In this book the Gentzen variant of the propositional logic is used to substantiate the space-time relations, including the Lorentz transformations, irreversible unidirectional time and metric space.

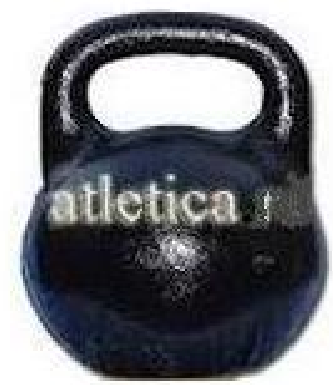
The logical foundations of probability theory, including Jacob Bernulli's Big Numbers Law and the statistical definition of probability, are also derived from this logic.

All concepts and statements of the Standard Model (except for the Higgs) are obtained as concepts and theorems of probability theory. The masses, spins, moments, energies of fermions are the parameters of the distribution of such a probability. The masses of the W and Z bosons are the results of the interaction of the probability flows into space-time.

Quark-gluon relations, including the phenomena of confinement and asymptotic freedom, are also a consequence of the properties of this probability.

The phenomenon of gravity with dark matter and dark energy is a continuation of these quark-gyonic relations.

For understanding of the maintenance of this book elementary knowledge in the field of linear algebra and the mathematical analysis is sufficient.



TIME-SPACE, PROBABILITY AND PHYSICS

GUNN QUZNETSOV

2020
PRESS

Contents

Introduction **v**

1 Truth **1**

2 Time-Space **3**
2.1. Recorders . . . 3
2.2. Time . . . 3
2.3. Space . . . 9
2.4. Relativity . . . 16
2.5. Matricies . . . 37

3 Logic and Probability **45**
3.1. Logic . . . 45
3.2. The Boole funcion . . . 50
3.3. Probability . . . 60
3.3.1. Events . . . 62
3.3.2. B-functions . . . 62
3.3.3. Independent tests . . . 64
3.3.4. The logic probability function . . . 66
3.3.5. Conditional probability . . . 67
3.3.6. Classical probability . . . 68
3.3.7. Probability and logic . . . 68
3.3.8. THE NONSTANDARD NUMBERS . . . 69
3.3.9. Model . . . 75

4 Physics **79**
4.1. Planck . . . 79
4.2. Quants . . . 80
4.3. Physical events . . . 81
4.4. Equations of moving . . . 84
4.5. Double-Slit Experiment . . . 89
4.6. Lepton Hamiltonian . . . 99
4.7. Masses . . . 107
4.8. One-Mass State . . . 112
4.9. Creating and Annihilation Operators . . . 116

4.10. Particles and Antiparticles . . . 120
4.11. Electroweak Fields . . . 123
4.12. Bi-mass state . . . 124
4.13. Electroweak Transformations . . . 128
4.14. Neutrinno . . . 141
4.15. Quarks and Gluons . . . 148
4.16. Asymptotic Freedom, Confinement, Gravitation . . . 162
4.16.1. Dark Energy . . . 164
4.16.2. Dark Matter . . . 167
4.16.3. Baryon Chrome . . . 171
4.17. Dimension of physical space . . . 186

Conclusion **191**

References **193**

Index **197**

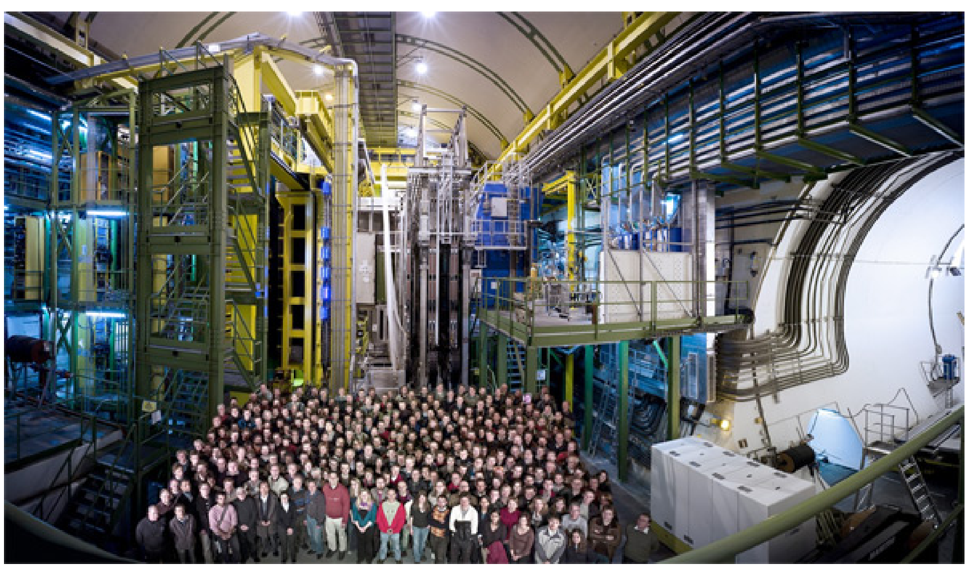
Introduction

> ”... They sawed dumb-bells ...
> ”What’s the matter?” Balaganov said suddenly, stopping work. ”I’ve been sawing away for three hours, and still it isn’t gold!” Panikovsky did not reply. He had made the discovery a half hour before, and had continued to move the saw only for the sake of appearance. ”Well, let’s saw some more,” redhaired Shura said gallantly. ”Of course we must saw,” remarked Panikovsky, trying to defer the moment of reckoning as long as possible. ... ”I can’t make it out,” said Shura, when he had sawed the dumbbell into two halves. ”This is not gold!” ”Go on sawing! Go on!” gabbled Panikovsky...”
> .
> Ilya Ilf, Yevgeny Petrov. ”The Little Golden Calf”. M., 1987.

The Manhattan Project began on September 17, 1943. It was attracted many outstanding physicists, many of whom were refugees from Europe. By the summer of 1945, the Americans had managed to build 3 atomic bombs, 2 of which were dropped on Hiroshima and Nagasaki, and a third had been tested shortly before. And the atomic race began. In the following years, the governments of many states allocated enormous sums of money to scientific organizations. Following these money, huge masses of easy luck seekers moved to physics. They invented SUSY, WIMP, BIG BANG, HIGGS and other theories of the same kind. Giant laboratory facilities were built and enormous human resources were attracted to experimentally confirm these theories

Results of the LHC and other science giant laboratory work are describe in [1] (since 10 September 2008 till 14 February 2013: RUNI) and [2] (from June 2015 to January 2018, RUNII) Large Hadron Collider (LHC) worked since 10 September 2008 till 14 February 2013 RUNI. RUNII works from June 2015 for today. Huge resources have been spent, but did not receive any fundamentally new results - no superpartners, no extra dimensions, or gravitons, or black holes. no dark matter or dark energy, etc. etc .. As for the Higgs, thefistly, there is no argument in favor of the fact that the particle 124.5 -126 GeV has some

relation to the Higgs mechanism. Secondly, the Higgs held permeates the vacuum of space, which means that the mass of the Higgs vacuum and stability are closely linked. For a particle of mass near 126 GeV - enough to destroy the cosmos. The Standard Model of particle physics has not given an answer to the question of why the universe did not collapse after the Big Bang. Moreover, Nothing in Standard Model gives a precise value for the Higgs???s own mass, and calculations from first principles, based on quantum theory, suggest it should be enormous???roughly a hundred million billion times higher than its measured value. Physicists have therefore introduced an ugly fudge factor into their equations (a process called ???fine-tuning???) to sidestep the problem. Third, all the known elementary bosons are gauge - it is photons, W- and Z-bosons and gluons[3]. It is likely that the 125-126 particle is of some hadron multiplet.



That is, in recent years, many theoretical physicists have studied what is not in the nature. It are SUSY, WIMP, Higgs, BIG BANG hypothesis, etc. On the other hand already in 2006 - 2007 the logic analysis of these subjects described in books [1], [2] it showed that all physical events are determined by well-known particles - leptons, quarks and gauge bosons.

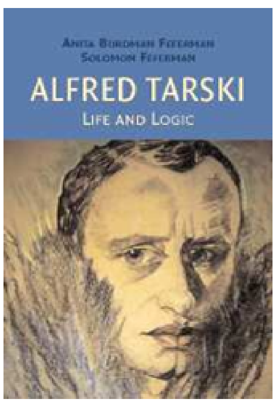
This book contains development and continuation of ideas of these books.

For understanding of the maintenance of this book elementary knowledge in the field of linear algebra and the mathematical analysis is sufficient.

Chapter 1

Truth

In the beginning was the Word.
Johnn, 1.1



Science presents its ideas and results with language texts. Therefore, we will begin by considering narrative sentences: By Alfred Tarski[1] [4]

A sentence Θ is true if and only if Θ .

For example, sentence It is raining is true if and only if it is raining.

A sentence Θ is false if and only if there is not that Θ .

For example, $2 + 3 = 4$.

Still an example[2]: Obviously, the following sentence isn't true and isn't false [5]:

This sentence is false.

Those sentences which can be either true, or false, are called as meaningful sentences. The previous example sentence is meaningless sentence.

Further, we consider only meaningful sentences which are either true, or false.

[1]Alfred Tarski January 14, 1901 October 26, 1983), born Alfred Teitelbaum, was a Polish-American logician and mathematician .

[2]Liar paradox, also called Epimenides paradox, paradox derived from the statement attributed to the Cretan prophet Epimenides (6th century BCE) that all Cretans are liars.

Chapter 2

Time-Space

”Do not expect answers before you have found clear meanings”
Hans Reichenbach, *The Direction of Time*, (1953)

2.1. Recorders

Any information, received from physical devices, can be expressed by a text, made of sentences.

Let $\widehat{\mathbf{a}}$ be some object which is able to receive, save, and/or transmit an information [**?**]. A set $\mathbf{a}$ of sentences, expressing an information of an object $\widehat{\mathbf{a}}$, is called *a recorder* of this object. Thus, statement: ”Sentence $\ll$A$\gg$ is an element of the set $\mathbf{a}$” denotes : ”$\widehat{\mathbf{a}}$ has information that the event, expressed by sentence $\ll A \gg$, took place.” In short: ”$\widehat{\mathbf{a}}$ knows that A.” Or by designation: ”$\mathbf{a}^\bullet \ll A \gg$”.

Obviously, the following conditions are satisfied:

I. For any $\mathbf{a}$ and for every A: false is that $\mathbf{a}^\bullet (A \& (\neg A))$, thus, any recorder doesn't contain a logical contradiction.

II. For every $\mathbf{a}$, every B, and all A: if B is a logical consequence from A, and $\mathbf{a}^\bullet A$, then $\mathbf{a}^\bullet B$.

*III. For all $\mathbf{a}$, $\mathbf{b}$ and for every A: if $\mathbf{a}^\bullet \ll \mathbf{b}^\bullet A \gg$ then $\mathbf{a}^\bullet A$.

For example, if device $\widehat{\mathbf{a}}$ has information that device $\widehat{\mathbf{b}}$ has information that mass of particle $\overleftarrow{\chi}$ equals to 7 then device $\widehat{\mathbf{a}}$ has information that mass of particle $\overleftarrow{\chi}$ equal to 7.

2.2. Time

There are many concepts of the theory of ”time” - in particular, quantum mechanical, relativistic, thermodynamic, causal, etc. All of them are based on unclearly defined concepts and in most cases contain a vicious circle.

The thermodynamic concept has the greatest favor. But if a sergeant has a platoon in a line, does this sergeant's wristwatch change direction?

Any subjects, connected with an information is called informational objects. For example, it can be a physics device, or computer disks and gramophone records, or people, carrying memory on events of their lifes, or trees, on cuts which annual rings tell on past climatic and ecological changes, or stones with imprints of long ago extincted plants and bestials, or minerals, telling on geological cataclysms, or celestial bodies, carrying an information on a remote distant past Universe, etc., etc.

It is clearly that an information, received from such information object, can be expressed by a text which made of sentences.

Let's consider finite (probably empty) path of symbols of form $\mathbf{q}^{\bullet}$.

Def. 1.3.1 A path α is *a subpath* of a path β (design.: $\alpha \prec \beta$) if α can be got from β by deletion of some (probably all) elements.

Designation: $(\alpha)^1$ is α, and $(\alpha)^{k+1}$ is $\alpha\,(\alpha)^k$.

Therefore, if $k \leq l$ then $(\alpha)^k \prec (\alpha)^l$.

Def. 1.3.2 A path α is *equivalent* to a path β (design.: $\alpha \sim \beta$) if α can be got from β by substitution of a subpath of form $(\mathbf{a}^{\bullet})^k$ by a path of the same form $((\mathbf{a}^{\bullet})^s)$.

In this case:

III. If $\beta \prec \alpha$ or $\beta \sim \alpha$ then for any K:

if $\mathbf{a}^{\bullet}K$ then $\mathbf{a}^{\bullet}\,(K\&\,(\alpha A \Rightarrow \beta A))$.

Obviously, III is a refinement of condition *III.

Def. 1.3.3 A natural number q is *instant*, at which a registrates B according to κ-*clock* $\{\mathbf{g}_0, A, \mathbf{b}_0\}$ (design.: q is $[\mathbf{a}^{\bullet}B \uparrow \mathbf{a}, \{\mathbf{g}_0, A, \mathbf{b}_0\}]$) if:

1. for any K: if $\mathbf{a}^{\bullet}K$ then

$$\mathbf{a}^{\bullet}\,(K\&\,(\mathbf{a}^{\bullet}B \Rightarrow \mathbf{a}^{\bullet}\,(\mathbf{g}_0^{\bullet}\mathbf{b}_0^{\bullet})^q\,\mathbf{g}_0^{\bullet}A))$$

and

$$\mathbf{a}^{\bullet}\left(K\&\left(\mathbf{a}^{\bullet}\,(\mathbf{g}_0^{\bullet}\mathbf{b}_0^{\bullet})^{q+1}\,\mathbf{g}_0^{\bullet}A \Rightarrow \mathbf{a}^{\bullet}B\right)\right).$$

2. $\mathbf{a}^{\bullet}\left(\mathbf{a}^{\bullet}B\&\left(\neg\mathbf{a}^{\bullet}\,(\mathbf{g}_0^{\bullet}\mathbf{b}_0^{\bullet})^{q+1}\,\mathbf{g}_0^{\bullet}A\right)\right)$.

Lm. 1.3.1 If

$$q \text{ is } [\mathbf{a}^{\bullet}\alpha B \uparrow \mathbf{a}, \{\mathbf{g}_0, A, \mathbf{b}_0\}], \tag{2.1}$$

$$p \text{ is } [\mathbf{a}^{\bullet}\beta B \uparrow \mathbf{a}, \{\mathbf{g}_0, A, \mathbf{b}_0\}], \tag{2.2}$$

$$\alpha \prec \beta, \tag{2.3}$$

then

$$q \leq p.$$

Proof of Lm. 1.3.1: From (2.2):

$$\mathbf{a}^{\bullet}\left(\left(\mathbf{a}^{\bullet}\beta B\right)\&\left(\neg\mathbf{a}^{\bullet}\left(\mathbf{g}_{0}^{\bullet}\mathbf{b}_{0}^{\bullet}\right)^{(p+1)}\mathbf{g}_{0}^{\bullet}A\right)\right). \tag{2.4}$$

From (2.3) according to III:

$$\mathbf{a}^{\bullet}\left(\left(\mathbf{a}^{\bullet}\beta B\&\left(\neg\mathbf{a}^{\bullet}\left(\mathbf{g}_{0}^{\bullet}\mathbf{b}_{0}^{\bullet}\right)^{(p+1)}\mathbf{g}_{0}^{\bullet}A\right)\right)\&\left(\mathbf{a}^{\bullet}\beta B\Rightarrow\mathbf{a}^{\bullet}\alpha B\right)\right). \tag{2.5}$$

Let us designate:
$R:=\mathbf{a}^{\bullet}\beta B$,
$S:=\left(\neg\mathbf{a}^{\bullet}\left(\mathbf{g}_{0}^{\bullet}\mathbf{b}_{0}^{\bullet}\right)^{(p+1)}\mathbf{g}_{0}^{\bullet}A\right)$,
$G:=\mathbf{a}^{\bullet}\alpha B$.
In that case a shape of formula (2.4) is

$$\mathbf{a}^{\bullet}\left(R\&S\right),$$

and a shape of formula (2.5) is

$$\mathbf{a}^{\bullet}\left(\left(R\&S\right)\&\left(R\Rightarrow G\right)\right).$$

Sentence $(G\&S)$ is a logical consequence from sentence $((R\&S)\&(R\Rightarrow G))$ (3.1). Hence

$$\mathbf{a}^{\bullet}\left(G\&S\right),$$

in accordance with II.
Hence

$$\mathbf{a}^{\bullet}\left(\mathbf{a}^{\bullet}\alpha B\&\left(\neg\mathbf{a}^{\bullet}\left(\mathbf{g}_{0}^{\bullet}\mathbf{b}_{0}^{\bullet}\right)^{(p+1)}\mathbf{g}_{0}^{\bullet}A\right)\right)$$

in accordance with the designation.
Hence from (2.1):

$$\mathbf{a}^{\bullet}\left(\left(\mathbf{a}^{\bullet}\alpha B\&\left(\neg\mathbf{a}^{\bullet}\left(\mathbf{g}_{0}^{\bullet}\mathbf{b}_{0}^{\bullet}\right)^{(p+1)}\mathbf{g}_{0}^{\bullet}A\right)\right)\&\left(\mathbf{a}^{\bullet}\alpha B\Rightarrow\mathbf{a}^{\bullet}\left(\mathbf{g}_{0}^{\bullet}\mathbf{b}_{0}^{\bullet}\right)^{q}\mathbf{g}_{0}^{\bullet}A\right)\right).$$

According to II:

$$\mathbf{a}^{\bullet}\left(\left(\neg\mathbf{a}^{\bullet}\left(\mathbf{g}_{0}^{\bullet}\mathbf{b}_{0}^{\bullet}\right)^{(p+1)}\mathbf{g}_{0}^{\bullet}A\right)\&\mathbf{a}^{\bullet}\left(\mathbf{g}_{0}^{\bullet}\mathbf{b}_{0}^{\bullet}\right)^{q}\mathbf{g}_{0}^{\bullet}A\right) \tag{2.6}$$

If $q>p$, i.e. $q\geq p+1$, then from (2.6) according to III

$$\mathbf{a}^{\bullet}\left(\begin{array}{c}\left(\left(\neg\mathbf{a}^{\bullet}\left(\mathbf{g}_{0}^{\bullet}\mathbf{b}_{0}^{\bullet}\right)^{(p+1)}\mathbf{g}_{0}^{\bullet}A\right)\&\mathbf{a}^{\bullet}\left(\mathbf{g}_{0}^{\bullet}\mathbf{b}_{0}^{\bullet}\right)^{q}\mathbf{g}_{0}^{\bullet}A\right)\&\\ \left(\mathbf{a}^{\bullet}\left(\mathbf{g}_{0}^{\bullet}\mathbf{b}_{0}^{\bullet}\right)^{q}\mathbf{g}_{0}^{\bullet}A\Rightarrow\mathbf{a}^{\bullet}\left(\mathbf{g}_{0}^{\bullet}\mathbf{b}_{0}^{\bullet}\right)^{(p+1)}\mathbf{g}_{0}^{\bullet}A\right)\end{array}\right).$$

According to II:

$$\mathbf{a}^{\bullet}\left(\left(\neg\mathbf{a}^{\bullet}\left(\mathbf{g}_{0}^{\bullet}\mathbf{b}_{0}^{\bullet}\right)^{(p+1)}\mathbf{g}_{0}^{\bullet}A\right)\&\mathbf{a}^{\bullet}\left(\mathbf{g}_{0}^{\bullet}\mathbf{b}_{0}^{\bullet}\right)^{(p+1)}\mathbf{g}_{0}^{\bullet}A\right).$$

It contradicts to condition I. Therefore, $q\leq p$ $\square$.
Lemma 1.3.1 proves that if

$$q \text{ is } \left[\mathbf{a}^{\bullet} B \uparrow \mathbf{a}, \{\mathbf{g}_0, A, \mathbf{b}_0\}\right],$$

and

$$p \text{ is } \left[\mathbf{a}^{\bullet} B \uparrow \mathbf{a}, \{\mathbf{g}_0, A, \mathbf{b}_0\}\right]$$

then

$$q = p.$$

That's why, expression "q is $[\mathbf{a}^{\bullet} B \uparrow \mathbf{a}, \{\mathbf{g}_0, A, \mathbf{b}_0\}]$" is equivalent to expression "$q = [\mathbf{a}^{\bullet} B \uparrow \mathbf{a}, \{\mathbf{g}_0, A, \mathbf{b}_0\}]$."

Def. 1.3.4 κ-clocks $\{\mathbf{g}_1, B, \mathbf{b}_1\}$ and $\{\mathbf{g}_2, B, \mathbf{b}_2\}$ have *the same direction* for $\mathbf{a}$ if the following condition is satisfied:

if

$$\begin{gathered} r = [\mathbf{a}^{\bullet} (\mathbf{g}_1^{\bullet} \mathbf{b}_1^{\bullet})^q \mathbf{g}_1^{\bullet} B \uparrow \mathbf{a}, \{\mathbf{g}_2, B, \mathbf{b}_2\}], \\ s = [\mathbf{a}^{\bullet} (\mathbf{g}_1^{\bullet} \mathbf{b}_1^{\bullet})^p \mathbf{g}_1^{\bullet} B \uparrow \mathbf{a}, \{\mathbf{g}_2, B, \mathbf{b}_2\}], \\ q < p, \end{gathered}$$

then

$$r \leq s.$$

Th. 1.3.1 All κ-clocks have the same direction.

Proof of Th. 1.3.1:

Let

$$\begin{gathered} r := [\mathbf{a}^{\bullet} (\mathbf{g}_1^{\bullet} \mathbf{b}_1^{\bullet})^q \mathbf{g}_1^{\bullet} B \uparrow \mathbf{a}, \{\mathbf{g}_2, B, \mathbf{b}_2\}], \\ s := [\mathbf{a}^{\bullet} (\mathbf{g}_1^{\bullet} \mathbf{b}_1^{\bullet})^p \mathbf{g}_1^{\bullet} B \uparrow \mathbf{a}, \{\mathbf{g}_2, B, \mathbf{b}_2\}], \end{gathered}$$

$$q < p.$$

In this case

$$(\mathbf{g}_1^{\bullet} \mathbf{b}_1^{\bullet})^q \prec (\mathbf{g}_1^{\bullet} \mathbf{b}_1^{\bullet})^p.$$

Consequently, according to Lm. 1.3.1

$$r \leq s$$

□

Consequently, a recorder orders its sentences with respect to instants. Moreover, this order is linear and it doesn't matter according to which κ-clock it is set.

Def. 1.3.5 κ-clock $\{\mathbf{g}_2, B, \mathbf{b}_2\}$ is k times more precise than κ-clock $\{\mathbf{g}_1, B, \mathbf{b}_1\}$ for recorder $\mathbf{a}$ if for every C the following condition is satisfied: if

$$q_1 = [\mathbf{a}^{\bullet} C \uparrow \mathbf{a}, \{\mathbf{g}_1, B, \mathbf{b}_1\}],$$
$$q_2 = [\mathbf{a}^{\bullet} C \uparrow \mathbf{a}, \{\mathbf{g}_2, B, \mathbf{b}_2\}],$$

then

$$q_1 < \tfrac{q_2}{k} < q_1 + 1.$$

Lm. 1.3.2 If for every n:

$$q_{n-1} < \frac{q_n}{k_n} < q_{n-1} + 1, \tag{2.7}$$

then the series

$$q_0 + \sum_{n=1}^{\infty} \frac{q_n - q_{n-1} k_n}{k_1 \dots k_n} \tag{2.8}$$

converges.

Proof of Lm. 1.3.2: According to (2.7):

$$0 \leq q_n - q_{n-1} k_n < k_n.$$

Consequently, series (2.8) is positive and majorizes next to

$$q_0 + 1 + \sum_{n=1}^{\infty} \frac{1}{k_1 \dots k_n},$$

convergence of which is checked by d'Alambert's criterion $\square$

Def. 1.3.6 A sequence $\widetilde{H}$ of κ-clocks:

$$\left\langle \{\mathbf{g}_0, A, \mathbf{b}_0\}, \ \{\mathbf{g}_1, A, \mathbf{b}_2\}, .., \left\{\mathbf{g}_j, A, \mathbf{b}_j\right\}, \ .. \right\rangle$$

is called *an absolutely precise* κ-clock of a recorder $\mathbf{a}$ if for every j exists a natural number k_j so that κ-clock$\left\{\mathbf{g}_j, A, \mathbf{b}_j\right\}$ is k_j times more precise than κ-clock$\left\{\mathbf{g}_{j-1}, A, \mathbf{b}_{j-1}\right\}$.

In this case if

$$q_j = \left[\mathbf{a}^{\bullet} C \uparrow \mathbf{a}, \left\{\mathbf{g}_j, A, \mathbf{b}_j\right\}\right]$$

and

$$t = q_0 + \sum_{j=1}^{\infty} \frac{q_j - q_{j-1} \cdot k_j}{k_1 \cdot k_2 \cdot .. \cdot k_j},$$

then

$$t \text{ is } \left[\mathbf{a}^{\bullet} C \uparrow \mathbf{a}, \widetilde{H}\right].$$

Lm. 1.3.3: If

$$q := q_0 + \sum_{j=1}^{\infty} \frac{q_j - q_{j-1} \cdot k_j}{k_1 \cdot k_2 \cdot .. \cdot k_j} \tag{2.9}$$

with

$$q_{n-1} \leq \frac{q_n}{k_n} < q_{n-1} + 1,$$

and

$$d := d_0 + \sum_{j=1}^{\infty} \frac{d_j - d_{j-1} \cdot k_j}{k_1 \cdot k_2 \cdot .. \cdot k_j} \tag{2.10}$$

with

$$d_{n-1} \leq \frac{d_n}{k_n} < d_{n-1} + 1$$

then if $q_n \leq d_n$ then $q \leq d$.
Proof of Lm. 1.3.3: A partial sum of series (2.9) is the following:

$$Q_u := q_0 + \frac{q_1 - q_0 k_1}{k_1} + \frac{q_2 - q_1 k_2}{k_1 k_2} + \cdots + \frac{q_u - q_{u-1} k_u}{k_1 k_2 \cdots k_u},$$
$$Q_u = q_0 + \frac{q_1}{k_1} - q_0 + \frac{q_2}{k_1 k_2} - \frac{q_1}{k_1} + \cdots + \frac{q_u}{k_1 k_2 \cdots k_u} - \frac{q_{u-1}}{k_1 k_2 \cdots k_{u-1}},$$

$$Q_u = \frac{q_u}{k_1 k_2 \cdots k_u}.$$

A partial sum of series (2.10) is the following:

$$D_u = \frac{d_u}{k_1 k_2 \cdots k_u}.$$

Consequently, according to the condition of Lemma: $Q_n \leq D_n$ □
Lm. 1.3.4 If

$$q \text{ is } \left[\mathbf{a}^\bullet \alpha C \uparrow \mathbf{a}, \widetilde{H}\right],$$
$$d \text{ is } \left[\mathbf{a}^\bullet \beta C \uparrow \mathbf{a}, \widetilde{H}\right],$$

and

$$\alpha \prec \beta$$

then

$$q \leq d.$$

Proof of Lm. 1.3.4 comes out of Lemmas 1.3.1 and 1.3.3 immediately □
Therefore, if $\alpha \sim \beta$ then $q = d$.

2.3. Space

Def. 1.4.1 A number t is called *a time, measured by a recorder* $\mathbf{a}$ *according to a* κ*-clock* $\widetilde{H}$, *during which a signal* C *did a path* $\mathbf{a}^{\bullet}\alpha\mathbf{a}^{\bullet}$ (design.:

$$t := \mathfrak{m}\left(\mathbf{a}\widetilde{H}\right)(\mathbf{a}^{\bullet}\alpha\mathbf{a}^{\bullet}C)),$$

if

$$t = \left[\mathbf{a}^{\bullet}\alpha\mathbf{a}^{\bullet}C \uparrow \mathbf{a}, \widetilde{H}\right] - \left[\mathbf{a}^{\bullet}C \uparrow \mathbf{a}, \widetilde{H}\right].$$

Th. 1.4.1

$$\mathfrak{m}\left(\mathbf{a}\widetilde{H}\right)(\mathbf{a}^{\bullet}\alpha\mathbf{a}^{\bullet}C) \geq 0.$$

Proof comes out straight of Lemma 1.3.4 □

Thus, any "signal", "sent" by the recorder, "will come back" to it not earlier than it was "sent".

Def. 1.4.2

1) for every recorder $\mathbf{a}$: $(\mathbf{a}^{\bullet})^{\dagger} = (\mathbf{a}^{\bullet})$;

2) for all paths α and β: $(\alpha\beta)^{\dagger} = (\beta)^{\dagger}(\alpha)^{\dagger}$.

Def. 1.4.3 A set $\mathfrak{R}$ of recorders is *an internally stationary system* for a recorder $\mathbf{a}$ with a κ-clock $\widetilde{H}$ (design.: $\mathfrak{R}$ is $ISS\left(\mathbf{a}, \widetilde{H}\right)$) if for all sentences B and C, for all elements $\mathbf{a}_1$ and $\mathbf{a}_2$ of set $\mathfrak{R}$, and for all paths α, made of elements of set $\mathfrak{R}$, the following conditions are satisfied:

1) $\left[\mathbf{a}^{\bullet}\mathbf{a}_2^{\bullet}\mathbf{a}_1^{\bullet}C \uparrow \mathbf{a}, \widetilde{H}\right] - \left[\mathbf{a}^{\bullet}\mathbf{a}_1^{\bullet}C \uparrow \mathbf{a}, \widetilde{H}\right] =$
$= \left[\mathbf{a}^{\bullet}\mathbf{a}_2^{\bullet}\mathbf{a}_1^{\bullet}B \uparrow \mathbf{a}, \widetilde{H}\right] - \left[\mathbf{a}^{\bullet}\mathbf{a}_1^{\bullet}B \uparrow \mathbf{a}, \widetilde{H}\right]$;

2) $\mathfrak{m}\left(\mathbf{a}\widetilde{H}\right)(\mathbf{a}^{\bullet}\alpha\mathbf{a}^{\bullet}C) = \mathfrak{m}\left(\mathbf{a}\widetilde{H}\right)\left(\mathbf{a}^{\bullet}\alpha^{\dagger}\mathbf{a}^{\bullet}C\right)$.

Th. 1.4.2

$$\{\mathbf{a}\} - ISS\left(\mathbf{a}, \widetilde{H}\right).$$

Proof:

1)As $\mathbf{a}^{\bullet} \sim \mathbf{a}^{\bullet}\mathbf{a}^{\bullet}$ then, according to Lemma 1.3.4 : if we symbolize

$$p := \left[\mathbf{a}^{\bullet}\mathbf{a}^{\bullet}B \uparrow \mathbf{a}, \widetilde{H}\right],$$
$$q := \left[\mathbf{a}^{\bullet}\mathbf{a}^{\bullet}\mathbf{a}^{\bullet}B \uparrow \mathbf{a}, \widetilde{H}\right],$$
$$r := \left[\mathbf{a}^{\bullet}\mathbf{a}^{\bullet}C \uparrow \mathbf{a}, \widetilde{H}\right],$$
$$s := \left[\mathbf{a}^{\bullet}\mathbf{a}^{\bullet}\mathbf{a}^{\bullet}C \uparrow \mathbf{a}, \widetilde{H}\right],$$

then $q = p$ and $s = r$.

That's why $q - p = s - r$.

2) Since any series α, made of elements of set $\{\mathbf{a}\}$ coincides with $\alpha^{\dagger}$ then

$$\mathfrak{m}\left(\mathbf{a}\widetilde{H}\right)\left(\mathbf{a}^{\bullet}\alpha\mathbf{a}^{\bullet}C\right)=\mathfrak{m}\left(\mathbf{a}\widetilde{H}\right)\left(\mathbf{a}^{\bullet}\alpha^{\dagger}\mathbf{a}^{\bullet}C\right).\ \square$$

Thus every singleton is an internally stationary systeminternally stationary system.
Lm. 1.4.1: If $\{\mathbf{a},\mathbf{a}_1,\mathbf{a}_2\}$ is$ISS\left(\mathbf{a},\widetilde{H}\right)$ then

$$\begin{aligned}&\left[\mathbf{a}^{\bullet}\mathbf{a}_2^{\bullet}\mathbf{a}_1^{\bullet}\mathbf{a}_2^{\bullet}C\uparrow\mathbf{a},\widetilde{H}\right]-\left[\mathbf{a}^{\bullet}\mathbf{a}_2^{\bullet}C\uparrow\mathbf{a},\widetilde{H}\right]=\\ &=\left[\mathbf{a}^{\bullet}\mathbf{a}_1^{\bullet}\mathbf{a}_2^{\bullet}\mathbf{a}_1^{\bullet}B\uparrow\mathbf{a},\widetilde{H}\right]-\left[\mathbf{a}^{\bullet}\mathbf{a}_1^{\bullet}B\uparrow\mathbf{a},\widetilde{H}\right]\end{aligned}$$

Proof: Let's symbolize

$$\begin{aligned}p&:=\left[\mathbf{a}^{\bullet}\mathbf{a}_1^{\bullet}B\uparrow\mathbf{a},\widetilde{H}\right],\\ q&:=\left[\mathbf{a}^{\bullet}\mathbf{a}_1^{\bullet}\mathbf{a}_2^{\bullet}\mathbf{a}_1^{\bullet}B\uparrow\mathbf{a},\widetilde{H}\right],\\ r&:=\left[\mathbf{a}^{\bullet}\mathbf{a}_2^{\bullet}C\uparrow\mathbf{a},\widetilde{H}\right],\\ s&:=\left[\mathbf{a}^{\bullet}\mathbf{a}_2^{\bullet}\mathbf{a}_1^{\bullet}\mathbf{a}_2^{\bullet}C\uparrow\mathbf{a},\widetilde{H}\right],\\ u&:=\left[\mathbf{a}^{\bullet}\mathbf{a}_2^{\bullet}\mathbf{a}_1^{\bullet}B\uparrow\mathbf{a},\widetilde{H}\right],\\ w&:=\left[\mathbf{a}^{\bullet}\mathbf{a}_1^{\bullet}\mathbf{a}_2^{\bullet}C\uparrow\mathbf{a},\widetilde{H}\right].\end{aligned}$$

Thus, according to statement 1.4.3

$$u-p=s-w,w-r=q-u.$$

Thus,

$$s-r=q-p$$

$\square$
Def. 1.4.4 A number l is called *an* $\mathbf{a}\widetilde{H}(B)$*-measure* of recorders $\mathbf{a}_1$ and $\mathbf{a}_2$ (design.:

$$l=\ell\left(\mathbf{a},\widetilde{H},B\right)\left(\mathbf{a}_1,\mathbf{a}_2\right)$$

if

$$l=0.5\cdot\left(\left[\mathbf{a}^{\bullet}\mathbf{a}_1^{\bullet}\mathbf{a}_2^{\bullet}\mathbf{a}_1^{\bullet}B\uparrow\mathbf{a},\widetilde{H}\right]-\left[\mathbf{a}^{\bullet}\mathbf{a}_1^{\bullet}B\uparrow\mathbf{a},\widetilde{H}\right]\right).$$

Lm. 1.4.2 If $\{\mathbf{a},\mathbf{a}_1,\mathbf{a}_2\}$ is $ISS\left(\mathbf{a},\widetilde{H}\right)$ then for all B and C:

$$\ell\left(\mathbf{a},\widetilde{H},B\right)\left(\mathbf{a}_1,\mathbf{a}_2\right)=\ell\left(\mathbf{a},\widetilde{H},C\right)\left(\mathbf{a}_1,\mathbf{a}_2\right).$$

Proof: Let us designate: Let us design:

$$p := \left[\mathbf{a}^\bullet \mathbf{a}_1^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right],$$
$$q := \left[\mathbf{a}^\bullet \mathbf{a}_1^\bullet \mathbf{a}_2^\bullet \mathbf{a}_1^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right],$$
$$r := \left[\mathbf{a}^\bullet \mathbf{a}_1^\bullet C \uparrow \mathbf{a}, \widetilde{H}\right],$$
$$s := \left[\mathbf{a}^\bullet \mathbf{a}_1^\bullet \mathbf{a}_2^\bullet \mathbf{a}_1^\bullet C \uparrow \mathbf{a}, \widetilde{H}\right],$$
$$u := \left[\mathbf{a}^\bullet \mathbf{a}_2^\bullet \mathbf{a}_1^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right],$$
$$w := \left[\mathbf{a}^\bullet \mathbf{a}_2^\bullet \mathbf{a}_1^\bullet C \uparrow \mathbf{a}, \widetilde{H}\right].$$

Thus, according to Def. 1.4.3:

$$u - p = w - r, q - u = s - w.$$

Thus,

$$q - p = s - r$$

□

Therefore, one can write expression of form "$\ell\left(\mathbf{a}, \widetilde{H}, B\right)(\mathbf{a}_1, \mathbf{a}_2)$" as the following: "$\ell\left(\mathbf{a}, \widetilde{H}\right)(\mathbf{a}_1, \mathbf{a}_2)$".

Th. 1.4.3: If $\{\mathbf{a}, \mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ is $ISS\left(\mathbf{a}, \widetilde{H}\right)$ then

1) $\ell\left(\mathbf{a}, \widetilde{H}\right)(\mathbf{a}_1, \mathbf{a}_2) \geq 0$;

2) $\ell\left(\mathbf{a}, \widetilde{H}\right)(\mathbf{a}_1, \mathbf{a}_1) = 0$;

3) $\ell\left(\mathbf{a}, \widetilde{H}\right)(\mathbf{a}_1, \mathbf{a}_2) = \ell\left(\mathbf{a}, \widetilde{H}\right)(\mathbf{a}_2, \mathbf{a}_1)$;

4) $\ell\left(\mathbf{a}, \widetilde{H}\right)(\mathbf{a}_1, \mathbf{a}_2) + \ell\left(\mathbf{a}, \widetilde{H}\right)(\mathbf{a}_2, \mathbf{a}_3) \geq \ell\left(\mathbf{a}, \widetilde{H}\right)(\mathbf{a}_1, \mathbf{a}_3)$.

Proof: 1) and 2) come out straight from Lemma 1.3.4 and 3) from Lemma 1.4.2.

Let's symbolize

$$p := \left[\mathbf{a}^\bullet \mathbf{a}_1^\bullet C \uparrow \mathbf{a}, \widetilde{H}\right],$$
$$q := \left[\mathbf{a}^\bullet \mathbf{a}_1^\bullet \mathbf{a}_2^\bullet \mathbf{a}_1^\bullet C \uparrow \mathbf{a}, \widetilde{H}\right],$$
$$r := \left[\mathbf{a}^\bullet \mathbf{a}_1^\bullet \mathbf{a}_3^\bullet \mathbf{a}_1^\bullet C \uparrow \mathbf{a}, \widetilde{H}\right],$$
$$s := \left[\mathbf{a}^\bullet \mathbf{a}_2^\bullet \mathbf{a}_1^\bullet C \uparrow \mathbf{a}, \widetilde{H}\right],$$
$$u := \left[\mathbf{a}^\bullet \mathbf{a}_2^\bullet \mathbf{a}_3^\bullet \mathbf{a}_2^\bullet \mathbf{a}_1^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right],$$
$$w = \left[\mathbf{a}^\bullet \mathbf{a}_1^\bullet \mathbf{a}_2^\bullet \mathbf{a}_3^\bullet \mathbf{a}_2^\bullet \mathbf{a}_1^\bullet C \uparrow \mathbf{a}, \widetilde{H}\right].$$

Thus, according to statement 1.4.3

$$w - u = q - s.$$

Therefore,

$$w - p = (q - p) + (u - s).$$

According to Lemma 1.3.4

$$w \geq r.$$

Consequently,

$$(q - p) + (u - s) \geq r - p$$

□

Thus, all four axioms of the metrical space [?] are accomplished for $\ell\left(\mathbf{a}, \widetilde{H}\right)$ in an internally stationary systeminternally stationary system of recorders.

Consequently, $\ell\left(\mathbf{a}, \widetilde{H}\right)$ is a distance length similitude in this space.

Def. 1.4.5 A set $\mathfrak{R}$ of recorders is *degenerated into a beam* $\mathbf{ab}_1$ *and point* $\mathbf{a}_1$ if there exists C such that the following conditions are satisfied:

1) For any sequence α, made of elements of set $\mathfrak{R}$, and for any K: if $\mathbf{a}^\bullet K$ then

$$\mathbf{a}^\bullet \left(K \& \left(\alpha \mathbf{a}_1^\bullet C \Rightarrow \alpha \mathbf{b}_1^\bullet \mathbf{a}_1^\bullet C\right)\right).$$

2) There is sequence β, made of elements of the set$\mathfrak{R}$, and there exist sentence S such that $\mathbf{a}^\bullet \left(\beta \mathbf{b}_1^\bullet C \& S\right)$ and it's false that $\mathbf{a}^\bullet \left(\beta \mathbf{a}_1^\bullet \mathbf{b}_1^\bullet C \& S\right)$

Further we'll consider only not degenerated sets of recorders.

Def. 1.4.6: B took place *in the same place as* $\mathbf{a}_1$ *for* $\mathbf{a}$ (design.: $\natural(\mathbf{a})(\mathbf{a}_1, B)$) if for every sequence α and for any sentence K the following condition is satisfied:

if $\mathbf{a}^\bullet K$ then $\mathbf{a}^\bullet \left(K \& \left(\alpha B \Rightarrow \alpha \mathbf{a}_1^\bullet B\right)\right)$.

Th. 1.4.4:

$$\natural(\mathbf{a})\left(\mathbf{a}_1, \mathbf{a}_1^\bullet B\right).$$

Proof: Since $\alpha \mathbf{a}_1^\bullet \sim \alpha \mathbf{a}_1^\bullet \mathbf{a}_1^\bullet$ then according to III: if $\mathbf{a}_1^\bullet K$ then

$$\mathbf{a}_1^\bullet \left(K \& \left(\alpha \mathbf{a}_1^\bullet B \Rightarrow \alpha \mathbf{a}_1^\bullet \mathbf{a}_1^\bullet B\right)\right)$$

□

Th. 1.4.5: If

$$\natural(\mathbf{a})(\mathbf{a}_1, B), \tag{2.11}$$

$$\natural(\mathbf{a})(\mathbf{a}_2, B), \tag{2.12}$$

then

$$\natural(\mathbf{a})(\mathbf{a}_2, \mathbf{a}_1^\bullet B).$$

Proof: Let $\mathbf{a}^\bullet K$.
In this case from (2.12):

$$\mathbf{a}^\bullet (K\&(\alpha\mathbf{a}_1^\bullet B \Rightarrow \alpha\mathbf{a}_1^\bullet\mathbf{a}_2^\bullet B)).$$

From (2.11):

$$\mathbf{a}^\bullet ((K\&(\alpha\mathbf{a}_1^\bullet B \Rightarrow \alpha\mathbf{a}_1^\bullet\mathbf{a}_2^\bullet B))\&(\alpha\mathbf{a}_1^\bullet\mathbf{a}_2^\bullet B \Rightarrow \alpha\mathbf{a}_1^\bullet\mathbf{a}_2^\bullet\mathbf{a}_1^\bullet B)).$$

According to II:

$$\mathbf{a}^\bullet (K\&(\alpha\mathbf{a}_1^\bullet B \Rightarrow \alpha\mathbf{a}_1^\bullet\mathbf{a}_2^\bullet\mathbf{a}_1^\bullet B)).$$

According to III:

$$\mathbf{a}^\bullet ((K\&(\alpha\mathbf{a}_1^\bullet B \Rightarrow \alpha\mathbf{a}_1^\bullet\mathbf{a}_2^\bullet\mathbf{a}_1^\bullet B))\&(\alpha\mathbf{a}_1^\bullet\mathbf{a}_2^\bullet\mathbf{a}_1^\bullet B \Rightarrow \alpha\mathbf{a}_2^\bullet\mathbf{a}_1^\bullet B)).$$

According to II:

$$\mathbf{a}^\bullet (K\&(\alpha\mathbf{a}_1^\bullet B \Rightarrow \alpha\mathbf{a}_2^\bullet\mathbf{a}_1^\bullet B))$$

□

Lm. 1.4.3: If

$$\natural(\mathbf{a})(\mathbf{a}_1, B), \tag{2.13}$$

$$t = \left[\mathbf{a}^\bullet\alpha B \uparrow \mathbf{a}, \widetilde{H}\right], \tag{2.14}$$

then

$$t = \left[\mathbf{a}^\bullet\alpha\mathbf{a}_1^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right].$$

Proof: Let's symbolize:

$$t_j := \left[\mathbf{a}^\bullet\alpha B \uparrow \mathbf{a}, \left\{\mathbf{g}_j, A, \mathbf{b}_j\right\}\right].$$

Therefore,

$$\mathbf{a}^\bullet \left(\mathbf{a}^\bullet\alpha B\&\left(\neg\mathbf{a}^\bullet\left(\mathbf{g}_j^\bullet\mathbf{b}_j^\bullet\right)^{t_j+1}\mathbf{g}_j^\bullet A\right)\right),$$

from (2.13):

$$\mathbf{a}^\bullet \left(\left(\mathbf{a}^\bullet\alpha B\&\left(\neg\mathbf{a}^\bullet\left(\mathbf{g}_j^\bullet\mathbf{b}_j^\bullet\right)^{t_j+1}\mathbf{g}_j^\bullet A\right)\right)\&(\mathbf{a}^\bullet\alpha B \Rightarrow \mathbf{a}^\bullet\alpha\mathbf{a}_1^\bullet B)\right).$$

According to II:

$$\mathbf{a}^\bullet \left(\mathbf{a}^\bullet\alpha\mathbf{a}_1^\bullet B\&\left(\neg\mathbf{a}^\bullet\left(\mathbf{g}_j^\bullet\mathbf{b}_j^\bullet\right)^{t_j+1}\mathbf{g}_j^\bullet A\right)\right), \tag{2.15}$$

Let $\mathbf{a}^{\bullet}K$. In this case from (2.14):

$$\mathbf{a}^{\bullet}\left(K\&\left(\mathbf{a}^{\bullet}\alpha B \Rightarrow \mathbf{a}^{\bullet}\left(\mathbf{g}_j^{\bullet}\mathbf{b}_j^{\bullet}\right)^{t_j}\mathbf{g}_j^{\bullet}A\right)\right).$$

Therefore, according to III:

$$\mathbf{a}^{\bullet}\left(\left(K\&\left(\mathbf{a}^{\bullet}\alpha B \Rightarrow \mathbf{a}^{\bullet}\left(\mathbf{g}_j^{\bullet}\mathbf{b}_j^{\bullet}\right)^{t_j}\mathbf{g}_j^{\bullet}A\right)\right)\&\left(\mathbf{a}^{\bullet}\alpha\mathbf{a}_1^{\bullet}B \Rightarrow \mathbf{a}^{\bullet}\alpha B\right)\right).$$

According to II:

$$\mathbf{a}^{\bullet}\left(K\&\left(\mathbf{a}^{\bullet}\alpha\mathbf{a}_1^{\bullet}B \Rightarrow \mathbf{a}^{\bullet}\left(\mathbf{g}_j^{\bullet}\mathbf{b}_j^{\bullet}\right)^{t_j}\mathbf{g}_j^{\bullet}A\right)\right). \tag{2.16}$$

From (2.13):

$$\mathbf{a}^{\bullet}\left(\left(K\&\left(\mathbf{a}^{\bullet}\left(\mathbf{g}_j^{\bullet}\mathbf{b}_j^{\bullet}\right)^{t_j+1}\mathbf{g}_j^{\bullet}A \Rightarrow \mathbf{a}^{\bullet}\alpha B\right)\right)\&\left(\mathbf{a}^{\bullet}\alpha B \Rightarrow \mathbf{a}^{\bullet}\alpha\mathbf{a}_1^{\bullet}B\right)\right).$$

according to II:

$$\mathbf{a}^{\bullet}\left(K\&\left(\mathbf{a}^{\bullet}\left(\mathbf{g}_j^{\bullet}\mathbf{b}_j^{\bullet}\right)^{t_j+1}\mathbf{g}_j^{\bullet}A \Rightarrow \mathbf{a}^{\bullet}\alpha\mathbf{a}_1^{\bullet}B\right)\right).$$

From (2.15), (2.16) for all j:

$$t_j = \left[\mathbf{a}^{\bullet}\alpha\mathbf{a}_1^{\bullet}B \uparrow \mathbf{a}, \left\{\mathbf{g}_j, A, \mathbf{b}_j\right\}\right].$$

Consequently,

$$t \;=\; \left[\mathbf{a}^{\bullet}\alpha\mathbf{a}_1^{\bullet}B \uparrow \mathbf{a}, \widetilde{H}\right]$$

□

Th. 1.4.6: If $\{\mathbf{a}, \mathbf{a}_1, \mathbf{a}_2\}$ is $ISS\left(\mathbf{a}, \widetilde{H}\right)$,

$$\natural\left(\mathbf{a}\right)\left(\mathbf{a}_1, B\right), \tag{2.17}$$

$$\natural\left(\mathbf{a}\right)\left(\mathbf{a}_2, B\right), \tag{2.18}$$

then

$$\ell\left(\mathbf{a}, \widetilde{H}\right)\left(\mathbf{a}_1, \mathbf{a}_2\right) = 0.$$

Proof: Let's symbolize:

$$t := \left[\mathbf{a}^{\bullet}B \uparrow \mathbf{a}, \widetilde{H}\right].$$

According to Lemma 1.4.3:
from (2.17):

$$t \;=\; \left[\mathbf{a}^{\bullet}\mathbf{a}_1^{\bullet}B \uparrow \mathbf{a}, \widetilde{H}\right],$$

from (2.18):

$$t=\left[\mathbf{a}^{\bullet}\mathbf{a}_1^{\bullet}\mathbf{a}_2^{\bullet}B\uparrow\mathbf{a},\widetilde{H}\right],$$

again from (2.17):

$$t=\left[\mathbf{a}^{\bullet}\mathbf{a}_1^{\bullet}\mathbf{a}_2^{\bullet}\mathbf{a}_1^{\bullet}B\uparrow\mathbf{a},\widetilde{H}\right].$$

Consequently,

$$\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a}_1,\mathbf{a}_2)=0.5\cdot(t-t)=0$$

□

Th. 1.4.7: If $\{\mathbf{a}_1,\mathbf{a}_2,\mathbf{a}_3\}$ is $ISS\left(\mathbf{a},\widetilde{H}\right)$ and there exists sentence B such that

$$\natural(\mathbf{a})(\mathbf{a}_1,B), \tag{2.19}$$

$$\natural(\mathbf{a})(\mathbf{a}_2,B), \tag{2.20}$$

then

$$\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a}_3,\mathbf{a}_2)=\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a}_3,\mathbf{a}_1).$$

Proof: According to Theorem 1.4.6 from (2.19) and (2.20):

$$\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a}_1,\mathbf{a}_2)=0; \tag{2.21}$$

according to Theorem 1.4.3:

$$\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a}_1,\mathbf{a}_2)+\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a}_2,\mathbf{a}_3)\geq\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a}_1,\mathbf{a}_3),$$

therefore, from (2.21):

$$\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a}_2,\mathbf{a}_3)\geq\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a}_1,\mathbf{a}_3),$$

i.e. according to Theorem 1.4.3:

$$\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a}_3,\mathbf{a}_2)\geq\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a}_1,\mathbf{a}_3). \tag{2.22}$$

From

$$\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a}_3,\mathbf{a}_1)+\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a}_1,\mathbf{a}_2)\geq\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a}_3,\mathbf{a}_2):$$

and from (2.21):

$$\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a}_3,\mathbf{a}_1)\geq\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a}_3,\mathbf{a}_2).$$

From (2.22):

$$\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a}_3,\mathbf{a}_1)=\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a}_3,\mathbf{a}_2)$$

$\square$

Def. 1.4.7 A real number t is an instant of a sentence B in *frame of reference* $\left(\mathfrak{R}\mathbf{a}\widetilde{H}\right)$ (design.: $t=\left[B\mid\mathfrak{R}\mathbf{a}\widetilde{H}\right]$) if

1) $\mathfrak{R}$ is *ISS* $\left(\mathbf{a},\widetilde{H}\right)$,

2) there exists a recorder $\mathbf{b}$ so that $\mathbf{b}\in\mathfrak{R}$ and $\natural(\mathbf{a})(\mathbf{b},B)$,

3) $t=\left[\mathbf{a}^{\bullet}B\uparrow\mathbf{a},\widetilde{H}\right]-\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a},\mathbf{b})$.

Lm. 1.4.4:

$$\left[\mathbf{a}^{\bullet}B\uparrow\mathbf{a},\widetilde{H}\right]=\left[\mathbf{a}^{\bullet}B\mid\mathfrak{R}\mathbf{a}\widetilde{H}\right].$$

Proof: Let $\mathfrak{R}$ is *ISS* $\left(\mathbf{a},\widetilde{H}\right)$, $\mathbf{a}_1\in\mathfrak{R}$ and

$$\natural(\mathbf{a})(\mathbf{a}_1,\mathbf{a}^{\bullet}B). \tag{2.23}$$

According to Theorem 1.4.4:

$$\natural(\mathbf{a})(\mathbf{a},\mathbf{a}^{\bullet}B).$$

From (2.23) according to Theorem 1.4.6:

$$\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a},\mathbf{a}_1)=0,$$

therefore

$$\left[\mathbf{a}^{\bullet}B\mid\mathfrak{R}\mathbf{a}\widetilde{H}\right]=\left[\mathbf{a}^{\bullet}B\uparrow\mathbf{a},\widetilde{H}\right]-\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a},\mathbf{a}_1)=\left[\mathbf{a}^{\bullet}B\uparrow\mathbf{a},\widetilde{H}\right]$$

$\square$

Def. 1.4.8 A real number z is *a distance length* between B and C in a frame of reference $\left(\mathfrak{R}\mathbf{a}\widetilde{H}\right)$ (design.: $z=\ell\left(\mathfrak{R}\mathbf{a}\widetilde{H}\right)(B,C)$) if

1) $\mathfrak{R}$ is *ISS* $\left(\mathbf{a},\widetilde{H}\right)$,

2) there exist recorders $\mathbf{a}_1$ and $\mathbf{a}_2$ so that $\mathbf{a}_1\in\mathfrak{R}$, $\mathbf{a}_2\in\mathfrak{R}$, $\natural(\mathbf{a})(\mathbf{a}_1,B))$ and $\natural(\mathbf{a})(\mathbf{a}_2,C))$,

3) $z=\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a}_2,\mathbf{a}_1)$.

According to Theorem 1.4.3 such distance length satisfies conditions of all axioms of a metric space.

2.4. Relativity

Def. 1.5.1: Recorders $\mathbf{a}_1$ and $\mathbf{a}_2$ *equally receive a signal* about B for a recorder $\mathbf{a}$ if

$$\ll\natural(\mathbf{a})(\mathbf{a}_2,\mathbf{a}_1^{\bullet}B)\gg=\ll\natural(\mathbf{a})(\mathbf{a}_1,\mathbf{a}_2^{\bullet}B)\gg.$$

Def. 1.5.2: Set of recorders are called *a homogeneous space of recorders*, if all its elements equally receive all signals.

Def. 1.5.3: A real number c is *an information velocity about B to the recorder* $\mathbf{a}_1$ in a frame of reference $\left(\Re\mathbf{a}\widetilde{H}\right)$ if

$$c = \frac{\ell\left(\Re\mathbf{a}\widetilde{H}\right)(B, \mathbf{a}_1^{\bullet}B)}{\left[\mathbf{a}_1^{\bullet}B \mid \Re\mathbf{a}\widetilde{H}\right] - \left[B \mid \Re\mathbf{a}\widetilde{H}\right]}.$$

Th. 1.5.1: In all homogeneous spaces:

$$c = 1.$$

Proof: Let c represents information velocity about B to a recorder $\mathbf{a}_1$ in a frame of reference $\left(\Re\mathbf{a}\widetilde{H}\right)$.

Thus, if

$$\Re \text{ is } ISS\left(\mathbf{a}, \widetilde{H}\right),$$

$$z := \ell\left(\Re\mathbf{a}\widetilde{H}\right)(B, \mathbf{a}_1^{\bullet}B), \tag{2.24}$$

$$t_1 := \left[B \mid \Re\mathbf{a}\widetilde{H}\right], \tag{2.25}$$

$$t_2 := \left[\mathbf{a}_1^{\bullet}B \mid \Re\mathbf{a}\widetilde{H}\right], \tag{2.26}$$

then

$$c = \frac{z}{t_2 - t_1}. \tag{2.27}$$

According to (2.24) there exist elements $\mathbf{b}_1$ and $\mathbf{b}_2$ of set $\Re$ such that:

$$\natural(\mathbf{a})(\mathbf{b}_1, B), \tag{2.28}$$

$$\natural(\mathbf{a})(\mathbf{b}_2, \mathbf{a}_2^{\bullet}B), \tag{2.29}$$

$$z = \ell\left(\mathbf{a}, \widetilde{H}\right)(\mathbf{b}_1, \mathbf{b}_2). \tag{2.30}$$

According to (2.25) and (2.26) there exist elements $\mathbf{b}_1'$ and $\mathbf{b}_2'$ of set $\Re$ such that:

$$\natural(\mathbf{a})\left(\mathbf{b}_1', B\right), \tag{2.31}$$

$$\natural(\mathbf{a})\left(\mathbf{b}_2', \mathbf{a}_2^{\bullet}B\right), \tag{2.32}$$

$$t_1 = \left[\mathbf{a}^{\bullet}B \uparrow \mathbf{a}, \widetilde{H}\right] - \ell\left(\mathbf{a}, \widetilde{H}\right)\left(\mathbf{a}, \mathbf{b}_1'\right), \tag{2.33}$$

$$t_2 = \left[\mathbf{a}^\bullet \mathbf{a}_2^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right] - \ell\left(\mathbf{a}, \widetilde{H}\right)\left(\mathbf{a}, \mathbf{b}_2'\right). \tag{2.34}$$

From (2.24), (2.28), (2.31) according to Theorem 1.4.7:

$$\ell\left(\mathbf{a}, \widetilde{H}\right)(\mathbf{a}, \mathbf{b}_1) = \ell\left(\mathbf{a}, \widetilde{H}\right)\left(\mathbf{a}, \mathbf{b}_1'\right). \tag{2.35}$$

Analogously from (2.24), (2.29), (2.32):

$$\ell\left(\mathbf{a}, \widetilde{H}\right)(\mathbf{a}, \mathbf{b}_2) = \ell\left(\mathbf{a}, \widetilde{H}\right)\left(\mathbf{a}, \mathbf{b}_2'\right). \tag{2.36}$$

Analogously from (2.33), (2.28), (2.35)according to Lemma 1.4.3:

$$t_1 = \left[\mathbf{a}^\bullet \mathbf{b}_1^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right] - \ell\left(\mathbf{a}, \widetilde{H}\right)(\mathbf{a}, \mathbf{b}_1). \tag{2.37}$$

From (2.29) according to Lemma 1.4.3:

$$\left[\mathbf{a}^\bullet \mathbf{a}_2^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right] = \left[\mathbf{a}^\bullet \mathbf{b}_2^\bullet \mathbf{a}_2^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right]. \tag{2.38}$$

According to Lemma 1.3.4:

$$\left[\mathbf{a}^\bullet \mathbf{b}_2^\bullet \mathbf{a}_2^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right] \geq \left[\mathbf{a}^\bullet \mathbf{b}_2^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right]. \tag{2.39}$$

From (2.29):

$$\natural(\mathbf{a})(\mathbf{a}_2, \mathbf{b}_2^\bullet B).$$

According to Lemma 1.4.3

$$\left[\mathbf{a}^\bullet \mathbf{a}_2^\bullet \mathbf{b}_2^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right] = \left[\mathbf{a}^\bullet \mathbf{b}_2^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right]. \tag{2.40}$$

Again according to Lemma 1.3.4:

$$\left[\mathbf{a}^\bullet \mathbf{a}_2^\bullet \mathbf{b}_2^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right] \geq \left[\mathbf{a}^\bullet \mathbf{a}_2^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right].$$

From (2.40), (2.38), (2.39):

$$\left[\mathbf{a}^\bullet \mathbf{a}_2^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right] \geq \left[\mathbf{a}^\bullet \mathbf{b}_2^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right] \geq \left[\mathbf{a}^\bullet \mathbf{a}_2^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right],$$

therefore,

$$\left[\mathbf{a}^\bullet \mathbf{a}_2^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right] = \left[\mathbf{a}^\bullet \mathbf{b}_2^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right].$$

From (2.34), (2.36):

$$t_2 = \left[\mathbf{a}^\bullet \mathbf{b}_2^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right] - \ell\left(\mathbf{a}, \widetilde{H}\right)(\mathbf{a}, \mathbf{b}_2).$$

From (2.28) according to Lemma 1.4.3

$$t_2 = \left[\mathbf{a}^\bullet \mathbf{b}_2^\bullet \mathbf{b}_1^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right] - \ell\left(\mathbf{a}, \widetilde{H}\right)(\mathbf{a}, \mathbf{b}_2). \tag{2.41}$$

Let's symbolize

$$u := \left[\mathbf{a}^{\bullet} C \uparrow \mathbf{a}, \widetilde{H}\right], \tag{2.42}$$

$$d := \left[\mathbf{a}^{\bullet} \mathbf{b}_1^{\bullet} \mathbf{a}^{\bullet} C \uparrow \mathbf{a}, \widetilde{H}\right], \tag{2.43}$$

$$w := \left[\mathbf{a}^{\bullet} \mathbf{b}_2^{\bullet} \mathbf{a}^{\bullet} C \uparrow \mathbf{a}, \widetilde{H}\right], \tag{2.44}$$

$$j := \left[\mathbf{a}^{\bullet} \mathbf{b}_2^{\bullet} \mathbf{b}_1^{\bullet} \mathbf{a}^{\bullet} C \uparrow \mathbf{a}, \widetilde{H}\right], \tag{2.45}$$

$$q := \left[\mathbf{a}^{\bullet} \mathbf{b}_1^{\bullet} \mathbf{b}_2^{\bullet} \mathbf{a}^{\bullet} C \uparrow \mathbf{a}, \widetilde{H}\right],$$

$$p := \left[\mathbf{a}^{\bullet} \mathbf{b}_1^{\bullet} \mathbf{b}_2^{\bullet} \mathbf{b}_1^{\bullet} \mathbf{a}^{\bullet} C \uparrow \mathbf{a}, \widetilde{H}\right], \tag{2.46}$$

$$r := \left[\mathbf{a}^{\bullet} \mathbf{b}_2^{\bullet} \mathbf{b}_1^{\bullet} \mathbf{b}_2^{\bullet} \mathbf{a}^{\bullet} C \uparrow \mathbf{a}, \widetilde{H}\right].$$

Since $\mathfrak{R}$ is $ISS\left(\mathbf{a}, \widetilde{H}\right)$ then

$$q - w = p - j, \tag{2.47}$$

$$j = q. \tag{2.48}$$

From (2.41), (2.37), (2.43), (2.45):

$$\left(t_2 + \ell\left(\mathbf{a}, \widetilde{H}\right)(\mathbf{a}, \mathbf{b}_2)\right) - \left(t_1 + \ell\left(\mathbf{a}, \widetilde{H}\right)(\mathbf{a}, \mathbf{b}_1)\right) = j - d,$$

therefore

$$t_2 - t_1 = j - d - \ell\left(\mathbf{a}, \widetilde{H}\right)(\mathbf{a}, \mathbf{b}_2) + \ell\left(\mathbf{a}, \widetilde{H}\right)(\mathbf{a}, \mathbf{b}_1). \tag{2.49}$$

From (2.42), (2.43), (2.44) according to Lemma 1.3.4:

$$\ell\left(\mathbf{a}, \widetilde{H}\right)(\mathbf{a}, \mathbf{b}_2) = 0.5 \cdot (w - u), \ell\left(\mathbf{a}, \widetilde{H}\right)(\mathbf{a}, \mathbf{b}_1) = 0.5 \cdot (d - u).$$

From (2.47), (2.48), (2.49):

$$t_2 - t_1 = 0.5 \cdot ((j - d) + (j - w)) = 0.5 \cdot (j - d + p - j) = 0.5 \cdot (p - d).$$

From (2.46), (2.43), (2.30):

$$z = 0.5 \cdot (p - d).$$

Consequently

$$z = t_2 - t_1$$

□

That is in every homogenous space a propagation velocity of every information to every recorder for every frame reference equals to 1.

Th. 1.5.2: If $\mathfrak{R}$ is a homogeneous space, then

$$\left[\mathbf{a}_1^{\bullet}B \mid \Re\mathbf{a}\widetilde{H}\right] \geq \left[B \mid \Re\mathbf{a}\widetilde{H}\right].$$

Proof comes out straight from Theorem 1.5.1.

Consequently, in any homogeneous space any recorder finds out that B "took place" not earlier than B "actually take place". "Time" is irreversible.

Th. 1.5.3 If $\mathbf{a}_1$ and $\mathbf{a}_2$ are elements of $\Re$,

$$\Re isISS\left(\mathbf{a},\widetilde{H}\right), \tag{2.50}$$

$$p := \left[\mathbf{a}_1^{\bullet}B \mid \Re\mathbf{a}\widetilde{H}\right], \tag{2.51}$$

$$q := \left[\mathbf{a}_2^{\bullet}\mathbf{a}_1^{\bullet}B \mid \Re\mathbf{a}\widetilde{H}\right], \tag{2.52}$$

$$z := \ell\left(\Re\mathbf{a}\widetilde{H}\right)(\mathbf{a}_1,\mathbf{a}_2),$$

then

$$z = q - p.$$

Proof: In accordance with Theorem 1.5.1
from (2.50), (2.51), (2.52):

$$q - p = \ell\left(\Re\mathbf{a}\widetilde{H}\right)(\mathbf{a}_1^{\bullet}B, \mathbf{a}_2^{\bullet}\mathbf{a}_1^{\bullet}B),$$

thus in accordance with Definition 1.4.8 there exist elements $\mathbf{b}_1$ and $\mathbf{b}_2$ of $\Re$ such that

$$\natural(\mathbf{a})(\mathbf{b}_1, \mathbf{a}_1^{\bullet}B), \tag{2.53}$$

$$\natural(\mathbf{a})(\mathbf{b}_2, \mathbf{a}_2^{\bullet}\mathbf{a}_1^{\bullet}B), \tag{2.54}$$

$$q - p = \ell\left(\Re\mathbf{a}\widetilde{H}\right)(\mathbf{b}_1, \mathbf{b}_2).$$

Moreover, in accordance with Theorem 1.4.4

$$\natural(\mathbf{a})(\mathbf{a}_1^{\bullet}, \mathbf{a}_1^{\bullet}B), \tag{2.55}$$

$$\natural(\mathbf{a})(\mathbf{a}_2^{\bullet}, \mathbf{a}_2^{\bullet}\mathbf{a}_1^{\bullet}B).$$

From (2.54) in accordance with Theorem 1.4.7:

$$\ell\left(\Re\mathbf{a}\widetilde{H}\right)(\mathbf{b}_1, \mathbf{b}_2) = \ell\left(\Re\mathbf{a}\widetilde{H}\right)(\mathbf{b}_1, \mathbf{a}_2). \tag{2.56}$$

In accordance with Theorem 1.4.3:

$$\ell\left(\Re\mathbf{a}\widetilde{H}\right)(\mathbf{b}_1, \mathbf{a}_2) = \ell\left(\Re\mathbf{a}\widetilde{H}\right)(\mathbf{a}_2, \mathbf{b}_1). \tag{2.57}$$

Again in accordance with Theorem 1.4.7 from (2.55), (2.53):

$$\ell\left(\Re\mathbf{a}\widetilde{H}\right)(\mathbf{a}_2,\mathbf{b}_1)=\ell\left(\Re\mathbf{a}\widetilde{H}\right)(\mathbf{a}_2,\mathbf{a}_1). \tag{2.58}$$

Again in accordance with Theorem 1.4.3:

$$\ell\left(\Re\mathbf{a}\widetilde{H}\right)(\mathbf{a}_2,\mathbf{a}_1)=\ell\left(\Re\mathbf{a}\widetilde{H}\right)(\mathbf{a}_1\mathbf{a}_2).$$

From (2.58), (2.57), (2.56):

$$\ell\left(\Re\mathbf{a}\widetilde{H}\right)(\mathbf{b}_1,\mathbf{b}_2)=\ell\left(\Re\mathbf{a}\widetilde{H}\right)(\mathbf{a}_1\mathbf{a}_2)$$

□

According to Urysohn‘s theorem[1] [6]: any homogeneous space is homeomorphic to some set of points of real Hilbert[2] space. If this homeomorphism is not Identical transformation, then $\Re$ will represent a non- Euclidean space.

In this case in this ”space-time” corresponding variant of General Relativity Theory can be constructed. Otherwise, $\Re$ is Euclidean space. In this case there exists *coordinates system* R^μ such that the following condition is satisfied: for all elements $\mathbf{a}_1$ and $\mathbf{a}_2$ of set $\Re$ there exist points $\mathbf{x}_1$ and $\mathbf{x}_2$ of system R^μ such that

$$\ell\left(\mathbf{a},\widetilde{H}\right)(\mathbf{a}_k,\mathbf{a}_s)=\left(\sum_{j=1}^{\mu}\left(x_{s,j}-x_{k,j}\right)^2\right)^{0.5}.$$

In this case R^μ is called *a coordinates system of frame of reference* $\left(\Re\mathbf{a}\widetilde{H}\right)$ and numbers $\langle x_{k,1},x_{k,2},\ldots,x_{k,\mu}\rangle$ are called *coordinates of recorder* $\mathbf{a}_k$ in R^μ.

A coordinates system of a frame of reference is specified accurate to transformations of shear, turn, and inversion.

Def. 1.5.4: Numbers $\langle x_1,x_2,\ldots,x_\mu\rangle$ are called *coordinates* of B in *a coordinate system* R^μ of *a frame of reference* $\left(\Re\mathbf{a}\widetilde{H}\right)$ if there exists a recorder $\mathbf{b}$ such that $\mathbf{b}\in\Re$, $\natural(\mathbf{a})(\mathbf{b},B)$ and these numbers are the coordinates in R^μ of this recorder.

Th. 1.5.4: In a coordinate system R^μ of a frame of reference $\left(\Re\mathbf{a}\widetilde{H}\right)$: if z is a distance length between B and C, coordinates of B are $(b_1,b_2,..,b_n)$, coordinates of C are $(c_1,c_2,..,c_3)$, then

$$z=\left(\sum_{j=1}^{\mu}(c_j-b_j)^2\right)^{0.5}.$$

[1]Pavel Samuilovich Urysohn, Pavel Uryson (February 3, 1898, Odessa - August 17, 1924, Batz-sur-Mer) was a Jewish mathematician who is best known for his contributions in the theory of dimension, and for developing Urysohn’s Metrization Theorem and Urysohn’s Lemma, both of which are fundamental results in topology.

[2]David Hilbert (23 January 1862 14 February 1943) was a German mathematician and one of the most influential and universal mathematicians of the 19th and early 20th centuries.

Proof came out straight from Definition 1.5.4 $\square$

Def. 1.5.5: Numbers $\langle x_1, x_2, \ldots, x_\mu \rangle$ are called *coordinates of the recor-der* $\mathbf{b}$ *in the coordinate system* R^μ *at the instant* t *of the frame of reference* $\left(\Re\mathbf{a}\widetilde{H}\right)$ if for every B the condition is satisfied: if

$$t = \left[\mathbf{b}^\bullet B \mid \Re\mathbf{a}\widetilde{H}\right]$$

then coordinates of $\ll \mathbf{b}^\bullet B \gg$ in coordinate system R^μ of frame of reference $\left(\Re\mathbf{a}\widetilde{H}\right)$ are the following:

$$\langle x_1, x_2, \ldots, x_\mu \rangle.$$

Lm. 1.5.1 If

$$\tau := [\mathbf{b}^\bullet C \uparrow \mathbf{b}, \{\mathbf{g}_0, B, \mathbf{b}_0\}], \tag{2.59}$$
$$p := \left[\mathbf{a}^\bullet b^\bullet (\mathbf{g}_0^\bullet b_0^\bullet)^\tau \mathbf{g}_0^\bullet B \uparrow \mathbf{a}, \{\mathbf{g}_1, A, \mathbf{b}_1\}\right], \tag{2.60}$$
$$q := \left[\mathbf{a}^\bullet b^\bullet (\mathbf{g}_0^\bullet b_0^\bullet)^{\tau+1} \mathbf{g}_0^\bullet B \uparrow \mathbf{a}, \{\mathbf{g}_1, A, \mathbf{b}_1\}\right], \tag{2.61}$$
$$t := [\mathbf{a}^\bullet b^\bullet C \uparrow \mathbf{a}, \{\mathbf{g}_1, A, \mathbf{b}_1\}] \tag{2.62}$$

then

$$p \leq t \leq q.$$

Proof

1) From (2.61):

$$\mathbf{a}^\bullet \left(\mathbf{a}^\bullet b^\bullet (\mathbf{g}_0^\bullet b_0^\bullet)^{\tau+1} \mathbf{g}_0^\bullet B \& \left(\neg \mathbf{a}^\bullet (\mathbf{g}_1^\bullet \mathbf{b}_1^\bullet)^{q+1} \mathbf{g}_1^\bullet A\right)\right). \tag{2.63}$$

Hence from (2.59):

$$\left(\mathbf{b}^\bullet (\mathbf{g}_0^\bullet \mathbf{b}_0^\bullet)^{\tau+1} \mathbf{g}_0^\bullet B \Rightarrow \mathbf{b}^\bullet C\right)$$

then from (2.63) according to II:

$$\mathbf{a}^\bullet \left(\mathbf{a}^\bullet b^\bullet C \& \left(\neg \mathbf{a}^\bullet (\mathbf{g}_1^\bullet \mathbf{b}_1^\bullet)^{q+1} \mathbf{g}_1^\bullet A\right)\right).$$

According to II, since from (2.62):

$$\left(\mathbf{a}^\bullet b^\bullet C \Rightarrow \mathbf{a}^\bullet (\mathbf{g}_1^\bullet \mathbf{b}_1^\bullet)^t \mathbf{g}_1^\bullet A\right)$$

then

$$\mathbf{a}^\bullet \left(\mathbf{a}^\bullet (\mathbf{g}_1^\bullet \mathbf{b}_1^\bullet)^t \mathbf{g}_1^\bullet A \& \left(\neg \mathbf{a}^\bullet (\mathbf{g}_1^\bullet \mathbf{b}_1^\bullet)^{q+1} \mathbf{g}_1^\bullet A\right)\right). \tag{2.64}$$

If $t > q$ then $t \geq q+1$. Hence according to III from (2.64):

$$\mathbf{a}^\bullet\left(\mathbf{a}^\bullet\left(\mathbf{g}_1^\bullet\mathbf{b}_1^\bullet\right)^{q+1}\mathbf{g}_1^\bullet A\&\left(\neg\mathbf{a}^\bullet\left(\mathbf{g}_1^\bullet\mathbf{b}_1^\bullet\right)^{q+1}\mathbf{g}_1^\bullet A\right)\right),$$

it contradicts to I. So $t \leq q$.

