On the parameters of SMBH 87 and Primordial Black Holes in String Theory and Inflation: New possible mathematical connections with some Ramanujan equations, ϕ, $\zeta(2)$ and Hausdorff dimension values

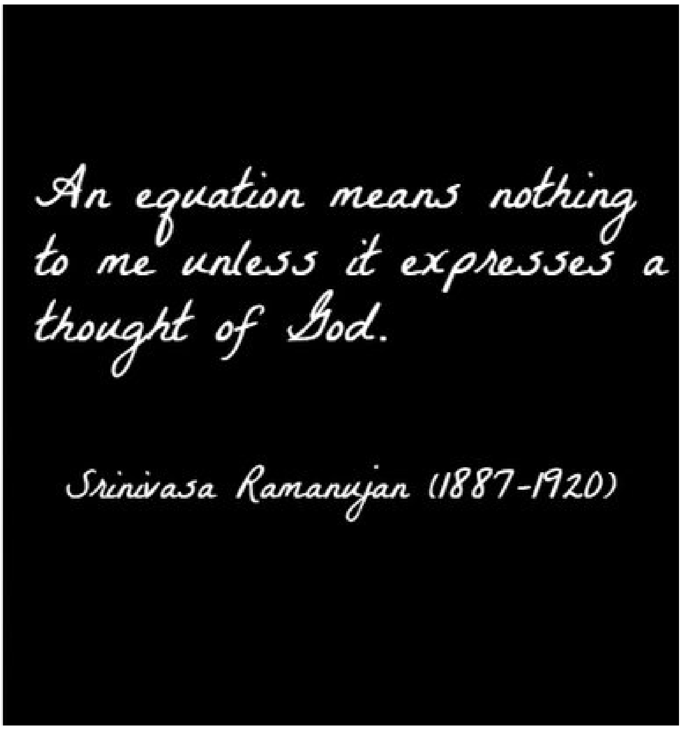
Michele Nardelli[1], **Antonio Nardelli**[2]

Abstract

In this paper we have described the parameters of SMBH 87 and some formulas concerning Primordial Black Holes in String Theory and Inflation. We described also new possible mathematical connections with some Ramanujan equations, ϕ, $\zeta(2)$ and Hausdorff dimension values

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

[2] A. Nardelli studies at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**



https://mobygeek.com/features/indian-mathematician-srinivasa-ramanujan-quotes-11012

From:

Universal interferometric signatures of a black hole's photon ring

Michael D. Johnson, Alexandru Lupsasca, Andrew Strominger, George N. Wong, Shahar Hadar, Daniel Kapec, Ramesh Narayan, Andrew Chael, Charles F. Gammie, Peter Galison, Daniel C. M. Palumbo, Sheperd S. Doeleman, Lindy Blackburn, Maciek Wielgus, Dominic W. Pesce, Joseph R. Farah, James M. Moran

Johnson *et al*., *Sci. Adv.* 2020; **6** : eaaz1310 18 March 2020

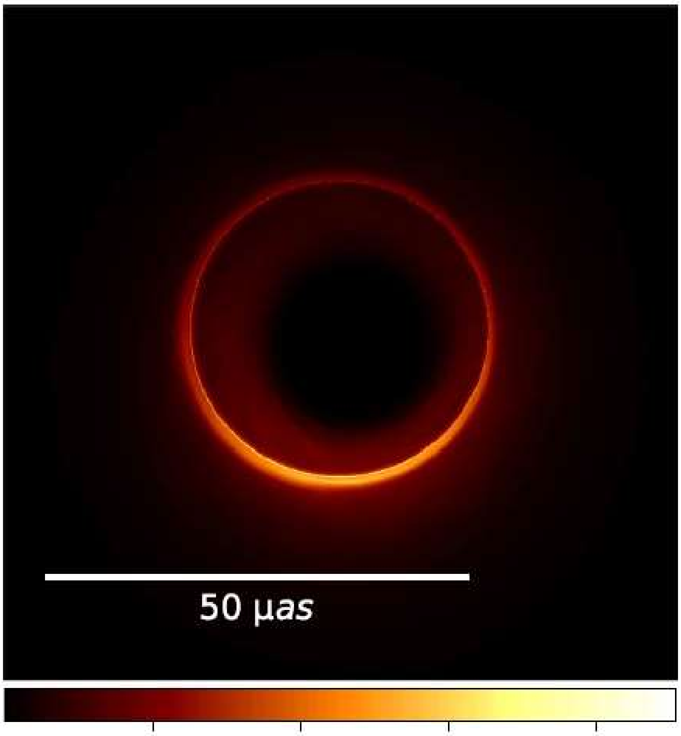


Fig. 1. Time-averaged image of a GRMHD simulation of M87. This model has parameters chosen to be consistent with the 2017 EHT data and corresponds to the high magnetic flux "magnetically arrested disk" accretion state. It has $M = 6.2 \times 10^9 M_\odot$, $a/M = 0.94$, $\theta_{obs} = 163°$, $r_{high} = 10$, and mass accretion rate matching the 1.3-mm flux density (5). The spin axis points left when projected onto the image. The time average was performed over 100 snapshots produced from uniformly spaced GRMHD fluid samples over a time range of 1000*M* (approximately 1 year). Although visually prominent, the thin, bright ring contains only ~20% of the total image flux density.

We have that:

The mass of black hole is $M = 6.2 * 10^9$ solar masses that is equal to

$M_{BH} = 6.2 * 10^9 * 1.9891 * 10^{30} = 1.233242 \times 10^{40} = 12.33242 * 10^{39}$ kg

The black hole spin is: $a / M = 0.94$ and $\theta_{obs} = 163°$

With regard the value of 163°, we have that:

Input interpretation:

163° (degrees)

Unit conversions:

2.84488668 radians

2.84488668

Unit conversions:

$\frac{163\pi}{180}$ radians

We note that:

1/[(163Pi) / 180]^1/24

Input:

$$\frac{1}{\sqrt[24]{\frac{163\pi}{180}}}$$

Exact result:

$$\sqrt[12]{6}\ \sqrt[24]{\frac{5}{163\pi}}$$

Decimal approximation:

0.957371789539716092539843002108088740028570823722423132235...

0.9573717895… result very near to the spectral index n_s , to the mesonic Regge slope, to the inflaton value at the end of the inflation 0.9402 (see Appendix) and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1}=1-\frac{e^{-\pi}}{1+\frac{e^{-2\pi}}{1+\frac{e^{-3\pi}}{1+\frac{e^{-4\pi}}{1+\ldots}}}}\approx 0.9568666373$$

From:

Astronomy & Astrophysics manuscript no. ms c ESO 2019 - September 24, 2019
Planck 2018 results. VI. Cosmological parameters

The primordial fluctuations are consistent with Gaussian purely adiabatic scalar perturbations characterized by a power spectrum with a spectral index n_s = 0.965 ± 0.004, consistent with the predictions of slow-roll, single-field, inflation.

We know that α' is the Regge slope (string tension). With regard the Omega mesons, the values are:

ω	6	$m_{u/d} = 0 - 60$	0.910 – 0.918
ω/ω_3	5 + 3	$m_{u/d} = 255 - 390$	0.988 – 1.18
ω/ω_3	5 + 3	$m_{u/d} = 240 - 345$	0.937 – 1.000

Property:

$\sqrt[12]{6}\,\sqrt[24]{\frac{5}{163\,\pi}}$ is a transcendental number

Alternative representations:

$$\frac{1}{\sqrt[24]{\frac{163\,\pi}{180}}} = \frac{1}{\sqrt[24]{\frac{29340\,°}{180}}}$$

$$\frac{1}{\sqrt[24]{\frac{163\,\pi}{180}}} = \frac{1}{\sqrt[24]{-\frac{163}{180}\,i\log(-1)}}$$

$$\frac{1}{\sqrt[24]{\frac{163\,\pi}{180}}} = \frac{1}{\sqrt[24]{\frac{163}{180}\cos^{-1}(-1)}}$$

Series representations:

$$\frac{1}{\sqrt[24]{\frac{163\,\pi}{180}}} = \sqrt[24]{\frac{5}{163}}\,\sqrt[12]{3}\,\sqrt[24]{\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}}}$$

$$\frac{1}{\sqrt[24]{\frac{163\,\pi}{180}}} = \sqrt[24]{\frac{5}{163}}\,\sqrt[12]{6}\,\sqrt[24]{\frac{1}{\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)}}$$

$$\frac{1}{\sqrt[24]{\frac{163\pi}{180}}} = \sqrt[24]{\frac{5}{163}} \sqrt[12]{3} \sqrt[24]{\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^{1+k} 1195^{-1-2k} \left(5^{1+2k} - 4 \times 239^{1+2k}\right)}{1+2k}}}$$

Integral representations:

$$\frac{1}{\sqrt[24]{\frac{163\pi}{180}}} = \sqrt[24]{\frac{10}{163}} \sqrt[12]{3} \sqrt[24]{\frac{1}{\int_0^{\infty} \frac{1}{1+t^2}\, dt}}$$

$$\frac{1}{\sqrt[24]{\frac{163\pi}{180}}} = \sqrt[24]{\frac{5}{163}} \sqrt[12]{3} \sqrt[24]{\frac{1}{\int_0^{1} \sqrt{1-t^2}\, dt}}$$

$$\frac{1}{\sqrt[24]{\frac{163\pi}{180}}} = \sqrt[24]{\frac{10}{163}} \sqrt[12]{3} \sqrt[24]{\frac{1}{\int_0^{1} \frac{1}{\sqrt{1-t^2}}\, dt}}$$

From r = 3M, we obtain: $3*M_{BH} = 3*12.33242 * 10^{39}$ kg $= 36.99726 * 10^{39}$

$r = 36.99726 * 10^{39}$

$a / M = 0.94$; $a / 12.33242 * 10^{39} = 0.94$; $a = 11.5924748 \times 10^{39}$

$J = a M = 14.2963268073016*10^{79}$

$M_{BH} = 12.33242 * 10^{39}$ kg

$r = 36.99726 * 10^{39}$

$a = 11.5924748 \times 10^{39}$

Photon shell

The photon shell, illustrated in Fig. 2, is the region of a black hole spacetime containing bound null geodesics or "bound orbits" that neither escape to infinity nor fall across the event horizon. For Schwarzschild, the photon shell is the two-dimensional sphere at $r = 3M$ and any θ, ϕ, and t. For Kerr, this two-dimensional sphere fattens to a three-dimensional spherical shell. It is best described using Boyer-Lindquist coordinates, in which the metric of a Kerr black hole of mass M and angular momentum $J = aM$ (with $0 \leq a \leq M$) is

$$ds^2 = -\frac{\Delta}{\Sigma}(dt - a\sin^2\theta d\phi)^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \frac{\sin^2\theta}{\Sigma}[(r^2 + a^2)d\phi - adt]^2 \tag{1A}$$

$$\Delta = r^2 - 2Mr + a^2, \quad \Sigma = r^2 + a^2\cos^2\theta \tag{1B}$$

These coordinates have the special property that each bound orbit lies at some fixed value of r in the range

$$r_-^\gamma \leq r \leq r_+^\gamma \tag{2A}$$

$$r_\pm^\gamma = 2M\left[1 + \cos\left(\frac{2}{3}\arccos\left(\pm\frac{a}{M}\right)\right)\right] \tag{2B}$$

Every point in the equatorial annulus $r_-^\gamma \leq r \leq r_+^\gamma$, $\theta = \pi/2$, has a unique bound orbit passing through it. On the boundaries $r = r_\pm^\gamma$, the orbits reside entirely in the equatorial plane. At generic points, on the other hand, they oscillate in the θ direction between polar angles

$$\theta_\pm = \arccos\left(\mp\sqrt{u_+}\right) \tag{3}$$

where

$$u_\pm = \frac{r}{a^2(r-M)^2}\left[-r^3 + 3M^2r - 2a^2M \pm 2\sqrt{M\Delta\left(2r^3 - 3Mr^2 + a^2M\right)}\right] \tag{4}$$

The bound orbit at radius r has the energy-rescaled angular momentum

$$\ell = \frac{M(r^2 - a^2) - r\Delta}{a(r-M)} \tag{6}$$

For:

$J = a M = 14.2963268073016*10^{79}$

$M_{BH} = 12.33242 * 10^{39}$ kg

$r = 36.99726 * 10^{39}$

$a = 11.5924748 \times 10^{39}$

From:

$$\Delta = r^2 - 2Mr + a^2,$$

we obtain:

(36.99726e+39)^2 – 2*(12.33242e+39)(36.99726e+39) + (11.5924748e+39)^2

Input interpretation:

$(36.99726 \times 10^{39})^2 - 2 \times 12.33242 \times 10^{39} \times 36.99726 \times 10^{39} + (11.5924748 \times 10^{39})^2$

Result:

590 651 221 157 835 040 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000

Scientific notation:

$5.9065122115783504 \times 10^{80}$

$5.9065122115783504 * 10^{80} = \Delta$

From:

$$r_{\pm}^{\gamma} = 2M\left[1 + \cos\left(\frac{2}{3}\arccos\left(\pm\frac{a}{M}\right)\right)\right]$$

we obtain:

2(12.33242e+39) [1+cos(2/3 acos((11.5924748e+39)/(12.33242e+39)))]

Input interpretation:

$$2\times 12.33242\times 10^{39}\left(1+\cos\left(\frac{2}{3}\cos^{-1}\left(\frac{11.5924748\times 10^{39}}{12.33242\times 10^{39}}\right)\right)\right)$$

$\cos^{-1}(x)$ is the inverse cosine function

Result:

$4.8668243941274112983724080481805715149868454675861063\ldots\times 10^{40}$

(result in radians)

$4.86682439412741\ldots{*}10^{40} = 48.6682439412741\ldots{*}10^{39} = r_{\pm}^{\gamma}$

From which:

18((ln((((2(12.33242e+39) [1+cos(2/3 acos((11.5924748e+39)/(12.33242e+39)))])))))+Pi))-11-3

Input interpretation:

$$18\left(\log\left(2\times 12.33242\times 10^{39}\left(1+\cos\left(\frac{2}{3}\cos^{-1}\left(\frac{11.5924748\times 10^{39}}{12.33242\times 10^{39}}\right)\right)\right)\right)+\pi\right)-11-3$$

$\cos^{-1}(x)$ is the inverse cosine function

$\log(x)$ is the natural logarithm

Result:

1728.8939...

(result in radians)

1728.8939…

[18((ln((((2(12.33242e+39) [1+cos(2/3 acos((11.5924748e+39)/(12.33242e+39)))])))))+Pi))-11-3]^1/15

Input interpretation:

$$\sqrt[15]{18\left(\log\left(2\times 12.33242\times 10^{39}\left(1+\cos\left(\frac{2}{3}\cos^{-1}\left(\frac{11.5924748\times 10^{39}}{12.33242\times 10^{39}}\right)\right)\right)\right)+\pi\right)-11-3}$$

$\cos^{-1}(x)$ is the inverse cosine function

$\log(x)$ is the natural logarithm

Result:

1.64380850...

(result in radians)

1.64380850…

[18((ln((((2(12.33242e+39) [1+cos(2/3 acos((11.5924748e+39)/(12.33242e+39)))])))))+Pi))-11-3]^1/15-(29-3)1/10^3

Input interpretation:

$$\sqrt[15]{18\left(\log\left(2\times 12.33242\times 10^{39}\left(1+\cos\left(\frac{2}{3}\cos^{-1}\left(\frac{11.5924748\times 10^{39}}{12.33242\times 10^{39}}\right)\right)\right)\right)+\pi\right)-11-3}-(29-3)\times\frac{1}{10^3}$$

$\cos^{-1}(x)$ is the inverse cosine function

$\log(x)$ is the natural logarithm

Result:

1.617808502725069686070753112705426455072877034214488871802...

