On some Ramanujan equations: new possible mathematical connections with ϕ, $\zeta(2)$, Hausdorff dimension values, several equations of Teleparallel Cosmology and Higher-Spin Interactions in String Theory

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Abstract

In this paper we have described some Ramanujan equations and obtained new possible mathematical connections with ϕ, $\zeta(2)$*, Hausdorff dimension values, several equations of Teleparallel Cosmology and Higher-Spin Interactions in String Theory*

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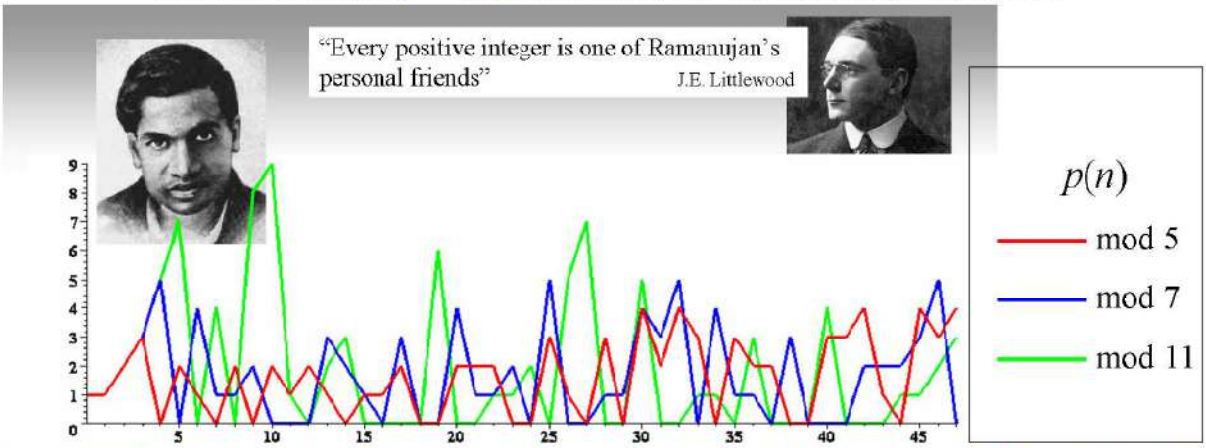
The Ramanujan Partition Congruences *Let n be a non-negative integer and let $p(n)$ denote the number of partitions of n (that is, the number of ways to write n as a sum of positive integers). Then $p(n)$ satisfies the congruence relations:*

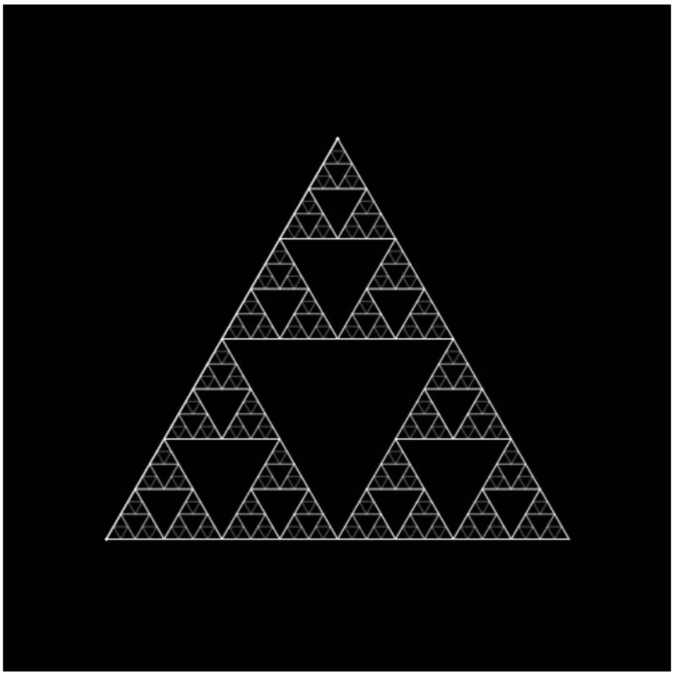
$$p(5t+4) \equiv 0 \pmod{5}, \qquad p(7t+5) \equiv 0 \pmod{7}, \quad \textit{and} \quad p(11t+6) \equiv 0 \pmod{11}.$$



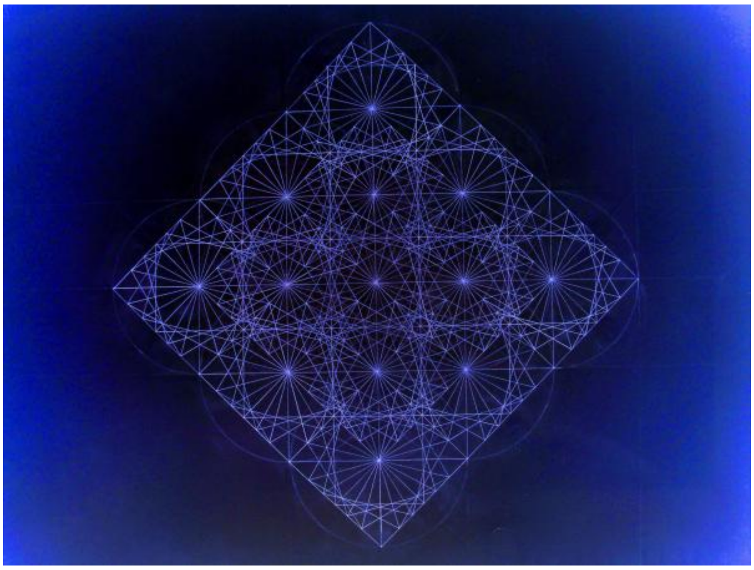
Ramanujan's congruences tell us that, in the set of values of n for which $p(n) \bmod q = 0$, when q is 5, 7 or 11, there is an infinite arithmetic progression of common difference q. Thus we see that, in the above plot, the three graphs touch the horizontal axis at intervals which appear quite irregular but are certainly constrained by this arithmetic progression property. The property extends to all primes $q \geq 5$, a deep result published in 2000 by Ken Ono, but the common differences will not generally be q: the set of values of n for which $p(n) \bmod 31 = 0$, for instance, contains an infinite arithmetic progression whose common difference is not 31 but 31×107^4, and which starts at $n = 30064597$. For $q = 3$, the situation is very different—it is not even known if the values of n for which $p(n) \bmod 3 = 0$ form an infinite set!

Ramanujan published proofs of the congruences for 5 and 7 in 1919. His proof for mod 11 remained unpublished at the time of his death in 1920 and was written up by G.H. Hardy.

https://www.theoremoftheday.org/NumberTheory/Ramanujan/TotDRamanujan.pdf



https://giphy.com/gifs/loop-oc-sierpinsky-EtFMAF8nmOmly https://proteviblog.typepad.com/.a/6a00d8341ef41d53ef016300401303970d-pi



From

Model-independent reconstruction of f(T) teleparallel cosmology
Salvatore Capozziello, Rocco D'Agostino and Orlando Luongo -
arXiv:1706.02962v1 [gr-qc] 9 Jun 2017

We have the following equations:

$$z(T)=\frac{1}{2\mathcal{Q}}\left[2(q_0^2-j_0)+\left(\frac{4\mathcal{M}(T)}{H_0^3}\right)^{1/3}+\left(\frac{16H_0^3}{\mathcal{M}(T)}\right)^{1/3}\left(j_0^2+q_0^2(6+12q_0+7q_0^2)-2j_0(3+7q_0+5q_0^2)-2s_0(1+q_0)\right)\right] \tag{47}$$

$$\mathcal{M}(T)\equiv H_0^2\sqrt{2\mathcal{P}(T)}\mathcal{Q}-2H_0^3\mathcal{N}+\sqrt{-6T}H_0^2\mathcal{Q}^2\ , \tag{49}$$

$$\mathcal{N}\equiv j_0^3+3j_0^2(1+q_0)(6+11q_0)-3j_0q_0^2\big(12+q_0(29+16q_0)\big)+q_0^4\big(18+q_0(36+17q_0)\big)-15q_0^2(1+q_0)s_0 \\ +3j_0(5+7q_0)s_0+3s_0^2\ . \tag{52}$$

For q_0 = -0.545; j_0 = 0.776 and s_0 = -0.192;

from (52):

0.776^3+3*0.776^2(1-0.545)(6-11*0.545)-3*0.776*0.545^2(12-0.545(29-16*0.545))+0.545^4(18-0.545(36-17*0.545))-15*0.545^2(1-0.545)*(-0.192)+3*0.776(5-7*0.545)*(-0.192)+3*0.192^2

we obtain:

Input:

$$0.776^3 + 3\times 0.776^2\,(1-0.545)\,(6-11\times 0.545) - 3\times 0.776\times 0.545^2\,(12-0.545\,(29-16\times 0.545)) + 0.545^4\,(18-0.545\,(36-17\times 0.545)) - 15\times 0.545^2\,((1-0.545)\times(-0.192)) + 3\times 0.776\,((5-7\times 0.545)\times(-0.192)) + 3\times 0.192^2$$

Result:

0.088999849049640625

0.088999849049640625 = $\mathcal{N}$

For:

T = -47526; H_0 = 71.47; q_0 = -0.545; j_0 = 0.776 and s_0 = -0.192;

$\mathcal{Q}$ = 0.038880875; $\mathcal{P}'(t)$ = -0.106423819; 2.00587*10^6 = $\mathcal{P}(T)$

7.5190294556…*10^{-7} = $\mathcal{P}''(T)$; 0.088999849049640625 = $\mathcal{N}$

From (49)

$$\mathcal{M}(T) \equiv H_0^2\sqrt{2\mathcal{P}(T)}\,\mathcal{Q} - 2H_0^3\mathcal{N} + \sqrt{-6T}H_0^2\mathcal{Q}^2$$

We obtain:

71.47^2*sqrt(2*2.00587e+6) * 0.038880875 – 2*71.47^3*0.088999849049640625 + sqrt(-6*-47526) * 71.47^2*0.038880875^2

Input interpretation:

$$71.47^2\sqrt{2\times 2.00587\times 10^6}\times 0.038880875 + 2\times 71.47^3\times(-0.088999849049640625) + \sqrt{-6\times(-47526)}\times 71.47^2\times 0.038880875^2$$

Result:

$3.36928... \times 10^5$

$3.36928 * 10^5 = \mathscr{M}(T)$

From which:

72*((((71.47^2*sqrt(2*2.00587e+6) * 0.038880875 – 2*71.47^3*0.08899849049640625 + sqrt(-6*-47526) * 71.47^2*0.038880875^2))))^1/4-5-(4/5)^2

Input interpretation:

$$72\left(71.47^2 \sqrt{2\times 2.00587\times 10^6}\times 0.038880875 + 2\times 71.47^3\times(-0.08899849049640625) + \sqrt{-6\times(-47526)}\times 71.47^2\times 0.038880875^2\right)\text{^}(1/4) - 5 - \left(\frac{4}{5}\right)^2$$

Result:

1729.03...

1729.03…

(((72*((((71.47^2*sqrt(2*2.00587e+6) * 0.038880875 – 2*71.47^3*0.08899849049640625 + sqrt(-6*-47526) * 71.47^2*0.038880875^2))))^1/4-5-(4/5)^2)))^1/15

Input interpretation:

$$\left(72\left(71.47^2 \sqrt{2\times 2.00587\times 10^6}\times 0.038880875 + 2\times 71.47^3\times(-0.08899849049640625) + \sqrt{-6\times(-47526)}\times 71.47^2\times 0.038880875^2\right)\text{^}(1/4) - 5 - \left(\frac{4}{5}\right)^2\right)\text{^}(1/15)$$

Result:

1.643817...

1.643817…

(((72*((((71.47^2*sqrt(2*2.00587e+6) * 0.038880875 –
2*71.47^3*0.0889984904964O625 + sqrt(-6*-47526) *
71.47^2*0.038880875^2))))^1/4-5-(4/5)^2)))^1/15-(21+5)/10^3

Input interpretation:

$$\left(72\left(71.47^2\sqrt{2\times 2.00587\times 10^6}\times 0.038880875+2\times 71.47^3\times(-0.08899849049640625)+\sqrt{-6\times(-47526)}\times 71.47^2\times 0.038880875^2\right)^\wedge(1/4)-5-\left(\frac{4}{5}\right)^2\right)^\wedge(1/15)-\frac{21+5}{10^3}$$

Result:

1.617817...

1.617817…

From (47)

$$z(T)=\frac{1}{2Q}\left[2(q_0^2-j_0)+\left(\frac{4\mathcal{M}(T)}{H_0^3}\right)^{1/3}+\left(\frac{16H_0^3}{\mathcal{M}(T)}\right)^{1/3}\left(j_0^2+q_0^2(6+12q_0+7q_0^2)-2j_0(3+7q_0+5q_0^2)-2s_0(1+q_0)\right)\right] \tag{47}$$

We obtain:

1/(2*0.038880875) * [2(0.545^2-0.776)+((4*3.36928e+5)/(71.47^3))^(1/3) + ((16*71.47^3)/(3.36928e+5))^(1/3) * (0.776^2-0.545^2(6-12*0.545-7*0.545^2)-2*0.776(3-7*0.545-5*0.545^2)-2*(-0.192)*(1-0.545))]

Input interpretation:

$$\frac{1}{2\times 0.038880875}\left(2\left(0.545^2-0.776\right)+\sqrt[3]{\frac{4\times 3.36928\times 10^5}{71.47^3}}+\sqrt[3]{\frac{16\times 71.47^3}{3.36928\times 10^5}}\left(0.776^2-0.545^2\left(6-12\times 0.545-7\times 0.545^2\right)-2\times 0.776\left(3-7\times 0.545-5\times 0.545^2\right)-2\times(-0.192)(1-0.545)\right)\right)$$

Result:

178.11832559402105787759862280763887908797287057452517904l7...

