SOLUTIONS OF SYSTEMS OF LINEAR DIOPHANTINE EQUATIONS

AND

PRIME NUMBERS

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Abstract:

A large number of systems of diophantine equations have been generated, paying particular attention to the solutions identified as prime numbers.
A statistical approach was taken to create such systems with random terms and coefficients. This paper gives a detailed view of the work carried out and the analysis of these solutions which are part of the set of prime numbers.
Data, observations and an analysis of results allowed to evaluate the frequency of prime number solutions, identify some structures in prime number solution scatter plots and some relationships between the number of prime solutions and the number of primes and finally establish two conjectures on the number of prime solutions generated by these linear diophantine equation systems.

Key Words : linear diophantine equation systems, prime numbers.

1- IT Tools:

- PC: AMD (tm) XP 2800+
  2.08 GHz. RAM: 1.00Go
- Software: Windows and Excel 2010
- Program: specifically developed for this study using the Visual Bsic for Applications *(VBA)* language.
- Program algorithm: see annex n°1.

2 Generation of systems of linear diophantine equations, and screening of their solutions belonging to the set of prime numbers.

2-1: systems of linear diophantine equations:
The systems generated are composed of *n* linear diophantines equations with

n unknowns.The lower bound of *n* is 2 and *n* can take any integer value greater than 2. However, systems with a large number of equations will be discussed more specifically later.

Example: a system of 2 linear diophantine equations with 2 unknowns and a pair of solutions which are prime numbers.

$8* x_1 + 2*x_2 = 178$ *Eq (1)*

$7* x_1 +3* x_2 = 172$ *Eq (2)*

The 2 solutions of this system are prime numbers, thus named *prime solutions* of this system: $x_{1\ prime} = 19$ and $x_{2\ prime} = 13$

Figure n°1 of annex-2 gives a geometric representation of this linear diophantine equation system. The pair of prime solutions is found at the intersection of the two lines.

To produce a set of systems of *n* linear diophantine equations with *n* unknowns, random terms and coefficients have been generated within specific ranges using the Visual Basic for Applications *(VBA)* functions « randomized » and « randbetween » as described below for a system of 2 linear diophantine equations and 2 unknowns :

$a*x_1 + b*x_2 = b_1$ *Eq (3)*

$c*x_1 + d*x_2 = b_2$ *Eq (4)*

a, b, c, d are the respective coefficients of the 2 equations and b_1 and b_2 their respective terms.

This study covers systems with 2, 3 and 4 equations with corresponding 2, 3 and 4 unknowns.
The coefficients a. b. c. and d on the left side of equations *Eq(3)* and *Eq (4)* have been varied within the range (1 to 10) and the terms on the right side of the equations in the following ranges :

100 to 300

1100 to 1300

10100 to 10300

100100 to 100300

1 000100 to 1 000300

10 000100 to 10 000300

100 000100 to 100 000300

and within a larger range : 1000000 to 3000000

<u>Notes</u> :

- *only equation systems with a matrix determinant different from zero have been considered for having a single n-uplet of solutions.*

- *only positive integer have been allocated to the coefficients and terms to systematically give n-uplets solutions some of which being n-uplets prime solutions. Indeed. for some sets of positive and/or negative integers coefficients and terms, n-uplets of prime solutions may not be obtained.*

- *The higner values on the right side of the equations also allows to produce high prime solutions.*

2-2: randomness of the terms and coefficients :

As it is known that computers can only produce pseudo-random numbers, we have tested the randomness of the coefficients and terms generated by the functions « randomize » and « randbetween ».
Three series of $2*10^4$ pseudo-random numbers have been produced in the range (1 to 10) to simulate the a, b, c, d coefficients of equations *Eq(3)* and *Eq(4)*.
Considering the three sets, -4.4 and 6.2 were the extreme values respectively found below and above the median. Figures n°2, n°3 and n°4 illustrate the histograms of these three sets.
Similarly. three sets of $2*10^4$ pseudo-random numbers have been produced in the respective ranges (100-300). (100000 -300000) and (1000000 - 3000000) to simulate the b_1 and b_2 terms of equations *Eq(3)* and *Eq(4)*.
Considering the three sets. -3.2 and 3.4 were the extreme values respectively found below and above the median. Figures n°5, n°6 and n°7 illustrate the histograms of these three sets.
So. this bias of the theoritical uniform distributions of the coefficients and terms is relatively small. However to minimize it further, sets of 100 runs of 20 000 linear diophantine equation systems each were performed to generate sets of 100 * 20 000 = $2*10^6$ linear diophantine equation systems.

2-3: sieving of solutions:

Each element of *n* uplet solutions of the selected equation systems can be a positive decimal or a positive integer the latter being or not a prime number. On the whole sets of *n* uplet solutions. the following sieves have been applied:

- Sieve -1 : retains equation systems of which all elements of the *n* uplets of solutions are = or > 2 *(the lowest prime number being 2).*Therefore, if only one element of a *n* uplet is < 2, then the corresponding equation system is eliminated by this first sieve.

- Sieve-2: from the equation systems retained by sieve-1, screen-2 retains equation systems of which all elements of the *n* uplets of solutions are integers. Therefore, if only one element of a *n* uplet is decimal, then the corresponding equation system is eliminated by sieve-2.

- Sieve-3 : from the equation systems retains by sieve-2, sieve -3 retains equation systems for which all elements of the *n*-uplets of solutions are prime numbers. Therefore, if ony one element of a *n* uplet is not a prime number, then the corresponding equation system is eliminated by sieve-3.

  So, at the end of the sieving process, only systems having *n*-uplets of solutions of which each element is a prime number are retained.

3- Results :

3-1 : systems of 2 linear diophantine equations with 2 unkowns :

3-1-1 number and frequency of prime solutions :

Seven sets of 100 runs, each of them containing $2*10^4$ equations systems, have been generated, the b_1 and b_2 terms varying as indicated above, within the following ranges: 100-300, 1100-1300, 10100-10300, 100100-100300, 1000100-1000300, 10000100-10000300 and 100000100-100000300.
The a, b,c and d coefficients vary within the range 1-10.
A first look to the histograms of figures n°8, n°9, n°10, n°11, n°12, n°13 and n°14 indicates that there is a trend for the number of pairs of prime solutions per run to decreases when the b_1 and b_2 terms increase. *(e.g : wheras, for the range b_1-b_2 : 100-300, over 100 runs 44 runs give 20 pairs of prime solutions per run and 47 runs give more than 20 pairs of prime solutions per run, for the range b_1-b_2 : 1100-1300, over 100 runs only one run gives 20 pairs of prime solutions.* As a consequence of this, the total number of pairs of prime solutions for a given number of runs decreases when the b_1 and b_2 terms increase. This is what shows

table n°1 which gives, for each range of b_1 and b_2 terms and different number of runs, the number of pairs of prime solutions . Indeed, the number of pairs of prime solutions logically increases while the number of runs increase and decreases while the b_1 and b_2 terms increase.

This is also illustrated by figures n°15 and n°16, the latter showing a good fit with the equation:

$y = - 931,1*Ln(x) + 1977,7$ ($R^2 = 0,973$) *Eq(5)*

More detailed statistics on the number of pairs of prime solutions per run are given in table n°2 and illustrated by figure n°17.

The corresponding frequency of pairs of prime solutions is calculated by dividing the number of pairs of prime solutions by the corresponding number of runs times $2*10^4$.
In line with the number of pairs of prime solutions, table n°3 shows that the frequency of pairs of prime solutions does not depend on the number of runs indicating that the results are not biased by a too important lack of randomness of either the coefficients or the terms and that the frequency of pairs of prime solutions decreases while the $b_1$and b_2 terms increase. This is shown by figure n°18.