(result in radians)

1.617808502725….

The ratio between r and $r_{\pm}^{\gamma}$ is equal to:

(((2(12.33242e+39) [1+cos(2/3 acos((11.5924748e+39)/(12.33242e+39)))]))) / (36.99726 * 10^39)

Input interpretation:

$$\frac{2\times 12.33242\times 10^{39}\left(1+\cos\left(\frac{2}{3}\cos^{-1}\left(\frac{11.5924748\times 10^{39}}{12.33242\times 10^{39}}\right)\right)\right)}{36.99726\times 10^{39}}$$

$\cos^{-1}(x)$ is the inverse cosine function

Result:

1.315455359161032816584906030387269628882475477261317832222...

(result in radians)

1.315455359….

From which, we obtain:

((((((2(12.33242e+39) [1+cos(2/3 acos((11.5924748e+39)/(12.33242e+39)))]))) / (36.99726 * 10^39))))^2+2/10^3

Input interpretation:

$$\left(\frac{2\times 12.33242\times 10^{39}\left(1+\cos\left(\frac{2}{3}\cos^{-1}\left(\frac{11.5924748\times 10^{39}}{12.33242\times 10^{39}}\right)\right)\right)}{36.99726\times 10^{39}}\right)^2+\frac{2}{10^3}$$

$\cos^{-1}(x)$ is the inverse cosine function

Result:

1.732422801945481844128889000385969344062189357662799872396...

(result in radians)

1.7324228.... ≈ √3

sqrt[2((((((2(12.33242e+39) [1+cos(2/3 acos((11.5924748e+39)/(12.33242e+39)))]))) / (36.99726 * 10^39))))] - 4/10^3

Input interpretation:

$$\sqrt{2\times\frac{2\times 12.33242\times 10^{39}\left(1+\cos\left(\frac{2}{3}\cos^{-1}\left(\frac{11.5924748\times 10^{39}}{12.33242\times 10^{39}}\right)\right)\right)}{36.99726\times 10^{39}}}-\frac{4}{10^3}$$

$\cos^{-1}(x)$ is the inverse cosine function

Result:

1.618008236206606206507303436468864127551941132320953132265...

(result in radians)

1.6180082362…..

(((sqrt[2((((((2(12.33242e+39) [1+cos(2/3 acos((11.5924748e+39)/(12.33242e+39)))]))) / (36.99726 * 10^39))))]+(18+4)1/10^3)))))))

Input interpretation:

$$\sqrt{2\times\frac{2\times 12.33242\times 10^{39}\left(1+\cos\left(\frac{2}{3}\cos^{-1}\left(\frac{11.5924748\times 10^{39}}{12.33242\times 10^{39}}\right)\right)\right)}{36.99726\times 10^{39}}}+(18+4)\times\frac{1}{10^3}$$

$\cos^{-1}(x)$ is the inverse cosine function

Result:

1.6440082362066062065073034364688641275519411323209531322 65...

(result in radians)

1.6440082362….

and:

sqrt(((((6(((sqrt[2((((((2(12.33242e+39) [1+cos(2/3 acos((11.5924748e+39)/(12.33242e+39)))])))) / (36.99726 * 10^39))))]+(18+4)1/10^3))))))))

Input interpretation:

$$\sqrt{6\left(\sqrt{2\times\frac{2\times 12.33242\times 10^{39}\left(1+\cos\left(\frac{2}{3}\cos^{-1}\left(\frac{11.5924748\times 10^{39}}{12.33242\times 10^{39}}\right)\right)\right)}{36.99726\times 10^{39}}}+(18+4)\times\frac{1}{10^3}\right)}$$

$\cos^{-1}(x)$ is the inverse cosine function

Result:

3.140708426014684547326967464072863097737529604761358238659...

(result in radians)

3.140708426…. ≈ π

We note that:

J = a M = 14.2963268073016*10^{79}

From which, we obtain:

ln(((1.15924748 × 10^40)(12.33242e+39)))

Input interpretation:

$\log(1.15924748\times 10^{40}\times 12.33242\times 10^{39})$

$\log(x)$ is the natural logarithm

Result:

184.56422...

184.56422…

And:

[ln(((1.15924748 × 10^40)(12.33242e+39)))]^1/11+11/10^3

Input interpretation:

$$\sqrt[11]{\log(1.15924748\times 10^{40}\times 12.33242\times 10^{39})}+\frac{11}{10^3}$$

log(x) is the natural logarithm

Result:

1.617990876866732558401998003890717677932314114074762681738...

1.61799087686.....

From:

$$\Delta = r^2 - 2Mr + a^2,$$

(36.99726e+39)^2 – 2*(12.33242e+39)(36.99726e+39) + (11.5924748e+39)^2

we obtain:

ln(((((36.99726e+39)^2 – 2*(12.33242e+39)(36.99726e+39) + (11.5924748e+39)^2))))

Input interpretation:

$$\log((36.99726\times 10^{39})^2 - 2\times 12.33242\times 10^{39}\times 36.99726\times 10^{39}+(11.5924748\times 10^{39})^2)$$

log(x) is the natural logarithm

Result:

185.98286...

185.98286…

[ln((((36.99726e+39)^2 – 2*(12.33242e+39)(36.99726e+39) + (11.5924748e+39)^2)))]^1/11+(8+2)1/10^3

Input interpretation:

$$\log((36.99726\times 10^{39})^2 - 2\times 12.33242\times 10^{39}\times 36.99726\times 10^{39}+ (11.5924748\times 10^{39})^2)\wedge(1/11)+(8+2)\times\frac{1}{10^3}$$

log(x) is the natural logarithm

Result:

1.618109882260330295447388812853063505467828646636824020861...

1.61810988226.....

Now, from:

$$u_{\pm} = \frac{r}{a^2(r-M)^2}\Big[-r^3 + 3M^2 r - 2a^2 M \pm 2\sqrt{M\Delta\left(2r^3 - 3Mr^2 + a^2 M\right)}\Big]$$

For:

J = a M = 14.2963268073016*10^{79}

M_{BH} = 12.33242 * 10^{39} kg

r = 36.99726 * 10^{39}

a = 11.5924748 × 10^{39}

5.9065122115783504 * 10^{80} = Δ , we obtain:

(36.99726e+39)/(((11.5924748e+39)^2*(36.99726e+39 – 12.33242e+39)^2))

Input interpretation:

$$\frac{36.99726\times 10^{39}}{(11.5924748\times 10^{39})^2\,(36.99726\times 10^{39} - 12.33242\times 10^{39})^2}$$

Result:

4.525438169130080509083642450084820457688824262639469... × 10^{-121}

4.525438169130080509083642450084820457688824262639469 × 10^{-121}

-(36.99726e+39)^3+3(12.33242e+39)^2(36.99726e+39)-2(11.5924748e+39)^2*(12.33242e+39)

Input interpretation:

$$-(36.99726\times10^{39})^3+3\,(12.33242\times10^{39})^2\times36.99726\times10^{39}-2\,(11.5924748\times10^{39})^2\times12.33242\times10^{39}$$

Result:

-37 075 761 267 139 517 863 993 600 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000

Result:

$-3.7075761267139517863993 6\times10^{121}$

$-3.70757612671395178639936 * 10^{121}$

2((((12.33242e+39*5.90651221e+80)*((2*(36.99726e+39)^3-3(12.33242e+39)(36.99726e+39)^2+(11.5924748e+39)^2*(12.33242e+39))))))^1/2

Input interpretation:

$$2\sqrt{(12.33242\times10^{39}\times5.90651221\times10^{80})\left(2\,(36.99726\times10^{39})^3-3\times12.33242\times10^{39}\,(36.99726\times10^{39})^2+(11.5924748\times10^{39})^2\times12.33242\times10^{39}\right)}$$

Result:

$3.903611461572848613643616970444881783277012997574676 5\ldots\times10^{121}$

$3.903611461572848613643616970444881783277012997 5\times10^{121}$

Thence:

$$u_\pm=\frac{r}{a^2(r-M)^2}\Big[-r^3+3M^2r-2a^2M\pm2\sqrt{M\Delta\left(2r^3-3Mr^2+a^2M\right)}\Big]$$

we obtain:

4.52543816913008050908 36 × 10^-121 * (-3.7075761267139517863993 * 10^121 + 3.9036114615728486136436 × 10^121)

Input interpretation:

$$\frac{4.5254381691300805090836}{10^{121}}\left(-3.7075761267139517863993\times 10^{121}+3.9036114615728486136436\times 10^{121}\right)$$

Result:

0.88714578686864830740117360414732738610632348

Repeating decimal:

0.887145786868648307401173604147327386106323480

0.88714578686…… = u_+

And also:

4.5254381691300805090836 × 10^-121 * (-3.7075761267139517863993 * 10^121 - 3.9036114615728486136436 × 10^121)

Input interpretation:

$$\frac{4.5254381691300805090836}{10^{121}}\left(-3.7075761267139517863993\times 10^{121}-3.9036114615728486136436\times 10^{121}\right)$$

Result:

−34.443958824442211005453733613849860567279686 44

-34.443958824…. = u_-

From:

$$\theta_{\pm} = \arccos\left(\mp\sqrt{u_+}\right)$$

we obtain:

((acos(sqrt(0.88714578686)))

Input interpretation:

$$\cos^{-1}\left(\sqrt{0.88714578686}\right)$$

$\cos^{-1}(x)$ is the inverse cosine function

Result:

0.3426007295...

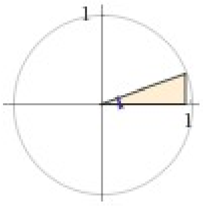
(result in radians)

0.3426007295…

Conversion from radians to degrees:

19.62957586°

Reference triangle for angle 0.3426007295 radians:



width	$\cos(0.3426007295) = 0.94188416849$
height	$\sin(0.3426007295) = 0.3359378114$

Alternative representations:

$$\cos^{-1}\left(\sqrt{0.887145786860000}\right) = \text{cd}^{-1}\left(\sqrt{0.887145786860000} \mid 0\right)$$

$$\cos^{-1}\left(\sqrt{0.887145786860000}\right) = \text{cn}^{-1}\left(\sqrt{0.887145786860000} \mid 0\right)$$

$$\cos^{-1}\left(\sqrt{0.887145786860000}\right) = \sec^{-1}\left(\frac{1}{\sqrt{0.887145786860000}}\right)$$

Series representations:

$$\cos^{-1}\left(\sqrt{0.887145786860000}\right) = \frac{\pi}{2} - \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k \sqrt{0.887145786860000}^{1+2k}}{k! + 2 k k!}$$

$$\cos^{-1}\left(\sqrt{0.887145786860000}\right) = \frac{1}{2}\pi \exp\left(i\pi \left\lfloor \frac{\arg\left(x - \sqrt{0.887145786860000}\right)}{2\pi} \right\rfloor\right) - \frac{1}{2}\exp\left(i\pi \left\lfloor \frac{\arg\left(x - \sqrt{0.887145786860000}\right)}{2\pi} \right\rfloor\right)\sqrt{\pi} \sum_{k=0}^{\infty} \frac{2^k x^{1-k}\, {}_3\tilde{F}_2\left(\frac{1}{2}, \frac{1}{2}, 1; 1 - \frac{k}{2}, \frac{3-k}{2}; x^2\right)\left(-x + \sqrt{0.887145786860000}\right)^k}{k!}$$

for $(x \in \mathbb{R}$ and $x > 1)$

$$\cos^{-1}\left(\sqrt{0.887145786860000}\right) = \frac{1}{2}\pi \exp\left(i\pi \left\lfloor \frac{\arg\left(-x + \sqrt{0.887145786860000}\right)}{2\pi} \right\rfloor\right) - 2\pi \left\lfloor \frac{\arg\left(-x + \sqrt{0.887145786860000}\right)}{2\pi} \right\rfloor - \frac{1}{2}\exp\left(i\pi \left\lfloor \frac{\arg\left(-x + \sqrt{0.887145786860000}\right)}{2\pi} \right\rfloor\right)\sqrt{\pi} \sum_{k=0}^{\infty} \frac{2^k x^{1-k}\, {}_3\tilde{F}_2\left(\frac{1}{2}, \frac{1}{2}, 1; 1 - \frac{k}{2}, \frac{3-k}{2}; x^2\right)\left(-x + \sqrt{0.887145786860000}\right)^k}{k!}$$

for $(x \in \mathbb{R}$ and $x < -1)$

Integral representations:

$$\cos^{-1}\left(\sqrt{0.887145786860000}\right) = \int_{\sqrt{0.887145786860000}}^{1} \frac{1}{\sqrt{1-t^2}}\, dt$$

$$\cos^{-1}\left(\sqrt{0.887145786860000}\right) = \frac{\pi}{2} - \frac{\sqrt{0.887145786860000}}{4 i \pi \sqrt{\pi}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \Gamma\left(\frac{1}{2} - s\right)^2 \Gamma(s)\,\Gamma\left(\frac{1}{2} + s\right) \left(1 - \sqrt{0.887145786860000}^{\,2}\right)^{-s} ds \quad \text{for } 0 < \gamma < \frac{1}{2}$$

Continued fraction representation:

$$\cos^{-1}\left(\sqrt{0.887145786860000}\right) = \frac{\pi}{2} - \frac{\sqrt{0.887145786860000}\sqrt{1-\sqrt{0.887145786860000}^2}}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-2\left\lfloor\frac{1+k}{2}\right\rfloor\left(-1+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\sqrt{0.887145786860000}^2}{1+2k}} = \frac{\pi}{2} - \cfrac{0.31641450617276}{1+-\cfrac{1.77429157372000}{3-\cfrac{1.77429157372000}{5-\cfrac{10.6457494423200}{7-\cfrac{10.6457494423200}{9+\ldots}}}}}$$

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}} a_k / b_k$ is a continued fraction

From which:

[4/(((acos(sqrt(0.88714578686)))))]^1/5-(18-2)1/10^3

Input interpretation:

$$\sqrt[5]{\frac{4}{\cos^{-1}\left(\sqrt{0.88714578686}\right)}} - (18-2)\times\frac{1}{10^3}$$

$\cos^{-1}(x)$ is the inverse cosine function

Result:

1.6187612758…

(result in radians)

1.6187612758…

Alternative representations:

$$\sqrt[5]{\frac{4}{\cos^{-1}\left(\sqrt{0.887145786860000}\right)}} - \frac{18-2}{10^3} = -\frac{16}{10^3} + \sqrt[5]{\frac{4}{\mathrm{cd}^{-1}\left(\sqrt{0.887145786860000}\,\middle|\,0\right)}}$$

$$\sqrt[5]{\frac{4}{\cos^{-1}\left(\sqrt{0.887145786860000}\right)}} - \frac{18-2}{10^3} = -\frac{16}{10^3} + \sqrt[5]{\frac{4}{\text{cn}^{-1}\left(\sqrt{0.887145786860000} \mid 0\right)}}$$

$$\sqrt[5]{\frac{4}{\cos^{-1}\left(\sqrt{0.887145786860000}\right)}} - \frac{18-2}{10^3} = -\frac{16}{10^3} + \sqrt[5]{\frac{4}{\sec^{-1}\left(\frac{1}{\sqrt{0.887145786860000}}\right)}}$$