178.118325594..... = z(T)

[1/(2*0.0388808) [2(0.545^2-0.776)+((4*3.36928e+5)/(71.47^3))^(1/3)+((16*71.47^3)/(3.36928e+5))^(1/3) (0.776^2-0.545^2(6-12*0.545-7*0.545^2)-2*0.776(3-7*0.545-5*0.545^2)-2*(-0.192)*(1-0.545))]]-34-5

Input interpretation:

$$\frac{1}{2\times 0.0388808}\left(2\left(0.545^2-0.776\right)+\sqrt[3]{\frac{4\times 3.36928\times 10^5}{71.47^3}}+\sqrt[3]{\frac{16\times 71.47^3}{3.36928\times 10^5}}\left(0.776^2-0.545^2\left(6-12\times 0.545-7\times 0.545^2\right)-2\times 0.776\left(3-7\times 0.545-5\times 0.545^2\right)-2\times(-0.192)(1-0.545)\right)\right)-34-5$$

Result:

139.119...

139.119...

[1/(2*0.0388808) [2(0.545^2-0.776)+((4*3.36928e+5)/(71.47^3))^(1/3)+((16*71.47^3)/(3.36928e+5))^(1/3) (0.776^2-0.545^2(6-12*0.545-7*0.545^2)-2*0.776(3-7*0.545-5*0.545^2)-2*(-0.192)*(1-0.545))]]-55+2

Input interpretation:

$$\frac{1}{2\times 0.0388808}\left(2\left(0.545^2-0.776\right)+\sqrt[3]{\frac{4\times 3.36928\times 10^5}{71.47^3}}+\sqrt[3]{\frac{16\times 71.47^3}{3.36928\times 10^5}}\left(0.776^2-0.545^2\left(6-12\times 0.545-7\times 0.545^2\right)-2\times 0.776\left(3-7\times 0.545-5\times 0.545^2\right)-2\times(-0.192)(1-0.545)\right)\right)-55+2$$

Result:

125.119...

125.119...

[1/(2*0.0388808) [2(0.545^2-0.776)+((4*3.36928e+5)/(71.47^3))^(1/3)+((16*71.47^3)/(3.36928e+5))^(1/3)(0.776^2-0.545^2(6-12*0.545-7*0.545^2)-2*0.776(3-7*0.545-5*0.545^2)-2*(-0.192)*(1-0.545))]]-47-3

Input interpretation:

$$\frac{1}{2\times 0.0388808}\left(2\left(0.545^2-0.776\right)+\sqrt[3]{\frac{4\times 3.36928\times 10^5}{71.47^3}}+\sqrt[3]{\frac{16\times 71.47^3}{3.36928\times 10^5}}\left(0.776^2-0.545^2\left(6-12\times 0.545-7\times 0.545^2\right)-2\times 0.776\left(3-7\times 0.545-5\times 0.545^2\right)-2\times(-0.192)(1-0.545)\right)\right)-47-3$$

Result:

128.119...

128.1186691794004624....

(27*1/2*128.1186691794004624)-1/2

Input interpretation:

$$27\times\frac{1}{2}\times 128.1186691794004624-\frac{1}{2}$$

Result:

1729.1020339219062424

1729.1020339219062424

([1/(2*0.0388)[2(0.545^2-0.776)+((4*3.36928e+5)/(71.47^3))^(1/3)+((16*71.47^3)/(3.36928e+5))^(1/3)(0.776^2-0.545^2(6-12*0.545-7*0.545^2)-2*0.776(3-7*0.545-5*0.545^2)-2(-0.192)(1-0.545))]])^1/10-0.034

Input interpretation:

$$\left(\frac{1}{2\times 0.0388}\left(2\left(0.545^2-0.776\right)+\sqrt[3]{\frac{4\times 3.36928\times 10^5}{71.47^3}}+\sqrt[3]{\frac{16\times 71.47^3}{3.36928\times 10^5}}\left(0.776^2-0.545^2\left(6-12\times 0.545-7\times 0.545^2\right)-2\times 0.776\left(3-7\times 0.545-5\times 0.545^2\right)-2\times(-0.192)(1-0.545)\right)\right)\right)^{\wedge}(1/10)-0.034$$

Result:

1.645427617985628311006450443359785667386737082984934875880...

1.64542761798...

([1/(2*0.0388)[2(0.545^2-0.776)+((4*3.36928e+5)/(71.47^3))^(1/3)+((16*71.47^3)/(3.36928e+5))^(1/3)(0.776^2-0.545^2(6-12*0.545-7*0.545^2)-2*0.776(3-7*0.545-5*0.545^2)-2(-0.192)(1-0.545))]])^1/11+0.016

Input interpretation:

$$\left(\frac{1}{2\times 0.0388}\left(2\left(0.545^2-0.776\right)+\sqrt[3]{\frac{4\times 3.36928\times 10^5}{71.47^3}}+\sqrt[3]{\frac{16\times 71.47^3}{3.36928\times 10^5}}\left(0.776^2-0.545^2\left(6-12\times 0.545-7\times 0.545^2\right)-2\times 0.776\left(3-7\times 0.545-5\times 0.545^2\right)-2\times(-0.192)(1-0.545)\right)\right)\right)^{\wedge}(1/11)+0.016$$

Result:

1.618109082833048917729495087236819874292150306896684936012...

1.6181090828...

Now, we take the three results:

178.118325594..... = z(T); 3.36928 * 10^5 = $\mathcal{M}(T)$; 0.088999849049640625 = $\mathcal{N}$

we obtain:

(178.118325594 + 3.36928 * 10^5 + 0.088999849049640625)^1/2

Input interpretation:

$$\sqrt{178.118325594 + 3.36928 \times 10^5 + 0.088999849049640625}$$

Result:

580.608...

580.608...

From the formula of coefficients of the '5th order' mock theta function ψ_1(q): (A053261 OEIS Sequence)

sqrt(golden ratio) * exp(Pi*sqrt(n/15)) / (2*5^(1/4)*sqrt(n)) for n = 143, we obtain:

sqrt(golden ratio) * exp(Pi*sqrt(143/15)) / (2*5^(1/4)*sqrt(143))

Input:

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{143}{15}}\right)}{2\sqrt[4]{5}\sqrt{143}}$$

ϕ is the golden ratio

Exact result:

$$\frac{e^{\sqrt{143/15}\,\pi}\sqrt{\frac{\phi}{143}}}{2\sqrt[4]{5}}$$

Decimal approximation:

580.3832861379661538177123618191380132499742461530883864161...

580.3832861379…

Property:

$$\frac{e^{\sqrt{143/15}\,\pi}\sqrt{\frac{\phi}{143}}}{2\sqrt[4]{5}}$$ is a transcendental number

Alternate forms:

$$\frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{1430}}\;e^{\sqrt{143/15}\,\pi}$$

$$\frac{\sqrt{\frac{1}{286}\left(1+\sqrt{5}\right)}\;e^{\sqrt{143/15}\,\pi}}{2\sqrt[4]{5}}$$

Series representations:

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{143}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{143}}=\frac{\exp\left(\pi\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{143}{15}-z_0\right)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}}{2\sqrt[4]{5}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(143-z_0)^k z_0^{-k}}{k!}}$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\le 0$))

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{143}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{143}}=\left(\exp\left(i\pi\left\lfloor\frac{\arg(\phi-x)}{2\pi}\right\rfloor\right)\right.$$
$$\exp\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{143}{15}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{143}{15}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$
$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k(\phi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Big/$$
$$\left(2\sqrt[4]{5}\,\exp\left(i\pi\left\lfloor\frac{\arg(143-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(143-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$

for ($x\in\mathbb{R}$ and $x<0$)

$$\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{143}{15}}\right)}{2\sqrt[4]{5}\sqrt{143}} =$$

$$\left(\exp\left(\pi\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{143}{15}-z_0\right)/(2\pi)\right\rfloor} z_0^{1/2\left(1+\left\lfloor\arg\left(\frac{143}{15}-z_0\right)/(2\pi)\right\rfloor\right)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{143}{15}-z_0\right)^k z_0^{-k}}{k!}\right)\right.$$
$$\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(143-z_0)/(2\pi)\rfloor+1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor} z_0^{-1/2\lfloor\arg(143-z_0)/(2\pi)\rfloor+1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}$$
$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right)\Bigg/\left(2\sqrt[4]{5}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(143-z_0)^k z_0^{-k}}{k!}\right)$$

And also:

3(178.118325594 + 3.36928 * 10^5 + 0.088999849)^1/2 – 13

Input interpretation:

$$3\sqrt{178.118325594 + 3.36928\times 10^5 + 0.088999849} - 13$$

Result:

1728.825440717004512084145621862939455893861741795392390614...

1728.8254407..... ≈ 1729

Now, we have:

$q_0 = -0.545$; $j_0 = 0.776$ and $s_0 = -0.192$;

Previous results

178.118325594..... = z(T); $3.36928 * 10^5 = \mathcal{M}(T)$; $0.088999849049640625 = \mathcal{N}$
T = -47526; $H_0 = 71.47$;

$-3.82117\ldots*10^{-7} = \mathcal{M}''(T)$; $-0.0539335\ldots = \mathcal{M}'(T)$; $2.00587*10^6 = \mathcal{P}(T)$;
$0.038880875 = \mathcal{Q}$; $7.5190294556\ldots*10^{-7} = \mathcal{P}''(T)$; $-0.106423819\ldots = \mathcal{P}'(T)$

$$(\mathcal{A},\mathcal{B},\mathcal{C}) = (-5.022,\ 8.577,\ 0.532)\ . \qquad (45)$$

$$G(T) \equiv -2H_0^2\mu + 2^{1/3}\mathcal{M}(T)^{2/3} \qquad (58)$$

$$\mu \equiv j_0^2 + q_0^2(6 + 12q_0 + 7q_0^2) - 2j_0(3 + 7q_0 + 5q_0^2) - 2s_0(1 + q_0) \qquad (59)$$

From (58), we obtain:

-2*71.47^2*0.194035454375+2^(1/3)*(3.36928*10^5)^(2/3)

Input interpretation:

$-2\times 71.47^2\times 0.194035454375+\sqrt[3]{2}\ (3.36928\times 10^5)^{2/3}$

Result:

4118.29...

4118.29.... = G(T)

From (59)

$$\mu \equiv j_0^2 + q_0^2(6 + 12q_0 + 7q_0^2) - 2j_0(3 + 7q_0 + 5q_0^2) - 2s_0(1 + q_0)$$

$q_0 = -0.545$; $j_0 = 0.776$ and $s_0 = -0.192$;

we obtain:

0.776^2+0.545^2(6-12*0.545+7*0.545^2)-2*0.776(3-7*0.545+5*0.545^2)-2*(-0.192)*(1-0.545)

Input:

$0.776^2+0.545^2\ (6-12\times 0.545+7\times 0.545^2)-2\times 0.776\ (3-7\times 0.545+5\times 0.545^2)-2\times(-0.192)\ (1-0.545)$

Result:

0.194035454375

0.194035454375 = μ

$$\xi(T) \equiv 2(q_0^2 - j_0) + \left(\frac{4\mathcal{M}(T)}{H_0^3}\right)^{1/3} + \left(\frac{16H_0^3}{\mathcal{M}(T)}\right)^{1/3} , \tag{55}$$

$$X(T) \equiv 2^{4/3}H_0^2\mu + 2H_0(q_0^2 - j_0)\mathcal{M}(T)^{1/3} + 2^{2/3}\mathcal{M}(T)^{2/3} , \tag{56}$$

$$Y(T) \equiv 2^{4/3}\mathcal{C}H_0^2\mu + 2H_0\left(2\mathcal{Q} + \mathcal{C}(q_0^2 - j_0)\right)\mathcal{M}(T)^{1/3} + 2^{2/3}\mathcal{C}\mathcal{M}(T)^{2/3} , \tag{57}$$

$$(\mathcal{A}, \mathcal{B}, \mathcal{C}) = (-5.022,\ 8.577,\ 0.532) . \qquad (45)$$

$q_0 = -0.545$; $j_0 = 0.776$ and $s_0 = -0.192$;

$178.118325594..... = z(T)$; $3.36928 * 10^5 = \mathcal{M}(T)$; $0.088999849049640625 = \mathcal{N}$
$T = -47526$; $H_0 = 71.47$;

$-3.82117...*10^{-7} = \mathcal{M}''(T)$; $-0.0539335... = \mathcal{M}'(T)$; $2.00587*10^6 = \mathcal{P}(T)$;
$0.038880875 = \mathcal{Q}$; $7.5190294556...*10^{-7} = \mathcal{P}''(T)$; $-0.106423819.... = \mathcal{P}'(T)$

From (55)

$$\xi(T) \equiv 2(q_0^2 - j_0) + \left(\frac{4\mathcal{M}(T)}{H_0^3}\right)^{1/3} + \left(\frac{16H_0^3}{\mathcal{M}(T)}\right)^{1/3} ,$$

we obtain:

2(0.545^2-0.776)+(((4*3.36928*10^5)/(71.47)^3))^(1/3) + (((16*71.47^3)/(3.36928*10^5)))^(1/3)

Input interpretation:

$$2\left(0.545^2 - 0.776\right) + \sqrt[3]{\frac{4\times 3.36928\times 10^5}{71.47^3}} + \sqrt[3]{\frac{16\times 71.47^3}{3.36928\times 10^5}}$$

Result:

3.175694190485744773330738970411538201769628596932473081142...

3.1756941904857.....= ξ(T)

From which:

(((2(0.545^2-0.776)+(((4*3.36928*10^5)/(71.47)^3))^(1/3) + (((16*71.47^3)/(3.36928*10^5)))^(1/3))))^(1/2.3296)

where 2.3296 is the Hausdorff dimension value of

Dodecahedron fractal

$$\frac{\log(20)}{\log(2+\varphi)}$$

ln(20) / ln((2+(sqrt5+1)/2))

Input:

$$\frac{\log(20)}{\log\left(2+\frac{1}{2}\left(\sqrt{5}+1\right)\right)}$$

$\log(x)$ is the natural logarithm

Decimal approximation:

2.329621716170345468969701854075147997056669056330913896234...

