Symmetry of the Lorentz Transformation equation in Dynamics

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Abstract

Lorentz transformation is a group of coordinate transformations based on the Principle of Special Relativity and the Principle of constant speed of light.By dealing with these basic coordinate transformations and combining with dynamics, we can also get many physical quantity transformations.In the process of derivation of Lorentz transformation, we do not stipulate which reference frames of relative motion is special, so the transformation obtained must also be equal weight.This requires that for the transformation of any physical quantity, the transformation equation of it must satisfy the symmetry, that is, the positive transformation f and the inverse transformation f' must keep the same mathematical form.Based on this principle, the symmetry of the obtained Lorentz transformation dynamics equation will be tested.

1. Symmetry of the Lorentz transformation.

The Lorentz transformation is a transformation with symmetry, which is also the embodiment of "all physical laws should have the same mathematical form in any inertial reference system" required by the Principle of Special Relativity.This requires that for Lorentz transformation of any physical quantity, its transformation formula should be symmetric, that is

$$p_i = f(p'_i, v_0) \Leftrightarrow p'_i = f(p_i, -v_0) \quad \textbf{(1)}$$

Here p is the physical quantity of transformation, v_0 is the relative speed of motion, $i \in x, y, z$,i.e. three dimensions in the corresponding space. (Note: Due to the directionality of the relative velocity, the relative velocity corresponding to the inverse transformation is $-v_0$.)

For example, for the Lorentz velocity transformation, here are

$$v_x = \frac{v'_x + v_0}{1 + \frac{v_0 v'_x}{c^2}}, v_y = \frac{v'_y \sqrt{1 - \frac{v_0^2}{c^2}}}{1 + \frac{v_0 v'_x}{c^2}}, v_z = \frac{v'_z \sqrt{1 - \frac{v_0^2}{c^2}}}{1 + \frac{v_0 v'_x}{c^2}} \quad \textbf{(2)}$$

The positive transformation equations can be used to solve the inverse transformation

$$v_x = \frac{v'_x + v_0}{1 + \frac{v_0 v'_x}{c^2}} \Leftrightarrow (1 + \frac{v_0 v'_x}{c^2})\ v_x = v'_x + v_0 \Leftrightarrow v'_x = \frac{v_x - v_0}{1 - \frac{v_0 v_x}{c^2}}$$

$$v_y = \frac{v'_y \sqrt{1 - \frac{v_0^2}{c^2}}}{1 + \frac{v_0 v'_x}{c^2}} \Leftrightarrow v'_y \sqrt{1 - \frac{v_0^2}{c^2}} = v_y (1 + \frac{v_0}{c^2} \cdot \frac{v_x - v_0}{1 - \frac{v_0 v_x}{c^2}}) \Leftrightarrow v'_y = \frac{v_y \sqrt{1 - \frac{v_0^2}{c^2}}}{1 - \frac{v_0 v_x}{c^2}}$$

Empathy

$$v'_z = \frac{v_z \sqrt{1 - \frac{v_0^2}{c^2}}}{1 - \frac{v_0 v_x}{c^2}}$$

That is, the transformation $v_i = f(v'_i, v_0)$ and the inverse transformation $v_i = f'(v'_i, v_0)$ keep the same mathematical form.

2.Symmetry of the Lorentz Transformation equation in Dynamics

In the paper ***on the Lorentz covariance of conservation law for moving bodies***, we get the Lorentz transformation equations of dynamic physical quantities (impulse, mass transformation, work)

$$dm = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}} (dm' + \frac{\vec{v}_0}{c^2} d\vec{I}') \quad \textbf{(3)}$$

$$dI_x = \frac{1}{\sqrt{1 - \frac{v_0^2}{c^2}}} (dm' v_0 + dI'_x), dI_y = dI'_y, dI_z = dI'_z \quad \textbf{(4)}$$

Now let's test the symmetry of the transformation equations (since $dW = dmc^2$, that is, dW and dm are equivalent in the equation, we only need to discuss dm):

$$dI_x = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}(dm'v_0 + dI'_x) \Leftrightarrow dI'_x = \sqrt{1-\frac{v_0^2}{c^2}}dI_x - dm'v_0 \qquad \textbf{(5)}$$

$$dm = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}(dm' + \frac{\vec{v}_0}{c^2}d\vec{I}') \Leftrightarrow dm' = \sqrt{1-\frac{v_0^2}{c^2}}dm - \frac{v_0}{c^2}dI'_x \qquad \textbf{(6)}$$

So it can be concluded that

$$dI'_x = \sqrt{1-\frac{v_0^2}{c^2}}dI_x - dm'v_0 = \sqrt{1-\frac{v_0^2}{c^2}}dI_x - \sqrt{1-\frac{v_0^2}{c^2}}dm\cdot v_0 + \frac{v_0^2}{c^2}dI'_x$$

Move the right end $\frac{v_0^2}{c^2}dI'_x$ of the equation to the left end, i.e

$$(1-\frac{v_0^2}{c^2})dI'_x = \sqrt{1-\frac{v_0^2}{c^2}}(dI_x - dm\cdot v_0) \Leftrightarrow dI'_x = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}(dI_x - dm\cdot v_0)$$

Substituting this equation into equation (6),it can be changed to

$$dm' = \sqrt{1-\frac{v_0^2}{c^2}}dm - \frac{v_0}{c^2}dI'_x = \sqrt{1-\frac{v_0^2}{c^2}}dm - \frac{v_0}{c^2}\frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}(dI_x - dm\cdot v_0) = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}(dm - \frac{v_0}{c^2}dI_x)$$

So the inverse transformation of this set of equations is

$$dm' = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}(dm - \frac{\vec{v}_0}{c^2}d\vec{I}) \qquad \textbf{(7)}$$

$$dI'_x = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}}(dI_x - dmv_0), dI'_y = dI_y, dI'_z = dI_z \qquad \textbf{(8)}$$

Therefore, the transformation $dI_i = f(dI'_i, v_0)$, $dm = f(dm', v_0)$ and the inverse transformation $dI_i = f'(dI'_i, v_0)$, $dm = f'(dm', v_0)$ keep the same mathematical form, that is,the Lorentz transformation of these dynamic equations have the symmetry.

Reference

[1]Ella Su,***on the Lorentz covariance of conservation law for moving bodies.***