$\text{cd}^{-1}(x \mid m)$ is the inverse of the Jacobi elliptic function cd

$\text{cn}^{-1}(x \mid m)$ is the inverse of the Jacobi elliptic function cn

$\sec^{-1}(x)$ is the inverse secant function

Series representations:

$$\sqrt[5]{\frac{4}{\cos^{-1}\left(\sqrt{0.887145786860000}\right)}} - \frac{18-2}{10^3} = -\frac{2}{125} + 2^{3/5}\left(1\Big/\left(\exp\left(i\pi\left\lfloor\frac{\arg\left(x-\sqrt{0.887145786860000}\right)}{2\pi}\right\rfloor\right)\left(\pi - \sqrt{\pi}\sum_{k=0}^{\infty}\frac{1}{k!}2^k x^{1-k}\,{}_3\tilde{F}_2\left(\frac{1}{2},\frac{1}{2},1;1-\frac{k}{2},\frac{3-k}{2};x^2\right)\left(-x+\sqrt{0.887145786860000}\right)^k\right)\right)\right)\text{^}(1/5) \text{ for } (x \in \mathbb{R} \text{ and } x > 1)$$

$$\sqrt[5]{\frac{4}{\cos^{-1}\left(\sqrt{0.887145786860000}\right)}} - \frac{18-2}{10^3} = -\frac{2}{125} + 2^{3/5}\left(-\left(1\Big/\left(4\pi\left\lfloor\frac{\arg\left(-x+\sqrt{0.887145786860000}\right)}{2\pi}\right\rfloor + \exp\left(i\pi\left\lfloor\frac{\arg\left(-x+\sqrt{0.887145786860000}\right)}{2\pi}\right\rfloor\right)\left(-\pi+\sqrt{\pi}\sum_{k=0}^{\infty}\frac{1}{k!}2^k x^{1-k}\,{}_3\tilde{F}_2\left(\frac{1}{2},\frac{1}{2},1;1-\frac{k}{2},\frac{3-k}{2};x^2\right)\left(-x+\sqrt{0.887145786860000}\right)^k\right)\right)\right)\right)^\wedge$$

$(1/5)$ for $(x \in \mathbb{R}$ and $x < -1)$

$$\sqrt[5]{\frac{4}{\cos^{-1}\left(\sqrt{0.887145786860000}\right)}} - \frac{18-2}{10^3} = -\frac{2}{125} + 2^{2/5}\left(1\Big/\left(-2\pi\left\lfloor\frac{\arg\left(-x+\sqrt{0.887145786860000}\right)}{2\pi}\right\rfloor + \exp\left(i\pi\left\lfloor\frac{\arg\left(-x+\sqrt{0.887145786860000}\right)}{2\pi}\right\rfloor\right)\left(\frac{\pi}{2}-\frac{1}{2}\sqrt{\pi}\sum_{k=0}^{\infty}\frac{1}{k!}2^k x^{1-k}\,{}_3\tilde{F}_2\left(\frac{1}{2},\frac{1}{2},1;1-\frac{k}{2},\frac{3-k}{2};x^2\right)\left(-x+\sqrt{0.887145786860000}\right)^k\right)\right)\right)^\wedge$$

$(1/5)$ for $(x \in \mathbb{R}$ and $x < -1)$

Integral representations:

$$\sqrt[5]{\frac{4}{\cos^{-1}\left(\sqrt{0.887145786860000}\right)}} - \frac{18-2}{10^3} = -\frac{2}{125} + 2^{2/5}\sqrt[5]{\frac{1}{\int_{\sqrt{0.887145786860000}}^{1}\frac{1}{\sqrt{1-t^2}}\,dt}}$$

$$\sqrt[5]{\frac{4}{\cos^{-1}\left(\sqrt{0.887145786860000}\right)}} - \frac{18-2}{10^3} =$$

$$-\frac{2}{125} + 2^{2/5}\left(1\Big/\left(\frac{\pi}{2} - \frac{\sqrt{0.887145786860000}}{4\,i\,\pi\,\sqrt{\pi}}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\Gamma\left(\frac{1}{2}-s\right)^2\Gamma(s)\,\Gamma\left(\frac{1}{2}+s\right)\left(1-\sqrt{0.887145786860000}^2\right)^{-s}ds\right)\right)\hat{}\,(1/5) \quad \text{for } 0<\gamma<\frac{1}{2}$$

Continued fraction representation:

$$\sqrt[5]{\frac{4}{\cos^{-1}\left(\sqrt{0.887145786860000}\right)}} - \frac{18-2}{10^3} =$$

$$-\frac{2}{125} + 2^{2/5}\sqrt[5]{\frac{1}{\frac{\pi}{2} - \frac{\sqrt{0.887145786860000}\,\sqrt{1-\sqrt{0.887145786860000}^2}}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-2\left\lfloor\frac{1+k}{2}\right\rfloor\left(-1+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)\sqrt{0.887145786860000}^2}{1+2k}}}} =$$

$$-\frac{2}{125} + 2^{2/5}\sqrt[5]{\frac{1}{\frac{\pi}{2} - \frac{0.31641450617276}{1+-\frac{1.77429157372000}{3-\frac{1.77429157372000}{5-\frac{10.6457494423200}{7-\frac{10.6457494423200}{9+\ldots}}}}}}}$$

Thence, we obtain:

$$2^{2/5}\sqrt[5]{\frac{1}{\frac{\pi}{2} - \frac{0.31641450617276}{1-\frac{1.77429157372000}{3-\frac{1.77429157372000}{5-\frac{10.6457494423200}{7-\frac{10.6457494423200}{\sum_{n=1}^{\infty}9}}}}}}} - \frac{2}{125} = 1.6187612758$$

Now, we analyze the continued fraction:

-2/125 + 2^(2/5) (1/(π/2 - 0.31641450617276/(1 -1.77429157372000/(3 - 1.77429157372000/(5 - 10.6457494423200/(7 - 10.6457494423200/(9 + ...)))))))^(1/5)

Input interpretation:

$$-\frac{2}{125}+2^{2/5}\sqrt[5]{\frac{1}{\frac{\pi}{2}-\frac{0.31641450617276}{1-\frac{1.77429157372000}{3-\frac{1.77429157372000}{5-\frac{10.6457494423200}{7-\frac{10.6457494423200}{9+\cdots}}}}}}}$$

Result:

$$2^{2/5}\sqrt[5]{\frac{1}{\frac{\pi}{2}-\frac{0.31641450617276}{1-\frac{1.77429157372000}{3-\frac{1.77429157372000}{5-\frac{10.6457494423200}{7-\frac{10.6457494423200}{\sum_{n=1}^{\infty}9}}}}}}}-\frac{2}{125}$$

Alternate forms:

$$2^{2/5}\sqrt[5]{\frac{5.658076347645-2.127676823962\times\sum_{n=1}^{\infty}9}{3.449791803414-1.0000000000000\times\sum_{n=1}^{\infty}9}}-\frac{2}{125}$$

$$2^{2/5}\sqrt[5]{\frac{1}{0.5945424381\times\sum_{n=1}^{\infty}9-2.0510476298}+2.12767682396}-\frac{2}{125}$$

$$\frac{1}{125}\left(125\times 2^{2/5}\sqrt[5]{\frac{1}{\frac{\pi}{2}-\frac{0.31641450617276}{1-\frac{1.77429157372000}{3-\frac{1.77429157372000}{5-\frac{10.6457494423200}{7-\frac{10.6457494423200}{\sum_{n=1}^{\infty}9}}}}}}}-2\right)$$

Alternate forms assuming n>0:

$$\frac{125\times 2^{2/5}-2\sqrt[5]{\frac{1.621389002579-0.4699961896177\times\sum_{n=1}^{\infty}9}{2.659274323959-1.0000000000000\times\sum_{n=1}^{\infty}9}}}{125\sqrt[5]{\frac{1.621389002579-0.4699961896177\times\sum_{n=1}^{\infty}9}{2.659274323959-1.0000000000000\times\sum_{n=1}^{\infty}9}}}$$

$$-\frac{2}{125}+\frac{2^{2/5}\exp\left(\frac{2}{5}\,i\,\pi\left\lfloor\frac{\arg\left(\frac{\pi}{2}-\frac{0.31641450617276}{1-\frac{1.77429157372000}{3-\frac{1.77429157372000}{5-\frac{10.6457494423200}{7-\frac{10.6457494423200}{\sum_{n=1}^{\infty}9}}}}}\right)}{2\pi}+\frac{1}{2}\right\rfloor\right)}{\sqrt[5]{\frac{\pi}{2}-\frac{0.31641450617276}{1-\frac{1.77429157372000}{3-\frac{1.77429157372000}{5-\frac{10.6457494423200}{7-\frac{10.6457494423200}{\sum_{n=1}^{\infty}9}}}}}}}$$

From

$$2^{2/5}\sqrt[5]{\frac{1}{\frac{\pi}{2}-\frac{0.31641450617276}{1-\frac{1.77429157372000}{3-\frac{1.77429157372000}{5-\frac{10.6457494423200}{7-\frac{10.6457494423200}{\sum_{n=1}^{\infty}9}}}}}}}-\frac{2}{125}$$

We can write also:

((((2^(2/5) (1/(π/2 - 0.31641450617276/(1 - 1.77429157372000/(3 - 1.77429157372000/(5 - 10.6457494423200/(7 - 10.6457494423200))))))^(1/5) - 2/125))))*159300326^(1/4)1/π^4

Input interpretation:

$$\left(2^{2/5}\sqrt[5]{\frac{1}{\frac{\pi}{2}-\frac{0.31641450617276}{1-\frac{1.77429157372000}{3-\frac{1.77429157372000}{5-\frac{10.6457494423200}{7-10.6457494423200}}}}}}-\frac{2}{125}\right)\sqrt[4]{159\,300\,326}\times\frac{1}{\pi^4}$$

Result:

1.61876127573432...

1.61876127573432....

Alternative representations:

$$\frac{\left(2^{2/5}\sqrt[5]{\frac{1}{\frac{\pi}{2}-\frac{0.316414506172760000}{1-\frac{1.774291573720000000}{3-\frac{1.774291573720000000}{5-\frac{10.645749442320000000}{7-10.645749442320000000}}}}}}-\frac{2}{125}\right)\sqrt[4]{159\,300\,326}}{\pi^4} = \frac{\sqrt[4]{159\,300\,326}\left(-\frac{2}{125}+2^{2/5}\sqrt[5]{\frac{1}{90^\circ-\frac{0.316414506172760000}{1-\frac{1.774291573720000000}{3-\frac{1.774291573720000000}{5--\frac{10.645749442320000000}{3.645749442320000000}}}}}}\right)}{(180^\circ)^4}$$

$$\frac{\left(2^{2/5}\sqrt[5]{\frac{1}{\frac{\pi}{2}-\frac{0.316414506172760000}{1-\frac{1.774291573720000000}{3-\frac{1.774291573720000000}{5-\frac{10.645749442320000000}{7-10.645749442320000000}}}}}}-\frac{2}{125}\right)\sqrt[4]{159\,300\,326}}{\pi^4} = \frac{\sqrt[4]{159\,300\,326}\left(-\frac{2}{125}+2^{2/5}\sqrt[5]{\frac{1}{E(0)-\frac{0.316414506172760000}{1-\frac{1.774291573720000000}{3-\frac{1.774291573720000000}{5--\frac{10.645749442320000000}{3.645749442320000000}}}}}}\right)}{(2\,E(0))^4}$$

$$\frac{\left(2^{2/5}\sqrt[5]{\frac{1}{\frac{\pi}{2}-\frac{0.316414506172760000}{1-\frac{1.774291573720000000}{3-\frac{1.774291573720000000}{5-\frac{10.64574944232000000}{7-10.64574944232000000}}}}}}-\frac{2}{125}\right)\sqrt[4]{159300326}}{\pi^4}=$$

$$\frac{\sqrt[4]{159300326}\left(-\frac{2}{125}+2^{2/5}\sqrt[5]{\frac{1}{K(0)-\frac{0.316414506172760000}{1-\frac{1.774291573720000000}{3-\frac{1.774291573720000000}{5-\frac{10.64574944232000000}{3.64574944232000000}}}}}}\right)}{(2K(0))^4}$$

$E(m)$ is the complete elliptic integral of the second kind with parameter $m = k^2$

$K(m)$ is the complete elliptic integral of the first kind with parameter $m = k^2$

Series representations:

$$\frac{\left(2^{2/5}\sqrt[5]{\frac{1}{\frac{\pi}{2}-\frac{0.316414506172760000}{1-\frac{1.774291573720000000}{3-\frac{1.774291573720000000}{5-\frac{10.64574944232000000}{7-10.64574944232000000}}}}}}-\frac{2}{125}\right)\sqrt[4]{159300326}}{\pi^4}=$$

$$\left(170.2833413876\left(-0.010556063286183+1.0000000000000\sqrt[5]{\frac{1}{-1.75376570257189184+\sum_{k=1}^{\infty}4^{-k}\left(-1+3^k\right)\zeta(1+k)}}\right)\right)/\left(\sum_{k=1}^{\infty}4^{-k}\left(-1+3^k\right)\zeta(1+k)\right)^4$$

$$\frac{\left(2^{2/5}\sqrt[5]{\frac{1}{\frac{\pi}{2}-\frac{0.316414506172760000}{1-\frac{1.774291573720000000}{3-\frac{1.774291573720000000}{5-\frac{10.64574944232000000}{7-10.64574944232000000}}}}}}-\frac{2}{125}\right)\sqrt[4]{159\,300\,326}}{\pi^4}=$$

$$\frac{\sqrt[4]{79\,650\,163}\left(-2+125\times 2^{2/5}\sqrt[5]{\frac{1}{-0.87688285128594592+2\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}}}\right)}{16\,000\times 2^{3/4}\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^4}$$

$$\frac{\left(2^{2/5}\sqrt[5]{\frac{1}{\frac{\pi}{2}-\frac{0.316414506172760000}{1-\frac{1.774291573720000000}{3-\frac{1.774291573720000000}{5-\frac{10.64574944232000000}{7-10.64574944232000000}}}}}}-\frac{2}{125}\right)\sqrt[4]{159\,300\,326}}{\pi^4}=$$

$$\frac{1}{\left(\sum_{k=0}^{\infty}\frac{2^{-k}(-6+50k)}{\binom{3k}{k}}\right)^4}170.2833413876\left(-0.010556063286183+\right.$$

$$\left.1.0000000000000\sqrt[5]{\frac{1}{-1.75376570257189184+\sum_{k=0}^{\infty}\frac{2^{-k}(-6+50k)}{\binom{3k}{k}}}}\right)$$

Now, from:

The bound orbit at radius r has the energy-rescaled angular momentum

$$\ell=\frac{M(r^2-a^2)-r\Delta}{a(r-M)} \tag{6}$$

For:

$M_{BH} = 12.33242 * 10^{39}$ kg

$r = 36.99726 * 10^{39}$

$a = 11.5924748 \times 10^{39}$

$5.9065122115783504 * 10^{80} = \Delta$, we obtain:

(((((12.33242e+39(((36.99726e+39)^2-(11.5924748e+39)^2))-(36.99726e+39(5.90651221157e+80))))))) / ((11.5924748e+39(36.99726e+39 – 12.33242e+39)))

Input interpretation:

$$(12.33242\times 10^{39}((36.99726\times 10^{39})^2-(11.5924748\times 10^{39})^2)-36.99726\times 10^{39}\times 5.90651221157\times 10^{80})/(11.5924748\times 10^{39}(36.99726\times 10^{39}-12.33242\times 10^{39}))$$

Result:

$-2.318494959989195059539831822623414285964201535292533... \times 10^{40}$

Repeating decimal:

$-2.318494959989195059539831822623414285964201535292533... \times 10^{40}$

(period 152490)

$-2.318494959989195....*10^{40}$

ln[-(((((12.33242e+39(((36.99726e+39)^2-(11.5924748e+39)^2))-(36.99726e+39(5.90651221157e+80))))))) / ((11.5924748e+39(36.99726e+39 – 12.33242e+39)))]+47-1/golden ratio

Input interpretation:

$$\log(-((12.33242\times 10^{39}((36.99726\times 10^{39})^2-(11.5924748\times 10^{39})^2)-36.99726\times 10^{39}\times 5.90651221157\times 10^{80})/(11.5924748\times 10^{39}(36.99726\times 10^{39}-12.33242\times 10^{39}))))+47-\frac{1}{\phi}$$

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

Result:

139.3263...