2) From (2.62):

$$\mathbf{a}^\bullet\left(\mathbf{a}^\bullet b^\bullet C\&\left(\neg\mathbf{a}^\bullet\left(\mathbf{g}_1^\bullet\mathbf{b}_1^\bullet\right)^{t+1}\mathbf{g}_1^\bullet A\right)\right). \tag{2.65}$$

Since from (2.59):

$$\left(\mathbf{b}^\bullet C \Rightarrow \mathbf{b}^\bullet\left(\mathbf{g}_0^\bullet\mathbf{b}_0^\bullet\right)^\tau\mathbf{g}_0^\bullet B\right)$$

then from (2.65) according to II:

$$\mathbf{a}^\bullet\left(\mathbf{a}^\bullet b^\bullet\left(\mathbf{g}_0^\bullet\mathbf{b}_0^\bullet\right)^\tau\mathbf{g}_0^\bullet B\&\left(\neg\mathbf{a}^\bullet\left(\mathbf{g}_1^\bullet\mathbf{b}_1^\bullet\right)^{t+1}\mathbf{g}_1^\bullet A\right)\right). \tag{2.66}$$

Since from (2.60):

$$\left(\mathbf{a}^\bullet b^\bullet\left(\mathbf{g}_0^\bullet b_0^\bullet\right)^\tau\mathbf{g}_0^\bullet B \Rightarrow \mathbf{a}^\bullet\left(\mathbf{g}_1^\bullet\mathbf{b}_1^\bullet\right)^p\mathbf{g}_1^\bullet A\right)$$

then according to II from (2.66):

$$\mathbf{a}^\bullet\left(\mathbf{a}^\bullet\left(\mathbf{g}_1^\bullet\mathbf{b}_1^\bullet\right)^p\mathbf{g}_1^\bullet A\&\left(\neg\mathbf{a}^\bullet\left(\mathbf{g}_1^\bullet\mathbf{b}_1^\bullet\right)^{t+1}\mathbf{g}_1^\bullet A\right)\right). \tag{2.67}$$

If $p > t$ then $p \geq t+1$. In that case from (2.67) according to III:

$$\mathbf{a}^\bullet\left(\mathbf{a}^\bullet\left(\mathbf{g}_1^\bullet\mathbf{b}_1^\bullet\right)^{t+1}\mathbf{g}_1^\bullet A\&\left(\neg\mathbf{a}^\bullet\left(\mathbf{g}_1^\bullet\mathbf{b}_1^\bullet\right)^{t+1}\mathbf{g}_1^\bullet A\right)\right),$$

it contradicts to I. So $p \leq t$ $\Box$

Th. 1.5.5 In a coordinates system R^μ of a frame of reference $\left(\Re\mathbf{a}\widetilde{H}\right)$: if in every instant t: coordinates of[3]:

$\mathbf{b}$: $\left\langle x_{\mathbf{b},1} + v\cdot t, x_{\mathbf{b},2}, x_{\mathbf{b},3}, \ldots, x_{\mathbf{b},\mu}\right\rangle$;
$\mathbf{g}_0$: $\left\langle x_{0,1} + v\cdot t, x_{0,2}, x_{0,3}, \ldots, x_{0,\mu}\right\rangle$;
$\mathbf{b}_0$: $\left\langle x_{0,1} + v\cdot t, x_{0,2} + l, x_{0,3}, \ldots, x_{0,\mu}\right\rangle$; and
$t_C = \left[\mathbf{b}^\bullet C \mid \Re\mathbf{a}\widetilde{H}\right]$;
$t_D = \left[\mathbf{b}^\bullet D \mid \Re\mathbf{a}\widetilde{H}\right]$;
$q_C = [\mathbf{b}^\bullet C \uparrow \mathbf{b}, \{\mathbf{g}_0, A, \mathbf{b}_0\}]$;
$q_D = [\mathbf{b}^\bullet D \uparrow \mathbf{b}, \{\mathbf{g}_0, A, \mathbf{b}_0\}]$,
then

$$\lim_{l\to 0} 2\cdot\frac{l}{\sqrt{(1-v^2)}}\cdot\frac{q_D - q_C}{t_D - t_C} = 1.$$

Proof: Let us designate:

[3]below v is a real positive number such that $|v| < 1$)

$$t_1 := \left[\mathbf{b}^\bullet \left(\mathbf{g_0}^\bullet b_0^\bullet\right)^{qc} \mathbf{g_0}^\bullet B \mid \Re\mathbf{a}\widetilde{H}\right], \tag{2.68}$$
$$t_2 := \left[\mathbf{b}^\bullet \left(\mathbf{g_0}^\bullet b_0^\bullet\right)^{qc+1} \mathbf{g_0}^\bullet B \mid \Re\mathbf{a}\widetilde{H}\right], \tag{2.69}$$
$$t_3 := \left[\left(\mathbf{g_0}^\bullet b_0^\bullet\right)^{qc} \mathbf{g_0}^\bullet B \mid \Re\mathbf{a}\widetilde{H}\right], \tag{2.70}$$
$$t_4 := \left[\left(\mathbf{g_0}^\bullet b_0^\bullet\right)^{qc+1} \mathbf{g_0}^\bullet B \mid \Re\mathbf{a}\widetilde{H}\right]. \tag{2.71}$$

In that case coordinates of:

$$\ll \mathbf{b}^\bullet \left(\mathbf{g_0}^\bullet b_0^\bullet\right)^{qc} \mathbf{g_0}^\bullet B \gg:$$
$$\left\langle x_{\mathbf{b},1} + v \cdot t_1, x_{\mathbf{b},2}, x_{\mathbf{b},3}, \ldots, x_{\mathbf{b},\mu}\right\rangle, \tag{2.72}$$
$$\ll \mathbf{b}^\bullet \left(\mathbf{g_0}^\bullet b_0^\bullet\right)^{qc+1} \mathbf{g_0}^\bullet B \gg:$$
$$\left\langle x_{\mathbf{b},1} + v \cdot t_2, x_{\mathbf{b},2}, x_{\mathbf{b},3}, \ldots, x_{\mathbf{b},\mu}\right\rangle, \tag{2.73}$$
$$\ll \left(\mathbf{g_0}^\bullet b_0^\bullet\right)^{qc} \mathbf{g_0}^\bullet B \gg: \left\langle x_{0,1} + v \cdot t_3, x_{0,2}, x_{0,3}, \ldots, x_{0,\mu}\right\rangle, \tag{2.74}$$
$$\ll \left(\mathbf{g_0}^\bullet b_0^\bullet\right)^{qc+1} \mathbf{g_0}^\bullet B \gg: \left\langle x_{0,1} + v \cdot t_4, x_{0,2}, x_{0,3}, \ldots, x_{0,\mu}\right\rangle, \tag{2.75}$$
$$\ll \mathbf{b}^\bullet C \gg: \left\langle x_{\mathbf{b},1} + v \cdot t_C, x_{\mathbf{b},2}, x_{\mathbf{b},3}, \ldots, x_{\mathbf{b},\mu}\right\rangle. \tag{2.76}$$

According to Theorem 1.5.1 and Lemma 1.4.4 from (2.68), (2.72), (2.69), (2.73), (2.76):

$$\begin{gathered}
\left[\mathbf{a}^\bullet b^\bullet \left(\mathbf{g_0}^\bullet b_0^\bullet\right)^{qc} \mathbf{g_0}^\bullet B \mid \Re\mathbf{a}\widetilde{H}\right] = \\
\left[\mathbf{a}^\bullet b^\bullet \left(\mathbf{g_0}^\bullet b_0^\bullet\right)^{qc} \mathbf{g_0}^\bullet B \uparrow \mathbf{a}, \widetilde{H}\right] = \\
t_1 + \left(\left(x_{b,1} + vt_1\right)^2 + \textstyle\sum_{j+2}^{\mu} x_{b,j}^2\right)^{0.5},
\end{gathered}$$

$$\begin{gathered}
\left[\mathbf{a}^\bullet b^\bullet \left(\mathbf{g_0}^\bullet b_0^\bullet\right)^{qc+1} B \mid \Re\mathbf{a}\widetilde{H}\right] = \\
\left[\mathbf{a}^\bullet b^\bullet \left(\mathbf{g_0}^\bullet b_0^\bullet\right)^{qc+1} B \uparrow \mathbf{a}, \widetilde{H}\right] = \\
t_2 + \left(\left(x_{b,1} + vt_2\right)^2 + \textstyle\sum_{j=2}^{\mu} x_{b,j}^2\right)^{0.5}.
\end{gathered}$$

According to Lemma 1.5.1:

$$\begin{aligned}
& t_1 + \left(\left(x_{b,1} + vt_1\right)^2 + \sum_{j=2}^{\mu} x_{b,j}^2\right)^{0.5} \\
\leq \quad & t_C + \left(\left(x_{b,1} + vt_C\right)^2 + \sum_{j=2}^{\mu} x_{b,j}^2\right)^{0.5} \\
\leq \quad & t_2 + \left(\left(x_{b,1} + vt_2\right)^2 + \sum_{j=2}^{\mu} x_{b,j}^2\right)^{0.5}.
\end{aligned} \tag{2.77}$$

According to Theorem 1.5.1 from (2.68), (2.70), (2.72), (2.74):

$$t_1 = t_3 + \left(\left(x_{0,1} + vt_3 - x_{b,1} - vt_1\right)^2 + \sum_{j=2}^{\mu} \left(x_{0,j} - x_{b,j}\right)^2 \right)^{0.5}.$$

From (2.69), (2.71), (2.73), (2.75):

$$t_2 = t_4 + \left(\left(x_{0,1} + vt_4 - x_{b,1} - vt_2\right)^2 + \sum_{j=2}^{\mu} \left(x_{0,j} - x_{b,j}\right)^2 \right)^{0.5}.$$

Hence:

$$\begin{aligned} (t_1 - t_3)^2 &= v^2 (t_1 - t_3)^2 - 2v (t_1 - t_3) (x_{0,1} - x_{b,1}) + \sum_{j=2}^{\mu} \left(x_{0,j} - x_{b,j}\right)^2, \\ (t_2 - t_4)^2 &= v^2 (t_2 - t_4)^2 - 2v (t_2 - t_4) (x_{0,1} - x_{b,1}) + \sum_{j=2}^{\mu} \left(x_{0,j} - x_{b,j}\right)^2. \end{aligned}$$

Therefore,

$$t_2 - t_4 = t_1 - t_3. \tag{2.78}$$

Let us designate:

$$t_5 := \left[\mathbf{b_0}^{\bullet} \left(\mathbf{g_0}^{\bullet} b_0{}^{\bullet}\right)^{qc} \mathbf{g_0}^{\bullet} B \mid \Re \mathbf{a} \widetilde{H} \right]. \tag{2.79}$$

In that case coordinates of:

$$\ll \mathbf{b_0}^{\bullet} \left(\mathbf{g_0}^{\bullet} b_0{}^{\bullet}\right)^{qc} \mathbf{g_0}^{\bullet} B \gg : \left\langle x_{0,1} + v \cdot t_5, x_{0,2} + l, x_{0,3}, \ldots, x_{0,\mu} \right\rangle .$$

hence from (2.70), (2.74) according to Theorem 1.5.1:

$$t_5 - t_3 = \left(\left(x_{0,1} + vt_5 - x_{0,1} - vt_3\right)^2 + \left(x_{0,2} + l - x_{0,2}\right)^2 + \sum_{j=3}^{\mu} \left(x_{0,j} - x_{0,j}\right)^2 \right)^{0.5},$$

hence:

$$t_5 - t_3 = \frac{l}{\sqrt{1 - v^2}}. \tag{2.80}$$

Analogously from (2.79), (2.71), (2.75):

$$t_4 - t_5 = \frac{l}{\sqrt{1 - v^2}}.$$

From (2.80):

$$t_4 - t_3 = \frac{2l}{\sqrt{1 - v^2}}.$$

From (2.78):

$$t_2 - t_1 = \frac{2l}{\sqrt{1 - v^2}}.$$

Hence from (2.77):

$$
\begin{aligned}
& t_1 + \left((x_{b,1} + v t_1)^2 + \sum_{j=2}^{\mu} x_{b,j}^2 \right)^{0.5} \\
\leq \quad & t_C + \left((x_{b,1} + v t_C)^2 + \sum_{j=2}^{\mu} x_{b,j}^2 \right)^{0.5} \\
\leq \quad & t_1 + \frac{2l}{\sqrt{1-v^2}} + \left(\left(x_{b,1} + v \left(t_1 + \frac{2l}{\sqrt{1-v^2}} \right) \right)^2 + \sum_{j=2}^{\mu} x_{b,j}^2 \right)^{0.5}.
\end{aligned}
$$

Or if $l \to 0$ then $t_2 \to t_1$, and

$$
\begin{aligned}
& \lim_{l \to 0} \left(t_1 + \left((x_{b,1} + v t_1)^2 + \sum_{j=2}^{\mu} x_{b,j}^2 \right)^{0.5} \right) \\
= \quad & t_C + \left((x_{b,1} + v t_C)^2 + \sum_{j=2}^{\mu} x_{b,j}^2 \right)^{0.5}.
\end{aligned}
$$

Since, if $v^2 < 1$ then function

$$
f(t) = t + \left((x_{b,1} + v t)^2 + \sum_{j=2}^{\mu} x_{b,j}^2 \right)^{0.5}
$$

is a monotonic one, then

$$
\lim_{l \to 0} t_1 = t_C,
$$

hence

$$
\lim_{l \to 0} \left[\mathbf{b}^{\bullet} \left(\mathbf{g_0}^{\bullet} b_0{}^{\bullet} \right)^{q_C} \mathbf{g_0}^{\bullet} B \mid \Re \mathbf{a} \widetilde{H} \right] = t_C. \tag{2.81}
$$

Analogously,

$$
\lim_{l \to 0} \left[\mathbf{b}^{\bullet} \left(\mathbf{g_0}^{\bullet} b_0{}^{\bullet} \right)^{q_D} \mathbf{g_0}^{\bullet} B \mid \Re \mathbf{a} \widetilde{H} \right] = t_D. \tag{2.82}
$$

According to Theorem 1.5.1 from (2.68) and (2.69):

$$
\begin{aligned}
& \left[\mathbf{b}^{\bullet} \left(\mathbf{g_0}^{\bullet} b_0{}^{\bullet} \right)^{q_D} \mathbf{g_0}^{\bullet} B \mid \Re \mathbf{a} \widetilde{H} \right] - \left[\mathbf{b}^{\bullet} \left(\mathbf{g_0}^{\bullet} b_0{}^{\bullet} \right)^{q_C} \mathbf{g_0}^{\bullet} B \mid \Re \mathbf{a} \widetilde{H} \right] \\
= \quad & \left(t_1 + \frac{2l}{\sqrt{1-v^2}} (q_D - q_C) \right) - t_1 \\
= \quad & \frac{2l (q_D - q_C)}{\sqrt{1-v^2}}.
\end{aligned}
$$

From (2.81) and (2.82):

$$\lim_{l\to 0}\frac{2l\left(q_D-q_C\right)}{t_D-t_C}=\sqrt{1-v^2}$$

$\square$

Corollary of Theorem 1.5.5: If designate: $q_D^{st}:=q_D$ and $q_C^{st}:=q_C$ for $v=0$, then

$$\lim_{l\to 0}2l\frac{q_D^{st}-q_C^{st}}{t_D-t_C}=1,$$

hence:

$$\lim_{l\to 0}\frac{q_D-q_C}{q_D^{st}-q_C^{st}}=\sqrt{1-v^2}.$$

For an absolutely precise κ-clock:

$$q_D^{st}-q_C^{st}=\frac{q_D-q_C}{\sqrt{1-v^2}}\square$$

Consequently, moving at speed v κ-clock are times slower than the one at rest.

Th. 1.5.6 Let: v ($|v|<1$) and l be real numbers and k_i be natural ones.

Let in a coordinates system R^μ of a frame of reference $\left(\Re\mathbf{a}\widetilde{H}\right)$: in each instant t coordinates of:

$\mathbf{b}$: $\left\langle x_{b,1}+v\cdot t,x_{b,2},x_{b,3},\ldots,x_{b,\mu}\right\rangle$,

$\mathbf{g}_j$: $\left\langle y_{j,1}+v\cdot t,y_{j,2},y_{j,3},\ldots,y_{j,\mu}\right\rangle$,

$\mathbf{u}_j$: $\left\langle y_{j,1}+v\cdot t,y_{j,2}+l/\left(k_1\cdot\ldots\cdot k_j\right),y_{j,3},\ldots,y_{j,\mu}\right\rangle$,

for all $\mathbf{b}_i$: if $\mathbf{b}_i\in\Im$, then coordinates of

$\mathbf{b}_i$: $\left\langle x_{i,1}+v\cdot t,x_{i,2},x_{i,3},\ldots,x_{i,\mu}\right\rangle$,

$\widetilde{T}$ is $\left\langle\left\{\mathbf{g}_1,A,\mathbf{u}_1\right\},\ \left\{\mathbf{g}_2,A,\mathbf{u}_2\right\},..,\left\{\mathbf{g}_j,A,\mathbf{u}_j\right\},\ ..\ \right\rangle$.

In that case: $\Im$ is $ISS\left(\mathbf{b},\widetilde{T}\right)$.

Proof

1) Let us designate:

$$p:=\left[\mathbf{b}^\bullet b_1^\bullet B\uparrow\mathbf{b},\widetilde{T}\right],$$
$$q:=\left[\mathbf{b}^\bullet b_2^\bullet b_1^\bullet B\uparrow\mathbf{b},\widetilde{T}\right],$$
$$r:=\left[\mathbf{b}^\bullet b_1^\bullet C\uparrow\mathbf{b},\widetilde{T}\right],$$
$$s:=\left[\mathbf{b}^\bullet b_2^\bullet b_1^\bullet C\uparrow\mathbf{b},\widetilde{T}\right],$$

$$t_p:=\left[\mathbf{b}^\bullet b_1^\bullet B\mid\Re\mathbf{a}\widetilde{H}\right],\tag{2.83}$$
$$t_q:=\left[\mathbf{b}^\bullet b_2^\bullet b_1^\bullet B\mid\Re\mathbf{a}\widetilde{H}\right],\tag{2.84}$$
$$t_r:=\left[\mathbf{b}^\bullet b_1^\bullet C\mid\Re\mathbf{a}\widetilde{H}\right],\tag{2.85}$$
$$t_s:=\left[\mathbf{b}^\bullet b_2^\bullet b_1^\bullet B\mid\Re\mathbf{a}\widetilde{H}\right].\tag{2.86}$$

According to Corollary of Theorem 1.5.5:

$$t_q - t_p = \frac{q-p}{\sqrt{1-v^2}}, \tag{2.87}$$
$$t_s - t_r = \frac{s-r}{\sqrt{1-v^2}}. \tag{2.88}$$

From (2.83-2.86) coordinates of:

$$\ll \mathbf{b}^\bullet b_1^\bullet B \gg: \left\langle x_{b,1} + vt_p, x_{b,2}, x_{b,3}, \ldots, x_{b,\mu} \right\rangle, \tag{2.89}$$
$$\ll \mathbf{b}^\bullet b_2^\bullet b_1^\bullet B \gg: \left\langle x_{b,1} + vt_q, x_{b,2}, x_{b,3}, \ldots, x_{b,\mu} \right\rangle,$$

$$\ll \mathbf{b}^\bullet b_1^\bullet C \gg: \left\langle x_{b,1} + vt_r, x_{b,2}, x_{b,3}, \ldots, x_{b,\mu} \right\rangle, \tag{2.90}$$
$$\ll \mathbf{b}^\bullet b_2^\bullet b_1^\bullet C \gg: \left\langle x_{b,1} + vt_s, x_{b,2}, x_{b,3}, \ldots, x_{b,\mu} \right\rangle.$$

Let us designate:

$$t_1 := \left[\mathbf{b}_1^\bullet B \mid \Re\mathbf{a}\widetilde{H}\right], \tag{2.91}$$
$$t_2 := \left[\mathbf{b}_1^\bullet C \mid \Re\mathbf{a}\widetilde{H}\right]. \tag{2.92}$$

Consequently, coordinates of:

$$\ll \mathbf{b}_1^\bullet B \gg: \left\langle x_{1,1} + vt_1, x_{1,2}, x_{1,3}, \ldots, x_{1,\mu} \right\rangle,$$
$$\ll \mathbf{b}_1^\bullet C \gg: \left\langle x_{1,1} + vt_2, x_{1,2}, x_{1,3}, \ldots, x_{1,\mu} \right\rangle.$$

According to Theorem 1.5.1 from (2.90), (2.92), (2.85):

$$t_r - t_2 = \left((x_{b,1} + vt_r - x_{1,1} - vt_2)^2 + \sum_{j=2}^{\mu} \left(x_{b,j} - x_{1,j}\right)^2 \right)^{0.5}.$$

Analogously from (2.89), (2.91), (2.83):

$$t_p - t_1 = \left((x_{b,1} + vt_p - x_{1,1} - vt_1)^2 + \sum_{j=2}^{\mu} \left(x_{b,j} - x_{1,j}\right)^2 \right)^{0.5}.$$

Hence,

$$t_r - t_2 = t_p - t_1. \tag{2.93}$$

Let us denote:

$$t_3 := \left[\mathbf{b_2}^\bullet b_1^\bullet B \mid \Re\mathbf{a}\widetilde{H}\right],$$
$$t_4 := \left[\mathbf{b_2}^\bullet b_1^\bullet C \mid \Re\mathbf{a}\widetilde{H}\right].$$

Hence, coordinates of:

$$\ll \mathbf{b_2}^\bullet b_1^\bullet B \gg: \left\langle x_{2,1} + vt_3, x_{2,2}, x_{2,3}, \ldots, x_{2,\mu}\right\rangle,$$
$$\ll \mathbf{b_2}^\bullet b_1^\bullet C \gg: \left\langle x_{2,1} + vt_4, x_{2,2}, x_{2,3}, \ldots, x_{2,\mu}\right\rangle.$$

According to Theorem 1.5.1:

$$t_3 - t_1 = \left((x_{2,1} + vt_3 - x_{1,1} - vt_1)^2 + \sum_{j=2}^{\mu}(x_{2,j} - x_{1,j})^2\right)^{0.5}.$$
$$t_4 - t_2 = \left((x_{2,1} + vt_4 - x_{1,1} - vt_2)^2 + \sum_{j=2}^{\mu}(x_{2,j} - x_{1,j})^2\right)^{0.5}.$$

Hence:

$$t_3 - t_4 = t_1 - t_2. \tag{2.94}$$

And analogously:

$$t_q - t_3 = t_s - t_4. \tag{2.95}$$

From (2.94), (2.95), (2.93):

$$t_q - t_p = t_s - t_r.$$

From (2.88), (2.87):

$$q - p = s - r. \tag{2.96}$$

2) Let us designate:

$$p' := \left[\mathbf{b}^\bullet C \uparrow \mathbf{b}, \widetilde{T}\right],$$
$$q' := \left[\mathbf{b}^\bullet \alpha \mathbf{b}^\bullet C \uparrow \mathbf{b}, \widetilde{T}\right],$$
$$r' := \left[\mathbf{b}^\bullet \alpha^\dagger \mathbf{b}^\bullet C \uparrow \mathbf{b}, \widetilde{T}\right];$$

here α is $\mathbf{b}_1^\bullet b_2^\bullet \ldots b_k^\bullet b_{k+1}^\bullet \ldots b_N^\bullet$.
Hence according Definition 1.4.1:

$$\mathfrak{m}\left(\mathbf{b}\widetilde{T}\right)(\mathbf{b}^{\bullet}\alpha\mathbf{b}^{\bullet}C) = q' - p', \tag{2.97}$$

$$\mathfrak{m}\left(\mathbf{b}\widetilde{T}\right)\left(\mathbf{b}^{\bullet}\alpha^{\dagger}\mathbf{b}^{\bullet}C\right) = r' - p'. \tag{2.98}$$

Let us designate:

$$\begin{aligned}
&t_0 := \left[\mathbf{b}^{\bullet}C \mid \Re\mathbf{a}\widetilde{H}\right],\\
&t_1 = \left[\mathbf{b}_1^{\bullet}b^{\bullet}C \mid \Re\mathbf{a}\widetilde{H}\right],\\
&t_2 := \left[\mathbf{b}_2^{\bullet}b_1^{\bullet}b^{\bullet}C \mid \Re\mathbf{a}\widetilde{H}\right],\\
&\cdots,\\
&t_k := \left[\mathbf{b}_k^{\bullet}\ldots\mathbf{b}_2^{\bullet}b_1^{\bullet}b^{\bullet}C \mid \Re\mathbf{a}\widetilde{H}\right],\\
&t_{k+1} := \left[\mathbf{b}_{k+1}^{\bullet}b_k^{\bullet}\ldots\mathbf{b}_2^{\bullet}b_1^{\bullet}b^{\bullet}C \mid \Re\mathbf{a}\widetilde{H}\right],\\
&\cdots,\\
&t_N := \left[\mathbf{b}_N^{\bullet}\ldots\mathbf{b}_{k+1}^{\bullet}b_k^{\bullet}\ldots\mathbf{b}_2^{\bullet}b_1^{\bullet}b^{\bullet}C \mid \Re\mathbf{a}\widetilde{H}\right],\\
&t_{N+1} := \left[\mathbf{b}^{\bullet}\alpha^{\dagger}\mathbf{b}^{\bullet}C \mid \Re\mathbf{a}\widetilde{H}\right].
\end{aligned} \tag{2.99}$$

Hence in accordance with this theorem condition coordinates of:

$$\begin{aligned}
\ll\ &\mathbf{b}^{\bullet}C \gg:\\
&\left\langle x_{b,1} + vt_0, x_{b,2}, x_{b,3}, \ldots, x_{b,\mu}\right\rangle,\\
\ll\ &\mathbf{b}_1^{\bullet}b^{\bullet}C \gg:\\
&\left\langle x_{1,1} + vt_1, x_{1,2}, x_{1,3}, \ldots, x_{1,\mu}\right\rangle,\\
\ll\ &\mathbf{b}_2^{\bullet}b_1^{\bullet}b^{\bullet}C \gg:\\
&\left\langle x_{2,1} + vt_2, x_{2,2}, x_{2,3}, \ldots, x_{2,\mu}\right\rangle,\\
&\cdots,\\
\ll\ &\mathbf{b}_k^{\bullet}\cdots\mathbf{b}_2^{\bullet}b_1^{\bullet}b^{\bullet}C \gg:\\
&\left\langle x_{k,1} + vt_k, x_{k,2}, x_{k,3}, \ldots, x_{k,\mu}\right\rangle,\\
\ll\ &\mathbf{b}_{k+1}^{\bullet}b_k^{\bullet}\cdots\mathbf{b}_2^{\bullet}b_1^{\bullet}b^{\bullet}C \gg:\\
&\left\langle x_{k+1,1} + vt_{k+1}, x_{k+1,2}, x_{k+1,3}, \ldots, x_{k+1,\mu}\right\rangle,\\
&\cdots,\\
\ll\ &\mathbf{b}_N^{\bullet}\cdots\mathbf{b}_{k+1}^{\bullet}b_k^{\bullet}\cdots\mathbf{b}_2^{\bullet}b_1^{\bullet}b^{\bullet}C \gg:\\
&\left\langle x_{N,1} + vt_N, x_{N,2}, x_{N,3}, \ldots, x_{N,\mu}\right\rangle,\\
\ll\ &\mathbf{b}^{\bullet}\alpha^{\dagger}\mathbf{b}^{\bullet}C \gg:\\
&\left\langle x_{N+1,1} + vt_{N+1}, x_{N+1,2}, x_{N+1,3}, \ldots, x_{N+1,\mu}\right\rangle.
\end{aligned}$$

Hence from (2.99) according Theorem 1.5.1:

$$
\begin{aligned}
& t_1 - t_0 \\
= \quad & \left((x_{1,1} + vt_1 - x_{b,1} - vt_0)^2 + \sum_{j=2}^{\mu} \left(x_{1,j} - x_{b,j} \right)^2 \right)^{0.5}, \\
& t_2 - t_1 \\
= \quad & \left((x_{2,1} + vt_2 - x_{1,1} - vt_1)^2 + \sum_{j=2}^{\mu} \left(x_{2,j} - x_{1,j} \right)^2 \right)^{0.5}, \\
& \ldots, \\
& t_{k+1} - t_k \\
= \quad & \left((x_{k+1,1} + vt_{k+1} - x_{k,1} - vt_k)^2 + \sum_{j=2}^{\mu} \left(x_{k+1,j} - x_{k,j} \right)^2 \right)^{0.5}, \\
& \ldots, \\
& t_{N+1} - t_N \\
= \quad & \left((x_{b,1} + vt_{N+1} - x_{N,1} - vt_N)^2 + \sum_{j=2}^{\mu} \left(x_{b,j} - x_{N,j} \right)^2 \right)^{0.5}.
\end{aligned}
$$

If designate:

$$
\rho^2_{a,b} := \sum_{j=1}^{\mu} (x_{b,1} - x_{a,1})^2,
$$

then for every k:

$$
\begin{aligned}
t_{k+1} - t_k = & \frac{v}{1 - v^2} (x_{k+1,1} - x_{k,1}) \\
& + \frac{1}{1 - v^2} \left(\rho^2_{k,k+1} - v^2 \sum_{j=2}^{\mu} \left(x_{k+1,j} - x_{k,j} \right)^2 \right)^{0.5}.
\end{aligned}
$$

Hence:

$$
\begin{aligned}
& t_{N+1} - t_0 = \\
= \quad & \frac{1}{1 - v^2} \left(\begin{array}{c}
\left(\rho^2_{b,1} - v^2 \sum_{j=2}^{\mu} \left(x_{1,j} - x_{b,j} \right)^2 \right)^{0.5} \\
+ \left(\rho^2_{N,b} - v^2 \sum_{j=2}^{\mu} \left(x_{b,j} - x_{N,j} \right)^2 \right)^{0.5} \\
+ \sum_{k=1}^{N-1} \left(\rho^2_{k,k+1} - v^2 \sum_{j=2}^{\mu} \left(x_{k+1,j} - x_{k,j} \right)^2 \right)^{0.5}
\end{array} \right).
\end{aligned}
$$

Analogously, if designate:

$$
\tau_{N+1} := \left[\mathbf{b}^{\bullet} \alpha \mathbf{b}^{\bullet} C \mid \Re \mathbf{a} \widetilde{H} \right]
$$

then

$$\tau_{N+1} - t_0 =$$

$$= \frac{1}{1-v^2}\left(\begin{array}{c} \left(\rho_{1,b}^2 - v^2\sum_{j=2}^{\mu}\left(x_{b,j}-x_{1,j}\right)^2\right)^{0.5} \\ +\left(\rho_{b,N}^2 - v^2\sum_{j=2}^{\mu}\left(x_{N,j}-x_{b,j}\right)^2\right)^{0.5} \\ +\sum_{k=1}^{N-1}\left(\rho_{k+1,k}^2 - v^2\sum_{j=2}^{\mu}\left(x_{k,j}-x_{k+1,j}\right)^2\right)^{0.5} \end{array}\right),$$

hence

$$t_{N+1} - t_0 = \tau_{N+1} - t_0. \qquad (2.100)$$

According to Theorem 1.5.5:

$$\tau_{N+1} - t_0 = \frac{q'-p'}{\sqrt{1-v^2}} \text{ and } t_{N+1} - t_0 = \frac{r'-p'}{\sqrt{1-v^2}}.$$

From (2.100), (2.97), (2.98):

$$\mathfrak{m}\left(\mathbf{b}\widetilde{T}\right)\left(\mathbf{b}^{\bullet}\alpha\mathbf{b}^{\bullet}C\right) = \mathfrak{m}\left(\mathbf{b}\widetilde{T}\right)\left(\mathbf{b}^{\bullet}\alpha^{\dagger}\mathbf{b}^{\bullet}C\right).$$

From (2.96) according to Definition 1.4.3: $\mathfrak{I}$ is $ISS\left(\mathbf{b},\widetilde{T}\right)$ □
Therefore, a inner stability survives on a uniform straight line motion.
Th. 1.5.7
Let:
1) in a coordinates system R^{μ} of a frame of reference $\left(\mathfrak{R}\mathbf{a}\widetilde{H}\right)$ in every instant t:
$\mathbf{b}: \langle x_{\mathbf{b},1}+v\cdot t, x_{\mathbf{b},2}, x_{\mathbf{b},3},\ldots,x_{\mathbf{b},\mu}\rangle$,
$\mathbf{g}_j$: $\langle y_{j,1}+v\cdot t, y_{j,2}, y_{j,3},\ldots,y_{j,\mu}\rangle$,
$\mathbf{u}_j$: $\langle y_{j,1}+v\cdot t, y_{j,2}+l/\left(k_1\cdot\ldots\cdot k_j\right), y_{j,3},\ldots,y_{j,\mu}\rangle$,
for every recorder $\mathbf{q}_i$: if $\mathbf{q}_i \in \mathfrak{I}$ then coordinates of
$\mathbf{q}_i: \langle x_{i,1}+v\cdot t, x_{i,2}, x_{i,3},\ldots,x_{i,\mu}\rangle$,
$\widetilde{T}$ is $\langle\{\mathbf{g}_1,A,\mathbf{u}_1\},\ \{\mathbf{g}_2,A,\mathbf{u}_2\},..,\{\mathbf{g}_j,A,\mathbf{u}_j\},\ ..\ \rangle$.
$C: \langle C_1,C_2,C_3,\ldots,C_{\mu}\rangle$,
$D: \langle D_1,D_2,D_3,\ldots,D_{\mu}\rangle$,
$t_C = \left[C \mid \mathfrak{R}\mathbf{a}\widetilde{H}\right]$,
$t_D = \left[D \mid \mathfrak{R}\mathbf{a}\widetilde{H}\right]$;
2) in a coordinates system $R^{\mu\prime}$ of a frame of reference $\left(\mathfrak{I}\mathbf{b}\widetilde{T}\right)$:
$C: \langle C_1',C_2',C_3',\ldots,C_{\mu}'\rangle$,
$D: \langle D_1',D_2',D_3',\ldots,D_{\mu}'\rangle$,
$t_C' = \left[C \mid \mathfrak{I}\mathbf{b}\widetilde{T}\right]$,
$t_D' = \left[D \mid \mathfrak{I}\mathbf{b}\widetilde{T}\right]$.
In that case:

$$
\begin{aligned}
t'_D - t'_C &= \frac{(t_D - t_C) - v(D_1 - C_1)}{\sqrt{1-v^2}}, \\
D'_1 - C'_1 &= \frac{(D_1 - C_1) - v(t_D - t_C)}{\sqrt{1-v^2}}.
\end{aligned}
$$

Proof:
Let us designate:

$$
\rho_{\mathbf{a},\mathbf{b}} := \left(\sum_{j=1}^{\mu} (b_j - a_j)^2 \right)^{0.5}.
$$

According to Definition 1.4.8 there exist elements $\mathbf{q}_C$ and $\mathbf{q}_D$ of set $\mathfrak{I}$ such that

$$
\natural(\mathbf{b})(\mathbf{q}_C, C)), \natural(\mathbf{b})(\mathbf{q}_D, D)
$$

and

$$
\ell\left(\mathfrak{I}\mathbf{b}\widetilde{T}\right)(C, D) = \ell\left(\mathbf{b}, \widetilde{T}\right)(\mathbf{q}_C, \mathbf{q}_D).
$$

In that case:

$$
\begin{aligned}
t'_C &= \left[C \mid \mathfrak{I}\mathbf{b}\widetilde{T}\right] = \left[\mathbf{q}_C^{\bullet} C \mid \mathfrak{I}\mathbf{b}\widetilde{T}\right], \\
t'_D &= \left[D \mid \mathfrak{I}\mathbf{b}\widetilde{T}\right] = \left[\mathbf{q}_D^{\bullet} D \mid \mathfrak{I}\mathbf{b}\widetilde{T}\right].
\end{aligned}
$$

According to Corollary of Theorem 1.5.5:

$$
\begin{aligned}
\left[\mathbf{q}_C^{\bullet} C \mid \mathfrak{R}\mathbf{a}\widetilde{H}\right] &= \left[C \mid \mathfrak{R}\mathbf{a}\widetilde{H}\right] = t_C, \\
\left[\mathbf{q}_D^{\bullet} D \mid \mathfrak{R}\mathbf{a}\widetilde{H}\right] &= \left[D \mid \mathfrak{R}\mathbf{a}\widetilde{H}\right] = t_D.
\end{aligned}
$$

Let us designate:

$$\tau_1 := \left[\mathbf{b}^\bullet C \uparrow \mathbf{b}, \widetilde{T}\right],$$
$$\tau_2 := \left[\mathbf{b}^\bullet D \uparrow \mathbf{b}, \widetilde{T}\right],$$
$$t_1 := \left[\mathbf{b}^\bullet C \mid \Re\mathbf{a}\widetilde{H}\right],$$
$$t_2 := \left[\mathbf{b}^\bullet D \mid \Re\mathbf{a}\widetilde{H}\right],$$
$$t_3 := \left[\mathbf{b}^\bullet B \mid \Re\mathbf{a}\widetilde{H}\right],$$
$$t_4 := \left[\mathbf{q}_C^\bullet \mathbf{b}^\bullet B \mid \Re\mathbf{a}\widetilde{H}\right],$$
$$t_5 := \left[\mathbf{b}^\bullet \mathbf{q}_C^\bullet \mathbf{b}^\bullet B \mid \Re\mathbf{a}\widetilde{H}\right],$$
$$t_6 := \left[\mathbf{q}_D^\bullet \mathbf{q}_C^\bullet \mathbf{b}^\bullet B \mid \Re\mathbf{a}\widetilde{H}\right],$$
$$t_7 := \left[\mathbf{q}_C^\bullet \mathbf{q}_D^\bullet \mathbf{q}_C^\bullet \mathbf{b}^\bullet B \mid \Re\mathbf{a}\widetilde{H}\right],$$
$$t_8 := \left[\mathbf{b}^\bullet \mathbf{q}_C^\bullet \mathbf{q}_D^\bullet \mathbf{q}_C^\bullet \mathbf{b}^\bullet B \mid \Re\mathbf{a}\widetilde{H}\right].$$

Under such designations:

$$\begin{aligned}
&t_8 - t_7 = t_5 - t_4 \text{ hence: } t_8 - t_5 = t_7 - t_4 \text{ and} \\
&\ell\left(\Im\mathbf{b}\widetilde{T}\right)(C,D) \\
&= 0.5\left(\left[\mathbf{b}^\bullet \mathbf{q}_C^\bullet \mathbf{q}_D^\bullet \mathbf{q}_C^\bullet \mathbf{b}^\bullet B \uparrow \mathbf{b}, \widetilde{T}\right] - \left[\mathbf{b}^\bullet \mathbf{q}_C^\bullet \mathbf{b}^\bullet B \uparrow \mathbf{b}, \widetilde{T}\right]\right),
\end{aligned}$$

hence:

$$\begin{aligned}
\ell\left(\Im\mathbf{b}\widetilde{T}\right)(C,D) = 0.5\,(t_8 - t_5)\sqrt{1-v^2} = 0.5\,(t_7 - t_4)\sqrt{1-v^2}, \\
(t_7 - t_6)^2 = (x_{C,1} + vt_7 - x_{D,1} - vt_6)^2 + \textstyle\sum_{j=2}^{\mu}(x_{C,j} - x_{D,j})^2, \\
(t_6 - t_4)^2 = (x_{D,1} + vt_6 - x_{C,1} - vt_4)^2 + \textstyle\sum_{j=2}^{\mu}(x_{C,j} - x_{D,j})^2,
\end{aligned}$$

hence:

$$\begin{aligned}
(t_7 - t_6)^2 = v^2(t_7 - t_6)^2 + 2v(x_{C,1} - x_{D,1})(t_7 - t_6) + \rho^2_{\mathbf{q}_C,\mathbf{q}_D}, \\
(t_6 - t_4)^2 = v^2(t_6 - t_4)^2 + 2v(x_{D,1} - x_{C,1})(t_6 - t_4) + \rho^2_{\mathbf{q}_D,\mathbf{q}_C}.
\end{aligned}$$

Sequencely:

$$t_7 - t_4 = \tfrac{2}{\sqrt{1-v^2}}\left(v^2(x_{D,1} - x_{C,1})^2 + \left(1 - v^2\right)\rho^2_{\mathbf{q}_C,\mathbf{q}_D}\right)^{0.5}.$$

Let us designate:

$$R_{\mathbf{a},\mathbf{b}} := \left(\rho_{\mathbf{a},\mathbf{b}}^2 - v^2 \sum_{j=2}^{\mu} (a_j - b_j)^2\right)^{0.5}.$$

Under such designation:

$$\ell\left(\Im\mathbf{b}\widetilde{T}\right)(C,D) = \frac{R_{\mathbf{q}_C,\mathbf{q}_D}}{\sqrt{1-v^2}}.$$

Since

$$\begin{gathered} C_1 = x_{C,1} + vt_C,\, D_1 = x_{D,1} + vt_D, \\ C_{j+1} = x_{C,j+1},\, D_{j+1} = x_{D,j+1} \end{gathered}$$

then

$$R_{\mathbf{q}_C,\mathbf{q}_D} = \left(\begin{array}{c} v^2 (D_1 - vt_D - C_1 + vt_C)^2 \\ +\left(1-v^2\right)\left(\begin{array}{c} (D_1 - vt_D - C_1 + vt_C)^2 \\ +\sum_{j=2}^{\mu} (D_j - C_j)^2 \end{array}\right) \end{array}\right)^{0.5},$$

hence:

$$R_{\mathbf{q}_C,\mathbf{q}_D} = \left(\begin{array}{c} v^2 (t_D - t_C)^2 - 2v (t_D - t_C)(D_1 - C_1) \\ +\rho_{C,D}^2 \\ -v^2 \sum_{j=2}^{\mu} (D_j - C_j)^2 \end{array}\right)^{0.5}. \tag{2.101}$$

Moreover, according to Definition 1.4.7:

$$\begin{gathered} t'_D - t'_C = (\tau_2 - \tau_1) - \\ -\left(\ell\left(\mathbf{b},\widetilde{T}\right)(\mathbf{b},\mathbf{q}_D) - \ell\left(\mathbf{b},\widetilde{T}\right)(\mathbf{b},\mathbf{q}_C)\right) \end{gathered} \tag{2.102}$$

According to Theorem 1.5.5:

$$\tau_2 - \tau_1 = (t_2 - t_1)\sqrt{1-v^2}. \tag{2.103}$$

According to Theorem 1.5.3:

$$\begin{gathered} (t_1 - t_C)^2 = (x_{\mathbf{b},1} + vt_1 - C_1)^2 + \sum_{j=2}^{\mu} (x_{\mathbf{b},j} - C_j)^2, \\ (t_2 - t_D)^2 = (x_{\mathbf{b},1} + vt_2 - D_1)^2 + \sum_{j=2}^{\mu} (x_{\mathbf{b},j} - D_j)^2. \end{gathered}$$

Therefore,

$$\begin{gathered} (t_1 - t_C)^2 = v^2 (t_1 - t_C)^2 + 2v (x_{\mathbf{b},1} - x_{C,1})(t_1 - t_C) + \rho_{\mathbf{b},\mathbf{q}_C}^2, \\ (t_2 - t_D)^2 = v^2 (t_2 - t_D)^2 + 2v (x_{\mathbf{b},1} - x_{D,1})(t_2 - t_D) + \rho_{\mathbf{b},\mathbf{q}_D}^2. \end{gathered}$$

Hence,

$$\begin{array}{c} t_2 - t_1 = \\ = (t_D - t_C) + \frac{v}{1-v^2} (x_{C,1} - x_{D,1}) \\ + \frac{1}{1-v^2} (R_{\mathbf{b},\mathbf{q}_D} - R_{\mathbf{b},\mathbf{q}_C}). \end{array}$$

Because

$$\ell\left(\mathbf{b}, \widetilde{T}\right)(\mathbf{b}, \mathbf{q}_D) = \frac{R_{\mathbf{b},\mathbf{q}_D}}{\sqrt{1-v^2}}, \ell\left(\mathbf{b}, \widetilde{T}\right)(\mathbf{b}, \mathbf{q}_C) = \frac{R_{\mathbf{b},\mathbf{q}_C}}{\sqrt{1-v^2}},$$

then from (2.102), (2.103), (2.104):

$$t'_D - t'_C = (t_D - t_C)\sqrt{1-v^2} - \frac{v}{\sqrt{1-v^2}} (x_{D,1} - x_{C,1}),$$

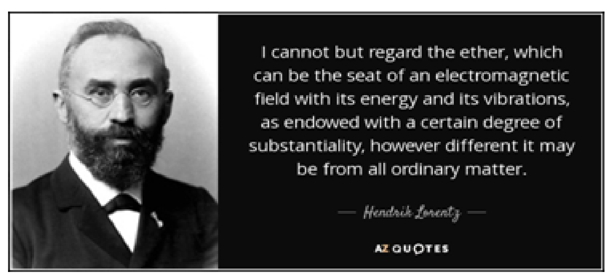
hence:

$$\begin{array}{rl} & t'_D - t'_C \\ = & (t_D - t_C)\sqrt{1-v^2} - \frac{v}{\sqrt{1-v^2}} ((D_1 - C_1) - v(t_D - t_C)), \end{array}$$

hence:

$$\begin{aligned} t'_D - t'_C &= \frac{(t_D - t_C) - v(D_1 - C_1)}{\sqrt{1-v^2}}, \\ D'_1 - C'_1 &= \frac{(D_1 - C_1) - v(t_D - t_C)}{\sqrt{1-v^2}}. \end{aligned} \tag{2.104}$$

It is the Lorentz spatial-temporal transformations[4] $\square$.



Thus, if you have some set of objects, dealing with information, then time and space are inevitable. And it doesnt matter whether this set is part our world or some other worlds, which dont have a space-time structure initially. I call such Time the Informational Time. Since, we get our time together with our information system all other notions of time (thermodynamical time, cosmological time, psychological time, quantum time etc.) should be defined by that Informational Time.

[4]Hendrik Antoon Lorentz (18 July 1853 - 4 February 1928) was a Dutch physicist who shared the 1902 Nobel Prize in Physics with Pieter Zeeman for the discovery and theoretical explanation of the Zeeman effect. He also derived the transformation equations subsequently used by Albert Einstein to describe space and time.

2.5. Matricies

Let 1_n be an identical $n \times n$ matrix and 0_n is a $n \times n$ zero matrix. If A and all $AB_{j,s}$ are $n \times n$ matricies then

$$\boxed{A \begin{bmatrix} B_{0,0} & B_{0,1} & \cdots & B_{0,n} \\ B_{1,0} & B_{1,1} & \cdots & B_{1,n} \\ \cdots & \cdots & \cdots & \cdots \\ B_{m,0} & B_{m,1} & \cdots & B_{m,n} \end{bmatrix} := \begin{bmatrix} AB_{0,0} & AB_{0,1} & \cdots & AB_{0,n} \\ AB_{1,0} & AB_{1,1} & \cdots & AB_{1,n} \\ \cdots & \cdots & \cdots & \cdots \\ AB_{m,0} & AB_{m,1} & \cdots & AB_{m,n} \end{bmatrix}}$$

and

$$\boxed{\begin{bmatrix} B_{0,0} & B_{0,1} & \cdots & B_{0,n} \\ B_{1,0} & B_{1,1} & \cdots & B_{1,n} \\ \cdots & \cdots & \cdots & \cdots \\ B_{m,0} & B_{m,1} & \cdots & B_{m,n} \end{bmatrix} A := \begin{bmatrix} B_{0,0}A & B_{0,1}A & \cdots & B_{0,n}A \\ B_{1,0}A & B_{1,1}A & \cdots & B_{1,n}A \\ \cdots & \cdots & \cdots & \cdots \\ B_{m,0}A & B_{m,1}A & \cdots & B_{m,n}A \end{bmatrix}}. \tag{2.105}$$

If A and all $B_{j,s}$ are $k \times k$ matrices then

$$\boxed{A + \begin{bmatrix} B_{0,0} & B_{0,1} & B_{0,2} & \cdots & B_{0,n} \\ B_{1,0} & B_{1,1} & B_{1,2} & \cdots & B_{1,n} \\ B_{2,0} & B_{2,1} & B_{2,2} & \cdots & B_{2,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ B_{n,0} & B_{n,1} & B_{n,2} & \cdots & B_{n,n} \end{bmatrix} :=}$$

Wolfgang Pauli

$$\boxed{:= A1_{nk} + \begin{bmatrix} B_{0,0} & B_{0,1} & B_{0,2} & \cdots & B_{0,n} \\ B_{1,0} & B_{1,1} & B_{1,2} & \cdots & B_{1,n} \\ B_{2,0} & B_{2,1} & B_{2,2} & \cdots & B_{2,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ B_{n,0} & B_{n,1} & B_{n,2} & \cdots & B_{n,n} \end{bmatrix} =}$$

$$\boxed{= \begin{bmatrix} B_{0,0}+A & B_{0,1} & B_{0,2} & \cdots & B_{0,n} \\ B_{1,0} & B_{1,1}+A & B_{1,2} & \cdots & B_{1,n} \\ B_{2,0} & B_{2,1} & B_{2,2}+A & \cdots & B_{2,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ B_{n,0} & B_{n,1} & B_{n,2} & \cdots & B_{n,n}+A \end{bmatrix}}. \tag{2.106}$$

Let

$$1_2 := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; 0_2 := \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}; \beta^{[0]} := -\begin{bmatrix} 1_2 & 0_2 \\ 0_2 & 1_2 \end{bmatrix} = -1_4.$$

The Pauli[5] matrices:

$$\sigma_1 := \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \sigma_2 := \begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}; \sigma_3 := -\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

A set $\widetilde{C}$ of complex $n \times n$ matrices is called *a Clifford set* [6] *of rank n* [7] if the following conditions are fulfilled:

if $\alpha_k \in \widetilde{C}$ and $\alpha_r \in \widetilde{C}$ then $\alpha_k\alpha_r + \alpha_r\alpha_k = 2\delta_{k,r}$;

if $\alpha_k\alpha_r + \alpha_r\alpha_k = 2\delta_{k,r}$ for all elements α_r of set $\widetilde{C}$ then $\alpha_k \in \widetilde{C}$.

If $n = 4$ then a Clifford set either contains 3 matrices (*a Clifford triplet*) or contains 5 matrices (*a Clifford pentad*).

Here exist only six Clifford pentads [7]: one *light pentad* β:

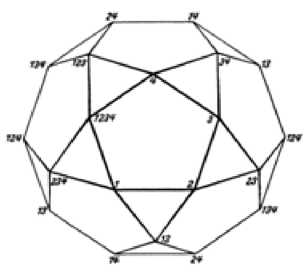


Abb. 1. Cliffordsche Zahlen.

$$\beta^{[1]} := \begin{bmatrix} \sigma_1 & 0_2 \\ 0_2 & -\sigma_1 \end{bmatrix}, \beta^{[2]} := \begin{bmatrix} \sigma_2 & 0_2 \\ 0_2 & -\sigma_2 \end{bmatrix}, \beta^{[3]} := \begin{bmatrix} \sigma_3 & 0_2 \\ 0_2 & -\sigma_3 \end{bmatrix}, \tag{2.107}$$

$$\gamma^{[0]} := \begin{bmatrix} 0_2 & 1_2 \\ 1_2 & 0_2 \end{bmatrix}, \tag{2.108}$$

$$\beta^{[4]} := \mathrm{i} \cdot \begin{bmatrix} 0_2 & 1_2 \\ -1_2 & 0_2 \end{bmatrix}; \tag{2.109}$$

three *chromatic* pentads:

the red pentad ζ:

$$\zeta^{[1]} = \begin{bmatrix} -\sigma_1 & 0_2 \\ 0_2 & \sigma_1 \end{bmatrix}, \zeta^{[2]} = \begin{bmatrix} \sigma_2 & 0_2 \\ 0_2 & \sigma_2 \end{bmatrix}, \zeta^{[3]} = \begin{bmatrix} -\sigma_3 & 0_2 \\ 0_2 & -\sigma_3 \end{bmatrix}, \tag{2.110}$$

$$\gamma_\zeta^{[0]} = \begin{bmatrix} 0_2 & -\sigma_1 \\ -\sigma_1 & 0_2 \end{bmatrix}, \zeta^{[4]} = \mathrm{i} \begin{bmatrix} 0_2 & \sigma_1 \\ -\sigma_1 & 0_2 \end{bmatrix}; \tag{2.111}$$

the green pentad η:

$$\eta^{[1]} = \begin{bmatrix} -\sigma_1 & 0_2 \\ 0_2 & -\sigma_1 \end{bmatrix}, \eta^{[2]} = \begin{bmatrix} -\sigma_2 & 0_2 \\ 0_2 & \sigma_2 \end{bmatrix}, \eta^{[3]} = \begin{bmatrix} \sigma_3 & 0_2 \\ 0_2 & \sigma_3 \end{bmatrix}, \tag{2.112}$$

$$\gamma_\eta^{[0]} = \begin{bmatrix} 0_2 & -\sigma_2 \\ -\sigma_2 & 0_2 \end{bmatrix}, \eta^{[4]} = \mathrm{i} \begin{bmatrix} 0_2 & \sigma_2 \\ -\sigma_2 & 0_2 \end{bmatrix}; \tag{2.113}$$

the blue pentad θ:

$$\theta^{[1]} = \begin{bmatrix} \sigma_1 & 0_2 \\ 0_2 & \sigma_1 \end{bmatrix}, \theta^{[2]} = \begin{bmatrix} -\sigma_2 & 0_2 \\ 0_2 & -\sigma_2 \end{bmatrix}, \theta^{[3]} = \begin{bmatrix} -\sigma_3 & 0_2 \\ 0_2 & \sigma_3 \end{bmatrix}, \tag{2.114}$$

[5] Wolfgang Ernst Pauli (German: 25 April 1900 15 December 1958) was an Austrian theoretical physicist

[6] William Kingdon Clifford (4 May 1845 3 March 1879) was an English mathematician and philosopher.

$$\gamma_{\theta}^{[0]} = \begin{bmatrix} 0_2 & -\sigma_3 \\ -\sigma_3 & 0_2 \end{bmatrix}, \theta^{[4]} = \mathrm{i}\begin{bmatrix} 0_2 & \sigma_3 \\ -\sigma_3 & 0_2 \end{bmatrix}; \tag{2.115}$$

two *gustatory* pentads:
the sweet pentad $\underline{\Delta}$:

$$\underline{\Delta}^{[1]} = \begin{bmatrix} 0_2 & -\sigma_1 \\ -\sigma_1 & 0_2 \end{bmatrix}, \underline{\Delta}^{[2]} = \begin{bmatrix} 0_2 & -\sigma_2 \\ -\sigma_2 & 0_2 \end{bmatrix}, \underline{\Delta}^{[3]} = \begin{bmatrix} 0_2 & -\sigma_3 \\ -\sigma_3 & 0_2 \end{bmatrix},$$

$$\underline{\Delta}^{[0]} = \begin{bmatrix} -1_2 & 0_2 \\ 0_2 & 1_2 \end{bmatrix}, \underline{\Delta}^{[4]} = \mathrm{i}\begin{bmatrix} 0_2 & 1_2 \\ -1_2 & 0_2 \end{bmatrix};$$

the bitter pentad $\underline{\Gamma}$:

$$\underline{\Gamma}^{[1]} = \mathrm{i}\begin{bmatrix} 0_2 & -\sigma_1 \\ \sigma_1 & 0_2 \end{bmatrix}, \underline{\Gamma}^{[2]} = \mathrm{i}\begin{bmatrix} 0_2 & -\sigma_2 \\ \sigma_2 & 0_2 \end{bmatrix}, \underline{\Gamma}^{[3]} = \mathrm{i}\begin{bmatrix} 0_2 & -\sigma_3 \\ \sigma_3 & 0_2 \end{bmatrix},$$

$$\underline{\Gamma}^{[0]} = \begin{bmatrix} -1_2 & 0_2 \\ 0_2 & 1_2 \end{bmatrix}, \underline{\Gamma}^{[4]} = \begin{bmatrix} 0_2 & 1_2 \\ 1_2 & 0_2 \end{bmatrix}.$$

Further we do not consider gustatory pentads since these pentads are not used yet in the contemporary physics.

Let in a coordinates system R^{μ} of a frame of reference $\left(\Re\mathbf{a}\widetilde{H}\right)$ in the instant x_0: the B coordinate be the following:
$\langle x \rangle = \langle x_0, x_1, x_2, x_3 \rangle = \langle \mathrm{c}t_0, \mathbf{x} \rangle$ (c = 299,792,458)

Let (t_A, x_A) be coordinates of event A and (t_B, x_B) be coordinates of event B.
In rhis case if

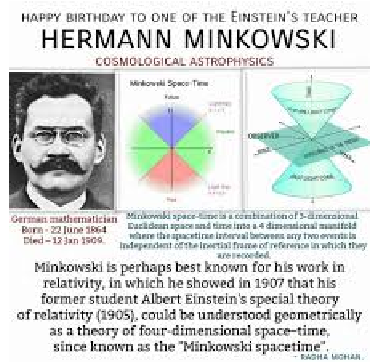
$$m(A;B) := \mathrm{c}^2 (t_B - t_A)^2 - (x_{B.1} - x_{A.1})^2 - (x_{B.2} - x_{A.2})^2 - (x_{B.3} - x_{A.3})^2$$

then $m(A;B)$ is called *the Minkovski interval*[7] between events A and B.
A Minkovski interval is invariant under the Cartesian[8] transformation:

[7]Hermann Minkowski 22 June 1864 12 January 1909) was a German mathematician and professor at K?nigsberg, Z?rich and G?ttingen.
[8]Ren? Descartes March 1596 11 February 1650) was a French philosopher, mathematician, and scientist .

$$
\begin{array}{l}
x_k \to x'_k := x_k \cos\lambda - x_j \sin\lambda, \\
x_j \to x'_j := x_j \cos\lambda + x_k \sin\lambda, \\
k \neq 0 \neq j.
\end{array}
$$

(turnabout of the coordinates system for angle λ)



And a Minkovski interval is invariant under the Lorentz transformation 2.104)

$$
\begin{array}{lcl}
x_0 & \to & x'_0 := x_0 \cosh\lambda - x_j \sinh\lambda; \\
x_j & \to & x'_j := x_j \cosh\lambda - x_0 \sinh\lambda;
\end{array}
$$

here:

$$
\cosh\lambda := \frac{1}{\sqrt{1-\frac{v^2}{c^2}}},\ \ \sinh\lambda := \frac{v}{c\sqrt{1-\frac{v^2}{c^2}}}
$$

If

$$
k(x) := \sum_{j=0}^{3} \beta^{[j]} x_j = \begin{bmatrix} -x_0 + x_3 & x_1 - \mathrm{i}x_2 & 0 & 0 \\ x_1 + \mathrm{i}x_2 & -x_0 - x_3 & 0 & 0 \\ 0 & 0 & -x_0 - x_3 & -x_1 + \mathrm{i}x_2 \\ 0 & 0 & -x_1 - \mathrm{i}x_2 & -x_0 + x_3 \end{bmatrix}
$$

then $k(x)$ is *the clift* of the point x.

Let

$$
U_{1,2} := \cos\lambda \cdot \beta^{[0]} + \sin\lambda \cdot \beta^{[1]}\beta^{[2]} \tag{2.116}
$$

Hence

$$
U_{1,2} = \begin{bmatrix} -\exp(-\mathrm{i}\lambda) & 0 & 0 & 0 \\ 0 & -\exp(\mathrm{i}\lambda) & 0 & 0 \\ 0 & 0 & -\exp(-\mathrm{i}\lambda) & 0 \\ 0 & 0 & 0 & -\exp(\mathrm{i}\lambda) \end{bmatrix}
$$

Hence

$$
U_{1,2}^{-1} k(x) U_{1,2} = \begin{bmatrix} x_3 - x_0 & x'_1 - ix'_2 & 0 & 0 \\ x'_1 + ix'_2 & -x_0 - x_3 & 0 & 0 \\ 0 & 0 & -x_0 - x_3 & -x'_1 + ix'_2 \\ 0 & 0 & -x'_1 - ix'_2 & x_3 - x_0 \end{bmatrix}
$$

Here

$$
\begin{array}{lcl}
x'_1 & = & (x_1 \cos 2\lambda + x_2 \sin 2\lambda); \\
x'_2 & = & (x_2 \cos 2\lambda - x_1 \sin 2\lambda).
\end{array}
$$

That is, $U_{1,2}$ mades the cartesian turnout of the $x_1 O\, x_2$ coordinate system on angle 2λ.

Similarly, if

$$U_{1,3} := \cos\lambda \cdot \beta^{[0]} + \sin\lambda \cdot \beta^{[1]}\beta^{[3]} \tag{2.117}$$

(that is

$$U_{1,3} = \begin{bmatrix} -\cos\lambda & -\sin\lambda & 0 & 0 \\ \sin\lambda & -\cos\lambda & 0 & 0 \\ 0 & 0 & -\cos\lambda & -\sin\lambda \\ 0 & 0 & \sin\lambda & -\cos\lambda \end{bmatrix}$$

) then $U_{1,3}$ mades the cartesian turnout of the $x_1 O\, x_3$ coordinate system on angle 2λ.
And if

$$U_{2,3} := \cos\lambda \cdot \beta[0] + \sin\lambda \cdot \beta^{[2]}\beta^{[3]} \tag{2.118}$$

(that is

$$U_{2,3} = \begin{bmatrix} -\cos\lambda & i\sin\lambda & 0 & 0 \\ i\sin\lambda & -\cos\lambda & 0 & 0 \\ 0 & 0 & -\cos\lambda & i\sin\lambda \\ 0 & 0 & i\sin\lambda & -\cos\lambda \end{bmatrix}$$

) then $U_{2,3}$ mades the cartesian turnout of the $x_2 O\, x_3$ coordinate system on angle 2λ.
Now if

$$U_{0,1} := \cosh\lambda \cdot \beta^{[0]} + \sinh\lambda \cdot \beta^{[0]}\beta^{[1]} \tag{2.119}$$

(that is

$$U_{0,1} = \begin{bmatrix} -\cosh\lambda & -\sinh\lambda & 0 & 0 \\ -\sinh\lambda & -\cosh\lambda & 0 & 0 \\ 0 & 0 & -\cosh\lambda & \sinh\lambda \\ 0 & 0 & \sinh\lambda & -\cosh\lambda \end{bmatrix}$$

) then

$$U_{0,1}^{\dagger} k(x)\, U_{0,1} = \begin{bmatrix} -x_0' + x_3 & x_1' - \mathrm{i}x_2 & 0 & 0 \\ x_1' + \mathrm{i}x_2 & -x_0' - x_3 & 0 & 0 \\ 0 & 0 & -x_0' - x_3 & -x_1' + \mathrm{i}x_2 \\ 0 & 0 & -x_1' - \mathrm{i}x_2 & -x_0' + x_3 \end{bmatrix}.$$

Here

$$\begin{aligned} x_0' &= x_0 \cosh 2\lambda - x_1 \sinh 2\lambda, \\ x_1' &= x_1 \cosh 2\lambda - x_0 \sinh 2\lambda. \end{aligned}$$

Hence, $U_{0,1}$ mades the Lorentz transformation betwin x_0 and x_1. .
Similarly, if

$$U_{0,2} := \cosh\lambda \cdot \beta^{[0]} + \sinh\lambda \cdot \beta^{[0]}\beta^{[2]} \tag{2.120}$$

(that is

$$U_{0,2} = \begin{bmatrix} -\cosh\lambda & i\sinh\lambda & 0 & 0 \\ -i\sinh\lambda & -\cosh\lambda & 0 & 0 \\ 0 & 0 & -\cosh\lambda & -i\sinh\lambda \\ 0 & 0 & i\sinh\lambda & -\cosh\lambda \end{bmatrix}$$

) then $U_{0,2}$ mades the Lorentz transformation betwin x_0 and x_2.
And if

$$U_{0,3} := \cosh\lambda \cdot \beta^{[0]} + \sinh\lambda \cdot \beta^{[0]}\beta^{[3]} \tag{2.121}$$

(that is

$$U_{0,3} = \begin{bmatrix} -e^{\lambda} & 0 & 0 & 0 \\ 0 & -e^{-\lambda} & 0 & 0 \\ 0 & 0 & -e^{-\lambda} & 0 \\ 0 & & 0 & -e^{\lambda} \end{bmatrix}$$

) then $U_{0,3}$ mades the Lorentz transformation betwin x_0 and x_3.
And if

$$U_{0,3} := \cosh\lambda \cdot \beta^{[0]} + \sinh\lambda \cdot \beta^{[0]}\beta^{[3]} \tag{2.122}$$

(that is

$$U_{0,3} = \begin{bmatrix} -e^{\lambda} & 0 & 0 & 0 \\ 0 & -e^{-\lambda} & 0 & 0 \\ 0 & 0 & -e^{-\lambda} & 0 \\ 0 & & 0 & -e^{\lambda} \end{bmatrix}$$

) then $U_{0,3}$ mades the Lorentz transformation betwin x_0 and x_3.
And if

$$U_{0,3} := \cosh\lambda \cdot \beta^{[0]} + \sinh\lambda \cdot \beta^{[0]}\beta^{[3]} \tag{2.123}$$

(that is

$$U_{0,3} = \begin{bmatrix} -e^{\lambda} & 0 & 0 & 0 \\ 0 & -e^{-\lambda} & 0 & 0 \\ 0 & 0 & -e^{-\lambda} & 0 \\ 0 & & 0 & -e^{\lambda} \end{bmatrix}$$

) then $U_{0,3}$ mades the Lorentz transformation betwin x_0 and x_3.
Two more matrices exist here. which do not change the Minkovski interval:

$$\widetilde{U} := \begin{bmatrix} e^{i\lambda} & 0 & 0 & 0 \\ 0 & e^{i\lambda} & 0 & 0 \\ 0 & 0 & e^{i2\lambda} & 0 \\ 0 & 0 & 0 & e^{i2\lambda} \end{bmatrix} \text{ and } \widehat{U} := \begin{bmatrix} e^{\lambda} & 0 & 0 & 0 \\ 0 & e^{\lambda} & 0 & 0 \\ 0 & 0 & e^{2\lambda} & 0 \\ 0 & 0 & 0 & e^{2\lambda} \end{bmatrix} \tag{2.124}$$

Here:

$$\widetilde{U}^{\dagger} k(x) \widetilde{U} = k(x), \widehat{U}^{-1} k(x) \widehat{U} = k$$

Chapter 3

Logic and Probability

3.1. Logic

> Logic as a scientific method is used for evidence of obvious and clear things that do not need any proof in view of their obviousness. The more obvious the thing, the more logic and less all the other in it, do not belong to the logic. It is clear that logic as such is most obvious of all things, since there is nothing at all except logic in it. It is for this reason that pure logic does not need any proofs or explanations and does not require any additional logic to understand it.
> A. S. Shlenski@, Short treatise on logic

Further I set out the version of the Gentzen[1] Natural Propositional calculus (NPC) [9]:

Def. 1.1.4: A sentence C is called *conjunction* of sentences A and B (design.: $C = (A\&B)$) if C is true, if and only if A and B are true.

Def. 1.1.5: A sentence C is called *negation* of sentences A (design.: $C = (\neg A)$) if C is true, if and only if A is not true.

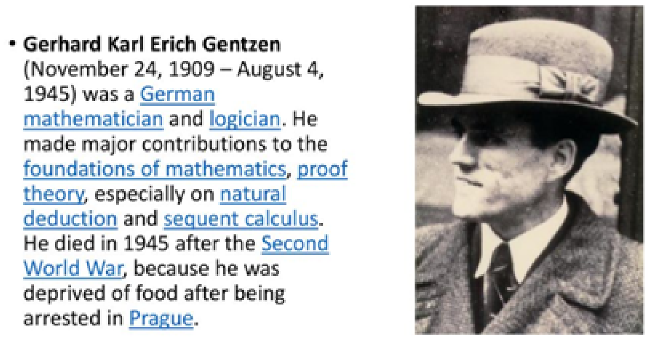
Def. 1.1.6: A sentence C is called *disjunction* of sentences A and B (design.: $C = (A \vee B)$) if C is true, if and only if A is true or B is true or both A and B are true.

Def. 1.1.7: A sentence C is called *implication* of sentences A and B (design.: $C = (A \Rightarrow B)$) if C is true, if and only if B is true and/or B is false.

A sentence is called *a simple sentence* if it isn't neither conjunction, nor a disjunction, neither implication, nor negation.

Th. 1.1.1:

[1] Gerhard Karl Erich Gentzen (November 24, 1909 August 4, 1945) was a German mathematician and logician. He made major contributions to the foundations of mathematics, proo theory, especially on natural deduction and sequent calculus. He died of starvation in a Soviet prison camp in Prague in 1945, having been interned as a German national after the Second World War



1) $(A\&A) = A$; $(A \vee A) = A$;
2) $(A\&B) = (B\&A)$; $(A \vee B) = (B \vee A)$;
3) $(A\&(B\&C)) = ((A\&B)\&C)$; $(A \vee (B \vee C)) = ((A \vee B) \vee C)$;
4) if T is a true sentence then for every sentence A: $(A\&T) = A$ and $(A \vee T) = T$.
5) if F is a false sentence then $(A\&F) = F$ and $(A \vee F) = A$.

Proof of Th. 1.1.1: This theorem directly follows from Def. 1.1.1, 1.2, 1.3, 1.4, 1.6. □

Further I set out the version of the Gentzen Natural Propositional calculus[2] (NPC) [9]:

Expression "Sentence C is a logical consequence of a list of sentences Γ." will be wrote as the following: "$\Gamma \vdash C$". Such expressions are called *sequences*. Elements of list Γ are called *hypothesizes*.

Def. 1.1.8

1. A sequence of form $C \vdash C$ is called *NPC axiom*.

2. A sequence of form $\Gamma \vdash A$ and $\Gamma \vdash B$ is obtained from sequences of form $\Gamma \vdash (A\&B)$ by *a conjunction removing rule* (design.: R&).

3. A sequence of form $\Gamma_1, \Gamma_2 \vdash (A\&B)$ is obtained from sequence of form $\Gamma_1 \vdash A$ and a sequence of form $\Gamma_2 \vdash B$ by *a conjunction inputting rule* (design: I&).

4. A sequence of form $\Gamma \vdash (A \vee B)$ is obtained from a sequence of form $\Gamma \vdash A$ or from a sequence of form $\Gamma \vdash B$ by *a disjunction inputting rule* (design.: I$\vee$).

5. A sequence of form $\Gamma_1[A], \Gamma_2[B], \Gamma_3 \vdash C$ is obtained from sequences of form $\Gamma_1 \vdash C$, $\Gamma_2 \vdash C$, snd $\Gamma_3 \vdash (A \vee B)$ by *a disjunction removing rule* (design.: R$\vee$) (Here and further: $\Gamma_1[A]$ is obtained from Γ_1 by removing of sentence A, and $\Gamma_2[B]$ is obtained from Γ_2 by removing of sentence B).

6. A sequence of form $\Gamma_1, \Gamma_2 \vdash B$ is obtained from a sequence of form $\Gamma_1 \vdash A$ and from a sequence of form $\Gamma_2 \vdash (A \Rightarrow B)$ by *a implication removing rule* (design.: R$\Rightarrow$).

7. A sequence of form $\Gamma[A] \vdash (A \Rightarrow B)$ is obtained from a sequence of form $\Gamma \vdash B$ by *an implication inputting rule* (design.: I$\Rightarrow$).

8. A sequence of form $\Gamma \vdash C$ is obtained from a sequence of form $\Gamma \vdash (\neg(\neg C))$ by *a negation removing* rule (design.: R$\neg$).

[2]Gerhard Karl Erich Gentzen (November 24, 1909, Greifswald, Germany August 4, 1945, Prague, Czechoslovakia) was a German mathematician and logician.

9. A sequence of form $\Gamma_1[C], \Gamma_2[C] \vdash (\neg C)$ is obtained from a sequence of form $\Gamma_1 \vdash A$ and from a sequence of form $\Gamma_2 \vdash (\neg A)$ by *negation inputting rule* (design.: I¬).

10. A finite string of sequences is called *a propositional natural deduction* if every element of this string either is a NPC axioms or is received from preceding sequences by one of the deduction rules (R&, I&, I∨, R∨, R⇒, I⇒, R¬, I¬).

Actually, these logical rules look naturally in light of the previous definitions.

Example 1: Let us consider the following string of sequences:

$$\begin{aligned}
&1. ((R\&S)\&(R \Rightarrow G)) \vdash ((R\&S)\&(R \Rightarrow G)) \text{ - NPC axiom.}\\
&2. ((R\&S)\&(R \Rightarrow G)) \vdash (R\&S) \text{ - R\& from 1.}\\
&3. ((R\&S)\&(R \Rightarrow G)) \vdash (R \Rightarrow G) \text{ - R\& from 1.}\\
&4. ((R\&S)\&(R \Rightarrow G)) \vdash R \text{ - R\& from 2.}\\
&5. ((R\&S)\&(R \Rightarrow G)) \vdash G \text{ - R} \Rightarrow \text{ from 3. and 4.}\\
&6. ((R\&S)\&(R \Rightarrow G)) \vdash S \text{ - R\& from 2.}\\
&7. ((R\&S)\&(R \Rightarrow G)) \vdash (G\&S) \text{ - I\& from 5. and 6.}
\end{aligned} \tag{3.1}$$

This string is a propositional natural deduction of sequence

$$((R\&S)\&(R \Rightarrow G)) \vdash (G\&S).$$

since it fulfills to all conditions of Def. 1.1.8.

Hence sentence $(G\&S)$ is logical consequence from sentence $((R\&S)\&(R \Rightarrow G))$.

Th. 1.1.2:

$$(A \vee B) = (\neg((\neg A)\&(\neg B))), \tag{3.2}$$

$$(A \Rightarrow B) = (\neg(A\&(\neg B))). \tag{3.3}$$

Proof of Th. 1.1.2:

The following string is a deduction of sequence $(A \vee B) \vdash (\neg((\neg A)\&(\neg B)))$:

1. $((\neg A)\&(\neg B)) \vdash ((\neg A)\&(\neg B))$, NPC axiom.
2. $((\neg A)\&(\neg B)) \vdash (\neg A)$, R& from 1.
3. $A \vdash A$, NPC axiom.
4. $A \vdash (\neg((\neg A)\&(\neg B)))$, I¬ from 2. and 3.
5. $((\neg A)\&(\neg B)) \vdash (\neg B)$, R& from 1.
6. $B \vdash B$, NPC axiom.
7. $B \vdash (\neg((\neg A)\&(\neg B)))$, I¬ from 5. and 6.
8. $(A \vee B) \vdash (A \vee B)$, NPC axiom.
9. $(A \vee B) \vdash (\neg((\neg A)\&(\neg B)))$, R∨ from 4., 7. and 8.

A deduction of sequence $(\neg((\neg A)\&(\neg B))) \vdash (A \vee B)$ is the following:

1. $(\neg A) \vdash (\neg A)$, NPC axiom.
2. $(\neg B) \vdash (\neg B)$, NPC axiom.

3. $(\neg A),(\neg B)\vdash((\neg A)\&(\neg B))$, I& from 1. and 2.
4. $(\neg((\neg A)\&(\neg B)))\vdash(\neg((\neg A)\&(\neg B)))$, NPC axiom.
5. $(\neg((\neg A)\&(\neg B))),(\neg B)\vdash(\neg(\neg A))$, I¬ from 3. and 4.
6. $(\neg((\neg A)\&(\neg B))),(\neg B)\vdash A$, R¬ from 5.
7. $(\neg((\neg A)\&(\neg B))),(\neg B)\vdash(A\vee B)$, i∨ from 6.
8. $(\neg(A\vee B))\vdash(\neg(A\vee B))$, NPC axiom.
9. $(\neg((\neg A)\&(\neg B))),(\neg(A\vee B))\vdash(\neg(\neg B))$, I¬ from 7. and 8.
10. $(\neg((\neg A)\&(\neg B))),(\neg(A\vee B))\vdash B$, R¬ from 9.
11. $(\neg((\neg A)\&(\neg B))),(\neg(A\vee B))\vdash(A\vee B)$, I∨ from 10.
12. $(\neg((\neg A)\&(\neg B)))\vdash(\neg(\neg(A\vee B)))$, I¬ from 8. and 11.
13. $(\neg((\neg A)\&(\neg B)))\vdash(A\vee B)$, R¬ from 12.
Therefore,

$$(\neg((\neg A)\&(\neg B)))=(A\vee B).$$

A deduction of sequence $(A\Rightarrow B)\vdash(\neg(A\&(\neg B)))$ is the following:
1. $(A\&(\neg B))\vdash(A\&(\neg B))$, NPC axiom.
2. $(A\&(\neg B))\vdash A$, R& from 1.
3. $(A\&(\neg B))\vdash(\neg B)$, R& from 1.
4. $(A\Rightarrow B)\vdash(A\Rightarrow B)$, NPC axiom.
5. $(A\&(\neg B)),(A\Rightarrow B)\vdash B$, R⇒ from 2. and 4.
6. $(A\Rightarrow B)\vdash(\neg(A\&(\neg B)))$, I¬ from 3. and 5.
A deduction of sequence $(\neg(A\&(\neg B)))\vdash(A\Rightarrow B)$ is the following:
1. $A\vdash A$, NPC axiom.
2. $(\neg B)\vdash(\neg B)$, NPC axiom.
3. $A,(\neg B)\vdash(A\&(\neg B))$, I& from 1. and 2.
4. $(\neg(A\&(\neg B)))\vdash(\neg(A\&(\neg B)))$, NPC axiom.
5. $A,(\neg(A\&(\neg B)))\vdash(\neg(\neg B))$, I¬ from 3. and 4.
6. $A,(\neg(A\&(\neg B)))\vdash B$, R¬ from 5.
7. $(\neg(A\&(\neg B)))\vdash(A\Rightarrow B)$, I⇒ from 6.
Therefore,

$$(\neg(A\&(\neg B)))=(A\Rightarrow B)\square$$

Example 2:
1. $A\vdash A$ - NPC axiom.
2. $(A\Rightarrow B)\vdash(A\Rightarrow B)$ - NPC axiom.
3. $A,(A\Rightarrow B)\vdash B$ - R⇒ from 1. and 2.
4. $(\neg B)\vdash(\neg B)$ - NPC axiom.
5. $(\neg B),(A\Rightarrow B)\vdash(\neg A)$ - I¬ from 3. and 4.
6. $(A\Rightarrow B)\vdash((\neg B)\Rightarrow(\neg A))$ - I⇒ from 5.
7. $\vdash((A\Rightarrow B)\Rightarrow((\neg B)\Rightarrow(\neg A)))$ - I⇒ from 6.
This string is a deduction of sentence of form

$$((A\Rightarrow B)\Rightarrow((\neg B)\Rightarrow(\neg A)))$$

from the empty list of sentences. I.e. sentences of such form are *logicaly provable*.

Th. 1.1.3: If sequence $\Gamma\rightarrow C$ is deduced and C is false then some false sentence is contained in Γ.

Proof of Th. 1.1.3: is received by induction of number of sequences in the deduction of sequence $\Gamma \to C$.

The recursion Basis: Let the deduction of sequence $\Gamma \to C$ contains single sentence. In accordance the definition of propositional natural deduction this deduction must be of the following type: $C \to C$. Obviously, in this case the lemma holds true.

The recursion Step:The recursion assumption: Let's admit that the lemma is carried out for any deduction which contains no more than n sequences.

Let deduction of $\Gamma \to C$ contains $n+1$ sequence. In accordance with the propositional natural deduction definition sequence $\Gamma \to C$ can be axiom NPC or can be received by the deduction rules from previous sequence.

a) If $\Gamma \to C$ is the NPC axiom then see the recursion basis.

b) Let $\Gamma \to C$ be received by R&. In this case sequence of type $\Gamma \to (C\&B)$ or sequence of type $\Gamma \to (B\&C)$ is contained among the previous sequences of this deduction. Hence, deductions of sequences $\Gamma \to (C\&B)$ and $\Gamma \to (B\&C)$ contains no more than n sequences. In accordance with the recursion assumption, these deductions submit to the lemma. Because C is false then $(C\&B)$ is false and $(B\&C)$ is false in accordance with the conjunction definition. Therefore, Γ contains some false sentence by the lemma. And in this case the lemma holds true.

c) Let $\Gamma \to C$ be received by I&. In this case sequence of type $\Gamma_1 \to A$ and sequence of type $\Gamma_2 \to B$ is contained among the previous sequences of this deduction, and $C = (A\&B)$ and $\Gamma = \Gamma_1, \Gamma_2$. Deductions of sequences $\Gamma_1 \to A$ and $\Gamma_2 \to B$ contains no more than n sequences. In accordance with the recursion assumption, these deductions submit to the lemma. Because C is false then A is false or B is false in accordance with the conjunction definition. Therefore, Γ contains some false sentence by the lemma. And in this case the lemma holds true.

d) Let $\Gamma \to C$ be received by R$\vee$. In this case sequences of type $\Gamma_1 \to (A \vee B)$, $\Gamma_2 [A] \to C$, and $\Gamma_3 [B] \to C$ are contained among the previous sequences of this deduction, and $\Gamma = \Gamma_1, \Gamma_2, \Gamma_3$. Because these previous deductions contain no more than n sequences then in accordance with the recursion assumption, these deductions submit to the lemma. Because C is false then $\Gamma_2 [A]$ contains some false sentence, and $\Gamma_3 [B]$ contains some false sentence. If A is true then the false sentence is contained in Γ_2. If B is true then the false sentence is contained in Γ_3. I.e. in these case some false sentence is contained in Γ. If A is false and B is false then $(A \vee B)$ is false in accordance with the disjunction definition. In this case Γ_1 contains some false sentence. And in all these cases the lemma holds true.

e) Let $\Gamma \to C$ be received by I$\vee$. In this case sequence of type $\Gamma \to A$ or sequence of type $\Gamma \to B$ is contained among the previous sequences of this deduction, and $C = (A \vee B)$. Deductions of sequences $\Gamma \to A$ and $\Gamma \to B$ contains no more than n sequences. In accordance with the recursion assumption, these deductions submit to the lemma. Because C is false then A is false and B is false in accordance with the disjunction definition. Therefore, Γ contains some false sentence by the lemma. And in this case the lemma holds true.

f) Let $\Gamma \to C$ be received by R$\Rightarrow$. In this case sequences of type $\Gamma_1 \to (A \Rightarrow C)$, $\Gamma_2 \to A$ are contained among the previous sequences of this deduction, and $\Gamma = \Gamma_1, \Gamma_2$. Because these previous deductions contain no more than n sequences then in accordance with the recursion assumption, these deductions submit to the lemma. If A is false then Γ_2 contains some false sentence. If A is true then $(A \Rightarrow C)$ is false in accordance with the implication defination since C is false. And in all these cases the lemma holds true.

g) Let $\Gamma \to C$ be received by I$\Rightarrow$. In this case sequences of type $\Gamma[A] \to B$ is contained among the previous sequences of this deduction, and $C = (A \Rightarrow B)$. Because deduction of $\Gamma[A] \to B$ contains no more than n sequences then in accordance with the recursion assumption, this deduction submit to the lemma. Because C is false then A is true in accordance with the implication definition. Hence, some false sentence is contained in Γ. Therefore, in this case the lemma holds true.

i) Let $\Gamma \to C$ be received by R$\neg$. In this case sequence of type $\Gamma \to (\neg(\neg C))$ is contained among the previous sequences of this deduction. This previous deduction contains no more than n sequences then in accordance with the recursion assumption, this deduction submit to the lemma Because C is false then $(\neg(\neg C))$ is false in accordance with the negation definition. Therefore, Γ contains some false sentence by the lemma. And in this case the lemma holds true.

j) Let $\Gamma \to C$ be received by I$\neg$. In this case sequences of type $\Gamma_1[A] \to B$, and $\Gamma_2[A] \to (\neg B)$ are contained among the previous sequences of this deduction, and $\Gamma = \Gamma_1, \Gamma_2$ and $C = (\neg A)$. Because these previous deductions contain no more than n sequences then in accordance with the recursion assumption, these deductions submit to the lemma. Because C is false then A is true. Hence, some false sentence is contained in Γ because B is false or $(\neg B)$ is false in accordance with the negation definition. Therefore, in all these cases the lemma holds true.

The recursion step conclusion: If the lemma holds true for deductions containing n sequences then the lemma holds true for deduction containing $n+1$ sequences.

The recursion conclusion: Lemma holds true for all deductions $\square$.

Def. 1.1.9 A sentence is *naturally propositionally provable* if there exists a prpositional natural deduction of this sentence from the empty list.

In accordance with Th. 1.1.3 all naturally propositionally provable sentences are true because otherwise the list would appear not empty.

But some true sentences are not naturally propositionally provable.

Alphabet of Propositional Calculations:

1. symbols p_k with natural k are called *PC-letters*;
2. symbols $\cap$, $\cup$, $\supset$, $\hat{}$ are called *PC-symbols*;
3. (,) are called *brackets*.

Formula of Propositional Calculations:

1. any PC-letter is *PC-formula*.
2. if q and r are PC-formulas then $(q \cap r)$, $(q \cup r)$,$(q \supset r)$,$(\hat{}q)$ are *PC-formula*s;
3. except listed by the two first points of this definition no *PC-formulas* are exist.

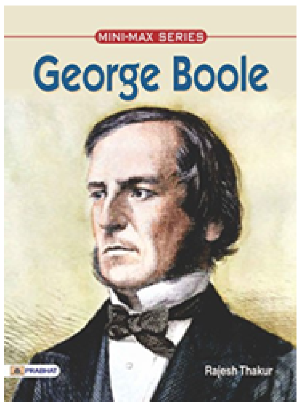
3.2. The Boole funcion

Def. 1.1.10 Let function $\mathfrak{g}$ has values on the double-elements set $\{0;1\}$ and has the set of PC-formulas as a domain. And let

1) $\mathfrak{g}(\hat{}q) = 1 - \mathfrak{g}(q)$ for every sentence q;
2) $\mathfrak{g}(q \cap r) = \mathfrak{g}(q) \cdot \mathfrak{g}(r)$ for all sentences q and r;
3) $\mathfrak{g}(q \cup r) = \mathfrak{g}(q) + \mathfrak{g}(r) - \mathfrak{g}(q) \cdot \mathfrak{g}(r)$ for all sentences q and r;

4) $\mathfrak{g}(q \supset r) = 1 - \mathfrak{g}(q) + \mathfrak{g}(q) \cdot \mathfrak{g}(r)$ for all sentences q and r.
In this case a function $\mathfrak{g}$ is called *a Boolean function* [3].
Hence if $\mathfrak{g}$ is a Boolean function then for every sentence q:

$$(\mathfrak{g}(q))^2 = \mathfrak{g}(q).$$



A Boolean function can be defined by a table:

$\mathfrak{g}(q)$	$\mathfrak{g}(r)$	$\mathfrak{g}(q \cap r)$	$\mathfrak{g}(q \cup r)$	$\mathfrak{g}(q \supset r)$	$\mathfrak{g}(\hat{\ } q)$
0	0	0	0	1	1
0	1	0	1	1	1
1	0	0	1	0	0
1	1	1	1	1	0

.

Such tables can be constructed for any sentence.
For example:

$\mathfrak{g}(q)$	$\mathfrak{g}(r)$	$\mathfrak{g}(s)$	$\mathfrak{g}(\hat{\ }((r \cap (\hat{\ } s)) \cap (\hat{\ } q)))$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

,

or:

$\mathfrak{g}(r)$	$\mathfrak{g}(s)$	$\mathfrak{g}(q)$	$\mathfrak{g}(((r \cap s) \cap (r \supset q)) \supset (q \cap s))$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

. (3.4)

Def. 1.1.11 A PC-formula q is called *a t-formula* if for any Boolean function $\mathfrak{g}$: $\mathfrak{g}(q) = 1$.

For example, formula $(((r \cap s) \cap (r \supset q)) \supset (q \cap s))$ is a t-formula by the table (3.4).