2.329621716…

Alternate forms:

$$\frac{\log(20)}{\log\left(\frac{1}{2}\left(5+\sqrt{5}\right)\right)}$$

$$\frac{2\log(2)+\log(5)}{\log\left(\frac{1}{2}\left(5+\sqrt{5}\right)\right)}$$

$$\frac{2\log(2)}{\log\left(2+\frac{1}{2}\left(1+\sqrt{5}\right)\right)}+\frac{\log(5)}{\log\left(2+\frac{1}{2}\left(1+\sqrt{5}\right)\right)}$$

Alternative representations:

$$\frac{\log(20)}{\log\left(2+\frac{1}{2}\left(\sqrt{5}+1\right)\right)}=\frac{\log_e(20)}{\log_e\left(2+\frac{1}{2}\left(1+\sqrt{5}\right)\right)}$$

$$\frac{\log(20)}{\log\left(2+\frac{1}{2}\left(\sqrt{5}+1\right)\right)}=\frac{\log(a)\log_a(20)}{\log(a)\log_a\left(2+\frac{1}{2}\left(1+\sqrt{5}\right)\right)}$$

$$\frac{\log(20)}{\log\left(2+\frac{1}{2}\left(\sqrt{5}+1\right)\right)} = \frac{-\text{Li}_1(-19)}{-\text{Li}_1\left(-1+\frac{1}{2}\left(-1-\sqrt{5}\right)\right)}$$

Series representations:

$$\frac{\log(20)}{\log\left(2+\frac{1}{2}\left(\sqrt{5}+1\right)\right)} = \frac{\log(19)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{19}\right)^k}{k}}{\log\left(1+\frac{1}{2}\left(1+\sqrt{5}\right)\right)-\sum_{k=1}^{\infty}\frac{\left(\frac{1}{2}\left(-3+\sqrt{5}\right)\right)^k}{k}}$$

$$\frac{\log(20)}{\log\left(2+\frac{1}{2}\left(\sqrt{5}+1\right)\right)} = \frac{\log(19)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{19}\right)^k}{k}}{\log\left(\frac{1}{2}\left(3+\sqrt{5}\right)\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{2}{3+\sqrt{5}}\right)^k}{k}}$$

$$\frac{\log(20)}{\log\left(2+\frac{1}{2}\left(\sqrt{5}+1\right)\right)} = \frac{2\pi\left\lfloor\frac{\arg(20-x)}{2\pi}\right\rfloor - i\log(x) + i\sum_{k=1}^{\infty}\frac{(-1)^k(20-x)^k x^{-k}}{k}}{2\pi\left\lfloor\frac{\arg\left(5+\sqrt{5}-2x\right)}{2\pi}\right\rfloor - i\log(x) + i\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(5+\sqrt{5}-2x\right)^k x^{-k}}{k}} \quad \text{for } x<0$$

Integral representations:

$$\frac{\log(20)}{\log\left(2+\frac{1}{2}\left(\sqrt{5}+1\right)\right)} = \frac{\int_1^{20}\frac{1}{t}\,dt}{\int_1^{\frac{1}{2}\left(5+\sqrt{5}\right)}\frac{1}{t}\,dt}$$

$$\frac{\log(20)}{\log\left(2+\frac{1}{2}\left(\sqrt{5}+1\right)\right)} = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{19^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\left(1+\frac{1}{2}\left(1+\sqrt{5}\right)\right)^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1<\gamma<0$$

Thence, we obtain:

Input interpretation:

$$\sqrt[2.3296]{2\left(0.545^2-0.776\right)+\sqrt[3]{\frac{4\times3.36928\times10^5}{71.47^3}}+\sqrt[3]{\frac{16\times71.47^3}{3.36928\times10^5}}}$$

Result:

1.642171032780368890808672719497521429332677009137103735355...

1.64217103278….

(((2(0.545^2-0.776)+(((4*3.36928*10^5)/(71.47)^3))^(1/3) + (((16*71.47^3)/(3.36928*10^5)))^(1/3))))^(1/2.3296)-(21+3)1/10^3

Input interpretation:

$$\sqrt[2.3296]{2\left(0.545^2-0.776\right)+\sqrt[3]{\frac{4\times 3.36928\times 10^5}{71.47^3}}+\sqrt[3]{\frac{16\times 71.47^3}{3.36928\times 10^5}}}-(21+3)\times\frac{1}{10^3}$$

Result:

1.6181710327803688908086727194975214293326770091371037353555...

1.61817103278…

From (56),

$$X(T)\equiv 2^{4/3}H_0^2\mu+2H_0(q_0^2-j_0)\mathcal{M}(T)^{1/3}+2^{2/3}\mathcal{M}(T)^{2/3}\ ,$$

For: q_0 = -0.545; j_0 = 0.776 and s_0 = -0.192; 0.194035454375 = μ

178.118325594..... = z(T); 3.36928 * 10^5 = $\mathcal{M}(T)$; T = -47526; H_0 = 71.47;

we obtain:

2^(4/3)*71.47^2*0.194035454375 + 2*[71.47(0.545^2-0.776)]*(3.36928 * 10^5)^(1/3)+2^(2/3)*(3.36928 * 10^5)^(2/3)

Input interpretation:

$$2^{4/3}\times 71.47^2\times 0.194035454375+2\left(71.47\left(0.545^2-0.776\right)\right)\sqrt[3]{3.36928\times 10^5}+2^{2/3}\left(3.36928\times 10^5\right)^{2/3}$$

Result:

5419.60...

5419.60.... = X(T)

From which:

((((2^(4/3)*71.47^2*0.194035454375 + 2*71.47[(0.545^2-0.776)]*(3.36928 * 10^5)^(1/3)+2^(2/3)*(3.36928 * 10^5)^(2/3)))))^1/18+(29+3)1/10^3

Input interpretation:

$$\left(2^{4/3} \times 71.47^2 \times 0.194035454375 + 2 \times 71.47 \left((0.545^2 - 0.776) \sqrt[3]{3.36928 \times 10^5}\right) + 2^{2/3} \left(3.36928 \times 10^5\right)^{2/3}\right) \wedge (1/18) + (29+3) \times \frac{1}{10^3}$$

Result:

1.644288...

1.644288…

((((2^(4/3)*71.47^2*0.194035454375 + 2*71.47[(0.545^2-0.776)]*(3.36928 * 10^5)^(1/3)+2^(2/3)*(3.36928 * 10^5)^(2/3)))))^1/18+(4+2)1/10^3

Input interpretation:

$$\left(2^{4/3} \times 71.47^2 \times 0.194035454375 + 2 \times 71.47 \left((0.545^2 - 0.776) \sqrt[3]{3.36928 \times 10^5}\right) + 2^{2/3} \left(3.36928 \times 10^5\right)^{2/3}\right) \wedge (1/18) + (4+2) \times \frac{1}{10^3}$$

Result:

1.618288...

1.618288…

From (57),

$$Y(T) \equiv 2^{4/3}\mathcal{C}H_0^2\mu + 2H_0\left(2\mathcal{Q} + \mathcal{C}(q_0^2 - j_0)\right)\mathcal{M}(T)^{1/3} + 2^{2/3}\mathcal{C}\mathcal{M}(T)^{2/3} ,$$

we obtain:

2^(4/3)*0.532*71.47^2*0.194035454375 + 2*71.47(2*0.038880875+0.532(0.545^2-0.776))*[3.36928e+5^(1/3)]+2^(2/3)*0.532*(3.36928*10^5)^(2/3)

Input interpretation:

$$2^{4/3}\times 0.532\times 71.47^2\times 0.194035454375 + 2\times 71.47\,(2\times 0.038880875 + 0.532\,(0.545^2 - 0.776))\sqrt[3]{3.36928\times 10^5} + 2^{2/3}\times 0.532\,(3.36928\times 10^5)^{2/3}$$

Result:

3656.67...

3656.67.... = Y(T)

From which:

(((2^(4/3)*0.532*71.47^2*0.194035454375 + 2*71.47(2*0.038880875+0.532(0.545^2-0.776))*[3.36928e+5^(1/3)]+2^(2/3)*0.532*(3.36928*10^5)^(2/3))))^1/17+(21+3)1/10^3

Input interpretation:

$$\left(2^{4/3}\times 0.532\times 71.47^2\times 0.194035454375 + 2\times 71.47\,(2\times 0.038880875 + 0.532\,(0.545^2 - 0.776))\sqrt[3]{3.36928\times 10^5} + 2^{2/3}\times 0.532\,(3.36928\times 10^5)^{2/3}\right) \wedge (1/17) + (21+3)\times \frac{1}{10^3}$$

Result:

1.64429...

1.64429…

(((2^(4/3)*0.532*71.47^2*0.194035454375 + 2*71.47(2*0.038880875+0.532(0.545^2-0.776))*[3.36928e+5^(1/3)]+2^(2/3)*0.532*(3.36928*10^5)^(2/3))))^1/17-2/10^3

Input interpretation:

$$\left(2^{4/3}\times 0.532\times 71.47^2\times 0.194035454375 + 2\times 71.47\,(2\times 0.038880875 + 0.532\,(0.545^2 - 0.776))\sqrt[3]{3.36928\times 10^5} + 2^{2/3}\times 0.532\,(3.36928\times 10^5)^{2/3}\right) \wedge (1/17) - \frac{2}{10^3}$$

Result:

1.61829...

1.61829….

Various results:

178.118325594..... = z(T); $3.36928 * 10^5 = \mathcal{M}(T)$; $0.088999849049640625 = \mathcal{N}$
T = -47526; $H_0 = 71.47$; $-3.82117\ldots*10^{-7} = \mathcal{M}''(T)$; $-0.0539335\ldots = \mathcal{M}'(T)$;
$2.00587*10^6 = \mathcal{P}(T)$; $0.038880875 = \mathcal{Q}$; $7.5190294556\ldots*10^{-7} = \mathcal{P}''(T)$;

$-0.106423819\ldots = \mathcal{P}'(T)$; 3656.67.... = Y(T); 5419.60.... = X(T);

3.1756941904857.....= ξ(T); 0.194035454375 = μ; 4118.29.... = G(T)

From the algebraic sum, we obtain:

(((178.118325 + (3.36928e+5) + 0.0889998 - 47526 + 71.47 - (3.82117e-7) - 0.0539335 + (2.00587e+6) + 0.03888 + (7.519029e-7) - 0.106423819+3656.67+5419.60+3.175694+0.1940354+4118.29)))

Input interpretation:

$$178.118325 + 3.36928\times 10^5 + 0.0889998 - 47526 + 71.47 - 3.82117\times 10^{-7} - 0.0539335 + 2.00587\times 10^6 + 0.03888 + 7.519029\times 10^{-7} - 0.106423819 + 3656.67 + 5419.60 + 3.175694 + 0.1940354 + 4118.29$$

Result:

$$2.30871948557725078590\times 10^6$$

$2.30871948557725078590*10^6$

From which:

(((178.118325+(3.36928e+5)+0.0889998-47526+71.47-(3.82117e-7)-0.0539335+(2.00587e+6)+0.03888+(7.519029e-7)-0.106423819+3656.67+5419.60+3.175694+0.1940354+4118.29)))^1/30+(11+4)1/10^3

Input interpretation:

$$\left(178.118325 + 3.36928\times 10^5 + 0.0889998 - 47526 + 71.47 - 3.82117\times 10^{-7} - 0.0539335 + 2.00587\times 10^6 + 0.03888 + 7.519029\times 10^{-7} - 0.106423819 + 3656.67 + 5419.60 + 3.175694 + 0.1940354 + 4118.29\right)\hat{}\,(1/30) + (11+4)\times\frac{1}{10^3}$$

Result:

1.644718...

1.644718…

(((178.118325 + (3.36928e+5) + 0.0889998 - 47526 + 71.47 - (3.82117e-7) - 0.0539335 + (2.00587e+6) + 0.03888 + (7.519029e-7) - 0.106423819+3656.67+5419.60+3.175694+0.1940354543+ 4118.29)))^1/30-11/10^3

Input interpretation:

$$\left(178.118325 + 3.36928\times 10^5 + 0.0889998 - 47526 + 71.47 - 3.82117\times 10^{-7} - 0.0539335 + 2.00587\times 10^6 + 0.03888 + 7.519029\times 10^{-7} - 0.106423819 + 3656.67 + 5419.60 + 3.175694 + 0.1940354543 + 4118.29\right)^\wedge(1/30) - \frac{11}{10^3}$$

Result:

1.618718...

1.618718….

And again, we obtain:

(((178.118325 + (3.36928e+5) + 0.0889998 - 47526 + 71.47 - (3.82117e-7) - 0.0539335 + (2.00587e+6) + 0.03888 + (7.519029e-7) - 0.106423819+3656.67+5419.60+3.175694+0.1940354+4118.29)))^1/3+7

Input interpretation:

$$\left(178.118325 + 3.36928\times 10^5 + 0.0889998 - 47526 + 71.47 - 3.82117\times 10^{-7} - 0.0539335 + 2.00587\times 10^6 + 0.03888 + 7.519029\times 10^{-7} - 0.106423819 + 3656.67 + 5419.60 + 3.175694 + 0.1940354 + 4118.29\right)^\wedge(1/3) + 7$$

Result:

139.167...

