{25\left(-1+\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}\right)}{\frac{1}{2}}\right)\right)=$$

$$\frac{1}{8}\left(4+16\frac{1}{2\left(\frac{1}{2}+\frac{1}{2\left(\frac{1}{2}+\frac{1}{2\left(\frac{1}{2}+\frac{1}{2\left(\frac{1}{2}+\ldots\right)}\right)}\right)}\right)}+\frac{5\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}{-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{1-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{3-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{5-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{7+\ldots}}}}}-\frac{5\left(-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{1-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{3-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{5-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{7+\ldots}}}}\right)}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

Multiple-argument formulas:

$$-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}=$$
$$\frac{1}{2}\sqrt{4+\frac{25\left(1-6\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+\tan^4\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{64\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\left(-1+\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}}+\frac{5\left(1-6\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+\tan^4\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{16\tan\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\left(-1+\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}$$

$$-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}=$$
$$\frac{1}{2}\sqrt{4+\frac{25\left(1-\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\left(1-\frac{4\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}{\left(1-\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}\right)^2}{64\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}}-\frac{5\left(1-\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)\left(1-\frac{4\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}{\left(1-\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}\right)}{16\tan\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}$$

$$-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}=$$
$$\frac{1}{2}\sqrt{\left(4+\left(25\left(-1+15\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)-15\tan^4\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+\tan^6\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\right)\Big/\left(16\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\left(1-3\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\left(-3+\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\right)\right)}+\frac{5\left(-1+15\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)-15\tan^4\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+\tan^6\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\left(3-10\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+3\tan^4\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}$$

$$-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4} =$$

$$\frac{1}{2}\sqrt{4+\frac{25\left(1-3\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\left(1-\frac{\left(3\tan\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)-\tan^3\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{\left(1-3\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}\right)^2}{16\left(3\tan\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)-\tan^3\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}} - \frac{5\left(1-3\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)\left(1-\frac{\left(3\tan\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)-\tan^3\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{\left(1-3\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}\right)}{8\left(3\tan\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)-\tan^3\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}$$

From which:

1/((-5(1- [tan(((1/2 * 2 tan^-1(1/(sqrt5)))))]^2)/((8(tan(((1/2*2 tan^-1(1/(sqrt5))))))))+1/2*sqrt[(25(1-[tan(((1/2*2 tan^-1(1/(sqrt5)))))]^2)^2)/(16(tan(((1/2*2 tan^-1(1/(sqrt5)))))^2))+4]))-1

Input:

$$\frac{1}{-5\times\frac{1-\tan^2\left(\frac{1}{2}\times 2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}{8\tan\left(\frac{1}{2}\times 2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{1}{2}\times 2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{1}{2}\times 2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}}-1$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}}-1$$

(result in radians)

Decimal approximation:

1.6180339887498948482045868343656381177203091798057628621354...

(result in radians)

1.6180339887…

Alternate forms:

$$\frac{1}{2}\left(1+\sqrt{5}\right)$$

$$\frac{\sqrt{5}}{2}+\frac{1}{2}$$

$$-1-\frac{2}{\sqrt{5}-3}$$

Alternative representations:

$$\frac{1}{-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}}-1=-1+\frac{1}{\frac{-5\left(1-\left(-\cot\left(-\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\right)}{-8\cot\left(-\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{4+\frac{25\left(1-\left(-\cot\left(-\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\right)^2}{16\left(-\cot\left(-\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}}}$$

$$\frac{1}{-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}}-1=-1+\frac{1}{-\frac{5\left(1-\left(-i+\frac{2i}{1+e^{2i\tan^{-1}(1/\sqrt{5})}}\right)^2\right)}{8\left(-i+\frac{2i}{1+e^{2i\tan^{-1}(1/\sqrt{5})}}\right)}+\frac{1}{2}\sqrt{4+\frac{25\left(1-\left(-i+\frac{2i}{1+e^{2i\tan^{-1}(1/\sqrt{5})}}\right)^2\right)^2}{16\left(-i+\frac{2i}{1+e^{2i\tan^{-1}(1/\sqrt{5})}}\right)^2}}}$$