[3]George Boole (2 November 1815 8 December 1864) was an English mathematician and philosopher.

Def. 1.1.12 Function $\varphi(x)$ which is defined on the PC-formulas set and which has the sentences set as a range of values, is called *an interpretation function* if the following conditions are carried out:

1. if p_k is a PC-letter then $\varphi(p_k) = A$ and here A is a simple sentence and if $\varphi(p_s) = B$ then if $s \neq k$ then $A \neq B$;

2. $\varphi(r \cap s) = (\varphi(r) \& \varphi(s))$, $\varphi(r \cup s) = (\varphi(r) \vee \varphi(s))$, $\varphi(r \supset s) = (\varphi(r) \Rightarrow \varphi(s))$, $\varphi(\hat{}r) = (\neg\varphi(r))$.

Def. 1.1.13 A sencence C is called *tautology* if the following condition is carried out:

if $\varphi(q) = C$ then q is a t-formula.

Lm. 1.1.1: If $\mathfrak{g}$ is a Boolean function then every natural propositional deduction of sequence $\Gamma \vdash A$ satisfy the following condition: if $\mathfrak{g}\left(\varphi^{-1}(A)\right) = 0$ then there exists a sentence C such that $C \in \Gamma$ and $\mathfrak{g}\left(\varphi^{-1}(C)\right) = 0$.

Proof of Lm. 1.1.1: is maked by a recursion on a number of sequences in the deduction of $\Gamma \vdash A$:

1. Basis of recursion: Let the deduction of $\Gamma \vdash A$ contains 1 sequence.

In that case a form of this sequence is $A \vdash A$ in accordance with the propositional natural deduction definition (Def. 1.1.8). Hence in this case the lemma holds true.

2. Step of recursion: The recursion assumption: Let the lemma holds true for every deduction, containing no more than n sequences.

Let the deduction of $\Gamma \vdash A$ contains $n+1$ sequences.

In that case either this sequence is a NPC-axiom or $\Gamma \vdash A$ is obtained from previous sequences by one of deduction rules.

If $\Gamma \vdash A$ is a NPC-axiom then the proof is the same as for the recursion basis.

a) Let $\Gamma \vdash A$ be obtained from a previous sequence by $R\&$.

In that case a form of this previous sequence is either the following $\Gamma \vdash (A\&B)$ or is the following $\Gamma \vdash (B\&A)$ in accordance with the definition of deduction. The deduction of this sequence contains no more than n elements. Hence the lemma holds true for this deduction in accordance with the recursion assumption.

If $\mathfrak{g}\left(\varphi^{-1}(A)\right) = 0$ then $\mathfrak{g}\left(\varphi^{-1}(A\&B)\right) = 0$ and $\mathfrak{g}\left(\varphi^{-1}(B\&A)\right) = 0$ in accordance with the Boolean function definition (Def. 1.1.10). Hence there exists sentence C such that $C \in \Gamma$ and $\mathfrak{g}\left(\varphi^{-1}(C)\right) = 0$ in accordance with the lemma.

Hence in that case the lemma holds true for the deduction of sequence $\Gamma \vdash A$.

b) Let $\Gamma \vdash A$ be obtained from previous sequences by I&.

In that case forms of these previous sequences are $\Gamma_1 \vdash B$ and $\Gamma_2 \vdash G$ with $\Gamma = \Gamma_1, \Gamma_2$ and $A = (B\&G)$ in accordance with the definition of deduction.

The lemma holds true for deductions of sequences $\Gamma_1 \vdash B$ and $\Gamma_2 \vdash G$ in accordance with the recursion assumption because these deductions contain no more than n elements.

In that case if $\mathfrak{g}\left(\varphi^{-1}(A)\right) = 0$ then $\mathfrak{g}\left(\varphi^{-1}(B)\right) = 0$ or $\mathfrak{g}\left(\varphi^{-1}(G)\right) = 0$ in accordance with the Boolean function definition. Hence there exists sentence C such that $\mathfrak{g}\left(\varphi^{-1}(C)\right) = 0$ and $C \in \Gamma_1$ or $C \in \Gamma_2$.

Hence in that case the lemma holds true for the deduction of sequence $\Gamma \vdash A$.

c) Let $\Gamma \vdash A$ be obtained from a previous sequence by R$\neg$.

In that case a form of this previous sequence is the following: $\Gamma \vdash (\neg(\neg A))$ in accordance with the definition of deduction. The lemma holds true for the deduction of this sequence in accordance with the recursion assumption because this deduction contains no more than n elements.

If $\mathfrak{g}\left(\varphi^{-1}(A)\right)=0$ then $\mathfrak{g}\left(\varphi^{-1}(\neg(\neg A))\right)=0$ in accordance with the Boolean function definition. Hence there exists sentence C such that $C\in\Gamma$ and $\mathfrak{g}\left(\varphi^{-1}(C)\right)=0$.

Hence the lemma holds true for the deduction of sequence $\Gamma\vdash A$.

d) Let $\Gamma\vdash A$ be obtained from previous sequences by I$\neg$.

In that case forms of these previous sequences are $\Gamma_1\vdash B$ and $\Gamma_2\vdash(\neg B)$ with $\Gamma=\Gamma_1[G],\Gamma_2[G]$ and $A=(\neg G)$ in accordance with the definition of deduction.

The lemma holds true for the deductions of sequences $\Gamma_1\vdash B$ and $\Gamma_2\vdash(\neg B)$ in accordance with the recursion assumption because these deductions contain no more than n elements.

If $\mathfrak{g}\left(\varphi^{-1}(A)\right)=0$ then $\mathfrak{g}\left(\varphi^{-1}(G)\right)=1$ in accordance with the Boolean function definition.

Either $\mathfrak{g}\left(\varphi^{-1}(B)\right)=0$ or $\mathfrak{g}\left(\varphi^{-1}(\neg B)\right)=0$ by the same definition. Hence there exists sentence C such that either $C\in\Gamma_1[G]$ or $C\in\Gamma_2[G]$ and$\mathfrak{g}\left(\varphi^{-1}(C)\right)=0$ in accordance with the recursion assumption.

Hence in that case the lemma holds true for the deduction of sequence $\Gamma\vdash A$.

e) Let $\Gamma\vdash A$ be obtained from a previous sequence by I$\vee$.

In that case a form of A is $(B\vee G)$ and a form of this previous sequence is either $\Gamma\vdash B$ or $\Gamma\vdash G$ in accordance with the definition of deduction. The lemma holds true for this previous sequence deduction in accordance with the recursion assumption because this deduction contains no more than n elements.

If $\mathfrak{g}\left(\varphi^{-1}(A)\right)=0$ then $\mathfrak{g}\left(\varphi^{-1}(B)\right)=0$ and $\mathfrak{g}\left(\varphi^{-1}(G)=0\right.$ in accordance with the Boolean function definition. Hence there exists sentence C such that $C\in\Gamma$ and $\mathfrak{g}\left(\varphi^{-1}(C)\right)=0$.

Hence in that case the lemma holds true for the deduction of sequence $\Gamma\vdash A$.

f) Let $\Gamma\vdash A$ be obtained from previous sequences by R$\vee$.

Forms of these previous sequences are $\Gamma_1\vdash A$, $\Gamma_2\vdash A$, and $\Gamma_3\vdash(B\vee G)$ with $\Gamma=\Gamma_1[B],\Gamma_2[G],\Gamma_3$ in accordance with the definition of deduction. The lemma holds true for the deductions of these sequences in accordance with the recursion assumption because these deductions contain no more than n elements.

If $\mathfrak{g}\left(\varphi^{-1}(A)\right)=0$ then there exists sentence C_1 such that $C_1\in\Gamma_1$ and $\mathfrak{g}\left(\varphi^{-1}(C_1)\right)=0$, and there exists sentence C_2 such that $C_2\in\Gamma_2$ and $\mathfrak{g}\left(\varphi^{-1}(C_2)\right)=0$ in accordance with the lemma.

If $\mathfrak{g}\left(\varphi^{-1}(B\vee G)\right)=0$ then there exists sentence C such that $C\in\Gamma_3$ and $\mathfrak{g}\left(\varphi^{-1}(C)\right)=0$ in accordance with the lemma. Hence in that case the lemma holds true for the deduction of sequence $\Gamma\vdash A$.

If $\mathfrak{g}\left(\varphi^{-1}(B\vee G)\right)=1$ then either $\mathfrak{g}\left(\varphi^{-1}(B)\right)=1$ or $\mathfrak{g}\left(\varphi^{-1}(G)\right)=1$ in accordance with the Boolean function definition.

If $\mathfrak{g}\left(\varphi^{-1}(B)\right)=1$ then $C_1\in\Gamma_1[B]$. Hence in that case the lemma holds true for the deduction of sequence $\Gamma\vdash A$.

If $\mathfrak{g}\left(\varphi^{-1}(G)\right)=1$ then a result is the same.

Hence the lemma holds true for the deduction of sequence $\Gamma\vdash A$ in all these cases.

g) Let $\Gamma\vdash A$ be obtained from previous sequences by R$\Rightarrow$.

Forms of these previous sequences are $\Gamma_1 \vdash (B \Rightarrow A)$ and $\Gamma_2 \vdash (B)$ with $\Gamma = \Gamma_1, \Gamma_2$ in accordance with the definitions of deduction. Hence the lemma holds true for these deduction in accordance with the recursion assumption because these deductions contain no more than n elements.

If $\mathfrak{g}\left(\varphi^{-1}(B \Rightarrow A)\right) = 0$ then there exists sentence C such that $C \in \Gamma_1$ and $\mathfrak{g}\left(\varphi^{-1}(C)\right) = 0$ in accordance with the lemma. Hence in that case the lemma holds true for the deduction of sequence $\Gamma \vdash A$.

If $\mathfrak{g}\left(\varphi^{-1}(B \Rightarrow A)\right) = 1$ then $\mathfrak{g}\left(\varphi^{-1}(B)\right) = 0$ in accordance with the Boolean function definition. Hence there exists sentence C such that $C \in \Gamma_2$ and $\mathfrak{g}\left(\varphi^{-1}(C)\right) = 0$.

Hence the lemma holds true for sequence $\Gamma \vdash A$ in all these cases.

h) Let $\Gamma \vdash A$ be obtained from a previous sequence by I$\Rightarrow$.

In that case a form of sentence A is $(B \Rightarrow G)$ and a form of this previous sequence is $\Gamma_1 \vdash G$ with $\Gamma = \Gamma_1 [B]$ in accordance with the definition of deduction. The lemma holds true for the deduction of this sequence in accordance the recursion assumption because this deduction contain no more than n elements.

If $\mathfrak{g}\left(\varphi^{-1}(A)\right) = 0$ then $\mathfrak{g}\left(\varphi^{-1}(G)\right) = 0$ and $\mathfrak{g}\left(\varphi^{-1}(B)\right) = 1$ in accordance with the Boolean function definition. Hence there exists sentence C such that $C \in \Gamma_1 [B]$ and $\mathfrak{g}\left(\varphi^{-1}(C)\right) = 0$.

The recursion step conclusion: Therefore, in each possible case, if the lemma holds true for a deduction, containing no more than n elements, then the lemma holds true for a deduction contained $n+1$ elements.

The recursion conclusion: Therefore the lemma holds true for a deduction of any length $\square$

Th. 1.1.4: Each naturally propositionally proven sentence is a tautology.

Proof of Th. 1.1.4: If a sentence A is naturally propositionally proven then there exists a natural propositional deduction of form $\vdash A$ in accordance with Def. 1.1.9. Hence for every Boolean function g: $\mathfrak{g}\left(\varphi^{-1}(A)\right) = 1$ in accordance with Lm. 1.1.1. Hence sentence A is a tautology in accordance with the tautology definition (Def. 1.1.13) $\square$

Designation 1: Let $\mathfrak{g}$ be a Boolean function. In that case for every sentence A:

$$A^{\mathfrak{g}} := \left\{ \begin{array}{c} A \text{ if } \mathfrak{g}\left(\varphi^{-1}(A)\right) = 1, \\ (\neg A) \text{ if } \mathfrak{g}\left(\varphi^{-1}(A)\right) = 0. \end{array} \right|$$

Lm. 1.1.2: Let $B_1, B_2, .., B_k$ be the simple sentences making sentence A by PC-symbols ($\neg$, &, $\vee$, $\Rightarrow$).

Let $\mathfrak{g}$ be any Boolean function.

In that case there exist a propositional natural deduction of sequence

$$B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash A^{\mathfrak{g}}.$$

Proof of Lm. 1.1.2: is received by a recursion on a number of PC-symbols in sentence A.

Basis of recursion Let A does not contain PC-symbols . In this case the string of one sequence:

1. $A^{\mathfrak{g}} \vdash A^{\mathfrak{g}}$, NPC-axiom.

is a fit deduction.

Step of recursion: The recursion assumption: Let the lemma holds true for every sentence, containing no more than n PC-symbols.

Let sentence A contains $n+1$ PC-symbol. Let us consider all possible cases.

a) Let $A = (\neg G)$. In that case the lemma holds true for G in accordance with the recursion assumption because G contains no more than n PC-symbols. Hence there exists a deduction of sequence

$$B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash G^{\mathfrak{g}}, \tag{3.5}$$

here $B_1, B_2, .., B_k$ are the simple sentences, making up sentence G. Hence $B_1, B_2, .., B_k$ make up sentence A.

If $\mathfrak{g}\left(\varphi^{-1}(A)\right) = 1$ then

$$A^{\mathfrak{g}} = A = (\neg G)$$

in accordance with Designation 1.

In that case $\mathfrak{g}\left(\varphi^{-1}(G)\right) = 0$ in accordance with the Boolean function definition.

Hence

$$G^{\mathfrak{g}} = (\neg G) = A$$

in accordance with Designation 1.

Hence in that case a form of sequence (3.5) is the following:

$$B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash A^{\mathfrak{g}}.$$

Hence in that case the lemma holds true.

If $\mathfrak{g}\left(\varphi^{-1}(A)\right) = 0$ then

$$A^{\mathfrak{g}} = (\neg A) = (\neg (\neg G)).$$

in accordance with Designation 1.

In that case $\mathfrak{g}\left(\varphi^{-1}(G)\right) = 1$ in accordance with the Boolean function definition.

Hence

$$G^{\mathfrak{g}} = G$$

in accordance with Designation 1.

Hence in that case a form of sequence (3.5) is

$$B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash G.$$

Let us continue the deduction of this sequence in the following way:

1. $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash G$.
2. $(\neg G) \vdash (\neg G)$, NPC-axiom.
3. $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash (\neg (\neg G))$, I$\neg$ from 1. and 2.

It is a deduction of sequence

$$B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash A^{\mathfrak{g}}.$$

Hence in that case the lemma holds true.

b) Let $A = (G\&R)$.

In that case the lemma holds true both for G and for R in accordance with the recursion assumption because G and R contain no more than n PC-symbols. Hence there exist deductions of sequences

$$B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash G^{\mathfrak{g}} \tag{3.6}$$

and

$$B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash R^{\mathfrak{g}}, \tag{3.7}$$

here $B_1, B_2, .., B_k$ are the simple sentences, making up sentences G and R. Hence $B_1, B_2, .., B_k$ make up sentence A.

If $\mathfrak{g}\left(\varphi^{-1}(A)\right) = 1$ then

$$A^{\mathfrak{g}} = A = (G\&R)$$

in accordance with Designation 1.

In that case $\mathfrak{g}\left(\varphi^{-1}(G)\right) = 1$ and $\mathfrak{g}\left(\varphi^{-1}(R)\right) = 1$ in accordance with the Boolean function definition.

Hence $G^{\mathfrak{g}} = G$ and $R^{\mathfrak{g}} = R$ in accordance with Designation 1.

Let us continue deductions of sequences (3.6) and (3.7) in the following way:

1. $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash G$, (3.6).
2. $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash R$, (3.7).
3. $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash (G\&R)$, I& from 1. and 2.

It is deduction of sequence $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash A^{\mathfrak{g}}$.

Hence in that case the lemma holds true.

If $\mathfrak{g}\left(\varphi^{-1}(A)\right) = 0$ then

$$A^{\mathfrak{g}} = (\neg A) = (\neg (G\&R))$$

in accordance with Designation 1.

In that case $\mathfrak{g}(G) = 0$ or $\mathfrak{g}(R) = 0$ in accordance with the Boolean function definition.

Hence $G^{\mathfrak{g}} = (\neg G)$ or $R^{\mathfrak{g}} = (\neg R)$ in accordance with Designation 1.

Let $G^{\mathfrak{g}} = (\neg G)$.

In that case let us continue a deduction of sequence (3.6) in the following way:

1. $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash (\neg G)$, (3.6).
2. $(G\&R) \vdash (G\&R)$, NPC-axiom.
3. $(G\&R) \vdash G$, R& from 2.
4. $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash (\neg (G\&R))$, I¬ from 1. and 3.

It is a deduction of sequence $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash A^{\mathfrak{g}}$.

Hence in that case the lemma holds true.

The same result is received if $R^{\mathfrak{g}} = (\neg R)$.

c) Let $A = (G \vee R)$.

In that case the lemma holds true both for G and for R in accordance with the recursion assumption because G and R contain no more than n PC-symbols. Hence there exist a deductions of sequences

$$B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash G^{\mathfrak{g}} \tag{3.8}$$

and

$$B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash R^{\mathfrak{g}}, \tag{3.9}$$

here $B_1, B_2, .., B_k$ are the simple sentences, making up sentences G and R. Hence $B_1, B_2, .., B_k$ make up sentence A.

If $\mathfrak{g}\left(\varphi^{-1}(A)\right) = 0$ then

$$A^{\mathfrak{g}} = (\neg A) = (\neg (G \vee R))$$

in accordance with Designation 1.

In that case $\mathfrak{g}\left(\varphi^{-1}(G)\right) = 0$ and $\mathfrak{g}\left(\varphi^{-1}(R)\right) = 0$ in accordance with the Boolean function definition.

Hence $G^{\mathfrak{g}} = (\neg G)$ and $R^{\mathfrak{g}} = (\neg R)$ in accordance with Designation 1.

Let us continue deductions of sequences (3.8) and (3.9) in the following way:

1. $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash (\neg G)$, (3.8).
2. $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash (\neg R)$, (3.9).
3. $G \vdash G$, NPC-axiom.
4. $R \vdash R$, NPC-axiom.
5. $(G \vee R) \vdash (G \vee R)$, NPC-axiom.
6. $G, B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash (\neg (G \vee R))$, I$\neg$ from 1. and 3.
7. $R, B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash (\neg (G \vee R))$, I$\neg$ from 2. and 4.
8. $(G \vee R), B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash (\neg (G \vee R))$, R$\vee$ from 5., 6., and 7.
9. $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash (\neg (G \vee R))$, I$\neg$ from 7. and 8.

It is a deduction of sequence $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash A^{\mathfrak{g}}$.

Hence in that case the lemma holds true.

If $\mathfrak{g}\left(\varphi^{-1}(A)\right) = 1$ then

$$A^{\mathfrak{g}} = A = (G \vee R)$$

in accordance with Designation 1.

In that case $\mathfrak{g}\left(\varphi^{-1}(G)\right) = 1$ or $\mathfrak{g}\left(\varphi^{-1}(R)\right) = 1$ in accordance with the Boolean function definition.

Hence $G^{\mathfrak{g}} = G$ or $R^{\mathfrak{g}} = R$ in accordance with Designation 1.

If $G^{\mathfrak{g}} = G$ then let us continue deduction of sequence (3.8) in the following way:

1. $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash G$, (3.8).
2. $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash (G \vee R)$, I$\vee$ from 1.

It is deduction of sequence $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash A^{\mathfrak{g}}$.

Hence in that case the lemma holds true.

The same result is received if $R^{\mathfrak{g}} = R$.

d) Let $A = (G \Rightarrow R)$.

In that case the lemma holds true both for G and for R in accordance with the recursion assumption because G and R contain no more than n PC-symbols. Hence there exist deductions of sequences

$$B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash G^{\mathfrak{g}} \tag{3.10}$$

and

$$B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash R^{\mathfrak{g}}, \tag{3.11}$$

here $B_1, B_2, .., B_k$ are the simple sentence, making up sentences G and R. Hence $B_1, B_2, .., B_k$ make up sentence A.

If $\mathfrak{g}\left(\varphi^{-1}(A)\right) = 0$ then

$$A^{\mathfrak{g}} = (\neg A) = (\neg (G \Rightarrow R))$$

in accordance with Designation 1.

In that case $\mathfrak{g}\left(\varphi^{-1}(G)\right) = 1$ and $\mathfrak{g}\left(\varphi^{-1}(R)\right) = 0$ in accordance with the Boolean function deduction.

Hence $G^{\mathfrak{g}} = G$ and $R^{\mathfrak{g}} = (\neg R)$ in accordance with Designation 1.

Let us continue deduction of sequences (3.10) and (3.11) in the following way:

1. $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash G$, (3.10).
2. $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash (\neg R)$, (3.11).
3. $(G \Rightarrow R) \vdash (G \Rightarrow R)$, NPC-axiom.
4. $(G \Rightarrow R), B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash R$, R$\Rightarrow$ from 1. and 3.
5. $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash (\neg (G \Rightarrow R))$, I$\neg$ from 2. and 4.

It is deduction of sequence $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash A^{\mathfrak{g}}$.

Hence in that case the lemma holds true.

If $\mathfrak{g}\left(\varphi^{-1}(A)\right) = 1$ then

$$A^{\mathfrak{g}} = A = (G \Rightarrow R)$$

in accordance with Designation 1.

In that case $\mathfrak{g}\left(\varphi^{-1}(G)\right) = 0$ or $\mathfrak{g}\left(\varphi^{-1}(R)\right) = 1$ in accordance with the Boolean function definition.

Hence $G^{\mathfrak{g}} = (\neg G)$ or $R^{\mathfrak{g}} = R$ in accordance with Designation 1.

If $G^{\mathfrak{g}} = (\neg G)$ then let us continue a deduction of sequence (3.10) in the following way:

1. $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash (\neg G)$, (3.10).
2. $G \vdash G$, NPC-axiom.
3. $G, B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash (\neg (\neg R))$, I$\neg$ from 1. and 2.
4. $G, B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash R$, R$\neg$ from 3.
5. $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash (G \Rightarrow R)$, I$\Rightarrow$ from 4.

It is deduction of sequence $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash A^{\mathfrak{g}}$.

Hence in that case the lemma holds true.

If $R^{\mathfrak{g}} = R$ then let us continue a deduction of sequence (3.11) in the following way:

1. $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash R$, (3.11).
2. $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash (G \Rightarrow R)$, I$\Rightarrow$ from 1.

It is deduction of sequence $B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash A^{\mathfrak{g}}$.

Hence in that case the lemma holds true.

The recursion step conclusion: If the lemma holds true for sentences, containing no more than n PC-symbols, then the lemma holds true for sentences, containing $n+1$ PC-symbols.

The recursion conclusion: The lemma holds true for sentences, containing any number PC-symbols □

Th. 1.1.5 (Laszlo Kalmar)[4]: Each tautology is a naturally propositionally proven sentence.

Proof of Th. 1.1.5: Let sentence A be a tautology. That is for every Boolean function $\mathfrak{g}$: $\mathfrak{g}\left(\varphi^{-1}(A)\right) = 1$ in accordance with Def. 1.1.13.

Hence there exists a deduction for sequence

$$B_1^{\mathfrak{g}}, B_2^{\mathfrak{g}}, .., B_k^{\mathfrak{g}} \vdash A \tag{3.12}$$

for every Boolean function $\mathfrak{g}$ in accordance with Lm. 1.1.2.

There exist Boolean functions $\mathfrak{g}_1$ and $\mathfrak{g}_2$ such that

$$\mathfrak{g}_1\left(\varphi^{-1}(B_1)\right) = 0, \mathfrak{g}_2\left(\varphi^{-1}(B_1)\right) = 1,$$
$$\mathfrak{g}_1\left(\varphi^{-1}(B_s)\right) = \mathfrak{g}_2\left(\varphi^{-1}(B_s)\right) \text{ for } s \in \{2, .., k\}.$$

Forms of sequences (3.12) for these Boolean functions are the following:

$$(\neg B_1), B_2^{\mathfrak{g}_1}, .., B_k^{\mathfrak{g}_1} \vdash A, \tag{3.13}$$

$$B_1, B_2^{\mathfrak{g}_2}, .., B_k^{\mathfrak{g}_2} \vdash A. \tag{3.14}$$

Let us continue deductions these sequence in the following way:

1. $(\neg B_1), B_2^{g_1}, .., B_k^{g_1} \vdash A$, (3.13).
2. $B_1, B_2^{g_1}, .., B_k^{g_1} \vdash A$, (3.14).
3. $(\neg A) \vdash (\neg A)$, NPC-axiom.
4. $(\neg A), B_2^{g_1}, .., B_k^{g_1} \vdash (\neg(\neg B_1))$, I¬ from 1. and 3.
5. $(\neg A), B_2^{g_1}, .., B_k^{g_1} \vdash (\neg B_1)$, I¬ from 2. and 3.
6. $B_2^{g_1}, .., B_k^{g_1} \vdash (\neg(\neg A))$, I¬ from 4. and 5.
7. $B_2^{g_1}, .., B_k^{g_1} \vdash A$, R¬ from 6.

It is deduction of sequence $B_2^{g_1}, .., B_k^{g_1} \vdash A$. This sequence is obtained from sequence (3.12) by deletion of first sentence from the hypothesizes list.

All rest hypothesizes are deleted from this list in the similar way.

Final sentence is the following:

$$\vdash A.$$

□

Therefore, in accordance with Th. 1.1.3, all tautologies are true sentences.

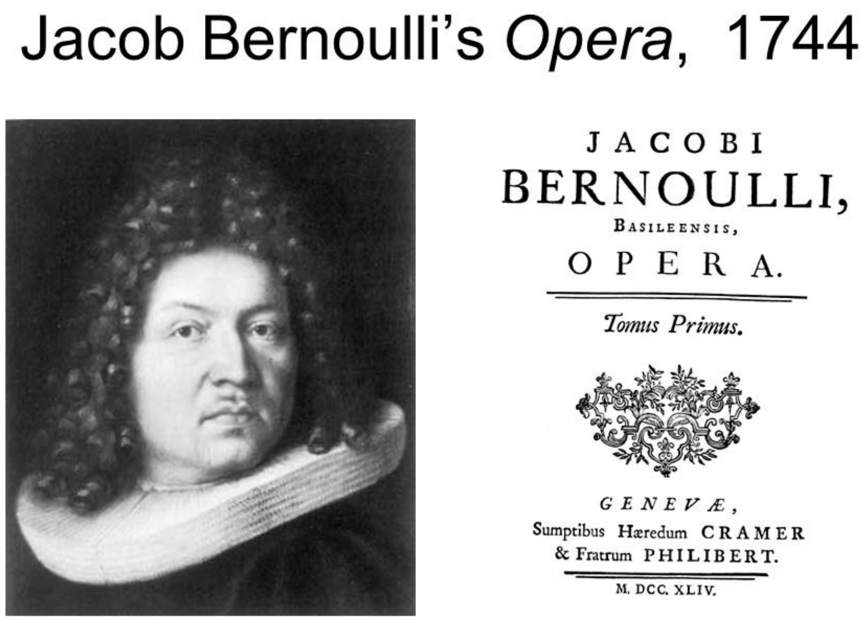
Therefore the natural propositional logic presents by Boolean functions.

[4]Laszlo Kalmar (March 27, 1905 August 2, 1976) was a Hungarian mathematician and Professor at the University of Szeged. Kalmar is considered the founder of mathematical logic and theoretical Computer Science in Hungary.

3.3. Probability

"The two greatest tyrants on earth: case and time"
Johann G. Herder

The first significant result in probability theory was obtained by the Swiss mathematician Jacob Bernoulli[5] in 1713 [10] (the Bernoulli Large Number law). Further, the development of the theory of probability went in two ways: using the axiomatic method, Soviet mathematician, Andrei Kolmogorov[6] embed this theory into mathematical analysis [11], and the American physicist Edwin Thompson Jaynes[7] began to develop the theory of probability from logic [12]. And we continue this way.



There is the evident nigh affinity between the classical probability function and the Boolean function of the classical propositional logic [**?**]. These functions are differed by the range of value, only. That is if the range of values of the Boolean function shall be

[5]Jacob Bernoulli (also known as James or Jacques; 6 January 1655 [O.S. 27 December 1654] 16 August 1705) was one of the many prominent mathematicians in the Bernoulli family.

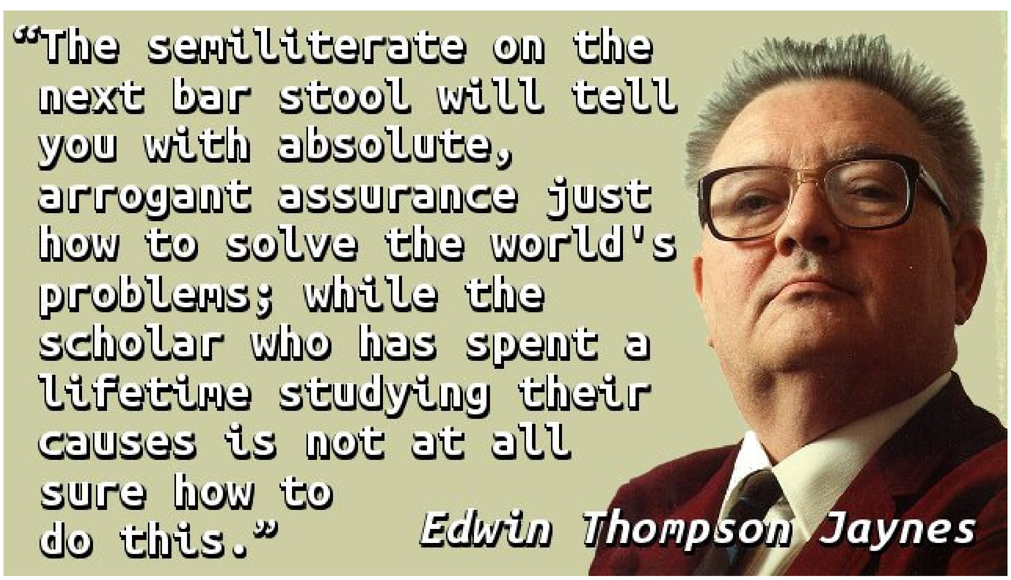
[6]Andrey Nikolaevich Kolmogorov, 25 April 1903 20 October 1987) was a 20th-century Soviet mathematician who made significant contributions to the mathematics of probability theory

[7]Edwin Thompson Jaynes (July 5, 1922 April 30, 1998) was the Wayman Crow Distinguished Professor of Physics at Washington University in St. Louis. He wrote extensively on statistical mechanics and on foundations of probability and statistical inference

Andrey Nikolaevich Kolmogorov

(1903, Tambov, Russia—1987 Moscow)

- Measure Theory
- Probability
- Analysis
- Intuitionistic Logic
- Cohomology
- Dynamical Systems
- Hydrodynamics
- Kolmogorov complexity



expanded from the two-elements set $\{0;1\}$ to the segment $[0;1]$ of the real numeric axis then the logical analog of the Bernoulli Large Number Law [10] can be deduced from the logical axioms. These topics is considered in this article.

Further we consider set of all meaningfull sentences.

3.3.1. Events

Def. 1.6.1.1: A set $\mathcal{B}$ of sentences is called *event, expressed by sentence C*, if the following conditions are fulfilled:

1. $C \in \mathcal{B}$;
2. if $A \in \mathcal{B}$ and $D \in \mathcal{B}$ then $A = D$;
3. if $D \in \mathcal{B}$ and $A = D$ then $A \in \mathcal{B}$.

In this case denote: $\mathcal{B} := {}^{\circ}C$.

Def. 1.6.1.2: An event $\mathcal{B}$ *occurs* if here exists a true sentence A such that $A \in \mathcal{B}$.

Def. 1.6.1.3: Events $\mathcal{A}$ and $\mathcal{B}$ *equal* (denote: $\mathcal{A} = \mathcal{B}$) if $\mathcal{A}$ occurs if and only if $\mathcal{B}$ occurs.

Def. 1.6.1.4: Event $\mathcal{C}$ is called *product* of event $\mathcal{A}$ and event $\mathcal{B}$ (denote: $\mathcal{C} = (\mathcal{A} \cdot \mathcal{B})$) if $\mathcal{C}$ occurs if and only if $\mathcal{A}$ occurs and $\mathcal{B}$ occurs.

Def. 1.6.1.5: Events $\mathcal{C}$ is called *complement* of event $\mathcal{A}$ (denote: $\mathcal{C} = (\#\mathcal{A})$) if $\mathcal{C}$ occurs if and only if $\mathcal{A}$ does not occur.

Def. 1.6.1.6: $(\mathcal{A} + \mathcal{B}) := (\#((\#\mathcal{A}) \cdot (\#\mathcal{B})))$. Event $(\mathcal{A} + \mathcal{B})$ is called *sum* of event $\mathcal{A}$ and event $\mathcal{B}$.

Therefore, the sum of event occurs if and only if there is at least one of the addends.

Def. 1.6.1.7: *The authentic event* (denote: $\mathcal{T}$) is the event which contains a tautology.

Hence, $\mathcal{T}$ occurs in accordance Def. 1.6.1.2:

The impossible event (denote: $\mathcal{F}$) is event which contains negation of a tautology.

Hence, $\mathcal{F}$ does not occur.

3.3.2. B-functions

Def. 1.6.2.1: Let $\mathfrak{b}(X)$ be a function defined on the set of events.

And let this function has values on he real numbers segment $[0;1]$.

Let there exists an event $\mathcal{C}_0$ such that $\mathfrak{b}(\mathcal{C}_0) = 1$.

Let for all events $\mathcal{A}$ and $\mathcal{B}$: $\mathfrak{b}(\mathcal{A} \cdot \mathcal{B}) + \mathfrak{b}(\mathcal{A} \cdot (\#\mathcal{B})) = \mathfrak{b}(\mathcal{A})$.

In that case function $\mathfrak{b}(X)$ is called *B-function*.

By this definition:

$$\mathfrak{b}(\mathcal{A} \cdot \mathcal{B}) \leq \mathfrak{b}(\mathcal{A}). \tag{3.15}$$

Hence, $\mathfrak{b}(\mathcal{T} \cdot \mathcal{C}_0) \leq \mathfrak{b}(\mathcal{T})$. Because $\mathcal{T} \cdot \mathcal{C}_0 = \mathcal{C}_0$ (by Def.1.6.1.4 and Def.1.6.1.7) then $\mathfrak{b}(\mathcal{C}_0) \leq \mathfrak{b}(\mathcal{T})$. Because $\mathfrak{b}(\mathcal{C}_0) = 1$then

$$\mathfrak{b}(\mathcal{T}) = 1. \tag{3.16}$$

From Def.1.6.2.1: $\mathfrak{b}(\mathcal{T} \cdot \mathcal{B}) + \mathfrak{b}(\mathcal{T} \cdot (\#\mathcal{B})) = \mathfrak{b}(\mathcal{T})$. Because $\mathcal{T}\mathcal{D} = \mathcal{D}$ for any $\mathcal{D}$ then $\mathfrak{b}(\mathcal{B}) + \mathfrak{b}(\#\mathcal{B}) = \mathfrak{b}(\mathcal{T})$. Hence, by (3.16): for any $\mathcal{B}$:

$$\mathfrak{b}(\mathcal{B})+b(\#\mathcal{B})=1. \tag{3.17}$$

Therefore, $\mathfrak{b}(\mathcal{T})+b(\#\mathcal{T})=1$. Hence, in accordance (3.16) : $1+b(\mathcal{F})=1$. Therefore,

$$\mathfrak{b}(\mathcal{F})=0. \tag{3.18}$$

In accordance with Def.1.6.2.1, Def.1.6.1.6, and (3.17):
$\mathfrak{b}(\mathcal{A}\cdot(\mathcal{B}+\mathcal{C}))=\mathfrak{b}(\mathcal{A}\cdot(\#((\#\mathcal{B})\cdot(\#\mathcal{C}))))=$
$=\mathfrak{b}(\mathcal{A})-\mathfrak{b}((\mathcal{A}\cdot(\#\mathcal{B}))\cdot(\#\mathcal{C}))=\mathfrak{b}(\mathcal{A})-\mathfrak{b}(\mathcal{A}\cdot(\#\mathcal{B}))+\mathfrak{b}((\mathcal{A}\cdot(\#\mathcal{B}))\cdot\mathcal{C})=$
$=\mathfrak{b}(\mathcal{A})-\mathfrak{b}(\mathcal{A})+\mathfrak{b}(\mathcal{A}\cdot\mathcal{B}))+\mathfrak{b}((\#\mathcal{B})\cdot(\mathcal{A}\cdot\mathcal{C}))=$
$=\mathfrak{b}(\mathcal{A}\cdot\mathcal{B}))+\mathfrak{b}(\mathcal{A}\cdot\mathcal{C})-\mathfrak{b}(\mathcal{B}\cdot\mathcal{A}\cdot\mathcal{C})$.
And $\mathfrak{b}((\mathcal{A}\cdot\mathcal{B})+(\mathcal{A}\cdot\mathcal{C}))=\mathfrak{b}(\#((\#(\mathcal{A}\cdot\mathcal{B}))\cdot(\#(\mathcal{A}\cdot\mathcal{C}))))=$
$=1-\mathfrak{b}((\#(\mathcal{A}\cdot\mathcal{B}))\cdot(\#(\mathcal{A}\cdot\mathcal{C})))=$
$=1-\mathfrak{b}(\#(\mathcal{A}\cdot\mathcal{B}))+\mathfrak{b}((\#(\mathcal{A}\cdot\mathcal{B}))\cdot(\mathcal{A}\cdot\mathcal{C}))=$
$=1-1+\mathfrak{b}(\mathcal{A}\cdot\mathcal{B})+\mathfrak{b}((\#(\mathcal{A}\cdot\mathcal{B}))\cdot(\mathcal{A}\cdot\mathcal{C}))=$
$=\mathfrak{b}(\mathcal{A}\cdot\mathcal{B})+\mathfrak{b}((\mathcal{A}\cdot\mathcal{C}))-\mathfrak{b}((\mathcal{A}\cdot\mathcal{B})\cdot(\mathcal{A}\cdot\mathcal{C}))=$
$=\mathfrak{b}(\mathcal{A}\cdot\mathcal{B})+\mathfrak{b}((\mathcal{A}\cdot\mathcal{C}))-\mathfrak{b}(\mathcal{A}\cdot\mathcal{B}\cdot\mathcal{C})$ because $\mathcal{A}\cdot\mathcal{A}=\mathcal{A}$.
Therefore:

$$\mathfrak{b}(\mathcal{A}\cdot(\mathcal{B}+\mathcal{C}))=\mathfrak{b}(\mathcal{A}\cdot\mathcal{B})+(\mathcal{A}\cdot\mathcal{C})-\mathfrak{b}(\mathcal{A}\cdot\mathcal{B}\cdot\mathcal{C})) \tag{3.19}$$

and

$$\mathfrak{b}((\mathcal{A}\cdot\mathcal{B})+(\mathcal{A}\cdot\mathcal{C}))=\mathfrak{b}(\mathcal{A}\cdot\mathcal{B}))+\mathfrak{b}(\mathcal{A}\cdot\mathcal{C})-\mathfrak{b}(\mathcal{A}\cdot\mathcal{B}\cdot\mathcal{C}). \tag{3.20}$$

Hence (*distributivity*):

$$\mathfrak{b}(\mathcal{A}\cdot(\mathcal{B}+\mathcal{C}))=\mathfrak{b}((\mathcal{A}\cdot\mathcal{B})+(\mathcal{A}\cdot\mathcal{C})). \tag{3.21}$$

If $\mathcal{A}=\mathcal{T}$ then from (3.19) and (3.20) (*the addition formula of probabilities*):

$$\mathfrak{b}(\mathcal{B}+\mathcal{C})=\mathfrak{b}(\mathcal{B})+\mathfrak{b}(\mathcal{C})-\mathfrak{b}(\mathcal{B}\cdot\mathcal{C}). \tag{3.22}$$

Def. 1.6.2.2– 19: Events $\mathcal{B}$ and $\mathcal{C}$ are *antithetical events* if $(\mathcal{B}\cdot\ \mathcal{C})=\mathcal{F}$.
From (3.22) and (3.18) for antithetical events $\mathcal{B}$ and $\mathcal{C}$:

$$\mathfrak{b}(\mathcal{B}+\mathcal{C})=\mathfrak{b}(\mathcal{B})+\mathfrak{b}(\mathcal{C}). \tag{3.23}$$

Def. 1.6.2.3-20: Events $\mathcal{B}$ and $\mathcal{C}$ are *independent for $\mathcal{B}$-function* $\mathfrak{b}$ *events* if $\mathfrak{b}(\mathcal{B}\cdot\mathcal{C})=\mathfrak{b}(\mathcal{B})\cdot\mathfrak{b}(\mathcal{B})$.

If events $\mathcal{B}$ and $\mathcal{C}$ are independent for B-function $\mathfrak{b}$ events then:
$\mathfrak{b}(\mathcal{B}\cdot(\#\mathcal{C}))=\mathfrak{b}(\mathcal{B})-\mathfrak{b}(\mathcal{B}\cdot\mathcal{C})=\mathfrak{b}(\mathcal{B})-\mathfrak{b}(\mathcal{B})\cdot\mathfrak{b}(\mathcal{C})=\mathfrak{b}(\mathcal{B})\cdot(1-\mathfrak{b}(\mathcal{C}))=\mathfrak{b}(\mathcal{B})\cdot\mathfrak{b}(\#\mathcal{C})$.
Hence, if events $\mathcal{B}$ and $\mathcal{C}$ are independent for $\mathcal{B}$-function b events then:

$$\mathfrak{b}(\mathcal{B}\cdot(\#\mathcal{C}))=\mathfrak{b}(\mathcal{B})\cdot\mathfrak{b}(\#\mathcal{C}). \tag{3.24}$$

Let calculate:

$$\mathfrak{b}(\mathcal{A}\cdot(\#\mathcal{A})\cdot\mathcal{C})=\mathfrak{b}(\mathcal{A}\cdot\mathcal{C})-\mathfrak{b}(\mathcal{A}\cdot\mathcal{A}\cdot\mathcal{C})=\mathfrak{b}(\mathcal{A}\cdot\mathcal{C})-\mathfrak{b}(\mathcal{A}\cdot\mathcal{C})=0. \tag{3.25}$$

3.3.3. Independent tests

Definition 1.6.3.1: Let $st(n)$ be a function such that $st(n)$ has domain on the set of natural numbers and has values in the events set.

In this case event $\mathcal{A}$ is a *[st]-series of range r* with *V- number k* if A, r and k fulfill to some one amongst the following conditions:

1) $r = 1$ and $k = 1$, $\mathcal{A} = st(1)$ or $k = 0$, $\mathcal{A} = (\# st(1))$;

2) $\mathcal{B}$ is [st]-series of range $r-1$ with V-number $k-1$ and

$$\mathcal{A} = (\mathcal{B} \cdot st(r)),$$

or $\mathcal{B}$ is [st]-series of range $r-1$ with V-number k and

$$\mathcal{A} = (\mathcal{B} \cdot (\# st(r))).$$

Let us denote a set of [st]-series of range r with V-number k as $[st](r,k)$.

For example, if $st(n)$ is a event $\mathcal{B}_n$ then the sentences:

$(\mathcal{B}_1 \cdot \mathcal{B}_2 \cdot (\#\mathcal{B}_3))$, $(\mathcal{B}_1 \cdot (\#\mathcal{B}_2) \cdot \mathcal{B}_3)$, $((\#\mathcal{B}_1) \cdot \mathcal{B}_2 \cdot \mathcal{B}_3)$

are the elements of $[st](3,2)$, and

$\left(\mathcal{B}_1 \cdot \mathcal{B}_2 \cdot (\#\mathcal{B}_3) \cdot \mathcal{B}_4 \cdot \overline{\mathcal{B}_5}\right) \in [st](5,3)$.

Definition 1.6.3.2: Function $st(n)$ is *independent* for B-function $\mathfrak{b}$ if for $\mathcal{A}$: if $\mathcal{A} \in [st](r,r)$ then:

$$\mathfrak{b}(\mathcal{A}) = \prod_{n=1}^{r} \mathfrak{b}(st(n)).$$

Definition 1.6.3.3: Let $st(n)$ be a function such that $st(n)$ has domain on the set of natural numbers and has values in the set of events.

In this case sentence $\mathcal{A}$ is *[st]-disjunction of range r* with *V-number k* (denote: $t[st](r,k)$) if $\mathcal{A}$ is the disjunction of all elements of $[st](r,k)$.

For example, if $st(n)$ is event $\mathcal{C}_n$ then:

$((\#\mathcal{C}_1) \cdot (\#\mathcal{C}_2) \cdot (\#\mathcal{C}_3)) = t[st](3,0)$,

$t[st](3,1) = ((\mathcal{C}_1 \cdot (\#\mathcal{C}_2) \cdot (\#\mathcal{C}_3)) + ((\#\mathcal{C}_1) \cdot \mathcal{C}_2 \cdot (\#\mathcal{C}_3)) + ((\#\mathcal{C}_1) \cdot (\#\mathcal{C}_2) \cdot \mathcal{C}_3))$,

$t[st](3,2) = ((\mathcal{C}_1 \cdot \mathcal{C}_2 \cdot (\#\mathcal{C}_3)) + ((\#\mathcal{C}_1) \cdot \mathcal{C}_2 \cdot \mathcal{C}_3) + (\mathcal{C}_1 \cdot (\#\mathcal{C}_2) \cdot \mathcal{C}_3))$,

$(\mathcal{C}_1 \cdot \mathcal{C}_2 \cdot \mathcal{C}_3) = t[st](3,3)$.

Definition 1.6.3.4: A rational number ω is called *frequency of sentence $\mathcal{A}$* in the [st]-series of r independent for B-function $\mathfrak{b}$ tests (designate: $\omega = \nu_r[st](\mathcal{A})$) if

1) $st(n)$ is independent for B-function $\mathfrak{b}$,

2) for all n: $\mathfrak{b}(st(n)) = \mathfrak{b}(\mathcal{A})$,

3) $t[st](r,k)$ is true and $\omega = k/r$.

Theorem: 1.6.3.1: (the J.Bernoulli[8] formula [10]**)** If $st(n)$ is independent for B-function $\mathfrak{b}$ and there exists a real number p such that for all n: $\mathfrak{b}(st(n)) = p$ then

$$\mathfrak{b}(t[st](r,k)) = \frac{r!}{k! \cdot (r-k)!} \cdot p^k \cdot (1-p)^{r-k}.$$

[8]Jacob Bernoulli (also known as James or Jacques) (27 December 1654 16 August 1705) was one of the many prominent mathematicians in the Bernoulli family.

Proof of the Theorem 1.6.3.1: By the Definition 1.6.3.2 and formula (3.24): if $\mathcal{B} \in [st](r,k)$ then:

$$\mathfrak{b}(\mathcal{B}) = p^k \cdot (1-p)^{r-k}.$$

Since $[st](r,k)$ contains $r!/(k! \cdot (r-k)!)$ elements then by the Theorems (3.24), (3.25) and (3.23) this Theorem is fulfilled.

Definition 1.6.3.5: Let function $st(n)$ has domain on the set of the natural numbers and has values in the set of the events.

Let function $f(r,k,l)$ has got the domain in the set of threes of the natural numbers and has got the range of values in the set of the events.

In this case $f(r,k,l) = T[st](r,k,l)$ if

1) $f(r,k,k) = t[st](r,k)$,

2) $f(r,k,l+1) = (f(r,k,l) + t[st](r,l+1))$.

Definition 1.6.3.6: If a and b are real numbers and $k-1 < a \leq k$ and $l \leq b < l+1$ then $T[st](r,a,b) = T[st](r,k,l)$.

Theorem: 1.6.3.2:

$$T[st](r,a,b) = ^{\circ}\ll \frac{a}{r} \leq \nu_r[st](\mathcal{A}) \leq \frac{b}{r} \gg.$$

Proof of the Theorem 1.6.3.2: By the Definition 1.6.3.6: there exist natural numbers r and k such that $k-1 < a \leq k$ and $l \leq b < l+1$.

The recursion on l:

1. Let $l = k$.

In this case by the Definition 1.6.3.4:

$$T[st](r,k,k) = t[st](r,k) = ^{\circ}\ll \nu_r[st](\mathcal{A}) = \frac{k}{r} \gg.$$

2. Let n be any natural number.

The recursive assumption: Let

$$T[st](r,k,k+n) = ^{\circ}\ll \frac{k}{r} \leq \nu_r[st](\mathcal{A}) \leq \frac{k+n}{r} \gg.$$

By the Definition 1.6.3.5:

$$T[st](r,k,k+n+1) = (T[st](r,k,k+n) + t[st](r,k+n+1)).$$

By the recursive assumption and by the Definition 1.6.3.4:

$$T[st](r,k,k+n+1) =$$

$$= (^{\circ}\ll \frac{k}{r} \leq \nu_r[st](\mathcal{A}) \leq \frac{k+n}{r} \gg + ^{\circ}\ll \nu_r[st](\mathcal{A}) = \frac{k+n+1}{r} \gg).$$

Hence, by the Definition 2.10:

$$T[st](r,k,k+n+1) = ^{\circ}\ll \frac{k}{r} \leq \nu_r[st](\mathcal{A}) \leq \frac{k+n+1}{r} \gg.$$

Theorem: 1.6.3.3 If $st(n)$ is independent for B-function $\mathfrak{b}$ and there exists a real number p such that $\mathfrak{b}(st(n)) = p$ for all n then

$$\mathfrak{b}(T[st](r,a,b)) = \sum_{a \leq k \leq b} \frac{r!}{k! \cdot (r-k)!} \cdot p^k \cdot (1-p)^{r-k}.$$

Proof of the Theorem 1.6.3.3: This is the consequence from the Theorem 1.6.3.1 by the Theorem 3.6.

Theorem: 1.6.3.4 If $st(n)$ is independent for the B-function $\mathfrak{b}$ and there exists a real number p such that $\mathfrak{b}(st(n)) = p$ for all n then

$$\mathfrak{b}(T[st](r, r \cdot (p-\varepsilon), r \cdot (p+\varepsilon))) \geq 1 - \frac{p \cdot (1-p)}{r \cdot \varepsilon^2}$$

for every positive real number ε.

Proof of the Theorem 1.6.3.4: Because

$$\sum_{k=0}^{r} (k - r \cdot p)^2 \cdot \frac{r!}{k! \cdot (r-k)!} \cdot p^k \cdot (1-p)^{r-k} = r \cdot p \cdot (1-p)$$

then if

$$J = \{k \in \mathbf{N} | 0 \leq k \leq r \cdot (p-\varepsilon)\} \cap \{k \in \mathbf{N} | r \cdot (p+\varepsilon) \leq k \leq r\}$$

then

$$\sum_{k \in J} \frac{r!}{k! \cdot (r-k)!} \cdot p^k \cdot (1-p)^{r-k} \leq \frac{p \cdot (1-p)}{r \cdot \varepsilon^2}.$$

Hence, by (3.17) this Theorem is fulfilled.

Hence

$$\lim_{r \to \infty} \mathfrak{b}(T[st](r, r \cdot (p-\varepsilon), r \cdot (p+\varepsilon))) = 1 \tag{3.26}$$

for all tiny positive numbers ε.

3.3.4. The logic probability function

Definition 1.6.4.1: B-function P is P-*function* if for every event $^\circ \ll \Theta \gg$:
If $P(^\circ \ll \Theta \gg) = 1$ then $\ll \Theta \gg$ is true sentence.

Hence from Theorem 1.6.3.2 and (3.26): if $\mathfrak{b}$ is a P-function then the sentence

$$\ll (p-\varepsilon) \leq \nu_r[st](\mathcal{A}) \leq (p+\varepsilon) \gg$$

is almost true sentence for large r and for all tiny ε. Therefore, it is almost truely that

$$\nu_r[st](\mathcal{A}) = p$$

for large r.

Therefore, it is almost true that

$$\mathfrak{b}(\mathcal{A}) = \nu_r[st](\mathcal{A})$$

for large r.

Therefore, the function, defined by the Definition 1.6.4.1 has got the statistical meaning. That is why I'm call such function as the logic probability function.

3.3.5. Conditional probability

Definition 1.6.5.1: *Conditional probability* $\mathcal{B}$ for $\mathcal{C}$ is the following function:

$$\mathfrak{b}(\mathcal{B}/\mathcal{C}) := \frac{\mathfrak{b}(\mathcal{C}\cdot\mathcal{B})}{\mathfrak{b}(\mathcal{C})}. \tag{3.27}$$

Theorem 1.6.5.1 The conditional probability function is a B-function.

Proof of Theorem 1.6.5.1 From Definition 1.6.5.1:

$$\mathfrak{b}(\mathcal{C}/\mathcal{C}) = \frac{\mathfrak{b}(\mathcal{C}\cdot\mathcal{C})}{\mathfrak{b}(\mathcal{C})}.$$

Hence by Theorem 1.1.1:

$$\mathfrak{b}(\mathcal{C}/\mathcal{C}) = \frac{\mathfrak{b}(\mathcal{C})}{\mathfrak{b}(\mathcal{C})} = 1.$$

Form Definition 1.6.5.1:

$$\mathfrak{b}((\mathcal{A}\cdot\mathcal{B})/\mathcal{C}) + \mathfrak{b}((\mathcal{A}\cdot(\#\mathcal{B}))/\mathcal{C}) = \frac{\mathfrak{b}(\mathcal{C}\cdot(\mathcal{A}\cdot\mathcal{B}))}{\mathfrak{b}(\mathcal{C})} + \frac{\mathfrak{b}(\mathcal{C}\cdot(\mathcal{A}\cdot(\#\mathcal{B})))}{\mathfrak{b}(\mathcal{C})}.$$

Hence:

$$\mathfrak{b}((\mathcal{A}\cdot\mathcal{B})/\mathcal{C}) + \mathfrak{b}((\mathcal{A}\cdot(\#\mathcal{B}))/\mathcal{C}) = \frac{\mathfrak{b}(\mathcal{C}\cdot(\mathcal{A}\cdot\mathcal{B})) + \mathfrak{b}(\mathcal{C}\cdot(\mathcal{A}\cdot(\#\mathcal{B})))}{\mathfrak{b}(\mathcal{C})}.$$

By Theorem 1.1.1:

$$\mathfrak{b}((\mathcal{A}\cdot\mathcal{B})/\mathcal{C}) + \mathfrak{b}((\mathcal{A}\cdot(\#\mathcal{B}))/\mathcal{C}) = \frac{\mathfrak{b}((\mathcal{C}\cdot\mathcal{A})\cdot\mathcal{B}) + \mathfrak{b}((\mathcal{C}\cdot\mathcal{A})\cdot(\#\mathcal{B}))}{\mathfrak{b}(\mathcal{C})}.$$

Hence by Definition 1.6.2.1:

$$\mathfrak{b}((\mathcal{A}\cdot\mathcal{B})/\mathcal{C}) + \mathfrak{b}((\mathcal{A}\cdot(\#\mathcal{B}))/\mathcal{C}) = \frac{\mathfrak{b}(\mathcal{C}\cdot\mathcal{A})}{\mathfrak{b}(\mathcal{C})}.$$

Hence by Definition 1.6.5.1:

$$\mathfrak{b}((\mathcal{A}\cdot\mathcal{B})/\mathcal{C}) + \mathfrak{b}((\mathcal{A}\cdot(\#\mathcal{B}))/\mathcal{C}) = \mathfrak{b}(\mathcal{A}/\mathcal{C}) \, \square$$

3.3.6. Classical probability

Let P be *P*-function.

Definition 1.6.6.1: $\{\mathcal{B}_1, \mathcal{B}_2, \ldots, \mathcal{B}_n\}$ is called as *complete set* if the following conditions are fulfilled:

1. if $k \neq s$ then $(\mathcal{B}_k \cdot \mathcal{B}_s)$ is a false sentence;

2. $(\mathcal{B}_1 + \mathcal{B}_2 + \ldots + \mathcal{B}_n)$ is a true sentence.

Definition 1.6.6.2: $\mathcal{B}$ is favorable for $\mathcal{A}$ if $(\mathcal{B} \cdot (\#\mathcal{A}))$ is a false sentence, and $\mathcal{B}$ is unfavorable for $\mathcal{A}$ if $(\mathcal{B} \wedge A)$ is a false sentence.

Let

1. $\{\mathcal{B}_1, \mathcal{B}_2, \ldots, \mathcal{B}_n\}$ be complete set;

2. for $k \in \{1, 2, \ldots, n\}$ and $s \in \{1, 2, \ldots, n\}$: $\mathrm{P}(\mathcal{B}_k) = \mathrm{P}(\mathcal{B}_s)$;

3. if $1 \leq k \leq m$ then $\mathcal{B}_k$ is favorable for $\mathcal{A}$, and if $m+1 \leq s \leq n$ then $\mathcal{B}_s$ is unfavorable for $\mathcal{A}$.

In that case from Theorem 1.1.1 and from (3.16) and (3.17):

$$\mathrm{P}((\#\mathcal{A}) \cdot \mathcal{B}_k) = 0$$

for $k \in \{1, 2, \ldots, m\}$ and

$$\mathrm{P}(\mathcal{A} \cdot \mathcal{B}_s) = 0$$

for $s \in \{m+1, m+2, \ldots, n\}$.

Hence from Definition 1.6.2.1:

$$\mathrm{P}(\mathcal{A} \cdot \mathcal{B}_k) = \mathrm{P}(\mathcal{B}_k)$$

for $k \in \{1, 2, \ldots, n\}$.

By point 4 of Theorem 1.1.1:

$$\mathcal{A} = (\mathcal{A} \cdot (\mathcal{B}_1 + \mathcal{B}_2 + \ldots + \mathcal{B}_m + \mathcal{B}_{m+1} \ldots + \mathcal{B}_n)).$$

Hence by formula (3.21):

$\mathrm{P}(\mathcal{A}) = \mathrm{P}(\mathcal{A} \cdot \mathcal{B}_1) + \mathrm{P}(\mathcal{A} \cdot \mathcal{B}_2) + \ldots +$
$+\mathrm{P}(\mathcal{A} \cdot \mathcal{B}_m) + \mathrm{P}(\mathcal{A} \cdot \mathcal{B}_{m+1}) + \ldots + \mathrm{P}(\mathcal{A} \cdot \mathcal{B}_n) =$
$= \mathrm{P}(\mathcal{B}_1) + \mathrm{P}(\mathcal{B}_2) + \ldots + \mathrm{P}(\mathcal{B}_m).$

Therefore

$$\mathrm{P}(\mathcal{A}) = \frac{m}{n}.$$

3.3.7. Probability and logic

Let P be the probability function and let B be the set of events A such that either A occurs or $(\#A)$ occurs.

In this case if $\mathrm{P}(A) = 1$ then A occurs, and $(A \cdot B) = B$ in accordance with Def. 1.6.1.4. Consequently, if $\mathrm{P}(B) = 1$ then $\mathrm{P}(A \cdot B) = 1$. Hence, in this case $\mathrm{P}(A \cdot B) = \mathrm{P}(A) \cdot \mathrm{P}(B)$.

If $\mathrm{P}(A) = 0$ then $\mathrm{P}(A \cdot B) = \mathrm{P}(A) \cdot \mathrm{P}(B)$ because $\mathrm{P}(A \cdot B) \leq \mathrm{P}(A)$ in accordance with (3.15).

Moreover in accordance with (3.17): $\mathrm{P}(\#A) = 1 - \mathrm{P}(A)$ since the function P is a B-function.

If event A occurs then $(A \cdot B) = B$ and $(A \cdot (\#B)) = (\#B)$ Hence, $\mathrm{P}(A \cdot B) + \mathrm{P}(A \cdot (\#B)) = \mathrm{P}(A) = \mathrm{P}(B) + \mathrm{P}(\#B) = 1$.

Consequently, if an element A of B occurs then $\mathrm{P}(A) = 1$. If does not occurs then $(\#A)$ occurs. Hence, $\mathrm{P}(\#A) = 1$ and because $\mathrm{P}(A) + \mathrm{P}(\#A) = 1$ then $\mathrm{P}(A) = 0$. Therefore, on B the range of values of is the two-element set $\{0; 1\}$ similar the Boolean function range of values. Hence, on set B the probability function obeys definition of a Boolean function (Def.1.1.10).

The logic probability function is the extension of the logic B-function. Therefore, **the probability is some generalization of the classic propositional logic.** That is the probability is the logic of events such that these events do not happen, yet.

3.3.8. THE NONSTANDARD NUMBERS

Here some modification of the Robinson[9] NONSTANDARD NUMBERS [**?**] is considered.

Let us consider the set $\mathbf{N}$ of natural numbers.

Definition A.1: The *n-part-set* $\mathbf{S}$ of $\mathbf{N}$ is defined recursively as follows:

1) $\mathbf{S}_1 = \{1\}$;

2) $\mathbf{S}_{(n+1)} = \mathbf{S}_n \cup \{n+1\}$.

Definition A.2: If $\mathbf{S}_n$ is the n-part-set of $\mathbf{N}$ and $\mathbf{A} \subseteq \mathbf{N}$ then $\|\mathbf{A} \cap \mathbf{S}_n\|$ is the quantity elements of the set $\mathbf{A} \cap \mathbf{S}_n$, and if

$$\varpi_n(\mathbf{A}) = \frac{\|\mathbf{A} \cap \mathbf{S}_n\|}{n},$$

then $\varpi_n(\mathbf{A})$ is *the frequency* of the set $\mathbf{A}$ on the n-part-set $\mathbf{S}_n$.

Theorem A.1:

1) $\varpi_n(\mathbf{N}) = 1$;

2) $\varpi_n(\emptyset) = 0$;

3) $\varpi_n(\mathbf{A}) + \varpi_n(\mathbf{N} - \mathbf{A}) = 1$;

4) $\varpi_n(\mathbf{A} \cap \mathbf{B}) + \varpi_n(\mathbf{A} \cap (\mathbf{N} - \mathbf{B})) = \varpi_n(\mathbf{A})$.

Proof of the Theorem A.1: From Definitions A.1 and A.2. **Definition A.3:** If "lim" is the Cauchy-Weierstrass "limit" then let us denote:

$$\mathbb{F} = \left\{ \mathbf{A} \subseteq \mathbf{N} | \lim_{n \to \infty} \varpi_n(\mathbf{A}) = 1 \right\}.$$

Theorem A.2: $\mathbb{F}$ is the filter [**?**], i.e.:

1) $\mathbf{N} \in \mathbb{F}$,

2) $\emptyset \notin \mathbb{F}$,

3) if $\mathbf{A} \in \mathbb{F}$ and $\mathbf{B} \in \mathbb{F}$ then $(\mathbf{A} \cap \mathbf{B}) \in \mathbb{F}$;

4) if $\mathbf{A} \in \mathbb{F}$ and $\mathbf{A} \subseteq \mathbf{B}$ then $\mathbf{B} \in \mathbb{F}$.

Proof of the Theorem A.2: From the point 3 of Theorem A.1:

[9] Abraham Robinson (born October 6, 1918 April 11, 1974) was a mathematician who is most widely known for development of non-standard analysis

$$\lim_{n\to\infty} \varpi_n(\mathbf{N}-\mathbf{B}) = 0.$$

From the point 4 of Theorem A.1:

$$\varpi_n(\mathbf{A}\cap(\mathbf{N}-\mathbf{B})) \le \varpi_n(\mathbf{N}-\mathbf{B}).$$

Hence,

$$\lim_{n\to\infty} \varpi_n\left(\mathbf{A}\cap(\mathbf{N}-\mathbf{B})\right) = 0.$$

Hence,

$$\lim_{n\to\infty} \varpi_n\left(\mathbf{A}\cap\mathbf{B}\right) = \lim_{n\to\infty} \varpi_n(\mathbf{A}).$$

In the following text we shall adopt to our topics the definitions and the proofs of the Robinson Nonstandard Analysis [**?**]:

Definition A.4: The sequences of the real numbers $\langle r_n\rangle$ and $\langle s_n\rangle$ are *Q-equivalent* (denote: $\langle r_n\rangle \sim \langle s_n\rangle$) if

$$\{n\in\mathbf{N}|r_n = s_n\} \in \blacksquare\mathbf{ix}.$$

Theorem A.3: If $\mathbf{r}$,$\mathbf{s}$,$\mathbf{u}$ are the sequences of the real numbers then

1) $\mathbf{r}\sim\mathbf{r}$,

2) if $\mathbf{r}\sim\mathbf{s}$ then $\mathbf{s}\sim\mathbf{r}$;

3) if $\mathbf{r}\sim\mathbf{s}$ and $\mathbf{s}\sim\mathbf{u}$ then $\mathbf{r}\sim\mathbf{u}$.

Proof of the Theorem A.3: By Definition A.4 from the Theorem A.2 is obvious.

Definition A.5: *The Q-number* is the set of the Q-equivalent sequences of the real numbers, i.e. if $\widetilde{a}$ is the Q-number and $\mathbf{r}\in\widetilde{a}$ and $\mathbf{s}\in\widetilde{a}$, then $\mathbf{r}\sim\mathbf{s}$; and if $\mathbf{r}\in\widetilde{a}$ and $\mathbf{r}\sim\mathbf{s}$ then $\mathbf{s}\in\widetilde{a}$.

Definition A.6: The Q-number $\widetilde{a}$ is *the standard Q-number a* if a is some real number and the sequence $\langle r_n\rangle$ exists, for which: $\langle r_n\rangle\in\widetilde{a}$ and

$$\{n\in\mathbf{N}|r_n = a\}\in\mathbb{F}.$$

Definition A.7: The Q-numbers $\widetilde{a}$ and $\widetilde{b}$ are *the equal Q-numbers* (denote: $\widetilde{a}=\widetilde{b}$) if a $\widetilde{a}\subseteq\widetilde{b}$ and $\widetilde{b}\subseteq\widetilde{a}$.

Theorem A.4: Let $\mathfrak{f}(x,y,z)$ be a function, which has got the domain in $\mathbf{R}\times\mathbf{R}\times\mathbf{R}$, has got the range of values in $\mathbf{R}$ ($\mathbf{R}$ is the real numbers set).

Let $\langle y_{1,n}\rangle$, $\langle y_{2,n}\rangle$, $\langle y_{3,n}\rangle$, $\langle z_{1,n}\rangle$, $\langle z_{2,n}\rangle$, $\langle z_{3,n}\rangle$ be any sequences of real numbers. $\rangle$.

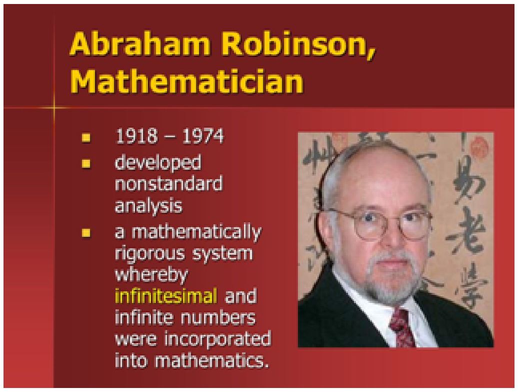
In this case if $\langle z_{i,n}\rangle\sim\langle y_{i,n}\rangle$ then $\langle\mathfrak{f}(y_{1,n},y_{2,n},y_{3,n})\rangle\sim\langle\mathfrak{f}(z_{1,n},z_{2,n},z_{3,n})\rangle$.

Proof of the Theorem A.4: Let us denote:

if $k=1$ or $k=2$ or $k=3$ then

$$\mathbf{A}_k = \{n\in\mathbf{N}|y_{k,n} = z_{k,n}\}.$$

In this case by Definition A.4 for all k:



$$\mathbf{A}_k \in \mathbb{F}.$$

Because

$$(\mathbf{A}_1 \cap \mathbf{A}_2 \cap \mathbf{A}_3) \subseteq \{n \in \mathbf{N} | \mathfrak{f}(y_{1,n}, y_{2,n}, y_{3,n}) = \mathfrak{f}(z_{1,n}, z_{2,n}, z_{3,n})\},$$

then by Theorem A.2:

$$\{n \in \mathbf{N} | \mathfrak{f}(y_{1,n}, y_{2,n}, y_{3,n}) = \mathfrak{f}(z_{1,n}, z_{2,n}, z_{3,n})\} \in \mathbb{F}.$$

Definition A.8: Let us denote: $Q\mathbf{R}$ is the set of the Q-numbers.

Definition A.9: The function $\widetilde{\mathfrak{f}}$, which has got the domain in $Q\mathbf{R} \times Q\mathbf{R} \times Q\mathbf{R}$, has got the range of values in $Q\mathbf{R}$, is *the Q-extension of the function* $\mathfrak{f}$, which has got the domain in $\mathbf{R} \times \mathbf{R} \times \mathbf{R}$, has got the range of values in $\mathbf{R}$, if the following condition is accomplished:

Let $\langle x_n \rangle$,$\langle y_n \rangle$,$\langle z_n \rangle$ be any sequences of real numbers. In this case: if

$\langle x_n \rangle \in \widetilde{x}$, $\langle y_n \rangle \in \widetilde{y}$, $\langle z_n \rangle \in \widetilde{z}$, $\widetilde{u} = \widetilde{\mathfrak{f}}(\widetilde{x}, \widetilde{y}, \widetilde{z})$,

then

$\langle \mathfrak{f}(x_n, y_n, z_n) \rangle \in \widetilde{u}$.

Theorem A.5: For all functions $\mathfrak{f}$, which have the domain in $\mathbf{R} \times \mathbf{R} \times \mathbf{R}$, have the range of values in $\mathbf{R}$, and for all real numbers a, b, c, d: if $\widetilde{\mathfrak{f}}$ is the Q-extension of $\mathfrak{f}$; $\widetilde{a}$, $\widetilde{b}$, $\widetilde{c}$, $\widetilde{d}$ are standard Q-numbers a, b, c, d, then:

if $d = \mathfrak{f}(a, b, c)$ then $\widetilde{d} = \widetilde{\mathfrak{f}}(\widetilde{a}, \widetilde{b}, \widetilde{c})$ and vice versa.

Proof of the Theorem A.5: If $\langle r_n \rangle \in \widetilde{a}$, $\langle s_n \rangle \in \widetilde{b}$, $\langle u_n \rangle \in \widetilde{c}$, $\langle t_n \rangle \in \widetilde{d}$ then by Definition A.6:

$$\begin{aligned} &\{n \in \mathbf{N} | r_n = a\} \in \mathbb{F}, \\ &\{n \in \mathbf{N} | s_n = b\} \in \mathbb{F}, \\ &\{n \in \mathbf{N} | u_n = c\} \in \mathbb{F}, \\ &\{n \in \mathbf{N} | t_n = d\} \in \mathbb{F}. \end{aligned}$$

1) Let $d = \mathfrak{f}(a, b, c)$.

In this case by Theorem A.2:

$$\{n \in \mathbf{N} | t_n = \mathfrak{f}(r_n, s_n, u_n)\} \in \mathbb{F}.$$

Hence, by Definition A.4:

$$\langle t_n \rangle \sim \langle \mathfrak{f}(r_n, s_n, u_n) \rangle.$$

Therefore by Definition A.5:

$$\langle \mathfrak{f}(r_n, s_n, u_n) \rangle \in \widetilde{d}.$$

Hence, by Definition A.9:

$$\widetilde{d} = \widetilde{\mathfrak{f}}(\widetilde{a}, \widetilde{b}, \widetilde{c}).$$

2) Let $\widetilde{d} = \widetilde{\mathfrak{f}}(\widetilde{a}, \widetilde{b}, \widetilde{c})$.

In this case by Definition A.9:

$$\langle \mathfrak{f}(r_n, s_n, u_n) \rangle \in \widetilde{d}.$$

Hence, by Definition A.5:

$$\langle t_n \rangle \sim \langle \mathfrak{f}(r_n, s_n, u_n) \rangle.$$

Therefore, by Definition A.4:

$$\{n \in \mathbf{N} | t_n = \mathfrak{f}(r_n, s_n, u_n)\} \in \mathbb{F}.$$

Hence, by the Theorem A.2:

$$\{n \in \mathbf{N} | t_n = \mathfrak{f}(r_n, s_n, u_n), r_n = a, s_n = b, u_n = c, t_n = d\} \in \mathbb{F}.$$

Hence, since this set does not empty, then

$$d = \mathfrak{f}(a, b, c).$$

By this Theorem: if $\widetilde{\mathfrak{f}}$ is the Q-extension of the function $\mathfrak{f}$ then the expression "$\widetilde{\mathfrak{f}}(\widetilde{x}, \widetilde{y}, \widetilde{z})$" will be denoted as "$\mathfrak{f}(\widetilde{x}, \widetilde{y}, \widetilde{z})$" and if $\widetilde{u}$ is the standard Q-number then the expression "$\widetilde{u}$" will be denoted as "u".

Theorem A.6: If for all real numbers a, b, c:

$$\varphi(a, b, c) = \psi(a, b, c)$$

then for all Q-numbers $\widetilde{x}$, $\widetilde{y}$, $\widetilde{z}$:

$$\varphi(\widetilde{x}, \widetilde{y}, \widetilde{z}) = \psi(\widetilde{x}, \widetilde{y}, \widetilde{z}).$$

Proof of the Theorem A.6: If $\langle x_n \rangle \in \widetilde{x}$, $\langle y_n \rangle \in \widetilde{y}$, $\langle z_n \rangle \in \widetilde{z}$, $\widetilde{u} = \varphi(\widetilde{x}, \widetilde{y}, \widetilde{z})$, then by Definition A.9: $\langle \varphi(x_n, y_n, z_n) \rangle \in \widetilde{u}$.

Because $\varphi(x_n, y_n, z_n) = \psi(x_n, y_n, z_n)$ then $\langle \psi(x_n, y_n, z_n) \rangle \in \widetilde{u}$.

If $\widetilde{v} = \psi(\widetilde{x}, \widetilde{y}, \widetilde{z})$ then by Definition A.9: $\langle \psi(x_n, y_n, z_n) \rangle \in \widetilde{v}$, too.

Therefore, for all sequences $\langle t_n \rangle$ of real numbers: if $\langle t_n \rangle \in \widetilde{u}$ then by Definition A.5: $\langle t_n \rangle \sim \langle \psi(x_n, y_n, z_n) \rangle$.

Hence, $\langle t_n \rangle \in \widetilde{v}$; and if $\langle t_n \rangle \in \widetilde{v}$ then $\langle t_n \rangle \sim \langle \varphi(x_n, y_n, z_n) \rangle$; hence, $\langle t_n \rangle \in \widetilde{u}$.

Therefore, $\widetilde{u} = \widetilde{v}$.

Theorem A.7: If for all real numbers a, b, c:

$$\mathfrak{f}(a, \varphi(b, c)) = \psi(a, b, c)$$

then for all Q-numbers $\widetilde{x}$, $\widetilde{y}$, $\widetilde{z}$:

$$\mathfrak{f}(\widetilde{x}, \varphi(\widetilde{y}, \widetilde{z})) = \psi(\widetilde{x}, \widetilde{y}, \widetilde{z}).$$

Consequences from Theorems A.6 and A.7: [**?**]: For all Q-numbers $\widetilde{x}$, $\widetilde{y}$, $\widetilde{z}$:
■**1:** $(\widetilde{x}+\widetilde{y})=(\widetilde{y}+\widetilde{x})$,
■**2:** $(\widetilde{x}+(\widetilde{y}+\widetilde{z}))=((\widetilde{x}+\widetilde{y})+\widetilde{z})$,
■**3:** $(\widetilde{x}+0)=\widetilde{x}$,
■**5:** $(\widetilde{x}\cdot\widetilde{y})=(\widetilde{y}\cdot\widetilde{x})$,
■**6:** $(\widetilde{x}\cdot(\widetilde{y}\cdot\widetilde{z}))=((\widetilde{x}\cdot\widetilde{y})\cdot\widetilde{z})$,
■**7:** $(\widetilde{x}\cdot 1)=\widetilde{x}$,
■**10:** $(\widetilde{x}\cdot(\widetilde{y}+\widetilde{z}))=((\widetilde{x}\cdot\widetilde{y})+(\widetilde{x}\cdot\widetilde{z}))$.

Proof of the Theorem A.7: Let $\langle w_n\rangle\in\widetilde{w}$, $\mathfrak{f}(\widetilde{x},\widetilde{w})=\widetilde{u}$, $\langle x_n\rangle\in\widetilde{x}$, $\langle y_n\rangle\in\widetilde{y}$, $\langle z_n\rangle\in\widetilde{z}$, $\varphi(\widetilde{y},\widetilde{z})=\widetilde{w}$, $\psi(\widetilde{x},\widetilde{y},\widetilde{z})=\widetilde{v}$.

By the condition of this Theorem: $\mathfrak{f}(x_n,\varphi(y_n,z_n))=\psi(x_n,y_n,z_n)$.

By Definition A.9: $\langle\psi(x_n,y_n,z_n)\rangle\in\widetilde{v}$, $\langle\varphi(x_n,y_n)\rangle\in\widetilde{w}$, $\langle\mathfrak{f}(x_n,w_n)\rangle\in\widetilde{u}$.

For all sequences $\langle t_n\rangle$ of real numbers:

1) If $\langle t_n\rangle\in\widetilde{v}$ then by Definition A.5: $\langle t_n\rangle\sim\langle\psi(x_n,y_n,z_n)\rangle$.

Hence $\langle t_n\rangle\sim\langle\mathfrak{f}(x_n,\varphi(y_n,z_n))\rangle$.

Therefore, by Definition A.4:

$$\{n\in\mathbf{N}|t_n=\mathfrak{f}(x_n,\varphi(y_n,z_n))\}\in\mathbb{F}$$

and

$$\{n\in\mathbf{N}|w_n=\varphi(y_n,z_n)\}\in\mathbb{F}.$$

Hence, by Theorem A.2:

$$\{n\in\mathbf{N}|t_n=\mathfrak{f}(x_n,w_n)\}\in\mathbb{F}.$$

Hence, by Definition A.4:

$$\langle t_n\rangle\sim\langle\mathfrak{f}(x_n,w_n)\rangle.$$

Therefore, by Definition A.5: $\langle t_n\rangle\in\widetilde{u}$.

2) If $\langle t_n\rangle\in\widetilde{u}$ then by Definition A.5: $\langle t_n\rangle\sim\langle\mathfrak{f}(x_n,w_n)\rangle$.

Because $\langle w_n\rangle\sim\langle\varphi(y_n,z_n)\rangle$ then by Definition A.4:

$$\{n\in\mathbf{N}|t_n=\mathfrak{f}(x_n,w_n)\}\in\mathbb{F},$$

$$\{n\in\mathbf{N}|w_n=\varphi(y_n,z_n)\}\in\mathbb{F}.$$

Therefore, by Theorem A.2:

$$\{n\in\mathbf{N}|t_n=\mathfrak{f}(x_n,\varphi(y_n,z_n))\}\in\mathbb{F}.$$

Hence, by Definition A.4:

$$\langle t_n\rangle\sim\langle\mathfrak{f}(x_n,\varphi(y_n,z_n))\rangle.$$

Therefore,

$$\langle t_n \rangle \sim \langle \psi(x_n, y_n, z_n) \rangle .$$

Hence, by Definition A.5: $\langle t_n \rangle \in \widetilde{v}$.

From above and from 1) by Definition A.7: $\widetilde{u} = \widetilde{v}$.

Theorem A.8: ■4: For every Q-number $\widetilde{x}$ the Q-number $\widetilde{y}$ exists, for which: $(\widetilde{x} + \widetilde{y}) = 0$.

Proof of the Theorem A.8: If $\langle x_n \rangle \in \widetilde{x}$ then $\widetilde{y}$ is the Q-number, which contains $\langle -x_n \rangle$.

Theorem A.9: ■9: There is not that $0 = 1$.

Proof of the Theorem A.9: is obvious from Definition A.6 and Definition A.7.

Definition A.10: The Q-number $\widetilde{x}$ is *Q-less* than the Q-number $\widetilde{y}$ (denote: $\widetilde{x} < \widetilde{y}$) if the sequences $\langle x_n \rangle$ and $\langle y_n \rangle$ of real numbers exist, for which: $\langle x_n \rangle \in \widetilde{x}$, $\langle y_n \rangle \in \widetilde{y}$ and

$$\{n \in \mathbf{N} | x_n < y_n\} \in \mathbb{F}.$$

Theorem A.10: For all Q-numbers $\widetilde{x}$, $\widetilde{y}$, $\widetilde{z}$: [**?**]

■**1**: there is not that $\widetilde{x} < \widetilde{x}$;

■**2**: if $\widetilde{x} < \widetilde{y}$ and $\widetilde{y} < \widetilde{z}$ then $\widetilde{x} < \widetilde{z}$;

■**4**: if $\widetilde{x} < \widetilde{y}$ then $(\widetilde{x} + \widetilde{z}) < (\widetilde{y} + \widetilde{z})$;

■**5**: if $0 < \widetilde{z}$ and $\widetilde{x} < \widetilde{y}$, then $(\widetilde{x} \cdot \widetilde{z}) < (\widetilde{y} \cdot \widetilde{z})$;

■**3**$'$: if $\widetilde{x} < \widetilde{y}$ then there is not, that $\widetilde{y} < \widetilde{x}$ or $\widetilde{x} = \widetilde{y}$ and vice versa;

■**3**$''$: for all standard Q-numbers x, y, z: $x < y$ or $y < x$ or $x = y$.

Proof of the Theorem A.10: is obvious from Definition A.10 by the Theorem A.2.

Theorem A.11: ■8: If $0 < |\widetilde{x}|$ then the Q-number $\widetilde{y}$ exists, for which $(\widetilde{x} \cdot \widetilde{y}) = 1$.

