R.H.

$$\xi(s) = \xi(0) \prod_{\operatorname{Im} \rho > 0} \left(1 - \frac{s}{\rho}\right)\left(1 - \frac{s}{1-\rho}\right)$$

Claim:- If ρ_0 is a zero of $\xi(s) = 0$ i.e
if $\xi(\rho_0) = 0$ then $\operatorname{Re} \rho_0 = \frac{1}{2}$.

Let, $\xi(\rho_0) = 0$

$\Rightarrow \xi(\overline{\rho_0}) = 0$

$$\xi(0) \prod_{\operatorname{Im} \rho > 0} \left(1 - \frac{\overline{\rho_0}}{\rho}\right)\left(1 - \frac{\overline{\rho_0}}{1-\rho}\right) = 0$$

Let ρ_0 be a zero of multiplicity k.

$$\xi(0) \left(1 - \frac{\overline{\rho_0}}{\rho_0}\right)^K \left(1 - \frac{\overline{\rho_0}}{1-\rho_0}\right)^K \prod_{\substack{\operatorname{Im} \rho > 0 \\ \rho \neq \rho_0}} \left(1 - \frac{\overline{\rho_0}}{\rho}\right)\left(1 - \frac{\overline{\rho_0}}{1-\rho}\right) = 0$$

$$\xi(0) \left[\frac{2i \operatorname{Im} \rho_0 (1 - 2 \operatorname{Re} \rho_0)}{\rho_0 (1-\rho_0)}\right]^K \prod_{\substack{\operatorname{Im} \rho > 0 \\ \rho \neq \rho_0}} \left(1 - \frac{\overline{\rho_0}}{\rho}\right)\left(1 - \frac{\overline{\rho_0}}{1-\rho}\right) = 0$$

$\because \operatorname{Im} \rho_0 > 0$ & $\because \xi(0) = \frac{1}{2}$

$$\Rightarrow (1 - 2 \operatorname{Re} \rho_0)^K \prod_{\substack{\operatorname{Im} \rho > 0 \\ \rho \neq \rho_0}} \left(1 - \frac{\overline{\rho_0}}{\rho}\right)\left(1 - \frac{\overline{\rho_0}}{1-\rho}\right) = 0$$

Let $\rho_0 = \sigma_0 + it_0$

$\rho_1 = \sigma_1 + it_1$

$\rho_0 = 1 - \rho_1$

$\sigma_0 + it_0 = 1 - \sigma_1 - it_1$

$t_0 = -t_1$

$\because \operatorname{Im} \rho_0 > 0$

$\Rightarrow t_0 > 0$

$\Rightarrow -t_1 > 0$

$\Rightarrow t_1 < 0$

$\Rightarrow \operatorname{Im} \rho_1 < 0$

Contradicts $\operatorname{Im} \rho_1 > 0$

So, our assumption that $I(\bar{\rho}_0) = 0$ is wrong.

$\therefore I(\bar{\rho}_0) \neq 0$

From ①, $(1 - 2 \operatorname{Re} \rho_0)^k \, I(\bar{\rho}_0) = 0$

$\Rightarrow (1 - 2 \operatorname{Re} \rho_0)^k = 0$

$\Rightarrow \operatorname{Re} \rho_0 = \frac{1}{2}$

$\therefore$ For any arbitrary zero ρ_0 of $\xi(s) = 0$

$\operatorname{Re} \rho_0 = \frac{1}{2}$.