3-1-2 : distributions of $x_{1\ prime}$ and $x_{2\ prime}$ solutions :

The $x_{1\ prime}$ values histograms of figures n° 19, n°20, n°21,
n° 22, n°23, n°24 and n°25 indicate a general drift to higher $x_{1\ prime}$ values while going from the 100-300 b_1-b_2 range to the 100000100-100000300 b_1-b_2 range.
The $x_{2\ prime}$ values histograms of figures n° 26, n°27, n°28, n° 29, n°30, n°31 and n°32 show the same trend.
Tables n°4 and n°5 give the respective statistics on the $x_{1\ prime}$ and $x_{2\ prime}$ solution values.
Figures n°33, and n°34 show the bar charts corresponding to these tables.
Note : For keeping the charts clear, labels have be allocated to the black bars of the maxima only.
Going into more details and back to figures n°19 and n°26, while there is a continuous decrease of $x_{1\ prime}$ and $x_{2\ prime\ values}$ for the b_1-b_2 range 100-300,well fitted by the respective equations :

for $x_{1\ prime}$: $y = - 10,425 *x^5 + 207,71* x^4 – 1611,5* x^3 + 6095,8 *x^2 – 11311* x + 8369$ *Eq(6)*
($R^2 = 1$)

for $x_{2\ prime}$:

$$y = -11{,}375*x^5 + 226{,}87*x^4 - 1759{,}5*x^3 + 6638{,}6*x^2 - 12240*x + 8936 \quad Eq(7)$$

(R^2 = 1)

for the b_1-b_2 range 1100-1300, a second distribution at higher $x_{1\ prime}$ and $x_{2\ prime}$ values is appearing on the right side of the histograms with a continum between the first and the second distribution. Both distributions are weel fitted by the following equations :

for $x_{1\ prime}$ solutions :

$$y = -1{,}144*x^3 + 31{,}04*x^2 - 265{,}4*x + 729{,}5 \quad Eq(8)$$

(R^2 = 0,961)

for $x_{2\ prime}$ solutions :

$$y = -0{,}020*x^5 + 0{,}949*x^4 - 17{,}09*x^3 + 151{,}8*x^2 - 664{,}3*x + 1153 \quad Eq(9)$$

(R^2 = 0,9982)

for the b_1-b_2 range : 10100 -10300, a gap between the two prime value distributions is visible and for the last four b_1-b_2 ranges;100100-100300, 1000100 - 1000300, 10000100 -10000300 and 100000100 -100000300, the width of the gaps and the importance of the second distribution increase.
figures n° 35 to n°41 give the $x_{1\ prime}$ solution values in the increasing order of their reference and similarly for $x_{2\ prime}$ solutions, figures n°42 to n°48 also show the gaps as the compiled figures n°49 and n° 50. Secondary gaps showing a discontinuity in the prime solutions, can also be seen on these figures.
So, identified above gaps do represent some discontinuity in the prime solution distributions.
Figure n° 51 shows for the $x_{1\ prime}$ and $x_{2\ prime}$ solutions the exponential increase of the width of the gaps *(difference between their maximum and minimum)* while the b_1-b_2 range increases, accordingly to the equations :

for $x_{1\ prime}$ solutions : $y = 9{,}1947*e^{2{,}5433*x}$ *(Eq10)*

(R^2 = 0,999)

for $x_{2\ prime}$ solutions : $y = 144{,}14*e^{2{,}3742*x}$ *(Eq11)*

(R^2 = 0,995)

Figure n°52 gives the same relationships on a semi-logarithmic y axis.

3-1-3 : Patterns in $x_{2\ prime}$ - $x_{1\ prime}$ solutions plots :

x-y coordinates scatter plots were produced to reveal in a different way the distribution of prime solutions and possible patterns. So, $x_{2\ prime}$ solutions were plotted versus $x_{1\ prime}$ solutions for the seven b_1-b_2 ranges. This is illustrated by figures n°53, n°54, n°55, n°56, n°57, n°58, n°59, n°60 and n°61.
All plots show an absence of points of $x_{1\ prime}$ and $x_{2\ prime}$ coordinates nearby the origin and a decrease of the density of them while the distance from the origin increases. This general pattern is in good agreement with the histograms of x_{1prime} and $x_{2\ prime}$ solutions previously described.
In figures n°53 and n°54 the latter being a zoom of figure n°53, we can see some arrangements of the points having $x_{1\ prime}$ and $x_{2\ prime}$ as coordinates :

- The horizontal and vertical bands which are free of solutions are easily explained by the exclusion of decimal solutions .

- A lot of solutions are vertically or horizontally aligned.This means that several $x_{2\ prime}$ solutions have been obtained for a given $x_{1\ prime}$ solution and vice-versa.

- It is also noticeable that other points are aligned in other directions.

These patterns can be seen in other $x_{2\ prime}$ - $x_{1\ prime}$ plots. Therefore, there is an evidence of some order in the prime solutions of these 2 linear diophantine equation systems.

To investigate this further, we performed one run of $2*10^4$ 2 linear diophantine equation systems with 2 unkowns, the coefficients a, b, c, d varying in the range 1-10 and the b_1-b_2 terms in the range 100-300. Then, $x_{2\ prime}$ - $x_{1\ prime}$ plots have been produced with all solutions, then with solutions > 2, then with integer solutions and finally with prime solutions. For having a representative comparison,
2096 solutions have been taken for each of these solution types.
Figures n°62, n° 63 *(zoom-1 of figure n°62)* and n°64 *(zoom-2 of figure n°62)* plot all solutions : no specific pattern can be seen, the solutions are randomly distributed. A similar observation can be made from figures n° 65, n° 66 *(zoom-1 of figure n°65)* and n° 67 *(zoom-2 of figure n°65)* plotting solutions > 2 which still include decimal solutions. Figures n°68, n°69 *(zoom-1 of figure n°68)*, n° 70 , *(zoom-2 of figure n°68)* which include interger solutions only, reveal again the free solution bands and the horizontal and vertical alignements of points. Some other points are also aligned in other directions. Figures n° 71 and n°72 also show the alignement of some prime solution points, confirming previous findings.

3-1-4 : relationships between the number of prime solutions and the number of primes :

It is well known that the number of prime numbers is decreasing while the size of them increases. In this paper, we have already shown that the number of prime solutions decreases with the size of them *(as shown by figures n°19 and n°26)*. This led to investigate the possible relationships between the number of prime solutions and the number of prime numbers.

Definition of the prime counting function :

If x is a positive number, then the prime counting function $\pi(x)$ gives the number of primes p, such that $p \leq x$.

e.g : $\pi(10) = 4$; the 4 prime numbers being 2,3,5 and 7.
$\pi(20) = 8$; the 8 prime number being 2, 3, 5, 7, 11, 13, 17 and19
and so on.

So using the $\pi(x)$ counting function, the number of prime numbers within each class of x_{1prime} and $x_{2\ prime}$ solutions corresponding to the seven ranges of b_1-b_2 terms *(100-300, 1100-1300, 10100-10300, 100100-100300, 1000100-1000300, 10000100-10000300, and100000100-100000300)* has been calculated. As shown by figures n°73 and n°74, this allowed to compare the number of $x_{1\text{-prime}}$ and $x_{2\text{-prime}}$ solutions of the seven sets described in paragraph 3-1-1 with the corresponding number of prime numbers.
The two diagrams show that the number of $x_{1\ prime}$ and $x_{2\ prime}$ solutions is logically inferior to the corresponding number of prime numbers, except fot the the first 100-300 b_1-b_2 range of figure n°74 which is explained by some duplicates or triplicates or *n*-plicates of $x_{1\text{-prime}}$-$x_{2\ prime}$ solutions.
Figure n° 75 gives the $\pi(x)$/number of prime solutions ratio for the seven ranges of b_1-b_2 terms.
Furthermore, 100 extra runs, each of them containing $2*10^4$ 2 linear equation systems with 2 unkowns were performed with the a, b, c, d coefficients varying in the range 1-10 and the b_1-b_2 terms varying in the range 1000000-3000000, thus allowing to explore a relatively large range of prime solutions.
Figure n° 76 shows the histogram of runs, with a second distribution containing only 34 pairs of $x_{1\ prime}$- $x_{2\ prime}$ solutions in 4 runs which is therefore relatively small.
Figures n°77 and n°78 show the histograms of the $x_{1\ prime}$ and $x_{2\ prime}$ solutions indicating again the drop of the number of prime solutions while their size increases.

The fitted equations are :

for $x_{1\ prime}$ solutions :

$y = 0{,}2086*x^4 – 5{,}2432*x^3 + 48{,}189*x^2 – 194{,}03*x + 298$ *Eq(12)*
(R^2 = 0,999)

for $x_{2\ prime}$ solutions :

$y = 0{,}0046*x^6 – 0{,}2006*x^5 + 3{,}4943*x^4 – 31{,}144*x^3 + 150{,}61*x^2 – 382{,}3*x + 418{,}04$ *Eq(13)*
(R^2 = 0,998)

Figures n°79 and n°80 *(zoom of figure n°79)* show the x_{1prime}-$x_{2\ prime}$ solution scatter plot and again some structure and alignement of points.