Proof of the Theorem A.11: If $\langle x_n \rangle \in \widetilde{x}$ then by Definition A.10: if

$$\mathbf{A} = \{n \in \mathbf{N} | 0 < |x_n|\}$$

then $\mathbf{A} \in \mathbb{F}$.

In this case: if for the sequence $\langle y_n \rangle$: if $n \in \mathbf{A}$ then $y_n = 1/x_n$

- then

$$\{n \in \mathbf{N} | x_n \cdot y_n = 1\} \in ■\mathbf{ix}.$$

Thus, Q-numbers are fulfilled to all properties of real numbers, except Ω3 [**?**]. The property Ω3 is accomplished by some weak meaning (Ω3' and Ω3").

Definition A.11: The Q-number $\widetilde{x}$ is *the infinitesimal Q-number* if the sequence of real numbers $\langle x_n \rangle$ exists, for which: $\langle x_n \rangle \in \widetilde{x}$ and for all positive real numbers ε:

$$\{n \in \mathbf{N} | |x_n| < \varepsilon\} \in \mathbb{F}.$$

Let the set of all infinitesimal Q-numbers be denoted as I.

Definition A.12: The Q-numbers $\widetilde{x}$ and $\widetilde{y}$ are t*he infinite closed Q-numbers* (denote: $\widetilde{x} \approx \widetilde{y}$) if $|\widetilde{x} - \widetilde{y}| = 0$ or $|\widetilde{x} - \widetilde{y}|$ is infinitesimal.

Definition A.13: The Q-number $\widetilde{x}$ is *the infinite Q-number* if the sequence $\langle r_n \rangle$ of real numbers exists, for which $\langle r_n \rangle \in \widetilde{x}$ and for every natural number m:

$$\{n \in \mathbf{N} | m < r_n\} \in \mathbb{F}.$$

3.3.9. Model

Let us define the propositional calculus like to ([**?**]), but the propositional forms shall be marked by the script greek letters.

Definition C1: A set $\Re$ of the propositional forms is *a U-world* if:

1) if $\alpha_1, \alpha_2, \ldots, \alpha_n \in \Re$ and $\alpha_1, \alpha_2, \ldots, \alpha_n \vdash \beta$ then $\beta \in \Re$,

2) for all propositional forms α: it is not that $(\alpha \& (\neg\alpha)) \in \Re$,

3) for every propositional form α: $\alpha \in \Re$ or $(\neg\alpha) \in \Re$.

Definition C2: The sequences of the propositional forms $\langle\alpha_n\rangle$ and $\langle\beta_n\rangle$ are *Q-equivalent* (denote: $\langle\alpha_n\rangle \sim \langle\beta_n\rangle$) if

$$\{n \in \mathbf{N} | \alpha_n \equiv \beta_n\} \in \mathbb{F}.$$

Let us define the notions of *the Q-extension of the functions* for like as in the Definitions A.5, A.2, A.9, A.5, A.6.

Definition C3: The Q-form $\widetilde{\alpha}$ is *Q-real* in the U-world $\Re$ if the sequence $\langle\alpha_n\rangle$ of the propositional forms exists, for which: $\langle\alpha_n\rangle \in \widetilde{\alpha}$ and

$$\{n \in \mathbf{N} | \alpha_n \in \Re\} \in \mathbb{F}.$$

Definition C4: The set $\widetilde{\Re}$ of the Q-forms is t*he Q-extension of the U-world* $\Re$ if $\widetilde{\Re}$ is the set of Q-forms $\widetilde{\alpha}$, which are Q-real in $\Re$.

Definition C5: The sequence $\left\langle \widetilde{\Re}_k \right\rangle$ of the Q-extensions is *the S-world*.

Definition C6: The Q-form $\widetilde{\alpha}$ is *S-real in the S-world* $\left\langle \widetilde{\Re}_k \right\rangle$ if

$$\left\{k \in \mathbf{N} | \widetilde{\alpha} \in \widetilde{\Re}_k\right\} \in \mathbb{F}.$$

Definition C7: The set $\mathbf{A}$ ($\mathbf{A} \subseteq \mathbf{N}$) is *the regular set* if for every real positive number ε the natural number n_0 exists, for which: for all natural numbers n and m, which are more or equal to n_0:

$$|w_n(\mathbf{A}) - w_m(\mathbf{A})| < \varepsilon.$$

Theorem C1: If $\mathbf{A}$ is the regular set and for all real positive ε:

$$\{k \in \mathbf{N} | w_k(\mathbf{A}) < \varepsilon\} \in \mathbb{F}.$$

then

$$\lim_{k \to \infty} w_k(\mathbf{A}) = 0.$$

Proof of theTheorem C1: Let be

$$\lim_{k \to \infty} w_k(\mathbf{A}) \neq 0.$$

That is the real number ε_0 exists, for which: for every natural number n' the natural number n exists, for which:

$$n > n' \text{ and } w_n(\mathbf{A}) > \varepsilon_0.$$

Let δ_0 be some positive real number, for which: $\varepsilon_0 - \delta_0 > 0$. Because $\mathbf{A}$ is the regular set then for δ_0 the natural number n_0 exists, for which: for all natural numbers n and m, which are more or equal to n_0:

$$|w_m(\mathbf{A}) - w_n(\mathbf{A})| < \delta_0.$$

That is

$$w_m(\mathbf{A}) > w_n(\mathbf{A}) - \delta_0.$$

Since $w_n(\mathbf{A}) \geq \varepsilon_0$ then $w_m(\mathbf{A}) \geq \varepsilon_0 - \delta_0$.

Hence, the natural number n_0 exists, for which: for all natural numbers m: if $m \geq n_0$ then $w_m(\mathbf{A}) \geq \varepsilon_0 - \delta_0$.

Therefore,

$$\{m \in \mathbf{N} | w_m(\mathbf{A}) \geq \varepsilon_0 - \delta_0\} \in \mathbb{F}.$$

and by this Theorem condition:

$$\{k \in \mathbf{N} | w_k(\mathbf{A}) < \varepsilon_0 - \delta_0\} \in \mathbb{F}.$$

Hence,

$$\{k \in \mathbf{N} | \varepsilon_0 - \delta_0 < \varepsilon_0 - \delta_0\} \in \mathbb{F}.$$

That is $\emptyset \notin \mathbb{F}$. It is the contradiction for the Theorem 2.2.

Definition C8: Let $\left\langle \widetilde{\mathfrak{R}}_k \right\rangle$ be a S-world.

In this case the function $\mathfrak{W}(\widetilde{\beta})$, which has got the domain in the set of the Q-forms, has got the range of values in $Q\mathbf{R}$, is defined as the following:

If $\mathfrak{W}(\widetilde{\beta}) = \widetilde{p}$ then the sequence $\langle p_n \rangle$ of the real numbers exists, for which: $\langle p_n \rangle \in \widetilde{p}$ and

$$p_n = w_n\left(\left\{k \in \mathbf{N} | \widetilde{\beta} \in \widetilde{\mathfrak{R}}_k\right\}\right).$$

Theorem C2: If $\left\{k \in \mathbf{N} | \widetilde{\beta} \in \widetilde{\mathfrak{R}}_k\right\}$ is the regular set and $\mathfrak{W}(\widetilde{\beta}) \approx 1$ then $\widetilde{\beta}$ is S-resl in $\left\langle \widetilde{\mathfrak{R}}_k \right\rangle$.

Proof of the Theorem C2: Since $\mathfrak{W}(\widetilde{\beta}) \approx 1$ then by Definitions.2.12 and 2.11: for all positive real ε:

$$\left\{n \in \mathbf{N} | w_n\left(\left\{k \in \mathbf{N} | \widetilde{\beta} \in \widetilde{\mathfrak{R}}_k\right\}\right) > 1 - \varepsilon\right\} \in \mathbb{F}.$$

Hence, by the point 3 of the Theorem 2.1: for all positive real ε:

$$\left\{n \in \mathbf{N} | \left(\mathbf{N} - w_n\left(\left\{k \in \mathbf{N} | \widetilde{\beta} \in \widetilde{\mathfrak{R}}_k\right\}\right)\right) < \varepsilon\right\} \in \mathbb{F}.$$

Therefore, by the Theorem C1:

$$\lim_{n\to\infty}\left(\mathbf{N}-w_n\left(\left\{k\in\mathbf{N}|\widetilde{\beta}\in\widetilde{\mathfrak{R}}_k\right\}\right)\right)=0.$$

That is:

$$\lim_{n\to\infty} w_n\left(\left\{k\in\mathbf{N}|\widetilde{\beta}\in\widetilde{\mathfrak{R}}_k\right\}\right)=1.$$

Hence, by Definition.2.3:

$$\left\{k\in\mathbf{N}|\widetilde{\beta}\in\widetilde{\mathfrak{R}}_k\right\}\in\mathbb{F}.$$

And by Definition C6: $\widetilde{\beta}$ is S-real in $\left\langle\widetilde{\mathfrak{R}}_k\right\rangle$.

Theorem C3: The P-function exists.

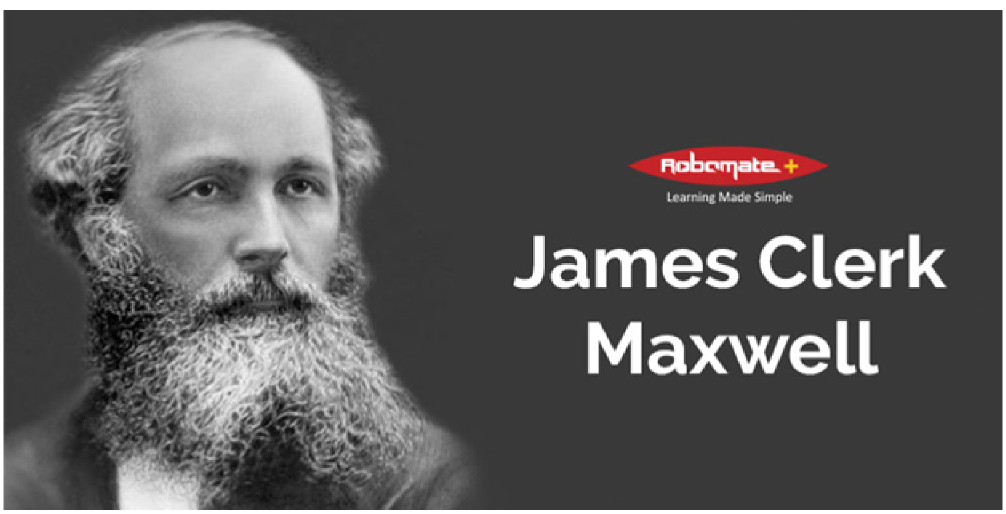
Proof of the Theorem C3: By the Theorems C2 and 2.1: $\mathfrak{W}(\widetilde{\beta})$ is the P-function in $\left\langle\widetilde{\mathfrak{R}}_k\right\rangle$.

Chapter 4

Physics

No group of people can claim power over
the thinking and views of others.
- Friedrich von Hayek

4.1. Planck



In 1865 James Clerk Maxwell[1] published book "A Dynamical Theory of the Electromagnetic Field". He proposed that light is an undulation in the same medium that is the cause of electric and magnetic phenomena. Thus, optics, electricity and magnetism turned out to be united by a unified theory. If we add Albert Einstein's[2] theory of space-time here, then we get a beautiful smooth picture of the world.

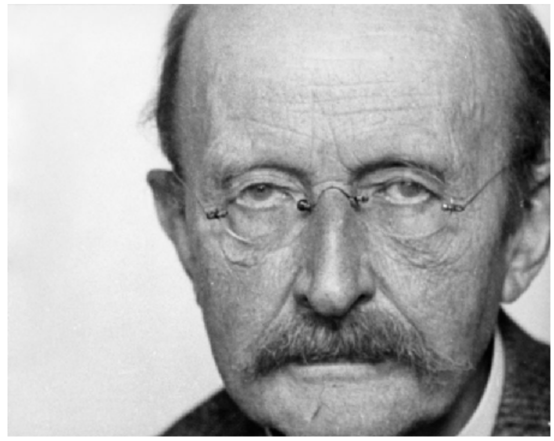
[1] James Clerk Maxwell (13 June 1831 5 November 1879) was a Scottish scientist in the field of mathematical physics.

[2] Albert Einstein ; (14 March 1879 18 April 1955) was a German-born theoretical physicist[5] who devel-

In 1900, Max Planck[3] discovered that our world is not continuous, but it is discrete [15]. This is a recognition of our limitations of our space .

$$|x| \leq \frac{\pi c}{h},$$

($h = 6.62607004427 \cdot 10^{34}$)). Therefore, functions describing the processes of our world are represented by Fourier[4] series by basis:,



Max Karl Ernst Ludwig Planck

$$\phi_{\mathbf{n}}(\mathbf{x}) := \left(\frac{h}{2\pi c}\right)^{\frac{3}{2}} \exp\left(-i\frac{h}{c}\mathbf{nx}\right) \, (hoh)$$

Hete

$$\mathbf{nx} := n_1x_1 + n_2x_2 + n_3x_3$$

(n_1, n_2, n_3 - integer numbers).

4.2. Quants

Quantum theory developed as a new branch of theoretical physics during the first few decades of the 20th century in an effort to understand the fundamental properties of matter in a

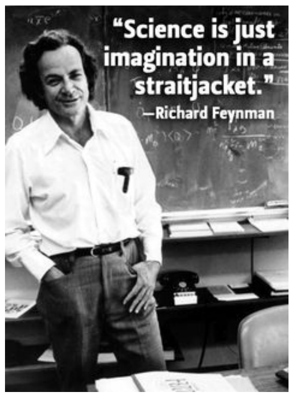
oped the theory of relativity

[3]Max Karl Ernst Ludwig Planck 23 April 1858 – 4 October 1947) was a German theoretical physicist whose discovery of energy quanta

[4]-Baptiste Joseph Fourier (21 March 1768 – 16 May 1830) was a French mathematician and physicist born in Auxerre and best known for initiating the investigation of Fourier series,

discrete world. Started by exploring interactions. matter and radiation. Some radiation effects cannot be explained by either classical mechanics or theory of electromagnetism.

Quantum theory was not the work of one individual, but the collaborative effort of some of the most brilliant physicists of the 20th century, among them Niels Bohr[5], Erwin Schrodinger[6], Wolfgang Pauli[7], and Max Born[8], Max Planck[9] and Werner Heisenberg[10].



Quantum Field Theory (QFT) is the mathematical and conceptual framework for contemporary elementary particle physics

(Eugene Wigner[11], Hans Bethe[12], Tomonaga[13], Schwinger[14], Feynman[15], Dyson[16], Yang[17] and Mills[18]).

Let $\langle X_{\mathsf{A},0}, X_{\mathsf{A},1}, X_{\mathsf{A},2}, X_{\mathsf{A},3} \rangle$ be random coordinates of event A.

Let F_{A} be a *Cumulative Distribution Function* i.e.:

$$F_{\mathsf{A}}\left(x_0, x_1, x_2, x_3\right) = \mathrm{P}\left(\left(X_{\mathsf{A},0} < x_0\right) \cdot \left(X_{\mathsf{A},1} < x_1\right) \cdot \left(X_{\mathsf{A},2} < x_2\right) \cdot \left(X_{\mathsf{A},3} < x_3\right)\right).$$

4.3. Physical events

If

[5]Niels Henrik David Bohr (7 October 1885 - 18 November 1962) was a Danish physicist

[6]Erwin Rudolf Josef Alexander Schrodinger (12 August 1887 - 4 January 1961) was an Austrian physicist and theoretical biologist who was one of the fathers of quantum mechanics

[7]Wolfgang Ernst Pauli (25 April 1900 15 December 1958) was an Austrian theoretical physicist

[8]Max Born (11 December 1882 5 January 1970) was a German-born physicist and mathematician

[9]Max Karl Ernst Ludwig Planck (April 23, 1858 October 4, 1947) was a German physicist

[10]Werner Karl Heisenberg (5 December 1901 1 February 1976) was a German theoretical physicist

[11]Eugene Paul Wigner (Hungarian Wigner Jeno Pal; November 17, 1902 - January 1, 1995) was a Hungarian American physicist and mathematician.

[12]Hans Albrecht Bethe (July 2, 1906 - March 6, 2005) [1] was a German-American nuclear physicist,

[13]Sin-Itiro Tomonaga (March 31, 1906 July 8, 1979) was a Japanese physicist

[14]Julian Seymour Schwinger (February 12, 1918 - July 16, 1994) was an American theoretical physicist.

[15]Richard Phillips Feynman (May 11, 1918 - February 15, 1988)[2] was an American physicist

[16]Freeman John Dyson FRS (born December 15, 1923) is a British-born American theoretical physicist and mathematician

[17]Chen-Ning Franklin Yang (born October 1, 1922) is a Chinese-American physicist

[18]Robert L. Mills (April 15, 1927 – October 27, 1999) was an English physicist

$$
\begin{aligned}
j_0 \quad &: \quad = \frac{\partial^3 F}{\partial x_1 \partial x_2 \partial x_3}, \\
j_1 \quad &: \quad = -\frac{\partial^3 F}{\partial x_0 \partial x_2 \partial x_3}, \\
j_2 \quad &: \quad = -\frac{\partial^3 F}{\partial x_0 \partial x_1 \partial x_3}, \\
j_3 \quad &: \quad = \frac{\partial^3 F}{\partial x_0 \partial x_1 \partial x_2}
\end{aligned}
$$

then $\langle j_0, j_1, j_2, j_3 \rangle$ is *a probability current vector of event*.

If $\rho := j_0/\mathrm{c}$ then ρ is *a a probability density function*.

If $\mathbf{u}_{\mathsf{A}} := \mathbf{j}_{\mathsf{A}}/\rho_{\mathsf{A}}$ then vector $\mathbf{u}_{\mathsf{A}}$ is *a velocity of the probability* of A propagation.

Erwin Schrodinger

(for example for u_2:

$$
u_2 = \frac{j_2}{\rho} = \frac{\left(-\frac{\partial^3 F}{\partial x_0 \partial x_1 \partial x_3}\right) c}{\left(\frac{\partial^3 F}{\partial x_1 \partial x_2 \partial x_3}\right)} = \left(-\frac{\Delta_{013} F}{\Delta_{123} F} \frac{\Delta x_2}{\Delta x_0}\right) c
$$

)

Probability, for which $u_1^2 + u_2^2 + u_3^2 \leq \mathrm{c}$ are called *traceable probability*.

Let us consider the following set of four real equations with eight real unknowns: b^2 with $b > 0$, α, β, χ, θ, γ, υ, λ:

$$
\left\{
\begin{array}{c}
b^2 = \rho, \\
b^2 \left(\cos^2(\alpha)\sin(2\beta)\cos(\theta - \gamma) - \sin^2(\alpha)\sin(2\chi)\cos(\upsilon - \lambda)\right) = -\frac{j_1}{\mathrm{c}}, \\
b^2 \left(\cos^2(\alpha)\sin(2\beta)\sin(\theta - \gamma) - \sin^2(\alpha)\sin(2\chi)\sin(\upsilon - \lambda)\right) = -\frac{j_2}{\mathrm{c}}, \\
b^2 \left(\cos^2(\alpha)\cos(2\beta) - \sin^2(\alpha)\cos(2\chi)\right) = -\frac{j_3}{\mathrm{c}}.
\end{array}
\right| \tag{4.1}
$$

This set has solutions for any traceable ρ and $j_{\mathcal{A},k}$. For example one of these solutions is the following:

1. A value of b^2 obtain from first equation.

2. Since

$$
u_k = \frac{j_k}{\rho}
$$

then

$$\left\{\begin{array}{c} \cos^2(\alpha)\sin(2\beta)\cos(\theta-\gamma)-\sin^2(\alpha)\sin(2\chi)\cos(\upsilon-\lambda)=-\frac{u_1}{c}, \\ \cos^2(\alpha)\sin(2\beta)\sin(\theta-\gamma)-\sin^2(\alpha)\sin(2\chi)\sin(\upsilon-\lambda)=-\frac{u_2}{c}, \\ \cos^2(\alpha)\cos(2\beta)-\sin^2(\alpha)\cos(2\chi)=-\frac{u_3}{c}. \end{array}\right|$$

3. Let $\beta=\chi$.
In that case:

$$\left\{\begin{array}{c} \left(\cos^2(\alpha)\cos(\theta-\gamma)-\sin^2(\alpha)\cos(\upsilon-\lambda)\right)\sin(2\beta)=-\frac{u_1}{c}, \\ \left(\cos^2(\alpha)\sin(\theta-\gamma)-\sin^2(\alpha)\sin(\upsilon-\lambda)\right)\sin(2\beta)=-\frac{u_2}{c}, \\ \left(\cos^2(\alpha)-\sin^2(\alpha)\right)\cos(2\beta)=-\frac{u_3}{c}. \end{array}\right|$$

4. Let $(\theta-\gamma)=(\upsilon-\lambda)$.
In that case:

$$\left\{\begin{array}{c} \cos(2\alpha)\cos(\theta-\gamma)\sin(2\beta)=-\frac{u_1}{c}, \\ \cos(2\alpha)\sin(\theta-\gamma)\sin(2\beta)=-\frac{u_2}{c}, \\ \cos(2\alpha)\cos(2\beta)=-\frac{u_3}{c}. \end{array}\right|$$

5. Let us raise to the second power the first and the second equations:

$$\left\{\begin{array}{c} \cos^2(2\alpha)\cos^2(\theta-\gamma)\sin^2(2\beta)=\left(-\frac{u_1}{c}\right)^2, \\ \cos^2(2\alpha)\sin^2(\theta-\gamma)\sin^2(2\beta)=\left(-\frac{u_2}{c}\right)^2, \\ \cos(2\alpha)\cos(2\beta)=-\frac{u_3}{c}. \end{array}\right|$$

and let us summat these two equations:

$$\left\{\begin{array}{c} \sin^2(2\beta)\cos^2(2\alpha)\left(\cos^2(\theta-\gamma)+\sin^2(\theta-\gamma)\right) \\ =\left(-\frac{u_1}{c}\right)^2+\left(-\frac{u_2}{c}\right)^2, \\ \cos(2\alpha)\cos(2\beta)=-\frac{u_3}{c}. \end{array}\right|$$

Hence:

$$\left\{\begin{array}{c} \sin^2(2\beta)\cos^2(2\alpha)=\left(-\frac{u_1}{c}\right)^2+\left(-\frac{u_2}{c}\right)^2, \\ \cos(2\alpha)\cos(2\beta)=-\frac{u_3}{c}. \end{array}\right|$$

6. Let us raise to the second power the second equation and add this equation to the previous one:

$$\left\{\begin{array}{c} \sin^2(2\beta)\cos^2(2\alpha)=\left(-\frac{u_1}{c}\right)^2+\left(-\frac{u_2}{c}\right)^2, \\ \cos^2(2\alpha)\cos^2(2\beta)=\left(-\frac{u_3}{c}\right)^2 \end{array}\right|$$

$$\left(\sin^2(2\beta)+\cos^2(2\beta)\right)\cos^2(2\alpha)=\left(-\frac{u_1}{c}\right)^2+\left(-\frac{u_2}{c}\right)^2+\left(-\frac{u_3}{c}\right)^2,$$

$$\cos^2(2\alpha)=\left(-\frac{u_1}{c}\right)^2+\left(-\frac{u_2}{c}\right)^2+\left(-\frac{u_3}{c}\right)^2, \tag{4.2}$$

We receive $\cos^2(2\alpha)$ (for a trackeable probabilities).

7. From

$$\cos^2(2\alpha)\cos^2(2\beta) = \left(-\frac{u_3}{\mathrm{c}}\right)^2$$

we receive $\cos^2(2\beta)$.
8. From

$$\cos^2(2\alpha)\cos^2(\theta-\gamma)\sin^2(2\beta) = \left(-\frac{u_1}{\mathrm{c}}\right)^2$$

we receive $\cos^2(\theta-\gamma)$.
———————————————————-
If

$$\begin{aligned}
\varphi_1 &:= b\exp(\mathrm{i}\gamma)\cos(\beta)\cos(\alpha), \\
\varphi_2 &:= b\exp(\mathrm{i}\theta)\sin(\beta)\cos(\alpha), \\
\varphi_3 &:= b\exp(\mathrm{i}\lambda)\cos(\chi)\sin(\alpha), \\
\varphi_4 &:= b\exp(\mathrm{i}\upsilon)\sin(\chi)\sin(\alpha)
\end{aligned} \tag{4.3}$$

then you can calculate that

$$\begin{aligned}
\rho &= \sum_{s=1}^{4}\varphi_s^*\varphi_s, \\
\frac{j_\alpha}{\mathrm{c}} &= -\sum_{k=1}^{4}\sum_{s=1}^{4}\varphi_s^*\beta_{s,k}^{[\alpha]}\varphi_k
\end{aligned} \tag{4.4}$$

4.4. Equations of moving

If $\varphi' := U_{0,2}(\phi)\varphi$ then

$$\rho' = \varphi'^{\dagger}\varphi' = \varphi^{\dagger}U_{0,2}^{\dagger}(\phi)U_{0,2}(\phi)\varphi = \rho\cosh 2\phi + \frac{j_2}{\mathrm{c}}\sinh 2\phi$$

and

$$\frac{j_2'}{\mathrm{c}} = -\varphi_s'^{\dagger}\beta^{[2]}\varphi_k' = -\varphi^{\dagger}U_{0,2}^{\dagger}(\phi)\beta^{[2]}U_{0,2}(\phi)\varphi = \frac{j_2}{\mathrm{c}}\cosh 2\phi + \rho\sinh 2\phi.$$

Similarly $U_{0,1}$ and $U_{0,3}$ transform the 3+1 vector $\langle \mathrm{c}\rho, \mathbf{j}\rangle$ by the Lorentz formulas and $U_{1,2}$, $U_{1,3}$, $U_{2,3}$ transform this vector by the cartesian formulas.

Because

$$\frac{\partial j_0}{\partial x_0} = \frac{\partial^4 F}{\partial x_0\partial x_1\partial x_2\partial x_3} = -\frac{\partial j_1}{\partial x_1} = -\frac{\partial j_2}{\partial x_2} = \frac{\partial j_3}{\partial x_3}$$

then (Continuity equation):

$$\frac{\partial\rho}{\partial x_0} + \frac{\partial j_1}{\partial x_1} + \frac{\partial j_2}{\partial x_2} + \frac{\partial j_3}{\partial x_3} = 0 \tag{4.5}$$

In that case:

$$\frac{\partial\left(\varphi^{\dagger}\varphi\right)}{\partial x_0}-\frac{\partial\left(\varphi^{\dagger}\beta^{[1]}\varphi\right)}{\partial x_1}-\frac{\partial\left(\varphi^{\dagger}\beta^{[2]}\varphi\right)}{\partial x_2}-\frac{\partial\left(\varphi^{\dagger}\beta^{[1]}\varphi\right)}{\partial x_3}=0$$

$$\begin{aligned}
&\frac{\partial\left(\varphi^{\dagger}\right)}{\partial x_0}\varphi+\varphi^{\dagger}\frac{\partial\left(\varphi\right)}{\partial x_0}\\
&-\frac{\partial\left(\varphi^{\dagger}\beta^{[1]}\right)}{\partial x_1}\varphi-\varphi^{\dagger}\frac{\partial\left(\beta^{[1]}\varphi\right)}{\partial x_1}\\
&-\frac{\partial\left(\varphi^{\dagger}\beta^{[2]}\right)}{\partial x_2}\varphi-\varphi^{\dagger}\frac{\partial\left(\beta^{[2]}\varphi\right)}{\partial x_2}\\
&-\frac{\partial\left(\varphi^{\dagger}\beta^{[3]}\right)}{\partial x_3}\varphi-\varphi^{\dagger}\frac{\partial\left(\beta^{[3]}\varphi\right)}{\partial x_3}\\
=\;&0
\end{aligned}$$

$$\begin{aligned}
&\varphi^{\dagger}\left(\frac{\partial}{\partial x_0}-\beta^{[1]}\frac{\partial}{\partial x_1}-\beta^{[2]}\frac{\partial}{\partial x_2}-\beta^{[3]}\frac{\partial}{\partial x_3}\right)^{\dagger}\varphi\\
&+\varphi^{\dagger}\left(\frac{\partial}{\partial x_0}-\beta^{[1]}\frac{\partial}{\partial x_1}-\beta^{[2]}\frac{\partial}{\partial x_2}-\beta^{[3]}\frac{\partial}{\partial x_3}\right)\varphi\\
=\;&0
\end{aligned}$$

Let

$$\widehat{Q}:=\frac{\partial}{x_0}-\sum_{s=1}^{3}\beta^{[s]}\frac{\partial}{x_s} \tag{4.6}$$

Hence,

$$\varphi^{\dagger}\left(\widehat{Q}^{\dagger}+\widehat{Q}\right)\varphi=0$$

$$\widehat{Q}^{\dagger}=-\widehat{Q} \tag{4.7}$$

Therefore, for every function φ_j here exists an operator $Q_{j,k}$ such that a dependence of φ_j on t is described by the following differential equations :

$$\partial_t\varphi_j=\mathrm{c}\sum_{k=1}^{4}\left(\beta_{j,k}^{[1]}\partial_1+\beta_{j,k}^{[2]}\partial_2+\beta_{j,k}^{[3]}\partial_3+Q_{j,k}\right)\varphi_k. \tag{4.8}$$

and $Q_{j,k}^{*}=-Q_{k,j}$.

In that case if

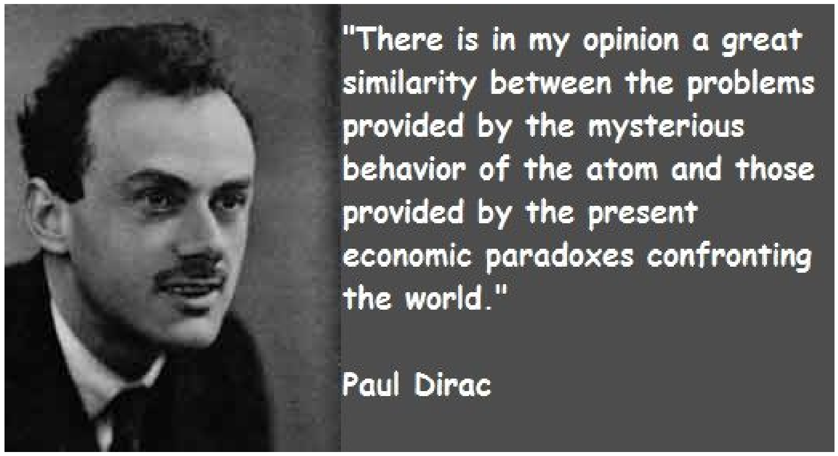
$$\widehat{H}_{j,k}:=\mathrm{ic}\left(\beta_{j,k}^{[1]}\partial_1+\beta_{j,k}^{[2]}\partial_2+\beta_{j,k}^{[3]}\partial_3+Q_{j,k}\right)$$

then $\widehat{H}$ is called *a Hamiltonian*[19] of a moving with equation (4.8).

[19]Sir William Rowan Hamilton (4 August 1805 2 September 1865) was an Irish physicist, astronomer, and mathematician, who made important contributions to classical mechanics, optics, and algebra.

A matrix form of formula (4.8) is the following:

$$\partial_t \varphi = \mathrm{c}\left(\beta^{[1]}\partial_1 + \beta^{[2]}\partial_2 + \beta^{[3]}\partial_3 + \widehat{Q}\right)\varphi \tag{4.9}$$



with

$$\varphi = \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{bmatrix}$$

and

$$\widehat{Q} = \begin{bmatrix} \mathrm{i}\vartheta_{1,1} & \mathrm{i}\vartheta_{1,2} - \varpi_{1,2} & \mathrm{i}\vartheta_{1,3} - \varpi_{1,3} & \mathrm{i}\vartheta_{1,4} - \varpi_{1,4} \\ \mathrm{i}\vartheta_{1,2} + \varpi_{1,2} & \mathrm{i}\vartheta_{2,2} & \mathrm{i}\vartheta_{2,3} - \varpi_{2,3} & \mathrm{i}\vartheta_{2,4} - \varpi_{2,4} \\ \mathrm{i}\vartheta_{1,3} + \varpi_{1,3} & \mathrm{i}\vartheta_{2,3} + \varpi_{2,3} & \mathrm{i}\vartheta_{3,3} & \mathrm{i}\vartheta_{3,4} - \varpi_{3,4} \\ \mathrm{i}\vartheta_{1,4} + \varpi_{1,4} & \mathrm{i}\vartheta_{2,4} + \varpi_{2,4} & \mathrm{i}\vartheta_{3,4} + \varpi_{3,4} & \mathrm{i}\vartheta_{4,4} \end{bmatrix} \tag{4.10}$$

with $\varpi_{s,k} = \mathrm{Re}\left(Q_{s,k}\right)$ and $\vartheta_{s,k} = \mathrm{Im}\left(Q_{s,k}\right)$. Matrix φ is called *a state vector* of the event $\mathcal{A}$ probability.

An operator $\widehat{U}\left(t,t_0\right)$ with a domain and with a range of values on the set of state vectors is called *an evolution operator* if each state vector φ fulfils the following condition:

$$\varphi\left(t\right) = \widehat{U}\left(t,t_0\right)\varphi\left(t_0\right). \tag{4.11}$$

Let us denote:

$$\widehat{H}_d := \mathrm{c}\sum_{s=1}^{3} \mathrm{i}\beta^{[s]}\partial_s.$$

In that case

$$\widehat{H} = \widehat{H}_d + \mathrm{ic}\widehat{Q}$$

according the Hamiltonian definition:

$$\widehat{H} = \mathrm{ic}\left(\beta^{[1]}\partial_1 + \beta^{[2]}\partial_2 + \beta^{[3]}\partial_3 + Q\right).$$

From (4.9):

$$\mathrm{i}\partial_t\varphi = \widehat{H}\varphi.$$

.

Hence:

$$\mathrm{i}\partial_t\varphi = \left(\widehat{H}_d + \mathrm{ic}\widehat{Q}\right)\varphi.$$

This differential equation has the following solution:

$$\frac{\partial\varphi}{\varphi} = -\mathrm{i}\left(\widehat{H}_d + \mathrm{ic}\widehat{Q}\right)\partial t,$$

$$\int_{t=t_0}^{t}\frac{\partial\varphi}{\varphi} = -\mathrm{i}\int_{t=t_0}^{t}\left(\widehat{H}_d + \mathrm{ic}\widehat{Q}\right)\partial t,$$

$$\ln\varphi(t) - \ln\varphi(t_0) = \left(-\mathrm{i}\int_{t=t_0}^{t}\widehat{H}_d\partial t - \mathrm{iic}\int_{t=t_0}^{t}\widehat{Q}\partial t\right).$$

Since $\widehat{H}_d$ does not depend on time then

$$\int_{t=t_0}^{t}\widehat{H}_d\partial t = \widehat{H}_d(t-t_0).$$

Hence, according logarithm properties:

$$\ln\frac{\varphi(t)}{\varphi(t_0)} = \left(-\mathrm{i}\widehat{H}_d(t-t_0) + \mathrm{c}\int_{t=t_0}^{t}\widehat{Q}\partial t\right).$$

Therefore,[20]:

$$\varphi(t) = \varphi(t_0)\exp\left(-\mathrm{i}\widehat{H}_d(t-t_0) + \mathrm{c}\int_{t=t_0}^{t}\widehat{Q}\partial t\right).$$

Hence, from (4.11):

$$\widehat{U}(t,t_0) = \exp\left(-\mathrm{i}\widehat{H}_d(t-t_0) + \mathrm{c}\int_{t=t_0}^{t}\widehat{Q}\partial t\right)$$

[20]For an operator $\widehat{S}$:

$$\exp\left(\widehat{S}\right) := \widehat{1} + \widehat{S} + \frac{1}{2}\widehat{S}^2 + \frac{1}{3!}\widehat{S}^3 + \cdots + \frac{1}{n!}\widehat{S}^n + \cdots$$

with $\widehat{S}^2 := \widehat{S}\widehat{S}$ and $\widehat{S}^{r+1} := \widehat{S}^r\,\widehat{S}$.
Here $\widehat{1}$ is *the unit operator* such that for every $\widetilde{u}$: $\widehat{1}\widetilde{u} = \widetilde{u}$.

A Fourier series for $\varphi_j(t,\mathbf{x})$ in $\Re_\Omega$ has the following shape: a

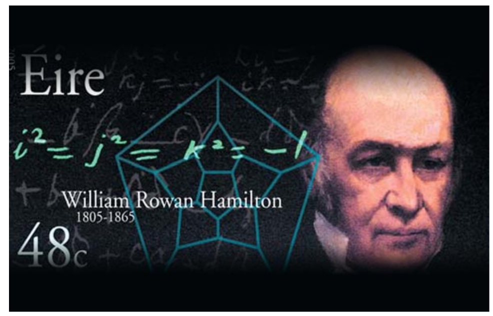
$$\varphi_j(t_0,\mathbf{x}) = \sum_{\mathbf{p}} c_{j,\mathbf{p}}(t_0)\,\varsigma_{\mathbf{p}}(t_0,\mathbf{x})$$

with

$$\varsigma_{\mathbf{p}}(\mathbf{x}) := \left\{ \begin{array}{c} \left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{\frac{3}{2}} \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{p}\mathbf{x}\right) \text{ if } \mathbf{x}\in\Omega; \\ 0, \text{ otherwise} \end{array} \right|$$

and with

$$c_{j,\mathbf{p}}(t_0) = \varsigma_{\mathbf{p}}(\mathbf{x}) * \varphi_j(t_0,\mathbf{x}).$$



That is in a matrix form:

$$c_{\mathbf{p}}(t_0) = \int_{(\Omega)} d\mathbf{x}_0 \cdot \left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{\frac{3}{2}} \exp\left(\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{p}\mathbf{x}_0\right) \varphi(t_0,\mathbf{x}_0)$$

Hence,

$$\varphi(t_0,\mathbf{x}) = \sum_{\mathbf{p}} \int_{(\blacksquare)} d\mathbf{x}_0 \cdot \left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{\frac{3}{2}} \exp\left(\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{p}\mathbf{x}_0\right) \varphi(t_0,\mathbf{x}_0) \left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{\frac{3}{2}} \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{p}\mathbf{x}\right).$$

That is:

$$\varphi(t_0,\mathbf{x}) = \int_{(\Omega)} d\mathbf{x}_0 \cdot \left(\sum_{\mathbf{p}} \left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{3} \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{p}(\mathbf{x}-\mathbf{x}_0)\right)\right) \varphi(t_0,\mathbf{x}_0).$$

Therefore,

$$\varphi(t,\mathbf{x}) = \int_{(\Omega)} d\mathbf{x}_0 \cdot \left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{3} \left(\begin{array}{c} \sum_{\mathbf{p}} \exp\left(-\mathrm{i}\widehat{H}_d(t-t_0) + \mathrm{c}\int_{t=t_0}^{t} \widehat{Q}\partial t\right) \cdot \\ \cdot \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{p}(\mathbf{x}-\mathbf{x}_0)\right) \end{array} \right) \varphi(t_0,\mathbf{x}_0).$$

An operator

$$K(t-t_0,\mathbf{x}-\mathbf{x}_0,t,t_0) := \left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{3} \left(\begin{array}{c} \sum_{\mathbf{p}} \exp\left(-\mathrm{i}\widehat{H}_d(t-t_0) + \mathrm{c}\int_{t=t_0}^{t} \widehat{Q}\partial t\right) \cdot \\ \cdot \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{p}(\mathbf{x}-\mathbf{x}_0)\right) \end{array} \right)$$

is called *propagator* of the event $\mathcal{A}$ probability.
Hence:

$$\varphi(t,\mathbf{x}) = \int_{(\Omega)} d\mathbf{x}_0 \cdot K(t-t_0,\mathbf{x}-\mathbf{x}_0,t,t_0)\,\varphi(t_0,\mathbf{x}_0). \tag{4.12}$$

A propagator has the following property:

$$K(t-t_0,\mathbf{x}-\mathbf{x}_0,t,t_0) = \int d\mathbf{x}_1 \cdot K(t-t_1,\mathbf{x}-\mathbf{x}_1,t,t_1)\,K(t_1-t_0,\mathbf{x}_1-\mathbf{x}_0,t_1,t_0).$$

4.5. Double-Slit Experiment

In a vacuum (Figure 1, Figure 2, Figure 3): Here transmitter s of electrons, wall w, and the electrons detecting black screen d are placed[13].

Electrons are emitted one by one from the source s. When an electron hits against screen d then a bright spot arises in the hit place of d..

1. Let slit a be opened in wall w (Figure 1). An electron flies out from s, passes by a, and is detected by d.

If such operation will be reiterated N of times then N bright spots shall arise on d against slit a in the vicinity of point y_a.

2. Let slit b be opened in wall w (Figure 2). An electron flies out from s, passes by b, and is detected by d.

If such operation will be reiterated N of times then N bright spots shall arise on d against slit b in the vicinity of point y_b.

3. Let both slits be opened. In that case do you expect a result as on fig. 3? But no. We get result as on Figure 4[21][14].

For instance, such experiment was realized at Hitachi by A. Tonomura, J. Endo, T. Matsuda, T. Kawasaki and H. Ezawa in 1989. Here was presumed that interference fringes are produced only when two electrons pass through both slits simultaneously. If there were two electrons from the source s at the same time, such interference might happen. But this cannot occur, because here is no more than one electron from this source at one time. Please keep watching the experiment a little longer. When a large number of electrons is accumulated, something like regular fringes begin to appear in the perpendicular direction as Figure 5(c) shows. Clear interference fringes can be seen in the last scene of the experiment after 20 minutes (Figure 5(d)). It should also be noted that the fringes are made up of bright spots, each of which records the detection of an electron. We have reached a mysterious conclusion. Although electrons were sent one by one, interference fringes could be observed. These interference fringes are formed only when electron waves pass through on both slits at the same time but nothing other than this. Whenever electrons are observed, they are always detected as individual particles. When accumulated, however, interference fringes are formed. Please recall that at any one instant here was at most one electron from s. We have reached a conclusion which is far from what our common sense tells us.

4. But nevertheless, across which slit the electron had slipped?

Let (Figure 6) two detectors d_a and d_b and a photon source sf be added to devices of Figure 4.

An electron, slipped across slit a, is lighten by source sf, and detector d_a snaps into action. And an electron, slipped across slit b, is lighten by source sf, and detector d_b snaps into action.

If photon source sf lights all N electrons, slipped across slits, then we received the picture of Figure 3.

If source sf is faint then only a little part of N electrons, slipped across slits, are noticed

[21] Single-electron events build up over a 20 minute exposure to form an interference pattern in this double-slit experiment by Akira Tonomura and co-workers. Figure 5(a) 8 electrons; Figure 5(b) 270 electrons; Figure 5(c) 2000 electrons; Figure 5(d) 60,000. A video of this experiment will soon be available on the web (www.hqrd.hitachi.co.jp/em/doubleslit.html).

by detectors d_a and d_b. In that case electrons, noticed by detectors d_a and d_b, make picture of Figure 3, and all unnoticed electrons make picture of Figure 4. In result here the Figure 6 picture is received.

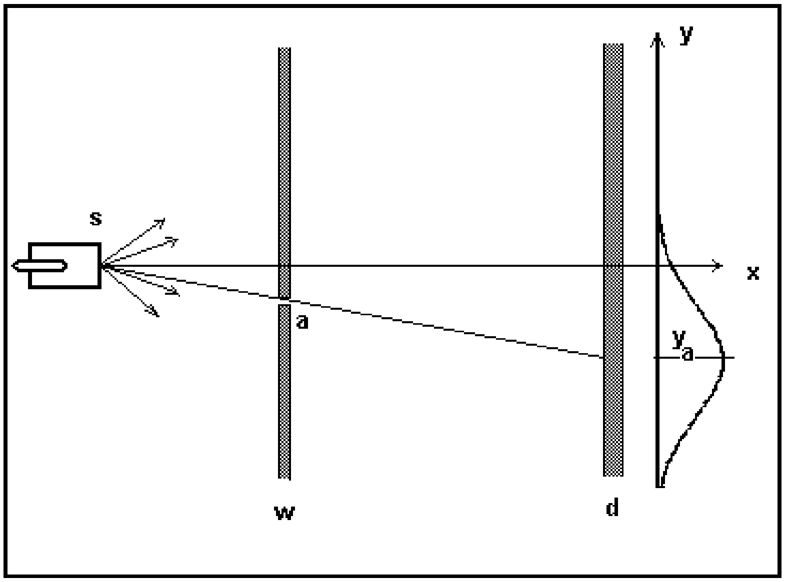


Figure 1:

Let us try to interpret these experiments by events and probabilities.

Denote the source s coordinates as $\langle x_0, y_0\rangle$, the slit a coordinates as $\langle x_a, y_a\rangle$, the slit b coordinates as $\langle x_b, y_b\rangle$. Here $x_a = x_b$, and the wall w equation is $x = x_a$. Denote the screen d equation as $x = x_d$.

Denote

an event, expressed by sentence: ≪electron is detected in point $\langle t, x, y\rangle$≫, as $\mathcal{C}(t,x,y)$,

an event, expressed by sentence ≪slit a is open≫, as $\mathcal{A}$,

and an event, expressed by sentence ≪slit b is open≫, as $\mathcal{B}$.

Let t_0 be an time instant of an electron emission from source s. Since s is a dotlike source then a state vector $\varphi_{\mathcal{C}}$ in instant t_0 has the following form:

$$\varphi_{\mathcal{C}}(t,x,y)|_{t=t_0} = \varphi_{\mathcal{C}}(t_0,x,y)\,\delta(x-x_0)\,\delta(y-y_0)\,. \tag{4.13}$$

Let t_w be an time instant such that if event $\mathcal{C}(t,x,y)$ occurs in that instant then $\mathcal{C}(t,x,y)$ occurs on wall w.

Let t_d be an time instant of a electron detecting by screen d.

1. Let slit a be opened in wall w (Figure 1).

In that case the $\mathcal{C}(t,x,y)$ probabilities propagator $K_{\mathcal{C}\mathcal{A}}(t-t_0, x-x_s, y-y_s)$ in instant t_w should be of the following shape:

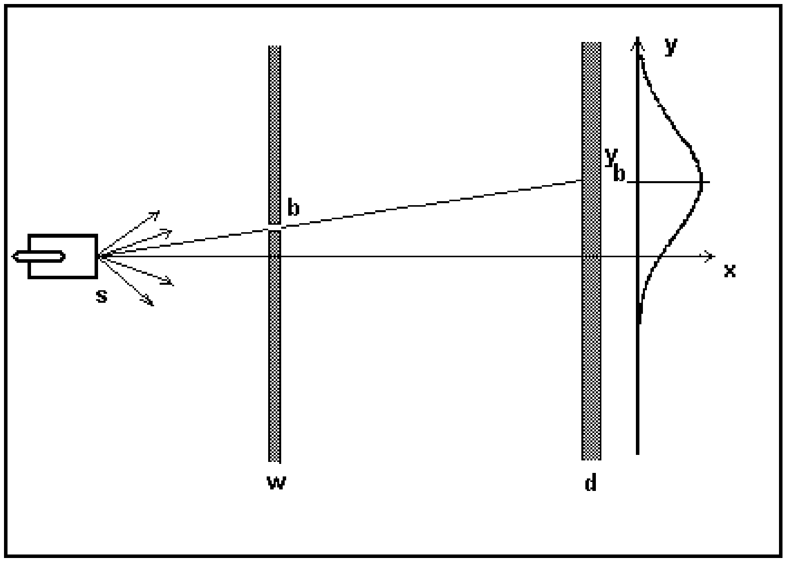


Figure 2:

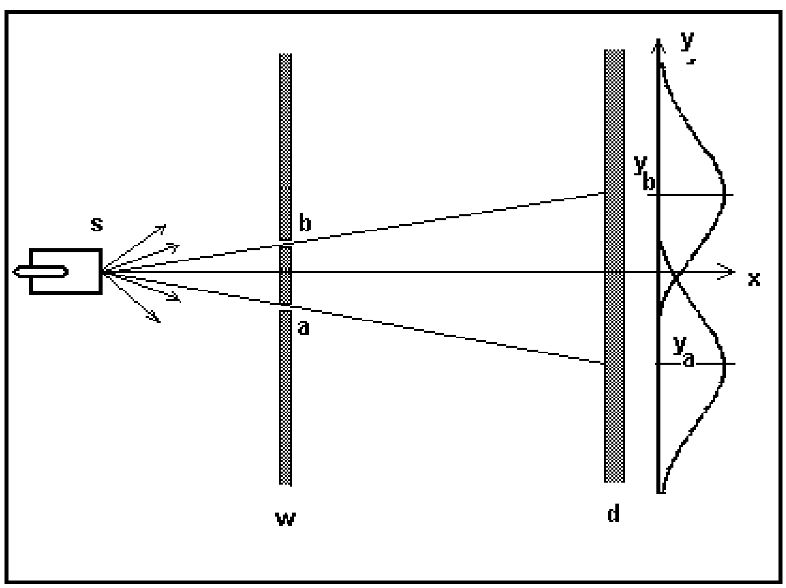


Figure 3:

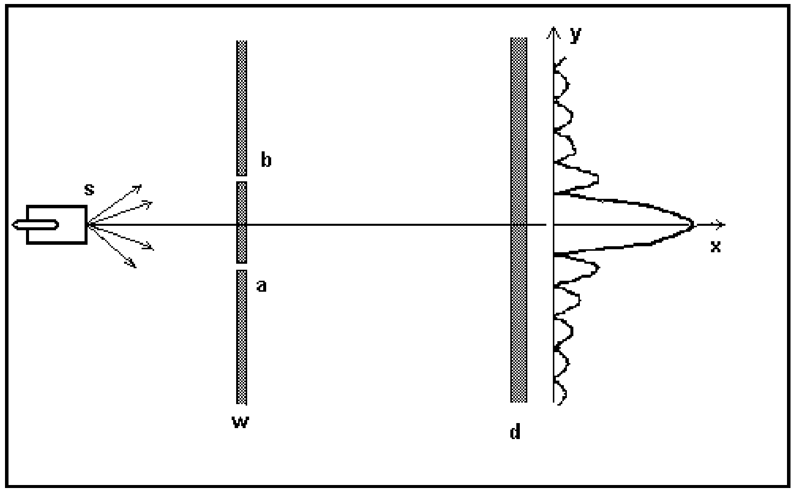


Figure 4:

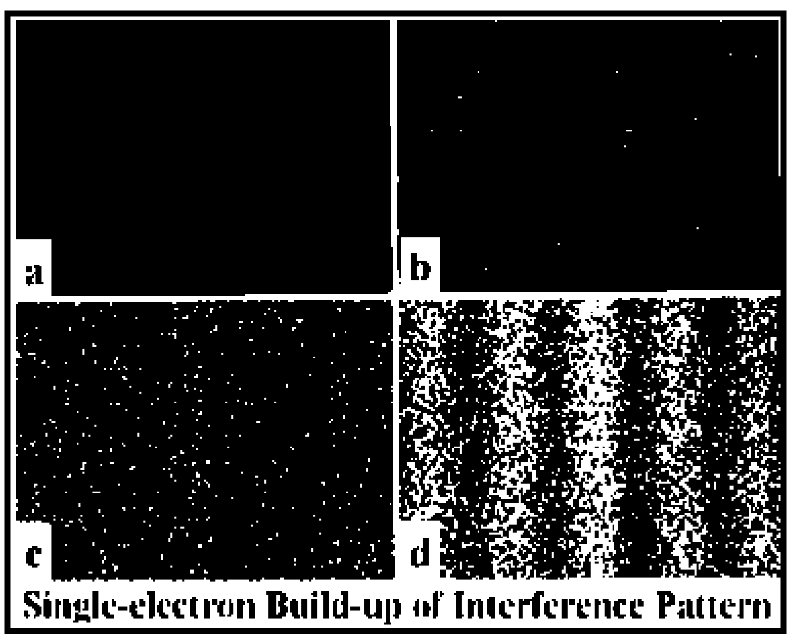


Figure 5:

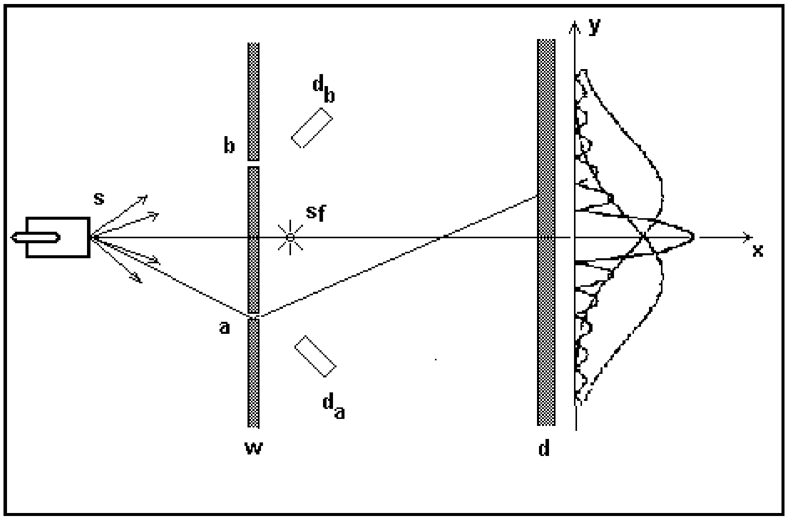


Figure 6:

$$
\begin{aligned}
& K_{\mathcal{CA}}\left(t-t_0, x-x_s, y-y_s\right)|_{t=t_w} \\
= \quad & K_{\mathcal{CA}}\left(t_w-t_0, x-x_s, y-y_s\right)\delta\left(x-x_a\right)\delta\left(y-y_a\right).
\end{aligned}
$$

According the propagator property:

$$
\begin{aligned}
& K\left(t-t_0, x-x_s, y-y_s\right)= \\
= \quad & \int_R dx_1 \int_R dy_1 \cdot K\left(t-t_1, x-x_1, y-y_1\right) K\left(t_1-t_0, x_1-x_s, y_1-y_s\right).
\end{aligned}
$$

Hence:

$$
\begin{aligned}
& K_{\mathcal{CA}}\left(t_d-t_0, x_d-x_s, y_d-y_s\right)= \\
= \quad & \int_R dx \int_R dy \cdot K_{\mathcal{CA}}\left(t_d-t_w, x_d-x, y_d-y\right) \\
& K_{\mathcal{CA}}\left(t_w-t_0, x-x_s, y-y_s\right)\delta\left(x-x_a\right)\delta\left(y-y_a\right).
\end{aligned}
$$

Therefore, according properties of δ-function:

$$
\begin{aligned}
& K_{\mathcal{CA}}\left(t_d-t_0, x_d-x_s, y_d-y_s\right)= \\
= \quad & K_{\mathcal{CA}}\left(t_d-t_w, x_d-x_a, y_d-y_a\right) K_{\mathcal{CA}}\left(t_w-t_0, x_a-x_s, y_a-y_s\right).
\end{aligned}
$$

The state vector for the event $\mathcal{C}(t,x,y)$ in condition $\mathcal{A}$ probability has the following form (4.12):

$$\varphi_{\mathcal{CA}}(t_d,x_d,y_d)=\int dx_s\int dy_s\cdot K_{\mathcal{CA}}(t_d-t_0,x_d-x_s,y_d-y_s)\,\varphi_{\mathcal{C}}(t_0,x_s,y_s)\,.$$

Hence, from (4.13):

$$\begin{aligned}\varphi_{\mathcal{CA}}(t_d,x_d,y_d) &= \int dx_s\int dy_s\cdot K_{\mathcal{CA}}(t_d-t_0,x_d-x_s,y_d-y_s)\\ &\quad \varphi_{\mathcal{C}}(t_0,x_s,y_s)\,\delta(x_s-x_0)\,\delta(y_s-y_0)\,.\end{aligned}$$

That is:

$$\begin{aligned}&\varphi_{\mathcal{CA}}(t_d,x_d,y_d)\\ =\ &\int dx_s\int dy_s\cdot K_{\mathcal{CA}}(t_d-t_w,x_d-x_a,y_d-y_a)\,K_{\mathcal{CA}}(t_w-t_0,x_a-x_s,y_a-y_s)\\ &\varphi_{\mathcal{C}}(t_0,x_s,y_s)\,\delta(x_s-x_0)\,\delta(y_s-y_0)\,.\end{aligned}$$

Hence, according properties of δ-function:

$$\begin{aligned}&\varphi_{\mathcal{CA}}(t_d,x_d,y_d)\\ =\ &K_{\mathcal{CA}}(t_d-t_w,x_d-x_a,y_d-y_a)\,K_{\mathcal{CA}}(t_w-t_0,x_a-x_0,y_a-y_0)\,\varphi_{\mathcal{C}}(t_0,x_0,y_0)\,.\end{aligned}$$

In accordance with (4.75):

$$\rho_{\mathcal{CA}}(t_d,x_d,y_d)=\varphi^{\dagger}_{\mathcal{CA}}(t_d,x_d,y_d)\,\varphi_{\mathcal{CA}}(t_d,x_d,y_d)\,.$$

Therefore, a probability to detect the electron in vicinity $\Delta x\Delta y$ of point $\langle x_d,y_d\rangle$ in instant t in condition $\mathcal{A}$ equals to the following:

$$P_a(t_d,x_d,y_d):=\mathrm{P}(\mathcal{C}(t_d,\Delta x\Delta y)/\mathcal{A})=\rho_{\mathcal{CA}}(t_d,x_d,y_d)\,\Delta x\Delta y.$$

2. Let slit b be opened in wall w (Figure 2).

In that case the $\mathcal{C}(t,x,y)$ probabilities propagator $K_{\mathcal{CB}}(t-t_0,x-x_s,y-y_s)$ in instant t_w should be of the following shape:

$$\begin{aligned}&K_{\mathcal{CB}}(t-t_0,x-x_s,y-y_s)\,|_{t=t_w}\\ =\ &K_{\mathcal{CB}}(t_w-t_0,x-x_s,y-y_s)\,\delta(x-x_b)\,\delta(y-y_b)\,.\end{aligned}$$

Hence, according the propagator property::

$$\begin{aligned}&K_{\mathcal{CB}}(t_d-t_0,x_d-x_s,y_d-y_s)=\\ =\ &\int_R dx\int_R dy\cdot K_{\mathcal{CB}}(t_d-t_w,x_d-x,y_d-y)\\ &K_{\mathcal{CB}}(t_w-t_0,x-x_s,y-y_s)\,\delta(x-x_b)\,\delta(y-y_b)\,.\end{aligned}$$

Therefore, according properties of δ-function:

$$\begin{aligned} & K_{\mathcal{CB}}(t_d - t_0, x_d - x_s, y_d - y_s) = \\ = \; & K_{\mathcal{CB}}(t_d - t_w, x_d - x_b, y_d - y_b)\, K_{\mathcal{CB}}(t_w - t_0, x_b - x_s, y_b - y_s)\,. \end{aligned}$$

The state vector for the event $\mathcal{C}(t,x,y)$ in condition $\mathcal{B}$ probability has the following form (4.12):

$$\varphi_{\mathcal{CB}}(t_d, x_d, y_d) = \int dx_s \int dy_s \cdot K_{\mathcal{CB}}(t_d - t_0, x_d - x_s, y_d - y_s)\, \varphi_{\mathcal{C}}(t_0, x_s, y_s)\,.$$

Hence, from (4.13):

$$\begin{aligned} \varphi_{\mathcal{CB}}(t_d, x_d, y_d) \; = \; & \int dx_s \int dy_s \cdot K_{\mathcal{CB}}(t_d - t_0, x_d - x_s, y_d - y_s) \\ & \varphi_{\mathcal{C}}(t_0, x_s, y_s)\, \delta(x_s - x_0)\, \delta(y_s - y_0)\,. \end{aligned}$$

That is:

$$\begin{aligned} & \varphi_{\mathcal{CB}}(t_d, x_d, y_d) \\ = \; & \int dx_s \int dy_s \cdot K_{\mathcal{CB}}(t_d - t_w, x_d - x_b, y_d - y_b)\, K_{\mathcal{CB}}(t_w - t_0, x_b - x_s, y_b - y_s) \\ & \varphi_{\mathcal{C}}(t_0, x_s, y_s)\, \delta(x_s - x_0)\, \delta(y_s - y_0)\,. \end{aligned}$$

Hence, according properties of δ-function:

$$\begin{aligned} & \varphi_{\mathcal{CB}}(t_d, x_d, y_d) \\ = \; & K_{\mathcal{CB}}(t_d - t_w, x_d - x_b, y_d - y_b)\, K_{\mathcal{CB}}(t_w - t_0, x_b - x_0, y_b - y_0)\, \varphi_{\mathcal{C}}(t_0, x_0, y_0)\,. \end{aligned}$$

In accordance with (4.75):

$$\rho_{\mathcal{CB}}(t_d, x_d, y_d) = \varphi^{\dagger}_{\mathcal{CB}}(t_d, x_d, y_d)\, \varphi_{\mathcal{CB}}(t_d, x_d, y_d)\,.$$

Therefore, a probability to detect the electron in vicinity $\Delta x \Delta y$ of point $\langle x_d, y_d \rangle$ in instant t in condition $\mathcal{B}$ equals to the following:

$$P_b(t_d, x_d, y_d) := \mathrm{P}(\mathcal{C}(t_d, \Delta x \Delta y) / \mathcal{B}) = \rho_{\mathcal{CB}}(t_d, x_d, y_d)\, \Delta x \Delta y.$$

3. Let both slits and a and b are opened (Figure 4).

In that case the $\mathcal{C}(t,x,y)$ probabilities propagator $K_{\mathcal{CAB}}(t - t_0, x - x_s, y - y_s)$ in instant t_w should be of the following shape:

$$\begin{aligned} & K_{\mathcal{CAB}}(t - t_0, x - x_s, y - y_s)\,|_{t=t_w} = \\ = \; & K_{\mathcal{CAB}}(t_w - t_0, x - x_s, y - y_s)\, (\delta(x - x_a)\, \delta(y - y_a) + \delta(x - x_b)\, \delta(y - y_b))\,. \end{aligned}$$

Hence, according the propagator property::

$$\begin{gathered}K_{\mathcal{CAB}}(t_d-t_0,x_d-x_s,y_d-y_s)=\\ =\int_R dx\int_R dy\cdot K_{\mathcal{CAB}}(t_d-t_w,x_d-x,y_d-y)\\ K_{\mathcal{CAB}}(t_w-t_0,x-x_s,y-y_s)\cdot\\ \cdot(\delta(x-x_a)\delta(y-y_a)+\delta(x-x_b)\delta(y-y_b)).\end{gathered}$$

Hence,

$$\begin{gathered}K_{\mathcal{CAB}}(t_d-t_0,x_d-x_s,y_d-y_s)=\\ \int_R dx\int_R dy\cdot K_{\mathcal{CAB}}(t_d-t_w,x_d-x,y_d-y)\ K_{\mathcal{CAB}}(t_w-t_0,x-x_s,y-y_s)\cdot\\ \cdot\delta(x-x_a)\delta(y-y_a)\\ +\int_R dx\int_R dy\cdot K_{\mathcal{CAB}}(t_d-t_w,x_d-x,y_d-y)\ K_{\mathcal{CAB}}(t_w-t_0,x-x_s,y-y_s)\cdot\\ \cdot\delta(x-x_b)\delta(y-y_b).\end{gathered}$$

Hence, according properties of δ-function:

$$\begin{gathered}K_{\mathcal{CAB}}(t_d-t_0,x_d-x_s,y_d-y_s)=\\ K_{\mathcal{CAB}}(t_d-t_w,x_d-x_a,y_d-y_a)\ K_{\mathcal{CAB}}(t_w-t_0,x_a-x_s,y_a-y_s)\\ +K_{\mathcal{CAB}}(t_d-t_w,x_d-x_b,y_d-y_b)\ K_{\mathcal{CAB}}(t_w-t_0,x_b-x_s,y_b-y_s)\end{gathered}\ .$$

The state vector for the event $\mathcal{C}(t,x,y)$ in condition $\mathcal{A}$ and $\mathcal{B}$ probability has the following form (4.12):

$$\varphi_{\mathcal{CAB}}(t_d,x_d,y_d)=\int dx_s\int dy_s\cdot K_{\mathcal{CAB}}(t_d-t_0,x_d-x_s,y_d-y_s)\,\varphi_{\mathcal{C}}(t_0,x_s,y_s).$$

Hence, from (4.13):

$$\begin{aligned}\varphi_{\mathcal{CAB}}(t_d,x_d,y_d)\ &=\ \int dx_s\int dy_s\cdot K_{\mathcal{CAB}}(t_d-t_0,x_d-x_s,y_d-y_s)\\ &\qquad\varphi_{\mathcal{C}}(t_0,x_s,y_s)\,\delta(x_s-x_0)\,\delta(y_s-y_0).\end{aligned}$$

That is:

$$\begin{gathered}\varphi_{\mathcal{CAB}}(t_d,x_d,y_d)=\int dx_s\int dy_s\cdot\\ \cdot\begin{pmatrix}K_{\mathcal{CAB}}(t_d-t_w,x_d-x_a,y_d-y_a)\ K_{\mathcal{CAB}}(t_w-t_0,x_a-x_s,y_a-y_s)\\ +K_{\mathcal{CAB}}(t_d-t_w,x_d-x_b,y_d-y_b)\ K_{\mathcal{CAB}}(t_w-t_0,x_b-x_s,y_b-y_s)\end{pmatrix}\\ \varphi_{\mathcal{C}}(t_0,x_s,y_s)\,\delta(x_s-x_0)\,\delta(y_s-y_0).\end{gathered}$$

Hence, according properties of δ-function:

$$\begin{gathered}\varphi_{\mathcal{CAB}}(t_d,x_d,y_d)=\\ =\begin{pmatrix}K_{\mathcal{CAB}}(t_d-t_w,x_d-x_a,y_d-y_a)\ K_{\mathcal{CAB}}(t_w-t_0,x_a-x_0,y_a-y_0)\\ +K_{\mathcal{CAB}}(t_d-t_w,x_d-x_b,y_d-y_b)\ K_{\mathcal{CAB}}(t_w-t_0,x_b-x_0,y_b-y_0)\end{pmatrix}\\ \varphi_{\mathcal{C}}(t_0,x_0,y_0).\end{gathered}$$

That is:

$$\begin{gathered}\varphi_{\mathcal{CAB}}(t_d,x_d,y_d)=\\ =K_{\mathcal{CAB}}(t_d-t_w,x_d-x_a,y_d-y_a)\ K_{\mathcal{CAB}}(t_w-t_0,x_a-x_0,y_a-y_0)\,\varphi_{\mathcal{C}}(t_0,x_0,y_0)\\ +K_{\mathcal{CAB}}(t_d-t_w,x_d-x_b,y_d-y_b)\ K_{\mathcal{CAB}}(t_w-t_0,x_b-x_0,y_b-y_0)\,\varphi_{\mathcal{C}}(t_0,x_0,y_0)\,.\end{gathered}$$

Therefore,

$$\varphi_{\mathcal{CAB}}(t_d,x_d,y_d)=\varphi_{\mathcal{CA}}(t_d,x_d,y_d)+\varphi_{\mathcal{CB}}(t_d,x_d,y_d)\,.$$

And in accordance with (4.75):

$$\rho_{\mathcal{CAB}}(t_d,x_d,y_d)=\varphi^{\dagger}_{\mathcal{CAB}}(t_d,x_d,y_d)\,\varphi_{\mathcal{CAB}}(t_d,x_d,y_d)\,,$$

i.e.

$$\rho_{\mathcal{CAB}}=(\varphi_{\mathcal{CA}}+\varphi_{\mathcal{CB}})^{\dagger}(\varphi_{\mathcal{CA}}+\varphi_{\mathcal{CB}})$$

Since state vectors $\varphi_{\mathcal{CA}}$ and $\varphi_{\mathcal{CB}}$ are not numbers with the same number signs then in the general case:

$$(\varphi_{\mathcal{CA}}+\varphi_{\mathcal{CB}})^{\dagger}(\varphi_{\mathcal{CA}}+\varphi_{\mathcal{CB}})\neq\varphi^{\dagger}_{\mathcal{CA}}\varphi_{\mathcal{CA}}+\varphi^{\dagger}_{\mathcal{CB}}\varphi_{\mathcal{CB}}.$$

Therefore, since a probability to detect the electron in vicinity $\Delta x\Delta y$ of point $\langle x_d,y_d\rangle$ in instant t in condition $\mathcal{AB}$ equals:

$$P_{ab}(t_d,x_d,y_d):=\mathrm{P}(\mathcal{C}(t_d,\Delta x\Delta y)/\mathcal{AB})=\rho_{\mathcal{CAB}}(t_d,x_d,y_d)\,\Delta x\Delta y$$

then

$$P_{ab}(t_d,x_d,y_d)\neq P_a(t_d,x_d,y_d)+P_b(t_d,x_d,y_d)\,.$$

Hence, we have the fig.23 picture instead of the Figure 3 picture.

4. Let us consider devices of Figure 6.

Denote event, expressed by sentence "detector d_a snaps into action", as $\mathcal{D}_a$ and event, expressed by sentence "detector d_b snaps into action", as $\mathcal{D}_b$. Since event $\mathcal{C}(t,x,y)$ is a dotlike event then events $\mathcal{D}_a$ and $\mathcal{D}_b$ are exclusive events.

According the property 10 of operations on events:

$$(\mathcal{D}_a+\mathcal{D}_b)+\overline{(\mathcal{D}_a+\mathcal{D}_b)}=\mathcal{T},$$

according the property 6 of operations on events:

$$\overline{(\mathcal{D}_a+\mathcal{D}_b)}=\overline{\mathcal{D}}_a\overline{\mathcal{D}}_b,$$

Hence:

$$\mathcal{D}_a+\mathcal{D}_b+\overline{\mathcal{D}}_a\overline{\mathcal{D}}_b=\mathcal{T}.$$

According the property 5 of operations on events:

$$\mathcal{C} = \mathcal{C}\mathcal{T} = \mathcal{C}\left(\mathcal{D}_a + \mathcal{D}_b + \overline{\mathcal{D}}_a\overline{\mathcal{D}}_b\right).$$

According the property 3 of operations on events:

$$\mathcal{C} = \mathcal{C}\mathcal{D}_a + \mathcal{C}\mathcal{D}_b + \mathcal{C}\overline{\mathcal{D}}_a\overline{\mathcal{D}}_b.$$

Therefore, according the probabilities addition formula for exclusive events:

$$\mathrm{P}(\mathcal{C}(t_d)) = \mathrm{P}(\mathcal{C}(t_d)\mathcal{D}_a) + \mathrm{P}(\mathcal{C}(t_d)\mathcal{D}_b) + \mathrm{P}\left(\mathcal{C}(t_d)\overline{\mathcal{D}}_a\overline{\mathcal{D}}_b\right).$$

But

$$\begin{aligned}
\mathrm{P}(\mathcal{C}(t_d)\mathcal{D}_a) &= P_a(t_d),\\
\mathrm{P}(\mathcal{C}(t_d)\mathcal{D}_b) &= P_b(t_d),\\
\mathrm{P}\left(\mathcal{C}(t_d)\overline{\mathcal{D}}_a\overline{\mathcal{D}}_b\right) &= P_{ab}(t_d),
\end{aligned}$$

and we receive the Figure 6 picture.

Thus, here are no paradoxes for the event-probability interpretation of these experiments. We should depart from notion of a continuously existing electron and consider an elementary particle an ensemble of events connected by probability. Its like the fact that physical particle exists only at the instant when it is involved in some event. A particle doesnt exist in any other time, but theres a probability that something will happen to it. Thus, if nothing happens with the particle between the event of creating it and the event of detecting it the behavior of the particle is the behavior of probability between the point of creating and the point of detecting it with the presence of interference.

But what is with Wilson cloud chamber where the particle has a clear trajectory and no interference?

In that case these trajectories are not totally continuous lines. Every point of ionization has neighboring point of ionization, and there are no events between these points.

Consequently, physical particle is moving because corresponding probability propagates in the space between points of ionization. Consequently, particle is an ensemble of events, connected by probability. And charges, masses, moments, etc. represent statistical parameters of these probability waves, propagated in the space-time. It explains all paradoxes of quantum physics. Schrodingers cat lives easy without any superposition of states until the micro event awaited by all occures. And the wave function disappears without any collapse in the moment when an event probability disappears after the event occurs.

Hence, entanglement concerns not particles but probabilities. That is when event of the measuring of spin of Alices electron occurs then probability for these entangled electrons is changed instantly on whole space. Therefore, nonlocality acts for probabilities, not for particles. But probabilities can not transmit any information

4.6. Lepton Hamiltonian

Let $\vartheta_{s,k}$ and $\varpi_{s,k}$ be terms of $\widehat{Q}$ (4.10) and let Θ_0, Θ_3, Υ_0 and Υ_3 be a solution of the following equations set:

$$\left\{\begin{array}{c} -\Theta_0+\Theta_3-\Upsilon_0+\Upsilon_3=\vartheta_{1,1}; \\ -\Theta_0-\Theta_3-\Upsilon_0-\Upsilon_3=\vartheta_{2,2}; \\ -\Theta_0-\Theta_3+\Upsilon_0+\Upsilon_3=\vartheta_{3,3}; \\ -\Theta_0+\Theta_3+\Upsilon_0-\Upsilon_3=\vartheta_{4,4} \end{array}\right|,$$

and Θ_1, Υ_1, Θ_2, Υ_2, M_0, M_4, $M_{\zeta,0}$, $M_{\zeta,4}$, $M_{\eta,0}$, $M_{\eta,4}$, $M_{\theta,0}$, $M_{\theta,4}$ be solutions of the following sets of equations:

$$\left\{\begin{array}{c} \Theta_1+\Upsilon_1=\vartheta_{1,2}; \\ -\Theta_1+\Upsilon_1=\vartheta_{3,4}; \end{array}\right|$$

$$\left\{\begin{array}{c} -\Theta_2-\Upsilon_2=\varpi_{1,2}; \\ \Theta_2-\Upsilon_2=\varpi_{3,4}; \end{array}\right|$$

$$\left\{\begin{array}{c} M_0+M_{\theta,0}=\vartheta_{1,3}; \\ M_0-M_{\theta,0}=\vartheta_{2,4}; \end{array}\right|$$

$$\left\{\begin{array}{c} M_4+M_{\theta,4}=\varpi_{1,3}; \\ M_4-M_{\theta,4}=\varpi_{2,4}; \end{array}\right|$$

$$\left\{\begin{array}{c} M_{\zeta,0}-M_{\eta,4}=\vartheta_{1,4}; \\ M_{\zeta,0}+M_{\eta,4}=\vartheta_{2,3}; \end{array}\right|$$

$$\left\{\begin{array}{c} M_{\zeta,4}-M_{\eta,0}=\varpi_{1,4}; \\ M_{\zeta,4}+M_{\eta,0}=\varpi_{2,3} \end{array}\right|.$$

Thus the columns of $\widehat{Q}$ are the following:
the first and the second columns:

$$\begin{array}{cc} -\mathrm{i}\Theta_0+\mathrm{i}\Theta_3-\mathrm{i}\Upsilon_0+\mathrm{i}\Upsilon_3 & \mathrm{i}\Theta_1+\mathrm{i}\Upsilon_1+\Theta_2+\Upsilon_2 \\ \mathrm{i}\Theta_1+\mathrm{i}\Upsilon_1-\Theta_2-\Upsilon_2 & -\mathrm{i}\Theta_0-\mathrm{i}\Theta_3-\mathrm{i}\Upsilon_0-\mathrm{i}\Upsilon_3 \\ \mathrm{i}M_0+\mathrm{i}M_{\theta,0}+M_4+M_{\theta,4} & \mathrm{i}M_{\zeta,0}+\mathrm{i}M_{\eta,4}+M_{\zeta,4}+M_{\eta,0} \\ \mathrm{i}M_{\zeta,0}-\mathrm{i}M_{\eta,4}+M_{\zeta,4}-M_{\eta,0} & \mathrm{i}M_0-\mathrm{i}M_{\theta,0}+M_4-M_{\theta,4} \end{array},$$

the third and the fourth columns:

$$\begin{array}{cc} \mathrm{i}M_0+\mathrm{i}M_{\theta,0}-M_4-M_{\theta,4} & \mathrm{i}M_{\zeta,0}-\mathrm{i}M_{\eta,4}-M_{\zeta,4}+M_{\eta,0} \\ \mathrm{i}M_{\zeta,0}+\mathrm{i}M_{\eta,4}-M_{\zeta,4}-M_{\eta,0} & \mathrm{i}M_0-\mathrm{i}M_{\theta,0}-M_4+M_{\theta,4} \\ -\mathrm{i}\Theta_0-\mathrm{i}\Theta_3+\mathrm{i}\Upsilon_0+\mathrm{i}\Upsilon_3 & -\mathrm{i}\Theta_1+\mathrm{i}\Upsilon_1-\Theta_2+\Upsilon_2 \\ -\mathrm{i}\Theta_1+\mathrm{i}\Upsilon_1+\Theta_2-\Upsilon_2 & -\mathrm{i}\Theta_0+\mathrm{i}\Theta_3+\mathrm{i}\Upsilon_0-\mathrm{i}\Upsilon_3 \end{array}.$$

Hence,

$$\begin{gathered}\widehat{Q}= \\ = \mathrm{i}\Theta_0\beta^{[0]}+\mathrm{i}\Upsilon_0\beta^{[0]}\gamma^{[5]}+ \\ +\mathrm{i}\Theta_1\beta^{[1]}+\mathrm{i}\Upsilon_1\beta^{[1]}\gamma^{[5]}+ \\ +\mathrm{i}\Theta_2\beta^{[2]}+\mathrm{i}\Upsilon_2\beta^{[2]}\gamma^{[5]}+ \\ +\mathrm{i}\Theta_3\beta^{[3]}+\mathrm{i}\Upsilon_3\beta^{[3]}\gamma^{[5]}+ \\ +\mathrm{i}M_0\gamma^{[0]}+\mathrm{i}M_4\beta^{[4]}- \\ -\mathrm{i}M_{\zeta,0}\gamma_{\zeta}^{[0]}+\mathrm{i}M_{\zeta,4}\zeta^{[4]}- \\ -\mathrm{i}M_{\eta,0}\gamma_{\eta}^{[0]}-\mathrm{i}M_{\eta,4}\eta^{[4]}+ \\ +\mathrm{i}M_{\theta,0}\gamma_{\theta}^{[0]}+\mathrm{i}M_{\theta,4}\theta^{[4]}.\end{gathered}$$

Therefore, from (4.9):

$$\frac{1}{\mathrm{c}}\partial_t\varphi-\left(\mathrm{i}\Theta_0\beta^{[0]}+\mathrm{i}\Upsilon_0\beta^{[0]}\gamma^{[5]}\right)\varphi=\left(\begin{gathered}\sum_{\nu=1}^{3}\beta^{[\nu]}\left(\partial_\nu+\mathrm{i}\Theta_\nu+\mathrm{i}\Upsilon_\nu\gamma^{[5]}\right)+ \\ +\mathrm{i}M_0\gamma^{[0]}+\mathrm{i}M_4\beta^{[4]}- \\ -\mathrm{i}M_{\zeta,0}\gamma_{\zeta}^{[0]}+\mathrm{i}M_{\zeta,4}\zeta^{[4]}- \\ -\mathrm{i}M_{\eta,0}\gamma_{\eta}^{[0]}-\mathrm{i}M_{\eta,4}\eta^{[4]}+ \\ +\mathrm{i}M_{\theta,0}\gamma_{\theta}^{[0]}+\mathrm{i}M_{\theta,4}\theta^{[4]}\end{gathered}\right)\varphi. \tag{4.14}$$

with

$$\gamma^{[5]}:=\left[\begin{array}{cc}1_2 & 0_2 \\ 0_2 & -1_2\end{array}\right]. \tag{4.15}$$

Because

$$\zeta^{[k]}+\eta^{[k]}+\theta^{[k]}=-\beta^{[k]}$$

with $k\in\{1,2,3\}$ then from (4.14):

$$\begin{gathered}\left(\begin{gathered}-\left(\partial_0+\mathrm{i}\Theta_0+\mathrm{i}\Upsilon_0\gamma^{[5]}\right)+\sum_{k=1}^{3}\beta^{[k]}\left(\partial_k+\mathrm{i}\Theta_k+\mathrm{i}\Upsilon_k\gamma^{[5]}\right) \\ +2\left(\mathrm{i}M_0\gamma^{[0]}+\mathrm{i}M_4\beta^{[4]}\right)\end{gathered}\right)\varphi+ \\ +\left(\begin{gathered}-\left(\partial_0+\mathrm{i}\Theta_0+\mathrm{i}\Upsilon_0\gamma^{[5]}\right)-\sum_{k=1}^{3}\zeta^{[k]}\left(\partial_k+\mathrm{i}\Theta_k+\mathrm{i}\Upsilon_k\gamma^{[5]}\right) \\ +2\left(-\mathrm{i}M_{\zeta,0}\gamma_{\zeta}^{[0]}+\mathrm{i}M_{\zeta,4}\zeta^{[4]}\right)\end{gathered}\right)\varphi+ \\ +\left(\begin{gathered}\left(\partial_0+\mathrm{i}\Theta_0+\mathrm{i}\Upsilon_0\gamma^{[5]}\right)-\sum_{k=1}^{3}\eta^{[k]}\left(\partial_k+\mathrm{i}\Theta_k+\mathrm{i}\Upsilon_k\gamma^{[5]}\right) \\ +2\left(-\mathrm{i}M_{\eta,0}\gamma_{\eta}^{[0]}-\mathrm{i}M_{\eta,4}\eta^{[4]}\right)\end{gathered}\right)\varphi+ \\ +\left(\begin{gathered}-\left(\partial_0+\mathrm{i}\Theta_0+\mathrm{i}\Upsilon_0\gamma^{[5]}\right)-\sum_{k=1}^{3}\theta^{[k]}\left(\partial_k+\mathrm{i}\Theta_k+\mathrm{i}\Upsilon_k\gamma^{[5]}\right) \\ +2\left(\mathrm{i}M_{\theta,0}\gamma_{\theta}^{[0]}+\mathrm{i}M_{\theta,4}\theta^{[4]}\right)\end{gathered}\right)\varphi=0.\end{gathered}$$

In (4.14) summands

$$-\mathrm{i}M_{\zeta,0}\gamma_{\zeta}^{[0]}+\mathrm{i}M_{\zeta,4}\zeta^{[4]}-$$
$$-\mathrm{i}M_{\eta,0}\gamma_{\eta}^{[0]}-\mathrm{i}M_{\eta,4}\eta^{[4]}+$$
$$+\mathrm{i}M_{\theta,0}\gamma_{\theta}^{[0]}+\mathrm{i}M_{\theta,4}\theta^{[4]}$$

contain elements of chromatic pentads and

$$\sum_{k=1}^{3}\beta^{[k]}\left(\partial_k+\mathrm{i}\Theta_k+\mathrm{i}\Upsilon_k\gamma^{[5]}\right)+\mathrm{i}M_0\gamma^{[0]}+\mathrm{i}M_4\beta^{[4]}$$

contains only elements of the light pentads. The following sum

$$\widehat{H}_l := \mathrm{c}\sum_{k=1}^{3}\beta^{[k]}\left(\mathrm{i}\partial_k-\Theta_k-\Upsilon_k\gamma^{[5]}\right)-\mathrm{c}M_0\gamma^{[0]}-\mathrm{c}M_4\beta^{[4]} \tag{4.16}$$

is called *lepton Hamiltonian*.
And the following equation:

$$\left(\sum_{k=0}^{3}\beta^{[k]}\left(\mathrm{i}\partial_k-\Theta_k-\Upsilon_k\gamma^{[5]}\right)-M_0\gamma^{[0]}-M_4\beta^{[4]}\right)\widetilde{\varphi}=0 \tag{4.17}$$

is called *lepton moving equation*
If like to (4.75):

$$\varphi^{\dagger}\gamma^{[0]}\varphi := -\tfrac{j_5}{\mathrm{c}} \text{ and } \varphi^{\dagger}\beta^{[4]}\varphi := -\tfrac{j_4}{\mathrm{c}}$$

and:

$$\rho u_4 := j_4 \text{ and } \rho u_5 := j_5 \tag{4.18}$$

then from (4.3):

$$-\frac{u_5}{\mathrm{c}}=\sin 2\alpha\left(\begin{array}{c}\sin\beta\sin\chi\cos\left(-\theta+\upsilon\right)\\+\cos\beta\cos\chi\cos\left(\gamma-\lambda\right)\end{array}\right),$$
$$-\frac{u_4}{\mathrm{c}}=\sin 2\alpha\left(\begin{array}{c}-\sin\beta\sin\chi\sin\left(-\theta+\upsilon\right)\\+\cos\beta\cos\chi\sin\left(\gamma-\lambda\right)\end{array}\right).$$

Hence, from (4.1):

$$u_1^2+u_2^2+u_3^2+u_{\mathcal{A},4}^2+u_5^2=\mathrm{c}^2.$$

Thus, of only all five elements of a Clifford pentad lends an entire kit of velocity components and, for completeness, yet two ”space” coordinates x_5 and x_4 should be added to our three x_1, x_2, x_3. These additional coordinates can be selected such that (4.1.)

$$-\frac{\pi\mathrm{c}}{\mathrm{h}}\leq x_5\leq\frac{\pi\mathrm{c}}{\mathrm{h}},-\frac{\pi\mathrm{c}}{\mathrm{h}}\leq x_4\leq\frac{\pi\mathrm{c}}{\mathrm{h}}.$$

Coordinates x_4 and x_5 are not of any events coordinates. Hence, our devices do not detect of its as space coordinates.