139.3263…

ln[-(((((12.33242e+39(((36.99726e+39)^2-(11.5924748e+39)^2))-(36.99726e+39(5.90651221157e+80))))))) / ((11.5924748e+39(36.99726e+39 – 12.33242e+39)))]+(47-11)-Pi-1/golden ratio

Input interpretation:

$$\log\left(-\left(\left(12.33242\times10^{39}\left(\left(36.99726\times10^{39}\right)^2-\left(11.5924748\times10^{39}\right)^2\right)-36.99726\times10^{39}\times5.90651221157\times10^{80}\right)/\left(11.5924748\times10^{39}\left(36.99726\times10^{39}-12.33242\times10^{39}\right)\right)\right)\right)+(47-11)-\pi-\frac{1}{\phi}$$

log(x) is the natural logarithm

ϕ is the golden ratio

Result:

125.1847...

125.1847…

27*1/2* (((ln[-(((((12.33242e+39(((36.99726e+39)^2-(11.59247e+39)^2))-(36.99726e+39(5.906512e+80))))))) / ((11.59247e+39(36.99726e+39 – 12.33242e+39)))]+(47-11)-1/golden ratio)))-Pi

Input interpretation:

$$27\times\frac{1}{2}\left(\log\left(-\left(\left(12.33242\times10^{39}\left(\left(36.99726\times10^{39}\right)^2-\left(11.59247\times10^{39}\right)^2\right)-36.99726\times10^{39}\times5.906512\times10^{80}\right)/\left(11.59247\times10^{39}\left(36.99726\times10^{39}-12.33242\times10^{39}\right)\right)\right)\right)+(47-11)-\frac{1}{\phi}\right)-\pi$$

log(x) is the natural logarithm

ϕ is the golden ratio

Result:

1729.263...

1729.263…

With regard 27 (From Wikipedia):

*"The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group **Z**/3**Z**, and its outer automorphism group is the cyclic group **Z**/2**Z**. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories".*

and:

[-(((((12.33242e+39(((36.99726e+39)^2-(11.5924748e+39)^2))-(36.99726e+39(5.90651221157e+80))))))) / ((11.5924748e+39(36.99726e+39 – 12.33242e+39)))]^1/192-4/10^3

Input interpretation:

$$(-((12.33242\times 10^{39}((36.99726\times 10^{39})^2-(11.5924748\times 10^{39})^2)-36.99726\times 10^{39}\times 5.90651221157\times 10^{80})/(11.5924748\times 10^{39}(36.99726\times 10^{39}-12.33242\times 10^{39}))))\,\hat{}\,(1/192)-\frac{4}{10^3}$$

Result:

1.618689585059880515274137379893817295787262138045683060201...

1.61868958505…

Now, we have that:

$$\gamma = \frac{4}{a}\sqrt{r^2-\frac{Mr\Delta}{(r-M)^2}}\int_0^1\frac{dt}{\sqrt{(1-t^2)(u_+t^2-u_-)}}$$

For:

-34.443958824…. = u_- 0.88714578686…… = u_+

M_{BH} = 12.33242 * 10^{39} kg   r = 36.99726 * 10^{39} a = 11.5924748 × 10^{39}

5.9065122115783504 * 10^{80} = Δ ,

we obtain:

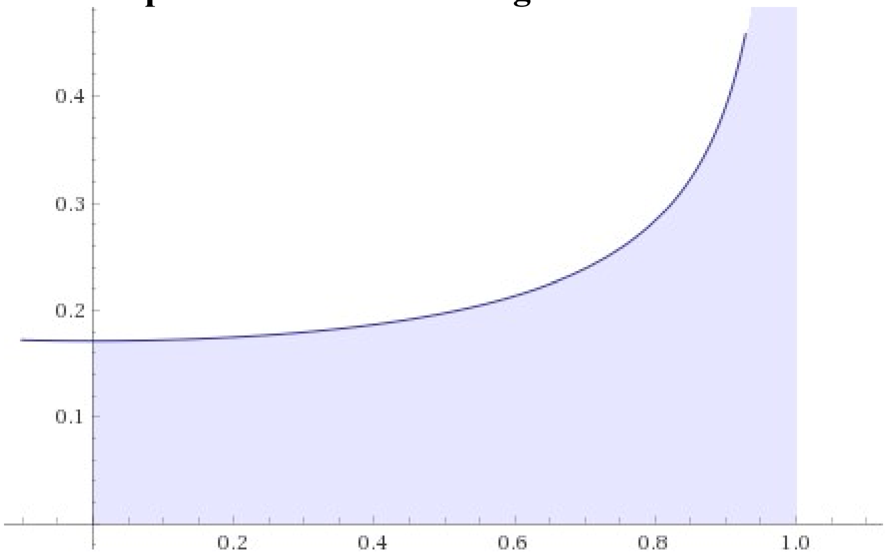
integrate (1/sqrt((1-x^2)(0.88714578686*x^2+34.443958824)))dx, x = 0..1

Definite integral:

$$\int_0^1 \frac{1}{\sqrt{(1-x^2)(34.443958824+0.88714578686\,x^2)}}\,dx = 0.265949$$

0.265949

Visual representation of the integral:



Thence:

4/(11.5924748e+39) * sqrt(((36.99726e+39)^2-(12.33242e+39*36.99726e+39*5.9065122115e+80)/(36.99726e+39-12.33242e+39)^2)) * 0.265949

Input interpretation:

$$\frac{4}{11.5924748\times 10^{39}}\sqrt{(36.99726\times 10^{39})^2-\frac{12.33242\times 10^{39}\times 36.99726\times 10^{39}\times 5.9065122115\times 10^{80}}{(36.99726\times 10^{39}-12.33242\times 10^{39})^2}}\times 0.265949$$

Result:

2.792176400962418653613725817206220636969224175698939603886...

2.792176400962….

From which:

((((4/(11.59247e+39) sqrt(((36.99726e+39)^2-(12.33242e+39*36.99726e+39*5.906512e+80)/(36.99726e+39-12.33242e+39)^2)) 0.265949))))^1/2-(21+5)1/10^3

Input interpretation:

$$\sqrt{\left(\frac{4}{11.59247\times 10^{39}}\sqrt{(36.99726\times 10^{39})^2-\frac{12.33242\times 10^{39}\times 36.99726\times 10^{39}\times 5.906512\times 10^{80}}{(36.99726\times 10^{39}-12.33242\times 10^{39})^2}}\times 0.265949\right)}-(21+5)\times\frac{1}{10^3}$$

Result:

1.644981023535685486460792422095041162228569049019211931939...

1.6449810235….

((((4/(11.59247e+39) sqrt(((36.99726e+39)^2-(12.33242e+39*36.99726e+39*5.906512e+80)/(36.99726e+39-12.33242e+39)^2)) 0.265949))))^1/2-(55-2)1/10^3

Input interpretation:

$$\sqrt{\left(\frac{4}{11.59247\times 10^{39}}\sqrt{(36.99726\times 10^{39})^2-\frac{12.33242\times 10^{39}\times 36.99726\times 10^{39}\times 5.906512\times 10^{80}}{(36.99726\times 10^{39}-12.33242\times 10^{39})^2}}\times 0.265949\right)}-(55-2)\times\frac{1}{10^3}$$

Result:

1.617981023535685486460792422095041162228569049019211931939...

1.617981023535……

From

Mechanisms for Primordial Black Hole Production in String Theory

Ogan Ozsoy, Susha Parameswaran , Gianmassimo Tasinato , Ivonne Zavala

arXiv:1803.07626v2 [hep-th] 2 Jul 2018

Now, we have that:

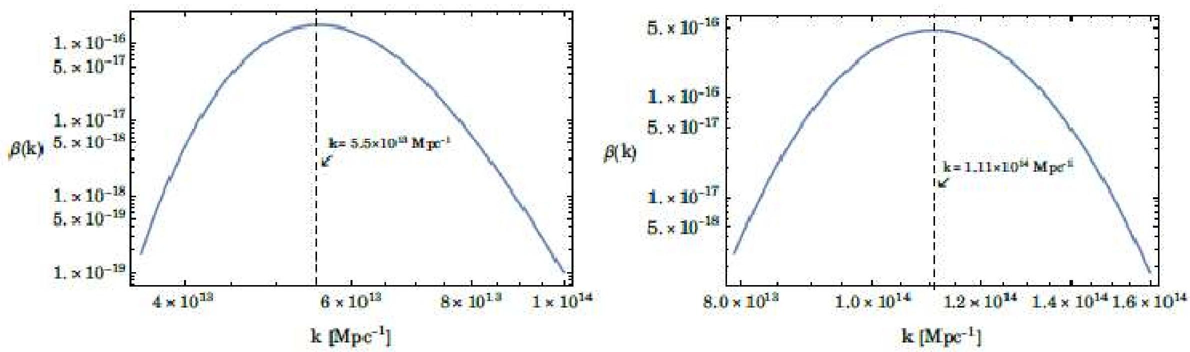


Figure 11. β as a function of the smoothing scale k. The values of k where $\beta(k)$ has a peak is also shown with dashed vertical lines.

Cases	δ_c	$M_{\rm peak}/M_\odot$	$\Omega^{\rm tot}_{\rm PBH}/\Omega_{\rm DM}$
$M_{\rm pl}/f = 1.6$	0.34	8×10^{-16}	0.113
$M_{\rm pl}/f = 1.7$	0.5	2×10^{-16}	0.514

Table 3. The two different choices of the critical threshold overdensity δ_c and the corresponding total abundance of PBHs for the models considered in Table 1. The peak value of the mass of the relevant PBHs which are obtained from the equation (3.14), is also shown.

(see equation (3.16)). In the following, to estimate the total PBH abundance with respect to Dark Matter abundance today, we will take values of δ_c within the range $\delta_c - 0.3 - 0.5$ as suggested in [56, 72, 73]. For these values of δ_c, one requires $\sigma^2(M) \sim 10^{-2} - 10^{-3}$ to reach the required level $\beta(M)$ on the relevant scale k (or M). This in turn arises from a power spectrum in equation (3.17) that is also of the order of 10^{-2}.

From

$$\beta(M(k)) \equiv \frac{\rho_{\rm PBH}}{\rho} = 2\int_{\delta_c}^{\infty} \frac{{\rm d}\delta}{\sqrt{2\pi}\sigma(M(k))} \exp\left(-\frac{\delta^2}{2\sigma^2(M(k))}\right), \\ = \sqrt{\frac{2}{\pi}}\frac{\sigma(M(k))}{\delta_c} \exp\left(-\frac{\delta_c^2}{2\sigma^2(M(k))}\right) \qquad (3.16)$$

For $\sigma^2(M(k)) = 1/12$ and $\delta_c = 0.5$, we obtain:

((sqrt(2/Pi) * sqrt(1/12)*1/(0.5))) * ((exp((-0.5^2)/(2*1/12))))

Input:

$$\left(\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{12}} \times \frac{1}{0.5}\right)\exp\left(-\frac{0.5^2}{2\times\frac{1}{12}}\right)$$

Result:

0.10278688653584618454081370518759576420343967179701570577l...

0.102786886535…

Series representations:

$$\frac{\exp\left(-\frac{0.5^2}{\frac{2}{12}}\right)\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{12}}}{0.5} = 2\exp(-1.5)\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{11}{12}\right)^{k_1}\left(-1+\frac{2}{\pi}\right)^{k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}}{k_1!\,k_2!}$$

$$\frac{\exp\left(-\frac{0.5^2}{\frac{2}{12}}\right)\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{12}}}{0.5} = 2\exp(-1.5)\sqrt{z_0}^2\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}\left(\frac{1}{12}-z_0\right)^{k_1}\left(\frac{2}{\pi}-z_0\right)^{k_2} z_0^{-k_1-k_2}}{k_1!\,k_2!}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{\exp\left(-\frac{0.5^2}{\frac{2}{12}}\right)\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{12}}}{0.5} = \frac{1}{\sqrt{\pi}^2} 0.5 \exp(-1.5)$$
$$\sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\left(\operatorname{Res}_{s=-j_1}\left(-\frac{11}{12}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)\left(\operatorname{Res}_{s=-j_2}\left(-1+\frac{2}{\pi}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)$$

From which:

1/(((sqrt(2/Pi) * sqrt(1/12)*1/(0.5) * exp((-0.5^2)/(2*1/12)))))

Input:

$$\frac{1}{\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{12}} \times \frac{1}{0.5}\exp\left(-\frac{0.5^2}{2\times\frac{1}{12}}\right)}$$

Result:

9.72887...

9.72887…

Series representations:

$$\frac{1}{\frac{\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{12}}\exp\left(-\frac{0.5^2}{\frac{2}{12}}\right)}{0.5}} = \frac{0.5}{\exp(-1.5)\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{11}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-1+\frac{2}{\pi}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{1}{\frac{\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{12}}\exp\left(-\frac{0.5^2}{\frac{2}{12}}\right)}{0.5}} = \frac{0.5}{\exp(-1.5)\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{1}{12}-z_0\right)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{2}{\pi}-z_0\right)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\frac{1}{\frac{\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{12}}\exp\left(-\frac{0.5^2}{\frac{2}{12}}\right)}{0.5}} = \frac{2\sqrt{\pi}^2}{\exp(-1.5)\left(\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\left(-\frac{11}{12}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\left(-1+\frac{2}{\pi}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}$$

1/6*1/(((sqrt(2/Pi) * sqrt(1/12)*1/(0.5) * exp((-0.5^2)/(2*1/12)))))-Pi/10^3

Input:

$$\frac{1}{6}\times\frac{1}{\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{12}}\times\frac{1}{0.5}\exp\left(-\frac{0.5^2}{2\times\frac{1}{12}}\right)}-\frac{\pi}{10^3}$$

Result:

1.618336324264760195190823030879800668398350507494414128825...

1.618336324….

Possible closed forms:

$\phi\approx 1.61803398$

$\frac{3\pi\zeta(3)}{7}\approx 1.61844562$

$\frac{17\pi}{33}\approx 1.618396215$

$\sqrt{\frac{55}{21}}\approx 1.618347187$

$\frac{300}{59\pi}\approx 1.61852484$

Series representations:

$$\frac{1}{\frac{\left(\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{12}}\exp\left(-\frac{0.5^2}{\frac{2}{12}}\right)\right)6}{0.5}}-\frac{\pi}{10^3}=$$

$$-\frac{0.001\left(-83.3333+\pi\exp(-1.5)\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{11}{12}\right)^{k_1}\left(-1+\frac{2}{\pi}\right)^{k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}}{k_1!\,k_2!}\right)}{\exp(-1.5)\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{11}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-1+\frac{2}{\pi}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{1}{\frac{\left(\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{12}}\exp\left(-\frac{0.5^2}{\frac{2}{12}}\right)\right)6}{0.5}} - \frac{\pi}{10^3} = -\left(\left(0.001\left(-83.3333 + \pi\exp(-1.5)\sqrt{z_0}^2 \sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\left(-\frac{1}{2}\right)_{k_1}\left(-\frac{1}{2}\right)_{k_2}\left(\frac{1}{12}-z_0\right)^{k_1}\left(\frac{2}{\pi}-z_0\right)^{k_2} z_0^{-k_1-k_2}}{k_1!\,k_2!}\right)\right)\Big/ \left(\exp(-1.5)\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{1}{12}-z_0\right)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{2}{\pi}-z_0\right)^k z_0^{-k}}{k!}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{1}{\frac{\left(\sqrt{\frac{2}{\pi}}\sqrt{\frac{1}{12}}\exp\left(-\frac{0.5^2}{\frac{2}{12}}\right)\right)6}{0.5}} - \frac{\pi}{10^3} =$$

$$-\left(\left(0.001\left(-333.333\sqrt{\pi}^2 + \pi\exp(-1.5)\sum_{j_1=0}^{\infty}\sum_{j_2=0}^{\infty}\left(\text{Res}_{s=-j_1}\left(-\frac{11}{12}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)\left(\text{Res}_{s=-j_2}\left(-1+\frac{2}{\pi}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)\right)\right)\Big/\left(\exp(-1.5)\left(\sum_{j=0}^{\infty}\text{Res}_{s=-j}\left(-\frac{11}{12}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)\sum_{j=0}^{\infty}\text{Res}_{s=-j}\left(-1+\frac{2}{\pi}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)\right)$$

From

Primordial Black Holes from String Inflation
Michele Cicoli, Victor A. Diaz, Francisco G. Pedro - arXiv:1803.02837v2 [hep-th] 16 Mar 2018

We have that:

$$\Delta N_{\rm CMB}^{\rm PBH} = \ln\left(\frac{a_{\rm PBH}H_{\rm inf}}{a_{\rm CMB}H_{\rm inf}}\right) = \ln\left(\frac{a_{\rm f}H_{\rm f}}{0.05\,{\rm Mpc}^{-1}}\right)$$
$$= 18.4 - \frac{1}{12}\ln\left(\frac{g_*}{g_{*0}}\right) + \frac{1}{2}\ln\gamma - \frac{1}{2}\ln\left(\frac{M}{M_\odot}\right). \qquad (3.10)$$

Setting again $\gamma = 1$, $g_{*0} = 3.36$ and $g_* = 106.75$ as in the SM case, the formation of PBHs with masses in the $[10^{-16}, 10^{-14}]\, M_\odot$ range implies that PBH scales leave the horizon approximately 34.2 to 36.5 efoldings after the CMB scales.