Over the 100 runs 266 pairs of $x_{1\ prime}$ and $x_{2\ prime}$ solutions were obtained. This corresponds to a frequency of : 0,000133. The minimum and maximum values for $x_{1\ prime}$ and $x_{2\ prime}$ were :
$x_{1\ prime}$:
minimum : 7, maximum : 222953
$x_{2\ prime}$:
minimum : 89, maximum : 280451
So, we have defined 12 classes between 25000 and 300000 with a width of 25000 and use the $\pi(x)$ counting function to calculate the number of prime number within each class.
Figure n°81 shows the coresponding histogram well fitted by the equation:

$y = – 0{,}084*x^5 + 3{,}076*x^4 – 42{,}99*x^3 + 288{,}5*x^2 – 963{,}2*x + 3468$ *Eq(14)*
(R^2 = 0,995)
The graph shows the expected decrease of the number of prime numbers when their size increases.

Figures n°82, n°83 show the correlations between the number of $x_{1\ prime}$ and $x_{2\ prime}$ solutions and the number of prime numbers. Figures n°84 and n°85 show the correlations expressed in percentage. To establish these correlations the number of $x_{1\ prime}$ and $x_{2\ prime}$ solutions and the number of prime have been allocated to the 12 classes used to calculate, as mentioned above, the number of prime numbers.The four curves are well fitted by the respective equations :

for the number of $x_{1\ prime}$ solutions = f (number of prime numbers)

$$y = -6*10^{-7}*x^3 + 0{,}004*x^2 - 10{,}90*x + 8412 \quad Eq(15)$$
(R^2 = 0,993)

for the number of $x_{2\ prime}$ solutions = f (number of prime numbers)

$$y = -4*10^{-7}*x^3 + 0{,}003*x^2 - 7{,}707*x + 6050 \quad Eq(16)$$
(R^2 = 0,998)

for the percentage of $x_{1\ prime}$ solutions = f (percentage of prime numbers)

$$y = -4{,}204*x^3 + 117{,}4*x^2 - 1066*x + 3162 \quad Eq(17)$$
(R^2 = 0,993)

for the percentage of $x_{2\ prime}$ solutions = f (percentage of prime numbers)

$$y = -2{,}797*x^3 + 81*x^2 - 753{,}2*x + 2274 \quad Eq(18)$$
(R^2 = 0,998)

A good agreement between the equations *Eq(15)* and *Eq(16)* and between the equations *Eq(17)* and *Eq(18)* is obsereved.

3-2: systems of 3 linear diophantine equations with 3 unkowns :

Following the same approach as above, systems of 3 linear diophantine equations with 3 unkowns have investigated.

System of 3 linear diophantine equations with 3 unkowns:

$$a*x + b*y + c*z = b_1 \quad Eq\ (19)$$

$$d*x + e*y + f*z = b_2 \quad Eq\ (20)$$

$$g*x + h*y + i*z = b_3 \quad Eq\ (21)$$

The coefficients a, b, c, d, e, f, g, h and i have been randomly generated in the range 1 to 10 and the b_1, b_2 and b_3 terms in the range 100 to 300.

Over 100 runs each of them containg $2*10^4$ systems, only 11 triplets of prime solutions have been obtained. This corresponds to a frequency of $5{,}50x10^{-6}$, so a much lower frequency compared to that of 0,001048 for the pairs of prime solutions from systems of 2 linear diophantine equations with 2 unkowns.

Figure n°: 86 gives the repartition of $x_{1\ prime}$, $x_{2\ prime}$, $x_{3\ prime}$ solutions and prime numbers, but the number of prime solutions is too small to extend the investigation further.

3-3: systems of 4 linear equations with 4 unkowns :

Finally, systems of 4 linear diophantine equations with 4 unkowns have been generated varying the a to p coefficients and the b_1, b_2, b_3, b_4 terms below within the respective ranges: 1 to 10 and 100 to 300.

System of 4 linear diophantine equations with 4 unkowns:

$a*x + b*y + c*z + d*w = b_1$ *Eq (22)*

$e*x + f*y + g*z + h*w = b_2$ *Eq (23)*

$i*x + j*y + k*z + l*w = b_3$ *Eq (24)*

$m*x + n*y + o*z + p*w = b_4$ *Eq (25)*

Over 10200000 systems, only eight 4-uplets of integer solutions have been found , thus a coresponding frequency of $7,8 * 10^{-7}$ with no prime solutions indicating a probabilty of less than $9,8*10^{-8}$ to find one .

4 – Conjectures :

From these results we assume that :

- If the coefficients and terms of the diophantine equations of the systems are constrained to vary in fixed intervals, then the number of prime solutions is defined.
- On the contrary, if these coefficients and terms are not constrained in fix intervals, thus can vary from 1 to + ∞, then the number of prime solutions is infinite, even if this number decreases rapidelly when the number of equations of the system and the size of the solutions increase.
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So, from the above the two following conjectures are formulated :

Conjecture CD1

There is finite number of linear diophantine equation systems of *n* equations with *n* unknowns, as written below, having a *n*-uplet of solutions which are all prime numbers , when the « a » coefficients and the « b» terms are positive integers *(0 excluded)* which vary within fixed intervals those endpoints are positive integers *(0 excluded)*.

$a_{11}*x_1 + a_{12}*x_2 + \ldots a_{1n}*x_n = b_1$ *Eq (26)*
$a_{21}*x_1 + a_{22}*x_2 + \ldots a_{2n}*x_n = b_2$ *Eq (27)*
$a_{n1}*x_1 + a_{n2}*x_2 + \ldots a_{nn}*x_n = b_n$ *Eq (28)*

Conjecture CD2 :

There is an infinite number of systems of *n* linear diophantine equations with *n* unkowns, as written below, having a *n*-uplet of solutions which are all prime numbers , when the « a » coefficients and the « b » terms are positive integers (0 excluded) which vary from 1 to + ∞.

$a_{11}*x_1 + a_{12}*x_2 + \ldots a_{1n}*x_n = b_1$ *Eq (29)*
$a_{21}*x_1 + a_{22}*x_2 + \ldots a_{2n}*x_n = b_2$ *Eq (30)*
$a_{n1}*x_1 + a_{n2}*x_2 + \ldots a_{nn}*x_n = b_n$ *Eq (31)*

5- Conclusions :

5-1: A Visual Basic for Applications *(VBA)* program has been written to :

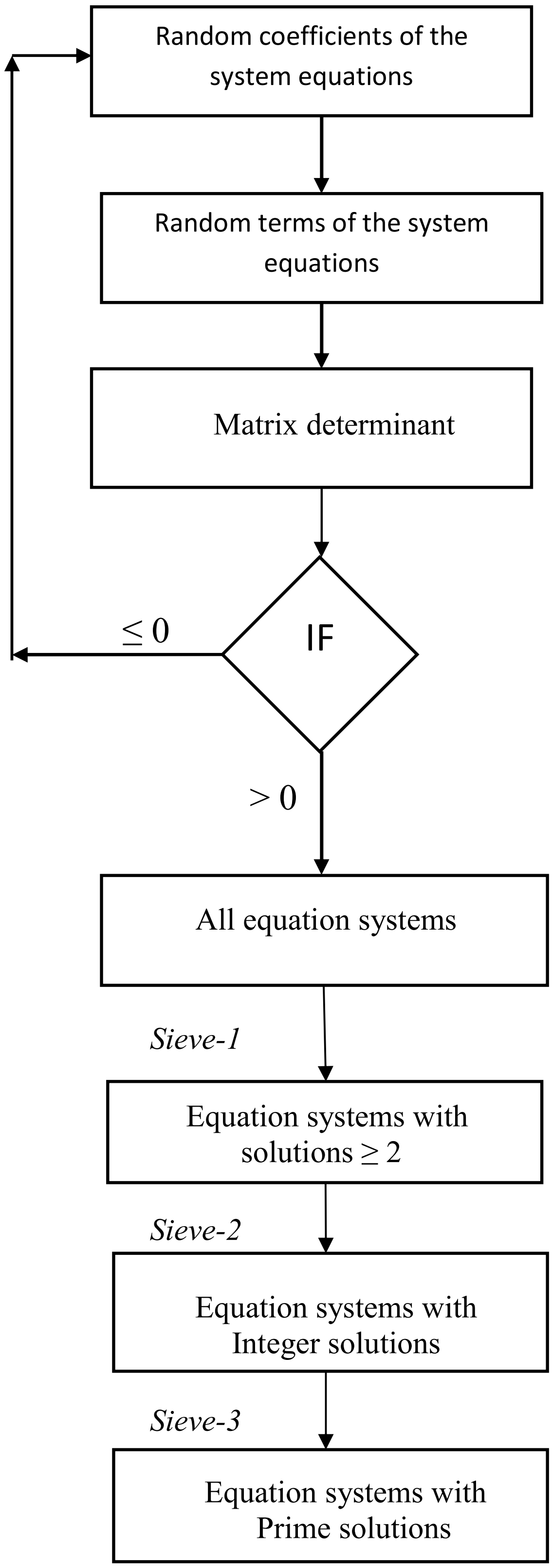
- solve linear systems of *n* linear equations with *n* unkowns and random coefficients and terms.
- Sieve among all solutions, the prime number ones.