Let us denote:

$$\widetilde{\varphi}(t,x_1,x_2,x_3,x_5,x_4) := \varphi(t,x_1,x_2,x_3)\cdot$$
$$\cdot(\exp(\mathrm{i}(x_5M_0(t,x_1,x_2,x_3)+x_4M_4(t,x_1,x_2,x_3)))).$$

In this case equation of moving with lepton Hamiltonian (4.16) shape is the following:

$$\left(\sum_{k=0}^{3}\beta^{[0]}\left(\mathrm{i}\partial_k-\Theta_k-\Upsilon_k\gamma^{[5]}\right)-\gamma^{[0]}\mathrm{i}\partial_5-\beta^{[4]}\mathrm{i}\partial_4\right)\widetilde{\varphi}=0 \tag{4.19}$$

Let g_1 be the positive real number and for $\mu\in\{0,1,2,3\}$: F_μ and B_μ be the solutions of the following system of the equations:

$$\left\{\begin{array}{c} -0.5g_1B_\mu+F_\mu=-\Theta_\mu-\Upsilon_\mu; \\ -g_1B_\mu+F_\mu=-\Theta_\mu+\Upsilon_\mu. \end{array}\right|$$

Let *charge matrix* be denoted as the following:

$$Y:=-\left[\begin{array}{cc} 1_2 & 0_2 \\ 0_2 & 2\cdot 1_2 \end{array}\right]. \tag{4.20}$$

Thus (2.106),(2.105) :

$$\begin{array}{cl} & -\Theta_\mu-\Upsilon_\mu\gamma^{[5]}= \\ = & -\Theta_\mu 1_4-\Upsilon_\mu\gamma^{[5]}= \\ = & -\Theta_\mu\left[\begin{array}{cc} 1_2 & 0_2 \\ 0_2 & 1_2 \end{array}\right]-\Upsilon_\mu\left[\begin{array}{cc} 1_2 & 0_2 \\ 0_2 & -1_2 \end{array}\right]= \\ = & -\left(\left[\begin{array}{cc} \Theta_\mu 1_2 & 0_2 \\ 0_2 & \Theta_\mu 1_2 \end{array}\right]+\left[\begin{array}{cc} \Upsilon_\mu 1_2 & 0_2 \\ 0_2 & -\Upsilon_\mu 1_2 \end{array}\right]\right)= \\ = & \left[\begin{array}{cc} (-\Theta_\mu-\Upsilon_\mu)1_2 & 0_2 \\ 0_2 & (-\Theta_\mu+\Upsilon_\mu)1_2 \end{array}\right]= \\ = & \left[\begin{array}{cc} (-0.5g_1B_\mu+F_\mu)1_2 & 0_2 \\ 0_2 & (-g_1B_\mu+F_\mu)1_2 \end{array}\right]. \end{array}$$

And

$$\begin{array}{l} F_\mu+0.5g_1YB_\mu= \\ =F_\mu 1_4+0.5g_1YB_\mu \\ =F_\mu\left[\begin{array}{cc} 1_2 & 0_2 \\ 0_2 & 1_2 \end{array}\right]+0.5g_1\left(-\left[\begin{array}{cc} 1_2 & 0_2 \\ 0_2 & 2\cdot 1_2 \end{array}\right]\right)B_\mu= \\ =\left[\begin{array}{cc} F_\mu 1_2 & 0_2 \\ 0_2 & F_\mu 1_2 \end{array}\right]-\left[\begin{array}{cc} 0.5g_1B_\mu 1_2 & 0_2 \\ 0_2 & 0.5g_1B_\mu 2\cdot 1_2 \end{array}\right]= \\ =\left[\begin{array}{cc} F_\mu 1_2-0.5g_1B_\mu 1_2 & 0_2 \\ 0_2 & F_\mu 1_2-g_1B_\mu\cdot 1_2 \end{array}\right]. \end{array}$$

Hence,

$$-\Theta_\mu - \Upsilon_\mu \gamma^{[5]} = F_\mu + 0.5 g_1 Y B_\mu$$

and from (4.19):

$$\left(\sum_{k=0}^{3} \beta^{[k]} \left(\mathrm{i}\partial_k + F_k + 0.5 g_1 Y B_k\right) - \gamma^{[0]} \mathrm{i}\partial_5 - \beta^{[4]} \mathrm{i}\partial_4\right) \widetilde{\varphi} = 0 \tag{4.21}$$

Let $\chi(t, x_1, x_2, x_3)$ be the real function and:

$$\widetilde{U}(\chi) := \begin{bmatrix} \exp\left(\mathrm{i}\frac{\chi}{2}\right) 1_2 & 0_2 \\ 0_2 & \exp(\mathrm{i}\chi) 1_2 \end{bmatrix}. \tag{4.22}$$

In that case for $\mu \in \{0, 1, 2, 3\}$:

$$\begin{aligned}
\partial_\mu \widetilde{U} &= \partial_\mu \begin{bmatrix} \exp\left(\mathrm{i}\frac{\chi}{2}\right) 1_2 & 0_2 \\ 0_2 & \exp(\mathrm{i}\chi) 1_2 \end{bmatrix} \\
&= \begin{bmatrix} \partial_\mu \exp\left(\mathrm{i}\frac{\chi}{2}\right) 1_2 & \partial_\mu 0_2 \\ \partial_\mu 0_2 & \partial_\mu \exp(\mathrm{i}\chi) 1_2 \end{bmatrix} \\
&= \begin{bmatrix} \mathrm{i}\frac{\partial_\mu \chi}{2} \exp\left(\mathrm{i}\frac{\chi}{2}\right) 1_2 & 0_2 \\ 0_2 & \mathrm{i}\partial_\mu \chi \exp(\mathrm{i}\chi) 1_2 \end{bmatrix} \\
&= \mathrm{i}\frac{\partial_\mu \chi}{2} \begin{bmatrix} \exp\left(\mathrm{i}\frac{\chi}{2}\right) 1_2 & 0_2 \\ 0_2 & 2\exp(\mathrm{i}\chi) 1_2 \end{bmatrix},
\end{aligned}$$

and

$$\begin{aligned}
Y\widetilde{U} &= -\begin{bmatrix} 1_2 & 0_2 \\ 0_2 & 2 \cdot 1_2 \end{bmatrix} \begin{bmatrix} \exp\left(\mathrm{i}\frac{\chi}{2}\right) 1_2 & 0_2 \\ 0_2 & \exp(\mathrm{i}\chi) 1_2 \end{bmatrix} \\
&= -\begin{bmatrix} \exp\left(\mathrm{i}\frac{\chi}{2}\right) 1_2 & 0_2 \\ 0_2 & 2\exp(\mathrm{i}\chi) 1_2 \end{bmatrix}.
\end{aligned}$$

Hence:

$$\partial_\mu \widetilde{U} = -\mathrm{i}\frac{\partial_\mu \chi}{2} Y \widetilde{U}. \tag{4.23}$$

Moreover you can calculate that:

$$\begin{aligned}
\widetilde{U}^\dagger \gamma^{[0]} \widetilde{U} &= \gamma^{[0]} \cos\frac{\chi}{2} + \beta^{[4]} \sin\frac{\chi}{2}, \\
\widetilde{U}^\dagger \beta^{[4]} \widetilde{U} &= \beta^{[4]} \cos\frac{\chi}{2} - \gamma^{[0]} \sin\frac{\chi}{2}, \\
\widetilde{U}^\dagger \widetilde{U} &= 1_4, \\
\widetilde{U}^\dagger Y \widetilde{U} &= Y, \\
\beta^{[k]} \widetilde{U} &= \widetilde{U} \beta^{[k]}
\end{aligned}$$

for $k \in \{0,1,2,3\}$
Let

$$\begin{aligned} x'_4 &= x_4 \cos\frac{\chi}{2} - x_5 \sin\frac{\chi}{2}, \\ x'_5 &= x_5 \cos\frac{\chi}{2} + x_4 \sin\frac{\chi}{2}. \end{aligned}$$

In that case by the partial derivate definition for any function u:

$$\begin{aligned} \partial_4 u &= \partial'_4 u \cdot \partial_4 x'_4 + \partial'_5 u \cdot \partial_4 x'_5 = \partial'_4 u \cdot \cos\frac{\chi}{2} + \partial'_5 u \cdot \sin\frac{\chi}{2}, \\ \partial_5 u &= \partial'_4 u \cdot \partial_5 x'_4 + \partial'_5 u \cdot \partial_5 x'_5 = \partial'_4 u \cdot \left(-\sin\frac{\chi}{2}\right) + \partial'_5 u \cdot \cos\frac{\chi}{2}. \end{aligned} \tag{4.24}$$

Let $\partial_4 \chi = 0$ and $\partial_5 \chi = 0$; hence, $\partial_4 U = U \partial_4$ and $\partial_5 U = U \partial_5$.
From (4.21):

$$\left(\sum_{s=0}^{3} \beta^{[s]} \left(\mathrm{i}\partial_s + F_s + 0.5 g_1 Y B_s\right) - \gamma^{[0]} \mathrm{i}\partial_5 - \beta^{[4]} \mathrm{i}\partial_4\right) \widetilde{\varphi} = 0. \tag{4.25}$$

Let

$$B'_\mu = B_\mu - \frac{1}{g_1} \partial_\mu \chi.$$

According to (4.24) and since $\widetilde{U}^\dagger \widetilde{U} = 1_4$ and $\widetilde{U}^\dagger Y \widetilde{U} = Y$ then

$$\left(\begin{array}{c} \sum_{s=0}^{3} \beta^{[s]} \left(\mathrm{i}\partial_s + F_s + 0.5 g_1 \widetilde{U}^\dagger Y \widetilde{U} \left(B'_s + \frac{1}{g_1}\partial_s \chi\right)\right) - \\ -\gamma^{[0]} \mathrm{i} \left(-\sin\frac{\chi}{2}\partial'_4 + \cos\frac{\chi}{2}\partial'_5\right) - \beta^{[4]} \mathrm{i} \left(\cos\frac{\chi}{2}\partial'_4 + \sin\frac{\chi}{2}\partial'_5\right) \end{array}\right) \widetilde{U}^\dagger \widetilde{U} \widetilde{\varphi} = 0.$$

Hence:

$$\left(\begin{array}{c} \sum_{s=0}^{3} \beta^{[s]} \left(\mathrm{i}\partial_s + F_s + 0.5 g_1 \widetilde{U}^\dagger Y \widetilde{U} \left(B'_s + \frac{1}{g_1}\partial_s \chi\right)\right) - \\ -\left(-\gamma^{[0]} \sin\frac{\chi}{2} + \beta^{[4]} \cos\frac{\chi}{2}\right) \mathrm{i}\partial'_4 - \left(\gamma^{[0]} \cos\frac{\chi}{2} + \beta^{[4]} \sin\frac{\chi}{2}\right) \mathrm{i}\partial'_5 \end{array}\right) \widetilde{U}^\dagger \widetilde{U} \widetilde{\varphi} = 0.$$

Since $\widetilde{U}$ is a linear operator then

$$\left(\begin{array}{c} \sum_{s=0}^{3} \beta^{[s]} \left(\mathrm{i}\partial_s + F_s + 0.5 g_1 \widetilde{U}^\dagger Y \widetilde{U} \left(B'_s + \frac{1}{g_1}\partial_s \chi\right)\right) \widetilde{U}^\dagger - \\ -\left(-\gamma^{[0]} \sin\frac{\chi}{2} + \beta^{[4]} \cos\frac{\chi}{2}\right) \mathrm{i}\partial'_4 \widetilde{U}^\dagger - \left(\gamma^{[0]} \cos\frac{\chi}{2} + \beta^{[4]} \sin\frac{\chi}{2}\right) \mathrm{i}\partial'_5 \widetilde{U}^\dagger \end{array}\right) \widetilde{U} \widetilde{\varphi} = 0$$

and since $\partial_4 U = U \partial_4$ and $\partial_5 U = U \partial_5$ then

$$\left(\begin{array}{c} \sum_{s=0}^{3} \beta^{[s]} \left(\mathrm{i}\partial_s \widetilde{U}^\dagger + F_s \widetilde{U}^\dagger + 0.5 g_1 \widetilde{U}^\dagger Y \widetilde{U} \widetilde{U}^\dagger \left(B'_s + \frac{1}{g_1}\partial_s \chi\right)\right) - \\ -\left(-\gamma^{[0]} \widetilde{U}^\dagger \sin\frac{\chi}{2} + \beta^{[4]} \widetilde{U}^\dagger \cos\frac{\chi}{2}\right) \mathrm{i}\partial'_4 \\ -\left(\gamma^{[0]} \widetilde{U}^\dagger \cos\frac{\chi}{2} + \beta^{[4]} \widetilde{U}^\dagger \sin\frac{\chi}{2}\right) \mathrm{i}\partial'_5 \end{array}\right) \widetilde{U} \widetilde{\varphi} = 0. \tag{4.26}$$

Since

$$\begin{aligned}\widetilde{U}^{\dagger}\gamma^{[0]}\widetilde{U} &= \gamma^{[0]}\cos\frac{\chi}{2}+\beta^{[4]}\sin\frac{\chi}{2},\\ \widetilde{U}^{\dagger}\beta^{[4]}\widetilde{U} &= \beta^{[4]}\cos\frac{\chi}{2}-\gamma^{[0]}\sin\frac{\chi}{2}\end{aligned}$$

then

$$\begin{aligned}\widetilde{U}^{\dagger}\gamma^{[0]}\widetilde{U}\widetilde{U}^{\dagger} &= \gamma^{[0]}\widetilde{U}^{\dagger}\cos\frac{\chi}{2}+\beta^{[4]}\widetilde{U}^{\dagger}\sin\frac{\chi}{2},\\ \widetilde{U}^{\dagger}\beta^{[4]}\widetilde{U}\widetilde{U}^{\dagger} &= \beta^{[4]}\widetilde{U}^{\dagger}\cos\frac{\chi}{2}-\gamma^{[0]}\widetilde{U}^{\dagger}\sin\frac{\chi}{2},\end{aligned}$$

Hence,

$$\begin{aligned}\widetilde{U}^{\dagger}\gamma^{[0]} &= \gamma^{[0]}\widetilde{U}^{\dagger}\cos\frac{\chi}{2}+\beta^{[4]}\widetilde{U}^{\dagger}\sin\frac{\chi}{2},\\ \widetilde{U}^{\dagger}\beta^{[4]} &= \beta^{[4]}\widetilde{U}^{\dagger}\cos\frac{\chi}{2}-\gamma^{[0]}\widetilde{U}^{\dagger}\sin\frac{\chi}{2}.\end{aligned}$$

Therefore,

$$\begin{aligned}\gamma^{[0]}\widetilde{U}^{\dagger} &= \widetilde{U}^{\dagger}\gamma^{[0]}\cos\tfrac{\chi}{2}-\widetilde{U}^{\dagger}\beta^{[4]}\sin\tfrac{\chi}{2},\\ \beta^{[4]}\widetilde{U}^{\dagger} &= \widetilde{U}^{\dagger}\gamma^{[0]}\sin\tfrac{\chi}{2}+\widetilde{U}^{\dagger}\beta^{[4]}\cos\tfrac{\chi}{2}.\end{aligned}$$

Thus, from (4.26):

$$\left(\begin{array}{c}\sum_{s=0}^{3}\beta^{[s]}\left(\mathrm{i}\partial_s\widetilde{U}^{\dagger}+F_s\widetilde{U}^{\dagger}+0.5g_1\widetilde{U}^{\dagger}Y\widetilde{U}\widetilde{U}^{\dagger}\left(B'_s+\frac{1}{g_1}\partial_s\chi\right)\right)-\\ -\left(\begin{array}{c}-\left(\widetilde{U}^{\dagger}\gamma^{[0]}\cos\frac{\chi}{2}-\widetilde{U}^{\dagger}\beta^{[4]}\sin\frac{\chi}{2}\right)\sin\frac{\chi}{2}\\ +\left(\widetilde{U}^{\dagger}\gamma^{[0]}\sin\frac{\chi}{2}+\widetilde{U}^{\dagger}\beta^{[4]}\cos\frac{\chi}{2}\right)\cos\frac{\chi}{2}\end{array}\right)\mathrm{i}\partial'_4\\ -\left(\begin{array}{c}\left(\widetilde{U}^{\dagger}\gamma^{[0]}\cos\frac{\chi}{2}-\widetilde{U}^{\dagger}\beta^{[4]}\sin\frac{\chi}{2}\right)\cos\frac{\chi}{2}\\ +\left(\widetilde{U}^{\dagger}\gamma^{[0]}\sin\frac{\chi}{2}+\widetilde{U}^{\dagger}\beta^{[4]}\cos\frac{\chi}{2}\right)\sin\frac{\chi}{2}\end{array}\right)\mathrm{i}\partial'_5\end{array}\right)\widetilde{U}\widetilde{\varphi}=0.$$

Hence:

$$\left(\begin{array}{c}\sum_{s=0}^{3}\beta^{[s]}\left(\mathrm{i}\partial_s\widetilde{U}^{\dagger}+F_s\widetilde{U}^{\dagger}+0.5g_1\widetilde{U}^{\dagger}Y\left(B'_s+\frac{1}{g_1}\partial_s\chi\right)\right)-\\ -\widetilde{U}^{\dagger}\beta^{[4]}\mathrm{i}\partial'_4-\ \widetilde{U}^{\dagger}\gamma^{[0]}\mathrm{i}\partial'_5\end{array}\right)\widetilde{U}\widetilde{\varphi}=0. \tag{4.27}$$

Since (4.23):

$$\partial_\mu\widetilde{U}=-\mathrm{i}\frac{\partial_\mu\chi}{2}Y\widetilde{U}$$

then for $s\in\{0,1,2,3\}$:

$$\partial_s \widetilde{U}^\dagger = \mathrm{i}\frac{\partial_s \chi}{2} \widetilde{U}^\dagger Y^\dagger = \mathrm{i}\frac{\partial_s \chi}{2} Y \widetilde{U}^\dagger.$$

Therefore,

$$\begin{gathered}\partial_s \left(\widetilde{U}^\dagger \widetilde{U} \widetilde{\varphi}\right) = \partial_s \left(\widetilde{U}^\dagger \left(\widetilde{U}\widetilde{\varphi}\right)\right) = \\ = \left(\partial_s \widetilde{U}^\dagger\right)\left(\widetilde{U}\widetilde{\varphi}\right) + \widetilde{U}^\dagger \partial_s \left(\widetilde{U}\widetilde{\varphi}\right) = \mathrm{i}\tfrac{\partial_s \chi}{2} Y \widetilde{U}^\dagger \left(\widetilde{U}\widetilde{\varphi}\right) + \widetilde{U}^\dagger \partial_s \left(\widetilde{U}\widetilde{\varphi}\right) = \\ = \left(\mathrm{i}\tfrac{\partial_s \chi}{2} Y \widetilde{U}^\dagger + \widetilde{U}^\dagger \partial_s\right)\left(\widetilde{U}\widetilde{\varphi}\right).\end{gathered}$$

Since $Y\widetilde{U}^\dagger = \widetilde{U}^\dagger Y$ then

$$\mathrm{i}\frac{\partial_s \chi}{2} Y \widetilde{U}^\dagger + \widetilde{U}^\dagger \partial_s = \widetilde{U}^\dagger \mathrm{i}\frac{\partial_s \chi}{2} Y + \widetilde{U}^\dagger \partial_s.$$

Hence,

$$\mathrm{i}\partial_s \widetilde{U}^\dagger = -\widetilde{U}^\dagger \frac{\partial_s \chi}{2} Y + \widetilde{U}^\dagger \mathrm{i}\partial_s.$$

Therefore, from (4.27):

$$\left(\begin{array}{c} \sum_{s=0}^{3} \beta^{[s]} \left(-\widetilde{U}^\dagger \frac{\partial_s \chi}{2} Y + \widetilde{U}^\dagger \mathrm{i}\partial_s + F_s \widetilde{U}^\dagger + 0.5 g_1 \widetilde{U}^\dagger Y \left(B'_s + \frac{1}{g_1}\partial_s \chi\right)\right) - \\ -\widetilde{U}^\dagger \beta^{[4]} \mathrm{i}\partial'_4 - \ \widetilde{U}^\dagger \gamma^{[0]} \mathrm{i}\partial'_5 \end{array}\right) \widetilde{U}\widetilde{\varphi} = 0.$$

Hence:

$$\left(\begin{array}{c} \sum_{s=0}^{3} \beta^{[s]} \left(\widetilde{U}^\dagger \mathrm{i}\partial_s + \widetilde{U}^\dagger F'_s + 0.5 g_1 \widetilde{U}^\dagger Y B'_s\right) - \\ -\widetilde{U}^\dagger \beta^{[4]} \mathrm{i}\partial'_4 - \ \widetilde{U}^\dagger \gamma^{[0]} \mathrm{i}\partial'_5 \end{array}\right) \widetilde{U}\widetilde{\varphi} = 0$$

with $F'_s := \widetilde{U} F_s \widetilde{U}^\dagger$.
Since $\beta^{[s]}\widetilde{U} = \widetilde{U}\beta^{[s]}$ for $s \in \{0,1,2,3\}$ then

$$\left(\begin{array}{c} \sum_{s=0}^{3} \widetilde{U}^\dagger \beta^{[s]} \left(\mathrm{i}\partial_s + \widetilde{U}^\dagger F'_s + 0.5 g_1 \widetilde{U}^\dagger Y B'_s\right) - \\ -\widetilde{U}^\dagger \beta^{[4]} \mathrm{i}\partial'_4 - \ \widetilde{U}^\dagger \gamma^{[0]} \mathrm{i}\partial'_5 \end{array}\right) \widetilde{U}\widetilde{\varphi} = 0.$$

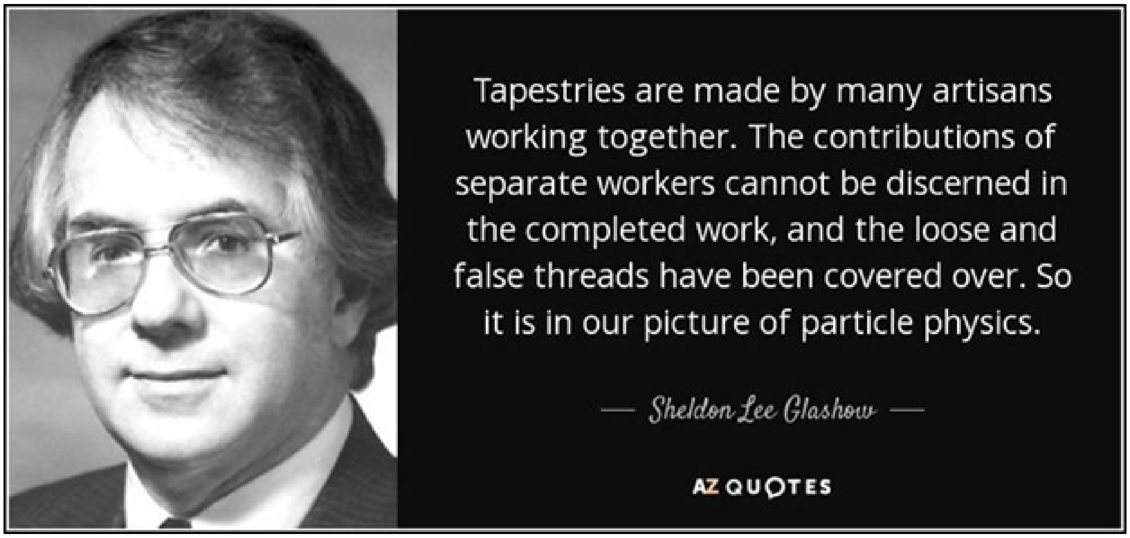
Hence, if denote: $\widetilde{\varphi}' := \widetilde{U}\widetilde{\varphi}$ then since $\widetilde{U}$ is a linear operator then:

$$\widetilde{U}^\dagger \left(\sum_{s=0}^{3} \beta^{[s]} \left(\mathrm{i}\partial_s + F'_s + 0.5 g_1 Y B'_s\right) - \beta^{[4]} \mathrm{i}\partial'_4 - \ \gamma^{[0]} \mathrm{i}\partial'_5\right) \widetilde{\varphi}' = 0.$$

That is

$$\left(\sum_{s=0}^{3} \beta^{[s]} \left(\mathrm{i}\partial_s + F'_s + 0.5 g_1 Y B'_s\right) - \beta^{[4]} \mathrm{i}\partial'_4 - \ \gamma^{[0]} \mathrm{i}\partial'_5\right) \widetilde{\varphi}' = 0.$$

Compare with (4.25).
Thus, this Equation of moving is invariant under the following transformations:



$$
\begin{aligned}
& x_4 \rightarrow x_4' = x_4 \cos\frac{\chi}{2} - x_5 \sin\frac{\chi}{2}; \\
& x_5 \rightarrow x_5' = x_5 \cos\frac{\chi}{2} + x_4 \sin\frac{\chi}{2}; \\
& x_\mu \rightarrow x_\mu' = x_\mu \text{ for } \mu \in \{0,1,2,3\}; \\
& \widetilde{\varphi} \rightarrow \widetilde{\varphi}' = \widetilde{U}\widetilde{\varphi}, \\
& B_\mu \rightarrow B_\mu' = B_\mu - \frac{1}{g_1}\partial_\mu \chi, \\
& F_\mu \rightarrow F_\mu' = \widetilde{U} F_s \widetilde{U}^\dagger.
\end{aligned} \tag{4.28}
$$

Therefore, B_μ is like to the B-boson field of Standard Model[22] [16]. field. (h = 6.6260700442710$^($34)).

4.7. Masses

The scalar product of the following shape:

$$
(\widetilde{\varphi}, \widetilde{\chi}) := \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{\mathrm{h}}} dx_5 \int_{-\frac{\pi c}{\mathrm{h}}}^{\frac{\pi c}{\mathrm{h}}} dx_4 \cdot \widetilde{\varphi}^\dagger \widetilde{\chi} \tag{4.29}
$$

In that case from (4.75):

$$
\begin{aligned}
(\widetilde{\varphi}, \widetilde{\varphi}) &= \rho_{\mathcal{A}}, \\
\left(\widetilde{\varphi}, \beta^{[s]} \widetilde{\varphi}\right) &= -\frac{j_k}{\mathrm{c}}.
\end{aligned} \tag{4.30}
$$

[22]Sheldon Lee Glashow (born December 5, 1932) is a American theoretical physicist.

for $s \in \{1,2,3\}$

Let

$$N_{\vartheta}(t,x_1,x_2,x_3) := \left(\frac{\mathrm{c}M_0}{\mathrm{h}}\right), N_{\varpi}(t,x_1,x_2,x_3) := \left(\frac{\mathrm{c}M_4}{\mathrm{h}}\right).$$

In that case :

$$\widetilde{\varphi} = \varphi(t,x_1,x_2,x_3) \cdot \exp\left(-\mathrm{i}\left(x_5 \frac{\mathrm{h}}{\mathrm{c}} N_{\vartheta}(t,x_1,x_2,x_3) + x_4 \frac{\mathrm{h}}{\mathrm{c}} N_{\varpi}(t,x_1,x_2,x_3)\right)\right)$$

and Fourier series for $\widetilde{\varphi}$ is of the following form:

$$\widetilde{\varphi}(t,x_1,x_2,x_3,x_5,x_4) = \varphi(t,x_1,x_2,x_3) \cdot \sum_{n,s} \delta_{-n,N_{\vartheta}(t,\mathbf{x})} \delta_{-s,N_{\varpi}(t,\mathbf{x})} \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(nx_5 + sx_4)\right)$$

with

$$\begin{aligned}
\delta_{-n,N_{\vartheta}} &= \frac{\mathrm{h}}{2\pi\mathrm{c}} \int_{-\frac{\pi\mathrm{c}}{\mathrm{h}}}^{\frac{\pi\mathrm{c}}{\mathrm{h}}} \exp\left(\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(nx_5)\right) \exp\left(\mathrm{i}N_{\vartheta}\frac{\mathrm{h}}{\mathrm{c}}x_5\right) dx_5 = \frac{\sin\pi(n+N_{\vartheta})}{\pi(n+N_{\vartheta})}, \\
\delta_{-s,N_{\varpi}} &= \frac{\mathrm{h}}{2\pi\mathrm{c}} \int_{-\frac{\pi\mathrm{c}}{\mathrm{h}}}^{\frac{\pi\mathrm{c}}{h}} \exp\left(\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(sx_4)\right) \exp\left(\mathrm{i}N_{\varpi}\frac{\mathrm{h}}{\mathrm{c}}x_4\right) dx_4 = \frac{\sin\pi(s+N_{\varpi})}{\pi(s+N_{\varpi})}
\end{aligned}$$

with integer n and s.

If denote:

$$f(t,\mathbf{x},-n,-s) := \varphi(t,\mathbf{x})\,\delta_{n,N_{\vartheta}(t,\mathbf{x})}\delta_{s,N_{\varpi}(t,\mathbf{x})}$$

then

$$\begin{gathered}\widetilde{\varphi}(t,\mathbf{x},x_5,x_4) = \\ = \sum_{n,s} f(t,\mathbf{x},n,s) \exp\left(-\mathrm{i}\tfrac{\mathrm{h}}{\mathrm{c}}(nx_5 + sx_4)\right).\end{gathered} \tag{4.31}$$

The integer numbers n and s are denoted *mass numbers*.

From properties of δ: in every point $\langle t,\mathbf{x}\rangle$: either

$$\widetilde{\varphi}(t,\mathbf{x},x_5,x_4) = 0$$

or integer numbers n_0 and s_0 exist for which:

$$\begin{gathered}\widetilde{\varphi}(t,\mathbf{x},x_5,x_4) = \\ = f(t,\mathbf{x},n_0,s_0) \exp\left(-\mathrm{i}\tfrac{\mathrm{h}}{\mathrm{c}}(n_0x_5 + s_0x_4)\right).\end{gathered} \tag{4.32}$$

That is for every space-time point: either this point is empty or single couple $(n_0;s_0)$ is placed in this point.

Let us consider a behaviour of the sum

$$\beta^{[4]}n_0 + \gamma^{[0]}s_9$$

under rotations:

Because

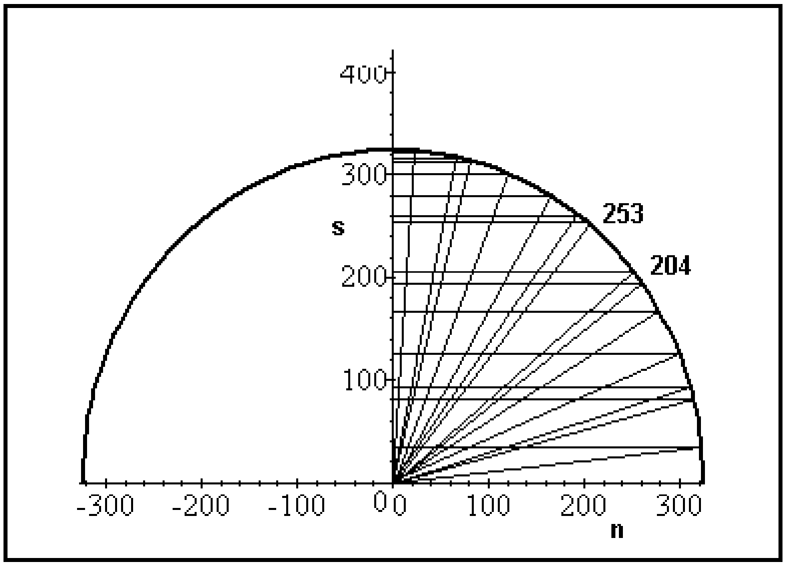


Figure 7:

$$U_{1,2}^{-1}\beta^{[4]}U_{1,2} = U_{1,3}^{-1}\beta^{[4]}U_{1,3} = U_{2,3}^{-1}\beta^{[4]}U_{2,3} = U_{0,1}^{\dagger}\beta^{[4]}U_{0,1} = U_{0,2}^{\dagger}\beta^{[4]}U_{0,2} = U_{0,3}^{\dagger}\beta^{[4]}U_{0,3} = \beta^{[4]}$$

and

$$U_{1,2}^{-1}\gamma^{[0]}U_{1,2} = U_{1,3}^{-1}\gamma^{[0]}U_{1,3} = U_{2,3}^{-1}\gamma^{[0]}U_{2,3} = U_{0,1}^{\dagger}\gamma^{[0]}U_{0,1} = U_{0,2}^{\dagger}\gamma^{[0]}U_{0,2} = U_{0,3}^{\dagger}\gamma^{[0]}U_{0,3} = \gamma^{[0]}$$

then this sum does not change under cartesian and Lorentz transformation.
But

$$\begin{aligned}
\widehat{U}^{-1}\gamma^{[0]}\widehat{U} &= \gamma^{[0]}\cos(\mathrm{i}\lambda) - \beta^{[4]}\sin(\mathrm{i}\lambda), \\
\widehat{U}^{-1}\beta^{[4]}\widehat{U} &= \left(\beta^{[4]}\cos(\mathrm{i}\lambda) + \gamma^{[0]}\sin(\mathrm{i}\lambda)\right).
\end{aligned}$$

and

$$\begin{aligned}
\widetilde{U}^{\dagger}\beta^{[4]}\widetilde{U} &= \left(\beta^{[4]}\cos\lambda - \gamma^{[0]}\sin\lambda\right), \\
\widetilde{U}^{\dagger}\gamma^{[0]}\widetilde{U} &= \left(\gamma^{[0]}\cos\lambda + \beta^{[4]}\sin\lambda\right).
\end{aligned}$$

Hence,

$$\begin{aligned}
\widetilde{U}^{\dagger}\left(\beta^{[4]}n_0 + \gamma^{[0]}s_0\right)\widetilde{U} &= \\
&= (n_0\cos\lambda + s_0\sin\lambda)\,\beta^{[4]} + (s_0\cos\lambda - n_0\sin\lambda)\,\gamma^{[0]}
\end{aligned}$$

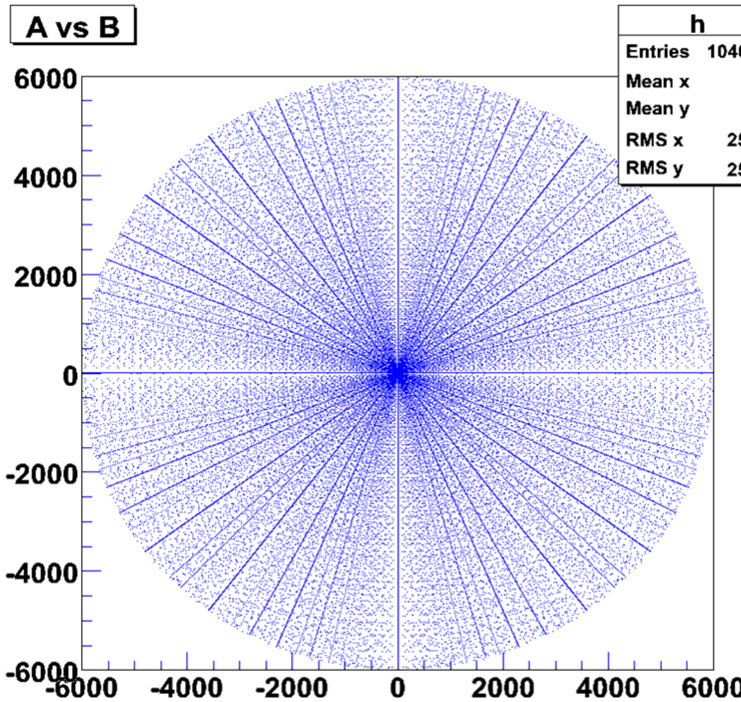
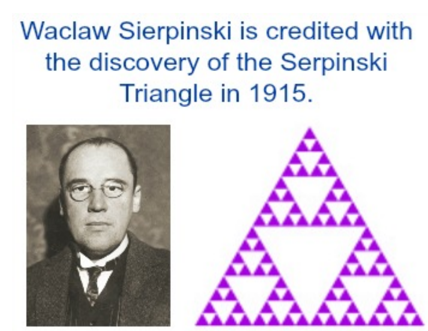


Figure 8:

Therefore, the $\widetilde{U}$ trancfurmation rotares vector $\overrightarrow{m}\{n_0;s_0\}$ into 2-dimension space $\left(\beta^{[4]};\gamma^{[0]}\right)$ on angle λ.

The $\widehat{U}$ trancfurmation rotares this vector into complex space $\left(\beta^{[4]};\gamma^{[0]}\right)$ on angle $i\lambda$.

Numbers n_0 and s_0 are integer but undet rotation this naturalness may fade. However, for some rotations, the coordinates of this ○vector remain integer: for example, Pythagorean[23] triples (Figure 7),



Let an integer number m is *a mass number* if for every angle α there some angle β ($\alpha - \frac{h}{2\pi c} \leq \beta \leq \alpha + \frac{h}{2\pi c}$) exist such that numbers $m\cos\beta$ and $m\sin\beta$ are integer.

[23] Pythagoras of Samos[a] (c.?570 c.?495 BC) was an ancient Ionian Greek philosopher and the eponymous founder of Pythagoreanism.

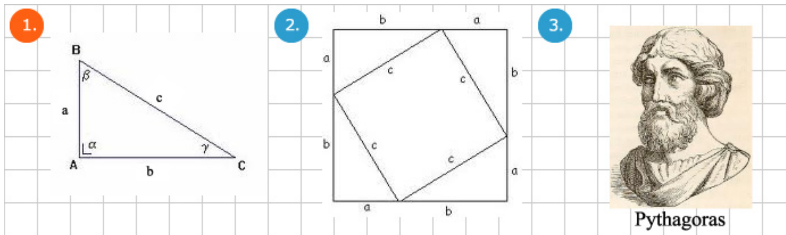


Figure 9:

Polish mathematician Waclaw Sierpinsk[24] in book, Pythagorean Triangles (1954) proofed the existing such numbers [17] (Figute 8).

Perhaps these mass numbers represent the possible masses of existing elementary particles

Here are three families (generations) according to the Standard Model of particle physics [16]:

$$\begin{bmatrix} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} & \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} & \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} \\ \begin{pmatrix} u \\ d \end{pmatrix} & \begin{pmatrix} c \\ s \end{pmatrix} & \begin{pmatrix} t \\ b \end{pmatrix} \end{bmatrix}.$$

Each generation is divided into two leptons:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix},$$

and two quarks:

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}.$$

The two leptons may be divided into one electron-like (e^- - electron, μ^- - μ–lepton, τ^- - τ-lepton) and neutrino (ν_e, ν_μ, ν_τ); the two quarks may be divided into one down-type (d, s, b) and one up-type (u, c, t). The first generation consists of the electron, electron neutrino and the down and up quarks. The second generation consists of the muon, muon neutrino and the strange and charm quarks. The third generation consists of the tau lepton, tau neutrino and the bottom and top quarks. Each member of a higher generation has greater mass than the corresponding particle of the previous generation. For example: the first-generation electron has a mass of only 0.511 MeV, the second-generation muon has a mass of 106 MeV, and the third-generation tau lepton has a mass of 1777 MeV (almost twice as heavy as a proton). All ordinary atoms are made of particles from the first generation. Electrons surround a nucleus made of protons and neutrons, which contain up and down quarks. The second and third generations of charged particles do not occur in normal matter and are only seen in extremely high-energy environments. Neutrinos of all generations stream throughout the universe but rarely interact with normal matter.

[24]Wac?aw Franciszek Sierpi?ski, (14 March 1882 – 21 October 1969) was a Polish mathematician.

4.8. One-Mass State

Let form of (4.31) be the following:

$$\widetilde{\varphi}(t,\mathbf{x},x_5,x_4) = \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}nx_5\right)\sum_{k=1}^{4} f_k(t,\mathbf{x},n,0).$$

In that case the Hamiltonian has the following form (from (4.21)):

$$\widehat{H} = \mathrm{c}\left(\sum_{k=1}^{3}\beta^{[k]}\mathrm{i}\partial_k + \frac{\mathrm{h}}{\mathrm{c}}n\gamma^{[0]} + \widehat{G}\right)$$

with

$$\widehat{G} := \sum_{\mu=0}^{3}\beta^{[\mu]}\left(F_\mu + 0.5g_1YB_\mu\right).$$

Let

$$\omega(\mathbf{k}) := \sqrt{\mathbf{k}^2+n^2} = \sqrt{k_1^2+k_2^2+k_3^2+n^2}$$

and

$$e_1(\mathbf{k}) := \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}}\begin{bmatrix}\omega(\mathbf{k})+n+k_3\\ k_1+\mathrm{i}k_2\\ \omega(\mathbf{k})+n-k_3\\ -k_1-\mathrm{i}k_2\end{bmatrix}. \tag{4.33}$$

Let

$$\widehat{H}_0 := \mathrm{c}\sum_{s=1}^{3}\beta^{[s]}\mathrm{i}\partial_s + \mathrm{h}n\gamma^{[0]}. \tag{4.34}$$

Let:

$$\mathrm{h}n = m\frac{\mathrm{c}^2}{\mathrm{h}}$$

then equation of moving with Hamiltonian $\widehat{H}_0$ has the following form:

$$\boxed{\frac{1}{\mathrm{c}}\mathrm{i}\partial_t\varphi = \left(\sum_{s=1}^{3}\beta^{[s]}\mathrm{i}\partial_s + m\frac{\mathrm{c}}{\mathrm{h}}\gamma^{[0]}\right)\varphi.} \tag{4.35}$$

This is the Dirac equation (Paul Dirac[25] formulated it in 1928).
Let us denote

$$\gamma^{[s]} := \gamma^{[0]}\beta^{[s]}$$

for $s \neq 0$.
Let us calculate:

[25] Paul Adrien Maurice Dirac (1902 – 1984) was an English theoretical physicist who made fundamental contributions to the early development of both quantum mechanics and quantum electrodynamics.

$$
\begin{aligned}
& \gamma^{[s]}\gamma^{[j]}+\gamma^{[j]}\gamma^{[s]} \\
= \quad & \gamma^{[0]}\beta^{[s]}\gamma^{[0]}\beta^{[j]}+\gamma^{[0]}\beta^{[j]}\gamma^{[0]}\beta^{[s]}= \\
= \quad & -\gamma^{[0]}\gamma^{[0]}\beta^{[s]}\beta^{[j]}-\gamma^{[0]}\gamma^{[0]}\beta^{[j]}\beta^{[s]}= \\
= \quad & -\left(\beta^{[s]}\beta^{[j]}+\beta^{[j]}\beta^{[s]}\right)=-2\delta_{j,s}
\end{aligned}
$$

for $s \neq 0$ and $j \neq 0$.
and

$$\gamma^{[s]}\gamma^{[0]}+\gamma^{[0]}\gamma^{[s]}=\gamma^{[0]}\beta^{[s]}\gamma^{[0]}+\gamma^{[0]}\gamma^{[0]}\beta^{[s]}=-\beta^{[s]}+\beta^{[s]}=0$$

for $s \neq 0$.
From (4.35):

$$\left(\frac{1}{\mathrm{c}}\mathrm{i}\gamma^{[0]}\partial_t-\sum_{s=1}^{3}\gamma^{[s]}\mathrm{i}\partial_s-m\frac{\mathrm{c}}{\mathrm{h}}\right)\varphi=0.$$

Let us multiply both parts of this equation on

$$\left(\frac{1}{\mathrm{c}}\mathrm{i}\gamma^{[0]}\partial_t-\sum_{s'=1}^{3}\gamma^{[s']}\mathrm{i}\partial_{s'}+m\frac{\mathrm{c}}{\mathrm{h}}\right):$$

$$\left(\frac{1}{\mathrm{c}}\mathrm{i}\gamma^{[0]}\partial_t-\sum_{s'=1}^{3}\gamma^{[s']}\mathrm{i}\partial_{s'}+m\frac{\mathrm{c}}{\mathrm{h}}\right)\left(\frac{1}{\mathrm{c}}\mathrm{i}\gamma^{[0]}\partial_t-\sum_{s=1}^{3}\gamma^{[s]}\mathrm{i}\partial_s-m\frac{\mathrm{c}}{\mathrm{h}}\right)\varphi=0.$$

Hence,

$$\left(\begin{array}{c}
-\frac{1}{\mathrm{c}^2}\partial_t^2 \\
-\sum_{s=1}^{3}\frac{1}{\mathrm{c}}\mathrm{i}\gamma^{[0]}\partial_t\gamma^{[s]}\mathrm{i}\partial_s-\sum_{s'=1}^{3}\gamma^{[s']}\mathrm{i}\partial_{s'}\frac{1}{\mathrm{c}}\mathrm{i}\gamma^{[0]}\partial_t \\
-\frac{1}{\mathrm{c}}\mathrm{i}\gamma^{[0]}\partial_t m\frac{\mathrm{c}}{\mathrm{h}}+m\frac{\mathrm{c}}{\mathrm{h}}\frac{1}{\mathrm{c}}\mathrm{i}\gamma^{[0]}\partial_t \\
+\sum_{s'=1}^{3}\gamma^{[s']}\mathrm{i}\partial_{s'}\sum_{s=1}^{3}\gamma^{[s]}\mathrm{i}\partial_s \\
+\sum_{s'=1}^{3}\gamma^{[s']}\mathrm{i}\partial_{s'}m\frac{\mathrm{c}}{\mathrm{h}}-\sum_{s=1}^{3}m\frac{\mathrm{c}}{\mathrm{h}}\gamma^{[s]}\mathrm{i}\partial_s \\
-\frac{m^2\mathrm{c}^2}{\mathrm{h}^2}
\end{array}\right)\varphi=0.$$

Hence,

$$\left(\begin{array}{c}
-\frac{1}{\mathrm{c}^2}\partial_t^2 \\
+\sum_{s'=1}^{3}\gamma^{[s']}\mathrm{i}\partial_{s'}\sum_{s=1}^{3}\gamma^{[s]}\mathrm{i}\partial_s \\
-\frac{m^2\mathrm{c}^2}{\mathrm{h}^2}
\end{array}\right)\varphi=0$$

Since

$$\begin{aligned}
&\sum_{s'=1}^{3}\gamma^{[s']}\mathrm{i}\partial_{s'}\sum_{s=1}^{3}\gamma^{[s]}\mathrm{i}\partial_{s}\\
= &\quad -\sum_{s=1}^{3}\sum_{s'=1}^{3}\gamma^{[s']}\gamma^{[s]}\partial_{s'}\partial_{s} =\\
= &\quad -\begin{pmatrix} \gamma^{[1]}\gamma^{[1]}\partial_1\partial_1+\gamma^{[2]}\gamma^{[1]}\partial_2\partial_1+\gamma^{[3]}\gamma^{[1]}\partial_3\partial_1\\ +\gamma^{[1]}\gamma^{[2]}\partial_1\partial_2+\gamma^{[2]}\gamma^{[2]}\partial_2\partial_2+\gamma^{[3]}\gamma^{[2]}\partial_3\partial_2\\ +\gamma^{[1]}\gamma^{[3]}\partial_1\partial_3+\gamma^{[2]}\gamma^{[3]}\partial_2\partial_3+\gamma^{[3]}\gamma^{[3]}\partial_3\partial_3 \end{pmatrix} =\\
= &\quad -\begin{pmatrix} -\partial_1\partial_1\\ +\gamma^{[2]}\gamma^{[1]}\partial_2\partial_1+\gamma^{[1]}\gamma^{[2]}\partial_1\partial_2\\ +\gamma^{[3]}\gamma^{[1]}\partial_3\partial_1+\gamma^{[1]}\gamma^{[3]}\partial_1\partial_3\\ -\partial_2\partial_2\\ +\gamma^{[3]}\gamma^{[2]}\partial_3\partial_2+\gamma^{[2]}\gamma^{[3]}\partial_2\partial_3\\ -\partial_3\partial_3 \end{pmatrix}.
\end{aligned}$$

Hence,

$$\sum_{s'=1}^{3}\gamma^{[s']}\mathrm{i}\partial_{s'}\sum_{s=1}^{3}\gamma^{[s]}\mathrm{i}\partial_{s} = \partial_1\partial_1+\partial_2\partial_2+\partial_3\partial_3 = \sum_{s=1}^{3}\partial_s^2.$$

Thus,

$$\boxed{\left(-\frac{1}{\mathrm{c}^2}\partial_t^2+\sum_{s=1}^{3}\partial_s^2-\frac{m^2\mathrm{c}^2}{\mathrm{h}^2}\right)\varphi=0.} \tag{4.36}$$

This is the Klein-Gordon[26][27] equation for a free particle with mass m.

Let us calculate:

$$\begin{gathered}
\widehat{H}_0 e_1(\mathbf{k})\left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{\frac{3}{2}}\exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)=\\
=\left(\mathrm{c}\sum_{s=1}^{3}\beta^{[s]}\mathrm{i}\partial_s+\mathrm{h}n\gamma^{[0]}\right)\left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{\frac{3}{2}}e_1(\mathbf{k})\exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)=\\
==\mathrm{c}\sum_{s=1}^{3}\beta^{[s]}\mathrm{i}\partial_s e_1(\mathbf{k})\left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{\frac{3}{2}}\exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)+\\
+\mathrm{h}n\gamma^{[0]}e_1(\mathbf{k})\left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{\frac{3}{2}}\exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)=\\
=\mathrm{c}\sum_{s=1}^{3}\beta^{[s]}\mathrm{i}e_1(\mathbf{k})\partial_s\left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{\frac{3}{2}}\exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)+\\
+\mathrm{h}n\left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{\frac{3}{2}}\exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)\gamma^{[0]}e_1(\mathbf{k})=\\
=\mathrm{c}\sum_{s=1}^{3}\beta^{[s]}\mathrm{i}e_1(\mathbf{k})\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}k_s\right)\left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{\frac{3}{2}}\exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}k_s\right)+\\
+\mathrm{h}n\left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{\frac{3}{2}}\exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)\gamma^{[0]}e_1(\mathbf{k})=\\
=\sum_{s=1}^{3}(-\mathrm{i}\mathrm{h}k_s)\beta^{[s]}\mathrm{i}e_1(\mathbf{k})\left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{\frac{3}{2}}\exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)+\\
+\mathrm{h}n\left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{\frac{3}{2}}\exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)\gamma^{[0]}e_1(\mathbf{k})=\\
=\mathrm{h}\left(\frac{\mathrm{h}}{2\pi\mathrm{c}}\right)^{\frac{3}{2}}\exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)\left(\sum_{s=1}^{3}k_s\beta^{[s]}+n\gamma^{[0]}\right)e_1(\mathbf{k})=
\end{gathered}$$

[26]Oskar Benjamin Klein; 15 September 1894 5 February 1977) was a Swedish theoretical physicist.
[27]Walter Gordon (13 August 1893 24 December 1939) was a German theoretical physicist.

$$
\begin{aligned}
&= \mathrm{h}\left(\frac{\mathrm{h}}{2\pi \mathrm{c}}\right)^{\frac{3}{2}} \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)\begin{bmatrix} k_3 & k_1-\mathrm{i}k_2 & n & 0 \\ k_1+\mathrm{i}k_2 & -k_3 & 0 & n \\ n & 0 & -k_3 & -k_1+\mathrm{i}k_2 \\ 0 & n & -k_1-\mathrm{i}k_2 & k_3 \end{bmatrix}. \\
&\quad \cdot\frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}}\begin{bmatrix} \omega(\mathbf{k})+n+k_3 \\ k_1+\mathrm{i}k_2 \\ \omega(\mathbf{k})+n-k_3 \\ -k_1-\mathrm{i}k_2 \end{bmatrix} \\
&= \mathrm{h}\left(\frac{\mathrm{h}}{2\pi \mathrm{c}}\right)^{\frac{3}{2}} \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)\begin{bmatrix} k_3\omega(\mathbf{k})+k_3^2+k_1^2+k_2^2+n\omega(\mathbf{k})+n^2 \\ k_1\omega(\mathbf{k})+ik_2\omega(\mathbf{k}) \\ n\omega(\mathbf{k})+n^2-k_3\omega(\mathbf{k})+k_3^2+k_1^2+k_2^2 \\ -k_1\omega(\mathbf{k})-ik_2\omega(\mathbf{k}) \end{bmatrix} \\
&= \omega(\mathbf{k})\frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}}\begin{bmatrix} k_3+n+\omega(\mathbf{k}) \\ k_1+ik_2 \\ n+\omega(\mathbf{k})-k_3 \\ -k_1-ik_2 \end{bmatrix}\mathrm{h}\left(\frac{\mathrm{h}}{2\pi \mathrm{c}}\right)^{\frac{3}{2}} \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right).
\end{aligned}
$$

Therefore,

$$
\widehat{H}_0 e_1(\mathbf{k})\left(\frac{\mathrm{h}}{2\pi \mathrm{c}}\right)^{\frac{3}{2}} \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right) = \mathrm{h}\omega(\mathbf{k})\, e_1(\mathbf{k})\left(\frac{\mathrm{h}}{2\pi \mathrm{c}}\right)^{\frac{3}{2}} \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right). \tag{4.37}
$$

Hence, function $e_1(\mathbf{k})\left(\frac{\mathrm{h}}{2\pi \mathrm{c}}\right)^{\frac{3}{2}} \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)$ is an eigenvector of $\widehat{H}_0$ with eigenvalue

$$
\mathrm{h}\omega(\mathbf{k}) = \mathrm{h}\sqrt{\mathbf{k}^2+n^2}.
$$

Similarly, function $e_2(\mathbf{k})\left(\frac{\mathrm{h}}{2\pi \mathrm{c}}\right)^{\frac{3}{2}} \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)$ with

$$
e_2(\mathbf{k}) := \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}}\begin{bmatrix} k_1-\mathrm{i}k_2 \\ \omega(\mathbf{k})+n-k_3 \\ -k_1+\mathrm{i}k_2 \\ \omega(\mathbf{k})+n+k_3 \end{bmatrix} \tag{4.38}
$$

is eigenvector of $\widehat{H}_0$ with eigenvalue $\mathrm{h}\omega(\mathbf{k}) = \mathrm{h}\sqrt{\mathbf{k}^2+n^2}$, too, and functions

$$
e_3(\mathbf{k})\left(\frac{\mathrm{h}}{2\pi \mathrm{c}}\right)^{\frac{3}{2}} \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right) \text{ and } e_4(\mathbf{k})\left(\frac{\mathrm{h}}{2\pi \mathrm{c}}\right)^{\frac{3}{2}} \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)
$$

with

$$
e_3(\mathbf{k}) := \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}}\begin{bmatrix} -\omega(\mathbf{k})-n+k_3 \\ k_1+\mathrm{i}k_2 \\ \omega(\mathbf{k})+n+k_3 \\ k_1+\mathrm{i}k_2 \end{bmatrix} \tag{4.39}
$$

and

$$e_4(\mathbf{k}) := \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}} \begin{bmatrix} k_1 - \mathrm{i}k_2 \\ -\omega(\mathbf{k}) - n - k_3 \\ k_1 - \mathrm{i}k_2 \\ \omega(\mathbf{k}) + n - k_3 \end{bmatrix} \tag{4.40}$$

are eigenvectors of $\widehat{H}_0$ with eigenvalue $-\mathrm{h}\omega(\mathbf{k})$.

Here $e_\mu(\mathbf{k})$ with $\mu \in \{1,2,3,4\}$ form an orthonormal basis in the space spanned on vectors ε_μ (**??**).

4.9. Creating and Annihilation Operators

Let $\mathfrak{H}$ be some unitary space. Let $\widetilde{0}$ be the zero element of $\mathfrak{H}$. That is any element $\widetilde{F}$ of $\mathfrak{H}$ obeys to the following conditions:

$0\widetilde{F} = \widetilde{0}$, $\widetilde{0} + \widetilde{F} = \widetilde{F}$, $\widetilde{0}^\dagger = \widetilde{0}$.

Let $\widehat{0}$ be the zero operator on $\mathfrak{H}$. That is any element $\widetilde{F}$ of $\mathfrak{H}$ obeys to the following condition:

$\widehat{0}\widetilde{F} = 0\widetilde{F}$, and if $\widehat{b}$ is any operator on $\mathfrak{H}$ then

$\widehat{0} + \widehat{b} = \widehat{b} + \widehat{0} = \widehat{b}$, $\widehat{0}\widehat{b} = \widehat{b}\widehat{0} = \widehat{0}$.

Let $\widehat{1}$ be the identy operator on $\mathfrak{H}$. That is any element $\widetilde{F}$ of $\mathfrak{H}$ obeys to the following condition:

$\widehat{1}\widetilde{F} = 1\widetilde{F} = \widetilde{F}$, and if $\widehat{b}$ is any operator on $\mathfrak{H}$ then
$\widehat{1}\widehat{b} = \widehat{b}\widehat{1} = \widehat{b}$.

Let linear operators $b_{s,\mathbf{k}}$ ($s \in \{1,2,3,4\}$) act on all elements of this space. And let these operators fulfill the following conditions:

$$\left\{b^\dagger_{s,\mathbf{k}}, b_{s',\mathbf{k}'}\right\} := b^\dagger_{s,\mathbf{k}} b_{s',\mathbf{k}'} + b_{s',\mathbf{k}'} b^\dagger_{s,\mathbf{k}} = \left(\frac{h}{2\pi}\right)^3 \delta_{\mathbf{k},\mathbf{k}'} \delta_{s,s'} \widehat{1},$$

$$\{b_{s,\mathbf{k}}, b_{s',\mathbf{k}'}\} = b_{s,\mathbf{k}} b_{s',\mathbf{k}'} + b_{s',\mathbf{k}'} b_{s,\mathbf{k}} = \left\{b^\dagger_{s,\mathbf{k}}, b^\dagger_{s',\mathbf{k}'}\right\} = \widehat{0}.$$

Hence,

$$b_{s,\mathbf{k}} b_{s,\mathbf{k}} = b^\dagger_{s,\mathbf{k}} b^\dagger_{s,\mathbf{k}} = \widehat{0}.$$

There exists element $\widetilde{F}_0$ of $\mathfrak{H}$ such that $\widetilde{F}^\dagger_0 \widetilde{F}_0 = 1$ and for any $b_{s,\mathbf{k}}$: $b_{s,\mathbf{k}} \widetilde{F}_0 = \widetilde{0}$. Hence, $\widetilde{F}^\dagger_0 b^\dagger_{s,\mathbf{k}} = \widetilde{0}$.

Let

$$\psi_s(\mathbf{x}) := \sum_{\mathbf{k}} \sum_{r=1}^{4} b_{r,\mathbf{k}} e_{r,s}(\mathbf{k}) \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}} \mathbf{k}\mathbf{x}\right).$$

Because

$$\sum_{r=1}^{4} e_{r,s}(\mathbf{k})\, e_{r,s'}(\mathbf{k}) = \delta_{s,s'}$$

and

$$\sum_{\mathbf{k}} \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{k}\left(\mathbf{x}-\mathbf{x}'\right)\right) = \left(\frac{2\pi \mathrm{c}}{\mathrm{h}}\right)^3 \delta\left(\mathbf{x}-\mathbf{x}'\right)$$

then

$$\begin{aligned}\left\{\psi_s^{\dagger}(\mathbf{x}), \psi_{s'}\left(\mathbf{x}'\right)\right\} &:= \psi_s^{\dagger}(\mathbf{x})\,\psi_{s'}\left(\mathbf{x}'\right) + \psi_{s'}\left(\mathbf{x}'\right)\psi_s^{\dagger}(\mathbf{x}) \\ &= \delta\left(\mathbf{x}-\mathbf{x}'\right)\delta_{s,s'}\widehat{1}.\end{aligned}$$

And these operators obey the following conditions:

$\psi_s(\mathbf{x})\,\widetilde{F}_0 = \widetilde{0}$, $\{\psi_s(\mathbf{x}), \psi_{s'}(\mathbf{x}')\} = \left\{\psi_s^{\dagger}(\mathbf{x}), \psi_{s'}^{\dagger}(\mathbf{x}')\right\} = \widehat{0}.$

Hence,

$\psi_s(\mathbf{x})\,\psi_{s'}(\mathbf{x}') = \psi_s^{\dagger}(\mathbf{x})\,\psi_{s'}^{\dagger}(\mathbf{x}') = \widehat{0}.$

Let

$$\Psi(t,\mathbf{x}) := \sum_{s=1}^{4} \varphi_s(t,\mathbf{x})\,\psi_s^{\dagger}(\mathbf{x})\,\widetilde{F}_0. \tag{4.41}$$

These function obey the following condition:

$$\Psi^{\dagger}\left(t,\mathbf{x}'\right)\Psi(t,\mathbf{x}) = \varphi^{\dagger}\left(t,\mathbf{x}'\right)\varphi(t,\mathbf{x})\,\delta\left(\mathbf{x}-\mathbf{x}'\right).$$

Hence,

$$\int d\mathbf{x}' \cdot \Psi^{\dagger}\left(t,\mathbf{x}'\right)\Psi(t,\mathbf{x}) = \rho(t,\mathbf{x}). \tag{4.42}$$

Let a Fourier series of $\varphi_s(t,\mathbf{x})$ has the following form:

$$\varphi_s(t,\mathbf{x}) = \sum_{\mathbf{p}} \sum_{r=1}^{4} c_r(t,\mathbf{p})\, e_{r,s}(\mathbf{p}) \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{p}\mathbf{x}\right).$$

In that case:

$$\underline{\Psi}(t,\mathbf{p}) := \left(\frac{2\pi \mathrm{c}}{\mathrm{h}}\right)^3 \sum_{r=1}^{4} c_r(t,\mathbf{p})\, b_{r,\mathbf{p}}^{\dagger}\widetilde{F}_0.$$

If

$$\mathcal{H}_0(\mathbf{x}) := \psi^{\dagger}(\mathbf{x})\,\widehat{H}_0\psi(\mathbf{x}) \tag{4.43}$$

then $\mathcal{H}_0(\mathbf{x})$ is called a Hamiltonian $\widehat{H}_0$ density.

Because

$$\widehat{H}_0\varphi(t,\mathbf{x}) = \mathrm{i}\frac{\partial}{\partial t}\varphi(t,\mathbf{x})$$

then

$$\int d\mathbf{x}' \cdot \mathcal{H}_0\left(\mathbf{x}'\right)\Psi(t,\mathbf{x}) = \mathrm{i}\frac{\partial}{\partial t}\Psi(t,\mathbf{x}). \tag{4.44}$$

then if

$$\widehat{\mathbb{H}} := \int d\mathbf{x}' \cdot \mathcal{H}_0\left(\mathbf{x}'\right)$$

then $\widehat{\mathbb{H}}$ acts similar to the Hamiltonian on space $\mathfrak{H}$.

And if

$$E_\Psi\left(\widetilde{F}_0\right) := \sum_{\mathbf{p}} \underline{\Psi}^\dagger(t,\mathbf{p})\,\widehat{\mathbb{H}}\underline{\Psi}(t,\mathbf{p})$$

then $E_\Psi\left(\widetilde{F}_0\right)$ is an energy of Ψ on vacuum $\widetilde{F}_0$.

Let us consider operator $\widehat{N}_a(\mathbf{x}_0) := \psi_a^\dagger(\mathbf{x}_0)\psi_a(\mathbf{x}_0)$.

Let us calculate an average value of this operator:

$$\left\langle \widehat{N}_a(\mathbf{x}_0)\right\rangle_\Psi := \int_\Omega d\mathbf{x} \cdot \widehat{N}_a(\mathbf{x}_0)\,\rho(t,\mathbf{x}).$$

In accordance with (4.42):

$$\left\langle \widehat{N}_a(\mathbf{x}_0)\right\rangle_\Psi = \int_\Omega d\mathbf{x}\int_\Omega d\mathbf{x}' \cdot \Psi^\dagger\left(t,\mathbf{x}'\right)\psi_a^\dagger(\mathbf{x}_0)\psi_a(\mathbf{x}_0)\Psi(t,\mathbf{x}).$$

Since in accordance with (4.41):

$$\Psi(t,\mathbf{x}) = \sum_{j=1}^{4}\varphi_j(t,\mathbf{x})\psi_j^\dagger(\mathbf{x})\widetilde{F}_0.$$

then

$$\begin{aligned}
&\left\langle \widehat{N}_a(\mathbf{x}_0)\right\rangle_\Psi = \\
= \quad & \int_\Omega d\mathbf{x}\int_\Omega d\mathbf{x}' \cdot \sum_{s=1}^{4}\varphi_s^*\left(t,\mathbf{x}'\right)\widetilde{F}_0^\dagger\psi_s\left(\mathbf{x}'\right)\psi_a^\dagger(\mathbf{x}_0)\psi_a(\mathbf{x}_0)\sum_{j=1}^{4}\varphi_j(t,\mathbf{x})\psi_j^\dagger(\mathbf{x})\widetilde{F}_0 = \\
= \quad & \int_\Omega d\mathbf{x}\int_\Omega d\mathbf{x}' \cdot \sum_{s=1}^{4}\sum_{j=1}^{4}\varphi_s^*\left(t,\mathbf{x}'\right)\varphi_j(t,\mathbf{x})\widetilde{F}_0^\dagger\psi_s\left(\mathbf{x}'\right)\psi_a^\dagger(\mathbf{x}_0)\psi_a(\mathbf{x}_0)\psi_j^\dagger(\mathbf{x})\widetilde{F}_0.
\end{aligned}$$

Since

$$\psi_a^\dagger(\mathbf{x}_0)\psi_s\left(\mathbf{x}'\right) + \psi_s\left(\mathbf{x}'\right)\psi_a^\dagger(\mathbf{x}_0) = \delta\left(\mathbf{x}_0 - \mathbf{x}'\right)\delta_{s,a}\widehat{1}$$

then

$$
\begin{aligned}
\left\langle \widehat{N}_a(\mathbf{x}_0)\right\rangle_{\Psi} &= \int_{\Omega} d\mathbf{x} \int_{\Omega} d\mathbf{x}' \cdot \sum_{s=1}^{4}\sum_{j=1}^{4} \varphi_s^*\left(t,\mathbf{x}'\right)\varphi_j(t,\mathbf{x})\cdot \\
&\cdot \widetilde{F}_0^{\dagger}\left(\delta\left(\mathbf{x}_0-\mathbf{x}'\right)\delta_{s,a}\widehat{1}-\psi_a^{\dagger}(\mathbf{x}_0)\psi_s\left(\mathbf{x}'\right)\right)\psi_a(\mathbf{x}_0)\psi_j^{\dagger}(\mathbf{x})\widetilde{F}_0 \\
&= \int_{\Omega} d\mathbf{x} \int_{\Omega} d\mathbf{x}' \cdot \sum_{s=1}^{4}\sum_{j=1}^{4} \varphi_s^*\left(t,\mathbf{x}'\right)\varphi_j(t,\mathbf{x})\cdot \\
&\cdot \left(\delta\left(\mathbf{x}_0-\mathbf{x}'\right)\delta_{s,a}\widetilde{F}_0^{\dagger}\widehat{1}-\widetilde{F}_0^{\dagger}\psi_a^{\dagger}(\mathbf{x}_0)\psi_s\left(\mathbf{x}'\right)\right)\psi_a(\mathbf{x}_0)\psi_j^{\dagger}(\mathbf{x})\widetilde{F}_0.
\end{aligned}
$$

Since $\widetilde{F}_0^{\dagger}\widehat{1}=\widetilde{F}_0^{\dagger}$ and $\widetilde{F}_0^{\dagger}\psi_a^{\dagger}(\mathbf{x}_0)=\widetilde{0}$ then

$$
\begin{aligned}
&\left\langle \widehat{N}_a(\mathbf{x}_0)\right\rangle_{\Psi} = \\
&= \int_{\Omega} d\mathbf{x} \int_{\Omega} d\mathbf{x}' \cdot \sum_{s=1}^{4}\sum_{j=1}^{4} \varphi_s^*\left(t,\mathbf{x}'\right)\varphi_j(t,\mathbf{x})\,\delta\left(\mathbf{x}_0-\mathbf{x}'\right)\delta_{s,a}\widetilde{F}_0^{\dagger}\psi_a(\mathbf{x}_0)\psi_j^{\dagger}(\mathbf{x})\widetilde{F}_0.
\end{aligned}
$$

According with properties of δ-function and δ:

$$
\left\langle \widehat{N}_a(\mathbf{x}_0)\right\rangle_{\Psi} = \int_{\Omega} d\mathbf{x} \cdot \sum_{j=1}^{4} \varphi_a^*(t,\mathbf{x}_0)\varphi_j(t,\mathbf{x})\widetilde{F}_0^{\dagger}\psi_a(\mathbf{x}_0)\psi_j^{\dagger}(\mathbf{x})\widetilde{F}_0.
$$

Since

$$
\psi_j^{\dagger}(\mathbf{x})\psi_a(\mathbf{x}_0)+\psi_a(\mathbf{x}_0)\psi_j^{\dagger}(\mathbf{x}) = \delta(\mathbf{x}_0-\mathbf{x})\delta_{j,a}\widehat{1}
$$

then

$$
\begin{aligned}
&\left\langle \widehat{N}_a(\mathbf{x}_0)\right\rangle_{\Psi} = \\
&= \int_{\Omega} d\mathbf{x} \cdot \sum_{j=1}^{4} \varphi_a^*(t,\mathbf{x}_0)\varphi_j(t,\mathbf{x})\widetilde{F}_0^{\dagger}\left(\delta(\mathbf{x}_0-\mathbf{x})\delta_{j,a}\widehat{1}-\psi_j^{\dagger}(\mathbf{x})\psi_a(\mathbf{x}_0)\right)\widetilde{F}_0 \\
&= \int_{\Omega} d\mathbf{x} \cdot \sum_{j=1}^{4} \varphi_a^*(t,\mathbf{x}_0)\varphi_j(t,\mathbf{x})\left(\delta(\mathbf{x}_0-\mathbf{x})\delta_{j,a}\widetilde{F}_0^{\dagger}\widehat{1}\widetilde{F}_0-\widetilde{F}_0^{\dagger}\psi_j^{\dagger}(\mathbf{x})\psi_a(\mathbf{x}_0)\widetilde{F}_0\right) \\
&= \int_{\Omega} d\mathbf{x} \cdot \sum_{j=1}^{4} \varphi_a^*(t,\mathbf{x}_0)\varphi_j(t,\mathbf{x})\left(\delta(\mathbf{x}_0-\mathbf{x})\delta_{j,a}\widetilde{F}_0^{\dagger}\widetilde{F}_0-\widetilde{0}^{\dagger}\widetilde{0}\right). \\
&= \int_{\Omega} d\mathbf{x} \cdot \sum_{j=1}^{4} \varphi_a^*(t,\mathbf{x}_0)\varphi_j(t,\mathbf{x})\left(\delta(\mathbf{x}_0-\mathbf{x})\delta_{j,a}1-0\right) \\
&= \int_{\Omega} d\mathbf{x} \cdot \sum_{j=1}^{4} \varphi_a^*(t,\mathbf{x}_0)\varphi_j(t,\mathbf{x})\,\delta(\mathbf{x}_0-\mathbf{x})\delta_{j,a}.
\end{aligned}
$$

Thus:

$$\boxed{\left\langle \widehat{N}_a(\mathbf{x}_0)\right\rangle_\Psi = \varphi_a^*(t,\mathbf{x}_0)\,\varphi_a(t,\mathbf{x}_0).} \tag{4.45}$$

That is operator $\widehat{N}_a(\mathbf{x}_0)$ brings the a-component of the event probability density.
Let $\Psi_a(t,\mathbf{x}) := \psi_a(\mathbf{x}_0)\,\Psi(t,\mathbf{x})$.
In that case

$$\begin{aligned}\left\langle \widehat{N}_a(\mathbf{x}_0)\right\rangle_{\Psi_a} &= \int_\Omega d\mathbf{x}\int_\Omega d\mathbf{x}'\cdot\Psi^\dagger(t,\mathbf{x})\,\psi_a^\dagger(\mathbf{x}_0)\,\psi_a^\dagger(\mathbf{x}_0)\\ &\quad \psi_a(\mathbf{x}_0)\,\psi_a(\mathbf{x}_0)\,\psi_a(\mathbf{x}_0)\,\Psi(t,\mathbf{x}).\end{aligned}$$

Since

$$\psi_a(\mathbf{x}_0)\,\psi_a(\mathbf{x}_0) = \widehat{0}$$

then

$$\left\langle \widehat{N}_a(\mathbf{x}_0)\right\rangle_{\Psi_a} = 0.$$

Therefore $\psi_a(\mathbf{x}_0)$ "annihilates" the a of the event-probability density.

4.10. Particles and Antiparticles

Operator $\widehat{\mathbb{H}}$ obeys the following condition:

$$\widehat{\mathbb{H}} = \left(\frac{2\pi c}{h}\right)^3 \sum_{\mathbf{k}} h\omega(\mathbf{k})\left(\sum_{r=1}^{2} b_{r,\mathbf{k}}^\dagger b_{r,\mathbf{k}} - \sum_{r=3}^{4} b_{r,\mathbf{k}}^\dagger b_{r,\mathbf{k}}\right).$$

This operator is not positive defined and in this case

$$E_\Psi\left(\widetilde{F}_0\right) = \left(\frac{2\pi c}{h}\right)^3 \sum_{\mathbf{p}} h\omega(\mathbf{p})\left(\sum_{r=1}^{2} |c_r(t,\mathbf{p})|^2 - \sum_{r=3}^{4} |c_r(t,\mathbf{p})|^2\right).$$

This problem is usually solved in the following way [18, p.54]:
Let:

$$\begin{aligned} v_1(\mathbf{k}) &: = \gamma^{[0]} e_3(\mathbf{k}),\\ v_2(\mathbf{k}) &: = \gamma^{[0]} e_4(\mathbf{k}),\\ d_{1,\mathbf{k}} &: = -b_{3,-\mathbf{k}}^\dagger,\\ d_{2,\mathbf{k}} &: = -b_{4,-\mathbf{k}}^\dagger. \end{aligned}$$

In that case:

$$\begin{aligned} e_3(\mathbf{k}) &= -v_1(-\mathbf{k}),\\ e_4(\mathbf{k}) &= -v_2(-\mathbf{k}),\\ b_{3,\mathbf{k}} &= -d_{1,-\mathbf{k}}^\dagger,\\ b_{4,\mathbf{k}} &= -d_{2,-\mathbf{k}}^\dagger. \end{aligned}$$

Therefore,

$$\begin{aligned}\psi_s(\mathbf{x}) \quad : \quad &= \sum_{\mathbf{k}} \sum_{r=1}^{2} \left(b_{r,\mathbf{k}} e_{r,s}(\mathbf{k}) \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right) + \right. \\ &\left. + d_{r,\mathbf{k}}^{\dagger} v_{r,s}(\mathbf{k}) \exp\left(\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)\right)\end{aligned}$$

$$\begin{aligned}\widehat{\mathbb{H}} \quad = \quad &\left(\frac{2\pi\mathrm{c}}{\mathrm{h}}\right)^3 \sum_{\mathbf{k}} \mathrm{h}\omega(\mathbf{k}) \sum_{r=1}^{2} \left(b_{r,\mathbf{k}}^{\dagger} b_{r,\mathbf{k}} + d_{r,\mathbf{k}}^{\dagger} d_{r,\mathbf{k}}\right) \\ &-2\sum_{\mathbf{k}} \mathrm{h}\omega(\mathbf{k})\widehat{1}.\end{aligned}$$

The first term on the right side of this equality is positive defined. This term is taken as the desired Hamiltonian. The second term of this equality is infinity constant. And this infinity is deleted (?!) [18, p.58]

But in this case $d_{r,\mathbf{k}}\widetilde{F}_0 \neq \widetilde{0}$. In order to satisfy such condition, the vacuum element $\widetilde{F}_0$ must be replaced by the following:

$$\widetilde{F}_0 \rightarrow \widetilde{\Phi}_0 := \prod_{\mathbf{k}} \prod_{r=3}^{4} \left(\frac{2\pi\mathrm{c}}{\mathrm{h}}\right)^3 b_{r,\mathbf{k}}^{\dagger} \widetilde{F}_0.$$

But in this case:

$$\psi_s(\mathbf{x})\widetilde{\Phi}_0 \neq \widetilde{0}.$$

And condition (4.44) isn't carried out.

In order to satisfy such condition, operators $\psi_s(\mathbf{x})$ must be replaced by the following:

$$\begin{aligned}&\psi_s(\mathbf{x}) \rightarrow \phi_s(\mathbf{x}) := \\ &:= \sum_{\mathbf{k}} \sum_{r=1}^{2} \left(b_{r,\mathbf{k}} e_{r,s}(\mathbf{k}) \exp\left(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right) + d_{r,\mathbf{k}} v_r(\mathbf{k}) \exp\left(\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}\mathbf{kx}\right)\right).\end{aligned}$$

Hence,

$$\begin{aligned}\widehat{\mathbb{H}} \quad &= \quad \int d\mathbf{x} \cdot \mathcal{H}(\mathbf{x}) = \int d\mathbf{x} \cdot \phi^{\dagger}(\mathbf{x}) \widehat{H}_0 \phi(\mathbf{x}) = \\ &= \quad \left(\frac{2\pi\mathrm{c}}{\mathrm{h}}\right)^3 \sum_{\mathbf{k}} \mathrm{h}\omega(\mathbf{k}) \sum_{r=1}^{2} \left(b_{r,\mathbf{k}}^{\dagger} b_{r,\mathbf{k}} - d_{r,\mathbf{k}}^{\dagger} d_{r,\mathbf{k}}\right).\end{aligned}$$

And again we get negative energy.

Let's consider the meaning of such energy: An event with positive energy transfers this energy photons which carries it on recorders observers. Observers know that this event occurs, not before it happens. But event with negative energy should absorb this energy from observers. Consequently, observers know that this event happens before it happens. This contradicts Theorem 1.5.2. Therefore, events with negative energy do not occur.

Hence, over vacuum $\widetilde{\Phi}_0$ single fermions can exist, but there is no single antifermions.