For M = 2 * 10^{-16} * 1.9891 * 10^{30}

18.4-1/12 ln(106.75/3.36)+1/2ln1-1/2 ln((2e-16*1.9891e+30)/(1.9891e+30))

Input interpretation:

$$18.4 - \frac{1}{12}\log\left(\frac{106.75}{3.36}\right) + \frac{1}{2}\log(1) - \frac{1}{2}\log\left(\frac{2\times 10^{-16}\times 1.9891\times 10^{30}}{1.9891\times 10^{30}}\right)$$

log(x) is the natural logarithm

Result:

36.1859...

36.1859…

From which:

48(((18.4-1/12 ln(106.75/3.36)+1/2ln1-1/2 ln((2e-16*1.9891e+30)/(1.9891e+30)))))-8

Input interpretation:

$$48\left(18.4 - \frac{1}{12}\log\left(\frac{106.75}{3.36}\right) + \frac{1}{2}\log(1) - \frac{1}{2}\log\left(\frac{2\times 10^{-16}\times 1.9891\times 10^{30}}{1.9891\times 10^{30}}\right)\right) - 8$$

log(x) is the natural logarithm

Result:

1728.92...

1728.92…

[48(((18.4-1/12 ln(106.75/3.36)+1/2ln1-1/2 ln((2e-16*1.9891e+30)/(1.9891e+30)))))-8]^1/15

Input interpretation:

$$\sqrt[15]{48\left(18.4 - \frac{1}{12}\log\left(\frac{106.75}{3.36}\right) + \frac{1}{2}\log(1) - \frac{1}{2}\log\left(\frac{2\times 10^{-16}\times 1.9891\times 10^{30}}{1.9891\times 10^{30}}\right)\right) - 8}$$

log(x) is the natural logarithm

Result:

1.643810...

1.643810…

[48(((18.4-1/12 ln(106.75/3.36)+1/2ln1-1/2 ln((2e-16*1.9891e+30)/(1.9891e+30)))))-8]^1/15 - (21+5)1/10^3

Input interpretation:

$$\sqrt[15]{48\left(18.4-\frac{1}{12}\log\left(\frac{106.75}{3.36}\right)+\frac{1}{2}\log(1)-\frac{1}{2}\log\left(\frac{2\times10^{-16}\times1.9891\times10^{30}}{1.9891\times10^{30}}\right)\right)-8}-(21+5)\times\frac{1}{10^3}$$

log(x) is the natural logarithm

Result:

1.6178103449702792593681158586553235459788775770436400655 50...

1.617810344970279….

sqrt(((6*[48(((18.4-1/12 ln(106.75/3.36)+1/2ln1-1/2 ln((2e-16*1.9891e+30)/(1.9891e+30)))))-8]^1/15)))

Input interpretation:

$$\sqrt{6\sqrt[15]{48\left(18.4-\frac{1}{12}\log\left(\frac{106.75}{3.36}\right)+\frac{1}{2}\log(1)-\frac{1}{2}\log\left(\frac{2\times10^{-16}\times1.9891\times10^{30}}{1.9891\times10^{30}}\right)\right)-8}}$$

log(x) is the natural logarithm

Result:

3.140519...

3.140519… ≈ π

From:

EVALUATIONS OF RAMANUJAN-WEBER CLASS INVARIANT g_n

S.Bhargava, K. R. Vasuki and B. R. Srivatsa Kumar - Journal of the Indian Mathematical Society · January 2005

Theorem 3.13. We have

$$(i) \qquad g_{490} = \left(\frac{A+\sqrt{A^2-4}}{2}\right)^{1/2}\left[A^2-2+\sqrt{A^4-4A^2+5}\right]^{1/6} = g_{2/245}^{-1},$$

and

$$(ii) \qquad g_{10/49} = \frac{\sqrt{2}\,\left[A^2-2+\sqrt{A^4-4A^2+5}\right]^{1/6}}{(A+\sqrt{A^2-4})^{1/2}} = g_{98/5}^{-1},$$

where

$$A = \frac{1}{3}\left[\left(14525+105\sqrt{30}\right)^{1/3} + \frac{595}{\left(14525+105\sqrt{30}\right)^{1/3}} + 20\right].$$

From:

$$A = \frac{1}{3}\left[\left(14525+105\sqrt{30}\right)^{1/3} + \frac{595}{\left(14525+105\sqrt{30}\right)^{1/3}} + 20\right].$$

we obtain:

1/3[(14525+105sqrt30)^(1/3) + 595/(14525+105sqrt30)^1/3 +20]

Input:

$$\frac{1}{3}\left(\sqrt[3]{14525+105\sqrt{30}} + \frac{595}{\sqrt[3]{14525+105\sqrt{30}}} + 20\right)$$

Decimal approximation:

22.92983238245319802791118792747045447888988816048964726921...

22.929832382…

Alternate forms:

$$\frac{20\sqrt[3]{415+3\sqrt{30}} + 5^{2/3}\sqrt[3]{7(34499+498\sqrt{30})} + 17\times 35^{2/3}}{3\sqrt[3]{415+3\sqrt{30}}}$$

root of $x^3-20x^2-65x-50$ near $x = 22.9298$

$$\frac{20}{3}+\frac{595}{3\sqrt[3]{14525+105\sqrt{30}}}+\frac{1}{3}\sqrt[3]{14525+105\sqrt{30}}$$

Minimal polynomial:

$x^3-20x^2-65x-50$

From:

$$g_{490} = \left(\frac{A+\sqrt{A^2-4}}{2}\right)^{1/2}\left[A^2-2+\sqrt{A^4-4A^2+5}\right]^{1/6} = g_{2/245}^{-1},$$

We obtain:

(1/2(22.9298+sqrt(22.9298^2-4)))^0.5 [22.9298^2-2+sqrt(22.9298^4-4*22.9298^2+5)]^(1/6)

Input interpretation:

$$\sqrt{\frac{1}{2}\left(22.9298+\sqrt{22.9298^2-4}\right)}\sqrt[6]{22.9298^2-2+\sqrt{22.9298^4-4\times 22.9298^2+5}}$$

Result:

15.2457…

15.2457…

From:

$$g_{10/49} = \frac{\sqrt{2}\left[A^2-2+\sqrt{A^4-4A^2+5}\right]^{1/6}}{(A+\sqrt{A^2-4})^{1/2}} = g_{98/5}^{-1},$$

we obtain:

sqrt2[22.9298^2-2+sqrt(22.9298^4-4*22.9298^2+5)]^(1/6) / (22.9298+sqrt(22.9298^2-4))^0.5

Input interpretation:

$$\sqrt{2}\times\frac{\sqrt[6]{22.9298^2-2+\sqrt{22.9298^4-4\times 22.9298^2+5}}}{\sqrt{22.9298+\sqrt{22.9298^2-4}}}$$

Result:

0.666157...

0.666157…

From the two results, we obtain:

(1/0.6661571294615327)^2(1/2(22.9298+sqrt(22.9298^2-4)))^0.5 [22.9298^2-2+sqrt(22.9298^4-4*22.9298^2+5)]^(1/6)

Input interpretation:

$$\left(\frac{1}{0.6661571294615327}\right)^2 \sqrt{\frac{1}{2}\left(22.9298+\sqrt{22.9298^2-4}\right)} \sqrt[6]{22.9298^2-2+\sqrt{22.9298^4-4\times 22.9298^2+5}}$$

Result:

34.3554...

34.3554...

and:

(1/0.6661571294615327)^2(1/2(22.9298+sqrt(22.9298^2-4)))^0.5 [22.9298^2-2+sqrt(22.9298^4-4*22.9298^2+5)]^(1/6)+Pi-sqrt2

Input interpretation:

$$\left(\frac{1}{0.6661571294615327}\right)^2 \sqrt{\frac{1}{2}\left(22.9298+\sqrt{22.9298^2-4}\right)} \sqrt[6]{22.9298^2-2+\sqrt{22.9298^4-4\times 22.9298^2+5}}+\pi-\sqrt{2}$$

Result:

36.0828...

36.0828…

Series representations:

$$\left(\frac{1}{0.66615712946153270000}\right)^2 \sqrt{\frac{1}{2}\left(22.9298+\sqrt{22.9298^2-4}\right)} \sqrt[6]{22.9298^2-2+\sqrt{22.9298^4-4\times 22.9298^2+5}} + \pi-\sqrt{2} = 1.5934250548963727128 \left(0.6275789356563332705\,\pi - 0.6275789356563332705\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} + 1.0000000000000000000 \sqrt{22.9298+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (521.776-z_0)^k z_0^{-k}}{k!}} \sqrt[6]{523.776+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (274342.-z_0)^k z_0^{-k}}{k!}}\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\left(\frac{1}{0.66615712946153270000}\right)^2 \sqrt{\frac{1}{2}\left(22.9298+\sqrt{22.9298^2-4}\right)} \sqrt[6]{22.9298^2-2+\sqrt{22.9298^4-4\times 22.9298^2+5}} + \pi-\sqrt{2} = 1.5934250548963727128 \left(0.6275789356563332705\,\pi - 0.6275789356563332705 \exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right) \sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!} + 1.0000000000000000000 \sqrt{\left(22.9298+\exp\left(i\pi\left\lfloor\frac{\arg(521.776-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (521.776-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)} \left(523.776+\exp\left(i\pi\left\lfloor\frac{\arg(274342.-x)}{2\pi}\right\rfloor\right)\sqrt{x} \sum_{k=0}^{\infty}\frac{(-1)^k (274342.-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\wedge(1/6)\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\left(\frac{1}{0.66615712946153270000}\right)^2 \sqrt{\frac{1}{2}\left(22.9298+\sqrt{22.9298^2-4}\right)} \sqrt[6]{22.9298^2-2+\sqrt{22.9298^4-4\times 22.9298^2+5}} + \pi - \sqrt{2} =$$

$$1.5934250548963727128 \left(0.6275789356563332705\,\pi - 0.6275789356563332705 \left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} + 1.0000000000000000000 \sqrt{\left(22.9298 + \left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(521.776-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(521.776-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (521.776-z_0)^k z_0^{-k}}{k!}\right)} \left(523.776 + \left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(274342.-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(274342.-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (274342.-z_0)^k z_0^{-k}}{k!}\right) \hat{}\ (1/6)\right)$$

48((((1/0.6661571294615327)^2(1/2(22.9298+sqrt(22.9298^2-4)))^0.5 [22.9298^2-2+sqrt(22.9298^4-4*22.9298^2+5)]^(1/6)+Pi-sqrt2)))-3

Input interpretation:

$$48\left(\left(\frac{1}{0.6661571294615327}\right)^2 \sqrt{\frac{1}{2}\left(22.9298+\sqrt{22.9298^2-4}\right)} \sqrt[6]{22.9298^2-2+\sqrt{22.9298^4-4\times 22.9298^2+5}} + \pi - \sqrt{2}\right) - 3$$

Result:

1728.97…

1728.97…

Series representations:

$$48\left(\left(\frac{1}{0.66615712946153270000}\right)^2 \sqrt{\frac{1}{2}\left(22.9298+\sqrt{22.9298^2-4}\right)} \sqrt[6]{22.9298^2-2+\sqrt{22.9298^4-4\times 22.9298^2+5}}+\pi-\sqrt{2}\right)-3 =$$

$$76.48440263502589022\left(-0.03922368347852082940 + 0.6275789356563332705\,\pi - 0.6275789356563332705\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} + 1.0000000000000000000 \sqrt{22.9298+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (521.776-z_0)^k z_0^{-k}}{k!}} \sqrt[6]{523.776+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (274342.-z_0)^k z_0^{-k}}{k!}}\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$48\left(\left(\frac{1}{0.66615712946153270000}\right)^2 \sqrt{\frac{1}{2}\left(22.9298+\sqrt{22.9298^2-4}\right)} \sqrt[6]{22.9298^2-2+\sqrt{22.9298^4-4\times 22.9298^2+5}}+\pi-\sqrt{2}\right)-3 =$$

$$76.48440263502589022\left(-0.03922368347852082940 + 0.6275789356563332705\,\pi - 0.6275789356563332705 \exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!} + 1.0000000000000000000 \sqrt{\left(22.9298 + \exp\left(i\pi\left\lfloor\frac{\arg(521.776-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (521.776-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)} \left(523.776+\exp\left(i\pi\left\lfloor\frac{\arg(274342.-x)}{2\pi}\right\rfloor\right)\sqrt{x} \sum_{k=0}^{\infty}\frac{(-1)^k (274342.-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^{\wedge}(1/6)\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$48\left(\left(\frac{1}{0.66615712946153270000}\right)^2 \sqrt{\frac{1}{2}\left(22.9298+\sqrt{22.9298^2-4}\right)} \sqrt[6]{22.9298^2-2+\sqrt{22.9298^4-4\times 22.9298^2+5}}+\pi-\sqrt{2}\right)-3 =$$

$$76.48440263502589022\left(-0.03922368347852082940+0.6275789356563332705\,\pi-0.6275789356563332705\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}+1.0000000000000000000\sqrt{\left(22.9298+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(521.776-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor\arg(521.776-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (521.776-z_0)^k z_0^{-k}}{k!}\right)}\left(523.776+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(274342.-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor\arg(274342.-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (274342.-z_0)^k z_0^{-k}}{k!}\right)\hat{}\,(1/6)\right)$$

((48((((1/0.66615712)^2(1/2(22.9298+sqrt(22.9298^2-4)))^0.5 [22.9298^2-2+sqrt(22.9298^4-4*22.9298^2+5)]^(1/6)+Pi-sqrt2))-3))^1/15

Input interpretation:

$$\left(48\left(\left(\frac{1}{0.66615712}\right)^2 \sqrt{\frac{1}{2}\left(22.9298+\sqrt{22.9298^2-4}\right)} \sqrt[6]{22.9298^2-2+\sqrt{22.9298^4-4\times 22.9298^2+5}}+\pi-\sqrt{2}\right)-3\right)\hat{}\,(1/15)$$

Result:

1.643814...

1.643814....