$$\frac{1}{-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}}-1=$$

$$-1+1\Bigg/\left(-\frac{5\left(1-\left(\frac{i\left(e^{-i\tan^{-1}(1/\sqrt{5})}-e^{i\tan^{-1}(1/\sqrt{5})}\right)}{e^{-i\tan^{-1}(1/\sqrt{5})}+e^{i\tan^{-1}(1/\sqrt{5})}}\right)^2\right)}{\frac{8i\left(e^{-i\tan^{-1}(1/\sqrt{5})}-e^{i\tan^{-1}(1/\sqrt{5})}\right)}{e^{-i\tan^{-1}(1/\sqrt{5})}+e^{i\tan^{-1}(1/\sqrt{5})}}}+\frac{1}{2}\sqrt{4+\frac{25\left(1-\left(\frac{i\left(e^{-i\tan^{-1}(1/\sqrt{5})}-e^{i\tan^{-1}(1/\sqrt{5})}\right)}{e^{-i\tan^{-1}(1/\sqrt{5})}+e^{i\tan^{-1}(1/\sqrt{5})}}\right)^2\right)^2}{16\left(\frac{i\left(e^{-i\tan^{-1}(1/\sqrt{5})}-e^{i\tan^{-1}(1/\sqrt{5})}\right)}{e^{-i\tan^{-1}(1/\sqrt{5})}+e^{i\tan^{-1}(1/\sqrt{5})}}\right)^2}}\right)$$

Series representations:

$$\frac{1}{-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}}-1=$$

$$-1+1\Bigg/\left(-\frac{5\left(1-i^2\left(1+2\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)^2\right)}{8\,i\left(1+2\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)}+\right.$$
$$\frac{1}{2}\exp\left(i\pi\left\lfloor\frac{\arg\left(4-x+\frac{25\left(1-\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}\right)}{2\pi}\right\rfloor\right)\sqrt{x}$$
$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(4-x+\frac{25\left(1-\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}\right)^k}{k!}\right)$$

for $\left(x\in\mathbb{R}\text{ and }x<0\text{ and }q=e^{i\tan^{-1}\left(1/\sqrt{5}\right)}\right)$

$$\frac{1}{-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}}-1=$$

$$-\left(\left(-5-8\,i\sum_{k=-\infty}^{\infty}(-1)^k e^{2ik\tan^{-1}\left(1/\sqrt{5}\right)}\operatorname{sgn}(k)+5\,i^2\left(\sum_{k=-\infty}^{\infty}(-1)^k e^{2ik\tan^{-1}\left(1/\sqrt{5}\right)}\operatorname{sgn}(k)\right)^2+4\,i\exp\left(i\pi\left\lfloor\frac{\arg\left(4-x+\frac{25\left(-1+\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k_1=-\infty}^{\infty}\sum_{k_2=0}^{\infty}\frac{1}{k_2!}(-1)^{k_1+k_2}e^{2i\tan^{-1}\left(1/\sqrt{5}\right)k_1}x^{-k_2}\left(-\frac{1}{2}\right)_{k_2}\operatorname{sgn}(k_1)\left(4-x+\frac{25\left(-1+\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}\right)^{k_2}\right)\Bigg/\left(-5+5\,i^2\left(\sum_{k=-\infty}^{\infty}(-1)^k e^{2ik\tan^{-1}\left(1/\sqrt{5}\right)}\operatorname{sgn}(k)\right)^2+4\,i\exp\left(i\pi\left\lfloor\frac{\arg\left(4-x+\frac{25\left(-1+\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k_1=-\infty}^{\infty}\sum_{k_2=0}^{\infty}\frac{1}{k_2!}(-1)^{k_1+k_2}e^{2i\tan^{-1}\left(1/\sqrt{5}\right)k_1}x^{-k_2}\left(-\frac{1}{2}\right)_{k_2}\operatorname{sgn}(k_1)\left(4-x+\frac{25\left(-1+\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}\right)^{k_2}\right)\right)\quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{1}{-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}}-1=$$

$$-\left(\left(-5-8i\sum_{k=-\infty}^{\infty}(-1)^k e^{2ik\tan^{-1}\left(1/\sqrt{5}\right)}\operatorname{sgn}(k)+\right.\right.$$