5-2 summary of results :

- The frequency of prime solutions rapidely decreases while the number of linear diophantine equations of the system increases.
- The number and frequency of prime solutions decreases while their size increases.
- Some structure and alignements have been observed in the $x_{1\ prime} - x_{2\ prime}$ scatter plots.
- Relationships between the number of prime solutions and the number of prime numbers have been found.
- Finally, two conjectures on the finite or infinite number of systems of *n* linear diophantine equations with *n* unkowns giving prime solutions have been developed from the observations and a detailed analysis of the results obtained.

Annex-1 : algorithm of the Visual Basic for applications *(VBA)* program.

A VBA program based on the below described algorithm has been developed to solve the *n* linear diophantine equation systems with *n* unkowns and sieve, among all solutions, the prime number ones.



Annex-2 : figures

Figure n° 1: system of two linear diophantine equations with two unknowns.

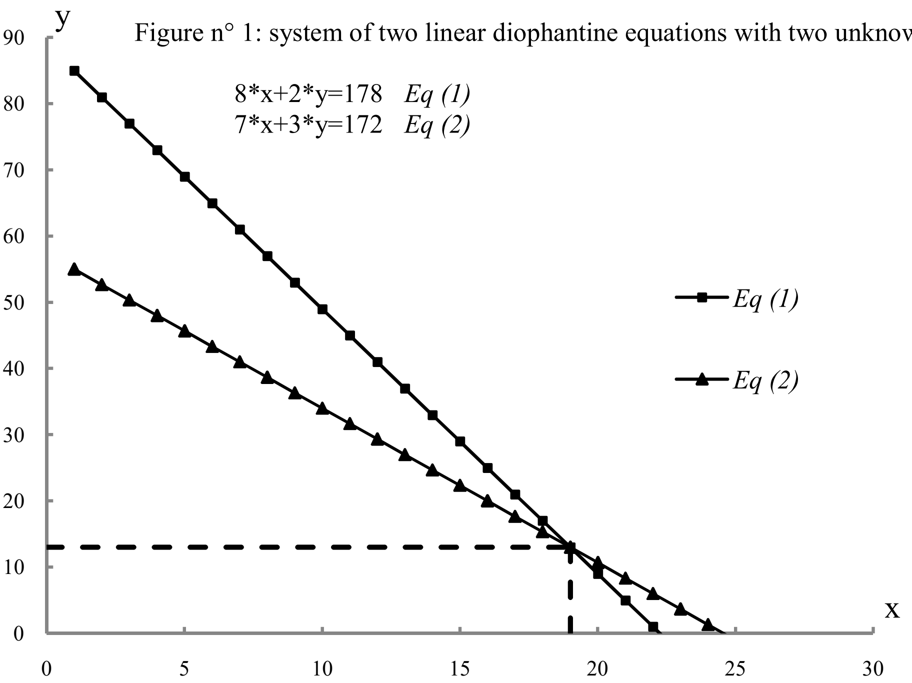


Figure n° 2: histogram of random a, b , c, d coefficients of 2 linear diophantine equation systems with 2 unkowns.
First trial
Sample size: 20000
Range of a, b, c, d coefficients: 1 -10

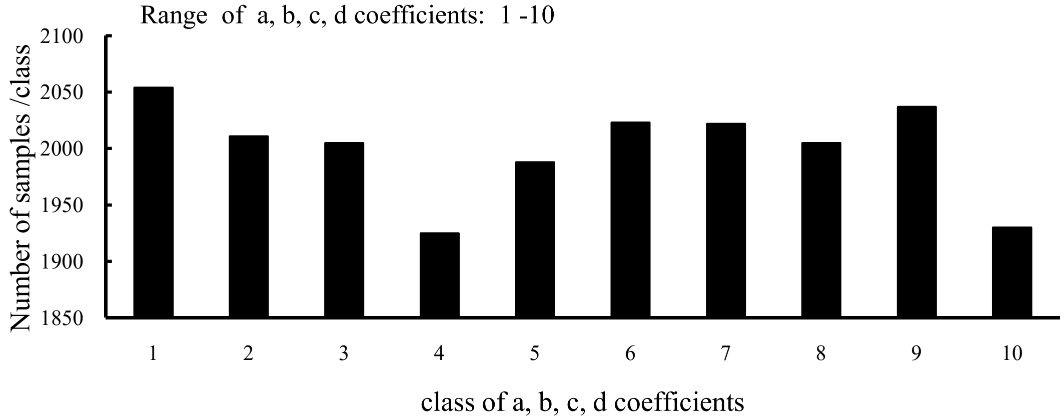


Figure n° 3: histogram of random a, b, c, d coefficients of 2 linear diophantine equation systems with 2 unkowns.
Second trial
Sample size: 20000
Range of varation: 1 - 10

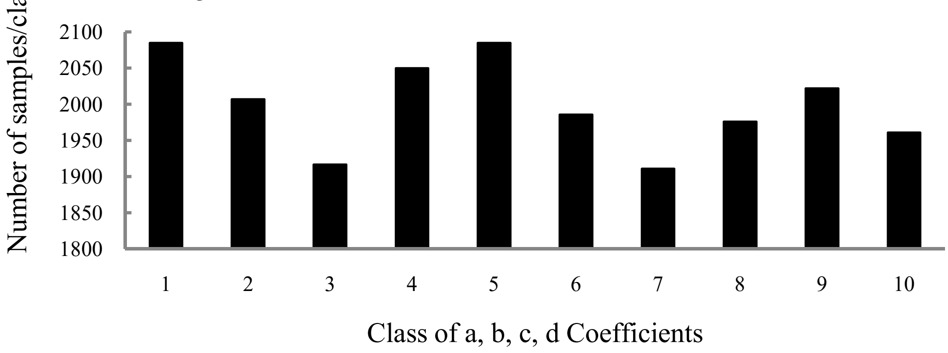


Figure n° 4: histogram of random coefficients a, b, c, d of 2 linear diophantine equation systems with 2 unkowns.
Third trial
Sample size: 20000
Range of variation: 1 - 10

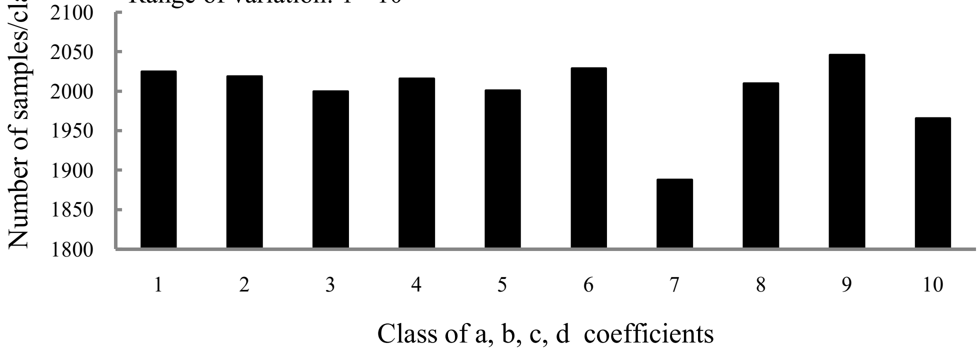


Figure n° 5: histogram of random b_1, b_2 terms of 2 linear diophantine equation systems with 2 unkowns.
Sample size: 20000
Range of b_1-b_2 terms : 100 - 300