139.167…

(((178.118325 + (3.36928e+5) + 0.0889998 - 47526 + 71.47 - (3.82117e-7) - 0.0539335 + (2.00587e+6) + 0.03888 + (7.519029e-7) - 0.106423819+3656.67+5419.60+3.175694+0.1940354+4118.29)))^1/3-7

Input interpretation:

$$(178.118325 + 3.36928\times 10^5 + 0.0889998 - 47526 + 71.47 - 3.82117\times 10^{-7} - 0.0539335 + 2.00587\times 10^6 + 0.03888 + 7.519029\times 10^{-7} - 0.106423819 + 3656.67 + 5419.60 + 3.175694 + 0.1940354 + 4118.29)\,\hat{}\,(1/3) - 7$$

Result:

125.167...

125.167…

27*1/2(((((178.118325 + (3.36928e+5) + 0.0889998 - 47526 + 71.47 - (3.82117e-7) - 0.0539335 + (2.00587e+6) + 0.03888 + (7.519029e-7) - 0.106423819+3656.67+5419.60+3.175694+0.1940354+4118.29)))^1/3-4))-1

Input interpretation:

$$27\times\frac{1}{2}\left((178.118325 + 3.36928\times 10^5 + 0.0889998 - 47526 + 71.47 - 3.82117\times 10^{-7} - 0.0539335 + 2.00587\times 10^6 + 0.03888 + 7.519029\times 10^{-7} - 0.106423819 + 3656.67 + 5419.60 + 3.175694 + 0.1940354 + 4118.29)\,\hat{}\,(1/3) - 4\right) - 1$$

Result:

1729.26...

1729.26…

With regard 27 (From Wikipedia):

*"The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group **Z**/3**Z**, and its outer automorphism group is the cyclic group **Z**/2**Z**. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories".*

Now, we have that:

178.118325594..... = z(T); 3.36928 * 10^5 = $\mathcal{M}(T)$; 0.088999849049640625 = $\mathcal{N}$
T = -47526; H_0 = 71.47; -3.82117…*10^{-7} = $\mathcal{M}''(T)$; -0.0539335... = $\mathcal{M}'(T)$;
2.00587*10^6 = $\mathcal{P}(T)$; 0.038880875 = $\mathcal{Q}$; 7.5190294556…*10^{-7} = $\mathcal{P}''(T)$;

-0.106423819.... = $\mathcal{P}'(T)$; 3656.67.... = Y(T); 5419.60.... = X(T);

3.1756941904857.....= ξ(T); 0.194035454375 = μ; 4118.29.... = G(T)

$$(\mathcal{A}, \mathcal{B}, \mathcal{C}) = (-5.022,\ 8.577,\ 0.532)\ .$$

α = 0.5

From

$$\rho_T = -\frac{1}{2}\left[T + \alpha\left(\mathcal{A} + \frac{\mathcal{B}\,\xi(T)}{4\mathcal{Q}^2}e^{\frac{\mathcal{C}\xi(T)}{2\mathcal{Q}}}\right)\right] + \frac{2^{1/3}\alpha\,\mathcal{B}\,T\,G(T)}{24H_0^3\mathcal{Q}^3\mathcal{M}(T)^2}X(T)\,Y(T)\,\mathcal{M}'(T)\,e^{\frac{\mathcal{C}\xi(T)}{2\mathcal{Q}}}\ , \tag{53}$$

we obtain:

-1/2[-47526+0.5((((-5.022+((8.577*(3.17569419)/(4*0.038880875^2))*exp(((0.532*3.17569419)/(2*0.038880875))))))]

Input interpretation:

$$-\frac{1}{2}\left(-47526 + 0.5\left(-5.022 + \left(8.577 \times \frac{3.17569419}{4 \times 0.038880875^2}\right)\exp\left(\frac{0.532 \times 3.17569419}{2 \times 0.038880875}\right)\right)\right)$$

Result:

$-3.07017... \times 10^{12}$

-3.07017…*10^{12}

(((2^(1/3)*0.5*8.577*(-47526)(4118.29))))/(((24*71.47^3*0.038880875^3*(3.36928*10^5)^2)))*5419.60*3656.67*(-0.0539335)* exp(((0.532*3.17569419)/(2*0.038880875)))

Input interpretation:

$$\frac{\sqrt[3]{2}\times 0.5\times 8.577\times(-47526)\times 4118.29}{24\times 71.47^3\times 0.038880875^3\,(3.36928\times 10^5)^2}\times 5419.60\times 3656.67\times(-0.0539335)\exp\left(\frac{0.532\times 3.17569419}{2\times 0.038880875}\right)$$

Result:

$5.27137... \times 10^{10}$

$5.27137…*10^{10}$

-1/2[-47526+0.5((-5.022+((8.577*(3.17569419)/(4*0.038880875^2))*exp(((0.532*3.17569419)/(2*0.038880875))))))]+5.27137 × 10^10

Input interpretation:

$$-\frac{1}{2}\left(-47526+0.5\left(-5.022+\left(8.577\times\frac{3.17569419}{4\times 0.038880875^2}\right)\exp\left(\frac{0.532\times 3.17569419}{2\times 0.038880875}\right)\right)\right)+5.27137\times 10^{10}$$

Result:

$-3.01745... \times 10^{12}$

$-3.01745...*10^{12}$ = final result

From which:

(((-(-1/2[-47526+0.5((-5.022+((8.577*(3.17569419)/(4*0.038880875^2))*exp(((0.532*3.17569419)/(2*0.038880875))))))]+5.27137 × 10^10))))^1/60+29/10^3

Input interpretation:

$$\left(-\left(-\frac{1}{2}\left(-47526+0.5\left(-5.022+\left(8.577\times\frac{3.17569419}{4\times 0.038880875^2}\right)\exp\left(\frac{0.532\times 3.17569419}{2\times 0.038880875}\right)\right)\right)+5.27137\times 10^{10}\right)\right)\text{^}(1/60)+\frac{29}{10^3}$$

Result:

1.643336278811445434211839828710457226554192681001604847547...

1.643336278….

(((-(-1/2[-47526+0.5((-5.022+((8.577*(3.17569419)/(4*0.038880875^2))*exp(((0.532*3.17569419)/(2*0.038880875))))))]+5.27137 × 10^10))))^1/60+4/10^3

Input interpretation:

$$\left(-\left(-\frac{1}{2}\left(-47526+0.5\left(-5.022+\left(8.577\times\frac{3.17569419}{4\times 0.038880875^2}\right)\exp\left(\frac{0.532\times 3.17569419}{2\times 0.038880875}\right)\right)\right)+5.27137\times 10^{10}\right)\right)\text{^}(1/60)+\frac{4}{10^3}$$

Result:

1.618336278811445434211839828710457226554192681001604847547...

1.618336278….

(((-(-1/2[-47526+0.5((-5.022+((8.577*(3.17569419)/(4*0.038880875^2))*exp(((0.532*3.17569419)/(2*0.038880875))))))]+5.27137 × 10^10))))^1/6+5

Input interpretation:

$$\left(-\left(-\frac{1}{2}\left(-47526+0.5\left(-5.022+\left(8.577\times\frac{3.17569419}{4\times 0.038880875^2}\right)\exp\left(\frac{0.532\times 3.17569419}{2\times 0.038880875}\right)\right)\right)+5.27137\times 10^{10}\right)\right)\text{^}(1/6)+5$$

Result:

125.210...

125.210...

(((-(-1/2[-47526+0.5((-5.022+((8.577*(3.17569419)/(4*0.038880875^2))*exp(((0.532*3.17569419)/(2*0.038880875))))))]+5.27137 × 10^10))))^1/6+18+golden ratio

Input interpretation:

$$\left(-\left(-\frac{1}{2}\left(-47526+0.5\left(-5.022+\left(8.577\times\frac{3.17569419}{4\times0.038880875^2}\right)\exp\left(\frac{0.532\times3.17569419}{2\times0.038880875}\right)\right)\right)+5.27137\times10^{10}\right)\right)\verb|^|(1/6)+18+\phi$$

ϕ is the golden ratio

Result:

139.828...

139.828...

27*1/2(((((-(-1/2[-47526+0.5((-5.022+((8.577*(3.17569419)/(4*0.038880875^2))*exp(((0.532*3.17569419)/(2*0.038880875))))))]+5.27137 × 10^10))))^1/6+8))-golden ratio

Input interpretation:

$$27\times\frac{1}{2}\left(\left(-\left(-\frac{1}{2}\left(-47526+0.5\left(-5.022+\left(8.577\times\frac{3.17569419}{4\times0.038880875^2}\right)\exp\left(\frac{0.532\times3.17569419}{2\times0.038880875}\right)\right)\right)+5.27137\times10^{10}\right)\right)\verb|^|(1/6)+8\right)-\phi$$

ϕ is the golden ratio

Result:

1729.21...

1729.21…

Now, from:

Notes on Strings and Higher Spins
A. Sagnotti - arXiv:1112.4285v4 [hep-th] 21 Jun 2012

We have that.

$$\mathcal{E}_0(D,s) \equiv \sum_{n=0}^{N} \rho_n(D-2,s) \frac{s!}{n!\,(s-2n)!\,2^n} \mathcal{J}^{[n]} \cdot \mathcal{J}^{[n]} , \tag{3.19}$$

$$\rho_{n+1}(D,s) = -\frac{\rho_n(D,s)}{D+2(s-n-2)} , \qquad \rho_0(D,s) = 1 , \tag{3.20}$$

which can be justified noticing that the recursive definition (3.20) of the ρ_n makes the sum (3.19) transverse and traceless, as in the preceding $s=3$ example. The first non–trivial case corresponds to $s=2$, where eq. (3.19) reduces to

$$\mathcal{E}_0(D,2) = T^{\mu\nu}\,T_{\mu\nu} - \frac{1}{D-2} \left(T^{\mu}{}_{\mu}\right)^2 \tag{3.21}$$

where, abiding to standard conventions, we have called $T_{\mu\nu}$ the corresponding current.

Let me stress that the amplitude $\mathcal{E}_0(D,s)$ depends on the spin s of the HS field determining the exchange and on the number D of space–time dimensions. If one insists on currents that are *conserved* in D dimensions, it is possible to adapt the analysis to *massive* spin–s fields. A harmonic dependence on an internal circle coordinate suffices in fact to introduce masses *à la* Stueckelberg, so that massive exchanges driven by currents that are still *conserved* in D dimensions and lack internal components obtain replacing D with $D+1$ in eq. (3.19). The difference between the exchanges for any given value of s computed in $D+1$ and D dimensions,

$$\Delta_0(D,s) = \sum_{n=0}^{N} \left[\rho_n(D-1,s) - \rho_n(D-2,s)\right] \frac{s!}{n!\,(s-2n)!\,2^n} \mathcal{J}^{[n]} \cdot \mathcal{J}^{[n]} , \tag{3.22}$$

thus encodes the generalization of the discontinuity originally found by van Dam, Veltman and Zakharov [52] for $s=2$, that can be computed starting from eq. (3.21).

This phenomenon has a very instructive counterpart in the presence of a cosmological constant λ that deforms Minkowski space to (A)dS backgrounds depending on its (negative)positive sign. Starting from the deformed equations of Section 3.1 and focussing on the dS case, for $s=2$ the resulting massive exchange for a generic mass M, computed some time ago in [54], reads[2]

$$\mathcal{E}_\lambda(D,2) = T^{\mu\nu}\,T_{\mu\nu} - \frac{1}{D-1}\,\frac{(ML)^2-(D-1)}{(ML)^2-(D-2)} \left(T^{\mu}{}_{\mu}\right)^2 , \tag{3.23}$$

where the relation between L and the cosmological constant is given in eq. (3.4). Notice that this exchange is a rational function of ML with a simple pole determined by the condition

$$(ML)^2 = D-2 . \tag{3.24}$$

....

More details can be found in [10], where eq. (3.23) was extended to symmetric tensors of arbitrary rank. The first terms, drawn from [10] but adapted to the index–free notation

of Section 2.1, read

$$\begin{aligned}
\mathcal{E}_\lambda(D,s) &= \mathcal{J}_s \;+\; \frac{g\,\mathcal{J}_s{}'}{2(\frac{5}{2}-\zeta)}\,\frac{(ML)^2+2(\frac{5}{2}-\zeta)}{(ML)^2-2(\zeta-3)} \\
&+ \frac{g^2\,\mathcal{J}_s{}^{[2]}}{8}\,\frac{(ML)^4+8(ML)^2(\frac{7}{2}-\zeta)+12\left(\frac{5}{2}-\zeta\right)_2}{\left(\frac{5}{2}-\zeta\right)_2[(ML)^2-2(\zeta-3)][(ML)^2-6(\zeta-4)]} \\
&+ \frac{g^3\,\mathcal{J}_s{}^{[3]}}{48}\,\frac{(ML)^6-(ML)^4(18\zeta-77)+92(ML)^2\left(\frac{7}{2}-\zeta\right)_2+120\left(\frac{5}{2}-\zeta\right)_3}{\left(\frac{5}{2}-\zeta\right)_3[(ML)^2-2(\zeta-3)][(ML)^2-6(\zeta-4)][(ML)^2-10(\zeta-5)]} \\
&+ \ldots\,,
\end{aligned} \tag{3.27}$$

where $\mathcal{J}_s{}^{[2,3]}$ are higher traces of $\mathcal{J}_s$, g is the background dS metric, $\zeta=\frac{D}{2}+s$ and

$$(a)_n = a(a+1)\ldots(a+n-1) \tag{3.28}$$

are Pochhammer symbols.

In the dS case the poles lie precisely at real values of ML where partially massless gauge transformations involving terms like the second in eq. (3.25), not protected by gradients and thus incompatible with generic conserved currents, emerge. For a rank–s tensor there are in fact $s-1$ partially massless points, for which

$$(ML)^2 = (s-1-r)(D+s+r-4)\,, \tag{3.29}$$

where $r=0,\ldots,s-2$, while $r=s-1$ corresponds to the more familiar massless point. For all $s\geq 2$ a first pole thus appears at $r=s-2$, and for $s>3$ others lie at alternate values of r below it. For instance, for $s=3$ the relevant value is $r=1$, since for $r=0$ all terms present in the partially massless gauge transformation contain gradients of the parameter and are thus manifestly compatible with conserved currents.