$$5i^2\left(\sum_{k=-\infty}^{\infty}(-1)^k e^{2ik\tan^{-1}\left(1/\sqrt{5}\right)}\operatorname{sgn}(k)\right)^2+$$

$$4i\exp\left(i\pi\left\lfloor\frac{\arg\left(4-x+\frac{25\left(1-\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}\right)}{2\pi}\right\rfloor\right)\sqrt{x}$$

$$\sum_{k_1=-\infty}^{\infty}\sum_{k_2=0}^{\infty}\frac{1}{k_2!}(-1)^{k_1+k_2}e^{2i\tan^{-1}\left(1/\sqrt{5}\right)k_1}x^{-k_2}\left(-\frac{1}{2}\right)_{k_2}$$

$$\left.\operatorname{sgn}(k_1)\left(4-x+\frac{25\left(1-\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}\right)^{k_2}\right)\Bigg/$$

$$\left(-5+5i^2\left(\sum_{k=-\infty}^{\infty}(-1)^k e^{2ik\tan^{-1}\left(1/\sqrt{5}\right)}\operatorname{sgn}(k)\right)^2+\right.$$

$$4i\exp\left(i\pi\left\lfloor\frac{\arg\left(4-x+\frac{25\left(1-\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}\right)}{2\pi}\right\rfloor\right)\sqrt{x}$$

$$\sum_{k_1=-\infty}^{\infty}\sum_{k_2=0}^{\infty}\frac{1}{k_2!}(-1)^{k_1+k_2}e^{2i\tan^{-1}\left(1/\sqrt{5}\right)k_1}x^{-k_2}\left(-\frac{1}{2}\right)_{k_2}\operatorname{sgn}(k_1)$$

$$\left.\left.\left(4-x+\frac{25\left(1-\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}\right)^{k_2}\right)\right) \quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

Continued fraction representations:

$$\frac{1}{-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}}-1=$$

$$-1+\frac{8}{\frac{5}{\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}}-5\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)+4\sqrt{4+\frac{25\left(-1+\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)^2\right)^2}{16\left(\mathop{\mathrm{K}}_{k=1}^{\infty}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)^2}}}=$$

$$-1+8\Bigg/\left(\frac{5}{-\frac{1}{-\frac{1}{-\frac{1}{-\frac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\ldots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}}-5\left(-\frac{1}{-\frac{1}{-\frac{1}{-\frac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\ldots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)+4\sqrt{\left(4+\left(25\left(-1+\left(-\left(1\Bigg/\left(-\left(1\Bigg/\left(-\frac{1}{-\frac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\ldots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)\right)\right)\right)+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)^2\right)^2\right)\Bigg/\left(16\left(-\left(1\Bigg/\left(-\left(1\Bigg/\left(-\frac{1}{-\frac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\ldots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)\right)\right)\right)+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)^2\right)\right)}\right)$$

$$\frac{1}{-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}}-1=$$

$$-1+\frac{8}{4+\frac{5}{\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}}-5\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)+16\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\frac{1}{16}\left(3+\frac{25\left(-1+\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}\right)}{\frac{1}{2}}\right)}=$$

$$-1+8\Bigg/\left(4+\frac{5}{-\frac{1}{-\frac{1}{-\frac{1}{-\frac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\dots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}}-5\left(-\frac{1}{-\frac{1}{-\frac{1}{-\frac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\dots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)+16\,\frac{1}{2\left(\frac{1}{2}+\frac{1}{2\left(\frac{1}{2}+\frac{1}{2\left(\frac{1}{2}+\frac{1}{2\left(\frac{1}{2}+\dots\right)}\right)}\right)}\right)}\right)$$

$$\frac{1}{-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}}-1=$$

$$-1+\frac{1}{\frac{5\left(1-\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)^2\right)}{8\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)}+\frac{1}{2}\sqrt{4+\frac{25\left(1-\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)^2\right)^2}{16\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)^2}}}=-1+$$

$$1\Bigg/\left(\left(5\left(1-\left(-1\Bigg/\left(-1\Bigg/\left(-\frac{1}{-\frac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\dots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)\right)^2\right)\right)\Bigg/\right.$$

$$\left(8\left(-1\Bigg/\left(-\frac{1}{-\frac{1}{-\frac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\dots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)\right)\right)+$$

$$\left.\frac{1}{2}\sqrt{\left(4+25\left(1-\left(-1\Bigg/\left(-1\Bigg/\left(-\frac{1}{-\frac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\dots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)\right)^2\right)^2\Bigg/\left(16\left(-1\Bigg/\left(-1\Bigg/\left(-\frac{1}{-\frac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\dots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)\right)^2\right)\right)}\right)$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