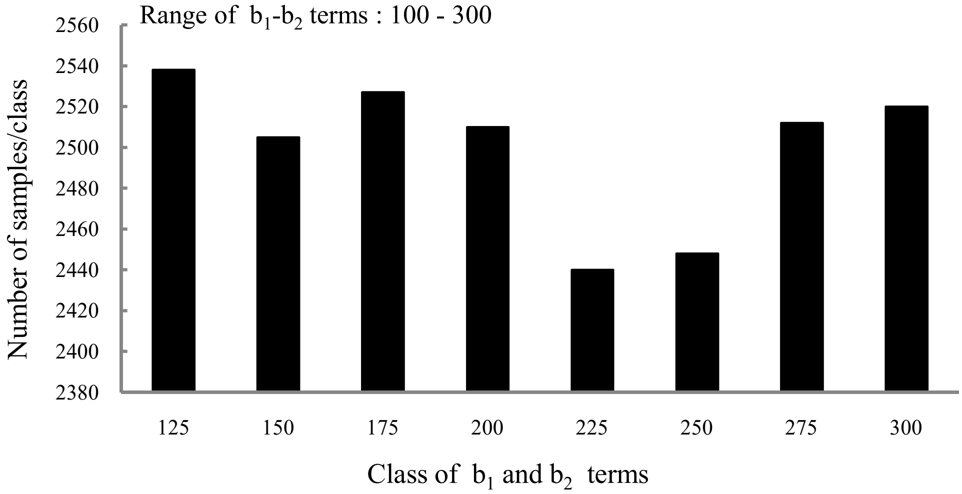


Figure n°6: histogram of random b_1,b_2 terms of 2 linear diophantine equation systems with 2 unkowns.
Sample size: 20000
Range of b_1-b_2 terms: 100000 - 300000

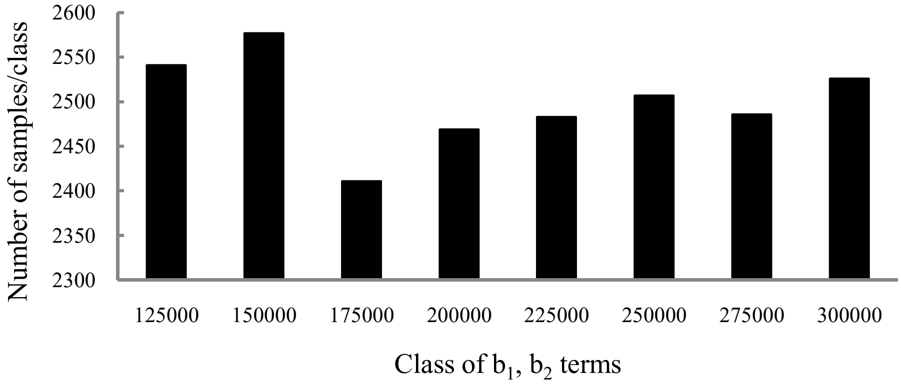


Figure n° 7: histogram of random b_1, b_2 terms of 2 linear diophantine equation systems with 2 unkowns.
Sample size: 20000
Range of b_1-b_2 terms: 1000000 - 3000000

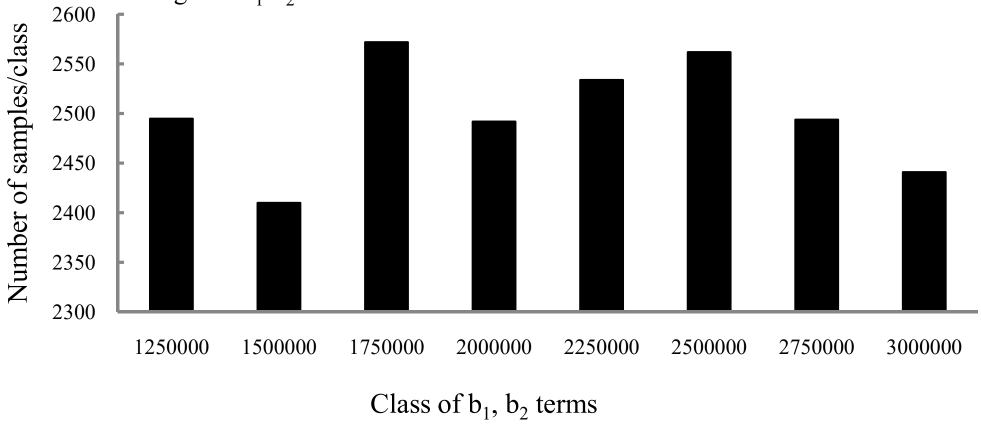


Figure n° 8: histogram of the number of runs/class over a total of 100 runs of $2*10^4$ systems of 2 linear diophantine equations with 2 unknows.
Range of b_1-b_2 terms: 100 - 300
Range of a, b, c, d coefficients: 1 - 10

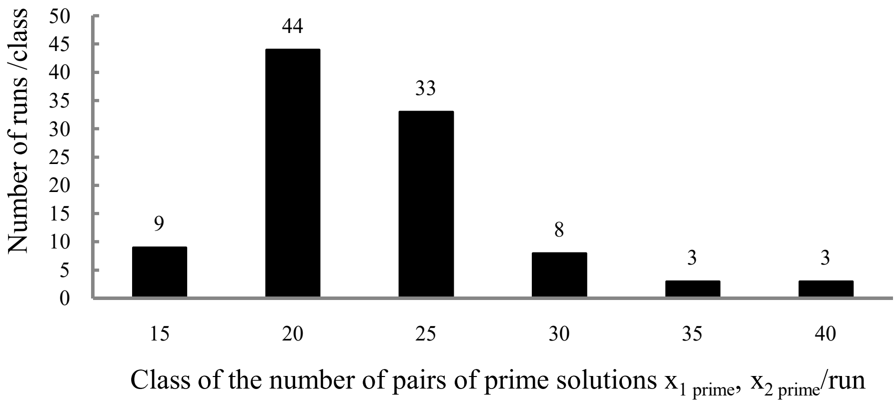


Figure n° 9: histogram of the number of runs/class over a total of 100 runs of $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns
Range of b_1-b_2 terms: 1100 - 1300
Range of a, b, c, d coefficients: 1 - 10

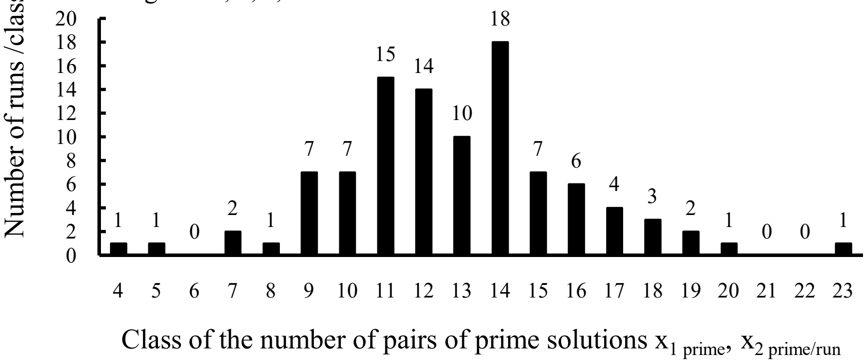


Figure n° 10: histogram of the number of runs/class over a total of 100 runs of $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns.
Range of b_1-b_2 terms: 10100 - 10300
Range of a, b, c, d coefficients: 1 - 10