For:

$D = 6$

$s \geq 2$, $s = 2$, $r = 0$ (for eq. (3.29)); $(ML)^2 = 4$

$\zeta=\frac{D}{2}+s$

$\zeta = 6/2+3 = 6$ for $s \geq 2$, $s = 3$ (<u>only for the value of ζ</u>)

From

$$\mathcal{E}_\lambda(D,s) = \mathcal{J}_s + \frac{g\,\mathcal{J}_s{}'}{2(\frac{5}{2}-\zeta)}\,\frac{(ML)^2+2(\frac{5}{2}-\zeta)}{(ML)^2-2(\zeta-3)}$$
$$+\frac{g^2\,\mathcal{J}_s{}^{[2]}}{8}\,\frac{(ML)^4+8(ML)^2(\frac{7}{2}-\zeta)+12\,(\frac{5}{2}-\zeta)_2}{(\frac{5}{2}-\zeta)_2\,[(ML)^2-2(\zeta-3)][(ML)^2-6(\zeta-4)]} \tag{3.27}$$
$$+\frac{g^3\,\mathcal{J}_s{}^{[3]}}{48}\,\frac{(ML)^6-(ML)^4(18\zeta-77)+92(ML)^2\,(\frac{7}{2}-\zeta)_2+120\,(\frac{5}{2}-\zeta)_3}{(\frac{5}{2}-\zeta)_3\,[(ML)^2-2(\zeta-3)][(ML)^2-6(\zeta-4)][(ML)^2-10(\zeta-5)]}$$
$$+\ldots,$$

we obtain:

1+1/(2(5/2-6))*(4+2(5/2-6))/(4-2(3))+1/8*((4^2+8*4(7/2-6)+12(5/2-6)))/(((5/2-6)(4-2(3))(4-6(2))))+1/48*((((4^3-4^2(18*6-77)+92*4(7/2-6)+120(5/2-6)))))/(((5/2-6)(4-2*(3))(4-6(2))(4-10)))

Input:

$$1+\frac{1}{2\left(\frac{5}{2}-6\right)}\times\frac{4+2\left(\frac{5}{2}-6\right)}{4-2\times3}+\frac{1}{8}\times\frac{4^2+8\times4\left(\frac{7}{2}-6\right)+12\left(\frac{5}{2}-6\right)}{\left(\frac{5}{2}-6\right)(4-2\times3)(4-6\times2)}+$$
$$\frac{1}{48}\times\frac{4^3-4^2(18\times6-77)+92\times4\left(\frac{7}{2}-6\right)+120\left(\frac{5}{2}-6\right)}{\left(\frac{5}{2}-6\right)(4-2\times3)(4-6\times2)(4-10)}$$

Exact result:

$$\frac{3679}{4032}$$

Decimal approximation:

0.912450396825396825396825396825396825396825396825396825396...

0.912450396825…. result very near to the spectral index n_s , to the mesonic Regge slope, to the inflaton value at the end of the inflation 0.9402 (see Appendix) and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}}-\varphi+1}=1-\cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\ldots}}}}\approx 0.9568666373$$

From:

Astronomy & Astrophysics manuscript no. ms c ESO 2019 - September 24, 2019
Planck 2018 results. VI. Cosmological parameters

The primordial fluctuations are consistent with Gaussian purely adiabatic scalar perturbations characterized by a power spectrum with a spectral index $n_s = 0.965 \pm 0.004$, consistent with the predictions of slow-roll, single-field, inflation.

We know that α' is the Regge slope (string tension). With regard the Omega mesons, the values are:

ω	6	$m_{u/d} = 0 - 60$	$0.910 - 0.918$
ω/ω_3	$5+3$	$m_{u/d} = 255 - 390$	$0.988 - 1.18$
ω/ω_3	$5+3$	$m_{u/d} = 240 - 345$	$0.937 - 1.000$

We have the following two Ramanujan continued fractions:

$$\frac{\sqrt{2}\,e^{-2\pi/8}}{1+\cfrac{e^{-2\pi}}{1+e^{-2\pi}+\cfrac{e^{-4\pi}}{1+e^{-4\pi}+\cfrac{e^{-6\pi}}{1+e^{-6\pi}+\cdots}}}} = \sqrt{\sqrt{2}-1}$$
$$= 0,6435942526\ldots$$

$$\frac{e^{-2\pi/5}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{\ 4\pi}}{1+\cfrac{e^{-6\pi}}{1+\cdots}}}} = \sqrt{\Phi+2}-\Phi = \left(\sqrt{\Phi\sqrt{5}}-\Phi\right)$$

$$= \sqrt{\frac{\sqrt{5}+1}{2}\sqrt{5}} - \frac{\sqrt{5}+1}{2} = 0,2840790438404122960\ldots$$

from the sum, we obtain:

(sqrt(sqrt2-1))+((((sqrt(golden ratio*sqrt5))-golden ratio)))

Input:

$$\sqrt{\sqrt{2}-1}+\left(\sqrt{\phi\sqrt{5}}-\phi\right)$$

ϕ is the golden ratio

Exact result:

$$\sqrt[4]{5}\sqrt{\phi}-\phi+\sqrt{\sqrt{2}-1}$$

Decimal approximation:

0.9276732967459949207637352698113359780152908308897452339 15...

0.927673296745… result very near to above solution

Minimal polynomial:

$$x^{16}+8x^{15}+8x^{14}-64x^{13}-48x^{12}+368x^{11}-112x^{10}-832x^9-680x^8-5792x^7+2656x^6-3584x^5+8512x^4+22336x^3-16192x^2-6400x+656$$

Alternate forms:

$$-\frac{1}{2}-\frac{\sqrt{5}}{2}+\sqrt{\sqrt{2}-1}+\frac{\sqrt[4]{5}\sqrt{1+\sqrt{5}}}{\sqrt{2}}$$

$$\frac{1}{2}\left(-1-\sqrt{5}+2\sqrt{\sqrt{2}-1}+\sqrt[4]{5}\sqrt{2\left(1+\sqrt{5}\right)}\right)$$

root of $x^{16}+8x^{15}+8x^{14}-64x^{13}-48x^{12}+368x^{11}-112x^{10}-832x^9-680x^8-5792x^7+2656x^6-3584x^5+8512x^4+22336x^3-16192x^2-6400x+656$ near $x=0.927673$

Series representations:

$$\sqrt{\sqrt{2}-1}+\left(\sqrt{\phi\sqrt{5}}-\phi\right)=-\phi+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\left(-1+\sqrt{2}-z_0\right)^k+\left(\phi\sqrt{5}-z_0\right)^k\right)z_0^{-k}}{k!}$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\le 0$))

$$\sqrt{\sqrt{2}-1}+\left(\sqrt{\phi\sqrt{5}}-\phi\right)=$$

$$-\phi+\sum_{k=0}^{\infty}\frac{1}{k!}(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(\exp\left(i\pi\left\lfloor\frac{\arg\left(-1-x+\sqrt{2}\right)}{2\pi}\right\rfloor\right)\left(-1-x+\sqrt{2}\right)^k+\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+\phi\sqrt{5}\right)}{2\pi}\right\rfloor\right)\left(-x+\phi\sqrt{5}\right)^k\right)\sqrt{x}\quad\text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\sqrt{\sqrt{2}-1}+\left(\sqrt{\phi\sqrt{5}}-\phi\right)=-\phi+\sum_{k=0}^{\infty}\frac{1}{k!}(-1)^k\left(-\frac{1}{2}\right)_k z_0^{1/2-k}\left(\left(-1+\sqrt{2}-z_0\right)^k\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(-1+\sqrt{2}-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2\left\lfloor\arg\left(-1+\sqrt{2}-z_0\right)/(2\pi)\right\rfloor}+\left(\phi\sqrt{5}-z_0\right)^k\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\phi\sqrt{5}-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2\left\lfloor\arg\left(\phi\sqrt{5}-z_0\right)/(2\pi)\right\rfloor}\right)$$

From:

Can. J. Math. Vol. 47 (5), 1995 pp. 897-914
SOME VALUES FOR THE ROGERS-RAMANUJAN CONTINUED FRACTION
BRUCE C. BERNDT AND HENG HUAT CHAN

We have the following expression:

$$G(-e^{-5\pi}) = -\frac{16\sqrt{2}}{(1+\sqrt{5})^2(2+\sqrt{3})\left\{\sqrt{4+\sqrt{15}}+(15)^{1/4}\right\}}$$

$$= -\frac{2\sqrt{2}(2-\sqrt{3})(3-\sqrt{5})}{(4+\sqrt{15})^{3/2}+3(4+\sqrt{15})(15)^{1/4}+3\sqrt{4+\sqrt{15}}\sqrt{15}+(15)^{3/4}}$$

$$= -\frac{(2-\sqrt{3})(3-\sqrt{5})}{\sqrt{2}\left(\sqrt{4+\sqrt{15}}(1+\sqrt{15})+(15)^{1/4}(3+\sqrt{15})\right)}.$$

Now

$$\sqrt{4+\sqrt{15}}(1+\sqrt{15}) = \sqrt{4+\sqrt{15}(1+\sqrt{15})^2}$$
$$= \sqrt{94+24\sqrt{15}}$$
$$= \sqrt{2}\sqrt{\frac{1}{4}(6\sqrt{3}+4\sqrt{5})^2}$$
$$= \frac{1}{\sqrt{2}}(6\sqrt{3}+4\sqrt{5}).$$

we obtain:

(1/sqrt2 (6sqrt3+4sqrt5))

Input:

$$\frac{1}{\sqrt{2}}\left(6\sqrt{3}+4\sqrt{5}\right)$$

Result:

$$\frac{6\sqrt{3}+4\sqrt{5}}{\sqrt{2}}$$

Decimal approximation:

13.673024548686292958589639312983111243336952720620444039 01...

13.67302454868…

Alternate forms:

$$3\sqrt{6}+2\sqrt{10}$$

$$\sqrt{2\left(47+12\sqrt{15}\right)}$$

$$\sqrt{2}\left(3\sqrt{3}+2\sqrt{5}\right)$$

Minimal polynomial:

$x^4 - 188 x^2 + 196$

Thence, from:

$$-\frac{(2-\sqrt{3})(3-\sqrt{5})}{\sqrt{2}\left(\sqrt{4+\sqrt{15}}(1+\sqrt{15})+(15)^{1/4}(3+\sqrt{15})\right)}.$$

we obtain:

-(((2-sqrt3)(3-sqrt5))) / [sqrt2*((((13.67302454)+15^0.25 (3+sqrt15)))]

Input interpretation:

$$-\frac{(2-\sqrt{3})(3-\sqrt{5})}{\sqrt{2}\left(13.67302454+15^{0.25}\left(3+\sqrt{15}\right)\right)}$$

Result:

$-0.00532157...$

-0.00532157...

From which:

(((-(-(((2-sqrt3)(3-sqrt5))) / [sqrt2*((((13.67302454)+15^0.25 (3+sqrt15))))])))))^1/57

Input interpretation:

$$\sqrt[57]{-\left(-\frac{(2-\sqrt{3})(3-\sqrt{5})}{\sqrt{2}\left(13.67302454+15^{0.25}\left(3+\sqrt{15}\right)\right)}\right)}$$

Result:

0.912233371658169466961470323254496767131312423055233556256...

0.912233371658..... result very near to the previous solution

Input interpretation:

0.9122333716581694669614703232544967671313124230552335

Rational approximation:

$$\frac{90195129239653116621511549}{98872867450250308980231544}$$

Continued fraction:

$$\cfrac{1}{1+\cfrac{1}{10+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{5+\cfrac{1}{1+\cfrac{1}{9+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{16+\cfrac{1}{24+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{\ldots}}}}}}}}}}}}}}}}}}}}}}$$

Possible closed forms:

$$\frac{8006217}{2793647\pi} \approx 0.91223337165816941041 8$$

$$\frac{12059273\pi^4}{1287699900} \approx 0.9122333716581694 79782$$

$$\frac{9}{5}\pi\tan^2\left(\frac{1062151}{2781059}\right) \approx 0.9122333716581694 49204$$

$$423-\frac{138}{\pi}+\frac{255}{4\sqrt{\pi}}-301\sqrt{\pi}+38\pi \approx 0.91223337165816946 55601$$

$$\frac{815\,442\,193\,\pi}{2\,808\,258\,591} \approx 0.91223337165816946691165 6$$

$$\pi \boxed{\text{root of } 3023x^5 + 528x^4 + 6x^3 + 234x^2 - 3x - 29 \text{ near } x = 0.290373} \approx 0.9122333716581694669 48934$$

$$e^{-\frac{11}{6} - \frac{41}{6e} + 5e - \frac{20}{3\pi} - \frac{8\pi}{3}} \pi^{3/2} (-\cos(e\pi))^{7/3} \csc^2(e\pi) \approx 0.91223337165816946 81144$$

$$\boxed{\text{root of } 27x^5 - 13x^4 + 572x^3 - 175x^2 - 756x + 393 \text{ near } x = 0.912233} \approx 0.9122333716581694669 74656$$

$$\boxed{\text{root of } 61\,113x^3 + 39\,330x^2 - 28\,392x - 53\,222 \text{ near } x = 0.912233} \approx 0.912233371658169466961 495429$$

$$\boxed{\text{root of } 141x^4 + 8343x^3 - 3783x^2 - 171x - 3127 \text{ near } x = 0.912233} \approx 0.912233371658169466961444706$$

$$\pi \boxed{\text{root of } 60\,349x^3 - 44\,824x^2 - 64\,917x + 21\,152 \text{ near } x = 0.290373} \approx 0.9122333716581694669630885$$

$$\frac{1}{\boxed{\text{root of } 53\,222x^3 + 28\,392x^2 - 39\,330x - 61\,113 \text{ near } x = 1.09621}} \approx 0.912233371658169466961495429$$

$$\pi \boxed{\text{root of } 4016x^4 - 3209x^3 - 3621x^2 + 4547x - 965 \text{ near } x = 0.290373} \approx 0.912233371658169466919463$$

$$\frac{1}{\boxed{\text{root of } 3127x^4 + 171x^3 + 3783x^2 - 8343x - 141 \text{ near } x = 1.09621}} \approx 0.912233371658169466961444706$$

$$\frac{2834}{3909} + \frac{4427}{3909\pi} - \frac{503\pi}{9121} \approx 0.912233371658169466924085$$

From Ramanujan paper:

THEOREM 6. *We have*

$$S(e^{-\pi/\sqrt{15}}) = \left(\frac{5\sqrt{5} - 3 + \sqrt{30(5 - \sqrt{5})}}{4}\right)^{1/5}. \tag{0.10}$$

COROLLARY. *We have*

$$S(e^{-\pi\sqrt{3/5}}) = \left(\frac{-5\sqrt{5} - 3 + \sqrt{30(5 + \sqrt{5})}}{4}\right)^{1/5}.$$

we obtain:

[1/4(((5sqrt5 – 3 + sqrt(30(5-sqrt5)))))]^(1/5)

Input:

$$\sqrt[5]{\frac{1}{4}\left(5\sqrt{5} - 3 + \sqrt{30\left(5 - \sqrt{5}\right)}\right)}$$

Result:

$$\frac{\sqrt[5]{-3 + 5\sqrt{5} + \sqrt{30(5 - \sqrt{5})}}}{2^{2/5}}$$

Decimal approximation:

1.340072390306138126503073359233198144708174724298375381579...