Multiple-argument formulas:

$$\frac{1}{-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}}-1=$$

$$-1+1\Bigg/\left(\frac{1}{2}\sqrt{4+\frac{25\left(1-6\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+\tan^4\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{64\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\left(-1+\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}}+\frac{5\left(1-6\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+\tan^4\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{16\tan\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\left(-1+\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}\right)$$

$$\frac{1}{-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}}-1=$$

$$-1+1\Bigg/\left(\frac{1}{2}\sqrt{4+\frac{25\left(1-\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\left(1-\frac{4\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}{\left(1-\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}\right)^2}{64\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}}-\frac{5\left(1-\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)\left(1-\frac{4\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}{\left(1-\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}\right)}{16\tan\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}\right)$$

$$\frac{1}{-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}}-1=$$

$$-1+1\Bigg/\left(\frac{1}{2}\sqrt{\left(4+\left(25\left(-1+15\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)-15\tan^4\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+\tan^6\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\right)\Big/\left(16\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\left(1-3\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\left(-3+\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\right)\right)}+\left(5\left(-1+15\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)-15\tan^4\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+\tan^6\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)\right)\Big/\left(8\tan\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\left(3-10\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+3\tan^4\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)\right)\right)$$

$$\frac{1}{-\frac{5\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}{8\tan\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+\frac{1}{2}\sqrt{\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4}}-1=$$

$$-1+1\Bigg/\left(\frac{1}{2}\sqrt{\left(4+\left(25\left(1-3\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\left(1-\frac{\left(3\tan\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)-\tan^3\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{\left(1-3\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}\right)^2\right)\Big/\left(16\left(3\tan\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)-\tan^3\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\right)\right)}-\frac{5\left(1-3\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)\left(1-\frac{\left(3\tan\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)-\tan^3\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{\left(1-3\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}\right)}{8\left(3\tan\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)-\tan^3\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)}\right)$$

From the left hand-side of the previous expression, we obtain:

$$\frac{25(1-t_1^2)^2}{16t_1^2}+4=\frac{9(1+v^2)^2}{(1+v-v^2)(1-11v-v^2)},$$

[25(1-(tan (((1/2 * 2 tan^-1(1/(sqrt5)))))^2))^2] / [16*(tan (((1/2 * 2 tan^-1(1/(sqrt5))))))^2] + 4

Input:

$$\frac{25\left(1-\tan^2\left(\frac{1}{2}\times 2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{1}{2}\times 2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact result:

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Alternative representations:

$$\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4=4+\frac{25\left(1-\left(-\cot\left(-\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\right)^2}{16\left(-\cot\left(-\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}$$

$$\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4=4+\frac{25\left(1-\left(-i+\frac{2i}{1+e^{2i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2\right)^2}{16\left(-i+\frac{2i}{1+e^{2i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2}$$

$$\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4=4+\frac{25\left(1-\left(\frac{i\left(e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}-e^{i\tan^{-1}\left(1/\sqrt{5}\right)}\right)}{e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}+e^{i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2\right)^2}{16\left(\frac{i\left(e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}-e^{i\tan^{-1}\left(1/\sqrt{5}\right)}\right)}{e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}+e^{i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2}$$