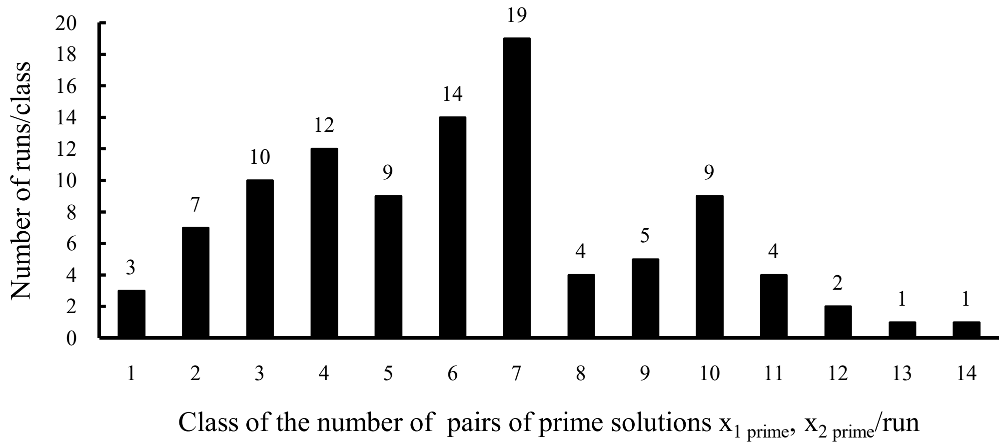


Figure n° 11: histogram of the number of runs/class over a total of 100 runs of $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns.
Range of b_1-b_2 terms: 100100 -100300
Range of a, b, c, d coefficients: 1 - 10

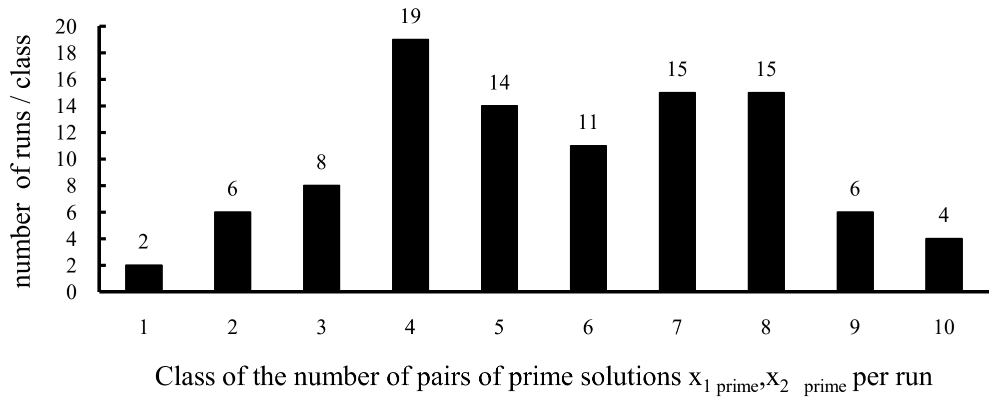


Figure n° 12: histogram of the number of runs/class over a total of per 100 runs, each of them containing of $2*10^4$ of 2 linear diophantine equations systems with 2 unknowns,
Range of b_1-b_2 terms: 1000100 - 1000300
Range of a, b, c, d coefficients: 1

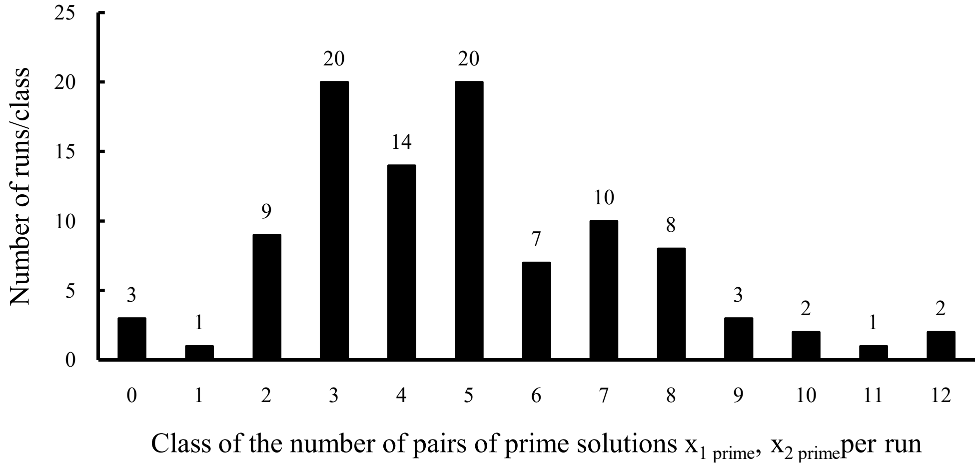


Figure n° 13: histogram of the number of runs over a total of 100 runs each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns.
Range of b_1-b_2 termss: 10000100-10000300
Range of a, b, c, d coefficients: 1 - 10

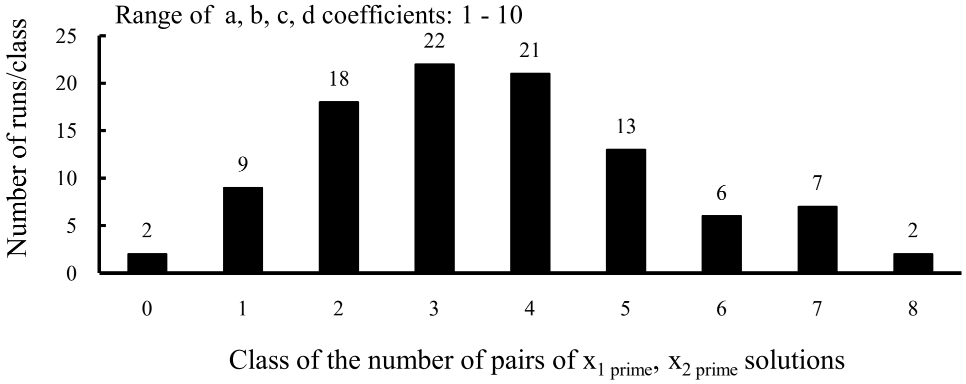
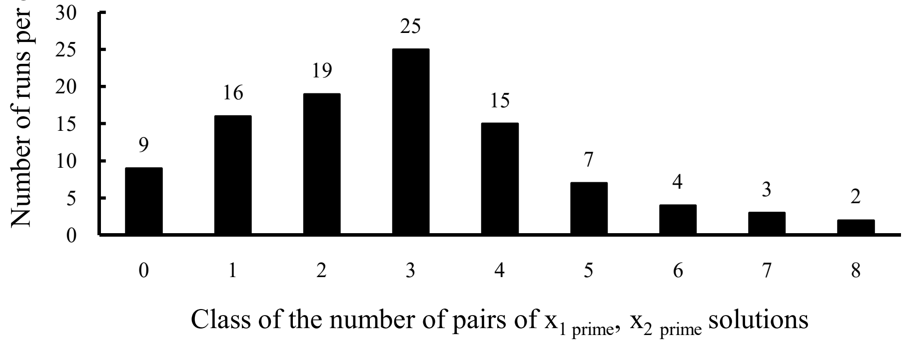


Figure n° 14: histogram of the number of runs over a total of 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns.
Range of b_1- b_2 terms: 100000100-100000300
Range of a, b, c, d coefficients: 1 - 10



Graph n° 15: number of pairs of prime solutions = f (number of runs) for each range of b_1-b_2 terms.
A run contains $2*10^4$ sytems of 2 linear diophantine equation systems with 2 unknowns.