((48((((1/0.66615712)^2(1/2(22.929+sqrt(22.929^2-4)))^0.5 [22.929^2-2+sqrt(22.929^4-4*22.929^2+5)]^(1/6)+Pi-sqrt2)))-3))^1/15-(21+5)1/10^3

Input interpretation:

$$\left(48\left(\left(\frac{1}{0.66615712}\right)^2\sqrt{\frac{1}{2}\left(22.929+\sqrt{22.929^2-4}\right)}\sqrt[6]{22.929^2-2+\sqrt{22.929^4-4\times 22.929^2+5}}+\pi-\sqrt{2}\right)-3\right)\text{^}(1/15)-(21+5)\times\frac{1}{10^3}$$

Result:

1.617810543899655186922498404182084630736273114381230937340...

1.61781054389965….

Series representations:

$$\left(48\left(\left(\frac{1}{0.666157}\right)^2\sqrt{\frac{1}{2}\left(22.929+\sqrt{22.929^2-4}\right)}\sqrt[6]{22.929^2-2+\sqrt{22.929^4-4\times 22.929^2+5}}+\pi-\sqrt{2}\right)-3\right)\text{^}(1/15)-\frac{21+5}{10^3}=$$

$$\frac{1}{500}\left(-13+500\left(-3+48\pi-48\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}+76.4844\sqrt{22.929+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(521.739-z_0)^k z_0^{-k}}{k!}}\sqrt[6]{523.739+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(274304.-z_0)^k z_0^{-k}}{k!}}\right)\text{^}(1/15)\right)$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0$))

$$\left(48\left(\left(\frac{1}{0.666157}\right)^2 \sqrt{\frac{1}{2}\left(22.929+\sqrt{22.929^2-4}\right)} \sqrt[6]{22.929^2-2+\sqrt{22.929^4-4\times 22.929^2+5}} + \pi-\sqrt{2}\right)-3\right)\hat{}\,(1/15)-\frac{21+5}{10^3} =$$

$$\frac{1}{500}\left(-13+500\left(-3+48\left(\pi-\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!} + 1.59343\sqrt{\left(22.929+\exp\left(i\pi\left\lfloor\frac{\arg(521.739-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(521.739-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)} \left(523.739+\exp\left(i\pi\left\lfloor\frac{\arg(274304.-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(274304.-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\hat{}\,(1/6)\right)\right)\hat{}\,(1/15)\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\left(48\left(\left(\frac{1}{0.666157}\right)^2 \sqrt{\frac{1}{2}\left(22.929+\sqrt{22.929^2-4}\right)} \sqrt[6]{22.929^2-2+\sqrt{22.929^4-4\times 22.929^2+5}}+\pi-\sqrt{2}\right)-3\right)\text{^}(1/15)-\frac{21+5}{10^3} =$$

$$\frac{1}{500}\left(-13+500\left(-3+48\left(\pi-\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}+1.59343\sqrt{\left(22.929+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(521.739-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor\arg(521.739-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (521.739-z_0)^k z_0^{-k}}{k!}\right)\left(523.739+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(274304.-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor\arg(274304.-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (274304.-z_0)^k z_0^{-k}}{k!}\right)}\text{^}(1/6)\right)\right)\text{^}(1/15)\right)$$

Note that:

sqrt(((288((((1/2(22.9298+sqrt(22.9298^2-4)))^0.5 [22.9298^2-2+sqrt(22.9298^4-4*22.9298^2+5)]^(1/6)-0.6661571294615327)))-89-13-1)))

Input interpretation:

$$\sqrt{\left(288\left(\sqrt{\frac{1}{2}\left(22.9298+\sqrt{22.9298^2-4}\right)} \sqrt[6]{22.9298^2-2+\sqrt{22.9298^4-4\times 22.9298^2+5}}-0.6661571294615327\right)-89-13-1\right)}$$

Result:

63.9994...

63.9994... ≈ 64

27sqrt(((288((((1/2(22.9298+sqrt(22.9298^2-4)))^0.5 [22.9298^2-2+sqrt(22.9298^4-4*22.9298^2+5)]^(1/6)-0.6661571294615327)))-89-13-1)))+1

Input interpretation:

$$27\sqrt{\left(288\left(\sqrt{\frac{1}{2}\left(22.9298+\sqrt{22.9298^2-4}\right)}\sqrt[6]{22.9298^2-2+\sqrt{22.9298^4-4\times 22.9298^2+5}}-0.6661571294615327\right)-89-13-1\right)}+1$$

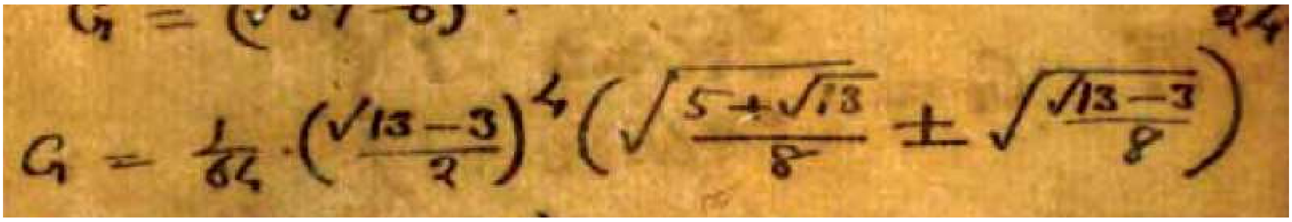
Result:

1728.98...

1728.98… ≈ 1729

From **Manuscript Book 2 of Srinivasa Ramanujan**

Page 298



$$G = \frac{1}{64}\left(\frac{\sqrt{13}-3}{2}\right)^4\left(\sqrt{\frac{5+\sqrt{13}}{8}} \pm \sqrt{\frac{\sqrt{13}-3}{8}}\right)^{24}$$

1/64 (1/2(sqrt13-3))^4 [(((1/8(5+sqrt13))^0.5+(1/8(sqrt13-3))^0.5))]^24

Input:

$$\frac{1}{64}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(\sqrt{13}-3\right)}\right)^{24}$$

Exact result:

$$\frac{\left(\sqrt{13}-3\right)^4\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{13}-3\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(5+\sqrt{13}\right)}\right)^{24}}{1024}$$

Decimal approximation:

0.089321319219383532717820532693319698548662535994269836508...

0.089321319219…..

Alternate forms:

$$\frac{1}{128}\left(-70+21\sqrt{13}+3\sqrt{546\sqrt{13}-1965}\right)$$

$$\frac{1}{128}\left(-70+21\sqrt{13}+64\sqrt{\frac{2457\sqrt{13}}{2048}-\frac{17685}{4096}}\right)$$

$$\frac{(\sqrt{13}-3)^4\left(\sqrt{\sqrt{13}-3}+\sqrt{5+\sqrt{13}}\right)^{24}}{70368744177664}$$

Minimal polynomial:

$$16777216x^4+36700160x^3+54583296x^2-5180224x+1$$

Possible closed forms:

$$\frac{8\pi}{141+\sqrt{19705}}\approx 0.0893213184149$$

$$\frac{13}{47}-\frac{77}{47\pi}+\frac{5\pi}{47}\approx 0.089321319882546$$

$$\frac{1+8\sqrt{\pi}-8\pi-5\pi^{3/2}+4\pi^2}{6\pi}\approx 0.08932131906623$$

$$\pi\ \boxed{\text{root of } 24x^4+36x^3+5x^2+35x-1 \text{ near } x=0.0284319}\approx 0.0893213187946$$

$$\log\left(\frac{1}{14}\left(7-\sqrt{2}-8e-4e^2+10\pi+3\pi^2\right)\right)\approx 0.0893213182542$$

$$\frac{4\sqrt[5]{\frac{2}{3}}\log^{9/5}(2)\log^{2/5}(3)}{3e^2}\approx 0.089321319887710$$

$$\frac{55\sqrt{\log(\pi)}}{147e^{3/2}}\approx 0.0893213222059$$

$$\frac{2(68+\pi^2)}{555\pi}\approx 0.0893213150057$$

From which:

Pi*1/(((1/64 (1/2(sqrt13-3))^4 [(((1/8(5+sqrt13))^0.5+(1/8(sqrt13-3))^0.5))]^24)))-1/golden ratio

Input:

$$\pi \times \frac{1}{\frac{1}{64}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4 \left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(\sqrt{13}-3\right)}\right)^{24}} - \frac{1}{\phi}$$

ϕ is the golden ratio

Exact result:

$$\frac{1024\,\pi}{\left(\sqrt{13}-3\right)^4 \left(\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{13}-3\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(5+\sqrt{13}\right)}\right)^{24}} - \frac{1}{\phi}$$

Decimal approximation:

34.55377808305433874201975958993199002884292745345582013896...

34.553778083…..

Property:

$$-\frac{1}{\phi}+\frac{1024\,\pi}{\left(-3+\sqrt{13}\right)^4 \left(\frac{1}{2}\sqrt{\frac{1}{2}\left(-3+\sqrt{13}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(5+\sqrt{13}\right)}\right)^{24}}$$

is a transcendental number

Alternate forms:

$$\pi \left(\text{root of } x^4 - 5\,180\,224\,x^3 + 54\,583\,296\,x^2 + 36\,700\,160\,x + 16\,777\,216 \text{ near } x = 11.1955\right) - \frac{1}{\phi}$$

$$\frac{70\,368\,744\,177\,664\,\pi}{\left(\sqrt{13}-3\right)^4 \left(\sqrt{\sqrt{13}-3}+\sqrt{5+\sqrt{13}}\right)^{24}} - \frac{1}{\phi}$$

$$\frac{1}{2}\left(1-\sqrt{5}\right)+16\left(80\,941+22\,449\sqrt{13}-3\sqrt{6\left(242\,644\,093+67\,297\,363\sqrt{13}\right)}\right)\pi$$

Series representations:

$$\frac{\pi}{\frac{1}{64}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(\sqrt{13}-3\right)}\right)^{24}}-\frac{1}{\phi}=-\frac{1}{\phi}+\frac{1024\pi}{\left(-3+\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}\right)^4\left(\frac{\sqrt{-3+\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}}}{2\sqrt{2}}+\frac{\sqrt{5+\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}}}{2\sqrt{2}}\right)^{24}}$$

$$\frac{\pi}{\frac{1}{64}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(\sqrt{13}-3\right)}\right)^{24}}-\frac{1}{\phi}=$$

$$-\frac{1}{\phi}+(1024\pi)\Bigg/\left(\left(-3+\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^4\left(\frac{\sqrt{-3+\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}}{2\sqrt{2}}+\frac{\sqrt{5+\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}}{2\sqrt{2}}\right)^{24}\right)$$

$$\frac{\pi}{\frac{1}{64}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(\sqrt{13}-3\right)}\right)^{24}}-\frac{1}{\phi}=$$

$$-\frac{1}{\phi}+(1024\,\pi)\Bigg/\left(\left(-3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}12^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)^4\left(\frac{\sqrt{-3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}12^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}}{2\sqrt{2}}+\frac{\sqrt{5+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}12^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}}{2\sqrt{2}}\right)^{24}\right)$$

We have also:

5*colog(((1/64 (1/2(sqrt13-3))^4 [(((1/8(5+sqrt13))^0.5+(1/8(sqrt13-3))^0.5))]^24)))

Input:

$$5\left(-\log\left(\frac{1}{64}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(\sqrt{13}-3\right)}\right)^{24}\right)\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$-5\log\left(\frac{\left(\sqrt{13}-3\right)^4\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{13}-3\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(5+\sqrt{13}\right)}\right)^{24}}{1024}\right)$$

Decimal approximation:

12.077575412668372371197396176973084209897640307709490793356...

12.07757541266…. result very near to the black hole entropy 12.1904, that is equal to ln(196884)

Property:

$$-5\log\left(\frac{(-3+\sqrt{13})^4\left(\frac{1}{2}\sqrt{\frac{1}{2}(-3+\sqrt{13})}+\frac{1}{2}\sqrt{\frac{1}{2}(5+\sqrt{13})}\right)^{24}}{1024}\right)$$

is a transcendental number

Alternate forms:

$$-5\log\left(\frac{1}{128}\left(-70+21\sqrt{13}+64\sqrt{\frac{2457\sqrt{13}}{2048}-\frac{17\,685}{4096}}\right)\right)$$

$$-5\left(-\log(1024)+4\log\left(\sqrt{13}-3\right)+24\log\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{13}-3\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(5+\sqrt{13}\right)}\right)\right)$$

$$10\left(6\log(8)+\log(32)-2\left(\log\left(\sqrt{13}-3\right)+6\log\left(\sqrt{\sqrt{13}-3}+\sqrt{5+\sqrt{13}}\right)\right)\right)$$

Alternative representations:

$$5(-1)\log\left(\frac{1}{64}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(\sqrt{13}-3\right)}\right)^{24}\right)=$$
$$-5\log_e\left(\frac{1}{64}\left(\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}\right)^{24}\left(\frac{1}{2}\left(-3+\sqrt{13}\right)\right)^4\right)$$

$$5(-1)\log\left(\frac{1}{64}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(\sqrt{13}-3\right)}\right)^{24}\right)=$$
$$-5\log(a)\log_a\left(\frac{1}{64}\left(\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}\right)^{24}\left(\frac{1}{2}\left(-3+\sqrt{13}\right)\right)^4\right)$$

$$5(-1)\log\left(\frac{1}{64}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(\sqrt{13}-3\right)}\right)^{24}\right)=$$
$$5\,\mathrm{Li}_1\left(1-\frac{1}{64}\left(\sqrt{\frac{1}{8}\left(-3+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}\right)^{24}\left(\frac{1}{2}\left(-3+\sqrt{13}\right)\right)^4\right)$$

and again:

48(((Pi / (((1/64 (1/2(sqrt13-3))^4 [(((1/8(5+sqrt13))^0.5+(1/8(sqrt13-3))^0.5))]^24)))+(89-3)/10^2)))-1/2

Input:

$$48\left(\frac{\pi}{\frac{1}{64}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(\sqrt{13}-3\right)}\right)^{24}}+\frac{89-3}{10^2}\right)-\frac{1}{2}$$

Exact result:

$$48\left(\frac{43}{50}+\frac{1024\pi}{\left(\sqrt{13}-3\right)^4\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{13}-3\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(5+\sqrt{13}\right)}\right)^{24}}\right)-\frac{1}{2}$$

Decimal approximation:

1729.026979446603212330768628366286151035035358396555984052...