Continued fraction representations:

$$\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4=4+\frac{25\left(-1+\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)^2\right)^2}{16\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)^2}=4+$$

$$\frac{25\left(-1+\left(-\frac{1}{-\frac{1}{-\frac{1}{-\frac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\ldots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)^2\right)^2}{16\left(-\frac{1}{-\frac{1}{-\frac{1}{-\frac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\ldots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)^2}$$

$$\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4=4+\frac{25\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2-\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{-1+2k}\right)^2\right)^2}{16\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{-1+2k}\right)^2}=$$

$$4+\frac{25\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2-\left(-\cfrac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{1-\cfrac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{3-\cfrac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{5-\cfrac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{7+\ldots}}}}\right)^2\right)^2}{16\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2\left(-\cfrac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{1-\cfrac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{3-\cfrac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{5-\cfrac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{7+\ldots}}}}\right)^2}$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

Multiple-argument formulas:

$$\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4=$$

$$4+\frac{25\left(1-6\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+\tan^4\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{64\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\left(-1+\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}$$

$$\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4=$$

$$4+\frac{25\left(-1+15\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)-15\tan^4\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+\tan^6\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\left(1-3\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\left(-3+\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}$$

From the right hand-side

$$\frac{25(1-t_1^2)^2}{16t_1^2}+4=\frac{9(1+v^2)^2}{(1+v-v^2)(1-11v-v^2)},$$

For v = 2, we obtain:

((9(1+4)^2)) / (((1+2-4)(1-22-4)))

Input:

$$\frac{9(1+4)^2}{(1+2-4)(1-22-4)}$$

Result:

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9

We note that:

((([25(1-(tan (((1/2 * 2 tan^-1(1/(sqrt5)))))^2))^2] / [16*(tan (((1/2 * 2 tan^-1(1/(sqrt5))))))^2] + 4)))^3 - 1

Input:

$$\left(\frac{25\left(1-\tan^2\left(\frac{1}{2}\times 2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{1}{2}\times 2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3-1$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact result:

728

728

Alternative representations:

$$\left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3-1 = -1+\left(4+\frac{25\left(1-\left(-\cot\left(-\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\right)^2}{16\left(-\cot\left(-\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}\right)^3$$

$$\left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3-1 = -1+\left(4+\frac{25\left(1-\left(-i+\frac{2i}{1+e^{2i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2\right)^2}{16\left(-i+\frac{2i}{1+e^{2i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2}\right)^3$$

$$\left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3-1 = -1+\left(4+\frac{25\left(1-\left(\frac{i\left(e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}-e^{i\tan^{-1}\left(1/\sqrt{5}\right)}\right)}{e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}+e^{i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2\right)^2}{16\left(\frac{i\left(e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}-e^{i\tan^{-1}\left(1/\sqrt{5}\right)}\right)}{e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}+e^{i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2}\right)^3$$

Continued fraction representations:

$$\left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3-1=-1+\left(4+\frac{25\left(-1+\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)^2\right)^2}{16\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)^2}\right)^3=$$

$$-1+\left(4+\left(25\left(-1+\left(-\left(1\middle/\left(-\left(1\middle/\left(-\frac{1}{-\frac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\dots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)\right)\right)+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)\right)^2\right)^2\right)\middle/\right.$$

$$\left.\left(16\left(-\left(1\middle/\left(-\left(1\middle/\left(-\frac{1}{-\frac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\dots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)\right)\right)+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)\right)^2\right)\right)^3$$

$$\left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3-1=$$

$$-1+\left(4+\frac{25\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2-\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{-1+2k}\right)^2\right)^2}{16\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{-1+2k}\right)^2}\right)^3=$$

$$-1+\left(4+\frac{25\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2-\left(-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{1-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{3-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{5-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{7+\ldots}}}}\right)^2\right)^2}{16\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2\left(-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{1-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{3-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{5-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{7+\ldots}}}}\right)^2}\right)^3$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k/b_k$ is a continued fraction

Multiple-argument formulas:

$$\left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3-1=$$
$$-1+\left(4+\frac{25\left(1-6\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+\tan^4\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{64\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\left(-1+\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}\right)^3$$

$$\left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3-1=-1+\left(4+\frac{25\left(1-3\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\left(1-\frac{\left(3\tan\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)-\tan^3\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{\left(1-3\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}\right)^2}{16\left(3\tan\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)-\tan^3\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}\right)^3$$

and:

10^3+((([25(1-(tan (((1/2 * 2 tan^-1(1/(sqrt5)))))^2))^2] / [16*(tan (((1/2 * 2 tan^-1(1/(sqrt5))))))^2] + 4)))^3