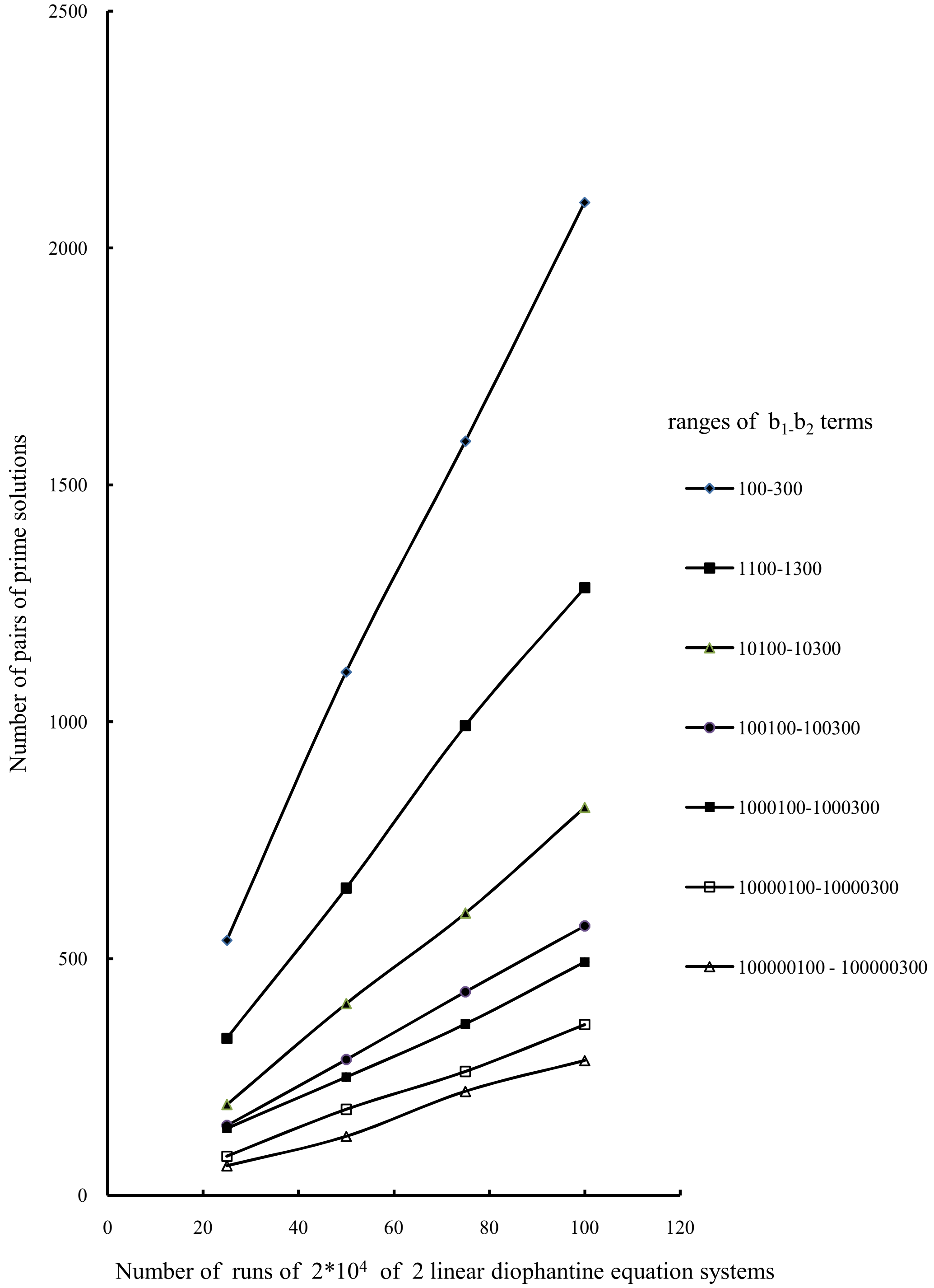


Figure n° 16: total number of pairs of prime solutions = f (b_1-b_2 range)
100 runs of $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns.
Range of a, b, c, d coefficients: 1 - 10

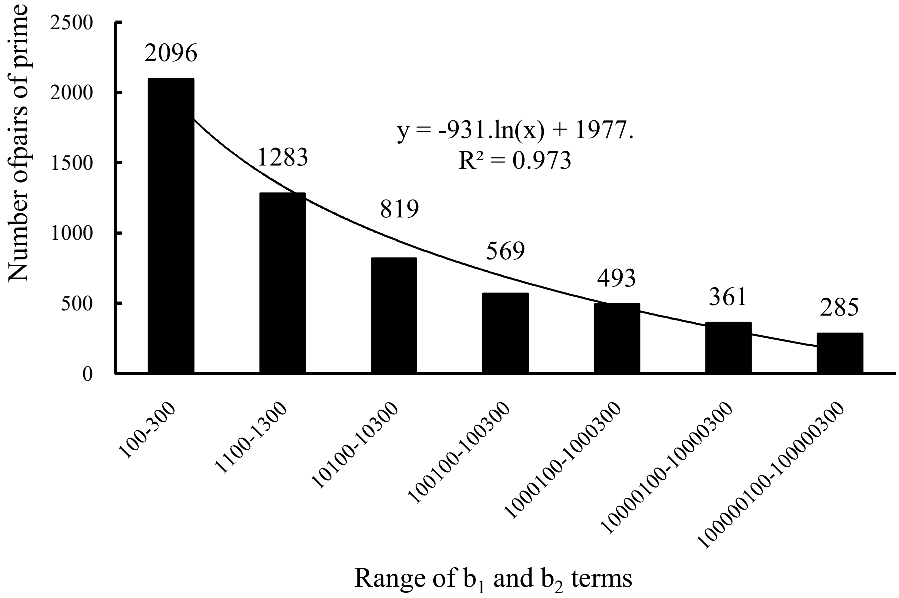
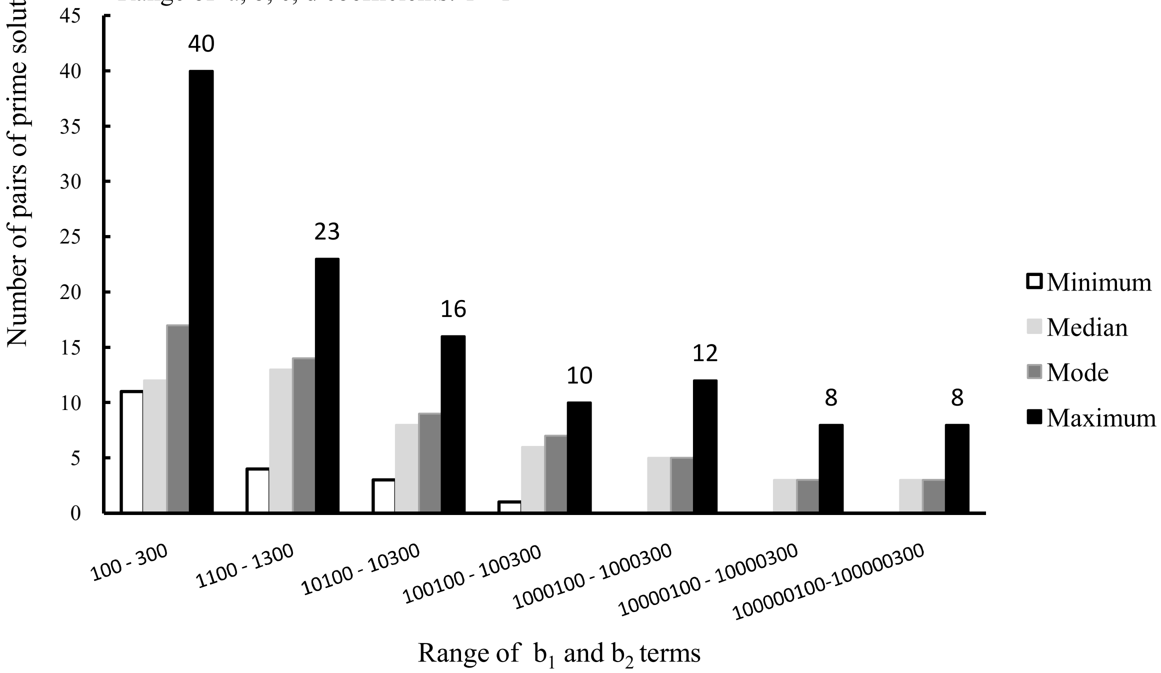


Figure n° 17: statistics on the number of pairs of solutions/run over a total of 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns for different ranges of b_1-b_2 terms.
Range of a, b, c, d coefficients: 1 - 1