A two-particle state is defined the following field operator [**?**]:

$$\psi_{s_1,s_2}(\mathbf{x},\mathbf{y}) := \left| \begin{array}{cc} \phi_{s_1}(\mathbf{x}) & \phi_{s_2}(\mathbf{x}) \\ \phi_{s_1}(\mathbf{y}) & \phi_{s_2}(\mathbf{y}) \end{array} \right|.$$

In that case:

$$\widehat{\mathbb{H}} = 2\mathrm{h}\left(\frac{2\pi\mathrm{c}}{\mathrm{h}}\right)^6 \left(\widehat{\mathbb{H}}_a + \widehat{\mathbb{H}}_b\right)$$

where

$$\begin{aligned}
\widehat{\mathbb{H}}_a \quad : \quad &= \sum_{\mathbf{k}}\sum_{\mathbf{p}} (\omega(\mathbf{k}) - \omega(\mathbf{p})) \sum_{r=1}^{2}\sum_{j=1}^{2} \times \\
&\times \Big\{ v_j^{\dagger}(-\mathbf{k})\, v_j(-\mathbf{p})\, e_r^{\dagger}(\mathbf{p})\, e_r(\mathbf{k}) \times \\
&\times \left(+b_{r,\mathbf{p}}^{\dagger} d_{j,-\mathbf{k}}^{\dagger} d_{j,-\mathbf{p}} b_{r,\mathbf{k}}\right) + \\
&+ \left(+d_{r,-\mathbf{p}}^{\dagger} b_{j,\mathbf{k}}^{\dagger} b_{j,\mathbf{k}} d_{r,-\mathbf{p}}\right) + \\
&+ v_j^{\dagger}(-\mathbf{p})\, v_j(-\mathbf{k})\, e_r^{\dagger}(\mathbf{k})\, e_r(\mathbf{p}) \times \\
&\times \left(-b_{r,\mathbf{k}}^{\dagger} d_{j,-\mathbf{p}}^{\dagger} d_{j,-\mathbf{k}} b_{r,\mathbf{p}}\right) + \\
&+ \left(-b_{r,\mathbf{p}}^{\dagger} d_{j,-\mathbf{k}}^{\dagger} d_{j,-\mathbf{k}} b_{r,\mathbf{p}}\right) \Big\}
\end{aligned}$$

and

$$\begin{aligned}
\widehat{\mathbb{H}}_b \quad : \quad &= \sum_{\mathbf{k}}\sum_{\mathbf{p}} (\omega(\mathbf{k}) + \omega(\mathbf{p})) \sum_{r=1}^{2}\sum_{j=1}^{2} \times \\
&\times \Big\{ v_j^{\dagger}(-\mathbf{p})\, v_j(-\mathbf{k})\, v_r^{\dagger}(-\mathbf{k})\, v_r(-\mathbf{p}) \times \\
&\times \left(-d_{r,-\mathbf{k}}^{\dagger} d_{j,-\mathbf{p}}^{\dagger} d_{j,-\mathbf{k}} d_{r,-\mathbf{p}}\right) + \\
&+ \left(-d_{r,-\mathbf{p}}^{\dagger} d_{j,-\mathbf{k}}^{\dagger} d_{j,-\mathbf{k}} d_{r,-\mathbf{p}}\right) \\
&+ e_r^{\dagger}(\mathbf{k})\, e_r(\mathbf{p})\, e_j^{\dagger}(\mathbf{p})\, e_j(\mathbf{k}) \times \\
&\times \left(+b_{r,\mathbf{k}}^{\dagger} b_{j,\mathbf{p}}^{\dagger} b_{j,\mathbf{k}} b_{r,\mathbf{p}}\right) + \\
&+ \left(+b_{r,\mathbf{p}}^{\dagger} b_{j,\mathbf{k}}^{\dagger} b_{j,\mathbf{k}} b_{r,\mathbf{p}}\right) \Big\}.
\end{aligned}$$

If velosities are small then the following formula is fair.

$$\widehat{\mathbb{H}} = 4\mathrm{h}\left(\frac{2\pi\mathrm{c}}{\mathrm{h}}\right)^6 \left(\widehat{\mathbb{H}}_a + \widehat{\mathbb{H}}_b\right)$$

where

$$\begin{aligned}
\widehat{\mathbb{H}}_a \quad : \quad &= \sum_{\mathbf{k}}\sum_{\mathbf{p}} (\omega(\mathbf{k}) - \omega(\mathbf{p})) \times \\
&\times \sum_{r=1}^{2}\sum_{j=1}^{2} \left(d_{j,-\mathbf{p}}^{\dagger} b_{r,\mathbf{k}}^{\dagger} b_{r,\mathbf{k}} d_{j,-\mathbf{p}} - b_{j,\mathbf{p}}^{\dagger} d_{r,-\mathbf{k}}^{\dagger} d_{r,-\mathbf{k}} b_{j,\mathbf{p}}\right)
\end{aligned}$$

and

$$
\begin{aligned}
\widehat{\mathbb{H}}_b \quad : \quad &= \sum_{\mathbf{k}} \sum_{\mathbf{p}} (\omega(\mathbf{k}) + \omega(\mathbf{p})) \times \\
&\times \sum_{j=1}^{2} \sum_{r=1}^{2} \left(b_{j,\mathbf{p}}^{\dagger} b_{r,\mathbf{k}}^{\dagger} b_{r,\mathbf{k}} b_{j,\mathbf{p}} - d_{j,-\mathbf{p}}^{\dagger} d_{r,-\mathbf{k}}^{\dagger} d_{r,-\mathbf{k}} d_{j,-\mathbf{p}} \right).
\end{aligned}
$$

Therefore, in any case events with pairs of fermions and events with fermion-antifermion pairs can occur, but events with pairs of antiftrmions can not happen.

Therefore, an antifermion can exists only with a fermion.

4.11. Electroweak Fields

In 1963 American physicist Sheldon Glashow[28] [20] proposed that the weak nuclear force and electricity and magnetism could arise from a partially unified electroweak theory. This was the beginning of the Standard Theory. But "... there is major problem: all the fermions and gauge bosons are massless, while experiment shows otherwise. Why not just add in mass terms explicitly? That will not work, since the associated terms break SU(2) or gauge invariances. For fermions, the mass term should be $m\overline{\psi}\psi$?

$$
\begin{aligned}
m\overline{\psi}\psi &= m\overline{\psi}(P_L + P_R)\psi = \\
&= m(\overline{\psi}(P_L P_L)\psi + \overline{\psi}(P_R P_R)\psi) \\
&= m(\overline{\psi}_R \psi_L + \overline{\psi}_L \psi_R).
\end{aligned}
$$

However, the left-handed fermion are put into SU(2) doublets and the right-handed ones into SU(2) singlets, so $\overline{\psi}_R\psi_L$ and $\overline{\psi}_L\psi_R$ are not SU(2) singlets and would not give an SU(2) invariant Lagrangian. Similarly, the expected mass terms for the gauge bosons,

$$
\frac{1}{2} m_B^2 B^{\mu} B_{\mu}
$$

plus similar terms for other, are clearly not invariant under gauge transformations $B_\mu \to B'_\mu = B_\mu - \partial_\mu \chi / g$, The only direct way to preserve the gauge invariance and SU(2) invariance of Lagrangian is to set $m = 0$ for all quarks, leptons and gauge bosons:. There is a way to solve this problem, called the Higgs mechanism" [19].

No. The Dirac Lagrangian for a free fermion can have of the following form:

$$
\mathcal{L}_f := \psi^{\dagger} \left(\beta^{[0]} \partial_0 + \beta^{[1]} \partial_1 + \beta^{[2]} \partial_2 + \beta^{[3]} \partial_3 + \mathrm{i} m \gamma^{[0]} \right) \psi.
$$

Indeed, this Lagrangian is not invariant under the SU(2) transformation. But it is beautiful and truncating its mass term is not good idea.

Further you will see, how it is possible to keep this beauty.

[28] Sheldon Lee Glashow (born December 5, 1932) is an American theoretical physicist.

4.12. Bi-mass state

Let U be a 8×8 matrix such that for every $\widetilde{\varphi}$: (4.29, 4.30):

$$\begin{aligned}\left(U\widetilde{\varphi},U\widetilde{\varphi}\right) &= \rho_{\mathcal{A}},\\ \left(U\widetilde{\varphi},\beta^{[s]}U\widetilde{\varphi}\right) &= -\frac{j_{\mathcal{A},s}}{\mathrm{c}}\end{aligned} \tag{4.46}$$

here (2.106)

$$\widetilde{\varphi}=\widetilde{\varphi}1_8;\ \beta^{[s]}=\beta^{[s]}1_8.$$

In that case:

$$U^{\dagger}\beta^{[\mu]}U=\beta^{[\mu]}$$

for $\mu\in\{0,1,2,3\}$.

Such transformation has a matrix of the following shape:

$$U:=\left[\begin{array}{cccc}(a"+b"\mathrm{i})\,1_2 & 0_2 & (c"+\mathrm{i}g")\,1_2 & 0_2\\ 0_2 & (a`+b`\mathrm{i})\,1_2 & 0_2 & (c`+\mathrm{i}g`)\,1_2\\ (u"+\mathrm{i}v")\,1_2 & 0_2 & (k"+\mathrm{i}s")\,1_2 & 0_2\\ 0_2 & (u`+\mathrm{i}v`)\,1_2 & 0_2 & (k`+\mathrm{i}s`)\,1_2\end{array}\right].$$

with real functions

$a"(t,\mathbf{x})$, $b"(t,\mathbf{x})$, $c"(t,\mathbf{x})$, $g"(t,\mathbf{x})$, $u"(t,\mathbf{x})$, $v"(t,\mathbf{x})$, $k"(t,\mathbf{x})$, $s"(t,\mathbf{x})$,
$a`(t,\mathbf{x})$, $b`(t,\mathbf{x})$, $c`(t,\mathbf{x})$, $g`(t,\mathbf{x})$, $u`(t,\mathbf{x})$, $v`(t,\mathbf{x})$, $k`(t,\mathbf{x})$, $s`(t,\mathbf{x})$.

These functions fulfil the following conditions:

$$\begin{aligned}v"^2+b"^2+u"^2+a"^2=1,\\ c"^2+g"^2+k"^2+s"^2=1,\end{aligned}$$

$$s"=-\frac{a"g"u"-u"b"c"+a"c"v"+b"g"v"}{u"^2+v"^2},$$

$$k"=\frac{-u"a"c"-u"b"g"+v"a"g"-b"c"v"}{u"^2+v"^2}.$$

$$\begin{aligned}v`^2+b`^2+u`^2+a`^2=1,\\ c`^2+g`^2+k`^2+s`^2=1,\end{aligned}$$

$$s`=-\frac{a`g`u`-u`b`c`+a`c`v`+b`g`v`}{u`^2+v`^2},$$

$$k`=\frac{-u`a`c`-u`b`g`+v`a`g`-b`c`v`}{u`^2+v`^2}.$$

U has 4 eigenvalues: $\exp(i\alpha_1)$, $\exp(i\alpha_2)$, $\exp(i\alpha_3)$, $\exp(i\alpha_4)$ for 8 orthonormalized eigenvectors:

$$\varepsilon_{1,1}, \varepsilon_{1,2}, \varepsilon_{2,1}, \varepsilon_{2,2}, \varepsilon_{3,1}, \varepsilon_{3,2}, \varepsilon_{4,1}, \varepsilon_{4,2}.$$

Let

$$K := \begin{bmatrix} \varepsilon_{1,1} & \varepsilon_{1,2} & \varepsilon_{2,1} & \varepsilon_{2,2} & \varepsilon_{3,1} & \varepsilon_{3,2} & \varepsilon_{4,1} & \varepsilon_{4,2} \end{bmatrix}.$$

Let θ_1, θ_2, θ_3, θ_4 be solution of the following system of equations:

$$\left\{ \begin{array}{l} \theta_1 + \theta_2 + \theta_3 + \theta_4 = \alpha_1, \\ \theta_1 + \theta_2 - \theta_3 - \theta_4 = \alpha_1, \\ \theta_1 - \theta_2 + \theta_3 - \theta_4 = \alpha_1, \\ \theta_1 - \theta_2 - \theta_3 + \theta_4 = \alpha_1. \end{array} \right|$$

and

$$U_1 := \exp(\mathrm{i}\theta_1),$$

$$U_2 := K \begin{bmatrix} \exp(\mathrm{i}\theta_2) 1_4 & 0_4 \\ 0_4 & \exp(-\mathrm{i}\theta_2) 1_4 \end{bmatrix} K^\dagger,$$

$$U_3 := K \begin{bmatrix} \exp(\mathrm{i}\theta_3) 1_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & \exp(-\mathrm{i}\theta_3) 1_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & \exp(\mathrm{i}\theta_3) 1_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & \exp(-\mathrm{i}\theta_3) 1_2 \end{bmatrix} K^\dagger,$$

$$U_4 := K \begin{bmatrix} \exp(\mathrm{i}\theta_4) 1_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & \exp(-\mathrm{i}\theta_4) 1_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & \exp(-\mathrm{i}\theta_4) 1_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & \exp(\mathrm{i}\theta_4) 1_2 \end{bmatrix} K^\dagger.$$

In this case:

$$U_1 U_2 U_3 U_4 = U$$

and

$$U_2 = \begin{bmatrix} \exp(\mathrm{i}\theta_2) 1_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & \exp(-\mathrm{i}\theta_2) 1_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & \exp(\mathrm{i}\theta_2) 1_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & \exp(-\mathrm{i}\theta_2) 1_2 \end{bmatrix}.$$

Besides

$$U_1 U_2 = \begin{bmatrix} e^{\mathrm{i}(\theta_1+\theta_2)} & 0 & 0 & 0 \\ 0 & e^{\mathrm{i}(\theta_1-\theta_2)} & 0 & 0 \\ 0 & 0 & e^{\mathrm{i}(\theta_1+\theta_2)} & 0 \\ 0 & 0 & 0 & e^{\mathrm{i}(\theta_1-\theta_2)} \end{bmatrix}.$$

Let χ and ς be thesolution of the following set of equations:

$$\left\{\begin{array}{c} 0.5\chi+\varsigma=\theta_1+\theta_2, \\ \chi+\varsigma=\theta_1-\theta_2, \end{array}\right|$$

i.e.:

$$\begin{array}{c} \chi=-4\theta_2, \\ \varsigma=\theta_1+3\theta_2. \end{array}$$

Let

$$U^{[e]}:=\exp\left(\mathrm{i}\varsigma\right)$$

and (4.22)

$$\widetilde{U}=\left[\begin{array}{cc} \exp\left(\mathrm{i}\frac{\chi}{2}\right)1_2 & 0_2 \\ 0_2 & \exp\left(\mathrm{i}\chi\right)1_2 \end{array}\right].$$

In that case:

$$\widetilde{U}U^{[e]}1_8=U_1U_2.$$

Here real functions
$a\left(t,\mathbf{x}\right)$, $b\left(t,\mathbf{x}\right)$, $c\left(t,\mathbf{x}\right)$, $g\left(t,\mathbf{x}\right)$, $u\left(t,\mathbf{x}\right)$, $v\left(t,\mathbf{x}\right)$, $k\left(t,\mathbf{x}\right)$,$s\left(t,\mathbf{x}\right)$
exist such that:

$$U_3U_4=\left[\begin{array}{cccc} (a+\mathrm{i}b)1_2 & 0_2 & (c+\mathrm{i}g)1_2 & 0_2 \\ 0_2 & (u+\mathrm{i}v)1_2 & 0_2 & (k+\mathrm{i}s)1_2 \\ (-c+\mathrm{i}g)1_2 & 0_2 & (a-\mathrm{i}b)1_2 & 0_2 \\ 0_2 & (-k+\mathrm{i}s)1_2 & 0_2 & (u-\mathrm{i}v)1_2 \end{array}\right]$$

and

$$\begin{array}{c} a^2+b^2+c^2+g^2=1, \\ u^2+v^2+r^2+s^2=1. \end{array}$$

If

$$U^{(+)}:=\left[\begin{array}{cccc} 1_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & (u+\mathrm{i}v)1_2 & 0_2 & (k+\mathrm{i}s)1_2 \\ 0_2 & 0_2 & 1_2 & 0_2 \\ 0_2 & (-k+\mathrm{i}s)1_2 & 0_2 & (u-\mathrm{i}v)1_2 \end{array}\right] \tag{4.47}$$

and

$$U^{(-)}:=\left[\begin{array}{cccc} (a+\mathrm{i}b)1_2 & 0_2 & (c+\mathrm{i}g)1_2 & 0_2 \\ 0_2 & 1_2 & 0_2 & 0_2 \\ (-c+\mathrm{i}g)1_2 & 0_2 & (a-\mathrm{i}b)1_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & 1_2 \end{array}\right] \tag{4.48}$$

then

$$U_3U_4 = U^{(-)}U^{(+)} = U^{(+)}U^{(-)}.$$

Let us consider $U^{(-)}$.

Let:

$$\ell_\circ := \frac{1}{2\sqrt{(1-a^2)}} \begin{bmatrix} \left(b+\sqrt{(1-a^2)}\right)1_4 & (q-\mathrm{i}c)\,1_4 \\ (q+\mathrm{i}c)\,1_4 & \left(\sqrt{(1-a^2)}-b\right)1_4 \end{bmatrix} \tag{4.49}$$

and

$$\ell_* := \frac{1}{2\sqrt{(1-a^2)}} \begin{bmatrix} \left(\sqrt{(1-a^2)}-b\right)1_4 & (-q+\mathrm{i}c)\,1_4 \\ (-q-\mathrm{i}c)\,1_4 & \left(b+\sqrt{(1-a^2)}\right)1_4 \end{bmatrix}. \tag{4.50}$$

These operators are fulfilled to the following conditions:

$$\begin{aligned} \ell_\circ\ell_\circ &= \ell_\circ,\, \ell_*\ell_* = \ell_*; \\ \ell_\circ\ell_* &= 0 = \ell_*\ell_\circ, \\ (\ell_\circ - \ell_*)(\ell_\circ - \ell_*) &= 1_8, \\ \ell_\circ + \ell_* &= 1_8, \end{aligned}$$

$$\begin{gathered} \ell_\circ\gamma^{[0]} = \gamma^{[0]}\ell_\circ,\, \ell_*\gamma^{[0]} = \gamma^{[0]}\ell_*, \\ \ell_\circ\beta^{[4]} = \beta^{[4]}\ell_\circ,\, \ell_*\beta^{[4]} = \beta^{[4]}\ell_* \end{gathered}$$

and

$$\begin{gathered} U^{(-)\dagger}\gamma^{[0]}U^{(-)} = a\gamma^{[0]} - (\ell_\circ - \ell_*)\sqrt{1-a^2}\beta^{[4]}, \\ U^{(-)\dagger}\beta^{[4]}U^{(-)} = a\beta^{[4]} + (\ell_\circ - \ell_*)\sqrt{1-a^2}\gamma^{[0]}. \end{gathered} \tag{4.51}$$

From (4.21) the lepton equation of motion is the following:

$$\left(\sum_{\mu=0}^{3}\beta^{[\mu]}\left(\mathrm{i}\partial_\mu + F_\mu + 0.5g_1YB_\mu\right) + \gamma^{[0]}\mathrm{i}\partial_5 + \beta^{[4]}\mathrm{i}\partial_4\right)U^{(-)\dagger}U^{(-)}\widetilde{\varphi} = 0.$$

If

$$\partial_k U^{(-)\dagger} = U^{(-)\dagger}\partial_k \tag{4.52}$$

for $k \in \{0,1,2,3,4,5\}$ then

$$\begin{pmatrix} U^{(-)\dagger}\mathrm{i}\sum_{\mu=0}^{3}\beta^{[\mu]}\left(\mathrm{i}\partial_\mu + F_\mu + 0.5g_1YB_\mu\right) \\ +\gamma^{[0]}U^{(-)\dagger}\mathrm{i}\partial_5 + \beta^{[4]}U^{(-)\dagger}\mathrm{i}\partial_4 \end{pmatrix} U^{(-)}\widetilde{\varphi} = 0.$$

Hence, from (4.51):

$$U^{(-)\dagger}\begin{pmatrix} \sum_{\mu=0}^{3}\beta^{[\mu]}\left(\mathrm{i}\partial_\mu + F_\mu + 0.5g_1YB_\mu\right) \\ +\gamma^{[0]}\mathrm{i}\left(a\partial_5 - (\ell_\circ - \ell_*)\sqrt{1-a^2}\partial_4\right) \\ +\beta^{[4]}\mathrm{i}\left(\sqrt{1-a^2}\,(\ell_\circ - \ell_*)\,\partial_5 + a\partial_4\right) \end{pmatrix} U^{(-)}\widetilde{\varphi} = 0.$$

Thus, if denote:

$$x_4' = (\ell_\circ + \ell_*)\, a x_4 + (\ell_\circ - \ell_*)\, \sqrt{1-a^2} x_5,$$
$$x_5' = (\ell_\circ + \ell_*)\, a x_5 - (\ell_\circ - \ell_*)\, \sqrt{1-a^2} x_4$$

then

$$\left(\sum_{\mu=0}^{3} \beta^{[\mu]} \left(\mathrm{i}\partial_\mu + F_\mu + 0.5 g_1 Y B_\mu\right) + \left(\gamma^{[0]} \mathrm{i}\partial_5' + \beta^{[4]} \mathrm{i}\partial_4'\right)\right) \widetilde{\varphi}' = 0 \tag{4.53}$$

with

$$\widetilde{\varphi}' = U^{(-)} \widetilde{\varphi}.$$

That is the lepton Hamiltonian is invariant for the following global transformation:

$$\begin{aligned}
&\widetilde{\varphi} \to \widetilde{\varphi}' = U^{(-)} \widetilde{\varphi}, \\
&x_4 \to x_4' = (\ell_\circ + \ell_*)\, a x_4 + (\ell_\circ - \ell_*)\, \sqrt{1-a^2} x_5, \\
&x_5 \to x_5' = (\ell_\circ + \ell_*)\, a x_5 - (\ell_\circ - \ell_*)\, \sqrt{1-a^2} x_4, \\
&x_\mu \to x_\mu' = x_\mu.
\end{aligned} \tag{4.54}$$

4.13. Electroweak Transformations

During the 1960s Sheldon Lee Glashow discovered that they could construct a gauge-invariant theory of the weak force, provided that they also included the electromagnetic force.

The existence of the force carriers, the neutral Z particles and the charged W particles, was verified experimentally in 1983 in high-energy proton-antiproton collisions at the European Organization for Nuclear Research (CERN).

Let (4.52) does not hold true, that is $U^{(-)}$ depends on $\underline{x}$. And let denote:

$$K := \sum_{\mu=0}^{3} \beta^{[\mu]} \left(F_\mu + 0.5 g_1 Y B_\mu\right). \tag{4.55}$$

In that case from (4.21) the equation of moving is of following form:

$$\left(K + \sum_{\mu=0}^{3} \beta^{[\mu]} \mathrm{i}\partial_\mu + \gamma^{[0]} \mathrm{i}\partial_5 + \beta^{[4]} \mathrm{i}\partial_4\right) \widetilde{\varphi} = 0. \tag{4.56}$$

Let us consider for this Hamiltonian the following transformations:

$$\begin{aligned}
&\widetilde{\varphi} \to \widetilde{\varphi}' := U^{(-)} \widetilde{\varphi}, \\
&x_4 \to x_4' := (\ell_\circ + \ell_*)\, a x_4 + (\ell_\circ - \ell_*)\, \sqrt{1-a^2} x_5, \\
&x_5 \to x_5' := (\ell_\circ + \ell_*)\, a x_5 - (\ell_\circ - \ell_*)\, \sqrt{1-a^2} x_4, \\
&x_\mu \to x_\mu' := x_\mu, \text{ for } \mu \in \{0,1,2,3\}, \\
&K \to K' = U^{(-)} K U^{(-)\dagger} - \mathrm{i} \sum_{\mu=0}^{3} \beta^{[\mu]} \left(\partial_\mu U^{(-)}\right) U^{(-)\dagger}
\end{aligned} \tag{4.57}$$

with

$$\partial_4 U^{(-)} = U^{(-)}\partial_4 \text{ and } \partial_5 U^{(-)} = U^{(-)}\partial_5 \text{:}$$

Since

$$(\ell_\circ - \ell_*)(\ell_\circ - \ell_*) = 1_8$$

then

$$\begin{aligned} x_4 &= a x_4' - (\ell_\circ - \ell_*)\sqrt{1-a^2}x_5' \text{ and} \\ x_5 &= (\ell_\circ - \ell_*)\sqrt{1-a^2}x_4' + a x_5'. \end{aligned}$$

Since for any f:

$$\begin{aligned} \partial_4' f &= \partial_4 f \cdot \partial_4' x_4 + \partial_5 f \cdot \partial_4' x_5, \\ \partial_5' f &= \partial_4 f \cdot \partial_5' x_4 + \partial_5 f \cdot \partial_5' x_5 \end{aligned}$$

then

$$\begin{aligned} \partial_4' f &= \partial_4 f \cdot a + \partial_5 f \cdot (\ell_\circ - \ell_*)\sqrt{1-a^2}, \\ \partial_5' f &= \partial_4 f \cdot \left(-(\ell_\circ - \ell_*)\sqrt{1-a^2}\right) + \partial_5 f \cdot a. \end{aligned}$$

Therefore, if

$$\left(K' + \sum_{\mu=0}^{3} \beta^{[\mu]} \mathrm{i}\partial_\mu + \gamma^{[0]}\mathrm{i}\partial_5' + \beta^{[4]}\mathrm{i}\partial_4'\right) U^{(-)}\widetilde{\varphi} = 0$$

then

$$\left(\begin{array}{c} U^{(-)}KU^{(-)\dagger} - \mathrm{i}\sum_{\mu=0}^{3}\beta^{[\mu]}\left(\partial_\mu U^{(-)}\right)U^{(-)\dagger} \\ +\sum_{\mu=0}^{3}\beta^{[\mu]}\mathrm{i}\partial_\mu + \gamma^{[0]}\mathrm{i}\left(\left(-(\ell_\circ - \ell_*)\sqrt{1-a^2}\right)\partial_4 + a\partial_5\right) \\ +\beta^{[4]}\mathrm{i}\left(a\partial_4 + (\ell_\circ - \ell_*)\sqrt{1-a^2}\partial_5\right) \end{array}\right) U^{(-)}\widetilde{\varphi} = 0.$$

Hence,

$$\left(\begin{array}{c} U^{(-)}KU^{(-)\dagger}U^{(-)} - \mathrm{i}\sum_{\mu=0}^{3}\beta^{[\mu]}\left(\partial_\mu U^{(-)}\right)U^{(-)\dagger}U^{(-)} \\ +\sum_{\mu=0}^{3}\beta^{[\mu]}\mathrm{i}\partial_\mu U^{(-)} + \gamma^{[0]}U^{(-)}\mathrm{i}\left(\left(-(\ell_\circ - \ell_*)\sqrt{1-a^2}\right)\partial_4 + a\partial_5\right) \\ +\beta^{[4]}U^{(-)}\mathrm{i}\left(a\partial_4 + (\ell_\circ - \ell_*)\sqrt{1-a^2}\partial_5\right) \end{array}\right)\widetilde{\varphi} = 0$$

since $U^{(-)}$ is a linear operator such that $\partial_4 U^{(-)} = U^{(-)}\partial_4$ and $\partial_5 U^{(-)} = U^{(-)}\partial_5$.

Since

$$U^{(-)\dagger}U^{(-)} = 1_8,$$

for any f:

$$\partial_\mu\left(U^{(-)}f\right) = \left(\partial_\mu U^{(-)}\right)f + \left(U^{(-)}\partial_\mu f\right) = \left(\left(\partial_\mu U^{(-)}\right) + U^{(-)}\partial_\mu\right)f,$$

and

$$\begin{array}{rcl}
\gamma^{[0]}U^{(-)} & = & U^{(-)}\left(a\gamma^{[0]} - (\ell_\circ - \ell_*)\sqrt{1-a^2}\beta^{[4]}\right), \\
\beta^{[4]}U^{(-)} & = & U^{(-)}\left(a\beta^{[4]} + (\ell_\circ - \ell_*)\sqrt{1-a^2}\gamma^{[0]}\right)
\end{array}$$

then

$$\left(\begin{array}{c}
U^{(-)}K - \mathrm{i}\sum_{\mu=0}^{3}\beta^{[\mu]}\left(\partial_\mu U^{(-)}\right) \\
+\sum_{\mu=0}^{3}\beta^{[\mu]}\mathrm{i}\left(\left(\partial_\mu U^{(-)}\right) + U^{(-)}\partial_\mu\right) \\
+U^{(-)}\left(a\gamma^{[0]} - (\ell_\circ - \ell_*)\sqrt{1-a^2}\beta^{[4]}\right)\times \\
\times\mathrm{i}\left(\left(-(\ell_\circ - \ell_*)\sqrt{1-a^2}\right)\partial_4 + a\partial_5\right) \\
+U^{(-)}\left(a\beta^{[4]} + (\ell_\circ - \ell_*)\sqrt{1-a^2}\gamma^{[0]}\right)\times \\
\times\mathrm{i}\left(a\partial_4 + (\ell_\circ - \ell_*)\sqrt{1-a^2}\partial_5\right)
\end{array}\right)\widetilde{\varphi} = 0.$$

Therefore,

$$\left(\begin{array}{c}
U^{(-)}K - \mathrm{i}\sum_{\mu=0}^{3}\beta^{[\mu]}\left(\partial_\mu U^{(-)}\right) \\
+\sum_{\mu=0}^{3}\beta^{[\mu]}\mathrm{i}\left(\left(\partial_\mu U^{(-)}\right) + U^{(-)}\partial_\mu\right) \\
+\mathrm{i}U^{(-)}\left(\begin{array}{c}
\left(a\gamma^{[0]} - (\ell_\circ - \ell_*)\sqrt{1-a^2}\beta^{[4]}\right)\times \\
\times\left(-(\ell_\circ - \ell_*)\sqrt{1-a^2}\partial_4 + a\partial_5\right) \\
+\left(a\beta^{[4]} + (\ell_\circ - \ell_*)\sqrt{1-a^2}\gamma^{[0]}\right)\times \\
\times\left(a\partial_4 + (\ell_\circ - \ell_*)\sqrt{1-a^2}\partial_5\right)
\end{array}\right)
\end{array}\right)\widetilde{\varphi} = 0,$$

$$\left(U^{(-)}K + \sum_{\mu=0}^{3}\beta^{[\mu]}\mathrm{i}U^{(-)}\partial_\mu + \mathrm{i}U^{(-)}\left(+\gamma^{[0]}\partial_5 + \beta^{[4]}\partial_4\right)\right)\widetilde{\varphi} = 0,$$

Hence,

$$U^{(-)}\left(K + \sum_{\mu=0}^{3}\beta^{[\mu]}\mathrm{i}\partial_\mu + \mathrm{i}\left(+\gamma^{[0]}\partial_5 + \beta^{[4]}\partial_4\right)\right)\widetilde{\varphi} = 0$$

since $\beta^{[\mu]}U^{(-)} = U^{(-)}\beta^{[\mu]}$ for $\mu \in \{0,1,2,3\}$.

Therefore, this equation of moving is invariant under the transformation (4.57).

Let g_2 be some positive real number.

If design (here: a,b,c,q form $U^{(-)}$ in (4.48)):

$$\begin{array}{c}W_{0,\mu}:=-2\frac{1}{g_2 q}\left(\begin{array}{c}q\left(\partial_\mu a\right)b-q\left(\partial_\mu b\right)a+\left(\partial_\mu c\right)q^2+\\+a\left(\partial_\mu a\right)c+b\left(\partial_\mu b\right)c+c^2\left(\partial_\mu c\right)\end{array}\right)\\W_{1,\mu}:=-2\frac{1}{g_2 q}\left(\begin{array}{c}\left(\partial_\mu a\right)a^2-bq\left(\partial_\mu c\right)+a\left(\partial_\mu b\right)b+\\+a\left(\partial_\mu c\right)c+q^2\left(\partial_\mu a\right)+c\left(\partial_\mu b\right)q\end{array}\right)\\W_{2,\mu}:=-2\frac{1}{g_2 q}\left(\begin{array}{c}q\left(\partial_\mu a\right)c-a\left(\partial_\mu a\right)b-b^2\left(\partial_\mu b\right)-\\-c\left(\partial_\mu c\right)b-\left(\partial_\mu b\right)q^2-\left(\partial_\mu c\right)qa\end{array}\right)\end{array}$$

and

$$W_\mu:=\left[\begin{array}{cccc}W_{0,\mu}1_2 & 0_2 & \left(W_{1,\mu}-\mathrm{i}W_{2,\mu}\right)1_2 & 0_2\\0_2 & 0_2 & 0_2 & 0_2\\\left(W_{1,\mu}+\mathrm{i}W_{2,\mu}\right)1_2 & 0_2 & -W_{0,\mu}1_2 & 0_2\\0_2 & 0_2 & 0_2 & 0_2\end{array}\right] \tag{4.58}$$

then

$$-\mathrm{i}\left(\partial_\mu U^{(-)}\right)U^{(-)\dagger}=\frac{1}{2}g_2 W_\mu, \tag{4.59}$$

and from (4.55), (4.56):

$$\left(\begin{array}{c}\sum_{\mu=0}^{3}\beta^{[\mu]}\mathrm{i}\left(\partial_\mu-\mathrm{i}0.5g_1B_\mu Y-\mathrm{i}\frac{1}{2}g_2W_\mu-\mathrm{i}F_\mu\right)\\+\gamma^{[0]}\mathrm{i}\partial'_5+\beta^{[4]}\mathrm{i}\partial'_4\end{array}\right)\widetilde{\varphi}'=0. \tag{4.60}$$

Let (4.48) $a'(t,\mathbf{x})$, $b'(t,\mathbf{x})$, $c'(t,\mathbf{x})$, $q'(t,\mathbf{x})$ are real functions and:

$$U':=\left[\begin{array}{cccc}\left(a'+\mathrm{i}b'\right)1_2 & 0_2 & \left(c'+\mathrm{i}g'\right)1_2 & 0_2\\0_2 & 1_2 & 0_2 & 0_2\\\left(-c'+\mathrm{i}g'\right)1_2 & 0_2 & \left(a'-\mathrm{i}b'\right)1_2 & 0_2\\0_2 & 0_2 & 0_2 & 1_2\end{array}\right].$$

In this case if

$$U'':=U'U^{(-)}$$

then there exist real functions $a''(t,\mathbf{x})$, $b''(t,\mathbf{x})$, $c''(t,\mathbf{x})$, $q''(t,\mathbf{x})$ such that U'' has the similar shape:

$$U''=\left[\begin{array}{cccc}\left(a''+\mathrm{i}b''\right)1_2 & 0_2 & \left(c''+\mathrm{i}g''\right)1_2 & 0_2\\0_2 & 1_2 & 0_2 & 0_2\\\left(-c''+\mathrm{i}g''\right)1_2 & 0_2 & \left(a''-\mathrm{i}b''\right)1_2 & 0_2\\0_2 & 0_2 & 0_2 & 1_2\end{array}\right].$$

Let:

$$W''_\mu:=-\frac{2\mathrm{i}}{g_2}\left(\partial_\mu\left(U'U^{(-)}\right)\right)\left(U'U^{(-)}\right)^\dagger,$$

Hence,

$$\begin{aligned}
W''_\mu &= -\frac{2\mathrm{i}}{g_2}\left(\partial_\mu U'\right)U^{(-)}\left(U'U^{(-)}\right)^\dagger - \frac{2\mathrm{i}}{g_2}U'\partial_\mu U^{(-)}\left(U'U^{(-)}\right)^\dagger \\
&= -\frac{2\mathrm{i}}{g_2}\left(\partial_\mu U'\right)U^{(-)}U^{(-)\dagger}U'^\dagger - \frac{2\mathrm{i}}{g_2}U'\left(\partial_\mu U^{(-)}\right)U^{(-)\dagger}U'^\dagger \\
&= -\frac{2\mathrm{i}}{g_2}\left(\partial_\mu U'\right)U'^\dagger - \frac{2\mathrm{i}}{g_2}U'\left(\partial_\mu U^{(-)}\right)U^{(-)\dagger}U'^\dagger .
\end{aligned}$$

Since from (4.59):

$$W_\mu = -\mathrm{i}\frac{2}{g_2}\left(\partial_\mu U^{(-)}\right)U^{(-)\dagger}$$

then

$$\begin{aligned}
W''_\mu &= -\frac{2\mathrm{i}}{g_2}\left(\partial_\mu U'\right)U'^\dagger - \frac{2\mathrm{i}}{g_2}U'\left(\left(\partial_\mu U^{(-)}\right)U^{(-)\dagger}\right)U'^\dagger \\
&= -\frac{2\mathrm{i}}{g_2}\left(\partial_\mu U'\right)U'^\dagger + U'W_\mu U'^\dagger .
\end{aligned}$$

Therefore, if

$$\ell''_\circ := \frac{1}{2\sqrt{(1-a''^2)}}\begin{bmatrix} \left(b''+\sqrt{(1-a''^2)}\right)1_4 & (q''-\mathrm{i}c'')\,1_4 \\ (q''+\mathrm{i}c'')\,1_4 & \left(\sqrt{(1-a''^2)}-b''\right)1_4 \end{bmatrix},$$
$$\ell''_* := \frac{1}{2\sqrt{(1-a''^2)}}\begin{bmatrix} \left(\sqrt{(1-a''^2)}-b''\right)1_4 & (-q''+\mathrm{i}c'')\,1_4 \\ (-q''-\mathrm{i}c'')\,1_4 & \left(b''+\sqrt{(1-a''^2)}\right)1_4 \end{bmatrix}.$$

then under the following transformation

$$\begin{aligned}
&\widetilde{\varphi} \to \widetilde{\varphi}'' := U''\widetilde{\varphi}, \\
&x_4 \to x''_4 := \left(\ell''_\circ + \ell''_*\right)a''x_4 + \left(\ell''_\circ - \ell''_*\right)\sqrt{1-a''^2}x_5, \\
&x_5 \to x''_5 := \left(\ell''_\circ + \ell''_*\right)a''x_5 - \left(\ell''_\circ - \ell''_*\right)\sqrt{1-a''^2}x_4, \\
&x_\mu \to x''_\mu := x_\mu, \text{ for } \mu \in \{0,1,2,3\}, \\
&K \to K'' := \sum_{\mu=0}^{3}\beta^{[\mu]}\left(F_\mu + 0.5g_1YB_\mu + \frac{1}{2}g_2W''_\mu\right)
\end{aligned} \tag{4.61}$$

fields W''_μ and W_μ are tied by the following equation

$$\boxed{W''_\mu = U'W_\mu U'^\dagger - \tfrac{2\mathrm{i}}{g_2}\left(\partial_\mu U'\right)U'^\dagger} \tag{4.62}$$

like in Standard Model.

From (4.59):

$$W_\mu = -\mathrm{i}\frac{2}{g_2}\left(\partial_\mu U^{(-)}\right)U^{(-)\dagger}.$$

Let us calculate:

$$\begin{aligned}
&\partial_\mu W_\nu - \partial_\nu W_\mu = \\
= \ & \partial_\mu\left(-\mathrm{i}\frac{2}{g_2}\left(\partial_\nu U^{(-)}\right)U^{(-)\dagger}\right) - \partial_\nu\left(-\mathrm{i}\frac{2}{g_2}\left(\partial_\mu U^{(-)}\right)U^{(-)\dagger}\right) = \\
= \ & -\mathrm{i}\frac{2}{g_2}\left(\begin{array}{c}\left(\partial_\mu\partial_\nu U^{(-)}\right)U^{(-)\dagger} + \left(\partial_\nu U^{(-)}\right)\left(\partial_\mu U^{(-)\dagger}\right)\\ -\left(\partial_\nu\partial_\mu U^{(-)}\right)U^{(-)\dagger} - \left(\partial_\mu U^{(-)}\right)\left(\partial_\nu U^{(-)\dagger}\right)\end{array}\right).
\end{aligned}$$

Since

$$\partial_\mu\partial_\nu U^{(-)} = \partial_\nu\partial_\mu U^{(-)}$$

then

$$\begin{aligned}
&\partial_\mu W_\nu - \partial_\nu W_\mu = \\
= \ & -\mathrm{i}\frac{2}{g_2}\left(\left(\partial_\nu U^{(-)}\right)\left(\partial_\mu U^{(-)\dagger}\right) - \left(\partial_\mu U^{(-)}\right)\left(\partial_\nu U^{(-)\dagger}\right)\right).
\end{aligned} \tag{4.63}$$

And let us calculate:

$$\begin{aligned}
&W_\mu W_\nu - W_\nu W_\mu = \\
= \ & \left(-\mathrm{i}\frac{2}{g_2}\left(\partial_\mu U^{(-)}\right)U^{(-)\dagger}\right)\left(-\mathrm{i}\frac{2}{g_2}\left(\partial_\nu U^{(-)}\right)U^{(-)\dagger}\right) - \\
& -\left(-\mathrm{i}\frac{2}{g_2}\left(\partial_\nu U^{(-)}\right)U^{(-)\dagger}\right)\left(-\mathrm{i}\frac{2}{g_2}\left(\partial_\mu U^{(-)}\right)U^{(-)\dagger}\right) \\
= \ & -\frac{4}{g_2^2}\left(\begin{array}{c}\left(\partial_\mu U^{(-)}\right)U^{(-)\dagger}\left(\partial_\nu U^{(-)}\right)U^{(-)\dagger}\\ -\left(\partial_\nu U^{(-)}\right)U^{(-)\dagger}\left(\partial_\mu U^{(-)}\right)U^{(-)\dagger}\end{array}\right).
\end{aligned}$$

Since

$$U^{(-)}U^{(-)\dagger} = 1_8$$

then

$$\partial_\mu\left(U^{(-)}U^{(-)\dagger}\right) = 0, \text{ and } \partial_\nu\left(U^{(-)}U^{(-)\dagger}\right) = 0,$$

Hence,

$$\left(\partial_\mu U^{(-)}\right)U^{(-)\dagger} + U^{(-)}\partial_\mu U^{(-)\dagger} = 0, \text{ and } \left(\partial_\nu U^{(-)}\right)U^{(-)\dagger} + U^{(-)}\partial_\nu U^{(-)\dagger} = 0$$

Hence,

$$\left(\partial_\mu U^{(-)}\right)U^{(-)\dagger} = -U^{(-)}\partial_\mu U^{(-)\dagger} \text{ and } \left(\partial_\nu U^{(-)}\right)U^{(-)\dagger} = -U^{(-)}\partial_\nu U^{(-)\dagger}.$$

Therefore,

$$\begin{array}{rl} & W_\mu W_\nu - W_\nu W_\mu = \\ = & -\frac{4}{g_2^2}\left(\begin{array}{c} -\left(\partial_\mu U^{(-)}\right)U^{(-)\dagger}U^{(-)}\partial_\nu U^{(-)\dagger}+ \\ +\left(\partial_\nu U^{(-)}\right)U^{(-)\dagger}U^{(-)}\partial_\mu U^{(-)\dagger} \end{array}\right) \\ = & -\frac{4}{g_2^2}\left(-\left(\partial_\mu U^{(-)}\right)\left(\partial_\nu U^{(-)\dagger}\right)+\left(\partial_\nu U^{(-)}\right)\left(\partial_\mu U^{(-)\dagger}\right)\right) \end{array}$$

since

$$U^{(-)\dagger}U^{(-)} = 1_8.$$

Therefore, in accordance with (4.63):

$$\boxed{\partial_\mu W_\nu - \partial_\nu W_\mu = \mathrm{i}\frac{g_2}{2}\left(W_\mu W_\nu - W_\nu W_\mu\right).} \tag{4.64}$$

In accordance with (4.58) matrix $W_\mu W_\nu - W_\nu W_\mu$ has the following columns:
the first and the second columns are the following:

$$\begin{array}{cc} 2\mathrm{i}W_{1,\mu}W_{2,\nu} - 2\mathrm{i}W_{2,\mu}W_{1,\nu} & 0_2 \\ 0_2 & 0_2 \\ 2W_{0,\nu}W_{1,\mu} + 2\mathrm{i}W_{0,\nu}W_{2,\mu} - 2W_{0,\mu}W_{1,\nu} - 2\mathrm{i}W_{0,\mu}W_{2,\nu} & 0_2 \\ 0_2 & 0_2 \end{array},$$

the third and the fourth columns are the following:

$$\begin{array}{cc} 2W_{0,\mu}W_{1,\nu} - 2\mathrm{i}W_{0,\mu}W_{2,\nu} - 2W_{0,\nu}W_{1,\mu} + 2\mathrm{i}W_{0,\nu}W_{2,\mu} & 0_2 \\ 0_2 & 0_2 \\ -2\mathrm{i}W_{1,\mu}W_{2,\nu} + 2\mathrm{i}W_{2,\mu}W_{1,\nu} & 0_2 \\ 0_2 & 0_2 \end{array}.$$

And matrix $\partial_\mu W_\nu - \partial_\nu W_\mu$ has the following columns:
the first and the second ones are the following:

$$\begin{array}{cc} \partial_\mu W_{0,\nu} - \partial_\nu W_{0,\mu} & 0_2 \\ 0_2 & 0_2 \\ \partial_\mu W_{1,\nu} + \mathrm{i}\partial_\mu W_{2,\nu} - \partial_\nu W_{1,\mu} - \mathrm{i}\partial_\nu W_{2,\mu} & 0_2 \\ 0_2 & 0_2 \end{array},$$

the third and the fourth columns are the following:

$$\begin{array}{cc} \partial_\mu W_{1,\nu} - \mathrm{i}\partial_\mu W_{2,\nu} - \partial_\nu W_{1,\mu} + \mathrm{i}\partial_\nu W_{2,\mu} & 0_2 \\ 0_2 & 0_2 \\ -\partial_\mu W_{0,\nu} + \partial_\nu W_{0,\mu} & 0_2 \\ 0_2 & 0_2 \end{array}.$$

Therefore, in accordance with (4.64):

$$
\begin{aligned}
& \mathrm{i}\frac{g_2}{2}\left(2\mathrm{i}W_{1,\mu}W_{2,\nu}-2\mathrm{i}W_{2,\mu}W_{1,\nu}\right) \\
= \;& \partial_\mu W_{0,\nu}-\partial_\nu W_{0,\mu}, \\
& \mathrm{i}\frac{g_2}{2}\left(2W_{0,\nu}W_{1,\mu}+2\mathrm{i}W_{0,\nu}W_{2,\mu}-2W_{0,\mu}W_{1,\nu}-2\mathrm{i}W_{0,\mu}W_{2,\nu}\right) \\
= \;& \partial_\mu W_{1,\nu}+\mathrm{i}\partial_\mu W_{2,\nu}-\partial_\nu W_{1,\mu}-\mathrm{i}\partial_\nu W_{2,\mu}, \\
& \mathrm{i}\frac{g_2}{2}\left(2W_{0,\mu}W_{1,\nu}-2\mathrm{i}W_{0,\mu}W_{2,\nu}-2W_{0,\nu}W_{1,\mu}+2\mathrm{i}W_{0,\nu}W_{2,\mu}\right) \\
= \;& \partial_\mu W_{1,\nu}-\mathrm{i}\partial_\mu W_{2,\nu}-\partial_\nu W_{1,\mu}+\mathrm{i}\partial_\nu W_{2,\mu}, \\
& \mathrm{i}\frac{g_2}{2}\left(-2\mathrm{i}W_{1,\mu}W_{2,\nu}+2\mathrm{i}W_{2,\mu}W_{1,\nu}\right) \\
= \;& -\partial_\mu W_{0,\nu}+\partial_\nu W_{0,\mu}.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\partial_\nu W_{0,\mu} &= \partial_\mu W_{0,\nu}-g_2\left(W_{1,\mu}W_{2,\nu}-W_{1,\nu}W_{2,\mu}\right), && (4.65)\\
\partial_\nu W_{1,\mu} &= \partial_\mu W_{1,\nu}-g_2\left(W_{2,\mu}W_{0,\nu}-W_{2,\nu}W_{0,\mu}\right), && (4.66)\\
\partial_\nu W_{2,\mu} &= \partial_\mu W_{2,\nu}-g_2\left(W_{0,\mu}W_{1,\nu}-W_{0,\nu}W_{1,\mu}\right). && (4.67)
\end{aligned}
$$

The derivative of (4.65) with respect to x_ν is of the following form:

$$
\begin{gathered}
\partial_\nu\partial_\nu W_{0,\mu}=\partial_\mu\partial_\nu W_{0,\nu}- \\
-g_2\left(\begin{array}{c}
\left(\partial_\nu W_{1,\mu}\right)W_{2,\nu}+W_{1,\mu}\left(\partial_\nu W_{2,\nu}\right) \\
-\left(\partial_\nu W_{1,\nu}\right)W_{2,\mu}-W_{1,\nu}\left(\partial_\nu W_{2,\mu}\right)
\end{array}\right).
\end{gathered}
$$

Let us substitute $\partial_\nu W_{1,\mu}$ and $\partial_\nu W_{2,\mu}$ for its expressions from (4.66) and (4.67):

$$
\begin{gathered}
\partial_\nu\partial_\nu W_{0,\mu}=\partial_\mu\partial_\nu W_{0,\nu}- \\
-g_2\left(\begin{array}{c}
\left(\partial_\mu W_{1,\nu}-g_2\left(W_{2,\mu}W_{0,\nu}-W_{2,\nu}W_{0,\mu}\right)\right)W_{2,\nu} \\
+W_{1,\mu}\left(\partial_\nu W_{2,\nu}\right)-\left(\partial_\nu W_{1,\nu}\right)W_{2,\mu} \\
-W_{1,\nu}\left(\partial_\mu W_{2,\nu}-g_2\left(W_{0,\mu}W_{1,\nu}-W_{0,\nu}W_{1,\mu}\right)\right)
\end{array}\right)= \\
=\partial_\mu\partial_\nu W_{0,\nu} \\
-g_2\left(\begin{array}{c}
\left(\partial_\mu W_{1,\nu}\right)W_{2,\nu}-g_2\left(W_{2,\mu}W_{0,\nu}W_{2,\nu}-W_{2,\nu}W_{0,\mu}W_{2,\nu}\right) \\
+W_{1,\mu}\left(\partial_\nu W_{2,\nu}\right)-\left(\partial_\nu W_{1,\nu}\right)W_{2,\mu} \\
-W_{1,\nu}\partial_\mu W_{2,\nu}+g_2\left(W_{1,\nu}W_{0,\mu}W_{1,\nu}-W_{1,\nu}W_{0,\nu}W_{1,\mu}\right)
\end{array}\right)= \\
=-g_2^2\left(W_{1,\nu}W_{1,\nu}+W_{2,\nu}W_{2,\nu}\right)W_{0,\mu}+ \\
+g_2^2\left(W_{1,\nu}W_{1,\mu}+W_{2,\mu}W_{2,\nu}\right)W_{0,\nu} \\
-g_2\left(\begin{array}{c}
\left(\partial_\mu W_{1,\nu}\right)W_{2,\nu}-W_{1,\nu}\partial_\mu W_{2,\nu} \\
+W_{1,\mu}\left(\partial_\nu W_{2,\nu}\right)-\left(\partial_\nu W_{1,\nu}\right)W_{2,\mu}
\end{array}\right)+ \\
+\partial_\mu\partial_\nu W_{0,\nu}
\end{gathered}
$$

Hence,

$$\begin{gathered}\partial_\nu \partial_\nu W_{0,\mu} = \\ = -g_2^2 \left(W_{1,\nu} W_{1,\nu} + W_{2,\nu} W_{2,\nu}\right) W_{0,\mu} + \\ + g_2^2 \left(W_{1,\nu} W_{1,\mu} + W_{2,\mu} W_{2,\nu}\right) W_{0,\nu} \\ -g_2 \left(\begin{array}{c} \left(\partial_\mu W_{1,\nu}\right) W_{2,\nu} - W_{1,\nu} \partial_\mu W_{2,\nu} \\ + W_{1,\mu} \left(\partial_\nu W_{2,\nu}\right) - \left(\partial_\nu W_{1,\nu}\right) W_{2,\mu} \end{array}\right) + \\ + \partial_\mu \partial_\nu W_{0,\nu}.\end{gathered}$$

Therefore:

$$\begin{gathered}\partial_\nu \partial_\nu W_{0,\mu} = \\ = -g_2^2 \left(W_{0,\nu} W_{0,\nu} + W_{1,\nu} W_{1,\nu} + W_{2,\nu} W_{2,\nu}\right) W_{0,\mu} + \\ + g_2^2 W_{0,\nu} W_{0,\nu} W_{0,\mu} \\ + g_2^2 \left(W_{1,\nu} W_{1,\mu} + W_{2,\mu} W_{2,\nu}\right) W_{0,\nu} \\ -g_2 \left(\begin{array}{c} \left(\partial_\mu W_{1,\nu}\right) W_{2,\nu} - W_{1,\nu} \partial_\mu W_{2,\nu} \\ + W_{1,\mu} \left(\partial_\nu W_{2,\nu}\right) - \left(\partial_\nu W_{1,\nu}\right) W_{2,\mu} \end{array}\right) + \\ + \partial_\mu \partial_\nu W_{0,\nu}.\end{gathered}$$

Thus,

$$\begin{gathered}\partial_\nu \partial_\nu W_{0,\mu} = \\ = -g_2^2 \left(W_{0,\nu} W_{0,\nu} + W_{1,\nu} W_{1,\nu} + W_{2,\nu} W_{2,\nu}\right) W_{0,\mu} + \\ + g_2^2 \left(W_{0,\nu} W_{0,\mu} + W_{1,\nu} W_{1,\mu} + W_{2,\mu} W_{2,\nu}\right) W_{0,\nu} \\ -g_2 \left(\begin{array}{c} \left(\partial_\mu W_{1,\nu}\right) W_{2,\nu} - W_{1,\nu} \partial_\mu W_{2,\nu} \\ + W_{1,\mu} \left(\partial_\nu W_{2,\nu}\right) - \left(\partial_\nu W_{1,\nu}\right) W_{2,\mu} \end{array}\right) + \\ + \partial_\mu \partial_\nu W_{0,\nu}.\end{gathered} \tag{4.68}$$

Since

$$\widetilde{W}_\nu^2 := W_{0,\nu} W_{0,\nu} + W_{1,\nu} W_{1,\nu} + W_{2,\nu} W_{2,\nu}$$

and

$$\left\langle \widetilde{W}_\nu | \widetilde{W}_\mu \right\rangle := W_{0,\nu} W_{0,\mu} + W_{1,\nu} W_{1,\mu} + W_{2,\mu} W_{2,\nu} = \left\langle \widetilde{W}_\nu | \widetilde{W}_\mu \right\rangle$$

for

$$\widetilde{W}_\mu = \left[\begin{array}{c} W_{0,\mu} \\ W_{1,\mu} \\ W_{2,\mu} \end{array}\right] \text{ and } \widetilde{W}_\nu = \left[\begin{array}{c} W_{0,\nu} \\ W_{1,\nu} \\ W_{2,\nu} \end{array}\right]$$

then

$$\begin{gathered}\partial_\nu \partial_\nu W_{0,\mu} = -\left(g_2 \widetilde{W}_\nu\right)^2 W_{0,\mu} + \\ + g_2^2 \left\langle \widetilde{W}_\nu | \widetilde{W}_\mu \right\rangle W_{0,\nu} \\ -g_2 \left(\begin{array}{c} \left(\partial_\mu W_{1,\nu}\right) W_{2,\nu} - W_{1,\nu} \partial_\mu W_{2,\nu} \\ + W_{1,\mu} \left(\partial_\nu W_{2,\nu}\right) - \left(\partial_\nu W_{1,\nu}\right) W_{2,\mu} \end{array}\right) + \\ + \partial_\mu \partial_\nu W_{0,\nu}.\end{gathered}$$

Hence,

$$\begin{array}{c}\partial_0\partial_0 W_{0,\mu} = -\left(g_2\widetilde{W}_0\right)^2 W_{0,\mu}+\\ +g_2^2\left\langle W_0|W_\mu\right\rangle W_{0,0}\\ -g_2\left(\begin{array}{c}\left(\partial_\mu W_{1,0}\right)W_{2,0}-W_{1,0}\partial_\mu W_{2,0}\\ +W_{1,\mu}\left(\partial_0 W_{2,0}\right)-\left(\partial_0 W_{1,0}\right)W_{2,\mu}\end{array}\right)+\\ +\partial_\mu\partial_0 W_{0,0}.\end{array}$$

Since $\partial_0 = \frac{1}{\mathrm{c}}\partial_t$ then

$$\begin{array}{c}\frac{1}{\mathrm{c}^2}\partial_t^2 W_{0,\mu} = -\left(g_2\widetilde{W}_0\right)^2 W_{0,\mu}+\\ +g_2^2\left\langle \widetilde{W}_0|\widetilde{W}_\mu\right\rangle W_{0,0}\\ -g_2\left(\begin{array}{c}\left(\partial_\mu W_{1,0}\right)W_{2,0}-W_{1,0}\partial_\mu W_{2,0}\\ +W_{1,\mu}\left(\partial_0 W_{2,0}\right)-\left(\partial_0 W_{1,0}\right)W_{2,\mu}\end{array}\right)+\\ +\partial_\mu\partial_0 W_{0,0}.\end{array}$$

And for $s \in \{1,2,3\}$:

$$\begin{array}{rcl}\partial_s\partial_s W_{0,\mu} & = & -\left(g_2\widetilde{W}_s\right)^2 W_{0,\mu}\\ & & +g_2^2\left\langle \widetilde{W}_s|\widetilde{W}_\mu\right\rangle W_{0,s}\\ & & -g_2\left(\begin{array}{c}\left(\partial_\mu W_{1,s}\right)W_{2,s}-W_{1,s}\partial_\mu W_{2,s}\\ +W_{1,\mu}\left(\partial_s W_{2,s}\right)-\left(\partial_s W_{1,s}\right)W_{2,\mu}\end{array}\right)\\ & & +\partial_\mu\partial_s W_{0,s}.\end{array}$$

Therefore,

$$\begin{array}{c}-\frac{1}{\mathrm{c}^2}\partial_t^2 W_{0,\mu}+\sum_{s=1}^3\partial_s^2 W_{0,\mu} =\\ -\left(\begin{array}{c}-\left(g_2\widetilde{W}_0\right)^2 W_{0,\mu}+g_2^2\left\langle \widetilde{W}_0|\widetilde{W}_\mu\right\rangle W_{0,0}\\ -g_2\left(\begin{array}{c}\left(\partial_\mu W_{1,0}\right)W_{2,0}-W_{1,0}\partial_\mu W_{2,0}\\ +W_{1,\mu}\left(\partial_0 W_{2,0}\right)-\left(\partial_0 W_{1,0}\right)W_{2,\mu}\end{array}\right)\\ +\partial_\mu\partial_0 W_{0,0}\end{array}\right)+\\ +\left(\begin{array}{c}\sum_{s=1}^3-\left(g_2\widetilde{W}_s\right)^2 W_{0,\mu}\\ +g_2^2\left\langle \widetilde{W}_s|\widetilde{W}_\mu\right\rangle W_{0,s}\\ -g_2\left(\begin{array}{c}\left(\partial_\mu W_{1,s}\right)W_{2,s}-W_{1,s}\partial_\mu W_{2,s}\\ +W_{1,\mu}\left(\partial_s W_{2,s}\right)-\left(\partial_s W_{1,s}\right)W_{2,\mu}\end{array}\right)\\ +\partial_\mu\partial_s W_{0,s}\end{array}\right)..\end{array}$$

Hence,

$$
\begin{gathered}
-\frac{1}{c^2}\partial_t^2 W_{0,\mu}+\sum_{s=1}^3 \partial_s^2 W_{0,\mu}= \\
\left(g_2\widetilde{W}_0\right)^2 W_{0,\mu}-\sum_{s=1}^3\left(g_2\widetilde{W}_s\right)^2 W_{0,\mu} \\
+g_2^2\sum_{s=1}^3\left\langle \widetilde{W}_s|\widetilde{W}_\mu\right\rangle W_{0,s}-g_2^2\left\langle \widetilde{W}_0|\widetilde{W}_\mu\right\rangle W_{0,0} \\
+g_2\left(\begin{array}{c}
\left(\begin{array}{c}
(\partial_\mu W_{1,0})W_{2,0}-W_{1,0}\partial_\mu W_{2,0} \\
+W_{1,\mu}(\partial_0 W_{2,0})-(\partial_0 W_{1,0})W_{2,\mu}
\end{array}\right) \\
-\sum_{s=1}^3\left(\begin{array}{c}
(\partial_\mu W_{1,s})W_{2,s}-W_{1,s}\partial_\mu W_{2,s} \\
+W_{1,\mu}(\partial_s W_{2,s})-(\partial_s W_{1,s})W_{2,\mu}
\end{array}\right)
\end{array}\right) \\
+\partial_\mu\sum_{s=1}^3\partial_s W_{0,s}-\partial_\mu\partial_0 W_{0,0}-.
\end{gathered}
$$

Hence,

$$
\boxed{
\begin{gathered}
\left(-\frac{1}{c^2}\partial_t^2+\sum_{s=1}^3\partial_s^2\right)W_{0,\mu}=g_2^2\left(\widetilde{W}_0^2-\sum_{s=1}^3\widetilde{W}_s^2\right)W_{0,\mu}+ \\
+g_2^2\left(\sum_{s=1}^3\left\langle \widetilde{W}_s|\widetilde{W}_\mu\right\rangle W_{0,s}-\left\langle \widetilde{W}_0|\widetilde{W}_\mu\right\rangle W_{0,0}\right) \\
+g_2\left(\begin{array}{c}
\left(\begin{array}{c}
(\partial_\mu W_{1,0})W_{2,0}-W_{1,0}\partial_\mu W_{2,0} \\
+W_{1,\mu}(\partial_0 W_{2,0})-(\partial_0 W_{1,0})W_{2,\mu}
\end{array}\right) \\
-\sum_{s=1}^3\left(\begin{array}{c}
(\partial_\mu W_{1,s})W_{2,s}-W_{1,s}\partial_\mu W_{2,s} \\
+W_{1,\mu}(\partial_s W_{2,s})-(\partial_s W_{1,s})W_{2,\mu}
\end{array}\right)
\end{array}\right) \\
+\partial_\mu\sum_{s=1}^3\partial_s W_{0,s}-\partial_\mu\partial_0 W_{0,0}.
\end{gathered}
}
\tag{4.69}
$$

This equation looks like to the Klein-Gordon[29][30] equation[31] of field $W_{0,\mu}$ with mass

$$
m=\frac{\mathrm{h}}{\mathrm{c}}g_2\sqrt{\widetilde{W}_0^2-\sum_{s=1}^3\widetilde{W}_s^2} \tag{4.70}
$$

and with additional terms of the $W_{0,\mu}$ interactions with others components of $\widetilde{W}$. You can receive similar equations for $W_{1,\mu}$ and for $W_{2,\mu}$.

If

$$
\widetilde{W}_0':=\frac{\widetilde{W}_0-\frac{v}{c}\widetilde{W}_k}{\sqrt{1-\left(\frac{v}{c}\right)^2}},\ \widetilde{W}_k':=\frac{\widetilde{W}_k-\frac{v}{c}\widetilde{W}_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}},\ \widetilde{W}_k':=\widetilde{W}_k,\text{ if } s\neq k
$$

then

[29] Walter Gordon (13 August 1893 – 24 December 1939) was a German theoretical physicist.

[30] Oskar Benjamin Klein (15 September 1894 – 5 February 1977) was a Swedish theoretical physicist.

[31] (4.36)

$$
\left(-\frac{1}{c^2}\partial_t^2+\sum_{s=1}^3\partial_s^2\right)\varphi=\frac{m^2c^2}{\mathrm{h}^2}\varphi
$$

$$
\begin{aligned}
&\widetilde{W}_0'^2 - \sum_{s=1}^{3} \widetilde{W}_s'^2 \\
= \quad & \frac{\left(\widetilde{W}_0 - \frac{v}{c}\widetilde{W}_k\right)^2}{1-\left(\frac{v}{c}\right)^2} - \frac{\left(\widetilde{W}_k - \frac{v}{c}\widetilde{W}_0\right)^2}{1-\left(\frac{v}{c}\right)^2} - \sum_{s\neq k} \widetilde{W}_s'^2 \\
= \quad & \frac{\widetilde{W}_0^2 + \left(\frac{v}{c}\right)^2 \widetilde{W}_k^2 - \widetilde{W}_k^2 - \left(\frac{v}{c}\right)^2 \widetilde{W}_0^2}{1-\left(\frac{v}{c}\right)^2} - \sum_{s\neq k} \widetilde{W}_s'^2 \\
= \quad & \frac{\left(1-\left(\frac{v}{c}\right)^2\right)\widetilde{W}_0^2 - \left(1-\left(\frac{v}{c}\right)^2\right)\widetilde{W}_k^2}{1-\left(\frac{v}{c}\right)^2} - \sum_{s\neq k} \widetilde{W}_s'^2.
\end{aligned}
$$

Hence,

$$
\widetilde{W}_0'^2 - \sum_{s=1}^{3} \widetilde{W}_s'^2 = \widetilde{W}_0^2 - \sum_{s=1}^{3} \widetilde{W}_s^2.
$$

Therefore, such ”mass” (4.70) is invariant for the Lorentz transformations:

You can calculate that it is invariant for the transformations of turns, too:

$$
\left\{
\begin{array}{l}
\widetilde{W}_r' = \widetilde{W}_r \cos\lambda - \widetilde{W}_s \sin\lambda. \\
\widetilde{W}_s' = \widetilde{W}_r \sin\lambda + \widetilde{W}_s \cos\lambda;
\end{array}
\right|
$$

with a real number λ, and $r \in \{1,2,3\}$, $s \in \{1,2,3\}$.

That is the form

$$
m = \frac{\mathrm{h}}{\mathrm{c}} g_2 \sqrt{\widetilde{W}_0^2 - \sum_{s=1}^{3} \widetilde{W}_s^2}
$$

is a mass.

A mass of the W-boson was measured, between 1996 and 2000 at LEP[32] [**?**].

Let[33]

$$
\begin{array}{r}
\alpha := \arctan \frac{g_1}{g_2}, \\
Z_\mu := (W_{0,\mu} \cos\alpha - B_\mu \sin\alpha), \\
A_\mu := (B_\mu \cos\alpha + W_{0,\mu} \sin\alpha).
\end{array}
\tag{4.71}
$$

In that case:

$$
\sum_\nu g_{\nu,\nu} \partial_\nu \partial_\nu W_{0,\mu} = \cos\alpha \cdot \sum_\nu g_{\nu,\nu} \partial_\nu \partial_\nu Z_\mu + \sin\alpha \cdot \sum_\nu g_{\nu,\nu} \partial_\nu \partial_\nu A_\mu.
$$

If

$$
\sum_\nu g_{\nu,\nu} \partial_\nu \partial_\nu A_\mu = 0
$$

[32]The Large Electron-Positron Collider (LEP) is largest particles accelerator (ring with a circumference of 27 kilometers built in a tunnel under the border of Switzerland and France.)

[33]here α is the Weinberg Angle. The experimental value of $\sin^2\alpha = 0.23124 \pm 0.00024$ [**?**].

then

$$m_Z = \frac{m_W}{\cos\alpha}$$

with m_W from (4.70). It is like Standard Model.

The equation of moving (4.60) under $F_\mu = 0$ has the following form:

$$\left(\begin{array}{c}\sum_{\mu=0}^{3}\beta^{[\mu]}\mathrm{i}\left(\partial_\mu - \mathrm{i}0.5g_1B_\mu Y - \mathrm{i}\frac{1}{2}g_2W_\mu\right)\\ +\gamma^{[0]}\mathrm{i}\partial_5+\beta^{[4]}\mathrm{i}\partial_4\end{array}\right)\widetilde{\varphi} = 0. \tag{4.72}$$

Hence, in accordance with (4.58) and (4.20):

$$\begin{array}{c}\sum_{\mu=0}^{3}\beta^{[\mu]}\mathrm{i}\times\\ \times\left(\begin{array}{c}\left(\begin{array}{c}\partial_\mu - \mathrm{i}0.5g_1B_\mu\left(-\left[\begin{array}{cc}1_2 & 0_2\\ 0_2 & 2\cdot 1_2\end{array}\right]\right)-\\ -\mathrm{i}\frac{1}{2}g_2\left[\begin{array}{cccc}W_{0,\mu}1_2 & 0_2 & (W_{1,\mu}-\mathrm{i}W_{2,\mu})1_2 & 0_2\\ 0_2 & 0_2 & 0_2 & 0_2\\ (W_{1,\mu}+\mathrm{i}W_{2,\mu})1_2 & 0_2 & -W_{0,\mu}1_2 & 0_2\\ 0_2 & 0_2 & 0_2 & 0_2\end{array}\right]\end{array}\right)\\ +\gamma^{[0]}\mathrm{i}\partial_5+\beta^{[4]}\mathrm{i}\partial_4\end{array}\right)\\ \cdot\widetilde{\varphi} = 0.\end{array}$$

In accordance with (4.71) [**?**]:

$$\begin{aligned}B_\mu &= \left(-Z_\mu\frac{g_1}{\sqrt{g_1^2+g_2^2}}+A_\mu\frac{g_2}{\sqrt{g_1^2+g_2^2}}\right),\\ W_{0,\mu} &= \left(Z_\mu\frac{g_2}{\sqrt{g_1^2+g_2^2}}+A_\mu\frac{g_1}{\sqrt{g_1^2+g_2^2}}\right).\end{aligned}$$

Let (e is the *elementary charge*[34]: e $= 1.60217733\times 10^{-19}$ C).

$$e := \frac{g_1g_2}{\sqrt{g_1^2+g_2^2}},$$

and let

$$\widehat{Z}_\mu := Z_\mu\frac{1}{\sqrt{g_2^2+g_1^2}}\left[\begin{array}{cccc}\left(g_2^2+g_1^2\right)1_2 & 0_2 & 0_2 & 0_2\\ 0_2 & 2g_1^21_2 & 0_2 & 0_2\\ 0_2 & 0_2 & \left(g_2^2-g_1^2\right)1_2 & 0_2\\ 0_2 & 0_2 & 0_2 & 2g_1^21_2\end{array}\right],$$

$$\widehat{W}_\mu := g_2\left[\begin{array}{cccc}0_2 & 0_2 & (W_{1,\mu}-\mathrm{i}W_{2,\mu})1_2 & 0_2\\ 0_2 & 0_2 & 0_2 & 0_2\\ (W_{1,\mu}+\mathrm{i}W_{2,\mu})1_2 & 0_2 & 0_2 & 0_2\\ 0_2 & 0_2 & 0_2 & 0_2\cdot 1_2\end{array}\right],$$

[34]Sir Joseph John "J. J." Thomson, (18 December 1856 — 30 August 1940) was a British physicist. He is credited for the discovery of the electron and of isotopes, and the invention of the mass spectrometer.

$$\widehat{A}_\mu := A_\mu \begin{bmatrix} 0_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & 1_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & 1_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & 1_2 \end{bmatrix}.$$

In that case from (4.72):

$$\boxed{\left(\sum_{\mu=0}^{3} \beta^{[\mu]} \mathrm{i} \left(\partial_\mu + \mathrm{i} e \widehat{A}_\mu - \mathrm{i} 0.5 \left(\widehat{Z}_\mu + \widehat{W}_\mu\right)\right) + \gamma^{[0]} \mathrm{i} \partial_5 + \beta^{[4]} \mathrm{i} \partial_4\right) \widetilde{\varphi} = 0.} \tag{4.73}$$

Let

$$\widetilde{\varphi} = \begin{bmatrix} \varphi_\nu \\ \overrightarrow{0}_2 \\ \varphi_{e,L} \\ \varphi_{e,R} \end{bmatrix}.$$

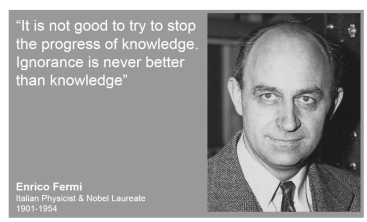
In that case

$$\boxed{\left(\begin{array}{c} \sum_{\mu=0}^{3} \beta^{[\mu]} \mathrm{i} \left(\partial_\mu \widetilde{\varphi} + \mathrm{i} A_\mu e \begin{bmatrix} \varphi_{e,L} \\ \varphi_{e,R} \end{bmatrix} - \mathrm{i} 0.5 \left(\widehat{Z}_\mu + \widehat{W}_\mu\right) \widetilde{\varphi}\right) \\ + \left(\gamma^{[0]} \mathrm{i} \partial_5 + \beta^{[4]} \mathrm{i} \partial_4\right) \widetilde{\varphi} \end{array}\right) = 0.} \tag{4.74}$$

Here the vector field A_μ is *the electromagnetic potential* [35]. And $\left(\widehat{Z}_\mu + \widehat{W}_\mu\right)$ is *the weak interaction potential* Evidently neutrinos do not involve in the electromagnetic interactions.

4.14. Neutrinno

Wolfgang Pauli postulated the neutrino in 1930 to explain the energy spectrum of beta decays, the decay of a neutron into a proton and an electron. Clyde Cowan, Frederick Reines found the neutrino experimentally in 1955. Enrico Fermi[36] developed the first theory describing neutrino interactions and denoted this particles as *neutrino* in 1933. In 1962 Leon M. Lederman, Melvin Schwartz and Jack Steinberger showed that more than one type of neutrino exists. Bruno Pontecorvo[37] suggested a practical method for investigating neutrino masses in 1957, over the subsequent 10 years he developed the mathematical formalism and the modern formulation of vacuum oscillations...



Let:

[35] James Clerk Maxwell of Glenlair (13 June 1831 — 5 November 1879) was a Scottish physicist and mathematician. His most prominent achievement was formulating classical electromagnetic theory.

[36] Enrico Fermi (29 September 1901 – 28 November 1954) was an Italian-born, naturalized American physicist particularly known for his work on the development of the first nuclear reactor, Chicago Pile-1, and for his contributions to the development of quantum theory, nuclear and particle physics, and statistical mechanics.

[37] Bruno Pontecorvo (Marina di Pisa, Italy, August 22, 1913 – Dubna, Russia, September 24, 1993) was an Italian-born atomic physicist, an early assistant of Enrico Fermi and then the author of numerous studies in high energy physics, especially on neutrinos.

$$\begin{aligned}&\widetilde{\varphi}(t,\mathbf{x},x_5,x_4) = \\ &= \exp(-\mathrm{i}hs_0x_4)\sum_{r=1}^{4}\phi_{4,r}(t,\mathbf{x},0,s_0)\varepsilon_r \\ &+ \exp(-\mathrm{i}hn_0x_5)\sum_{r=1}^{4}\phi_{5,r}(t,\mathbf{x},n_0,0)\varepsilon_r\end{aligned}$$

and

$$\widehat{H}_{0,4} \overset{Def.}{=} \sum_{r=1}^{3}\beta^{[r]}\mathrm{i}\partial_r + h\left(n_0\gamma^{[0]} + s_0\beta^{[4]}\right).$$

$$\underline{u}_1(\mathbf{k},n) \overset{Def}{=} \frac{1}{2\sqrt{\omega(\mathbf{k},n)(\omega(\mathbf{k},n)+n)}}\begin{bmatrix}0\\0\\0\\0\\\omega(\mathbf{k},n)+n+k_3\\k_1+\mathrm{i}k_2\\\omega(\mathbf{k},n)+n-k_3\\-k_1-\mathrm{i}k_2\end{bmatrix}$$

and

$$\underline{u}_2(\mathbf{k},n) \overset{Def}{=} \frac{1}{2\sqrt{\omega(\mathbf{k},n)(\omega(\mathbf{k},n)+n)}}\begin{bmatrix}0\\0\\0\\0\\k_1-\mathrm{i}k_2\\\omega(\mathbf{k},n)+n-k_3\\-k_1+\mathrm{i}k_2\\\omega(\mathbf{k},n)+n+k_3\end{bmatrix}$$

correspond to eigenvectors of $\widehat{H}_{0,4}$ with eigenvalue

$$\omega(\mathbf{k},n) = \sqrt{\mathbf{k}^2 + n^2}$$

and 8-vectors

$$\underline{u}_3(\mathbf{k},n) \overset{Def}{=} \frac{1}{2\sqrt{\omega(\mathbf{k},n)(\omega(\mathbf{k},n)+n)}}\begin{bmatrix}0\\0\\0\\0\\-\omega(\mathbf{k},n)-n+k_3\\k_1+\mathrm{i}k_2\\\omega(\mathbf{k},n)+n+k_3\\k_1+\mathrm{i}k_2\end{bmatrix}$$

and

$$\underline{u}_4(\mathbf{k},n) \overset{Def}{=} \frac{1}{2\sqrt{\omega(\mathbf{k},n)(\omega(\mathbf{k},n)+n)}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ k_1 - \mathrm{i}k_2 \\ -\omega(\mathbf{k},n) - n - k_3 \\ k_1 - \mathrm{i}k_2 \\ \omega(\mathbf{k},n) + n - k_3 \end{bmatrix}$$

correspond to eigenvectors of $\widehat{H}_{0,4}$ with eigenvalue $-\omega(\mathbf{k},n)$.
Let

$$\widehat{H}'_{0,4} \overset{Def}{=} U^{(-)}\widehat{H}_{0,4}U^{(-)\dagger},$$
$$\underline{u}'_\mu(\mathbf{k},n) \overset{Def}{=} U^{(-)}\underline{u}_\mu(\mathbf{k},n).$$

That is

$$\underline{u}'_1(\mathbf{k},n) = \frac{1}{2\sqrt{\omega(\mathbf{k},n)(\omega(\mathbf{k},n)+n)}} \begin{bmatrix} (c+\mathrm{i}q)(\omega(\mathbf{k},n)+n+k_3) \\ (c+\mathrm{i}q)(k_1+\mathrm{i}k_2) \\ 0 \\ 0 \\ (a-\mathrm{i}b)(\omega(\mathbf{k},n)+n+k_3) \\ (a-\mathrm{i}b)(k_1+\mathrm{i}k_2) \\ \omega(\mathbf{k},n)+n-k_3 \\ -k_1-\mathrm{i}k_2 \end{bmatrix},$$

$$\underline{u}'_2(\mathbf{k},n) = \frac{1}{2\sqrt{\omega(\mathbf{k},n)(\omega(\mathbf{k},n)+n)}} \begin{bmatrix} (c+\mathrm{i}q)(k_1-\mathrm{i}k_2) \\ (c+\mathrm{i}q)(\omega(\mathbf{k},n)+n-k_3) \\ 0 \\ 0 \\ (a-\mathrm{i}b)(k_1-\mathrm{i}k_2) \\ (a-\mathrm{i}b)(\omega(\mathbf{k},n)+n-k_3) \\ -k_1+\mathrm{i}k_2 \\ \omega(\mathbf{k},n)+n+k_3 \end{bmatrix},$$

$$\underline{u}'_3(\mathbf{k},n) = \frac{1}{2\sqrt{\omega(\mathbf{k},n)(\omega(\mathbf{k},n)+n)}} \begin{bmatrix} -(c+\mathrm{i}q)(\omega(\mathbf{k},n)+n-k_3) \\ (c+\mathrm{i}q)(k_1+\mathrm{i}k_2) \\ 0 \\ 0 \\ -(a-\mathrm{i}b)(\omega(\mathbf{k},n)+n-k_3) \\ (a-\mathrm{i}b)(k_1+\mathrm{i}k_2) \\ \omega(\mathbf{k},n)+n+k_3 \\ k_1+\mathrm{i}k_2 \end{bmatrix},$$

$$\underline{u}'_4(\mathbf{k},n) = \frac{1}{2\sqrt{\omega(\mathbf{k},n)(\omega(\mathbf{k},n)+n)}} \begin{bmatrix} (c+\mathrm{i}q)(k_1-\mathrm{i}k_2) \\ -(c+\mathrm{i}q)(\omega(\mathbf{k},n)+n+k_3) \\ 0 \\ 0 \\ (a-\mathrm{i}b)(k_1-\mathrm{i}k_2) \\ -(a-\mathrm{i}b)(\omega(\mathbf{k},n)+n+k_3) \\ k_1-\mathrm{i}k_2 \\ \omega(\mathbf{k},n)+n-k_3 \end{bmatrix}.$$

Here $\underline{u}'_1(\mathbf{k},n)$ and $\underline{u}'_2(\mathbf{k},n)$ correspond to eigenvectors of $\widehat{H}'_{0,4}$ with eigenvalue $\omega(\mathbf{k},n) = \sqrt{\mathbf{k}^2+n^2}$, and $\underline{u}'_3(\mathbf{k},n)$ and $\underline{u}'_4(\mathbf{k},n)$ correspond to eigenvectors of $\widehat{H}'_{0,4}$ with eigenvalue $-\omega(\mathbf{k},n)$.