Input:

$$10^3+\left(\frac{25\left(1-\tan^2\left(\frac{1}{2}\times 2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{1}{2}\times 2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact result:

1729

1729

Alternative representations:

$$10^3+\left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3=$$
$$10^3+\left(4+\frac{25\left(1-\left(-\cot\left(-\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\right)^2}{16\left(-\cot\left(-\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}\right)^3$$

$$10^3+\left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3=$$

$$10^3+\left(4+\frac{25\left(1-\left(-i+\frac{2i}{1+e^{2i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2\right)^2}{16\left(-i+\frac{2i}{1+e^{2i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2}\right)^3$$

$$10^3+\left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3=$$

$$10^3+\left(4+\frac{25\left(1-\left(\frac{i\left(e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}-e^{i\tan^{-1}\left(1/\sqrt{5}\right)}\right)}{e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}+e^{i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2\right)^2}{16\left(\frac{i\left(e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}-e^{i\tan^{-1}\left(1/\sqrt{5}\right)}\right)}{e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}+e^{i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2}\right)^3$$

Continued fraction representations:

$$10^3+\left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3=$$

$$1000+\left(4+\frac{25\left(-1+\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)^2\right)^2}{16\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)^2}\right)^3=$$

$$1000+\left(4+\left(25\left(-1+\left(-1\middle/\left(-1\middle/\left(-\frac{1}{-\frac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\dots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)\right)+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)\right)^2\right)^2\right)\middle/\right.$$

$$\left(16\left(-1\middle/\left(-1\middle/\left(-\frac{1}{-\frac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\dots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}+\frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)\right)+\frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)\right)^2\right)\right)^3$$

$$10^3+\left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3=$$

$$1000+\left(4+\frac{25\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2-\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{-1+2k}\right)^2\right)^2}{16\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{-1+2k}\right)^2}\right)^3=$$

$$1000+\left(4+\frac{25\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2-\left(-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{1-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{3-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{5-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{7+\ldots}}}}\right)^2\right)^2}{16\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2\left(-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{1-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{3-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{5-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{7+\ldots}}}}\right)^2}\right)^3$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k/b_k$ is a continued fraction

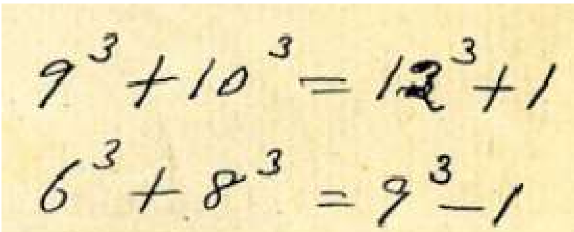
Multiple-argument formulas:

$$10^3+\left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3=$$

$$1000+\left(4+\frac{25\left(1-6\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+\tan^4\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{64\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\left(-1+\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}\right)^3$$

$$10^3+\left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3=1000+\left(4+\frac{25\left(1-3\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\left(1-\frac{\left(3\tan\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)-\tan^3\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{\left(1-3\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}\right)^2}{16\left(3\tan\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)-\tan^3\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}\right)^3$$

We note that, from the Ramanujan taxicab numbers:



that are 1729 and 728 respectively

In conclusion, we obtain:

(((10^3+((([25(1-(tan (((1/2 * 2 tan^-1(1/(sqrt5)))))^2))^2] / [16*(tan (((1/2 * 2 tan^-1(1/(sqrt5))))))^2] + 4)))^3)))^1/15

Input:

$$\sqrt[15]{10^3+\left(\frac{25\left(1-\tan^2\left(\frac{1}{2}\times 2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{1}{2}\times 2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3}$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\sqrt[15]{1729}$$

(result in radians)

Decimal approximation:

1.643815228748728130580088031324769514329283143699940172645...

(result in radians)

1.643815228748...