1.340072390306…

Alternate forms:

$$\frac{1}{2}\sqrt[5]{\sqrt{30\left(5 - \sqrt{5}\right)} + 5\sqrt{5} - 3}\; 2^{3/5}$$

root of $x^{20} + 3x^{15} - 31x^{10} - 3x^5 + 1$ near $x = 1.34007$

Minimal polynomial:

$x^{20} + 3x^{15} - 31x^{10} - 3x^5 + 1$

From which:

1/((([1/4(((5sqrt5 – 3 + sqrt(30(5-sqrt5)))))]^(1/5))))^1/(Pi)

Input:

$$\frac{1}{\sqrt[\pi]{\sqrt[5]{\frac{1}{4}\left(5\sqrt{5}-3+\sqrt{30\left(5-\sqrt{5}\right)}\right)}}}$$

Exact result:

$$2^{2/(5\pi)}\left(-3+5\sqrt{5}+\sqrt{30\left(5-\sqrt{5}\right)}\right)^{-1/(5\pi)}$$

Decimal approximation:

0.9110323909473981721165218888513219004644324807679748870 48...

0.911032390947…. result very near to the previous solution

Continued fraction:

$$\cfrac{1}{1+\cfrac{1}{10+\cfrac{1}{4+\cfrac{1}{6+\cfrac{1}{34+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{16+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{13+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{3+\cfrac{1}{11+\cfrac{1}{11+\cfrac{1}{2+\cfrac{1}{\ldots}}}}}}}}}}}}}}}}}}}}}$$

Series representations:

$$\frac{1}{\sqrt[\pi]{\sqrt[5]{\frac{1}{4}\left(5\sqrt{5}-3+\sqrt{30\left(5-\sqrt{5}\right)}\right)}}} = 2^{2/(5\pi)}\left(-3+\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(5\times 4^{-k}\sqrt{4}+\left(149-30\sqrt{5}\right)^{-k}\sqrt{149-30\sqrt{5}}\right)\right)^{-1/(5\pi)}$$

$$\frac{1}{\sqrt[\pi]{\sqrt[5]{\frac{1}{4}\left(5\sqrt{5}-3+\sqrt{30\left(5-\sqrt{5}\right)}\right)}}} = 2^{2/(5\pi)}\left(-3+\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)_k\left(5\left(-\frac{1}{4}\right)^k\sqrt{4}+(-1)^k\left(149-30\sqrt{5}\right)^{-k}\sqrt{149-30\sqrt{5}}\right)}{k!}\right)^{-1/(5\pi)}$$

$$\frac{1}{\sqrt[\pi]{\sqrt[5]{\frac{1}{4}\left(5\sqrt{5}-3+\sqrt{30\left(5-\sqrt{5}\right)}\right)}}} = 2^{2/(5\pi)}\left(-3+\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\sqrt{z_0}\left(5\left(5-z_0\right)^k+\left(-30\left(-5+\sqrt{5}\right)-z_0\right)^k\right)z_0^{-k}}{k!}\right)^{-1/(5\pi)}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

Now, we have that:

$$\begin{aligned} Q &= \sqrt{5}2^{1/4}g_{25}^2G_{25} \\ &= \sqrt{5}\left(\frac{1+\sqrt{5}}{2}\right)^{3/2}(3+2\cdot 5^{1/4})^{1/2} \\ &= \sqrt{5}\frac{\sqrt{2}}{\sqrt{5}-1}(1+\sqrt{5})^{1/2}(3+2\cdot 5^{1/4})^{1/2} \\ &= \sqrt{5}\frac{\{(1+5^{1/4})^4\}^{1/2}}{\sqrt{5}-1} \\ &= \sqrt{5}\frac{5^{1/4}+1}{5^{1/4}-1}. \end{aligned}$$

we obtain:

sqrt5* (((5^(1/4) + 1))) / (((5^(1/4) – 1)))

Input:

$$\sqrt{5} \times \frac{\sqrt[4]{5} + 1}{\sqrt[4]{5} - 1}$$

Result:

$$\frac{\sqrt{5} \left(1 + \sqrt[4]{5}\right)}{\sqrt[4]{5} - 1}$$

Decimal approximation:

11.26432468174379095940159067159581768253966491258524836212...

11.2643246817…

Alternate forms:

$$\frac{\sqrt{5} + 5^{3/4}}{\sqrt[4]{5} - 1}$$

$$2 + 2\sqrt[4]{5} + \sqrt{5} + \frac{1}{\frac{\sqrt[4]{5}}{2} - \frac{1}{2}}$$

$$\frac{5}{2} + \frac{3\sqrt{5}}{2} + \sqrt{\frac{5}{2}\left(5 + 3\sqrt{5}\right)}$$

Minimal polynomial:

$$x^4 - 10x^3 - 10x^2 - 50x + 25$$

From which:

((36Pi)/11) 1/ ((((sqrt5* (((5^(1/4) + 1))) / (((5^(1/4) – 1)))))))

Input:

$$\frac{36\pi}{11} \times \frac{1}{\sqrt{5} \times \frac{\sqrt[4]{5}+1}{\sqrt[4]{5}-1}}$$

Result:

$$\frac{36\left(\sqrt[4]{5}-1\right)\pi}{11\sqrt{5}\left(1+\sqrt[4]{5}\right)}$$

Decimal approximation:

0.9127556464939631556336831784268254726191701659908449067 50...

0.9127556464939…. result very near to the previous solution

Property:

$\frac{36\left(-1+\sqrt[4]{5}\right)\pi}{11\sqrt{5}\left(1+\sqrt[4]{5}\right)}$ is a transcendental number

Alternate forms:

$$\frac{36\pi\left(5^{3/4}-\sqrt{5}\right)}{55\left(1+\sqrt[4]{5}\right)}$$

$$\frac{36\left(\sqrt[4]{5}-1\right)\pi}{11\left(\sqrt{5}+5^{3/4}\right)}$$

$$\frac{1}{22}\left(36+\frac{108}{\sqrt{5}}-11\sqrt{\frac{2592}{121}+\frac{7776}{121\sqrt{5}}}\right)\pi$$

Series representations:

$$\frac{36\pi}{\frac{\sqrt{5}\left(\sqrt[4]{5}+1\right)11}{\sqrt[4]{5}-1}}=\frac{36\left(-1+\sqrt[4]{5}\right)\pi}{11\left(1+\sqrt[4]{5}\right)\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{36\pi}{\frac{\sqrt{5}\left(\sqrt[4]{5}+1\right)11}{\sqrt[4]{5}-1}}=\frac{36\left(-1+\sqrt[4]{5}\right)\pi}{11\left(1+\sqrt[4]{5}\right)\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{36\pi}{\frac{\sqrt{5}\left(\sqrt[4]{5}+1\right)11}{\sqrt[4]{5}-1}}=\frac{72\left(-1+\sqrt[4]{5}\right)\pi\sqrt{\pi}}{11\left(1+\sqrt[4]{5}\right)\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}$$

Now, we have that:

$$\varphi_1 \left(p_1, \partial_\xi \pm \sqrt{\frac{\alpha'}{2}}\, p_{31} \right) \varphi_2 \left(p_2, \xi + \partial_\xi \pm \sqrt{\frac{\alpha'}{2}}\, p_{23} \right) \varphi_3 \left(p_3, \xi \pm \sqrt{\frac{\alpha'}{2}}\, p_{12} \right) \Bigg|_{\xi=0} \qquad (3.48)$$

More details on how these results lead directly to the general cubic couplings can be found in [42]. This expression is to be computed at $\xi = 0$, $p_{ij} = p_i - p_j$ and the notation is as in eq. (3.43)

$$A(\xi) \;=\; \sum_{k=0}^{\infty} A_{\mu_1 \ldots \mu_k} \, \frac{\xi^{\mu_1} \ldots \xi^{\mu_k}}{k!} \qquad (3.43)$$

It is instructive to expand eq. (3.48) for the first few cases,

$$\begin{aligned}
\mathcal{A}^{\pm}_{0-0-s} &= \left(\pm\sqrt{\frac{\alpha'}{2}} \right)^{s} \varphi_1\, \varphi_2\, \varphi_3 \cdot p_{12}^{\,s} \,, \\
\mathcal{A}^{\pm}_{1-1-s} &= \left(\pm\sqrt{\frac{\alpha'}{2}} \right)^{s-2} s(s-1)\, A_{1\,\mu}\, A_{2\,\nu}\, \varphi^{\mu\nu\ldots}\, p_{12}^{\,s-2} \\
&+ \left(\pm\sqrt{\frac{\alpha'}{2}} \right)^{s} \Big[A_1 \cdot A_2\, \varphi \cdot p_{12}^{\,s} + s\, A_1 \cdot p_{23}\, A_{2\,\nu}\, \varphi^{\nu\ldots}\, p_{12}^{\,s-1} \\
&+ s\, A_2 \cdot p_{31}\, A_{1\,\nu}\, \varphi^{\nu\ldots}\, p_{12}^{\,s-1} \Big] \\
&+ \left(\pm\sqrt{\frac{\alpha'}{2}} \right)^{s+2} A_1 \cdot p_{23}\, A_2 \cdot p_{31}\, \varphi \cdot p_{12}^{\,s} \,,
\end{aligned} \qquad (3.50)$$

the first of which corresponds to the Wigner currents (3.31) used in [53]. Notice that the

From:

String Lessons for Higher-Spin Interactions
A. Sagnotti and M. Taronna - arXiv:1006.5242v2 [hep-th] 31 Aug 2010

We have that:

$$\begin{aligned}
\mathcal{A}^{\pm} &= \exp\left[\partial_{\xi_1} \cdot \partial_{\xi_2} + \partial_{\xi_2} \cdot \partial_{\xi_3} + \partial_{\xi_3} \cdot \partial_{\xi_1} \right] \\
&\times \phi_1 \left(p_1,\, \xi_1 \pm \sqrt{\frac{\alpha'}{2}}\, p_{23} \right) \phi_2 \left(p_2,\, \xi_2 \pm \sqrt{\frac{\alpha'}{2}}\, p_{31} \right) \phi_3 \left(p_3,\, \xi_3 \pm \sqrt{\frac{\alpha'}{2}}\, p_{12} \right) \Bigg|_{\xi_i=0} \,,
\end{aligned} \qquad (3.16)$$

We can now turn to the spin–s current constructed from a pair of spin–2 fields that is embodied in eq. (3.16). Collecting terms that contain identical numbers of momenta, the result can be cast in the form

The starting point is the expansion of eq. (3.16), computed using the standard multinomial formula,

$$\mathcal{A}^{\pm} = \phi_1 \phi_2 \phi_3 \sum_{i\,,j\,,k \in \mathcal{I}} \left(\pm \sqrt{\frac{\alpha'}{2}} \right)^{s_1+s_2+s_3-2i-2j-2k} \times \left[\frac{s_1!\, s_2!\, s_3!\, p_{23}^{\,s_1-j-k}\; p_{31}^{\,s_2-k-i}\; p_{12}^{\,s_3-i-j}}{i!\, j!\, k!\, (s_1\,-\,j\,-\,k)!\, (s_2\,-\,k\,-\,i)!\, (s_3\,-\,i\,-\,j)!}\, \delta_{23}^{\,i}\, \delta_{31}^{\,j}\, \delta_{12}^{\,k} \right] \,, \tag{4.1}$$

where ϕ_1, ϕ_2 and ϕ_3 are totally symmetric momentum–space fields of spins s_1, s_2 and s_3 respectively, the p_{ij} are defined in eq. (3.6) and

$$\mathcal{I} = \Big\{ i\,,j\,,k \in \mathbb{N} \,\Big|\, s_1 - j - k > 0\,;\; s_2 - k - i > 0\,;\; s_3 - i - j > 0 \Big\}\,. \tag{4.2}$$