Let as in (**??**):

$$\begin{aligned}
\underline{v}_{(1)}(\mathbf{k},n) &\stackrel{Def}{=} \underline{\gamma^{[0]}}\underline{u}'_3(\mathbf{k},n), \\
\underline{v}_{(2)}(\mathbf{k},n) &\stackrel{Def}{=} \underline{\gamma^{[0]}}\underline{u}'_4(\mathbf{k},n), \\
\underline{u}_{(1)}(\mathbf{k},n) &\stackrel{Def}{=} \underline{u}'_1(\mathbf{k},n), \\
\underline{u}_{(2)}(\mathbf{k},n) &\stackrel{Def}{=} \underline{u}'_2(\mathbf{k},n).
\end{aligned}$$

Hence

$$\underline{v}_{(1)}(\mathbf{k},n) = \frac{1}{2\sqrt{\omega(\mathbf{k},n)(\omega(\mathbf{k},n)+n)}} \begin{bmatrix} 0 \\ 0 \\ -(c+\mathrm{i}q)(\omega(\mathbf{k},n)+n-k_3) \\ (c+\mathrm{i}q)(k_1+\mathrm{i}k_2) \\ \omega(\mathbf{k},n)+n+k_3 \\ k_1+\mathrm{i}k_2 \\ -(a-\mathrm{i}b)(\omega(\mathbf{k},n)+n-k_3) \\ (a-\mathrm{i}b)(k_1+\mathrm{i}k_2) \end{bmatrix}$$

and

$$\underline{v}_{(2)}(\mathbf{k},n) = \frac{1}{2\sqrt{\omega(\mathbf{k},n)(\omega(\mathbf{k},n)+n)}} \begin{bmatrix} 0 \\ 0 \\ (c+\mathrm{i}q)(k_1-\mathrm{i}k_2) \\ -(c+\mathrm{i}q)(\omega(\mathbf{k},n)+n+k_3) \\ k_1-\mathrm{i}k_2 \\ \omega(\mathbf{k},n)+n-k_3 \\ (a-\mathrm{i}b)(k_1-\mathrm{i}k_2) \\ -(a-\mathrm{i}b)(\omega(\mathbf{k},n)+n+k_3) \end{bmatrix}.$$

$\underline{u}'_{(\alpha)}(\mathbf{k},n)$ are denoted as *bi-n-leptonn* and $\underline{v}_{(\alpha)}(\mathbf{k},n)$ is are denoted as *bi-anti-n-leptonn* basic vectors with momentum $\mathbf{k}$ and spin index α.

Hence bi-anti-n-leptonn basic vectors are a result of acting of $U^{(+)}$ (4.47).
Vectors

$$
\begin{aligned}
l_{n,(1)}(\mathbf{k},n) &= \begin{bmatrix} (a-\mathrm{i}b)(\omega(\mathbf{k},n)+n+k_3) \\ (a-\mathrm{i}b)(k_1+\mathrm{i}k_2) \\ \omega(\mathbf{k},n)+n-k_3 \\ -k_1-\mathrm{i}k_2 \end{bmatrix} \text{ and} \\
l_{n,(2)}(\mathbf{k},n) &= \begin{bmatrix} (a-\mathrm{i}b)(k_1-\mathrm{i}k_2) \\ (a-\mathrm{i}b)(\omega(\mathbf{k},n)+n-k_3) \\ -k_1+\mathrm{i}k_2 \\ \omega(\mathbf{k},n)+n+k_3 \end{bmatrix}
\end{aligned}
$$

are denoted as *leptonn components* of bi-n-leptonn basic vectors, and vectors

$$
\nu_{n,(1)}(\mathbf{k},n) = \begin{bmatrix} \omega(\mathbf{k},n)+n+k_3 \\ k_1+\mathrm{i}k_2 \\ 0 \\ 0 \end{bmatrix} \text{ and } \nu_{n,(2)}(\mathbf{k},n) = \begin{bmatrix} k_1-\mathrm{i}k_2 \\ \omega(\mathbf{k},n)+n-k_3 \\ 0 \\ 0 \end{bmatrix}
$$

are denoted as *neutrinno components* of bi-n-leptonn basic vectors.
Vectors

$$
\bar{l}_{n,(1)}(\mathbf{k},n) = \begin{bmatrix} \omega(\mathbf{k},n)+n+k_3 \\ k_1+\mathrm{i}k_2 \\ -(a-\mathrm{i}b)(\omega(\mathbf{k},n)+n-k_3) \\ (a-\mathrm{i}b)(k_1+\mathrm{i}k_2) \end{bmatrix}
$$

and

$$
\bar{l}_{n,(2)}(\mathbf{k},n) = \begin{bmatrix} k_1-\mathrm{i}k_2 \\ \omega(\mathbf{k},n)+n-k_3 \\ (a-\mathrm{i}b)(k_1-\mathrm{i}k_2) \\ -(a-\mathrm{i}b)(\omega(\mathbf{k},n)+n+k_3) \end{bmatrix}
$$

are denoted as *leptonn components* of anti-bi-n-leptonn basic vectors, and vectors

$$
\bar{\nu}_{n,(1)}(\mathbf{k},n) = \begin{bmatrix} 0 \\ 0 \\ -(\omega(\mathbf{k},n)+n-k_3) \\ k_1+\mathrm{i}k_2 \end{bmatrix}
$$

and

$$
\bar{\nu}_{n,(2)}(\mathbf{k},n) = \begin{bmatrix} 0 \\ 0 \\ k_1-\mathrm{i}k_2 \\ -(\omega(\mathbf{k},n)+n+k_3) \end{bmatrix}
$$

are denoted as *neutrinno components* of anti-bi-n-leptonn basic vectors.

Vectors

$$\nu_{n,(1)}(\mathbf{k},n)=\left[\begin{array}{c}\omega(\mathbf{k},n)+n+k_3\\ k_1+\mathrm{i}k_2\\ 0\\ 0\end{array}\right] \text{ and } \nu_{n,(2)}(\mathbf{k},n)=\left[\begin{array}{c}k_1-\mathrm{i}k_2\\ \omega(\mathbf{k},n)+n-k_3\\ 0\\ 0\end{array}\right]$$

$$\nu_{n,(3)}(\mathbf{k},n)=\left[\begin{array}{c}-(n+\omega(\mathbf{k},n)-k_3)\\ (k_1+ik_2)\\ 0\\ 0\end{array}\right],\ \nu_{n,(4)}(\mathbf{k},n)=\left[\begin{array}{c}(k_1-ik_2)\\ -(n+\omega(\mathbf{k},n)+k_3)\\ 0\\ 0\end{array}\right]$$

are denoted as *neutrinno components* of bi-n-leptonn basic vectors. These vectors form a linear space of functions of the type

$$\varphi:=\left[\begin{array}{c}\varphi_1\\ \varphi_2\\ 0\\ 0\end{array}\right].$$

Because

$$\begin{aligned}\rho &= \sum_{s=1}^{4}\varphi_s^*\varphi_s,\\ \frac{j_\alpha}{\mathrm{c}} &= -\sum_{k=1}^{4}\sum_{s=1}^{4}\varphi_s^*\beta_{s,k}^{[\alpha]}\varphi_k\end{aligned} \tag{4.75}$$

then

$$\begin{aligned}\rho &= \varphi^\dagger\varphi=\varphi_1\varphi_1^*+\varphi_2\varphi_2^*,\\ \frac{j_1}{\mathrm{c}} &= -\varphi^\dagger\beta^{[1]}\varphi=-(\varphi_1\varphi_2^*+\varphi_2\varphi_1^*),\\ \frac{j_2}{\mathrm{c}} &= -\varphi^\dagger\beta^{[2]}\varphi=-\mathrm{i}(\varphi_1\varphi_2^*-\varphi_2\varphi_1^*),\\ \frac{j_3}{\mathrm{c}} &= -\varphi^\dagger\beta^{[3]}\varphi==-(\varphi_1\varphi_1^*-\varphi_2\varphi_2^*).\end{aligned}$$

Because velocities:

$$\begin{aligned}u_1 &= \frac{j_1}{\rho}=\frac{-\mathrm{c}(\varphi_1\varphi_2^*+\varphi_2\varphi_1^*)}{\varphi_1\varphi_1^*+\varphi_2\varphi_2^*},\\ u_2 &= \frac{j_2}{\rho}=\frac{-\mathrm{ci}(\varphi_1\varphi_2^*-\varphi_2\varphi_1^*)}{\varphi_1\varphi_1^*+\varphi_2\varphi_2^*},\\ u_3 &= \frac{j_3}{\rho}=\frac{-\mathrm{c}(\varphi_1\varphi_1^*-\varphi_2\varphi_2^*)}{\varphi_1\varphi_1^*+\varphi_2\varphi_2^*}.\end{aligned}$$

then

$$u_1^2 + u_1^2 + u_1^2 = c^2.$$

Hence, the neutrimo velocity eqaual to the light velocity.

———————————————————————————————————————

—-

Because the neutrino mass:

$$m = \varphi^\dagger \left(\beta^{[4]} n_0 + \gamma^{[0]} s_0 \right) \varphi.$$

and

$$\varphi^\dagger \beta^{[4]} \varphi = 0 \text{ and } \varphi^\dagger \gamma^{[0]} \varphi = 0.$$

then $m = 0$.

Hence, a neutrino has ZERO mass.

————————————————————————-

Because the electromagnetic potencial:

$$\widehat{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

then :

$$\begin{aligned} \widehat{A} \underline{u}'_1 (\mathbf{k}, n) &= \underline{u}_1 (\mathbf{k}, n), \\ \widehat{A} \underline{u}'_2 (\mathbf{k}, n) &= \underline{u}_2 (\mathbf{k}, n), \\ \widehat{A} \underline{u}'_3 (\mathbf{k}, n) &= \underline{u}_3 (\mathbf{k}, n), \\ \widehat{A} \underline{u}'_4 (\mathbf{k}, n) &= \underline{u}_4 (\mathbf{k}, n). \end{aligned}$$

Hence, the neutrino does not interact with the electomagnetic field.

———————————————————————————————————————

Linear space of vectors $\nu_{n,(s)} (\mathbf{k}, n)$ can be representated as linear space of vectors $\nu_{m,(s)} (\mathbf{k}, m)$. Therefore, neitrinos of lepton with mass n oscillirues on neutrinos of lepton with mass m.

Therefore,

A netrino is derivative from a lepton under some unitar transformation.

the neutrino does not interact with the electomagnetic field,

a neutrino has ZERO mass,

the neutrimo velocity eqaual to the light velocity,

neitrinos of lepton with mass n oscillirues on neutrinos of lepton with mass m.

4.15. Quarks and Gluons

The quark model was independently proposed by physicists Murray Gell-Mann[38] and George Zweig[39] in 1964.

The first direct experimental evidence of gluons was found in 1979 when three-jet events were observed at t he electron-positron collider PETRA. However, just before PETRA[40] appeared on the scene, the PLUTO experiment at DORIS[41] showed event topologies suggestive of a three-gluon decay.

The following part of (4.14):

$$\left(\begin{array}{c}\sum_{k=0}^{3}\beta^{[k]}\left(-\mathrm{i}\partial_k+\Theta_k+\Upsilon_k\gamma^{[5]}\right)-\\ -M_{\zeta,0}\gamma_{\zeta}^{[0]}+M_{\zeta,4}\zeta^{[4]}+\\ -M_{\eta,0}\gamma_{\eta}^{[0]}-M_{\eta,4}\eta^{[4]}+\\ +M_{\theta,0}\gamma_{\theta}^{[0]}+M_{\theta,4}\theta^{[4]}\end{array}\right)\varphi=0. \tag{4.76}$$

is called *the chromatic equation of moving*.

Here (2.111), (2.113), (2.115):

$$\gamma_{\zeta}^{[0]}=-\begin{bmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{bmatrix},\ \zeta^{[4]}=\begin{bmatrix}0&0&0&\mathrm{i}\\0&0&\mathrm{i}&0\\0&-\mathrm{i}&0&0\\-\mathrm{i}&0&0&0\end{bmatrix}$$

are mass elements of red pentad;

$$\gamma_{\eta}^{[0]}=\begin{bmatrix}0&0&0&\mathrm{i}\\0&0&-\mathrm{i}&0\\0&\mathrm{i}&0&0\\-\mathrm{i}&0&0&0\end{bmatrix},\ \eta^{[4]}=\begin{bmatrix}0&0&0&1\\0&0&-1&0\\0&-1&0&0\\1&0&0&0\end{bmatrix}$$

[38]Murray Gell-Mann (born September 15, 1929) is an American physicist and linguist

[39]George Zweig (born on May 30, 1937 in Moscow, Russia into a Jewish family) was originally trained as a particle physicist under Richard Feynman and later turned his attention to neurobiology. He spent a number of years as a Research Scientist at Los Alamos National Laboratory and MIT, but as of 2004, has gone on to work in the financial services industry.

[40]PETRA (or the Positron-Electron Tandem Ring Accelerator) is one of the particle accelerators at DESY in Hamburg, Germany.

[41]DORIS (Doppel-Ring-Speicher, "double-ring storage"), built between 1969 and 1974, was DESY's second circular accelerator and its first storage ring with a circumference of nearly 300 m.

are mass elements of green pentad;

$$\gamma_{\theta}^{[0]} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad \theta^{[4]} = \begin{bmatrix} 0 & 0 & -\mathrm{i} & 0 \\ 0 & 0 & 0 & \mathrm{i} \\ -\mathrm{i} & 0 & 0 & 0 \\ 0 & \mathrm{i} & 0 & 0 \end{bmatrix}$$

are mass elements of blue pentad.

I call:

- $M_{\zeta,0}$, $M_{\zeta,4}$ *red lower and upper mass members*;
- $M_{\eta,0}$, $M_{\eta,4}$ *green lower and upper mass members*;
- $M_{\theta,0}$, $M_{\theta,4}$ *blue lower and upper mass members*.

The mass members of this equation form the following matrix sum:

$$\widehat{M} := \begin{pmatrix} -M_{\zeta,0}\gamma_{\zeta}^{[0]} + M_{\zeta,4}\zeta^{[4]} - \\ -M_{\eta,0}\gamma_{\eta}^{[0]} - M_{\eta,4}\eta^{[4]} + \\ +M_{\theta,0}\gamma_{\theta}^{[0]} + M_{\theta,4}\theta^{[4]} \end{pmatrix} =$$

$$= \begin{bmatrix} 0 & 0 & -M_{\theta,0} & M_{\zeta,\eta,0} \\ 0 & 0 & M_{\zeta,\eta,0}^{*} & M_{\theta,0} \\ -M_{\theta,0} & M_{\zeta,\eta,0} & 0 & 0 \\ M_{\zeta,\eta,0}^{*} & M_{\theta,0} & 0 & 0 \end{bmatrix} + \mathrm{i} \begin{bmatrix} 0 & 0 & M_{\theta,4} & M_{\zeta,\eta,4}^{*} \\ 0 & 0 & M_{\zeta,\eta,4} & -M_{\theta,4} \\ -M_{\theta,4} & -M_{\zeta,\eta,4}^{*} & 0 & 0 \\ -M_{\zeta,\eta,4} & M_{\theta,4} & 0 & 0 \end{bmatrix}$$

with $M_{\zeta,\eta,0} := M_{\zeta,0} - \mathrm{i}M_{\eta,0}$ and $M_{\zeta,\eta,4} := M_{\zeta,4} - \mathrm{i}M_{\eta,4}$.

Elements of these matrices can be turned by formula of shape:

$$\begin{bmatrix} \cos\frac{\theta}{2} & \mathrm{i}\sin\frac{\theta}{2} \\ \mathrm{i}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} \begin{bmatrix} Z & X - \mathrm{i}Y \\ X + \mathrm{i}Y & -Z \end{bmatrix} \begin{bmatrix} \cos\frac{\theta}{2} & -\mathrm{i}\sin\frac{\theta}{2} \\ -\mathrm{i}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} =$$

$$= \begin{bmatrix} Z\cos\theta - Y\sin\theta & X - \mathrm{i}\begin{pmatrix} Y\cos\theta \\ +Z\sin\theta \end{pmatrix} \\ X + \mathrm{i}\begin{pmatrix} Y\cos\theta \\ +Z\sin\theta \end{pmatrix} & -Z\cos\theta + Y\sin\theta \end{bmatrix}.$$

Hence, if:

$$U_{2,3}(\alpha) := \begin{bmatrix} \cos\alpha & \mathrm{i}\sin\alpha & 0 & 0 \\ \mathrm{i}\sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & \cos\alpha & \mathrm{i}\sin\alpha \\ 0 & 0 & \mathrm{i}\sin\alpha & \cos\alpha \end{bmatrix}$$

and

$$\widehat{M}' := \begin{pmatrix} -M'_{\zeta,0}\gamma_\zeta^{[0]} + M'_{\zeta,4}\zeta^{[4]} - \\ -M'_{\eta,0}\gamma_\eta^{[0]} - M'_{\eta,4}\eta^{[4]} + \\ +M'_{\theta,0}\gamma_\theta^{[0]} + M'_{\theta,4}\theta^{[4]} \end{pmatrix} := U_{2,3}^{\dagger}(\alpha)\,\widehat{M}U_{2,3}(\alpha)$$

then

$$\begin{aligned}
&M'_{\zeta,0} = M_{\zeta,0}\,,\\
&M'_{\eta,0} = M_{\eta,0}\cos 2\alpha + M_{\theta,0}\sin 2\alpha\,,\\
&M'_{\theta,0} = M_{\theta,0}\cos 2\alpha - M_{\eta,0}\sin 2\alpha\,,\\
&M'_{\zeta,4} = M_{\zeta,4}\,,\\
&M'_{\eta,4} = M_{\eta,4}\cos 2\alpha + M_{\theta,4}\sin 2\alpha\,,\\
&M'_{\theta,4} = M_{\theta,4}\cos 2\alpha - M_{\eta,4}\sin 2\alpha\,.
\end{aligned}$$

Therefore, matrix $U_{2,3}(\alpha)$ makes an oscillation between green and blue chromatics.

Let us consider equation (4.14) under transformation $U_{2,3}(\alpha)$ where α is an arbitrary real function of time-space variables ($\alpha = \alpha(t, x_1, x_2, x_3)$):

$$\begin{aligned}
&U_{2,3}^{\dagger}(\alpha)\left(\frac{1}{\mathrm{c}}\partial_t + \mathrm{i}\Theta_0 + \mathrm{i}\Upsilon_0\gamma^{[5]}\right)U_{2,3}(\alpha)\varphi = \\
&= U_{2,3}^{\dagger}(\alpha)\begin{pmatrix} \sum\limits_{\nu=1}^{3}\beta^{[\nu]}\left(\partial_\nu + \mathrm{i}\Theta_\nu + \mathrm{i}\Upsilon_\nu\gamma^{[5]}\right) + \\ +\mathrm{i}M_0\gamma^{[0]} + \mathrm{i}M_4\beta^{[4]} + \widehat{M} \end{pmatrix} U_{2,3}(\alpha)\varphi\,.
\end{aligned}$$

Because

$$\begin{gathered}
U_{2,3}^{\dagger}(\alpha)\,U_{2,3}(\alpha) = 1_4\,,\\
U_{2,3}^{\dagger}(\alpha)\,\gamma^{[5]}U_{2,3}(\alpha) = \gamma^{[5]}\,,\\
U_{2,3}^{\dagger}(\alpha)\,\gamma^{[0]}U_{2,3}(\alpha) = \gamma^{[0]}\,,\\
U_{2,3}^{\dagger}(\alpha)\,\beta^{[4]}U_{2,3}(\alpha) = \beta^{[4]}\,,\\
U_{2,3}^{\dagger}(\alpha)\,\beta^{[1]} = \beta^{[1]}U_{2,3}^{\dagger}(\alpha)\,,\\
U_{2,3}^{\dagger}(\alpha)\,\beta^{[2]} = \left(\beta^{[2]}\cos 2\alpha + \beta^{[3]}\sin 2\alpha\right)U_{2,3}^{\dagger}(\alpha)\,,\\
U_{2,3}^{\dagger}(\alpha)\,\beta^{[3]} = \left(\beta^{[3]}\cos 2\alpha - \beta^{[2]}\sin 2\alpha\right)U_{2,3}^{\dagger}(\alpha)\,,
\end{gathered}$$

then

$$\left(\frac{1}{\mathrm{c}}\partial_t+U_{2,3}^{\dagger}(\alpha)\frac{1}{\mathrm{c}}\partial_t U_{2,3}(\alpha)+\mathrm{i}\Theta_0+\mathrm{i}\Upsilon_0\gamma^{[5]}\right)\varphi=$$

$$=\left(\begin{array}{c}\beta^{[1]}\left(\partial_1+U_{2,3}^{\dagger}(\alpha)\partial_1 U_{2,3}(\alpha)+\mathrm{i}\Theta_1+\mathrm{i}\Upsilon_1\gamma^{[5]}\right)+\\ +\beta^{[2]}\left(\begin{array}{c}(\cos 2\alpha\cdot\partial_2-\sin 2\alpha\cdot\partial_3)\\ +U_{2,3}^{\dagger}(\alpha)(\cos 2\alpha\cdot\partial_2-\sin 2\alpha\cdot\partial_3)U_{2,3}(\alpha)\\ +\mathrm{i}(\Theta_2\cos 2\alpha-\Theta_3\sin 2\alpha)\\ +\mathrm{i}\left(\Upsilon_2\gamma^{[5]}\cos 2\alpha-\Upsilon_3\gamma^{[5]}\sin 2\alpha\right)\end{array}\right)\\ +\beta^{[3]}\left(\begin{array}{c}(\cos 2\alpha\cdot\partial_3+\sin 2\alpha\cdot\partial_2)\\ +U_{2,3}^{\dagger}(\alpha)(\cos 2\alpha\cdot\partial_3+\sin 2\alpha\cdot\partial_2)U_{2,3}(\alpha)\\ +\mathrm{i}(\Theta_2\sin 2\alpha+\Theta_3\cos 2\alpha)\\ +\mathrm{i}\left(\Upsilon_3\gamma^{[5]}\cos 2\alpha+\Upsilon_2\gamma^{[5]}\sin 2\alpha\right)\end{array}\right)\\ +\mathrm{i}M_0\gamma^{[0]}+\mathrm{i}M_4\beta^{[4]}+\widehat{M}'\end{array}\right)\varphi. \tag{4.77}$$

Let x_2' and x_3' be elements of other coordinate system such that 2.118:

$$\begin{aligned}\partial_2' &:= (\cos 2\alpha\cdot\partial_2-\sin 2\alpha\cdot\partial_3),\\ \partial_3' &:= (\cos 2\alpha\cdot\partial_3+\sin 2\alpha\cdot\partial_2).\end{aligned} \tag{4.78}$$

Therefore, from (4.77):

$$\left(\frac{1}{\mathrm{c}}\partial_t+U_{2,3}^{\dagger}(\alpha)\frac{1}{\mathrm{c}}\partial_t U_{2,3}(\alpha)+\mathrm{i}\Theta_0+\mathrm{i}\Upsilon_0\gamma^{[5]}\right)\varphi=$$

$$=\left(\begin{array}{c}\beta^{[1]}\left(\partial_1+U_{2,3}^{\dagger}(\alpha)\partial_1 U_{2,3}(\alpha)+\mathrm{i}\Theta_1+\mathrm{i}\Upsilon_1\gamma^{[5]}\right)\\ +\beta^{[2]}\left(\partial_2'+U_{2,3}^{\dagger}(\alpha)\partial_2' U_{2,3}(\alpha)+\mathrm{i}\Theta_2'+\mathrm{i}\Upsilon_2'\gamma^{[5]}\right)\\ +\beta^{[3]}\left(\partial_3'+U_{2,3}^{\dagger}(\alpha)\partial_3' U_{2,3}(\alpha)+\mathrm{i}\Theta_3'+\mathrm{i}\Upsilon_3'\gamma^{[5]}\right)\\ +\mathrm{i}M_0\gamma^{[0]}+\mathrm{i}M_4\beta^{[4]}+\widehat{M}'\end{array}\right)\varphi.$$

with

$$\begin{aligned}\Theta_2' &:= \Theta_2\cos 2\alpha-\Theta_3\sin 2\alpha,\\ \Theta_3' &:= \Theta_2\sin 2\alpha+\Theta_3\cos 2\alpha,\\ \Upsilon_2' &:= \Upsilon_2\cos 2\alpha-\Upsilon_3\sin 2\alpha,\\ \Upsilon_3' &:= \Upsilon_3\cos 2\alpha+\Upsilon_2\sin 2\alpha.\end{aligned}$$

Therefore, the oscillation between blue and green chromatics curves the space in the x_2, x_3 directions.

Similarly, matrix

$$U_{1,3}(\vartheta):=\left[\begin{array}{cccc}\cos\vartheta & \sin\vartheta & 0 & 0\\ -\sin\vartheta & \cos\vartheta & 0 & 0\\ 0 & 0 & \cos\vartheta & \sin\vartheta\\ 0 & 0 & -\sin\vartheta & \cos\vartheta\end{array}\right]$$

with an arbitrary real function $\vartheta(t,x_1,x_2,x_3)$ describes the oscillation between blue and red chromatics which curves the space in the x_1, x_3 directions. And matrix

$$U_{1,2}(\varsigma) := \left[\begin{array}{cccc} e^{-\mathrm{i}\varsigma} & 0 & 0 & 0 \\ 0 & e^{\mathrm{i}\varsigma} & 0 & 0 \\ 0 & 0 & e^{-\mathrm{i}\varsigma} & 0 \\ 0 & 0 & 0 & e^{\mathrm{i}\varsigma} \end{array}\right]$$

with an arbitrary real function $\varsigma(t,x_1,x_2,x_3)$ describes the oscillation between green and red chromatics which curves the space in the x_1, x_2 directions.

Now, let

$$U_{0,1}(\sigma) := \left[\begin{array}{cccc} \cosh\sigma & -\sinh\sigma & 0 & 0 \\ -\sinh\sigma & \cosh\sigma & 0 & 0 \\ 0 & 0 & \cosh\sigma & \sinh\sigma \\ 0 & 0 & \sinh\sigma & \cosh\sigma \end{array}\right].$$

and

$$\widehat{M}'' := \left(\begin{array}{c} -M''_{\zeta,0}\gamma_{\zeta}^{[0]} + M''_{\zeta,4}\zeta^{[4]} - \\ -M''_{\eta,0}\gamma_{\eta}^{[0]} - M''_{\eta,4}\eta^{[4]} + \\ +M''_{\theta,0}\gamma_{\theta}^{[0]} + M''_{\theta,4}\theta^{[4]} \end{array}\right) := U_{0,1}^{\dagger}(\sigma)\widehat{M}U_{0,1}(\sigma)$$

then:

$$\begin{aligned}
M''_{\zeta,0} &= M_{\zeta,0}\,, \\
M''_{\eta,0} &= (M_{\eta,0}\cosh 2\sigma - M_{\theta,4}\sinh 2\sigma)\,, \\
M''_{\theta,0} &= M_{\theta,0}\cosh 2\sigma + M_{\eta,4}\sinh 2\sigma\,, \\
M''_{\zeta,4} &= M_{\zeta,4}\,, \\
M''_{\eta,4} &= M_{\eta,4}\cosh 2\sigma + M_{\theta,0}\sinh 2\sigma\,, \\
M''_{\theta,4} &= M_{\theta,4}\cosh 2\sigma - M_{\eta,0}\sinh 2\sigma\,.
\end{aligned}$$

Therefore, matrix $U_{0,1}(\sigma)$ makes an oscillation between green and blue chromatics with an oscillation between upper and lower mass members.

Let us consider equation (4.14) under transformation $U_{0,1}(\sigma)$ where σ is an arbitrary real function of time-space variables ($\sigma = \sigma(t,x_1,x_2,x_3)$):

$$\begin{aligned}
&U_{0,1}^{\dagger}(\sigma)\left(\frac{1}{\mathrm{c}}\partial_t + \mathrm{i}\Theta_0 + \mathrm{i}\Upsilon_0\gamma^{[5]}\right)U_{0,1}(\sigma)\varphi = \\
&= U_{0,1}^{\dagger}(\sigma)\left(\begin{array}{c} \sum\limits_{\nu=1}^{3}\beta^{[\nu]}\left(\partial_\nu + \mathrm{i}\Theta_\nu + \mathrm{i}\Upsilon_\nu\gamma^{[5]}\right) + \\ +\mathrm{i}M_0\gamma^{[0]} + \mathrm{i}M_4\beta^{[4]} + \widehat{M} \end{array}\right)U_{0,1}(\sigma)\varphi\,.
\end{aligned}$$

Since:

$$
\begin{aligned}
&U_{0,1}^{\dagger}(\sigma)\,U_{0,1}(\sigma) = \left(\cosh 2\sigma - \beta^{[1]}\sinh 2\sigma\right),\\
&U_{0,1}^{\dagger}(\sigma) = \left(\cosh 2\sigma + \beta^{[1]}\sinh 2\sigma\right) U_{0,1}^{-1}(\sigma),\\
&U_{0,1}^{\dagger}(\sigma)\,\beta^{[1]} = \left(\beta^{[1]}\cosh 2\sigma - \sinh 2\sigma\right) U_{0,1}^{-1}(\sigma),\\
&U_{0,1}^{\dagger}(\sigma)\,\beta^{[2]} = \beta^{[2]} U_{0,1}^{-1}(\sigma),\\
&U_{0,1}^{\dagger}(\sigma)\,\beta^{[3]} = \beta^{[3]} U_{0,1}^{-1}(\sigma),\\
&U_{0,1}^{\dagger}(\sigma)\,\gamma^{[0]} U_{0,1}(\sigma) = \gamma^{[0]},\\
&U_{0,1}^{\dagger}(\sigma)\,\beta^{[4]} U_{0,1}(\sigma) = \beta^{[4]},\\
&U_{0,1}^{-1}(\sigma)\,U_{0,1}(\sigma) = 1_4,\\
&U_{0,1}^{-1}(\sigma)\,\gamma^{[5]} U_{0,1}(\sigma) = \gamma^{[5]},\\
&U_{0,1}^{\dagger}(\sigma)\,\gamma^{[5]} U_{0,1}(\sigma) = \gamma^{[5]}\left(\cosh 2\sigma - \beta^{[1]}\sinh 2\sigma\right),
\end{aligned}
$$

then

$$
\left(
\begin{array}{c}
U_{0,1}^{-1}(\sigma)\left(\cosh 2\sigma \cdot \frac{1}{\mathrm{c}}\partial_t + \sinh 2\sigma \cdot \partial_1\right) U_{0,1}(\sigma)\\
+\left(\cosh 2\sigma \cdot \frac{1}{\mathrm{c}}\partial_t + \sinh 2\sigma \cdot \partial_1\right)\\
+\mathrm{i}\left(\Theta_0 \cosh 2\sigma + \Theta_1 \sinh 2\sigma\right)\\
+\mathrm{i}\left(\Upsilon_0 \cosh 2\sigma + \sinh 2\sigma \cdot \Upsilon_1\right)\gamma^{[5]} -\\
-\beta^{[1]}\left(
\begin{array}{c}
U_{0,1}^{-1}(\sigma)\left(\cosh 2\sigma \cdot \partial_1 + \sinh 2\sigma \cdot \frac{1}{\mathrm{c}}\partial_t\right) U_{0,1}(\sigma)\\
+\left(\cosh 2\sigma \cdot \partial_1 + \sinh 2\sigma \cdot \frac{1}{\mathrm{c}}\partial_t\right)\\
+\mathrm{i}\left(\Theta_1 \cosh 2\sigma + \Theta_0 \sinh 2\sigma\right)\\
+\mathrm{i}\left(\Upsilon_1 \cosh 2\sigma + \Upsilon_0 \sinh 2\sigma\right)\gamma^{[5]}
\end{array}
\right)\\
-\beta^{[2]}\left(\partial_2 + U_{0,1}^{-1}(\sigma)\left(\partial_2 U_{0,1}(\sigma)\right) + \mathrm{i}\Theta_2 + \mathrm{i}\Upsilon_2\gamma^{[5]}\right)\\
-\beta^{[3]}\left(\partial_3 + U_{0,1}^{-1}(\sigma)\left(\partial_3 U_{0,1}(\sigma)\right) + \mathrm{i}\Theta_3 + \mathrm{i}\Upsilon_3\gamma^{[5]}\right)\\
-\mathrm{i}M_0\gamma^{[0]} - \mathrm{i}M_4\beta^{[4]} - \widehat{M}''
\end{array}
\right)\varphi = 0. \qquad (4.79)
$$

Let t' and x'_1 be elements of other coordinate system such that:

$$\left.\begin{array}{l}\dfrac{\partial x_1}{\partial x'_1}=\cosh 2\sigma \\ \dfrac{\partial t}{\partial x'_1}=\dfrac{1}{\mathrm{c}}\sinh 2\sigma \\ \dfrac{\partial x_1}{\partial t'}=\mathrm{c}\sinh 2\sigma \\ \dfrac{\partial t}{\partial t'}=\cosh 2\sigma \\ \dfrac{\partial x_2}{\partial t'}=\dfrac{\partial x_3}{\partial t'}=\dfrac{\partial x_2}{\partial x'_1}=\dfrac{\partial x_3}{\partial x'_1}=0\end{array}\right\}. \tag{4.80}$$

Hence:

$$\begin{aligned}\partial'_t &:= \frac{\partial}{\partial t'}=\frac{\partial}{\partial t}\frac{\partial t}{\partial t'}+\frac{\partial}{\partial x_1}\frac{\partial x_1}{\partial t'}+\frac{\partial}{\partial x_2}\frac{\partial x_2}{\partial t'}+\frac{\partial}{\partial x_3}\frac{\partial x_3}{\partial t'}= \\ &=\cosh 2\sigma\cdot\frac{\partial}{\partial t}+\mathrm{c}\sinh 2\sigma\cdot\frac{\partial}{\partial x_1}= \\ &=\cosh 2\sigma\cdot\partial_t+\mathrm{c}\sinh 2\sigma\cdot\partial_1\,,\end{aligned}$$

that is

$$\frac{1}{\mathrm{c}}\partial'_t=\frac{1}{\mathrm{c}}\cosh 2\sigma\cdot\partial_t+\sinh 2\sigma\cdot\partial_1$$

and

$$\begin{aligned}\partial'_1 &:= \frac{\partial}{\partial x'_1}= \\ &=\frac{\partial}{\partial t}\frac{\partial t}{\partial x'_1}+\frac{\partial}{\partial x_1}\frac{\partial x_1}{\partial x'_1}+\frac{\partial}{\partial x_2}\frac{\partial x_2}{\partial x'_1}+\frac{\partial}{\partial x_3}\frac{\partial x_3}{\partial x'_1}= \\ &=\cosh 2\sigma\cdot\frac{\partial}{\partial x_1}+\sinh 2\sigma\cdot\frac{1}{\mathrm{c}}\frac{\partial}{\partial t}= \\ &=\cosh 2\sigma\cdot\partial_1+\sinh 2\sigma\cdot\frac{1}{\mathrm{c}}\partial_t\,.\end{aligned}$$

Therefore, from (4.79):

$$\left(\begin{array}{c}\beta^{[0]}\left(\frac{1}{\mathrm{c}}\partial'_t+U_{0,1}^{-1}(\sigma)\frac{1}{\mathrm{c}}\partial'_t U_{0,1}(\sigma)+\mathrm{i}\Theta''_0+\mathrm{i}\Upsilon''_0\gamma^{[5]}\right) \\ +\beta^{[1]}\left(\partial'_1+U_{0,1}^{-1}(\sigma)\partial'_1 U_{0,1}(\sigma)+\mathrm{i}\Theta''_1+\mathrm{i}\Upsilon''_1\gamma^{[5]}\right) \\ +\beta^{[2]}\left(\partial_2+U_{0,1}^{-1}(\sigma)\partial_2 U_{0,1}(\sigma)+\mathrm{i}\Theta_2+\mathrm{i}\Upsilon_2\gamma^{[5]}\right) \\ +\beta^{[3]}\left(\partial_3+U_{0,1}^{-1}(\sigma)\partial_3 U_{0,1}(\sigma)+\mathrm{i}\Theta_3+\mathrm{i}\Upsilon_3\gamma^{[5]}\right) \\ +\mathrm{i}M_0\gamma^{[0]}+\mathrm{i}M_4\beta^{[4]}+\widehat{M}''\end{array}\right)\varphi=0$$

with

$$\begin{aligned}
\Theta_0'' &:= \Theta_0 \cosh 2\sigma + \Theta_1 \sinh 2\sigma\,, \\
\Theta_1'' &:= \Theta_1 \cosh 2\sigma + \Theta_0 \sinh 2\sigma\,, \\
\Upsilon_0'' &:= \Upsilon_0 \cosh 2\sigma + \sinh 2\sigma \cdot \Upsilon_1\,, \\
\Upsilon_1'' &:= \Upsilon_1 \cosh 2\sigma + \Upsilon_0 \sinh 2\sigma\,.
\end{aligned}$$

Therefore, the oscillation between blue and green chromatics with the oscillation between upper and lower mass members curves the space in the t, x_1 directions.

Similarly, matrix

$$U_{0,2}(\phi) := \begin{bmatrix} \cosh\phi & \mathrm{i}\sinh\phi & 0 & 0 \\ -\mathrm{i}\sinh\phi & \cosh\phi & 0 & 0 \\ 0 & 0 & \cosh\phi & -\mathrm{i}\sinh\phi \\ 0 & 0 & \mathrm{i}\sinh\phi & \cosh\phi \end{bmatrix}$$

with an arbitrary real function $\phi(t,x_1,x_2,x_3)$ describes the oscillation between blue and red chromatics with the oscillation between upper and lower mass members curves the space in the t, x_2 directions. And matrix

$$U_{0,3}(\iota) := \begin{bmatrix} e^{\iota} & 0 & 0 & 0 \\ 0 & e^{-\iota} & 0 & 0 \\ 0 & 0 & e^{-\iota} & 0 \\ 0 & 0 & 0 & e^{\iota} \end{bmatrix}$$

with an arbitrary real function $\iota(t,x_1,x_2,x_3)$ describes the oscillation between green and red chromatics with the oscillation between upper and lower mass members curves the space in the t, x_3 directions.

Now let

$$\widetilde{U}(\chi) := \begin{bmatrix} e^{\mathrm{i}\chi} & 0 & 0 & 0 \\ 0 & e^{\mathrm{i}\chi} & 0 & 0 \\ 0 & 0 & e^{2\mathrm{i}\chi} & 0 \\ 0 & 0 & 0 & e^{2\mathrm{i}\chi} \end{bmatrix}$$

and

$$\widehat{M}' := \begin{pmatrix} -M'_{\zeta,0}\gamma_\zeta^{[0]} + M'_{\zeta,4}\zeta^{[4]} - \\ -M'_{\eta,0}\gamma_\eta^{[0]} - M'_{\eta,4}\eta^{[4]} + \\ +M'_{\theta,0}\gamma_\theta^{[0]} + M'_{\theta,4}\theta^{[4]} \end{pmatrix} := \widetilde{U}^\dagger(\chi)\widehat{M}\widetilde{U}(\chi)$$

then:

$$\begin{aligned}
M'_{\zeta,0} &= \left(M_{\zeta,0}\cos\chi - M_{\zeta,4}\sin\chi\right), \\
M'_{\zeta,4} &= \left(M_{\zeta,4}\cos\chi + M_{\zeta,0}\sin\chi\right), \\
M'_{\eta,4} &= \left(M_{\eta,4}\cos\chi - M_{\eta,0}\sin\chi\right), \\
M'_{\eta,0} &= \left(M_{\eta,0}\cos\chi + M_{\eta,4}\sin\chi\right), \\
M'_{\theta,0} &= \left(M_{\theta,0}\cos\chi + M_{\theta,4}\sin\chi\right), \\
M'_{\theta,4} &= \left(M_{\theta,4}\cos\chi - M_{\theta,0}\sin\chi\right).
\end{aligned}$$

Therefore, matrix $\widetilde{U}(\chi)$ makes an oscillation between upper and lower mass members.

Let us consider equation (4.76) under transformation $\widetilde{U}(\chi)$ where χ is an arbitrary real function of time-space variables ($\chi = \chi(t, x_1, x_2, x_3)$):

$$\begin{aligned}
&\widetilde{U}^{\dagger}(\chi)\left(\frac{1}{\mathrm{c}}\partial_t + \mathrm{i}\Theta_0 + \mathrm{i}\Upsilon_0\gamma^{[5]}\right)\widetilde{U}(\chi)\varphi = \\
&= \widetilde{U}^{\dagger}(\chi)\left(\sum_{\nu=1}^{3}\beta^{[\nu]}\left(\partial_\nu + \mathrm{i}\Theta_\nu + \mathrm{i}\Upsilon_\nu\gamma^{[5]}\right) + \widehat{M}\right)\widetilde{U}(\chi)\varphi.
\end{aligned}$$

Because

$$\gamma^{[5]}\widetilde{U}(\chi) = \widetilde{U}(\chi)\gamma^{[5]},$$

$$\begin{aligned}
\beta^{[1]}\widetilde{U}(\chi) &= \widetilde{U}(\chi)\beta^{[1]},\\
\beta^{[2]}\widetilde{U}(\chi) &= \widetilde{U}(\chi)\beta^{[2]},\\
\beta^{[3]}\widetilde{U}(\chi) &= \widetilde{U}(\chi)\beta^{[3]},
\end{aligned}$$

$$\widetilde{U}^{\dagger}(\chi)\widetilde{U}(\chi) = 1_4,$$

then

$$\begin{aligned}
&\left(\frac{1}{\mathrm{c}}\partial_t + \frac{1}{\mathrm{c}}\widetilde{U}^{\dagger}(\chi)\left(\partial_t\widetilde{U}(\chi)\right) + \mathrm{i}\Theta_0 + \mathrm{i}\Upsilon_0\gamma^{[5]}\right)\varphi = \\
&= \left(\begin{array}{c} \sum_{\nu=1}^{3}\beta^{[\nu]}\left(\partial_\nu + \widetilde{U}^{\dagger}(\chi)\left(\partial_\nu\widetilde{U}(\chi)\right) + \mathrm{i}\Theta_\nu + \mathrm{i}\Upsilon_\nu\gamma^{[5]}\right) \\ + \widetilde{U}^{\dagger}(\chi)\widehat{M}\widetilde{U}(\chi) \end{array}\right)\varphi.
\end{aligned}$$

Now let:

$$\widehat{U}(\kappa) := \left[\begin{array}{cccc} e^{\kappa} & 0 & 0 & 0 \\ 0 & e^{\kappa} & 0 & 0 \\ 0 & 0 & e^{2\kappa} & 0 \\ 0 & 0 & 0 & e^{2\kappa} \end{array}\right]$$

and

$$\widehat{M}' := \left(\begin{array}{c} -M'_{\zeta,0}\gamma_{\zeta}^{[0]} + M'_{\zeta,4}\zeta^{[4]} - \\ -M'_{\eta,0}\gamma_{\eta}^{[0]} - M'_{\eta,4}\eta^{[4]} + \\ + M'_{\theta,0}\gamma_{\theta}^{[0]} + M'_{\theta,4}\theta^{[4]} \end{array}\right) := \widehat{U}^{-1}(\kappa)\widehat{M}\widehat{U}(\kappa)$$

then:

$$\begin{aligned}
M'_{\theta,0} &= \left(M_{\theta,0}\cosh\kappa - \mathrm{i}M_{\theta,4}\sinh\kappa\right),\\
M'_{\theta,4} &= \left(M_{\theta,4}\cosh\kappa + \mathrm{i}M_{\theta,0}\sinh\kappa\right),\\
M'_{\eta,0} &= \left(M_{\eta,0}\cosh\kappa - \mathrm{i}M_{\eta,4}\sinh\kappa\right),\\
M'_{\eta,4} &= \left(M_{\eta,4}\cosh\kappa + \mathrm{i}M_{\eta,0}\sinh\kappa\right),\\
M'_{\zeta,0} &= \left(M_{\zeta,0}\cosh\kappa + \mathrm{i}M_{\zeta,4}\sinh\kappa\right),\\
M'_{\zeta,4} &= \left(M_{\zeta,4}\cosh\kappa - \mathrm{i}M_{\zeta,0}\sinh\kappa\right).
\end{aligned}$$

Therefore, matrix $\widehat{U}(\kappa)$ makes an oscillation between upper and lower mass members, too.

Let us consider equation (4.76) under transformation $\widehat{U}(\kappa)$ where κ is an arbitrary real function of time-space variables ($\kappa = \kappa(t, x_1, x_2, x_3)$):

$$\widehat{U}^{-1}(\kappa)\left(\frac{1}{\mathrm{c}}\partial_t + \mathrm{i}\Theta_0 + \mathrm{i}\Upsilon_0\gamma^{[5]}\right)\widehat{U}(\kappa)\varphi =$$
$$= \widehat{U}^{-1}(\kappa)\left(\sum_{\nu=1}^{3}\beta^{[\nu]}\left(\partial_\nu + \mathrm{i}\Theta_\nu + \mathrm{i}\Upsilon_\nu\gamma^{[5]}\right) + \widehat{M}\right)\widehat{U}(\kappa)\varphi.$$

Because

$$\gamma^{[5]}\widehat{U}(\kappa) = \widehat{U}(\kappa)\gamma^{[5]},$$

$$\widehat{U}^{-1}(\kappa)\beta^{[1]} = \beta^{[1]}\widehat{U}^{-1}(\kappa),$$
$$\widehat{U}^{-1}(\kappa)\beta^{[2]} = \beta^{[2]}\widehat{U}^{-1}(\kappa),$$
$$\widehat{U}^{-1}(\kappa)\beta^{[3]} = \beta^{[3]}\widehat{U}^{-1}(\kappa),$$

$$\widehat{U}^{-1}(\kappa)\widehat{U}(\kappa) = 1_4,$$

then

$$\left(\frac{1}{\mathrm{c}}\partial_t + \widehat{U}^{-1}(\kappa)\left(\frac{1}{\mathrm{c}}\partial_t\widehat{U}(\kappa)\right) + \mathrm{i}\Theta_0 + \mathrm{i}\Upsilon_0\gamma^{[5]}\right)\varphi =$$

$$= \left(\begin{array}{c}\sum_{\nu=1}^{3}\beta^{[\nu]}\left(\partial_\nu + \widehat{U}^{-1}(\kappa)\left(\partial_\nu\widehat{U}(\kappa)\right) + \mathrm{i}\Theta_\nu + \mathrm{i}\Upsilon_\nu\gamma^{[5]}\right) + \\ +\widehat{U}^{-1}(\kappa)\widehat{M}\widehat{U}(\kappa)\end{array}\right)\varphi.$$

If denote:

$$\Lambda_1 := \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$\Lambda_2 := \begin{bmatrix} 0 & \mathrm{i} & 0 & 0 \\ \mathrm{i} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathrm{i} \\ 0 & 0 & \mathrm{i} & 0 \end{bmatrix},$$

$$\Lambda_3 := \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix},$$

$$\Lambda_4 := \begin{bmatrix} 0 & \mathrm{i} & 0 & 0 \\ -\mathrm{i} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\mathrm{i} \\ 0 & 0 & \mathrm{i} & 0 \end{bmatrix},$$

$$\Lambda_5 := \begin{bmatrix} -\mathrm{i} & 0 & 0 & 0 \\ 0 & \mathrm{i} & 0 & 0 \\ 0 & 0 & -\mathrm{i} & 0 \\ 0 & 0 & 0 & \mathrm{i} \end{bmatrix},$$

$$\Lambda_6 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\Lambda_7 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix},$$

$$\Lambda_8 := \begin{bmatrix} \mathrm{i} & 0 & 0 & 0 \\ 0 & \mathrm{i} & 0 & 0 \\ 0 & 0 & 2\mathrm{i} & 0 \\ 0 & 0 & 0 & 2\mathrm{i} \end{bmatrix},$$

then

$$\begin{aligned}
U_{0,1}^{-1}(\sigma)\left(\partial_s U_{0,1}(\sigma)\right) &= \Lambda_1 \partial_s \sigma, \\
U_{2,3}^{-1}(\alpha)\left(\partial_s U_{2,3}(\alpha)\right) &= \Lambda_2 \partial_s \alpha, \\
U_{1,3}^{-1}(\vartheta)\left(\partial_s U_{1,3}(\vartheta)\right) &= \Lambda_3 \partial_s \vartheta, \\
U_{0,2}^{-1}(\phi)\left(\partial_s U_{0,2}(\phi)\right) &= \Lambda_4 \partial_s \phi, \\
U_{1,2}^{-1}(\varsigma)\left(\partial_s U_{1,2}(\varsigma)\right) &= \Lambda_5 \partial_s \varsigma, \\
U_{0,3}^{-1}(\iota)\left(\partial_s U_{0,3}(\iota)\right) &= \Lambda_6 \partial_s \iota, \\
\widehat{U}^{-1}(\kappa)\left(\partial_s \widehat{U}(\kappa)\right) &= \Lambda_7 \partial_s \kappa, \\
\widetilde{U}^{-1}(\chi)\left(\partial_s \widetilde{U}(\chi)\right) &= \Lambda_8 \partial_s \chi.
\end{aligned}$$

Let $\grave{U}$ be the following set:

$$\grave{U} := \left\{U_{0,1}, U_{2,3}, U_{1,3}, U_{0,2}, U_{1,2}, U_{0,3}, \widehat{U}, \widetilde{U}\right\}.$$

Because

$U_{2,3}^{-1}(\alpha)\Lambda_1 U_{2,3}(\alpha) = \Lambda_1$
$U_{1,3}^{-1}(\vartheta)\Lambda_1 U_{1,3}(\vartheta) = (\Lambda_1 \cos 2\vartheta + \Lambda_6 \sin 2\vartheta)$
$U_{0,2}^{-1}(\phi)\Lambda_1 U_{0,2}(\phi) = (\Lambda_1 \cosh 2\phi - \Lambda_5 \sinh 2\phi)$
$U_{1,2}^{-1}(\varsigma)\Lambda_1 U_{1,2}(\varsigma) = \Lambda_1 \cos 2\varsigma - \Lambda_4 \sin 2\varsigma$
$U_{0,3}^{-1}(\iota)\Lambda_1 U_{0,3}(\iota) = \Lambda_1 \cosh 2\iota + \Lambda_3 \sinh 2\iota$
$\widehat{U}^{-1}(\kappa)\Lambda_1 \widehat{U}(\kappa) = \Lambda_1$

$\widetilde{U}^{-1}(\chi)\,\Lambda_1\widetilde{U}(\chi) = \Lambda_1$

========

$\widetilde{U}^{-1}(\chi)\,\Lambda_2\widetilde{U}(\chi) = \Lambda_2$

$\widehat{U}^{-1}(\kappa)\,\Lambda_2\widehat{U}(\kappa) = \Lambda_2$

$U_{0,3}^{-1}(\iota)\,\Lambda_2 U_{0,3}(\iota) = \Lambda_2\cosh 2\iota - \Lambda_4\sinh 2\iota$

$U_{1,2}^{-1}(\varsigma)\,\Lambda_2 U_{1,2}(\varsigma) = \Lambda_2\cos 2\varsigma - \Lambda_3\sin 2\varsigma$

$U_{0,2}^{-1}(\phi)\,\Lambda_2 U_{0,2}(\phi) = \Lambda_2\cosh 2\phi + \Lambda_6\sinh 2\phi$

$U_{1,3}^{-1}(\vartheta)\,\Lambda_2 U_{1,3}(\vartheta) = \Lambda_2\cos 2\vartheta + \Lambda_5\sin 2\vartheta$

$U_{0,1}^{-1}(\sigma)\,\Lambda_2 U_{0,1}(\sigma) = \Lambda_2$

=========

$U_{0,1}^{-1}(\sigma)\ \Lambda_3 U_{0,1}(\sigma) = \Lambda_3\cosh 2\sigma - \Lambda_6\sinh 2\sigma$

$U_{2,3}^{-1}(\alpha)\ \Lambda_3 U_{2,3}(\alpha) = \Lambda_3\cos 2\alpha - \Lambda_5\sin 2\alpha$

$U_{0,2}^{-1}(\phi)\ \Lambda_3 U_{0,2}(\phi) = \Lambda_3$

$U_{1,2}^{-1}(\varsigma)\ \Lambda_3 U_{1,2}(\varsigma) = \Lambda_3\cos 2\varsigma + \Lambda_2\sin 2\varsigma$

$U_{0,3}^{-1}(\iota)\ \Lambda_3 U_{0,3}(\iota) = \ \Lambda_3\cosh 2\iota + \Lambda_1\sinh 2\iota$

$\widehat{U}^{-1}(\kappa)\ \Lambda_3\widehat{U}(\kappa) = \Lambda_3$

$\widetilde{U}^{-1}(\chi)\ \Lambda_3\widetilde{U}(\chi) = \Lambda_3$

==========

$\widetilde{U}^{-1}(\chi)\ \Lambda_4\widetilde{U}(\chi) = \Lambda_4$

$\widehat{U}^{-1}(\kappa)\ \Lambda_4\widehat{U}(\kappa) = \Lambda_4$

$U_{0,3}^{-1}(\iota)\ \Lambda_4 U_{0,3}(\iota) = \Lambda_4\cosh 2\iota - \Lambda_2\sinh 2\iota$

$U_{1,2}^{-1}(\varsigma)\ \Lambda_4 U_{1,2}(\varsigma) = \Lambda_4\cos 2\varsigma + \Lambda_1\sin 2\varsigma$

$U_{1,3}^{-1}(\vartheta)\ \Lambda_4 U_{1,3}(\vartheta) = \Lambda_4$

$U_{2,3}^{-1}(\alpha)\ \Lambda_4 U_{2,3}(\alpha) = \Lambda_4\cos 2\alpha - \Lambda_6\sin 2\alpha$

$U_{0,1}^{-1}(\sigma)\ \Lambda_4 U_{0,1}(\sigma) = \Lambda_4\cosh 2\sigma + \Lambda_5\sinh 2\sigma$

==========

$U_{0,1}^{-1}(\sigma)\ \Lambda_5 U_{0,1}(\sigma)\ = \Lambda_5\cosh 2\sigma + \Lambda_4\sinh 2\sigma$

$U_{2,3}^{-1}(\alpha)\ \Lambda_5 U_{2,3}(\alpha)\ = \Lambda_5\cos 2\alpha + \Lambda_3\sin 2\alpha$

$U_{1,3}^{-1}(\vartheta)\ \Lambda_5 U_{1,3}(\vartheta)\ = (\Lambda_5\cos 2\vartheta - \Lambda_2\sin 2\vartheta)$

$U_{0,2}^{-1}(\phi)\ \Lambda_5 U_{0,2}(\phi)\ = \Lambda_5\cosh 2\phi - \Lambda_1\sinh 2\phi$

$U_{0,3}^{-1}(\iota)\ \Lambda_5 U_{0,3}(\iota)\ = \Lambda_5$

$\widehat{U}^{-1}(\kappa)\ \Lambda_5\widehat{U}(\kappa)\ = \Lambda_5$

$\widetilde{U}^{-1}(\chi)\,\Lambda_5\widetilde{U}(\chi)\ = \Lambda_5$

===========

$\widetilde{U}^{-1}(\chi)\,\Lambda_6\widetilde{U}(\chi)\ = \Lambda_6$

$\widehat{U}^{-1}(\kappa)\Lambda_6\widehat{U}(\kappa) = \Lambda_6$

$U_{1,2}^{-1}(\varsigma)\Lambda_6 U_{1,2}(\varsigma) = \Lambda_6$

$U_{0,2}^{-1}(\phi)\Lambda_6 U_{0,2}(\phi) = \Lambda_6\cosh 2\phi + \Lambda_2\sinh 2\phi$

$U_{1,3}^{-1}(\vartheta)\Lambda_6 U_{1,3}(\vartheta) = \Lambda_6\cos 2\vartheta - \Lambda_1\sin 2\vartheta$

$U_{2,3}^{-1}(\alpha)\Lambda_6 U_{2,3}(\alpha) = \Lambda_6\cos 2\alpha + \Lambda_4\sin 2\alpha$

$U_{0,1}^{-1}(\sigma)\Lambda_6 U_{0,1}(\sigma) = \Lambda_6\cosh 2\sigma - \Lambda_3\sinh 2\sigma$

========

$\widetilde{U}^{-1}(\chi)\ \Lambda_7\widetilde{U}(\chi) = \Lambda_7$

$U_{0,3}^{-1}(\iota)\ \Lambda_7 U_{0,3}(\iota) = \Lambda_7$

$U_{1,2}^{-1}(\varsigma)\ \Lambda_7 U_{1,2}(\varsigma) = \Lambda_7$

$U_{0,2}^{-1}(\phi)\ \Lambda_7 U_{0,2}(\phi) = \Lambda_7$

$U_{1,3}^{-1}(\vartheta)\ \Lambda_7 U_{1,3}(\vartheta) = \Lambda_7$

$U_{2,3}^{-1}(\alpha)\ \Lambda_7 U_{2,3}(\sigma) = \Lambda_7$

$U_{0,1}^{-1}(\sigma)\ \Lambda_7 U_{0,1}(\sigma) = \Lambda_7$

=========

$U_{0,1}^{-1}(\sigma)\ \Lambda_8 U_{0,1}(\sigma) = \Lambda_8$

$U_{2,3}^{-1}(\alpha)\ \Lambda_8 U_{2,3}(\alpha) = \Lambda_8$

$U_{1,3}^{-1}(\vartheta)\ \Lambda_8 U_{1,3}(\vartheta) = \Lambda_8$

$U_{0,2}^{-1}(\phi)\ \Lambda_8 U_{0,2}(\phi) = \Lambda_8$

$U_{1,2}^{-1}(\varsigma)\ \Lambda_8 U_{1,2}(\varsigma) = \Lambda_8$

$U_{0,3}^{-1}(\iota)\ \Lambda_8 U_{0,3}(\iota) = \Lambda_8$

$\widehat{U}^{-1}(\kappa)\ \Lambda_8\widehat{U}(\kappa) = \Lambda_8$

then for every product U of $\grave{U}$'s elements real functions $G_s^r(t,x_1,x_2,x_3)$ exist such that

$$U^{-1}(\partial_s U) = \frac{g_3}{2}\sum_{r=1}^{8}\Lambda_r G_s^r$$

with some real constant g_3 (similar to 8 gluons).

The chrome states equations of moving are the following [21, p.86], (4.74):

$$\begin{aligned}
\left(\begin{array}{c}\sum_{\mu=0}^{3}\left(-\zeta^{[k]}\right)\partial_\mu + \left(-\gamma_\zeta^{[0]}\partial_y^\zeta + \zeta^{[4]}\partial_z^\zeta\right) \\ +\sum_{\mu=0}^{3}\beta^{[\mu]}\left(-s\widehat{A}_\mu + 0.5\left(\widehat{Z}_\mu + \widehat{W}_\mu\right)\right)\end{array}\right)\xi &= 0, \qquad (4.81)\\
\left(\begin{array}{c}\sum_{\mu=0}^{3}\left(-\eta^{[k]}\right)\partial_\mu + \left(-\gamma_\eta^{[0]}\partial_y^\eta - \eta^{[4]}\partial_z^\eta\right) \\ +\sum_{\mu=0}^{3}\beta^{[\mu]}\left(-s\widehat{A}_\mu + 0.5\left(\widehat{Z}_\mu + \widehat{W}_\mu\right)\right)\end{array}\right)\xi &= 0,\\
\left(\begin{array}{c}\sum_{\mu=0}^{3}\left(-\theta^{[k]}\right)\partial_\mu + \left(\gamma_\theta^{[0]}\partial_y^\theta + \theta^{[4]}\partial_z^\theta\right) \\ +\sum_{\mu=0}^{3}\beta^{[\mu]}\left(-s\widehat{A}_\mu + 0.5\left(\widehat{Z}_\mu + \widehat{W}_\mu\right)\right)\end{array}\right)\xi &= 0;
\end{aligned}$$

here:

$$\xi := \begin{bmatrix} d_L \\ d_R \\ u_L \\ u_R \end{bmatrix}$$

($\begin{bmatrix} d_L \\ d_R \end{bmatrix}$ is a lower chrome state, and $\begin{bmatrix} u_L \\ u_R \end{bmatrix}$ is a upper chrome state);

s is an electric charge, and [21, p.145]

$$\widehat{A}_\mu := A_\mu \begin{bmatrix} 0_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & 1_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & 1_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & 1_2 \end{bmatrix}.$$

Hence,

$$\begin{aligned} s\widehat{A}_\mu \xi &= sA_\mu \begin{bmatrix} 0_2 & 0_2 & 0_2 & 0_2 \\ 0_2 & 1_2 & 0_2 & 0_2 \\ 0_2 & 0_2 & 1_2 & 0_2 \\ 0_2 & 0_2 & 0_2 & 1_2 \end{bmatrix} \begin{bmatrix} d_L \\ d_R \\ u_L \\ u_R \end{bmatrix} = sA_\mu \begin{bmatrix} \overrightarrow{0}_2 \\ d_R \\ u_L \\ u_R \end{bmatrix}, \\ \overrightarrow{0}_2 \quad : &= \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \end{aligned}$$

Sum of the equations (4.81) is the following:

$$\left(\begin{array}{c} \sum_{\mu=0}^{3} \left(-\left(\zeta^{[k]} + \eta^{[k]} + \theta^{[k]} \right) \right) \partial_\mu + \\ + \left(\begin{array}{c} -\gamma_\zeta^{[0]} \partial_y^\zeta + \zeta^{[4]} \partial_z^\zeta \\ -\gamma_\eta^{[0]} \partial_y^\eta - \eta^{[4]} \partial_z^\eta \\ +\gamma_\theta^{[0]} \partial_y^\theta + \theta^{[4]} \partial_z^\theta \end{array} \right) + \left(\begin{array}{c} \sum_{\mu=0}^{3} \beta^{[\mu]} (-3s) \widehat{A}_\mu \\ +0.5 \cdot 3 \sum_{\mu=0}^{3} \beta^{[\mu]} \left(\widehat{Z}_\mu + \widehat{W}_\mu \right) \end{array} \right) \end{array} \right) \xi = 0.$$

Because $\beta^{[k]} = -\left(\zeta^{[k]} + \eta^{[k]} + \theta^{[k]} \right)$ rhen

$$\left(\begin{array}{c} \sum_{\mu=0}^{3} \beta^{[k]} \partial_\mu + \\ + \left(\begin{array}{c} -\gamma_\zeta^{[0]} \partial_y^\zeta + \zeta^{[4]} \partial_z^\zeta \\ -\gamma_\eta^{[0]} \partial_y^\eta - \eta^{[4]} \partial_z^\eta \\ +\gamma_\theta^{[0]} \partial_y^\theta + \theta^{[4]} \partial_z^\theta \end{array} \right) + \left(\begin{array}{c} \sum_{\mu=0}^{3} \beta^{[\mu]} (-3s) \widehat{A}_\mu \\ +0.5 \cdot 3 \sum_{\mu=0}^{3} \beta^{[\mu]} \left(\widehat{Z}_\mu + \widehat{W}_\mu \right) \end{array} \right) \end{array} \right) \xi = 0.$$

Hence, from (4.73):

$$(-3s) = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}$$

that is:

$$s = -\frac{1}{3}\frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} = -\frac{1}{3}e$$

(e is a lepton electric charge).
Because

$$s\widehat{A}_\mu \xi = -\frac{1}{3}eA_\mu \begin{bmatrix} \overrightarrow{0}_2 \\ d_R \\ u_L \\ u_R \end{bmatrix}$$

then a charge of $\begin{bmatrix} u_L \\ u_R \end{bmatrix}$ is $-\frac{2}{3}e$ and a module of charge of $\begin{bmatrix} \overrightarrow{0}_2 \\ d_R \end{bmatrix}$ is $\frac{1}{3}e$.

4.16. Asymptotic Freedom, Confinement, Gravitation

The Quarks Asymptotic Freedom phenomenon and the Quarks Confinement phenomenon has been was discovered by J. Friedman[42], H. Kendall[43], R. Taylor[44] at SLAC in the late 1960s and early 1970s.

Researches of the phenomenon of gravitation were spent by Galileo Galilei[45] in the late 16th and early 17th centuries, by Isaac Newton[46] in 17th centuries, by A. Einstein[47] in 20th centuries.

From (4.80):

$$\begin{aligned} \frac{\partial t}{\partial t'} &= \cosh 2\sigma, \\ \frac{\partial x}{\partial t'} &= c \sinh 2\sigma. \end{aligned} \tag{4.82}$$

Hence, if v is the velocity of a coordinate system $\{t', x'\}$ in the coordinate system $\{t, x\}$ then

$$\sinh 2\sigma = \frac{\left(\frac{v}{c}\right)}{\sqrt{1 - \left(\frac{v}{c}\right)^2}},\ \cosh 2\sigma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}.$$

Therefore,

[42] Jerome Isaac Friedman (born March 28, 1930) is an American physicist.

[43] Henry Way Kendall (December 9, 1926 – February 15, 1999) was an American particle physicist

[44] Richard Edward Taylor (born November 2, 1929 in Medicine Hat, Alberta) is a Canadian-American professor (Emeritus) at Stanford University.

[45] Galileo Galilei (15 February 1564[4] – 8 January 1642), was an Italian physicist, mathematician, astronomer, and philosopher

[46] Sir Isaac Newton PRS (25 December 1642 – 20 March 1727 was an English physicist, mathematician, astronomer, natural philosopher, alchemist, and theologian

[47] Albert Einstein (14 March 1879 – 18 April 1955) was a German-born theoretical physicist

$$v = \mathrm{c} \tanh 2\sigma. \tag{4.83}$$

Let

$$2\sigma := \omega(x) \frac{t}{x}$$

with

$$\omega(x) = \frac{\lambda}{|x|},$$

where λ is a real constant with positive numerical value.

In that case

$$v(t, x) = \mathrm{c} \tanh\left(\frac{\lambda}{|x|} \frac{t}{|x|}\right). \tag{4.84}$$

and if g is an acceleration of system $\{t', x_1'\}$ as respects to system $\{t, x_1\}$ then

$$\mathrm{g}(t, x_1) = \frac{\partial v}{\partial t} = \frac{\mathrm{c}\omega(x)}{\left(\cosh^2 \omega(x) \frac{t}{|x|}\right)|x|}.$$

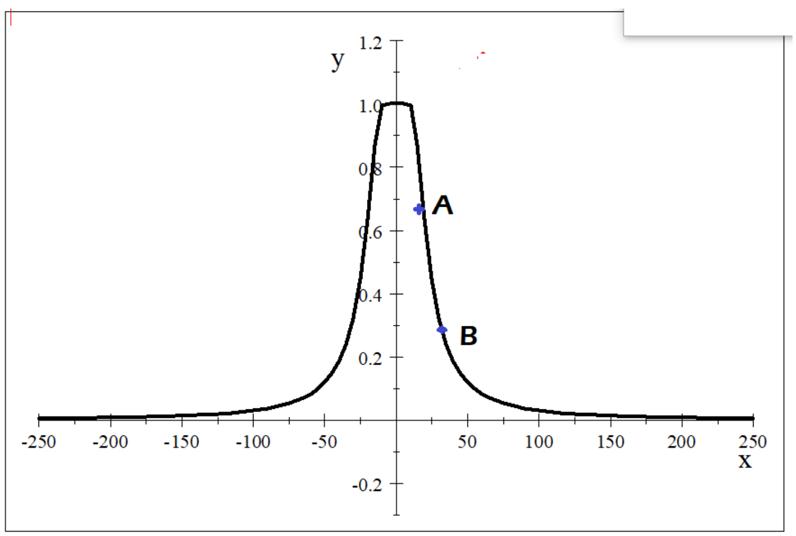
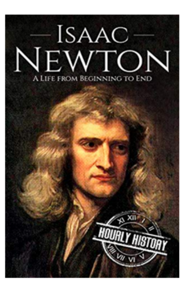


Figure 10: the dependency of a system $\{t', x_1'\}$ velocity $v(t, x_1)$ on x_1 in system $\{t, x_1\}$.



This velocity in point A is not equal to one in point B. Hence, an oscillator, placed in B, has a nonzero velocity in respect to an observer, placed in point A. Therefore, from the Lorentz transformations, this oscillator frequency for observer, placed in point A, is less than own frequency of this oscillator (*red shift*).

If an object immovable in system $\{t, x_1\}$ is placed in point K then in system $\{t', x_1'\}$ this object must move to the left with acceleration g and $\mathrm{g} \simeq \frac{\lambda}{x_1^2}$.

I call:

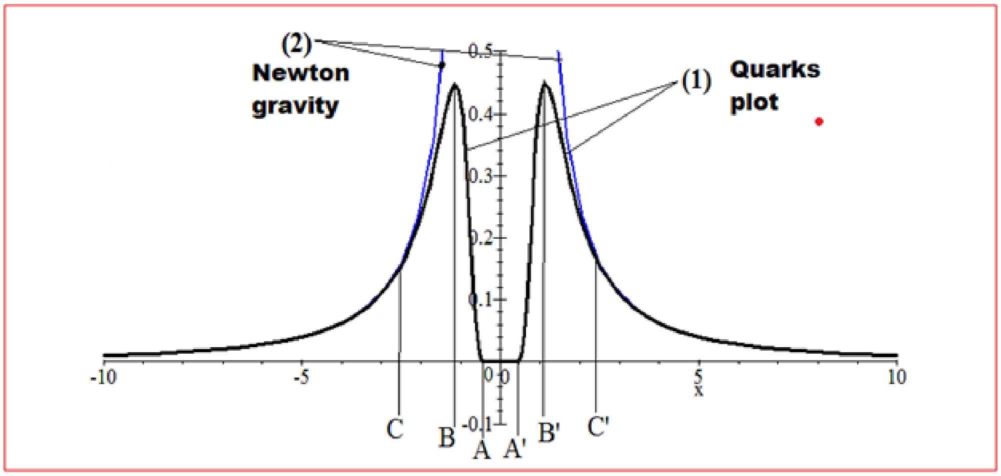


Figure 11: a dependency of a system $\{t', x_1'\}$ acceleration $g(t, x_1)$ on x_1 in system $\{t, x_1\}$.

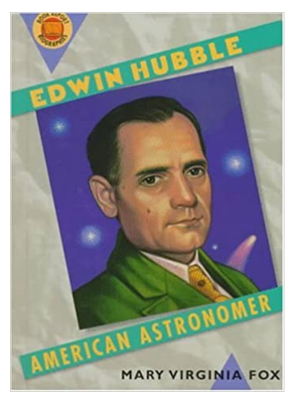
- interval from C' to ∞ the *Newton Gravity Zone*,
- interval from B' to C' the *the Confinement Force Zone*.
- and interval from A to A' *the Asymptotic Freedom Zone*,

4.16.1. Dark Energy

In 1998 observations of Type Ia supernovae suggested that the expansion of the universe is accelerating [25]. In the past few years, these observations have been corroborated by several independent sources [26]. This expansion is defined by the Hubble[48] rule [27]:

$$V(r) = Hr, \tag{4.85}$$

here $V(r)$ is the velocity of expansion on the distance r, H is the Hubble's constant ($H \approx 2.3 \times 10^{-18} c^{-1}$ [28]).



Let a black hole be placed in a point O. Then a tremendous number of quarks oscillate in this point. These oscillations bend time-space and if t has some fixed volume, $x > 0$, and $\Lambda := \lambda t$ then

$$v(x) = c \tanh\left(\frac{\Lambda}{x^2}\right). \tag{4.86}$$

[48]Edwin Powell Hubble (November 20, 1889 September 28, 1953)[1] was an American astronomer

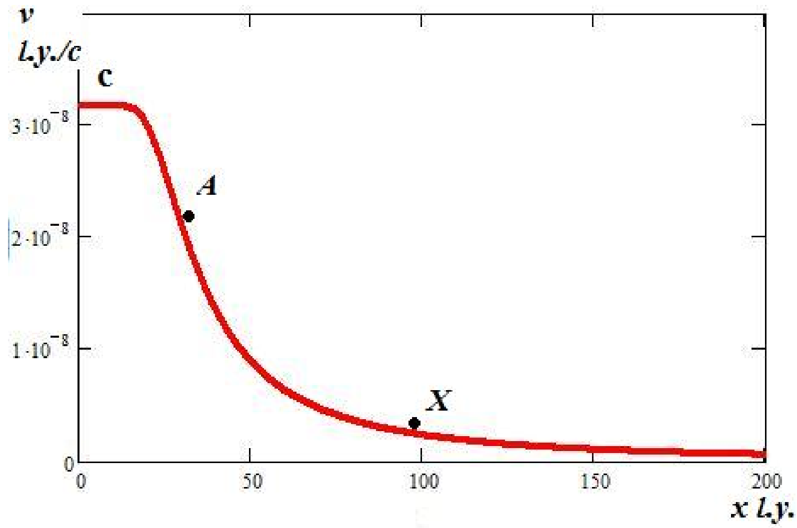


Figure 12: Dependence of v (light year/c) on x (light year) with $\Lambda = 741.907$

A dependency of $v(x)$ (light years/c) from x (light years) with $\Lambda = 741.907$ is shown in Figure 12.

Let a placed in a point A observer be stationary in the coordinate system $\{t, x\}$. Hence, in the coordinate system $\{t', x'\}$ this observer is flying to the left to the point O with velocity $-v(x_A)$. And point X is flying to the left to the point O with velocity $-v(x)$.

Consequently, the observer A sees that the point X flies away from him to the right with velocity

$$V_A(x) = \mathrm{c} \tanh\left(\frac{\Lambda}{x_A^2} - \frac{\Lambda}{x^2}\right) \tag{4.87}$$

in accordance with the relativistic rule of addition of velocities.

Let $r := x - x_A$ (i.e. r is distance from A to X), and

$$V_A(r) := \mathrm{c} \tanh\left(\frac{\Lambda}{x_A^2} - \frac{\Lambda}{(x_A + r)^2}\right). \tag{4.88}$$

In that case Figure 13 demonstrates the dependence of $V_A(r)$ on r with $x_A = 25 \times 10^3$ l.y.

Hence, X runs from A with almost constant acceleration:

$$\frac{V_A(r)}{r} = H. \tag{4.89}$$

Figure 14 demonstrates the dependence of H on r. (the Hubble constant.).

Therefore, the phenomenon of the accelerated expansion of Universe is explained by oscillations of chromatic states.

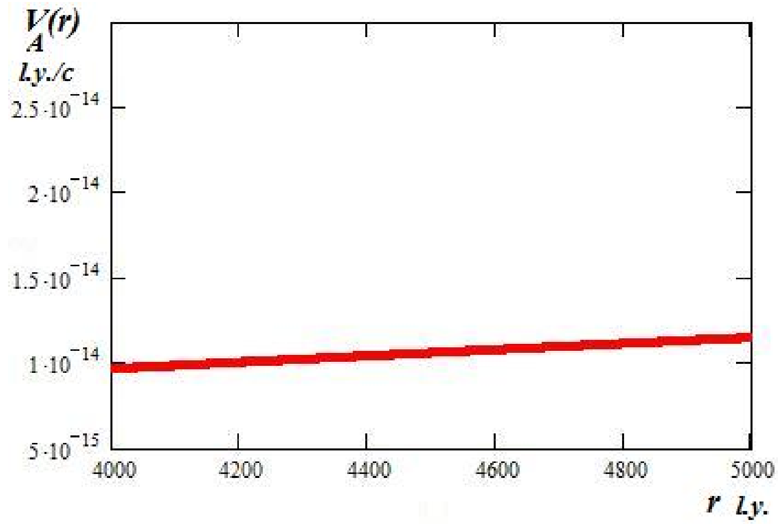


Figure 13: Dependence of $V_A\left(r\right)$ on r with $x_A = 25 \times 10^3$ l.y.

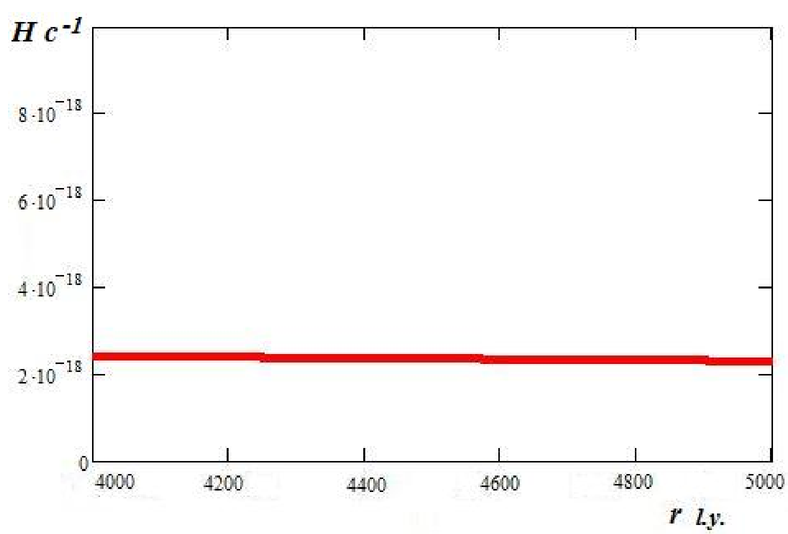


Figure 14: Dependence of H on r

4.16.2. Dark Matter

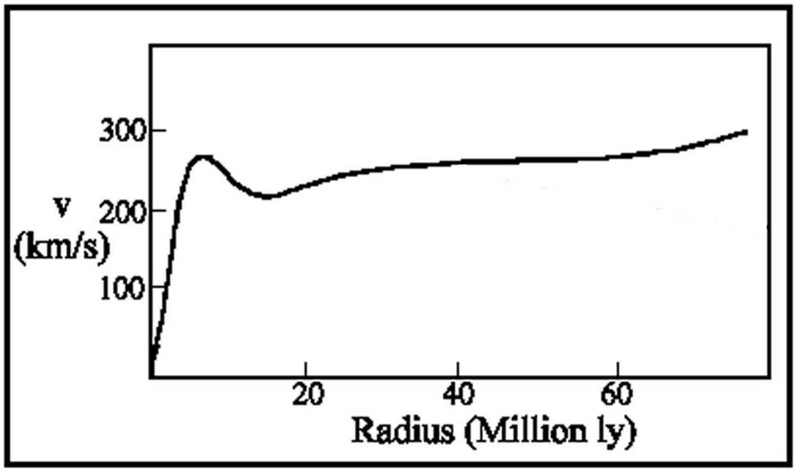


Figure 15: A rotation curve for a typical spiral galaxy. The solid line shows actual measurements (Hawley and Holcomb., 1998, p. 390) [30]

”In 1933, the astronomer Fritz Zwicky[49] was studying the motions of distant galaxies. Zwicky estimated the total mass of a group of galaxies by measuring their brightness. When he used a different method to compute the mass of the same cluster of galaxies, he came up with a number that was 400 times his original estimate. This discrepancy in the observed and computed masses is now known as ”the missing mass problem.” Nobody did much with Zwicky’s finding until the 1970’s, when scientists began to realize that only large amounts of hidden mass could explain many of their observations. Scientists also realize that the existence of some unseen mass would also support theories regarding the structure of the universe. Today, scientists are searching for the mysterious dark matter not only to explain the gravitational motions of galaxies, but also to validate current theories about the origin and the fate of the universe” [29] (15 [30], 16 [31]).

Some oscillations of chromatic states bend space-time as follows (2.118):

$$\begin{aligned}\frac{\partial}{\partial x'} &= \cos 2\alpha\cdot\frac{\partial}{\partial x} - \sin 2\alpha\cdot\frac{\partial}{\partial y},\\ \frac{\partial}{\partial y'} &= \cos 2\alpha\cdot\frac{\partial}{\partial y} + \sin 2\alpha\cdot\frac{\partial}{\partial x}.\end{aligned} \tag{4.90}$$

[49]Fritz Zwicky (February 14, 1898 – February 8, 1974) was a Swiss astronomer.

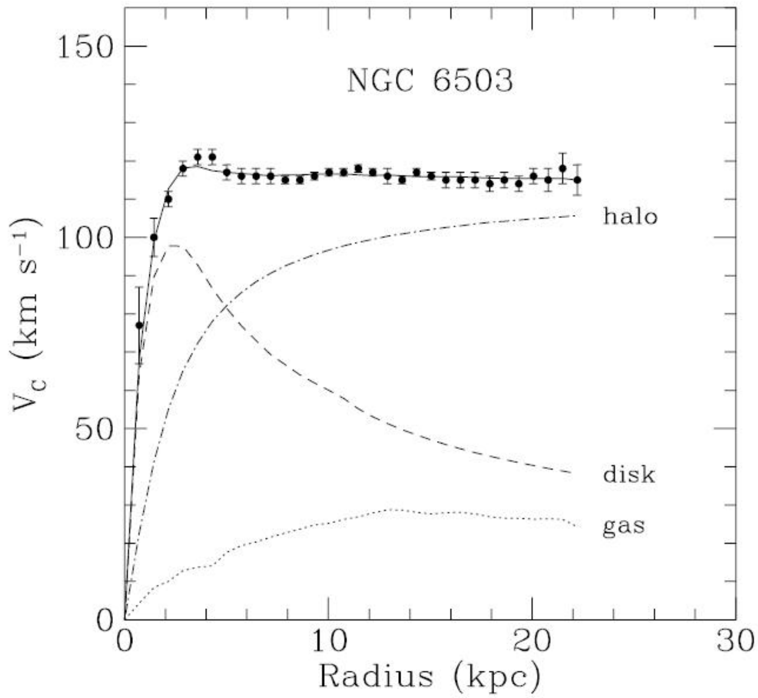


Figure 16: Rotation curve of NGC 6503. The dotted, dashed and dash-dotted lines are the contributions of gas, disk and dark matter, respectively.