1729.026979…

Property:

$$-\frac{1}{2}+48\left(\frac{43}{50}+\frac{1024\pi}{\left(-3+\sqrt{13}\right)^4\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(-3+\sqrt{13}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(5+\sqrt{13}\right)}\right)^{24}}\right)$$ is a transcendental number

Alternate forms:

$$768\pi\left(\text{root of } x^4-323764x^3+213216x^2+8960x+256 \text{ near } x=0.699721\right)+\frac{2039}{50}$$

$$48\left(\frac{43}{50}+16\left(80941+22449\sqrt{13}-3\sqrt{6\left(242644093+67297363\sqrt{13}\right)}\right)\pi\right)-\frac{1}{2}$$

$$\frac{2039}{50}+\frac{49152\pi}{\left(\sqrt{13}-3\right)^4\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{13}-3\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(5+\sqrt{13}\right)}\right)^{24}}$$

Series representations:

$$48\left(\frac{\pi}{\frac{1}{64}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(\sqrt{13}-3\right)}\right)^{24}}+\frac{89-3}{10^2}\right)-\frac{1}{2}=\frac{2039}{50}+\frac{49152\,\pi}{\left(-3+\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}\right)^4\left(\frac{\sqrt{-3+\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}}}{2\sqrt{2}}+\frac{\sqrt{5+\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}}}{2\sqrt{2}}\right)^{24}}$$

$$48\left(\frac{\pi}{\frac{1}{64}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(\sqrt{13}-3\right)}\right)^{24}}+\frac{89-3}{10^2}\right)-\frac{1}{2}=$$
$$\frac{2039}{50}+(49152\,\pi)\Bigg/\left(\left(-3+\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^4\left(\frac{\sqrt{-3+\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}}{2\sqrt{2}}+\frac{\sqrt{5+\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}}{2\sqrt{2}}\right)^{24}\right)$$

$$48\left(\frac{\pi}{\frac{1}{64}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(\sqrt{13}-3\right)}\right)^{24}}+\frac{89-3}{10^2}\right)-\frac{1}{2}=$$

$$\frac{2039}{50}+(49\,152\,\pi)\Bigg/\left(\left(-3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}12^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)^4\left(\frac{\sqrt{-3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}12^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}}{2\sqrt{2}}+\frac{\sqrt{5+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}12^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}}{2\sqrt{2}}\right)^{24}\right)$$

((((48(((Pi / (((1/64 (1/2(sqrt13-3))^4 [(((1/8(5+sqrt13))^0.5+(1/8(sqrt13-3))^0.5))]^24)))+(89-3)/10^2)))-1/2))))^1/15-(21+5)1/10^3

Input:

$$\sqrt[15]{48\left(\frac{\pi}{\frac{1}{64}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(\sqrt{13}-3\right)}\right)^{24}}+\frac{89-3}{10^2}\right)-\frac{1}{2}}-(21+5)\times\frac{1}{10^3}$$

Exact result:

$$\sqrt[15]{48\left(\frac{43}{50}+\frac{1024\,\pi}{\left(\sqrt{13}-3\right)^4\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{13}-3\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(5+\sqrt{13}\right)}\right)^{24}}\right)-\frac{1}{2}}-\frac{13}{500}$$

Decimal approximation:

1.6178169387507426327255083424967668130910428469861690419 99...

1.61781693875…

Property:

$$-\frac{13}{500}+\sqrt[15]{-\frac{1}{2}+48\left(\frac{43}{50}+\frac{1024\pi}{\left(-3+\sqrt{13}\right)^4\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(-3+\sqrt{13}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(5+\sqrt{13}\right)}\right)^{24}}\right)}$$

is a transcendental number

Alternate forms:

$$\left(48\left(\frac{43}{50}+16\left(80941+22449\sqrt{13}-3\sqrt{6\left(242644093+67297363\sqrt{13}\right)}\right)\pi\right)-\frac{1}{2}\right)\char`^(1/15)-\frac{13}{500}$$

$$\sqrt[15]{\frac{2039}{50}+\frac{3377699720527872\pi}{\left(\sqrt{13}-3\right)^4\left(\sqrt{\sqrt{13}-3}+\sqrt{5+\sqrt{13}}\right)^{24}}}-\frac{13}{500}$$

$$\sqrt[15]{\frac{2039}{50}+\frac{49152\pi}{\left(\sqrt{13}-3\right)^4\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(\sqrt{13}-3\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(5+\sqrt{13}\right)}\right)^{24}}}-\frac{13}{500}$$

Series representations:

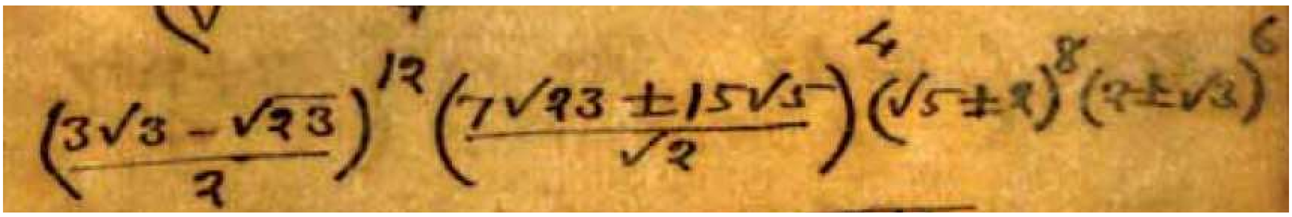
$$\sqrt[15]{48\left(\frac{\pi}{\frac{1}{64}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(\sqrt{13}-3\right)}\right)^{24}}+\frac{89-3}{10^2}\right)-\frac{1}{2}-\frac{21+5}{10^3}} = -\frac{13}{500}+\left(-\frac{1}{2}+48\left(\frac{43}{50}+(1024\,\pi)\Big/\left(\left(-3+\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}\right)^4\left(\frac{\sqrt{-3+\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}}}{2\sqrt{2}}+\frac{\sqrt{5+\sqrt{12}\sum_{k=0}^{\infty}12^{-k}\binom{\frac{1}{2}}{k}}}{2\sqrt{2}}\right)^{24}\right)\right)\right)\verb|^|(1/15)$$

$$\sqrt[15]{48\left(\frac{\pi}{\frac{1}{64}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(\sqrt{13}-3\right)}\right)^{24}}+\frac{89-3}{10^2}\right)-\frac{1}{2}-\frac{21+5}{10^3}} = -\frac{13}{500}+\left(-\frac{1}{2}+48\left(\frac{43}{50}+(1024\,\pi)\Big/\left(\left(-3+\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^4\left(\frac{\sqrt{-3+\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}}{2\sqrt{2}}+\frac{\sqrt{5+\sqrt{12}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}}{2\sqrt{2}}\right)^{24}\right)\right)\right)\verb|^|(1/15)$$

$$\sqrt[15]{48\left(\frac{\pi}{\frac{1}{64}\left(\frac{1}{2}\left(\sqrt{13}-3\right)\right)^4\left(\sqrt{\frac{1}{8}\left(5+\sqrt{13}\right)}+\sqrt{\frac{1}{8}\left(\sqrt{13}-3\right)}\right)^{24}}+\frac{89-3}{10^2}\right)-\frac{1}{2}}-\frac{21+5}{10^3}=$$

$$-\frac{13}{500}+\left(-\frac{1}{2}+48\left(\frac{43}{50}+(1024\,\pi)\Bigg/\left(\left(-3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}12^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)^4\left(\frac{\sqrt{-3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}12^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}}{2\sqrt{2}}+\frac{\sqrt{5+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}12^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}}{2\sqrt{2}}\right)^{24}\right)\right)\right)\hat{}\,(1/15)$$

Page 300



(1/2(3sqrt3-sqrt23))^12 (1/(sqrt2)(7sqrt23+15sqrt5))^4 (sqrt5+2)^8 (2+sqrt3)^6

Input:

$$\left(\frac{1}{2}\left(3\sqrt{3}-\sqrt{23}\right)\right)^{12}\left(\frac{1}{\sqrt{2}}\left(7\sqrt{23}+15\sqrt{5}\right)\right)^4\left(\sqrt{5}+2\right)^8\left(2+\sqrt{3}\right)^6$$

Result:

$$\frac{\left(2+\sqrt{3}\right)^6\left(2+\sqrt{5}\right)^8\left(3\sqrt{3}-\sqrt{23}\right)^{12}\left(15\sqrt{5}+7\sqrt{23}\right)^4}{16\,384}$$

Decimal approximation:

$5.875765692817611398000252648843108068037012620132919 63\ldots \times 10^6$

5.87576569281761….*10^6

Alternate forms:

$$\left(120\,901\,249 - 14\,554\,800\sqrt{69}\right)\left(2\,535\,751 + 236\,460\sqrt{115}\right)\left(51\,841 + 23\,184\sqrt{5}\right)\left(1351 + 780\sqrt{3}\right)$$

$$5\,886\,002\,618\,885\,809 - 3\,393\,670\,530\,775\,980\sqrt{3} + 2\,628\,727\,383\,670\,416\sqrt{5} - 1\,519\,758\,478\,667\,520\sqrt{15} + 1\,225\,649\,766\,124\,800\sqrt{23} - 708\,591\,446\,362\,800\sqrt{69} + 548\,872\,262\,953\,140\sqrt{115} - 316\,461\,588\,534\,000\sqrt{345}$$

root of $x^8 - 47\,088\,020\,951\,086\,472\,x^7 + 1\,504\,445\,469\,592\,605\,048\,410\,836\,619\,548\,x^6 - 20\,095\,493\,903\,394\,515\,998\,718\,222\,812\,887\,593\,935\,544\,x^5 + 118\,024\,473\,255\,471\,838\,186\,044\,828\,220\,966\,075\,354\,132\,876\,870\,x^4 - 20\,095\,493\,903\,394\,515\,998\,718\,222\,812\,887\,593\,935\,544\,x^3 + 1\,504\,445\,469\,592\,605\,048\,410\,836\,619\,548\,x^2 - 47\,088\,020\,951\,086\,472\,x + 1$ near $x = 5.87577 \times 10^6$

Minimal polynomial:

$$x^8 - 47\,088\,020\,951\,086\,472\,x^7 + 1\,504\,445\,469\,592\,605\,048\,410\,836\,619\,548\,x^6 - 20\,095\,493\,903\,394\,515\,998\,718\,222\,812\,887\,593\,935\,544\,x^5 + 118\,024\,473\,255\,471\,838\,186\,044\,828\,220\,966\,075\,354\,132\,876\,870\,x^4 - 20\,095\,493\,903\,394\,515\,998\,718\,222\,812\,887\,593\,935\,544\,x^3 + 1\,504\,445\,469\,592\,605\,048\,410\,836\,619\,548\,x^2 - 47\,088\,020\,951\,086\,472\,x + 1$$

From which:

((((1/2(3sqrt3-sqrt23))^12 (1/(sqrt2)(7sqrt23+15sqrt5))^4 (sqrt5+2)^8 (2+sqrt3)^6)))^1/4-11-4

Input:

$$\sqrt[4]{\left(\frac{1}{2}\left(3\sqrt{3} - \sqrt{23}\right)\right)^{12}\left(\frac{1}{\sqrt{2}}\left(7\sqrt{23} + 15\sqrt{5}\right)\right)^4\left(\sqrt{5} + 2\right)^8\left(2 + \sqrt{3}\right)^6} - 11 - 4$$

Result:

$$\frac{(2+\sqrt{3})^{3/2}(2+\sqrt{5})^2(3\sqrt{3} - \sqrt{23})^3(15\sqrt{5} + 7\sqrt{23})}{8\sqrt{2}} - 15$$

Decimal approximation:

34.23411290893838248005139396869080383321532785887379590586...

34.2341129089….

Alternate forms:

$$\left(\left(1351+780\sqrt{3}\right)\left(51841+23184\sqrt{5}\right)\left(120901249-14554800\sqrt{69}\right)\left(2535751+236460\sqrt{115}\right)\right)\hat{}\,(1/4)-15$$

$$\frac{(2+\sqrt{3})^{3/2}(9+4\sqrt{5})(36\sqrt{3}-13\sqrt{23})(15\sqrt{5}+7\sqrt{23})}{\sqrt{2}}-15$$

$$\frac{(2+\sqrt{3})^{3/2}(2+\sqrt{5})^2(3\sqrt{3}-\sqrt{23})^3(15\sqrt{5}+7\sqrt{23})-120\sqrt{2}}{8\sqrt{2}}$$

We note that:

ln((((1/2(3sqrt3-sqrt23))^12 (1/(sqrt2)(7sqrt23+15sqrt5))^4 (sqrt5+2)^8 (2+sqrt3)^6)))

Input:

$$\log\left(\left(\frac{1}{2}\left(3\sqrt{3}-\sqrt{23}\right)\right)^{12}\left(\frac{1}{\sqrt{2}}\left(7\sqrt{23}+15\sqrt{5}\right)\right)^4\left(\sqrt{5}+2\right)^8\left(2+\sqrt{3}\right)^6\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\log\left(\frac{(2+\sqrt{3})^6(2+\sqrt{5})^8(3\sqrt{3}-\sqrt{23})^{12}(15\sqrt{5}+7\sqrt{23})^4}{16384}\right)$$

Decimal approximation:

15.58634694019454932827072867347950949730836185513534329375...

15.58634694… result very near to the Bekenstein-Hawking entropy value 15.6730

Property:

$$\log\left(\frac{(2+\sqrt{3})^6(2+\sqrt{5})^8(3\sqrt{3}-\sqrt{23})^{12}(15\sqrt{5}+7\sqrt{23})^4}{16384}\right)$$

is a transcendental number

Alternate forms:

$$4\log\left(15\sqrt{5}+7\sqrt{23}\right)+\log\left(\frac{120\,901\,249}{4}-3\,638\,700\sqrt{69}\right)+8\sinh^{-1}(2)+6\cosh^{-1}(2)$$

$$\log\left(\text{root of } x^8-47\,088\,020\,951\,086\,472\,x^7+1\,504\,445\,469\,592\,605\,048\,410\,836\,619\,548\,x^6-20\,095\,493\,903\,394\,515\,998\,718\,222\,812\,887\,593\,935\,544\,x^5+118\,024\,473\,255\,471\,838\,186\,044\,828\,220\,966\,075\,354\,132\,876\,870\,x^4-20\,095\,493\,903\,394\,515\,998\,718\,222\,812\,887\,593\,935\,544\,x^3+1\,504\,445\,469\,592\,605\,048\,410\,836\,619\,548\,x^2-47\,088\,020\,951\,086\,472\,x+1 \text{ near } x=5.87577\times10^6\right)$$

$$2\left(-7\log(2)+3\log\left(2+\sqrt{3}\right)+4\log\left(2+\sqrt{5}\right)+6\log\left(3\sqrt{3}-\sqrt{23}\right)+2\log\left(15\sqrt{5}+7\sqrt{23}\right)\right)$$

$\cosh^{-1}(x)$ is the inverse hyperbolic cosine function

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Alternative representations:

$$\log\left(\left(\frac{1}{2}\left(3\sqrt{3}-\sqrt{23}\right)\right)^{12}\left(\frac{7\sqrt{23}+15\sqrt{5}}{\sqrt{2}}\right)^4\left(\sqrt{5}+2\right)^8\left(2+\sqrt{3}\right)^6\right)=\log_e\left(\left(2+\sqrt{3}\right)^6\left(2+\sqrt{5}\right)^8\left(\frac{1}{2}\left(3\sqrt{3}-\sqrt{23}\right)\right)^{12}\left(\frac{15\sqrt{5}+7\sqrt{23}}{\sqrt{2}}\right)^4\right)$$

$$\log\left(\left(\frac{1}{2}\left(3\sqrt{3}-\sqrt{23}\right)\right)^{12}\left(\frac{7\sqrt{23}+15\sqrt{5}}{\sqrt{2}}\right)^4\left(\sqrt{5}+2\right)^8\left(2+\sqrt{3}\right)^6\right)=\log(a)\log_a\left(\left(2+\sqrt{3}\right)^6\left(2+\sqrt{5}\right)^8\left(\frac{1}{2}\left(3\sqrt{3}-\sqrt{23}\right)\right)^{12}\left(\frac{15\sqrt{5}+7\sqrt{23}}{\sqrt{2}}\right)^4\right)$$

$$\log\left(\left(\frac{1}{2}\left(3\sqrt{3}-\sqrt{23}\right)\right)^{12}\left(\frac{7\sqrt{23}+15\sqrt{5}}{\sqrt{2}}\right)^4\left(\sqrt{5}+2\right)^8\left(2+\sqrt{3}\right)^6\right)=-\text{Li}_1\left(1-\left(2+\sqrt{3}\right)^6\left(2+\sqrt{5}\right)^8\left(\frac{1}{2}\left(3\sqrt{3}-\sqrt{23}\right)\right)^{12}\left(\frac{15\sqrt{5}+7\sqrt{23}}{\sqrt{2}}\right)^4\right)$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

We obtain also:

Pi^2*((((1/2(3sqrt3-sqrt23))^12 (1/(sqrt2)(7sqrt23+15sqrt5))^4 (sqrt5+2)^8 (2+sqrt3)^6)))^1/3 - 55+3

Input:

$$\pi^2 \sqrt[3]{\left(\frac{1}{2}\left(3\sqrt{3}-\sqrt{23}\right)\right)^{12}\left(\frac{1}{\sqrt{2}}\left(7\sqrt{23}+15\sqrt{5}\right)\right)^4\left(\sqrt{5}+2\right)^8\left(2+\sqrt{3}\right)^6} - 55 + 3$$

Result:

$$\frac{(2+\sqrt{3})^2 (2+\sqrt{5})^{8/3} (3\sqrt{3}-\sqrt{23})^4 (15\sqrt{5}+7\sqrt{23})^{4/3} \pi^2}{16 \times 2^{2/3}} - 52$$

Decimal approximation:

1728.961652765012195735661032033486725993703375492634783520...