4.3 2–2–s Couplings

We can now turn to the spin–s current constructed from a pair of spin–2 fields that is embodied in eq. (3.16). Collecting terms that contain identical numbers of momenta, the result can be cast in the form

$$\begin{aligned}
\mathcal{A}^{\pm}_{2-2-s} \;=\;& \left(\pm\sqrt{\frac{\alpha'}{2}}\right)^{s-4} s(s-1)(s-2)(s-3)\, h_{1\,\alpha_1\alpha_2}\, h_{2\,\beta_1\beta_2}\, \phi^{\alpha_1\alpha_2\beta_1\beta_2\ldots}\cdot p_{12}^{s-4} \\
+& \left(\pm\sqrt{\frac{\alpha'}{2}}\right)^{s-2} \Big[4s(s-1)\, h_{1\,\alpha_1\alpha_2}\, h_2^{\alpha_1}{}_{\beta_2}\, \phi^{\alpha_2\beta_2\ldots}\cdot p_{12}^{s-2} \\
&+ 2s(s-1)(s-2)\, h_{1\,\alpha_1\alpha_2}\, h_{2\,\beta_2}\cdot p_{31}\, \phi^{\alpha_1\alpha_2\beta_2\ldots}\cdot p_{12}^{s-3} \\
&+ 2s(s-1)(s-2)\, h_{1\,\alpha_2}\cdot p_{23}\, h_{2\,\beta_1\beta_2}\, \phi^{\alpha_2\beta_1\beta_2\ldots}\cdot p_{12}^{s-3}\Big] \\
+& \left(\pm\sqrt{\frac{\alpha'}{2}}\right)^{s} \Big[2h_{1\,\alpha_1\alpha_2}\, h_2^{\alpha_1\alpha_2}\, \phi\cdot p_{12}^{s} + 4s\, \Big(h_{1\,\alpha_1\alpha_2}\, h_2^{\alpha_2}\cdot p_{31}\, \phi^{\alpha_1\ldots}\cdot p_{12}^{s-1} \\
&+ h_{1\,\alpha_1}\cdot p_{23}\, h_2^{\alpha_1}{}_{\beta_2}\, \phi^{\beta_2\ldots}\cdot p_{12}^{s-1}\Big) \\
&+ s(s-1)\, \Big(h_{1\,\alpha_1\alpha_2}\, h_2\cdot p_{31}^2\, \phi^{\alpha_1\alpha_2\ldots}\cdot p_{12}^{s-2} \\
&+ h_1\cdot p_{23}^2\, h_{2\,\beta_1\beta_2}\, \phi^{\beta_1\beta_2\ldots}\cdot p_{12}^{s-2} \\
&+ 4\, h_{1\,\alpha_1}\cdot p_{23}\, h_{2\,\beta_1}\cdot p_{31}\, \phi^{\alpha_1\beta_1\ldots}\cdot p_{12}^{s-2}\Big)\Big] \\
+& \left(\pm\sqrt{\frac{\alpha'}{2}}\right)^{s+2} \Big[2s\, h_{1\,\alpha_1}\cdot p_{23}\, h_2\cdot p_{31}^2\, \phi^{\alpha_1\ldots}\cdot p_{12}^{s-1} \\
&+ 2s\, h_1\cdot p_{23}^2\, h_{2\,\beta_1}\cdot p_{31}\, \phi^{\beta_1\ldots}\cdot p_{12}^{s-1} + 4h_{1\,\alpha_1}\cdot p_{23}\, h_2^{\alpha_1}\cdot p_{31}\, \phi\cdot p_{12}^{s}\Big] \\
+& \left(\pm\sqrt{\frac{\alpha'}{2}}\right)^{s+4} h_1\cdot p_{23}^2\, h_2\cdot p_{31}^2\, \phi\cdot p_{12}^{s}\,,
\end{aligned} \tag{4.21}$$

where h_1, h_2 and ϕ are the two spin–2 and the spin–s momentum–space fields. The special form chosen for eq. (4.21) guarantees, once more, that all terms where the gauge parameter is contracted solely with momenta disappear from the gauge variation. In this fashion, in the *massless limit* one recovers the gauge invariant couplings

$$
\begin{aligned}
\mathcal{A}^{[0]\pm}_{2-2-s} \;=\;& 2s(s-1)\left(\pm\sqrt{\frac{\alpha'}{2}}\right)^{s}\Big[h_{1\,\alpha_1\alpha_2}\,h_2^{\alpha_1\alpha_2}\,\phi\cdot p_{12}^{s}+\;2\,\Big(h_{1\,\alpha_1\alpha_2}\,h_2^{\alpha_2}\cdot p_{31}\,\phi^{\alpha_1\cdots}\cdot p_{12}^{s-1}\\
&\qquad+h_{1\,\alpha_1}\cdot p_{23}\,h_2^{\alpha_1}{}_{\beta_2}\,\phi^{\beta_2\cdots}\cdot p_{12}^{s-1}\Big)\\
&\qquad+\Big(h_{1\,\alpha_1\alpha_2}\,h_2\cdot p_{31}^2\,\phi^{\alpha_1\alpha_2\cdots}\cdot p_{12}^{s-2}\\
&\qquad+h_1\cdot p_{23}^2\,h_{2\,\beta_1\beta_2}\,\phi^{\beta_1\beta_2\cdots}\cdot p_{12}^{s-2}\\
&\qquad+\;2\,h_{1\,\alpha_1}\cdot p_{23}\,h_{2\,\beta_1}\cdot p_{31}\,\phi^{\alpha_1\beta_1\cdots}\cdot p_{12}^{s-2}\Big)\Big]\\
&+4s\left(\pm\sqrt{\frac{\alpha'}{2}}\right)^{s+2}\Big[h_{1\,\alpha_1}\cdot p_{23}\,h_2\cdot p_{31}^2\,\phi^{\alpha_1\cdots}\cdot p_{12}^{s-1}+h_1\cdot p_{23}^2\,h_{2\,\beta_1}\cdot p_{31}\,\phi^{\beta_1\cdots}\cdot p_{12}^{s-1}\\
&\qquad+\;h_{1\,\alpha_1}\cdot p_{23}\,h_2^{\alpha_1}\cdot p_{31}\,\phi\cdot p_{12}^{s}\Big]\\
&+\left(\pm\sqrt{\frac{\alpha'}{2}}\right)^{s+4}h_1\cdot p_{23}^2\,h_2\cdot p_{31}^2\,\phi\cdot p_{12}^{s}\,,
\end{aligned}
\tag{4.22}
$$

$s \geq 1$

We have: $s = 2$; $h_1 = 3$; $h_2 = 5$; $\phi = 8$; $p_{23} = p_{31} = p_{12} = 0.968458i$; $\alpha' = 1.0662$

$\alpha = -3$

$$\alpha = -6(s-\frac{3}{2})\,,\quad M^2 = -p^2 = \frac{s-1}{\alpha'}$$

$\alpha = -6*(2-3/2) = -3$

-x^2 = (2-1)/(1.0662)

Complex solutions:

$x = -0.968458\,i$

$x = 0.968458\,i$

p = 0.968458i

(2-1)/(-6(2-3/2))

-x^2 = (2-1)/(-6(2-3/2)) = p = 1/√3

$$-x^2 = -\frac{2-1}{6\left(2-\frac{3}{2}\right)}$$

$$-x^2 = -\frac{1}{3} \quad x = -\frac{1}{\sqrt{3}}$$

$$x = \frac{1}{\sqrt{3}}$$

As in the previous case, it is interesting to stress the type of pattern that is emerging for the gauge invariant results that obtain in the massless limit. The massless 2–2–s coupling can be nicely expressed in the form

$$\begin{aligned} \mathcal{A}^{[0]\pm}_{2-2-s} &= \left[1 \pm \mathcal{G} + \frac{1}{2}\,\mathcal{G}^2\right] \\ &\times h_1 \cdot \left(\xi_1 + \sqrt{\frac{\alpha'}{2}}\,p_{23}\right)^2 h_2 \cdot \left(\xi_2 + \sqrt{\frac{\alpha'}{2}}\,p_{31}\right)^2 \phi_3 \cdot \left(\xi_3 + \sqrt{\frac{\alpha'}{2}}\,p_{12}\right)^s \Bigg|_{\xi_i = 0}, \end{aligned} \tag{4.23}$$

so that the three types of contributions involving different powers of the external momenta arise as iterations induced by the same operator $\mathcal{G}$ that was presented in eq. (4.18).

h_1 , h_2 and φ are the two spin–2 and the spin–s momentum–space fields.

From:

$$\begin{aligned} \mathcal{A}^{[0]\pm}_{2-2-s} \;=\; & 2s(s-1)\left(\pm\sqrt{\frac{\alpha'}{2}}\right)^s \Big[h_{1\,\alpha_1\alpha_2}\, h_2^{\alpha_1\alpha_2}\, \phi \cdot p_{12}^s + 2\,\Big(h_{1\,\alpha_1\alpha_2}\, h_2^{\alpha_2} \cdot p_{31}\, \phi^{\alpha_1 \cdots} \cdot p_{12}^{s\;\;1} \\ & \qquad + h_{1\,\alpha_1} \cdot p_{23}\, h_2^{\alpha_1}{}_{\beta_2}\, \phi^{\beta_2 \cdots} \cdot p_{12}^{s-1}\Big) \\ & \qquad + \Big(h_{1\,\alpha_1\alpha_2}\, h_2 \cdot p_{31}^2\, \phi^{\alpha_1\alpha_2 \cdots} \cdot p_{12}^{s-2} \\ & \qquad + h_1 \cdot p_{23}^2\, h_{2\,\beta_1\beta_2}\, \phi^{\beta_1\beta_2 \cdots} \cdot p_{12}^{s-2} \\ & \qquad + 2\, h_{1\,\alpha_1} \cdot p_{23}\, h_{2\,\beta_1} \cdot p_{31}\, \phi^{\alpha_1\beta_1 \cdots} \cdot p_{12}^{s-2}\Big)\Big] \\ & + 4s\left(\pm\sqrt{\frac{\alpha'}{2}}\right)^{s+2} \Big[h_{1\,\alpha_1} \cdot p_{23}\, h_2 \cdot p_{31}^2\, \phi^{\alpha_1 \cdots} \cdot p_{12}^{s-1} + h_1 \cdot p_{23}^2\, h_{2\,\beta_1} \cdot p_{31}\, \phi^{\beta_1 \cdots} \cdot p_{12}^{s-1} \\ & \qquad + h_{1\,\alpha_1} \cdot p_{23}\, h_2^{\alpha_1} \cdot p_{31}\, \phi \cdot p_{12}^s \Big] \\ & + \left(\pm\sqrt{\frac{\alpha'}{2}}\right)^{s+4} h_1 \cdot p_{23}^2\, h_2 \cdot p_{31}^2\, \phi \cdot p_{12}^s\,, \end{aligned} \tag{4.22}$$

for: $s = 2$; $h_1 = 3$; $h_2 = 5$; $\phi = 8$; $p_{23} = p_{31} = p_{12} = 0.968458i$; $\alpha' = 1.0662$
$\alpha = -3$;

we obtain:

4(sqrt(1.0662/2))^2*[3*5*8*0.968458+2(3*5*8*0.968458^2+15*8*0.968458^2)+(15*8*0.968458^2+15*8*0.968458^2+2(3*5*8*0.968458^2+15*8*0.968458^2)]

Input interpretation:

$$4\sqrt{\frac{1.0662}{2}}^2 (3\times5\times8\times0.968458 + 2(3\times5\times8\times0.968458^2 + 15\times8\times0.968458^2) + (15\times8\times0.968458^2 + 15\times8\times0.968458^2 + 2(3\times5\times8\times0.968458^2 + 15\times8\times0.968458^2)))$$

Result:

2647.81821877434432

2647.81821877434432 result very near to the rest mass of charmed Xi baryon 2645.9

8(sqrt(1.0662/2))^4*[15*8*0.968458^4+15*8*0.968458^4+15*8*0.968458^4]

Input interpretation:

$$8\sqrt{\frac{1.0662}{2}}^4 (15\times8\times0.968458^4 + 15\times8\times0.968458^4 + 15\times8\times0.968458^4)$$

Result:

720.0008628424650977894015804195328

720.00086284…

(sqrt(1.0662/2))^6 * 15*8*0.968458^6

Input interpretation:

$$\sqrt{\frac{1.0662}{2}}^6 \times 15\times8\times0.968458^6$$

Result:

15.00002696383511261506459104682361389470228949248

15.000026963835...

Thence, we obtain:

(2647.81821877434432+720.000862842465097+15.000026963835112615)

Input interpretation:

$2647.81821877434432 + 720.000862842465097 + 15.000026963835112615$

Result:

3382.819108580644529615

3382.819108580644529615 (final result)

From which:

(2647.8182187+720.0008628 +15.0000269)+199+29+11

Input interpretation:

$(2647.8182187 + 720.0008628 + 15.0000269) + 199 + 29 + 11$

Result:

3621.8191084

3621.8191084 result practically equal to the rest mass of double charmed Xi baryon 3621.40

Further, we obtain also:

1/2(2647.8182187+720.0008628 +15.0000269)+34+3+1/golden ratio

Input interpretation:

$$\frac{1}{2}(2647.8182187 + 720.0008628 + 15.0000269) + 34 + 3 + \frac{1}{\phi}$$

ϕ is the golden ratio

Result:

1729.027588...

1729.027588…

Alternative representations:

$$\frac{1}{2}(2647.81821870000+720.001+15.)+34+3+\frac{1}{\phi}=1728.41+\frac{1}{2\sin(54\,°)}$$

$$\frac{1}{2}(2647.81821870000+720.001+15.)+34+3+\frac{1}{\phi}=1728.41+-\frac{1}{2\cos(216\,°)}$$

$$\frac{1}{2}(2647.81821870000+720.001+15.)+34+3+\frac{1}{\phi}=1728.41+-\frac{1}{2\sin(666\,°)}$$

We obtain also:

1/24(2647.8182187+720.0008628 +15.0000269)-golden ratio

Input interpretation:

$$\frac{1}{24}(2647.8182187+720.0008628+15.0000269)-\phi$$

ϕ is the golden ratio

Result:

139.3327622...