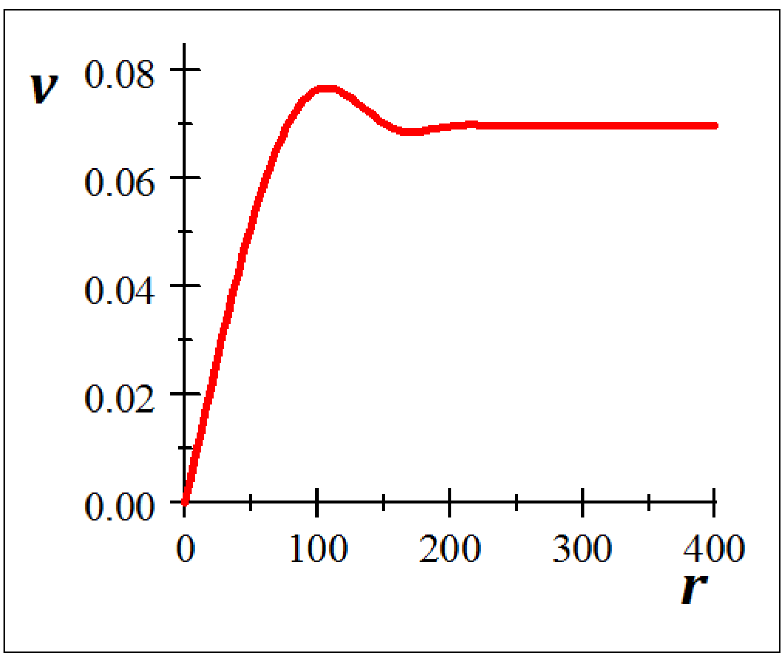


Figure 17: For $t = 10000$, $\theta = 13\pi/14$:

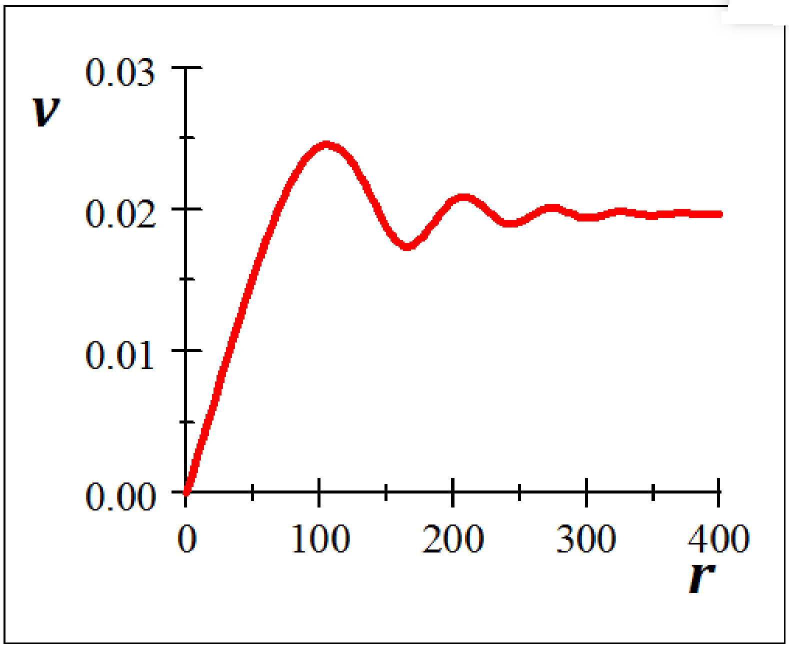


Figure 18: For $t = 10000$, $\theta = 0.98\pi$:

Let

$$\begin{aligned} z &: = x + \mathrm{i}y, z = re^{\mathrm{i}\theta}. \\ z' &: = x' + \mathrm{i}y'. \end{aligned}$$

Because linear velocity of the curved coordinate system $\langle x', y' \rangle$ into the initial system $\langle x, y \rangle$ is the following[50]:

$$v = \sqrt{\left(\overset{\bullet}{x'}\right)^2 + \left(\overset{\bullet}{y'}\right)^2}$$

then in thic case:

$$v = \left|\overset{\bullet}{z'}\right|.$$

Let function z' be a holomorphic function. Hence, in accordance with the Cauchy-Riemann conditions the following equations are fulfilled:

$$\begin{aligned} \frac{\partial x'}{\partial x} &= \frac{\partial y'}{\partial y}, \\ \frac{\partial x'}{\partial y} &= -\frac{\partial y'}{\partial x}. \end{aligned}$$

[50] $\overset{\bullet}{x'} := \frac{\partial x'}{\partial t}$, $\overset{\bullet}{y'} := \frac{\partial y'}{\partial t}$.

Therefore, in accordance with (4.90):

$$dz' = e^{-\mathrm{i}(2\alpha)} dz$$

where 2α is an holomorphic function, too. For example, let

$$2\alpha := \frac{1}{\mathrm{i}}\left((x+y)+\mathrm{i}\,(y-x)\right)^2.$$

In this case:

$$z' = \int \exp\left(\frac{\left((x+y)+i\,(y-x)\right)^2}{t}\right)dx + i\int \exp\left(\frac{\left((x+y)+i\,(y-x)\right)^2}{t}\right)dy.$$

Let $k := y/x$.
Hence,

$$z' = \int \exp\left(\frac{\left((x+kx)+i\,(kx-x)\right)^2}{t}\right)dx + i\int \exp\left(\frac{\left(\left(\frac{y}{k}+y\right)+i\left(y-\frac{y}{k}\right)\right)^2}{t}\right)dy.$$

Calculate:

$$\int \exp\left(\frac{\left((x+kx)+i\,(kx-x)\right)^2}{t}\right)dx = \frac{1}{2}\sqrt{\pi}\frac{\operatorname{erf}\left(x\sqrt{-\frac{1}{t}\left(2ik^2+4k-2i\right)}\right)}{\sqrt{-\frac{1}{t}\left(2ik^2+4k-2i\right)}},$$

$$i\int \exp\left(\frac{\left(\left(\frac{y}{k}+y\right)+i\left(y-\frac{y}{k}\right)\right)^2}{t}\right)dy = \frac{1}{2}i\sqrt{\pi}\frac{\operatorname{erf}\left(y\sqrt{-\frac{1}{k^2t}\left(2ik^2+4k-2i\right)}\right)}{\sqrt{-\frac{1}{k^2t}\left(2ik^2+4k-2i\right)}}.$$

Calculate:

$$\frac{\partial z'}{\partial t} = \frac{1}{-8\sqrt{t}\,i\,(k-i)^3\sqrt{-2i}}\left(\begin{array}{c} -4y\,(k-i)^2\sqrt{-\frac{1}{t}2i\,(k-i)^2}\exp\left(\frac{1}{k^2t}y^2 2i\,(k-i)^2\right) \\ +4ikx\,(k-i)^2\sqrt{-\frac{1}{k^2t}2i\,(k-i)^2}\exp\left(\frac{1}{t}x^2 2i\,(k-i)^2\right) \\ +i\sqrt{\pi}k^2t2i\,(k-i)^2\sqrt{\frac{1}{t}}\sqrt{\frac{1}{k^2}}\operatorname{erf}\left(y\sqrt{-\frac{1}{k^2t}2i\,(k-i)^2}\right) \\ +\sqrt{\pi}kt2i\,(k-i)^2\sqrt{\frac{1}{t^2}}\sqrt{\frac{1}{k^2}}\operatorname{erf}\left(x\sqrt{-\frac{1}{t}2i\,(k-i)^2}\right) \end{array}\right).$$

For large t:

$$\frac{\partial z'}{\partial t} \approx \frac{1}{-8\sqrt{t}\,i\,(k-i)^3\sqrt{-2i}}\,i\sqrt{\pi}k^2t2i\,(k-i)^2\sqrt{\frac{1}{t}}\sqrt{\frac{1}{k^2}}\operatorname{erf}\left(y\sqrt{-\frac{1}{k^2t}2i\,(k-i)^2}\right)$$

Hence,

$$v \approx \left| \frac{1}{8}(1-i)\,k\sqrt{\pi}\frac{1}{k-i}\mathrm{erf}\left(x\sqrt{-\frac{1}{t}2i\,(k-i)^2}\right)\right|.$$

Because

$$k = \tan\theta,\, x = r\cos\theta$$

then

$$v \approx \left| \frac{1}{8}(1-i)(\tan\theta)\sqrt{\pi}\frac{1}{\tan\theta - i}\mathrm{erf}\left(r(\cos\theta)\sqrt{-\frac{1}{t}2i\,((\tan\theta)-i)^2}\right)\right|$$

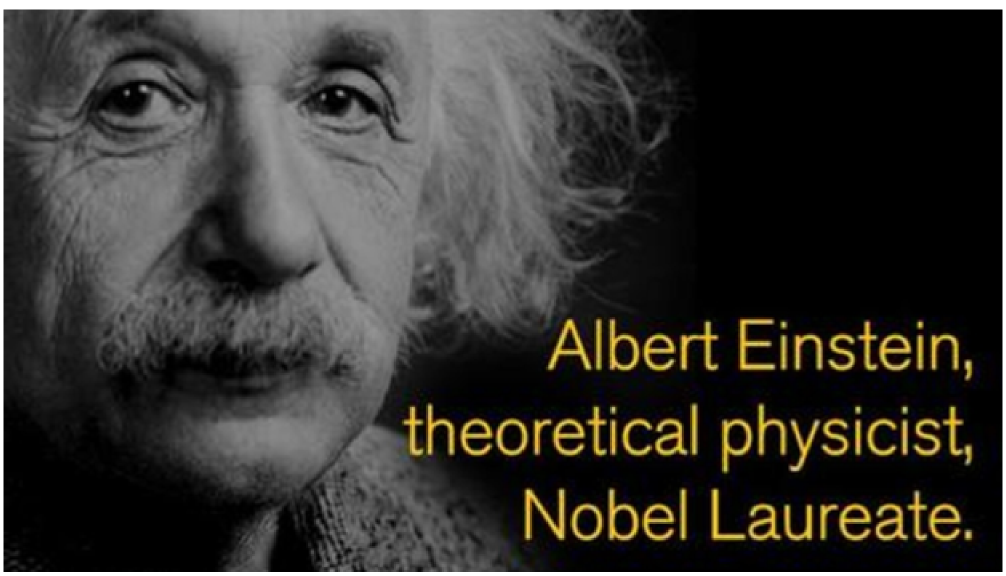
.

Figure 17 shows the dependence of velocity v on the radius r at large $t \sim 10^4$ and at $\theta = 13\pi/14$. Compare with 15

Figure 18 shows the dependence of velocity v on the radius r at large $t \sim 10^4$ and at $\theta = 0.98\pi$. Compare with 16

Hence, Dark Matter and Dark Energy can be mirages in the space-time, which is curved by oscillations of chromatic states.

The idea of curved time-space belongs to Albert Einstein (the General Relatively Theory. 1913).



4.16.3. Baryon Chrome

According to the quark model, [22] the properties of hadrons are primarily determined by their so-called valence quarks. For example, a proton is composed of two up quarks and one down quark. Although quarks also carry color charge, hadrons must have zero total color charge because of a phenomenon called color Confinement. That is, hadrons must be “colorless” or “white”. These are the simplest of the two ways: three quarks of different colors, or a quark of one color and an antiquark carrying the corresponding anticolor. Hadrons with the first arrangement are called baryons, and those with the second arrangement are mesons.

Let α be any real number and

$$\begin{aligned}
&x'_0 := x_0, \\
&x'_1 := x_1 \cos(\alpha) - x_2 \sin(\alpha); \\
&x'_2 := x_1 \sin(\alpha) + x_2 \cos(\alpha); \\
&x'_3 := x_3;
\end{aligned} \tag{4.91}$$

Since j_A is a 3+1-vector then from [21, p.59]:

$$\begin{aligned}
j'_{A,0} &= -\varphi^\dagger \beta^{[0]} \varphi, \\
j'_{A,1} &= -\varphi^\dagger \left(\beta^{[1]} \cos(\alpha) - \beta^{[2]} \sin(\alpha)\right) \varphi; \\
j'_{A,2} &= -\varphi^\dagger \left(\beta^{[1]} \sin(\alpha) + \beta^{[2]} \cos(\alpha)\right) \varphi; \\
j'_{A,3} &= -\varphi^\dagger \beta^{[3]} \varphi.
\end{aligned} \tag{4.92}$$

Hence if for φ':

$$\begin{aligned}
j'_{A,0} = -\varphi'^\dagger \beta^{[0]} \varphi', \\
j'_{A,1} = -\varphi'^\dagger \beta^{[1]} \varphi'; \\
j'_{A,2} = -\varphi'^\dagger \beta^{[2]} \varphi'; \\
j'_{A,3} = -\varphi'^\dagger \beta^{[3]} \varphi',
\end{aligned}$$

and

$$\varphi' := U_{1,2}(\alpha) \varphi$$

then

$$\begin{aligned}
U^\dagger_{1,2}(\alpha) \beta^{[0]} U_{1,2}(\alpha) &= \beta^{[0]}, \\
U^\dagger_{1,2}(\alpha) \beta^{[1]} U_{1,2}(\alpha) &= \beta^{[1]} \cos\alpha - \beta^{[2]} \sin\alpha; \\
U^\dagger_{1,2}(\alpha) \beta^{[2]} U_{1,2}(\alpha) &= \beta^{[2]} \cos\alpha + \beta^{[1]} \sin\alpha; \\
U^\dagger_{1,2}(\alpha) \beta^{[3]} U_{1,2}(\alpha) &= \beta^{[3]};
\end{aligned} \tag{4.93}$$

from [21, p.62]: because

$$\rho_A = \varphi^\dagger \varphi = \varphi'^\dagger \varphi',$$

then

$$U^\dagger_{1,2}(\alpha) U_{1,2}(\alpha) = 1_4. \tag{4.94}$$

If

$$U_{1,2}(\alpha) := \cos\frac{\alpha}{2} \cdot 1_4 - \sin\frac{\alpha}{2} \cdot \beta^{[1]}\beta^{[2]}$$

i.e. 2.116:

$$U_{1,2}(\alpha) = \left[\begin{array}{cccc} e^{-i\frac{1}{2}\alpha} & 0 & 0 & 0 \\ 0 & e^{i\frac{1}{2}\alpha} & 0 & 0 \\ 0 & 0 & e^{-i\frac{1}{2}\alpha} & 0 \\ 0 & 0 & 0 & e^{i\frac{1}{2}\alpha} \end{array}\right] \tag{4.95}$$

then $U_{1,2}(\alpha)$ fulfils to all these conditions (4.93), (4.94).

Then let

$$\begin{aligned} x'_0 &:= x_0, \\ x'_1 &:= x_1\cos(\alpha) - x_3\sin(\alpha), \\ x'_2 &:= x_2, \\ x'_3 &:= x_1\sin(\alpha) + x_3\cos(\alpha). \end{aligned} \tag{4.96}$$

Let

$$U_{1,3}(\alpha) := \cos\frac{\alpha}{2} \cdot 1_4 - \sin\frac{\alpha}{2} \cdot \beta^{[1]}\beta^{[3]}.$$

In rhis case 2.117:

$$U_{1,3}(\alpha) = \left[\begin{array}{cccc} \cos\frac{1}{2}\alpha & \sin\frac{1}{2}\alpha & 0 & 0 \\ -\sin\frac{1}{2}\alpha & \cos\frac{1}{2}\alpha & 0 & 0 \\ 0 & 0 & \cos\frac{1}{2}\alpha & \sin\frac{1}{2}\alpha \\ 0 & 0 & -\sin\frac{1}{2}\alpha & \cos\frac{1}{2}\alpha \end{array}\right] \tag{4.97}$$

and

$$\begin{aligned} U^{\dagger}_{1,3}(\alpha)\beta^{[0]}U_{1,3}(\alpha) &= \beta^{[0]}, \\ U^{\dagger}_{1,3}(\alpha)\beta^{[1]}U_{1,3}(\alpha) &= \beta^{[1]}\cos\alpha - \beta^{[3]}\sin\alpha, \\ U^{\dagger}_{1,3}(\alpha)\beta^{[2]}U_{1,3}(\alpha) &= \beta^{[2]}, \\ U^{\dagger}_{1,3}(\alpha)\beta^{[3]}U_{1,3}(\alpha) &= \beta^{[3]}\cos\alpha + \beta^{[1]}\sin\alpha. \end{aligned} \tag{4.98}$$

If

$$\varphi' := U_{1,3}(\alpha)\varphi$$

and

$$j'_{A,k} := \varphi'^{\dagger}\beta^{[k]}\varphi'$$

where ($k \in \{0,1,2,3\}$) then

$$\begin{aligned} j'_{A,0} &= j_{A,0}, && (4.99) \\ j'_{A,1} &= j_{A,1}\cos\alpha - j_{A,3}\sin\alpha, && (4.100) \\ j'_{A,2} &= j_{A,2}, \\ j'_{A,3} &= j_{A,3}\cos\alpha + j_{A,1}\sin\alpha, \end{aligned}$$

Then let

$$
\begin{aligned}
x'_0 &: = x_0, \\
x'_1 &: = x_1, \\
x'_2 &= \cos\alpha \cdot x_2 + \sin\alpha \cdot x_3, \\
x'_3 &= \cos\alpha \cdot x_3 - \sin\alpha \cdot x_2.
\end{aligned} \tag{4.101}
$$

Let

$$U_{3,2}(\alpha) = \cos\frac{\alpha}{2} \cdot 1_4 - \sin\frac{\alpha}{2} \cdot \beta^{[3]}\beta^{[2]}$$

In this case:

$$U_{3,2}(\alpha) = \begin{bmatrix} \cos\frac{1}{2}\alpha & i\sin\frac{1}{2}\alpha & 0 & 0 \\ i\sin\frac{1}{2}\alpha & \cos\frac{1}{2}\alpha & 0 & 0 \\ 0 & 0 & \cos\frac{1}{2}\alpha & i\sin\frac{1}{2}\alpha \\ 0 & 0 & i\sin\frac{1}{2}\alpha & \cos\frac{1}{2}\alpha \end{bmatrix}, \tag{4.102}$$

and

$$
\begin{aligned}
U^{\dagger}_{3,2}(\alpha)\beta^{[0]}U_{3,2}(\alpha) &= \beta^{[0]}, \\
U^{\dagger}_{3,2}(\alpha)\beta^{[1]}U_{3,2}(\alpha) &= \beta^{[1]}, \\
U^{\dagger}_{3,2}(\alpha)\beta^{[0]}U_{3,2}(\alpha) &= \beta^{[0]}\cos\alpha + \beta^{[3]}\sin\alpha, \\
U^{\dagger}_{3,2}(\alpha)\beta^{[3]}U_{3,2}(\alpha) &= \beta^{[3]}\cos\alpha - \beta^{[2]}\sin\alpha
\end{aligned} \tag{4.103}
$$

If

$$\varphi' := U_{3,2}(\alpha)\varphi$$

and

$$j'_{A,k} := \varphi'^{\dagger}\beta^{[k]}\varphi'$$

where ($k \in \{0,1,2,3\}$) then

$$
\begin{aligned}
j'_{A,0} &= j_{A,0}, \\
j'_{A,1} &= j_{A,1}, \\
j'_{A,2} &= j_{A,2}\cos\alpha + j_{A,3}\sin\alpha, \\
j'_{A,3} &= j_{A,3}\cos\alpha - j_{A,1}\sin\alpha,
\end{aligned} \tag{4.104}
$$

Let v be any real number such that $-1 < v < 1$.
And let:

$$\alpha := \frac{1}{2} \ln \frac{1-v}{1+v}.$$

In this case:

$$\begin{aligned} \cosh \alpha &= \frac{1}{\sqrt{1-v^2}}, \\ \sinh \alpha &= -\frac{v}{\sqrt{1-v^2}}. \end{aligned} \tag{4.105}$$

Let

$$\begin{aligned} x'_0 &: = x_0 \cosh \alpha - x_1 \sinh \alpha, \qquad (4.106) \\ x'_1 &: = x_1 \cosh \alpha - x_0 \sinh \alpha, \\ x'_2 &: = x_2, \\ x'_3 &: = x_3. \end{aligned}$$

Let

$$U_{1,0}(\alpha) = \cosh \frac{\alpha}{2} \cdot 1_4 - \sinh \frac{\alpha}{2} \cdot \beta^{[1]} \beta^{[0]}.$$

That is 2.119:

$$U_{1,0}(\alpha) := \begin{bmatrix} \cosh \frac{1}{2}\alpha & \sinh \frac{1}{2}\alpha & 0 & 0 \\ \sinh \frac{1}{2}\alpha & \cosh \frac{1}{2}\alpha & 0 & 0 \\ 0 & 0 & \cosh \frac{1}{2}\alpha & -\sinh \frac{1}{2}\alpha \\ 0 & 0 & -\sinh \frac{1}{2}\alpha & \cosh \frac{1}{2}\alpha \end{bmatrix}. \tag{4.107}$$

In rhis case:

$$\begin{aligned} U^{\dagger}_{1,0}(\alpha) \beta^{[0]} U_{1,0}(\alpha) &= \beta^{[0]} \cosh \alpha - \beta^{[1]} \sinh \alpha, \qquad (4.108) \\ U^{\dagger}_{1,0}(\alpha) \beta^{[1]} U_{1,0}(\alpha) &= \beta^{[1]} \cosh \alpha - \beta^{[0]} \sinh \alpha, \\ U^{\dagger}_{1,0}(\alpha) \beta^{[2]} U_{1,0}(\alpha) &= \beta^{[2]}, \\ U^{\dagger}_{1,0}(\alpha) \beta^{[3]} U_{1,0}(\alpha) &= \beta^{[3]}. \end{aligned}$$

If

$$\varphi' := U_{1,0}(\alpha) \varphi$$

and

$$j'_{A,k} := \varphi'^{\dagger} \beta^{[k]} \varphi'$$

where ($k \in \{0,1,2,3\}$) then

$$
\begin{aligned}
j'_{A,0} &= j_{A,0}\cosh\alpha - j_{A,1}\sinh\alpha, \\
j'_{A,1} &= j_{A,1}\cosh\alpha - j_{A,0}\sinh\alpha, \\
j'_{A,2} &= j_{A,2}, \\
j'_{A,3} &= j_{A,3}.
\end{aligned} \tag{4.109}
$$

Then let

$$
\begin{aligned}
x'_0 &: = x_0\cosh\alpha - x_2\sinh\alpha, \\
x'_1 &: = x_1, \\
x'_2 &: = x_2\cosh\alpha - x_0\sinh\alpha, \\
x'_3 &: = x_3.
\end{aligned} \tag{4.110}
$$

Let

$$U_{2,0}(\alpha) := \cosh\frac{\alpha}{2}\cdot 1_4 - \sinh\frac{\alpha}{2}\cdot\beta^{[2]}\beta^{[0]}. \tag{4.111}$$

That is:

$$U_{2,0}(\alpha) = \begin{bmatrix} \cosh\frac{1}{2}\alpha & -i\sinh\frac{1}{2}\alpha & 0 & 0 \\ i\sinh\frac{1}{2}\alpha & \cosh\frac{1}{2}\alpha & 0 & 0 \\ 0 & 0 & \cosh\frac{1}{2}\alpha & i\sinh\frac{1}{2}\alpha \\ 0 & 0 & -i\sinh\frac{1}{2}\alpha & \cosh\frac{1}{2}\alpha \end{bmatrix}.$$

In rhis case:

$$
\begin{aligned}
U^{\dagger}_{2,0}(\alpha)\beta^{[0]}U_{2,0}(\alpha) &= \beta^{[0]}\cosh\alpha - \beta^{[2]}\sinh\alpha, \\
U^{\dagger}_{2,0}(\alpha)\beta^{[1]}U_{1,0}(\alpha) &= \beta^{[1]}, \\
U^{\dagger}_{2,0}(\alpha)\beta^{[2]}U_{1,0}(\alpha) &= \beta^{[2]}\cosh\alpha - \beta^{[0]}\sinh\alpha, \\
U^{\dagger}_{2,0}(\alpha)\beta^{[3]}U_{2,0}(\alpha) &= \beta^{[3]}.
\end{aligned} \tag{4.112}
$$

If

$$\varphi' := U_{2,0}(\alpha)\varphi$$

and

$$j'_{A,k} := \varphi'^{\dagger}\beta^{[k]}\varphi'$$

where ($k \in \{0,1,2,3\}$) then

$$\begin{aligned} j'_{A,0} &= j_{A,0}\cosh\alpha - j_{A,1}\sinh\alpha, \\ j'_{A,1} &= j_{A,1}, \\ j'_{A,2} &= j_{A,2}\cosh\alpha - j_{A,0}\sinh\alpha, \\ j'_{A,3} &= j_{A,3}. \end{aligned} \tag{4.113}$$

Then let

$$\begin{aligned} x'_0 &: = x_0\cosh\alpha - x_3\sinh\alpha, \\ x'_1 &: = x_1, \\ x'_2 &: = x_2, \\ x'_3 &: = x_3\cosh\alpha - x_0\sinh\alpha. \end{aligned} \tag{4.114}$$

Let

$$U_{3,0}(\alpha) := \cosh\frac{\alpha}{2}\cdot 1_4 - \sinh\frac{\alpha}{2}\cdot\beta^{[3]}\beta^{[0]}.$$

That is:

$$U_{3,0}(\alpha) = \begin{bmatrix} e^{\frac{1}{2}\alpha} & 0 & 0 & 0 \\ 0 & e^{-\frac{1}{2}\alpha} & 0 & 0 \\ 0 & 0 & e^{-\frac{1}{2}\alpha} & 0 \\ 0 & 0 & 0 & e^{\frac{1}{2}\alpha} \end{bmatrix}. \tag{4.115}$$

In rhis case:

$$\begin{aligned} U^{\dagger}_{3,0}(\alpha)\beta^{[0]}U_{3,0}(\alpha) &= \beta^{[0]}\cosh\alpha - \beta^{[3]}\sinh\alpha, \\ U_{3,0}(\alpha)\beta^{[1]}U_{3,0}(\alpha) &= \beta^{[1]}, \\ U_{3,0}(\alpha)\beta^{[2]}U_{3,0}(\alpha) &= \beta^{[2]}, \\ U_{3,0}(\alpha)\beta^{[3]}U_{3,0}(\alpha) &= \beta^{[3]}\cosh\alpha - \beta^{[0]}\sinh\alpha. \end{aligned} \tag{4.116}$$

If

$$\varphi' := U_{3,0}(\alpha)\varphi$$

and

$$j'_{A,k} := \varphi'^{\dagger}\beta^{[k]}\varphi'$$

where ($k \in \{0,1,2,3\}$) then

$$\begin{aligned} j'_{A,0} &= j_{A,0}\cosh\alpha - j_{A,3}\sinh\alpha, \\ j'_{A,1} &= j_{A,1}, \\ j'_{A,2} &= j_{A,2}. \\ j'_{A,3} &= j_{A,3}\cosh\alpha - j_{A,0}\sinh\alpha. \end{aligned} \tag{4.117}$$

Function φ submits to the following equation:[21, p.82]

$$\frac{1}{c}\partial_t\varphi - \left(i\Theta_0\beta^{[0]} + i\Upsilon_0\beta^{[0]}\gamma^{[5]}\right)\varphi = \\ = \begin{array}{c} (\sum_{\nu=1}^{3}\beta^{[\nu]}\left(\partial_\nu + i\Theta_\nu + i\Upsilon_\nu\gamma^{[5]}\right) + \\ +iM_0\gamma^{[0]} + iM_4\beta^{[4]} - \\ -iM_{\zeta,0}\gamma_\zeta^{[0]} + iM_{\zeta,4}\zeta^{[4]} - \\ -iM_{\eta,0}\gamma_\eta^{[0]} - iM_{\eta,4}\eta^{[4]} + \\ +iM_{\theta,0}\gamma_\theta^{[0]} + iM_{\theta,4}\theta^{[4]})\varphi \end{array}.$$

That is:

$$\begin{array}{c} (\sum_{\nu=0}^{3}\beta^{[\nu]}\left(\partial_\nu + i\Theta_\nu + i\Upsilon_\nu\gamma^{[5]}\right) + \\ +iM_0\gamma^{[0]} + iM_4\beta^{[4]} - \\ -iM_{\zeta,0}\gamma_\zeta^{[0]} + iM_{\zeta,4}\zeta^{[4]} - \\ -iM_{\eta,0}\gamma_\eta^{[0]} - iM_{\eta,4}\eta^{[4]} + \\ +iM_{\theta,0}\gamma_\theta^{[0]} + iM_{\theta,4}\theta^{[4]})\varphi = 0. \end{array} \tag{4.118}$$

Like coordinates x_5 and x_4 [21, p.83] here are entered new coordinates y^β, z^β, y^ζ, z^ζ, y^η, z^η, y^θ, z^θ such that

$$\begin{array}{rcl} -\frac{\pi c}{h} & \leq & y^\beta \leq \frac{\pi c}{h}, -\frac{\pi c}{h} \leq z^\beta \leq \frac{\pi c}{h}, \\ -\frac{\pi c}{h} & \leq & y^\zeta \leq \frac{\pi c}{h}, -\frac{\pi c}{h} \leq z^\zeta \leq \frac{\pi c}{h}, \\ -\frac{\pi c}{h} & \leq & y^\eta \leq \frac{\pi c}{h}, -\frac{\pi c}{h} \leq z^\eta \leq \frac{\pi c}{h}, \\ -\frac{\pi c}{h} & \leq & y^\theta \leq \frac{\pi c}{h}, -\frac{\pi c}{h} \leq z^\theta \leq \frac{\pi c}{h}. \end{array}$$

and like $\widetilde{\varphi}$, [21, p.83] let:

$$\begin{array}{rl} & [\varphi]\left(t, \mathbf{x}, y^\beta, z^\beta, y^\zeta, z^\zeta, y^\eta, z^\eta, y^\theta, z^\theta\right) := \\ : & = \varphi(t,\mathbf{x}) \times \exp(i(y^\beta M_0 + z^\beta M_4 + y^\zeta M_{\zeta,0} + z^\zeta M_{\zeta,4} + \\ & +y^\eta M_{\eta,0} + z^\eta M_{\eta,4} + y^\theta M_{\theta,0} + z^\theta M_{\theta,4})), \end{array} \tag{4.119}$$

In fhis case if

$$\begin{array}{c} ([\varphi],[\chi]) := \\ := \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dy^\beta \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dz^\beta \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dy^\zeta \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dz^\zeta \times \\ \times \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dy^\eta \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dz^\eta \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dy^\theta \int_{-\frac{\pi c}{h}}^{\frac{\pi c}{h}} dz^\theta \times \\ \times \quad [\varphi]^\dagger [\chi] \end{array} \tag{4.120}$$

then

$$\begin{aligned}\left(\left[\varphi\right],\left[\varphi\right]\right) &= \rho_{\mathcal{A}},\\ \left(\left[\varphi\right],\beta^{[s]}\left[\varphi\right]\right) &= -\frac{j_{\mathcal{A},k}}{\mathrm{c}}.\end{aligned} \tag{4.121}$$

and in this case from (4.118):

$$\begin{array}{c}(\sum\limits_{\nu=0}^{3}\beta^{[\nu]}\left(\partial_{\nu}+\mathrm{i}\Theta_{\nu}+\mathrm{i}\Upsilon_{\nu}\gamma^{[5]}\right)+\\ +\gamma^{[0]}\partial_{y}^{\beta}+\beta^{[4]}\partial_{z}^{\beta}-\\ -\gamma_{\zeta}^{[0]}\partial_{y}^{\zeta}+\zeta^{[4]}\partial_{z}^{\zeta}-\\ -\gamma_{\eta}^{[0]}\partial_{y}^{\eta}-\eta^{[4]}\partial_{z}^{\eta}+\\ +\gamma_{\theta}^{[0]}\partial_{y}^{\theta}+\theta^{[4]}\partial_{z}^{\theta})\left[\varphi\right]=0\end{array}. \tag{4.122}$$

Because

$$\gamma_{\eta}^{[0]}=\begin{bmatrix}0 & 0 & 0 & i\\ 0 & 0 & -i & 0\\ 0 & i & 0 & 0\\ -i & 0 & 0 & 0\end{bmatrix},\eta^{[4]}=\mathrm{i}\begin{bmatrix}0 & 0 & 0 & -i\\ 0 & 0 & i & 0\\ 0 & i & 0 & 0\\ -i & 0 & 0 & 0\end{bmatrix}; \tag{4.123}$$

$$\gamma_{\theta}^{[0]}=\begin{bmatrix}0 & 0 & -1 & 0\\ 0 & 0 & 0 & 1\\ -1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix},\theta^{[4]}=\mathrm{i}\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & -1\\ -1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix} \tag{4.124}$$

$$\gamma_{\zeta}^{[0]}=\begin{bmatrix}0 & 0 & 0 & -1\\ 0 & 0 & -1 & 0\\ 0 & -1 & 0 & 0\\ -1 & 0 & 0 & 0\end{bmatrix},\zeta^{[4]}=\mathrm{i}\begin{bmatrix}0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\\ 0 & -1 & 0 & 0\\ -1 & 0 & 0 & 0\end{bmatrix}; \tag{4.125}$$

then from (4.122):

$$\begin{array}{c}\sum\limits_{\nu=0}^{3}\beta^{[\nu]}\left(\partial_{\nu}+\mathrm{i}\Theta_{\nu}+\mathrm{i}\Upsilon_{\nu}\gamma^{[5]}\right)\left[\varphi\right]\\ +\gamma^{[0]}\partial_{y}^{\beta}\left[\varphi\right]+\beta^{[4]}\partial_{z}^{\beta}\left[\varphi\right]+\\ (\begin{bmatrix}0 & 0 & -\partial_{y}^{\theta} & \partial_{y}^{\zeta}-i\partial_{y}^{\eta}\\ 0 & 0 & \partial_{y}^{\zeta}+i\partial_{y}^{\eta} & \partial_{y}^{\theta}\\ -\partial_{y}^{\theta} & \partial_{y}^{\zeta}-i\partial_{y}^{\eta} & 0 & 0\\ \partial_{y}^{\zeta}+i\partial_{y}^{\eta} & \partial_{y}^{\theta} & 0 & 0\end{bmatrix}+\\ \mathrm{i}\begin{bmatrix}0 & 0 & \partial_{z}^{\theta} & \partial_{z}^{\zeta}+i\partial_{z}^{\eta}\\ 0 & 0 & \partial_{z}^{\zeta}-i\partial_{z}^{\eta} & -\partial_{z}^{\theta}\\ -\partial_{z}^{\theta} & -\partial_{z}^{\zeta}-i\partial_{z}^{\eta} & 0 & 0\\ -\partial_{z}^{\zeta}+i\partial_{z}^{\eta} & \partial_{z}^{\theta} & 0 & 0\end{bmatrix})\\ \times\left[\varphi\right]=0.\end{array} \tag{4.126}$$

Let a Fourier transformation of

$$[\varphi]\left(t,\mathbf{x},y^{\beta},z^{\beta},y^{\zeta},z^{\zeta},y^{\eta},z^{\eta},y^{\theta},z^{\theta}\right)$$

be the following;

$$\begin{aligned}
&[\varphi]\left(t,\mathbf{x},y^{\beta},z^{\beta},y^{\zeta},z^{\zeta},y^{\eta},z^{\eta},y^{\theta},z^{\theta}\right)=\\
=\ &\sum_{w,p_1,p_2,p_3,n^{\beta},s^{\beta},n^{\zeta},s^{\zeta},n^{\eta},s^{\eta},n^{\theta},s^{\theta}} c(w,p_1,p_2,p_3,n^{\beta},s^{\beta},\\
&n^{\zeta},s^{\zeta},n^{\eta},s^{\eta},n^{\theta},s^{\theta})\times\\
&\times\exp(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(wx_0+p_1x_1+p_2x_2+p_3x_3+\\
&+n^{\beta}y^{\beta}+s^{\beta}z^{\beta}\ +n^{\zeta}y^{\zeta}+s^{\zeta}z^{\zeta}+\\
&+n^{\eta}y^{\eta}+s^{\eta}z^{\eta}+n^{\theta}y^{\theta}+s^{\theta}z^{\theta})).
\end{aligned} \tag{4.127}$$

Let in (4.126) $\Theta_{\nu}=0$ and $\Upsilon_{\nu}=0$.

Let us designe:

$$\begin{aligned}
G_0 := (\sum_{\nu=0}^{3}\beta^{[\nu]}\partial_{\nu}+\gamma^{[0]}\partial_y^{\beta}+\beta^{[4]}\partial_z^{\beta}-\\
-\gamma_{\zeta}^{[0]}\partial_y^{\zeta}+\zeta^{[4]}\partial_z^{\zeta}-\\
-\gamma_{\eta}^{[0]}\partial_y^{\eta}-\eta^{[4]}\partial_z^{\eta}+\\
+\gamma_{\theta}^{[0]}\partial_y^{\theta}+\theta^{[4]}\partial_z^{\theta}).
\end{aligned} \tag{4.128}$$

that is:

$$\begin{aligned}
&G_0=\\
&\begin{bmatrix}
-\partial_0+\partial_3 & \partial_1-\mathrm{i}\partial_2 & \partial_y^{\beta}-\partial_y^{\theta} & \partial_y^{\zeta}-i\partial_y^{\eta}\\
\partial_1+\mathrm{i}\partial_2 & -\partial_0-\partial_3 & \partial_y^{\zeta}+i\partial_y^{\eta} & \partial_y^{\beta}+\partial_y^{\theta}\\
\partial_y^{\beta}-\partial_y^{\theta} & \partial_y^{\zeta}-i\partial_y^{\eta} & -\partial_0-\partial_3 & -\partial_1+\mathrm{i}\partial_2\\
\partial_y^{\zeta}+i\partial_y^{\eta} & \partial_y^{\beta}+\partial_y^{\theta} & -\partial_1-\mathrm{i}\partial_2 & -\partial_0+\partial_3
\end{bmatrix}\\
&+\mathrm{i}\begin{bmatrix}
0 & 0 & \partial_z^{\beta}+\partial_z^{\theta} & \partial_z^{\zeta}+i\partial_z^{\eta}\\
0 & 0 & \partial_z^{\zeta}-i\partial_z^{\eta} & \partial_z^{\beta}-\partial_z^{\theta}\\
-\partial_z^{\beta}-\partial_z^{\theta} & -\partial_z^{\zeta}-i\partial_z^{\eta} & 0 & 0\\
-\partial_z^{\zeta}+i\partial_z^{\eta} & -\partial_z^{\beta}+\partial_z^{\theta} & 0 & 0
\end{bmatrix}
\end{aligned} \tag{4.129}$$

$$G_0[\varphi] = -\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}} \sum_{w,p_1,p_2,p_3,n^\beta,s^\beta,n^\zeta,s^\zeta,n^\eta,s^\eta,n^\theta,s^\theta} \breve{g}(w,$$
$$p_1,p_2,p_3,n^\beta,s^\beta,n^\zeta,s^\zeta,n^\eta,s^\eta,n^\theta,s^\theta)$$
$$\sum_{k=0}^{3} c_k(w,p_1,p_2,p_3,n^\beta,s^\beta,n^\zeta,s^\zeta,n^\eta,s^\eta,n^\theta,s^\theta)\times$$
$$\times\exp(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(wx_0+p_1x_1+p_2x_2+p_3x_3+ \tag{4.130}$$
$$+n^\beta y^\beta + s^\beta z^\beta \ + n^\zeta y^\zeta + s^\zeta z^\zeta +$$
$$+n^\eta y^\eta + s^\eta z^\eta + n^\theta y^\theta + s^\theta z^\theta)).$$

here

$$c_k(w,p_1,p_2,p_3,n^\beta,s^\beta,n^\zeta,s^\zeta,n^\eta,s^\eta,n^\theta,s^\theta)$$

is an eigenvector of

$$\breve{g}(w,p_1,p_2,p_3,n^\beta,s^\beta,n^\zeta,s^\zeta,n^\eta,s^\eta,n^\theta,s^\theta)$$

and

$$\breve{g}(w,p_1,p_2,p_3,n^\beta,s^\beta,n^\zeta,s^\zeta,n^\eta,s^\eta,n^\theta,s^\theta) := \tag{4.131}$$
$$:= \beta^{[0]}w+\beta^{[1]}p_1+\beta^{[2]}p_2+\beta^{[3]}p_3+$$
$$+\gamma^{[0]}n^\beta+\beta^{[4]}s^\beta-\gamma_\zeta^{[0]}n^\zeta+\zeta^{[4]}s^\zeta-$$
$$-\gamma_\eta^{[0]}n^\eta-\eta^{[4]}s^\eta+\gamma_\theta^{[0]}n^\theta+\theta^{[4]}s^\theta.$$

Here

$$\{c_0,c_1,c_2,c_3\}$$

is an orthonormalized basis of the complex4-vectors space.

Functions

$$c_k(w,p_1,p_2,p_3,n^\beta,s^\beta,n^\zeta,s^\zeta,n^\eta,s^\eta,n^\theta,s^\theta)\times \tag{4.132}$$
$$\times\exp(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(wx_0+p_1x_1+p_2x_2+p_3x_3+$$
$$+n^\beta y^\beta+s^\beta z^\beta \ +$$
$$+n^\zeta y^\zeta+s^\zeta z^\zeta++n^\eta y^\eta+s^\eta z^\eta+n^\theta y^\theta+s^\theta z^\theta))$$

are eigenvectors of operator G_0.

$$\varphi_y^\zeta := c(w,\mathbf{p},f)\exp(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(wx_0+\mathbf{px}+\gamma_\zeta^{[0]}fy^\zeta))$$

is *a red lower chrome function,*

$$\varphi_z^\zeta := c(w,\mathbf{p},f)\exp(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(wx_0+\mathbf{px}-\mathrm{i}\zeta^{[4]}fz^\zeta))$$

is *a red upper chrome function,*

$$\varphi_y^{\eta} := c(w,\mathbf{p},f)\exp(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(wx_0+\mathbf{px}+\gamma_{\eta}^{[0]}fy^{\eta}))$$

is *a green lower chrome function,*

$$\varphi_z^{\eta} := c(w,\mathbf{p},,f)\exp(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(wx_0+\mathbf{px}-\mathrm{i}\eta^{[4]}fz^{\eta}))$$

is *a green upper chrome function,*

$$\varphi_y^{\theta} := c(w,\mathbf{p},f)\exp(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(wx_0+\mathbf{px}+\gamma_{\theta}^{[0]}fy^{\theta}))$$

is *a blue lower function,*

$$\varphi_z^{\theta} := c(w,\mathbf{p},s^{\theta})\exp(-\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}(wx_0+\mathbf{px}-\mathrm{i}\theta^{[4]}fz^{\theta}))$$

is *a blue upper chrome function.*

Operator $-\partial_y^{\zeta}\partial_y^{\zeta}$ is called *a red lower chrome operator*, $-\partial_z^{\zeta}\partial_z^{\zeta}$ is *a red upper chrome operator,*

$-\partial_y^{\eta}\partial_y^{\eta}$ is called *a green lower chrome operator*, $-\partial_z^{\eta}\partial_z^{\eta}$ is *a green upper chrome operator,*

$-\partial_y^{\theta}\partial_y^{\theta}$ is called *a blue lower chrome operator*, $-\partial_z^{\theta}\partial_z^{\theta}$ is *a blue upper chrome operator*

For example, if φ_z^{ζ} is a red upper chrome function then

$$\begin{aligned}-\partial_y^{\zeta}\partial_y^{\zeta}\varphi_z^{\zeta} &= -\partial_y^{\eta}\partial_y^{\eta}\varphi_z^{\zeta} = -\partial_z^{\eta}\partial_z^{\eta}\varphi_z^{\zeta} = \\ &= -\partial_y^{\theta}\partial_y^{\theta}\varphi_z^{\zeta} = -\partial_z^{\theta}\partial_z^{\theta}\varphi_z^{\zeta} = 0\end{aligned}$$

but

$$-\partial_z^{\zeta}\partial_z^{\zeta}\varphi_z^{\zeta} = -\left(\frac{\mathrm{h}}{\mathrm{c}}f\right)^2\varphi_z^{\zeta}.$$

Because

$$G_0[\varphi] = 0$$

then

$$UG_0U^{-1}U[\varphi] = 0$$

If $U = U_{1,2}(\alpha)$ then $G_0 \to U_{1,2}(\alpha)G_0U_{1,2}^{-1}(\alpha)$ and $[\varphi] \to U_{1,2}(\alpha)[\varphi]$.

In this case:

$\partial_1 \to \partial_1' := (\cos\alpha\cdot\partial_1 - \sin\alpha\cdot\partial_2)$,

$\partial_2 \to \partial_2' := (\cos\alpha\cdot\partial_2 + \sin\alpha\cdot\partial_1)$,

$\partial_0 \to \partial_0' := \partial_0$,

$\partial_3 \to \partial_3' := \partial_3$,

$\partial_y^{\beta} \to \partial_y^{\beta\prime} := \partial_y^{\beta}$,

$\partial_z^{\beta} \to \partial_z^{\beta\prime} := \partial_z^{\beta}$,

$\partial_y^{\zeta} \to \partial_y^{\zeta\prime} := \left(\cos\alpha\cdot\partial_y^{\zeta} - \sin\alpha\cdot\partial_y^{\eta}\right)$,

$\partial_y^\eta \to \partial_y^{\eta\prime} := \left(\cos\alpha\cdot\partial_y^\eta + \sin\alpha\cdot\partial_y^\zeta\right)$,
$\partial_z^\zeta \to \partial_z^{\zeta\prime} := \left(\cos\alpha\cdot\partial_z^\zeta + \sin\alpha\cdot\partial_z^\eta\right)$,
$\partial_z^\eta \to \partial_z^{\eta\prime} := \left(\cos\alpha\cdot\partial_z^\eta - \sin\alpha\cdot\partial_z^\zeta\right)$,
$\partial_y^\theta \to \partial_y^{\theta\prime} := \partial_y^\theta$,
$\partial_z^\theta \to \partial_z^{\theta\prime} := \partial_z^\theta$.
Therefore,

$$\begin{aligned}
-\partial_z^{\zeta\prime}\partial_z^{\zeta\prime}\varphi_z^\zeta &= \left(f\frac{\mathrm{h}}{\mathrm{c}}\cos\alpha\right)^2\cdot\varphi_z^\zeta,\\
-\partial_z^{\eta\prime}\partial_z^{\eta\prime}\varphi_z^\zeta &= \left(-\sin\alpha\cdot f\frac{\mathrm{h}}{\mathrm{c}}\right)^2\varphi_z^\zeta.
\end{aligned}$$

If $\alpha = -\frac{\pi}{2}$ then

$$\begin{aligned}
-\partial_z^{\zeta\prime}\partial_z^{\zeta\prime}\varphi_z^\zeta &= 0,\\
-\partial_z^{\eta\prime}\partial_z^{\eta\prime}\varphi_z^\zeta &= \left(f\frac{\mathrm{h}}{\mathrm{c}}\right)^2\varphi_z^\zeta.
\end{aligned}$$

That is under such rotation the red state becomes the green state.
If $U = U_{3,2}(\alpha)$ then $G_0 \to U_{3,2}(\alpha)\,G_0U_{3,2}^{-1}(\alpha)$ and $[\varphi] \to U_{3,2}(\alpha)[\varphi]$.
In this case:
$\partial_0 \to \partial_0' := \partial_0$,
$\partial_1 \to \partial_1' := \partial_1$,
$\partial_2 \to \partial_2' := (\cos\alpha\cdot\partial_2 + \sin\alpha\cdot\partial_3)$,
$\partial_3 \to \partial_3' := (\cos\alpha\cdot\partial_3 - \sin\alpha\cdot\partial_2)$,
$\partial_y^\beta \to \partial_y^{\beta\prime} := \partial_y^\beta$,
$\partial_y^\zeta \to \partial_y^{\zeta\prime} := \partial_y^\zeta$,
$\partial_y^\eta \to \partial_y^{\eta\prime} := \left(\cos\alpha\cdot\partial_y^\eta - \sin\alpha\cdot\partial_y^\theta\right)$,
$\partial_y^\theta \to \partial_y^{\theta\prime} := \left(\cos\alpha\cdot\partial_y^\theta + \sin\alpha\cdot\partial_y^\eta\right)$,
$\partial_z^\beta \to \partial_z^{\beta\prime} := \partial_z^\beta$,
$\partial_z^\zeta \to \partial_z^{\zeta\prime} := \partial_z^\zeta$,
$\partial_z^\eta \to \partial_z^{\eta\prime} := \left(\cos\alpha\cdot\partial_z^\eta - \sin\alpha\cdot\partial_z^\theta\right)$,
$\partial_z^\theta \to \partial_z^{\theta\prime} := \left(\cos\alpha\cdot\partial_z^\theta + \sin\alpha\cdot\partial_z^\eta\right)$,
Therefore, if φ_y^η is a green lower chrome function then

$$\begin{aligned}
-\partial_z^{\eta\prime}\partial_z^{\eta\prime}\varphi_y^\eta &= \left(\frac{\mathrm{h}}{\mathrm{c}}\cos\alpha\cdot f\right)^2\cdot\varphi_y^\eta,\\
-\partial_y^{\theta\prime}\partial_y^{\theta\prime}\varphi_y^\eta &= \left(\frac{\mathrm{h}}{\mathrm{c}}\sin\alpha\cdot f\right)^2\cdot\varphi_y^\eta.
\end{aligned}$$

If $\alpha = \pi/2$ then

$$
\begin{aligned}
-\partial_z^{\eta\prime}\partial_z^{\eta\prime}\varphi_y^{\eta} &= 0, \\
-\partial_y^{\theta\prime}\partial_y^{\theta\prime}\varphi_y^{\eta} &= \left(\frac{\mathrm{h}}{\mathrm{c}}f\right)^2\cdot\varphi_y^{\eta}.
\end{aligned}
$$

That is under such rotation the green state becomes blue state.

If $U=U_{3,1}(\alpha)$ then $G_0\to U_{3,1}(\alpha)G_0U_{3,1}^{-1}(\alpha)$ and $[\varphi]\to U_{3,1}(\alpha)[\varphi]$.

In this case:

$\partial_0\to\partial_0' := \partial_0$,
$\partial_1\to\partial_1' := (\cos\alpha\cdot\partial_1-\sin\alpha\cdot\partial_3)$,
$\partial_2\to\partial_2' := \partial_2$,
$\partial_3\to\partial_3' := (\cos\alpha\cdot\partial_3+\sin\alpha\cdot\partial_1)$,
$\partial_y^{\beta}\to\partial_3' := \partial_y^{\beta}$,
$\partial_y^{\zeta}\to\partial_y^{\zeta\prime} := \left(\cos\alpha\cdot\partial_y^{\zeta}+\sin\alpha\cdot\partial_y^{\theta}\right)$,
$\partial_y^{\eta}\to\partial_y^{\eta\prime} := \partial_y^{\eta}$,
$\partial_y^{\theta}\to\partial_y^{\theta\prime} := \left(\cos\alpha\cdot\partial_y^{\theta}-\sin\alpha\cdot\partial_y^{\zeta}\right)$,
$\partial_z^{\beta}\to\partial_z^{\beta\prime} := \partial_z^{\beta}$,
$\partial_z^{\zeta}\to\partial_z^{\zeta\prime} := \left(\cos\alpha\cdot\partial_z^{\zeta}-\sin\alpha\cdot\partial_z^{\theta}\right)$,
$\partial_z^{\eta}\to\partial_z^{\eta\prime} := \partial_z^{\eta}$,
$\partial_z^{\theta}\to\partial_z^{\theta\prime} := \left(\cos\alpha\cdot\partial_z^{\theta}+\sin\alpha\cdot\partial_z^{\zeta}\right)$.

Therefore,

$$
\begin{aligned}
-\partial_z^{\zeta\prime}\partial_z^{\zeta\prime}\varphi_z^{\zeta} &= -\left(f\frac{\mathrm{h}}{\mathrm{c}}\cos\alpha\right)^2\cdot\varphi_z^{\zeta}, \\
-\partial_z^{\theta\prime}\partial_z^{\theta\prime}\varphi_z^{\zeta} &= -\left(\sin\alpha\cdot f\frac{\mathrm{h}}{\mathrm{c}}\right)^2\varphi_z^{\zeta}.
\end{aligned}
$$

If $\alpha=\pi/2$ then

$$
\begin{aligned}
-\partial_z^{\zeta\prime}\partial_z^{\zeta\prime}\varphi_z^{\zeta} &= 0, \\
-\partial_z^{\theta\prime}\partial_z^{\theta\prime}\varphi_z^{\zeta} &= -\left(f\frac{\mathrm{h}}{\mathrm{c}}\right)^2\varphi_z^{\zeta}.
\end{aligned}
$$

That is under such rotation the red state becomes the blue state. Thus at the Cartesian turns chrome of a state is changed.

One of ways of elimination of this noninvariancy consists in the following. Calculations in [21, p.156] give the grounds to assume that some oscillations of quarks states bend time-space in such a way that acceleration of the bent system in relation to initial system submits to the following law (Figure 19):

$$g(t,\mathbf{x}) = \mathrm{c}\lambda/\left(\mathbf{x}^2\cosh^2\left(\lambda t/\mathbf{x}^2\right)\right).$$

Here the acceleration plot is line (1) and the line (2) is plot of $\lambda/\mathbf{x}^2$.

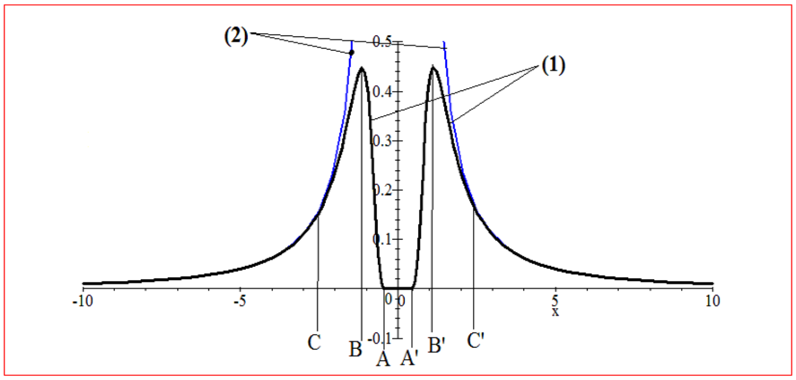


Figure 19:

Hence, to the right from point C' and to the left from poin C t*he Newtonian gravitation law* is carried out.

AA' is *the Asymptotic Freedom Zone*.

CB and $B'C'$ is *the Confinement Zone.*

Let in the potential hole AA' there are three quarks φ_y^ζ, φ_y^η, φ_y^θ. Their general state function is determinant with elements of the following type: $\varphi_y^{\zeta\eta\theta} := \varphi_y^\zeta\varphi_y^\eta\varphi_y^\theta$. In this case:

$$-\partial_y^\zeta\partial_y^\zeta\varphi_y^{\zeta\eta\theta} = \left(\frac{\mathrm{h}}{\mathrm{c}}f\right)^2 \varphi_y^{\zeta\eta\theta}$$

and under rotation $U_{1,2}(\alpha)$:

$$\begin{aligned}
-\partial_y^{\zeta\prime}\partial_y^{\zeta\prime}\varphi_y^{\zeta\eta\theta} &= \\
&= \left(\frac{\mathrm{h}}{\mathrm{c}}f\right)^2 \left(\gamma_\zeta^{[0]}\cos\alpha - \gamma_\eta^{[0]}\sin\alpha\right)^2 \left(\varphi_y^\zeta\varphi_y^\eta\varphi_y^\theta\right) = \\
&= \left(\frac{\mathrm{h}}{\mathrm{c}}f\right)^2 \varphi_y^{\zeta\eta\theta}.
\end{aligned}$$

That is at such turns the quantity of red chrome remains.

As and for all other Cartesian turns and for all other chromes.

Baryons $\Delta^- = ddd$, $\Delta^{++} = uuu$, $\Omega^- = sss$ belong to such structures.

If $U = U_{1,0}(\alpha)$ then $G_0 \to U_{1,0}^{-1\ddagger}(\alpha)\, G_0 U_{1,0}^{-1}(\alpha)$ and $[\varphi] \to U_{1,0}(\alpha)[\varphi]$.

In this case:

$\partial_0 \to \partial_0' := (\cosh\alpha\cdot\partial_0 + \sinh\alpha\cdot\partial_1)$,

$\partial_1 \to \partial_1' := (\cosh\alpha\cdot\partial_1 + \sinh\alpha\cdot\partial_0)$,

$\partial_2 \to \partial_2' := \partial_2$,

$\partial_3 \to \partial_3' := \partial_3$,

$\partial_y^\beta \to \partial_y^{\beta\prime} := \partial_y^\beta$,

$\partial_y^\zeta \to \partial_y^{\zeta\prime} := \partial_y^\zeta$,

$\partial_y^\eta \to \partial_y^{\eta\prime} := \left(\cosh\alpha\cdot\partial_y^\eta - \sinh\alpha\cdot\partial_z^\theta\right)$,

$\partial_y^\theta \to \partial_y^{\theta\prime} := \left(\cosh\alpha \cdot \partial_y^\theta + \sinh\alpha \cdot \partial_z^\eta\right),$
$\partial_z^\beta \to \partial_z^{\beta\prime} := \partial_z^\beta,$
$\partial_z^\zeta \to \partial_z^{\zeta\prime} := \partial_z^\zeta,$
$\partial_z^\eta \to \partial_z^{\eta\prime} := \left(\cosh\alpha \cdot \partial_z^\eta + \sinh\alpha \cdot \partial_y^\theta\right),$
$\partial_z^\theta \to \partial_z^{\theta\prime} := \left(\cosh\alpha \cdot \partial_z^\theta - \sinh\alpha \cdot \partial_y^\eta\right).$

Therefore,

$$\begin{aligned} -\partial_y^{\eta\prime}\partial_y^{\eta\prime}\varphi_y^\eta &= \left(1+\sinh^2\alpha\right)\cdot\left(\frac{\mathrm{h}}{\mathrm{c}}f\right)^2\varphi_y^\eta, \\ -\partial_z^{\theta\prime}\partial_z^{\theta\prime}\varphi_y^\eta &= \sinh^2\alpha\cdot\left(\frac{\mathrm{h}}{\mathrm{c}}f\right)^2\varphi_y^\eta. \end{aligned}$$

Similarly chromes and grades change for other states and under other Lorentz transformation.

One of ways of elimination of this noninvariancy is the following:

Let

$$\varphi_{yz}^{\zeta\eta\theta} := \varphi_y^\zeta\varphi_y^\eta\varphi_y^\theta\varphi_z^\zeta\varphi_z^\eta\varphi_z^\theta.$$

Under transformation $U_{1,0}(\alpha)$:

$$-\partial_z^{\theta\prime}\partial_z^{\theta\prime}\varphi_{yz}^{\zeta\eta\theta} = -\left(\mathrm{i}\frac{\mathrm{h}}{\mathrm{c}}f\right)^2\varphi_{yz}^{\zeta\eta\theta}.$$

That is a magnitude of red chrome of this state doesn't depend on angle α.

This condition is satisfied for all chromes and under all Lorentz's transformations.

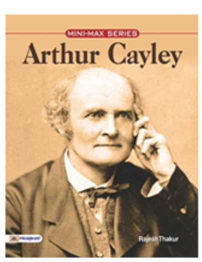
Pairs of baryons

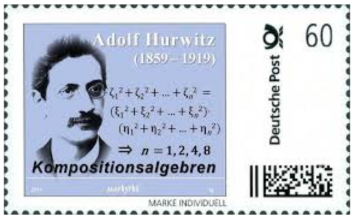
$$\begin{aligned} &\{p = uud, n = ddu\}, \\ &\left\{\Sigma^+ = uus, \Xi^0 = uss\right\}, \\ &\left\{\Delta^+ = uud, \Delta^0 = udd\right\} \end{aligned}$$

belong to such structures.

Baryons represent one of ways of elimination of the chrome noninvariancy under Cartesian's and under Lorentz's transformations.

4.17. Dimension of physical space





Further I use Cayley-Dickson algebras [23, 24]:

Let $1, i, j, k, E, I, J, K$ be basis elements of a 8-dimensional algebra Cayley (*the octavians algebra*) [23, 24]. A product of this algebra is defined the following way [23]:

1. for every basic element e:

$$\mathrm{ee} = -1;$$

2. If u_1, u_2, v_1, v_2 are real number then

$$(u_1 + u_2\mathrm{i})(v_1 + v_2\mathrm{i}) = (u_1v_1 - v_2u_2) + (v_2u_1 + u_2v_1)\,\mathrm{i}.$$

3. If u_1, u_2, v_1, v_2 are numbers of shape $w = w_1 + w_2\mathrm{i}$ (w_s, and $s \in \{1, 2\}$ are real numbers, and $\overline{w} = w_1 - w_2\mathrm{i}$) then

$$(u_1 + u_2\mathrm{j})(v_1 + v_2\mathrm{j}) = (u_1v_1 - \overline{v}_2u_2) + (v_2u_1 + u_2\overline{v}_1)\,\mathrm{j} \tag{4.133}$$

and $\mathrm{ij} = \mathrm{k}$.

4. If u_1, u_2, v_1, v_2 are number of shape $w = w_1 + w_2\mathrm{i} + w_3\mathrm{j} + w_4\mathrm{k}$ (w_s, and $s \in \{1, 2, 3, 4\}$ are real numbers, and $\overline{w} = w_1 - w_2\mathrm{i} - w_3\mathrm{j} - w_4\mathrm{k}$) then

$$(u_1 + u_2\mathrm{E})(v_1 + v_2\mathrm{E}) = (u_1v_1 - \overline{v}_2u_2) + (v_2u_1 + u_2\overline{v}_1)\,\mathrm{E} \tag{4.134}$$

and

$$\begin{aligned} \mathrm{iE} &= \mathrm{I}, \\ \mathrm{jE} &= \mathrm{J}, \\ \mathrm{kE} &= \mathrm{K}. \end{aligned}$$

Therefore, in according with point 2.: the real numbers field (**R**) is extended to the complex numbers field (**R**), and in according with point 3.: the complex numbers field is expanded to the quaternions field (**K**), and point 4. expands the quaternions fields to the octavians field (**O**). This method of expanding of fields is called *a Dickson doubling procedure* [23].

If

$$u = a + b\mathrm{i} + c\mathrm{j} + d\mathrm{k} + A\mathrm{E} + B\mathrm{I} + C\mathrm{J} + \mathrm{K}$$

with real a, b, c, d, A, B, C, D then a real number

$$\|u\| \overset{def}{=} \sqrt{u\overline{u}} = \left(a^2 + b^2 + c^2 + d^2 + A^2 + B^2 + C^2 + D^2\right)^{0.5}$$

is called *a norm* of octavian u [23].

For each octavians u and v:

$$\|uv\| = \|u\|\,\|v\|\,. \tag{4.135}$$

Algebras with this conditions are called *normalized algebras* [23, 24].

Any 3+1-vector of a probability density can be represented by the following equations in matrix form (4.75)

$$\rho = \varphi^{\dagger}\varphi,$$
$$j_k = \varphi^{\dagger}\beta^{[k]}\varphi$$

with $k \in \{1,2,3\}$.

There $\beta^{[k]}$ are complex 2-diagonal 4×4-matrices of Clifford's set of rank 4, and φ is matrix columns with four complex components. The light and colored pentads of Clifford's set of such rank contain in threes 2-diagonal matrices, corresponding to 3 space coordinates in according with Dirac's equation. Hence, a space of these events is 3-dimensional.

Let $\rho(t,\mathbf{x})$ be a probability density of event $A(t,\mathbf{x})$, and

$$\rho_c(t,\mathbf{x}|t_0,\mathbf{x}_0)$$

be a probability density of event $A(t,\mathbf{x})$ on condition that event $B(t_0,\mathbf{x}_0)$.

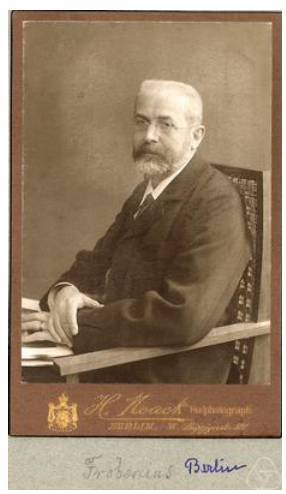
In that case if function $q(t,\mathbf{x}|t_0,\mathbf{x}_0)$ is fulfilled to condition:

$$\rho_c(t,\mathbf{x}|t_0,\mathbf{x}_0) = q(t,\mathbf{x}|t_0,\mathbf{x}_0)\rho(t,\mathbf{x}), \tag{4.136}$$

then one is called *a disturbance function* B to A.

If $q = 1$ then B does not disturbance to A.

A conditional probability density of event $A(t,\mathbf{x})$ on condition that event $B(t_0,\mathbf{x}_0)$ is presented as:



$$\rho_c = \varphi_c^{\dagger}\varphi_c$$

like to a probability density of event $A(t,\mathbf{x})$.

Let

$$\varphi = \begin{bmatrix} \varphi_{1,1}+\mathrm{i}\varphi_{1,2} \\ \varphi_{2,1}+\mathrm{i}\varphi_{2,2} \\ \varphi_{3,1}+\mathrm{i}\varphi_{3,2} \\ \varphi_{4,1}+\mathrm{i}\varphi_{4,2} \end{bmatrix}$$

and

$$\varphi_c = \begin{bmatrix} \varphi_{c,1,1}+\mathrm{i}\varphi_{c,1,2} \\ \varphi_{c,2,1}+\mathrm{i}\varphi_{c,2,2} \\ \varphi_{c,3,1}+\mathrm{i}\varphi_{c,3,2} \\ \varphi_{c,4,1}+\mathrm{i}\varphi_{c,4,2} \end{bmatrix}$$

(all $\varphi_{r,s}$ and $\varphi_{c,r,s}$ are real numbers).

In that case octavian

$$u = \varphi_{1,1}+\varphi_{1,2}\mathrm{i}+\varphi_{2,1}\mathrm{j}+\varphi_{2,2}\mathrm{k}+\varphi_{3,1}\mathrm{E}+\varphi_{3,2}\mathrm{I}+\varphi_{4,1}\mathrm{J}+\varphi_{4,2}\mathrm{K}$$

is called *a Caylean* of φ. Therefore, octavian

$u_c = \varphi_{c,1,1} + \varphi_{c,1,2}\mathrm{i} + \varphi_{c,2,1}\mathrm{j} + \varphi_{c,2,2}\mathrm{k} + \varphi_{c,3,1}\mathrm{E} + \varphi_{c,3,2}\mathrm{I} + \varphi_{c,4,1}\mathrm{J} + \varphi_{c,4,2}\mathrm{K}$
is Caylean[51] of φ_c.

In accordance with the octavian norm definition:

$$\begin{aligned} \|u_c\|^2 &= \rho_c \\ \|u\|^2 &= \rho \end{aligned} \tag{4.137}$$

Because the octavian algebra is a division algebra [23, 24] then for each octavians u and u_c there exists an octavian w such that

$$u_c = wu,$$

Because the octavians algebra is normalized then

$$\|u_c\|^2 = \|w\|^2 \|u\|^2 .$$

Hence, from (4.136) and (4.137):

$$q = \|w\|^2 .$$

Therefore, in a 3+1-dimensional space-time there exists an octavian-Caylean for a disturbance function of any event to any event.

In order to increase a space dimensionality the octavian algebra can be expanded by a Dickson doubling procedure:

Another 8 elements should be added to basic octavians:

$$\mathrm{z}_1, \mathrm{z}_2, \mathrm{z}_3, \mathrm{z}_4, \mathrm{z}_5, \mathrm{z}_6, \mathrm{z}_7, \mathrm{z}_8,$$

such that:

$$\begin{aligned} \mathrm{z}_2 &= \mathrm{i}\mathrm{z}_1, \\ \mathrm{z}_3 &= \mathrm{j}\mathrm{z}_1, \\ \mathrm{z}_4 &= \mathrm{k}\mathrm{z}_1, \\ \mathrm{z}_5 &= \mathrm{E}\mathrm{z}_1, \\ \mathrm{z}_6 &= \mathrm{I}\mathrm{z}_1, \\ \mathrm{z}_7 &= \mathrm{J}\mathrm{z}_1, \\ \mathrm{z}_8 &= \mathrm{K}\mathrm{z}_1, \end{aligned}$$

and for every octavians u_1, u_2, v_1, v_2:

$$(u_1 + u_2\mathrm{z}_1)(v_1 + v_2\mathrm{z}_1) = (u_1v_1 - \overline{v}_2u_2) + (v_2u_1 + u_2\overline{v}_1)\mathrm{z}_1$$

(here: if $w = w_1 + w_2\mathrm{i} + w_3\mathrm{j} + w_4\mathrm{k} + w_5\mathrm{E} + w_6\mathrm{I} + w_7\mathrm{J} + w_8\mathrm{K}$ with real w_s then $\overline{w} = w_1 - w_2\mathrm{i} - w_3\mathrm{j} - w_4\mathrm{k} - w_5\mathrm{E} - w_6\mathrm{I} - w_7\mathrm{J} - w_8\mathrm{K}$).

It is a 16-dimensional Cayley-Dickson algebra.

In according with [32]: for any natural number z there exists a Clifford set of rank 2^z. In considering case for $z = 3$ there is Clifford's seven:

[51] Arthur Cayley FRS (16 August 1821 26 January 1895) was a prolific British mathematician who worked mostly on alge

$$\begin{aligned}
\underline{\beta}^{[1]} &= \begin{bmatrix} \beta^{[1]} & 0_4 \\ 0_4 & -\beta^{[1]} \end{bmatrix}, \underline{\beta}^{[2]} = \begin{bmatrix} \beta^{[2]} & 0_4 \\ 0_4 & -\beta^{[2]} \end{bmatrix}, \\
\underline{\beta}^{[3]} &= \begin{bmatrix} \beta^{[3]} & 0_4 \\ 0_4 & -\beta^{[3]} \end{bmatrix}, \underline{\beta}^{[4]} = \begin{bmatrix} \beta^{[4]} & 0_4 \\ 0_4 & -\beta^{[4]} \end{bmatrix}, \\
\underline{\beta}^{[5]} &= \begin{bmatrix} \gamma^{[0]} & 0_4 \\ 0_4 & -\gamma^{[0]} \end{bmatrix},
\end{aligned} \tag{4.138}$$

$$\underline{\beta}^{[6]} = \begin{bmatrix} 0_4 & 1_4 \\ 1_4 & 0_4 \end{bmatrix}, \underline{\beta}^{[7]} = \mathrm{i} \begin{bmatrix} 0_4 & -1_4 \\ 1_4 & 0_4 \end{bmatrix}, \tag{4.139}$$

Therefore, in this seven five 4-diagonal matrices (4.138) define a 5-dimensio-nal space of events, and two 4-antidiagonal matrices (4.139) defined a 2-dimensi-onal space for the electroweak transformations.

It is evident that such procedure of dimensions building up can be continued endlessly. But in accordance with the Hurwitz[52] theorem[53] and with the generalized Frobenius[54] theorem[55] a more than 8-dimensional Cayley-Dickson algebra does not a division algebra. Hence, there in a more than 3-dimensional space exist events such that a disturbance function between these events does not hold a Caylean. I call such disturbance *supernatural*.

Therefore, supernatural disturbance do not exist in a 3-dimensional space, but in a more than 3-dimensional space such supernatural disturbance act.

[52]Adolf Hurwitz (26 March 1859 18 November 1919) was a German mathematician who worked on algebra, analysis, geometry and number theory.

[53]Every normalized algebra with unit is isomorphous to one of the following: the real numbers algebra **R**, the complex numbers algebra **C**, the quaternions algebra **K**, the octavians algebra **O** [23]

[54]Ferdinand Georg Frobenius (26 October 1849 3 August 1917) was a German mathematician, best known for his contributions to the theory of elliptic functions, differential equations, number theory, and to group theory.

[55]A division algebra can be only either 1 or 2 or 4 or 8-dimensional [24]

Conclusion

Models are not needed, because fundamental theoretical physics is part of probability theory.

Physics is a game of probabilities in space-time. Irreversible unidirectional time and metric space is an essential attribute of any information system, and probability is the logic of events that have not yet occurred.

References

[1] Gunn Quznetsov (2017), Farewell to Higgs

[2] Gunn Quznetsov (2018), Runii: no Susy, no Wimp, no Higgs, no New Physics

[3] Gunn Quznetsov, All Elementary Bosons are Gaugehttp: , American Journal of Modern Physics. Special Issue: Physics Without Higgs and Without Supersymmetry. Vol. 5, No. 1-1, 2016, pp. 1-7. doi: 10.11648/j.ajmp.s.2016050101.11

[4] 4. Tarski, A. *The Semantic Conception of Truth and the Foundations of Semantics*, Philosophy and Phenomenological Research, 4, 1944.

[5] Eubulides of Miletus,it Liar paradox, . 4th century BCE

[6] Urysohn P. *Zum Metrisationsproblem*. Math. Ann., 1925, v. 94, 309315.

[7] For instance, Madelung, E., *Die Mathematischen Hilfsmittel des Physikers.* Springer Verlag, (1957) p.29

[8] Edwin Thompson Jaynes. *Probability Theory: The Logic of Science*. Cambridge University Press, (2003). ISBN 0-521-592 71

[9] 12. Gentzen, *Untersuchungen Uber das Logische Schliessen*, Math. Z. 39, (1934), pp. 176-210, 405-431

[10] Bernoulli J., *Ars Conjectandi*, (BASILEA, Impenfis THURNISORUM, Fratrum, 1713)

[11] A. Kolmogoroff (1933). *Grundbegriffe der Wahrscheinlichkeitsrechnung*. An English translation by Nathan Morrison appeared under the title *Foundations of the Theory of Probability* (Chelsea, New York) in 1950, with a second edition in 1956

[12] Edwin Thompson Jaynes. *Probability Theory: The Logic of Science*. Cambridge University Press, (2003)

[13] Feynman, R. P., *The Theory of Fundamentaln Processes*, W. A. Benjamin, INC., NY, (1961), Feynman, R. P., Leighton, R. B., Sands, M., *The Feynman Lectures on Physics*, vol.3, ch.37 (Quantum behaviour) Addison-Wesley Publishing Company, INC., (1963)

[14] Tonomura, A., Endo, J., Matsuda, T., Kawasaki, T. and Ezawa, H. Demonstration of single-electron buildup of an interference pattern, *Am. J. Phys.* (1989), 57, 117-120
Double-slit experiment, http://www.hqrd.hitachi.co.jp/em/doubleslit.cfm Hitachi

[15] Planck, M. (1901). *Ueber das Gesetz der Energieverteilung im Normalspektrum*. Annalen der Physik. 309 (3): 553?563.

[16] For instance, Peak L. and Varvell, K., *The Physics of The Standard Model*, part 2. (2002), p.37

[17] Sierpinski Waclaw, Pythagorean Triangles, Dover, 2003.

[18] For instance, Peskin M. E., Schroeder D. V. *An Introduction to Quantum Field Theory*, Perseus Books Publishing, L.L.C., 1995.

[19] Gordon Kane, *Modern Elementary Particle Physics*, Addison-Wesley Publ. Comp., (1993), p.93

[20] Sheldon Lee Glashow, A Guide to Weak Interactions, in *Subnuclear Phenomena*, ed. A.Zichichi (Academic Press, 1970) B-330

[21] Quznetsov G 2013 Logical foundation of fundamental theoretical physics (Lambert Academic Publ.)

[22] Amsler C. et al. (Particle Data Group). Review of particle physics — quark model. *Physics Letters B*, 2008, v. 667, 1.

[23] {M. L. Cantor, A. S. Solodovnikov, *Hipercomplex numbers*, Moscow, (1973), p.99}, Kantor, I. L.; Solodownikow, A. S. (1978), *Hyperkomplexe Zahlen*, Leipzig: B.G. Teubner

[24] O. V. Mel'nikov, V. N. Remeslennikov, et el., *General Algebra*, Moscow, (1990), p.396

[25] Riess, A. et al. *Astronomical Journal*, 1998, v.116, 1009-1038.

[26] Spergel, D. N., et al. *The Astrophysical Journal Supplement Series*, 2003 September, v.148, 175-194. Chaboyer, B. and L. M. Krauss, *Astrophys. J. Lett.*, 2002, L45, 567. Astier, P., et al. *Astronomy and Astrophysics*, 2006, v.447, 31-48. Wood-Vasey, W. M., et al., *The Astrophysical Journal*, 2007, v. 666, Issue 2, 694-715.

[27] Peter Coles, ed., Routledge Critical Dictionary of the New Cosmology. Routledge, 2001, 202.

[28] "Chandra Confirms the Hubble Constant". 2006-08-08. chandra/2006/08/08/ chandra-confirms-the-hubble-constant/ . Retrieved 2007-03-07.

[29] Miller, Chris. Cosmic Hide and Seek: the Search for the Missing Mass. DM. And see Van Den Bergh, Sidney. The early history of Dark Matter. preprint, astro-ph/9904251.

[30] Hawley, J.F. and K.A. Holcomb. Foundations of modem cosmology. Oxford University Press, New York, 1998.

[31] K. G. Begeman, A. H. Broeils and R. H. Sanders, 1991, MNRAS, 249, 523

[32] V. A. Zhelnorovich, *Theory of spinors. Application to mathematics and physics*, Moscow, (1982), p.21.

Index

B-boson field, 107, 140
W-boson, 137, 139, 140
Z-boson, 140
3+1 vector, 84
3+1-vector, 172, 188

Alfred Tarski, 1
algebra Cayley, 187
antithetical events, 63
Asymptotic Freedom, 162, 185

Baryon, 172, 185, 186
basis, 116
Bernoulli, Jacob, 64
Bethe, Hans, 81
Bohr, Niels, 81
Boole, George, 51
Boolean function, 51–60, 69
Born, Max, 81
boson, 139

cartesian, 84
Cartesian turns, 184, 185
Cauchy-Weierstrass ”limit” , 69
Caylean, 188–190
Cayley-Dickson algebras, 187, 189, 190
charge, 102, 111, 140
chromatic, 38, 101, 148, 165, 167, 171
chromatic pentad, 38, 101
chrome, 160, 161, 181–184, 186
Clifford pentad, 38, 101
Clifford set, 38
Clifford, W. , 38
clock, 4, 6, 7, 9, 27
Confinement, 162, 164, 185
confinement, 171
conjunction, 45, 46, 49
continuity, 84
coordinates, 90, 101
Cowan, Clyde, 141
Cumulative Distribution Function, 81

density, 120
direction, 6
disjunction, 45, 46, 49, 64
distance length, 12, 16, 21
disturbance, 188–190
division algebra, 189, 190
DORIS, 148
Dyson, Freeman, 81

eigenvalue, 115, 116, 124
eigenvector, 115, 116, 124
Einstein, Albert, 162
electric charge, 161, 162
electromagnetic, 141
elementary charge, 140
energy, 111, 141
entangle, 98
Epimenides paradox, 1
equation of motion, 127
Equation of moving, 106
equation of moving, 128, 130, 138, 140, 148
event, 3, 62–66, 68, 69, 86, 88–90, 93, 95–98, 101, 120, 121, 123, 148

false, 1, 3, 12, 45, 48, 49, 68
Fermi, E., 141
Fermi, Enrico, 141
Feynman, Richard, 81
Fourier, 88, 108, 117
Fourier series, 88

frame of reference, 16, 17, 19, 21–23, 27, 32, 39
frequency, 69
Friedman, Jerome, 162

Galilei, Galileo, 162
Gell-Mann, Murray, 148
Gentzen, 45, 46
Glashow, S. L., 107
Glashow, Sheldon, 123, 128
gluon, 148, 160
Gordon, W, 114
gravitation, 162, 167, 185
gustatory pentad, 39

Hamilton, W., 85
Hamiltonian, 85, 87, 99, 101, 102, 112, 128
Heisenberg, Werner, 81
HIGGS, v, 123
Hubble, Edwin, 164

implication, 45, 46, 49
independent, 63, 64, 66
infinite, 74
infinitesimal, 74
instant, 4, 6, 16, 22, 23, 27, 32, 39, 89, 90, 94, 95, 97, 98
internally stationary system, 9

Kalmar, Laszlo, 59
Kendall, Henry, 162
Klein, O, 114
Klein-Gordon equation, 114, 138

Large Number Law, 62
lepton, 99, 101, 102, 111, 127, 128, 162
light pentad, 38, 101
logical rules, 46, 47, 49, 52
Lorentz transformation, 36, 139, 163
Lorentz, Hrndrik, 36
lower, 161

mass, 111, 112, 114, 138, 139, 141
mass number, 108
matrix, 37, 86, 88, 102, 124, 134
Maxwell, 79, 141
Maxwell, J. C., 141
meaningful, 62
metrical space, 12
Mills, Robert, 81

negation, 45–47, 50, 62
neutrino, 111, 141
Newton, Isaak, 162
Newton Gravity, 164
NONSTANDARD NUMBERS, 69
normalized, 124

octavian, 187–190
operator, 85–88, 104, 106, 118, 120, 127, 129
orthonormal, 116

part-set, 69
Pauli, W., 141
Pauli, Wolfgang, 81
PETRA, 148
Planck, Max, 81
PLUTO, 148
Pontecorvo, Bruno, 141
probabilities addition formula, 98
probability density, 82
probability function, 68, 69
Pythagor, 110, 111

Q-equivalent, 70, 75
Q-extension, 72, 75
Q-number, 70–74
quark, 111, 123, 148, 162, 164, 171, 184, 185

rank, 38
recorder, 3, 6, 7, 9, 10, 12, 16, 17, 19, 21, 32
red chrome, 185, 186
Reines, Frederick, 141
Robinson, 70

scalar product, 107
Schrodinger, Erwin, 81
Schwinger, Julian, 81
sentence, 3, 5, 6, 9, 12, 15, 16, 45–59, 62, 64, 66, 68, 90, 97

sequence, 12, 46–50, 52–59, 70–76
standard, 70, 72, 74
standard , 71
Standard Model, 107, 111, 132, 140
state vector, 86, 90, 93, 95–97
supernatural, 190
SUSY, v

tautology, 52, 54, 59, 62
Taylor, Richard, 162
Thomson, J. J., 140
time, 87, 89, 90, 108
Tomonaga, Sin-Itiro, 81
trackeable, 82, 83
true, 1, 45, 49, 50, 52–58, 128
truth, 45, 46, 50, 59, 62, 64, 66, 68

unit operator, 87
unitary, 107
upper, 161
Urysohn, Pavel, 21

weak interaction, 141
Weinberg, S., 139
Wigner, Eugene, 81

Yang, Chen-Ning, 81

Zweig, George, 148
Zwicky, Fritz, 167