(((10^3+((([25(1-(tan (((1/2 * 2 tan^-1(1/(sqrt5)))))^2))^2] / [16*(tan (((1/2 * 2 tan^-1(1/(sqrt5))))))^2] + 4)))^3)))^1/15 -26/10^3

Input:

$$\sqrt[15]{10^3+\left(\frac{25\left(1-\tan^2\left(\frac{1}{2}\times 2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{1}{2}\times 2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3}-\frac{26}{10^3}$$

$\tan^{-1}(x)$ is the inverse tangent function

Exact Result:

$$\sqrt[15]{1729}-\frac{13}{500}$$

(result in radians)

Decimal approximation:

1.617815228748728130580088031324769514329283143699940172645...

(result in radians)

1.617815228748...

Alternate forms:

$$\frac{1}{500}\left(500\sqrt[15]{1729}-13\right)$$

$$\frac{1}{500}\left(\left(500\ \text{root of } 31\,250\,000\,000\,000\,x^5+686\,562\,500\,000\,000\,x^4+6\,033\,511\,250\,000\,000\,x^3+26\,511\,248\,432\,500\,000\,x^2+58\,245\,212\,806\,202\,500\,x-52\,764\,892\,578\,124\,999\,999\,999\,999\,948\,814\,106\,985\,909\,243 \text{ near } x=1.11045\times10^6+2197\right)\hat{}\,(1/3)-13\right)$$

Alternative representations:

$$\sqrt[15]{10^3+\left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3}-\frac{26}{10^3}=$$

$$-\frac{26}{10^3}+\sqrt[15]{10^3+\left(4+\frac{25\left(1-\left(-\cot\left(-\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\right)^2}{16\left(-\cot\left(-\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}\right)^3}$$

$$\sqrt[15]{10^3+\left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3}-\frac{26}{10^3}=$$

$$-\frac{26}{10^3}+\sqrt[15]{10^3+\left(4+\frac{25\left(1-\left(-i+\frac{2i}{1+e^{2i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2\right)^2}{16\left(-i+\frac{2i}{1+e^{2i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2}\right)^3}$$

$$\sqrt[15]{10^3+\left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3}-\frac{26}{10^3}=$$

$$-\frac{26}{10^3}+\sqrt[15]{10^3+\left(4+\frac{25\left(1-\left(\frac{i\left(e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}-e^{i\tan^{-1}\left(1/\sqrt{5}\right)}\right)}{e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}+e^{i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2\right)^2}{16\left(\frac{i\left(e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}-e^{i\tan^{-1}\left(1/\sqrt{5}\right)}\right)}{e^{-i\tan^{-1}\left(1/\sqrt{5}\right)}+e^{i\tan^{-1}\left(1/\sqrt{5}\right)}}\right)^2}\right)^3}$$

Series representations:

$$\sqrt[15]{10^3+\left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3}-\frac{26}{10^3}=$$

$$-\frac{13}{500}+\sqrt[15]{1000+\left(4+\frac{25\left(1-i^2\left(1+2\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)^2\right)^2}{16\,i^2\left(1+2\sum_{k=1}^{\infty}(-1)^k q^{2k}\right)^2}\right)^3}\quad \text{for } q=e^{i\tan^{-1}\left(1/\sqrt{5}\right)}$$

$$\sqrt[15]{10^3 + \left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)} + 4\right)^3} - \frac{26}{10^3} =$$

$$\frac{1}{500}\left(-13 + 500\sqrt[15]{1000 + \left(4 + \frac{25\left(-1 + i^2\left(\sum_{k=-\infty}^{\infty}(-1)^k e^{2 i k \tan^{-1}\left(1/\sqrt{5}\right)}\operatorname{sgn}(k)\right)^2\right)^2}{16 i^2\left(\sum_{k=-\infty}^{\infty}(-1)^k e^{2 i k \tan^{-1}\left(1/\sqrt{5}\right)}\operatorname{sgn}(k)\right)^2}\right)^3}\right)$$

$$\sqrt[15]{10^3 + \left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)} + 4\right)^3} - \frac{26}{10^3} =$$

$$\frac{1}{500}\left(-13 + 500\sqrt[15]{1000 + \left(4 + \frac{25\left(1 - i^2\left(\sum_{k=-\infty}^{\infty}(-1)^k e^{2 i k \tan^{-1}\left(1/\sqrt{5}\right)}\operatorname{sgn}(k)\right)^2\right)^2}{16 i^2\left(\sum_{k=-\infty}^{\infty}(-1)^k e^{2 i k \tan^{-1}\left(1/\sqrt{5}\right)}\operatorname{sgn}(k)\right)^2}\right)^3}\right)$$