1728.961652765….

Property:

$$-52 + \frac{(2+\sqrt{3})^2 (2+\sqrt{5})^{8/3} (3\sqrt{3}-\sqrt{23})^4 (15\sqrt{5}+7\sqrt{23})^{4/3} \pi^2}{16 \times 2^{2/3}}$$ is a transcendental number

Alternate forms:

$$\left(\left(1351+780\sqrt{3}\right)\left(51841+23184\sqrt{5}\right)\left(120901249-14554800\sqrt{69}\right)\left(2535751+236460\sqrt{115}\right)\right) \hat{}\ (1/3)\,\pi^2 - 52$$

$$\pi^2 \left(\text{root of } x^8 - 47088020951086472 x^7 + 1504445469592605048410836619548 x^6 - 20095493903394515998718222812887593935544 x^5 + 118024473255471838186044828220966075354132876870 x^4 - 20095493903394515998718222812887593935544 x^3 + 1504445469592605048410836619548 x^2 - 47088020951086472 x + 1 \text{ near } x = 5.87577 \times 10^6\right) \hat{}\ (1/3) - 52$$

$$\frac{(2+\sqrt{3})^2 (2+\sqrt{5})^{8/3} (3\sqrt{3}-\sqrt{23})^4 (15\sqrt{5}+7\sqrt{23})^{4/3} \pi^2 - 832 \times 2^{2/3}}{16 \times 2^{2/3}}$$

((((Pi^2*((((1/2(3sqrt3-sqrt23))^12 (1/(sqrt2)(7sqrt23+15sqrt5))^4 (sqrt5+2)^8 (2+sqrt3)^6)))^1/3 - 55+3))))^1/15-(21+5)1/10^3

Input:

$$\sqrt[15]{\pi^2 \sqrt[3]{\left(\frac{1}{2}\left(3\sqrt{3}-\sqrt{23}\right)\right)^{12}\left(\frac{1}{\sqrt{2}}\left(7\sqrt{23}+15\sqrt{5}\right)\right)^4\left(\sqrt{5}+2\right)^8\left(2+\sqrt{3}\right)^6}-55+3} - (21+5)\times\frac{1}{10^3}$$

Exact result:

$$\sqrt[15]{\frac{(2+\sqrt{3})^2 (2+\sqrt{5})^{8/3} (3\sqrt{3}-\sqrt{23})^4 (15\sqrt{5}+7\sqrt{23})^{4/3}\pi^2}{16\times 2^{2/3}}-52} - \frac{13}{500}$$

Decimal approximation:

1.617812798194600845324465750709477175842761334433118704916...

1.6178127981946…

Property:

$$-\frac{13}{500} + \sqrt[15]{-52 + \frac{(2+\sqrt{3})^2 (2+\sqrt{5})^{8/3} (3\sqrt{3}-\sqrt{23})^4 (15\sqrt{5}+7\sqrt{23})^{4/3}\pi^2}{16\times 2^{2/3}}}$$

is a transcendental number

Alternate forms:

$$\frac{\sqrt[15]{(2+\sqrt{3})^2 (2+\sqrt{5})^{8/3} (3\sqrt{3}-\sqrt{23})^4 (15\sqrt{5}+7\sqrt{23})^{4/3}\pi^2 - 832\times 2^{2/3}}}{2^{14/45}} - \frac{13}{500}$$

$$\left(\pi^2 \left(\text{root of } x^8 - 47088020951086472x^7 + 1504445469592605048410836619548x^6 - 20095493903394515998718222812887593935544x^5 + 118024473255471838186044828220966075354132876870x^4 - 20095493903394515998718222812887593935544x^3 + 1504445469592605048410836619548x^2 - 47088020951086472x + 1 \text{ near } x = 5.87577\times 10^6\right)\text{^}(1/3) - 52\right)\text{^}(1/15) - \frac{13}{500}$$

$$\frac{1}{500\times 2^{4/15}}\left(250\times 2^{43/45}\sqrt[15]{\left(2+\sqrt{3}\right)^2\left(2+\sqrt{5}\right)^{8/3}\left(3\sqrt{3}-\sqrt{23}\right)^4\left(15\sqrt{5}+7\sqrt{23}\right)^{4/3}\pi^2-832\times 2^{2/3}}-13\times 2^{4/15}\right)$$

Observations

Figs.

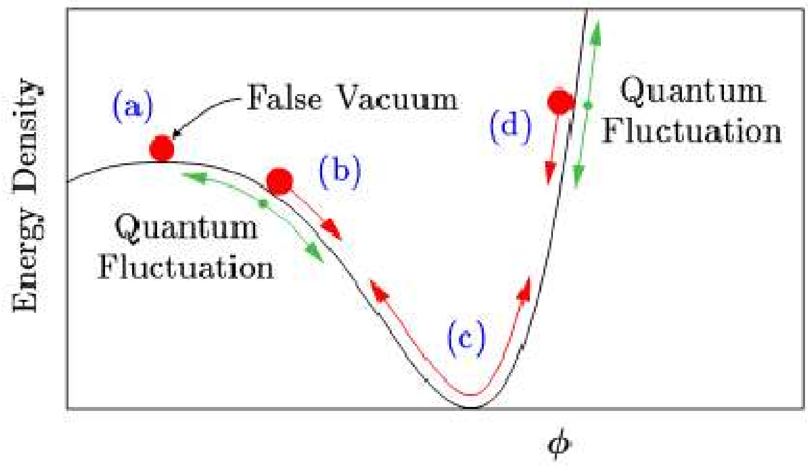
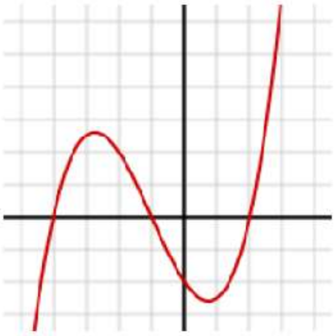


FIG. 1: In simple inflationary models, the universe at early times is dominated by the potential energy density of a scalar field, ϕ. Red arrows show the classical motion of ϕ. When ϕ is near region (a), the energy density will remain nearly constant, $\rho \cong \rho_f$, even as the universe expands. Moreover, cosmic expansion acts like a frictional drag, slowing the motion of ϕ: Even near regions (b) and (d), ϕ behaves more like a marble moving in a bowl of molasses, slowly creeping down the side of its potential, rather than like a marble sliding down the inside of a polished bowl. During this period of "slow roll," ρ remains nearly constant. Only after ϕ has slid most of the way down its potential will it begin to oscillate around its minimum, as in region (c), ending inflation.



Graph of a cubic function with 3 real roots (where the curve crosses the horizontal axis at $y = 0$). The case shown has two critical points. Here the function is $f(x) = (x^3 + 3x^2 - 6x - 8)/4$.

The ratio between M_0 and q

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{\left(3\sqrt{3}\right) M_s}{2}.$$

i.e. the gravitating mass M_0 and the Wheelerian mass q of the wormhole, is equal to:

$$\frac{\sqrt{3\,(2.17049 \times 10^{37})^2 - 0.001^2}}{\frac{1}{2}\left(\left(3\sqrt{3}\right)\left(4.2 \times 10^6 \times 1.9891 \times 10^{30}\right)\right)}$$

1.732050787905194420703947625671018160083566548802082460520...

1.7320507879

$1.7320507879 \approx \sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q of the wormhole

We note that:

$$\left(-\frac{1}{2} + \frac{i}{2}\sqrt{3}\right) - \left(-\frac{1}{2} - \frac{i}{2}\sqrt{3}\right)$$

i is the imaginary unit

$i\sqrt{3}$

1.7320508075688772935274463415058723669428052538103806280558... i

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

1.73205

This result is very near to the ratio between M_0 and q, that is equal to 1.7320507879 $\approx \sqrt{3}$

With regard $\sqrt{3}$, we note that is a fundamental value of the formula structure that we need to calculate a Cubic Equation

We have that the previous result

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) = i\sqrt{3} =$$

$=$ 1.7320508075688772935274463415058723669428052538103806280558... i

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

can be related with:

$$u^2(-u)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)+v^2(-v)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)=q$$

Considering:

$$(-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q$$

$= i\sqrt{3} =$ 1.7320508075688772935274463415058723669428052538103806280558... i

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

Thence:

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) \Rightarrow$$

$$\Rightarrow (-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q=1.73205\approx\sqrt{3}$$

We observe how the graph above, concerning the cubic function, is very similar to the graph that represent the scalar field (in red). It is possible to hypothesize that cubic functions and cubic equations, with their roots, are connected to the scalar field.

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between *the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.*

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

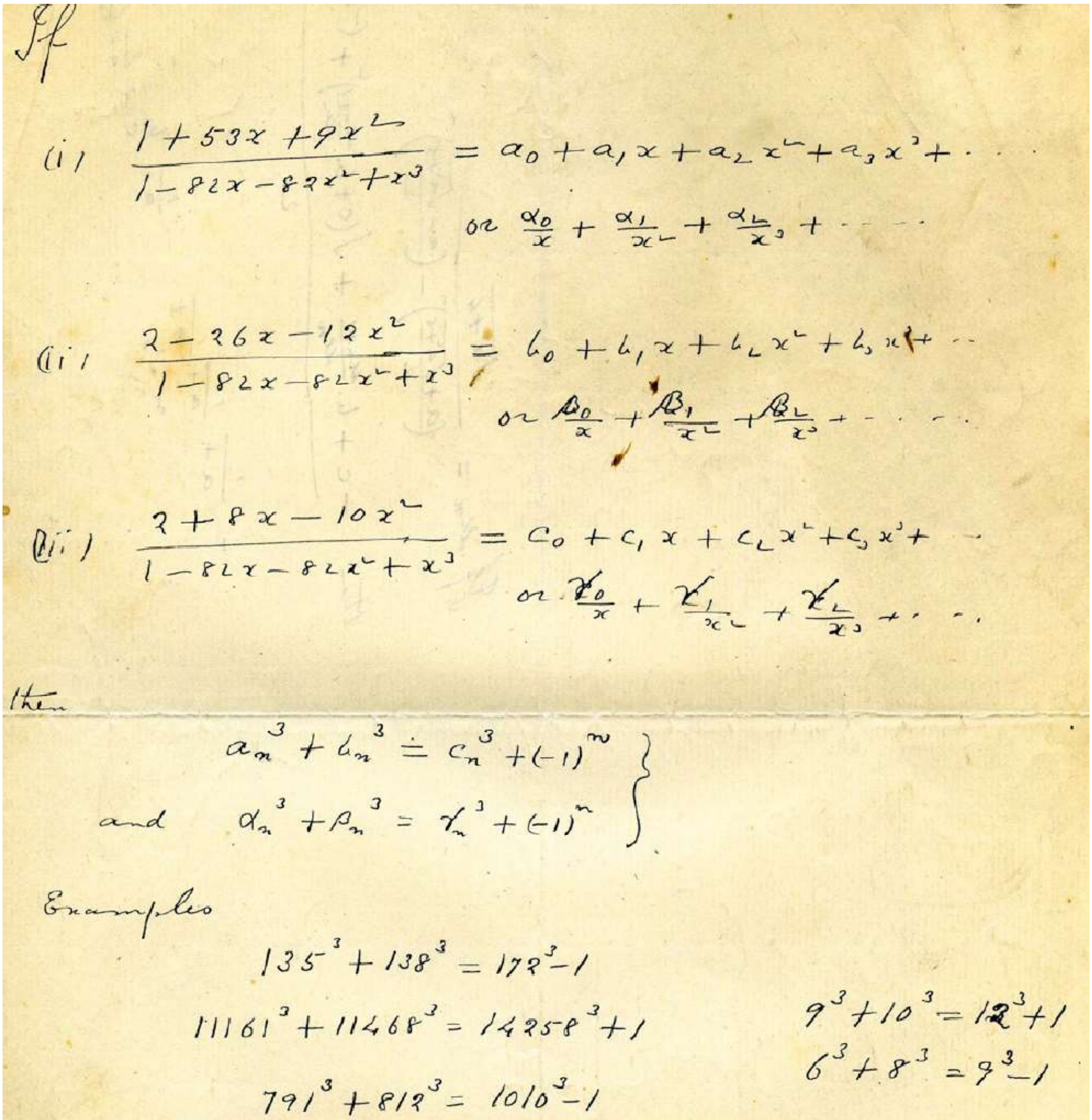
In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

https://plus.maths.org/content/ramanujan



If

(i) $$\frac{1+53x+9x^2}{1-82x-82x^2+x^3} = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$
or $$\frac{\alpha_0}{x} + \frac{\alpha_1}{x^2} + \frac{\alpha_2}{x^3} + \cdots$$

(ii) $$\frac{2-26x-12x^2}{1-82x-82x^2+x^3} = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + \cdots$$
or $$\frac{\beta_0}{x} + \frac{\beta_1}{x^2} + \frac{\beta_2}{x^3} + \cdots$$

(iii) $$\frac{2+8x-10x^2}{1-82x-82x^2+x^3} = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$$
or $$\frac{\gamma_0}{x} + \frac{\gamma_1}{x^2} + \frac{\gamma_2}{x^3} + \cdots$$

then

$$a_n^3 + b_n^3 = c_n^3 + (-1)^n$$

and $$\alpha_n^3 + \beta_n^3 = \gamma_n^3 + (-1)^n$$

Examples

$$135^3 + 138^3 = 172^3 - 1$$
$$11161^3 + 11468^3 = 14258^3 + 1$$
$$791^3 + 812^3 = 1010^3 - 1$$
$$9^3 + 10^3 = 12^3 + 1$$
$$6^3 + 8^3 = 9^3 - 1$$

Ramanujan's manuscript. The representations of 1729 as the sum of two cubes appear in the bottom right corner. The equation expressing the near counter examples to Fermat's last theorem appears further up: $\alpha^3 + \beta^3 = \gamma^3 + (-1)^n$. Image courtesy Trinity College library.

References

Universal interferometric signatures of a black hole's photon ring

Michael D. Johnson, Alexandru Lupsasca, Andrew Strominger, George N. Wong, Shahar Hadar, Daniel Kapec, Ramesh Narayan, Andrew Chael, Charles F. Gammie, Peter Galison, Daniel C. M. Palumbo, Sheperd S. Doeleman, Lindy Blackburn, Maciek Wielgus, Dominic W. Pesce, Joseph R. Farah, James M. Moran

Johnson *et al*., *Sci. Adv.* 2020; **6** : eaaz1310 18 March 2020

Mechanisms for Primordial Black Hole Production in String Theory
Ogan Ozsoy, Susha Parameswaran , Gianmassimo Tasinato , Ivonne Zavala
arXiv:1803.07626v2 [hep-th] 2 Jul 2018

Primordial Black Holes from String Inflation
Michele Cicoli, Victor A. Diaz, Francisco G. Pedro - arXiv:1803.02837v2 [hep-th] 16 Mar 2018

EVALUATIONS OF RAMANUJAN-WEBER CLASS INVARIANT g_n
S.Bhargava, K. R. Vasuki and B. R. Srivatsa Kumar - Journal of the Indian Mathematical Society · January 2005

Manuscript Book 2 of Srinivasa Ramanujan