139.3327622…

Alternative representations:

$$\frac{1}{24}(2647.81821870000+720.001+15.)-\phi=\frac{3382.82}{24}-2\sin(54\,°)$$

$$\frac{1}{24}(2647.81821870000+720.001+15.)-\phi=2\cos(216\,°)+\frac{3382.82}{24}$$

$$\frac{1}{24}(2647.81821870000+720.001+15.)-\phi=\frac{3382.82}{24}+2\sin(666\,°)$$

1/24(2647.8182187+720.0008628 +15.0000269)-13-golden ratio^2

Input interpretation:

$$\frac{1}{24}(2647.8182187 + 720.0008628 + 15.0000269) - 13 - \phi^2$$

ϕ is the golden ratio

Result:

125.3327622...

125.3327622…

Alternative representations:

$$\frac{1}{24}(2647.81821870000 + 720.001 + 15.) - 13 - \phi^2 = -13 + \frac{3382.82}{24} - (2\sin(54°))^2$$

$$\frac{1}{24}(2647.81821870000 + 720.001 + 15.) - 13 - \phi^2 = -13 + \frac{3382.82}{24} - (-2\cos(216°))^2$$

$$\frac{1}{24}(2647.81821870000 + 720.001 + 15.) - 13 - \phi^2 = -13 + \frac{3382.82}{24} - (-2\sin(666°))^2$$

1/24(2647.8182187+720.0008628 +15.0000269)-Pi-1/golden ratio

Input interpretation:

$$\frac{1}{24}(2647.8182187 + 720.0008628 + 15.0000269) - \pi - \frac{1}{\phi}$$

ϕ is the golden ratio

Result:

137.1911695...

137.1911695…

This result is very near to the inverse of fine-structure constant 137,035

Alternative representations:

$$\frac{1}{24}(2647.81821870000+720.001+15.)-\pi-\frac{1}{\phi}=-\pi+\frac{3382.82}{24}--\frac{1}{2\cos(216\,^\circ)}$$

$$\frac{1}{24}(2647.81821870000+720.001+15.)-\pi-\frac{1}{\phi}=-180\,^\circ+\frac{3382.82}{24}--\frac{1}{2\cos(216\,^\circ)}$$

$$\frac{1}{24}(2647.81821870000+720.001+15.)-\pi-\frac{1}{\phi}=-\pi+\frac{3382.82}{24}-\frac{1}{2\cos\left(\frac{\pi}{5}\right)}$$

Series representations:

$$\frac{1}{24}(2647.81821870000+720.001+15.)-\pi-\frac{1}{\phi}=140.951-\frac{1}{\phi}-4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\frac{1}{24}(2647.81821870000+720.001+15.)-\pi-\frac{1}{\phi}=142.951-\frac{1}{\phi}-2\sum_{k=1}^{\infty}\frac{2^k}{\binom{2k}{k}}$$

$$\frac{1}{24}(2647.81821870000+720.001+15.)-\pi-\frac{1}{\phi}=140.951-\frac{1}{\phi}-\sum_{k=0}^{\infty}\frac{2^{-k}(-6+50k)}{\binom{3k}{k}}$$

Integral representations:

$$\frac{1}{24}(2647.81821870000+720.001+15.)-\pi-\frac{1}{\phi}=140.951-\frac{1}{\phi}-2\int_0^\infty\frac{1}{1+t^2}\,dt$$

$$\frac{1}{24}(2647.81821870000+720.001+15.)-\pi-\frac{1}{\phi}=140.951-\frac{1}{\phi}-4\int_0^1\sqrt{1-t^2}\,dt$$

$$\frac{1}{24}(2647.81821870000+720.001+15.)-\pi-\frac{1}{\phi}=140.951-\frac{1}{\phi}-2\int_0^\infty\frac{\sin(t)}{t}\,dt$$

And again:

(2647.8182187+720.0008628 +15.0000269)^1/16-18/10^3

Input interpretation:

$$\sqrt[16]{2647.8182187 + 720.0008628 + 15.0000269} - \frac{18}{10^3}$$

Result:

1.64380447239...

1.64380447239...

(2647.8182187+720.0008628 +15.0000269)^1/17+5/10^3

Input interpretation:

$$\sqrt[17]{2647.8182187 + 720.0008628 + 15.0000269} + \frac{5}{10^3}$$

Result:

1.61788957127...

1.61788957127…

sqrt[6((((2647.8182187+720.0008628 +15.0000269)^1/16-18/10^3)))]

Input interpretation:

$$\sqrt{6\left(\sqrt[16]{2647.8182187 + 720.0008628 + 15.0000269} - \frac{18}{10^3}\right)}$$

Result:

3.14051378509...

3.14051378509... ≈ π

In conclusion, we obtain also:

(2647.8182187+720.0008628 +15.0000269)^1/15+13/10^3

Input interpretation:

$$\sqrt[15]{2647.8182187 + 720.0008628 + 15.0000269} + \frac{13}{10^3}$$

Result:

1.73203710795...

1.73203710795.... ≈ $\sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{\left(3\sqrt{3}\right) M_s}{2}.$$

(see: Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019)

Possible closed forms:

$\sqrt{3} \approx 1.732050807$

$\mathcal{T}_T \approx 1.732050807$

$\frac{5\pi^4}{14 e^3} \approx 1.732040383$

Appendix

From the following Ramanujan Taxicab Numbers

$$135^3 + 138^3 = 172^3 - 1$$

$$9^3 + 10^3 = 12^3 + 1$$

we obtain:

135^3+138^3 – 9^3+10^3 = 5088718

Input:

$$135^3 + 138^3 - 9^3 + 10^3$$

Result:

5 088 718

5088718

From which:

(5088718 - 135^3+9^3 - 10^3)^1/3+golden ratio

Input:

$$\sqrt[3]{5\,088\,718 - 135^3 + 9^3 - 10^3} + \phi$$

ϕ is the golden ratio

Result:

$\phi + 138$

Decimal approximation:

139.6180339887498948482045868343656381177203091798057628621...

139.61803398….

Alternate forms:

$$\frac{1}{2}\left(277+\sqrt{5}\right)$$

$$\frac{277}{2}+\frac{\sqrt{5}}{2}$$

$$138+\frac{1}{2}\left(1+\sqrt{5}\right)$$

Alternative representations:

$$\sqrt[3]{5088718-135^3+9^3-10^3}+\phi=\sqrt[3]{5088718+9^3-10^3-135^3}+2\sin(54°)$$

$$\sqrt[3]{5088718-135^3+9^3-10^3}+\phi=-2\cos(216°)+\sqrt[3]{5088718+9^3-10^3-135^3}$$

$$\sqrt[3]{5088718-135^3+9^3-10^3}+\phi=\sqrt[3]{5088718+9^3-10^3-135^3}-2\sin(666°)$$

(5088718 - 138^3+9^3 - 10^3)^1/3-10+1/golden ratio

Input:

$$\sqrt[3]{5088718-138^3+9^3-10^3}-10+\frac{1}{\phi}$$

ϕ is the golden ratio

Result:

$$\frac{1}{\phi}+125$$

Decimal approximation:

125.6180339887498948482045868343656381177203091798057628621...

125.61803398….

Alternate forms:

$$\frac{1}{2}\left(249+\sqrt{5}\right)$$

$$\frac{125\,\phi+1}{\phi}$$

$$\frac{\sqrt{5}}{2}+\frac{249}{2}$$

Alternative representations:

$$\sqrt[3]{5\,088\,718-138^3+9^3-10^3}-10+\frac{1}{\phi}=$$
$$-10+\sqrt[3]{5\,088\,718+9^3-10^3-138^3}+\frac{1}{2\sin(54\,°)}$$

$$\sqrt[3]{5\,088\,718-138^3+9^3-10^3}-10+\frac{1}{\phi}=$$
$$-10+-\frac{1}{2\cos(216\,°)}+\sqrt[3]{5\,088\,718+9^3-10^3-138^3}$$

$$\sqrt[3]{5\,088\,718-138^3+9^3-10^3}-10+\frac{1}{\phi}=$$
$$-10+\sqrt[3]{5\,088\,718+9^3-10^3-138^3}+-\frac{1}{2\sin(666\,°)}$$

135^3+138^3 – x = 5086718

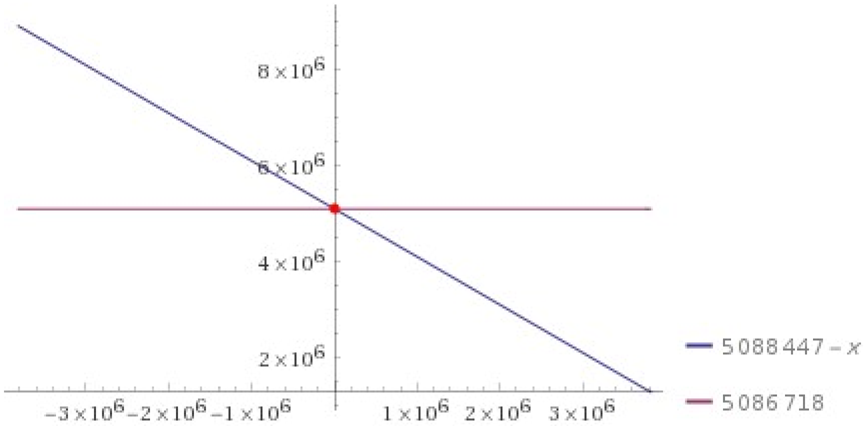
Input:

$$135^3+138^3-x=5\,086\,718$$

Result:

$$5\,088\,447-x=5\,086\,718$$

Plot:



Alternate form:

$1729 - x = 0$

Solution:

$x = 1729$

1729

Observations

Figs.

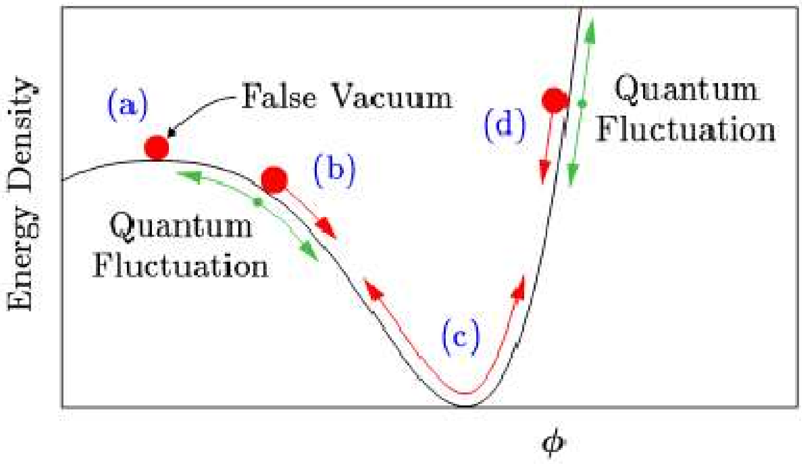
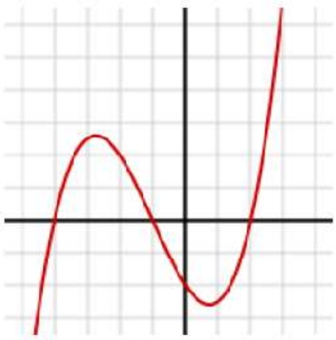


FIG. 1: In simple inflationary models, the universe at early times is dominated by the potential energy density of a scalar field, ϕ. Red arrows show the classical motion of ϕ. When ϕ is near region (a), the energy density will remain nearly constant, $\rho \cong \rho_f$, even as the universe expands. Moreover, cosmic expansion acts like a frictional drag, slowing the motion of ϕ: Even near regions (b) and (d), ϕ behaves more like a marble moving in a bowl of molasses, slowly creeping down the side of its potential, rather than like a marble sliding down the inside of a polished bowl. During this period of "slow roll," ρ remains nearly constant. Only after ϕ has slid most of the way down its potential will it begin to oscillate around its minimum, as in region (c), ending inflation.



Graph of a cubic function with 3 real roots (where the curve crosses the horizontal axis at $y = 0$). The case shown has two critical points. Here the function is $f(x) = (x^3 + 3x^2 - 6x - 8)/4$.

The ratio between M_0 and q

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3})\, M_s}{2}.$$

i.e. the gravitating mass M_0 and the Wheelerian mass q of the wormhole, is equal to:

$$\frac{\sqrt{3\,(2.17049 \times 10^{37})^2 - 0.001^2}}{\frac{1}{2}\,((3\sqrt{3})\,(4.2 \times 10^6 \times 1.9891 \times 10^{30}))}$$

1.7320507879051944207039476256710181600835665488020824605 20...

1.7320507879

$1.7320507879 \approx \sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q of the wormhole

We note that:

$$\left(-\frac{1}{2} + \frac{i}{2}\sqrt{3}\right) - \left(-\frac{1}{2} - \frac{i}{2}\sqrt{3}\right)$$

i is the imaginary unit

$i\sqrt{3}$

1.7320508075688772935274463415058723669428052538103806280 55... i

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

1.73205

This result is very near to the ratio between M_0 and q, that is equal to 1.7320507879 $\approx \sqrt{3}$

With regard $\sqrt{3}$, we note that is a fundamental value of the formula structure that we need to calculate a Cubic Equation

We have that the previous result

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) = i\sqrt{3} =$$

$$= 1.7320508075688772935274463415058723669428052538103806280558... i$$

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

can be related with:

$$u^2(-u)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)+v^2(-v)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)=q$$

Considering:

$$(-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q$$

$$= i\sqrt{3} = 1.7320508075688772935274463415058723669428052538103806280558... i$$

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

Thence:

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) \Rightarrow$$

$$\Rightarrow (-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q=1.73205\approx\sqrt{3}$$

We observe how the graph above, concerning the cubic function, is very similar to the graph that represent the scalar field (in red). It is possible to hypothesize that cubic functions and cubic equations, with their roots, are connected to the scalar field.

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.

Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

References

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