Continued fraction representations:

$$\sqrt[15]{10^3 + \left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3} - \frac{26}{10^3} =$$

$$-\frac{13}{500} + \sqrt[15]{1000 + \left(4 + \frac{25\left(-1+\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)^2\right)^2}{16\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-1}{\frac{-1+2k}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}}\right)^2}\right)^3} =$$

$$-\frac{13}{500} + \left(1000 + \left(4 + \left(25\left(-1 + \left(-\left(1\Big/\left(-\left(1\Big/\left(-\frac{1}{-\frac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\ldots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}} + \frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)\right) + \frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)\right)\right)^2\right)^2\right)\Big/\right.\right.$$

$$\left.\left.\left(16\left(-\left(1\Big/\left(-\left(1\Big/\left(-\frac{1}{-\frac{1}{\frac{7}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}+\ldots}+\frac{5}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}} + \frac{3}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)\right) + \frac{1}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)}\right)\right)\right)^2\right)\right)^3\right)\hat{}\,(1/15)$$

$$\sqrt[15]{10^3+\left(\frac{25\left(1-\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}+4\right)^3}-\frac{26}{10^3}=$$

$$-\frac{13}{500}+\sqrt[15]{1000+\left(4+\frac{25\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2-\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{-1+2k}\right)^2\right)^2}{16\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{-1+2k}\right)^2}\right)^3}=$$

$$-\frac{13}{500}+\sqrt[15]{1000+\left(4+\frac{25\left(\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2-\left(-\cfrac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{1-\cfrac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{3-\cfrac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{5-\cfrac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{7+\ldots}}}}\right)^2\right)^2}{16\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2\left(-\cfrac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{1-\cfrac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{3-\cfrac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{5-\cfrac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{7+\ldots}}}}\right)^2}\right)^3}$$

$$\sqrt[15]{10^3 + \left(\frac{25\left(1 - \tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)} + 4\right)^3} - \frac{26}{10^3} =$$

$$-\frac{13}{500} + \sqrt[15]{1000 + \left(4 + \frac{25\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2\left(1 - \frac{\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{-1+2k}\right)^2}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}\right)^2}{16\left(\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{-1+2k}\right)^2}\right)^3} =$$

$$-\frac{13}{500} + \sqrt[15]{1000 + \left(4 + \frac{25\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2\left(1 - \frac{\left(-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{1-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{3-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{5-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{7+\dots}}}}\right)^2}{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}\right)^2}{16\left(-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{1-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{3-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{5-\frac{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)^2}{7+\dots}}}}\right)^2}\right)^3}$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

Multiple-argument formulas:

$$\sqrt[15]{10^3 + \left(\frac{25\left(1 - \tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)} + 4\right)^3} - \frac{26}{10^3} =$$

$$-\frac{13}{500} + \sqrt[15]{1000 + \left(4 + \frac{25\left(1 - \tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2 \left(1 - \frac{4\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}{\left(1-\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}\right)^2}{64\tan^2\left(\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)}\right)^3}$$

$$\sqrt[15]{10^3 + \left(\frac{25\left(1 - \tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{16\tan^2\left(\frac{2}{2}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)} + 4\right)^3} - \frac{26}{10^3} =$$

$$-\frac{13}{500} + \left(1000 + \left(4 + \left(25\left(1 - 3\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2 \left(1 - \frac{\left(3\tan\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right) - \tan^3\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}{\left(1 - 3\tan^2\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2}\right)^2\right) \Big/ \left(16\left(3\tan\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right) - \tan^3\left(\frac{1}{3}\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)^2\right)\right)^3\right) \hat{}\ (1/15)$$

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the *golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.*

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the *second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.*

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:
2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

References

INCOMPLETE ELLIPTIC INTEGRALS IN RAMANUJAN'S LOST NOTEBOOK
BRUCE C. BERNDT, HENG HUAT CHAN, AND SEN–SHAN HUANG