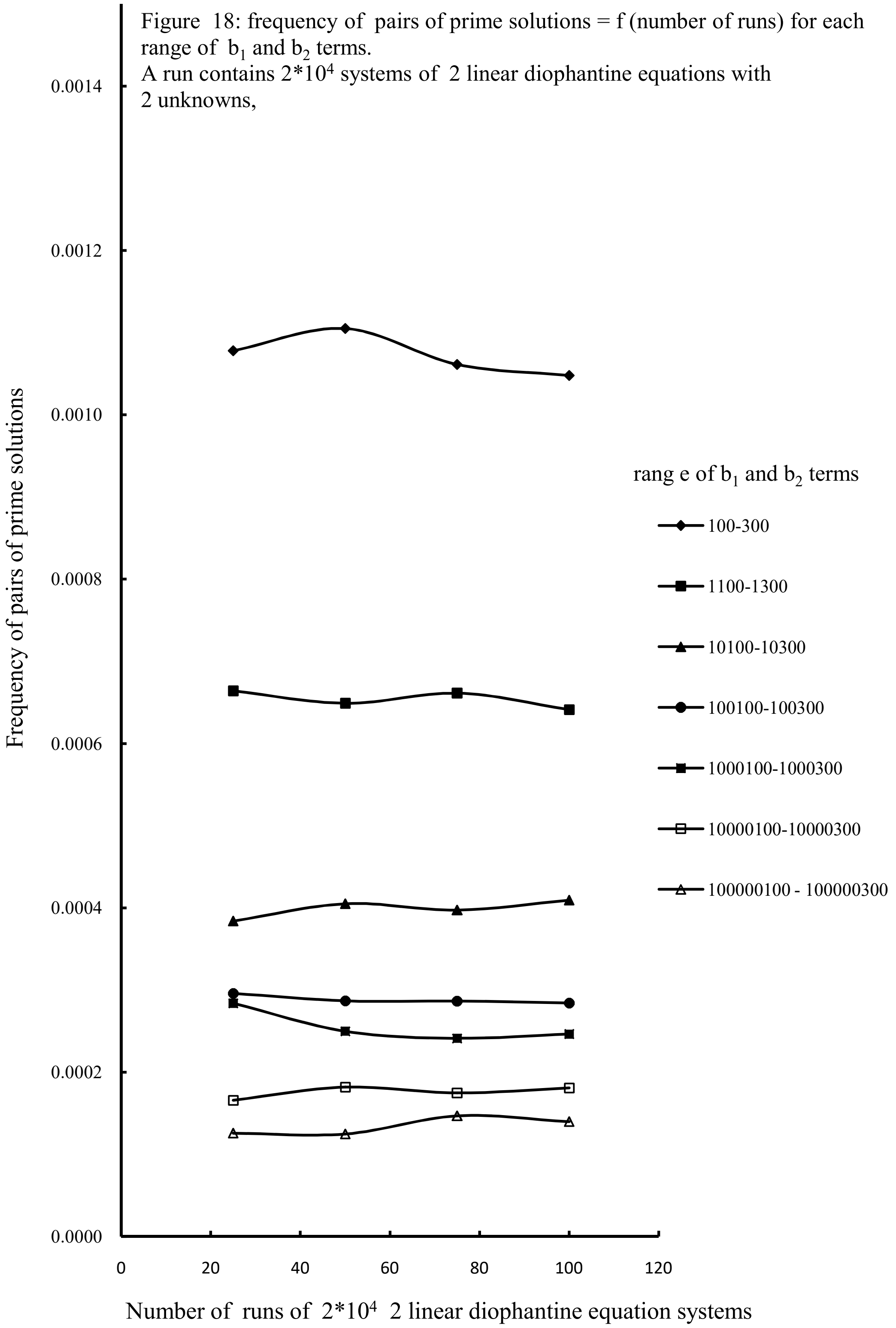


Figure 18: frequency of pairs of prime solutions = f (number of runs) for each range of b_1 and b_2 terms.
A run contains $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns,
Frequency of pairs of prime solutions
0.0014
0.0012
0.0010
0.0008
0.0006
0.0004
0.0002
0.0000
0
20
40
60
80
100
120
rang e of b_1 and b_2 terms
100-300
1100-1300
10100-10300
100100-100300
1000100-1000300
10000100-10000300
100000100 - 100000300
Number of runs of $2*10^4$ 2 linear diophantine equation systems

Figure n°19: histogram of prime solutions $x_{1\ prime}$ over a total of 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unknows.
Range of b_1 and b_2 terms: 100 - 300
Range of a, b, c, d coefficients: 1 - 10

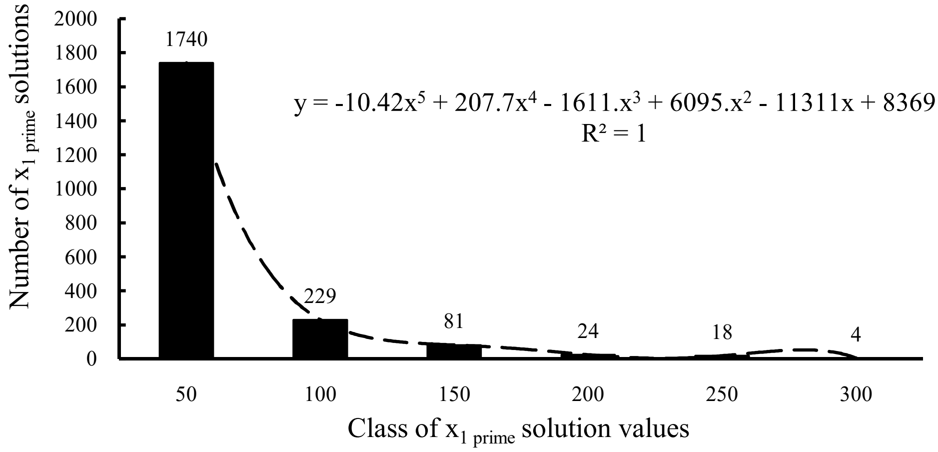


Figure n° 20: histogram of prime solutions $x_{1\ prime}$ over a total of 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns,.
Range of b_1 and b_2 terms: 1100 - 1300

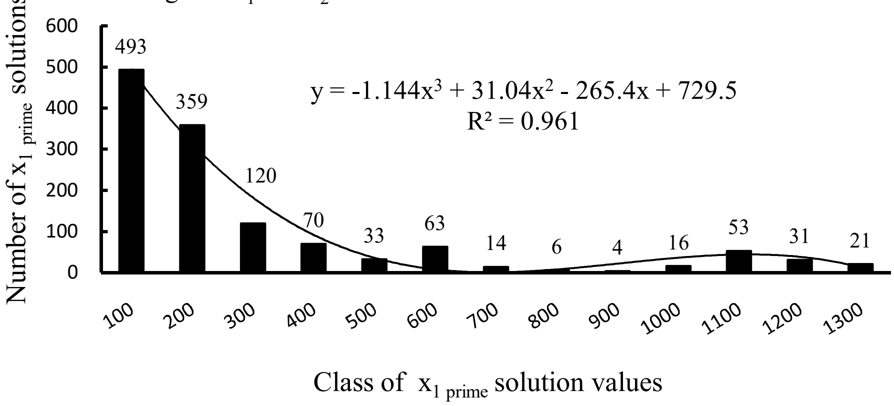


Figure n° 21: histogram of prime solutions $x_{1\ prime}$ over a total of 100 runs of $2*10^4$ systems of 2 linear diophantine equations with 2 unknows.
Range of b_1 and b_2 terms: 10100 -10300
Range of a, b, c, d coefficients: 1 - 10

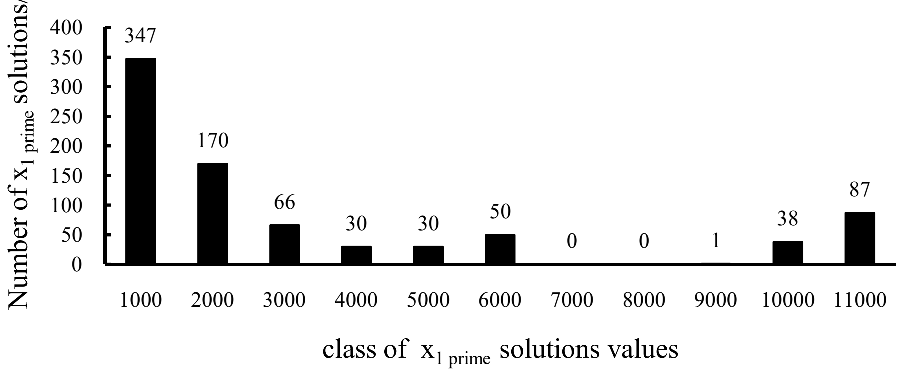


Figure n° 22: histogram of prime solutions $x_{1\ prime}$ over a total of 100 runs of $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns
Range of b_1 and b_2 terms: 100100 - 100300
Range of a, b, c, d coefficients: 1 - 10

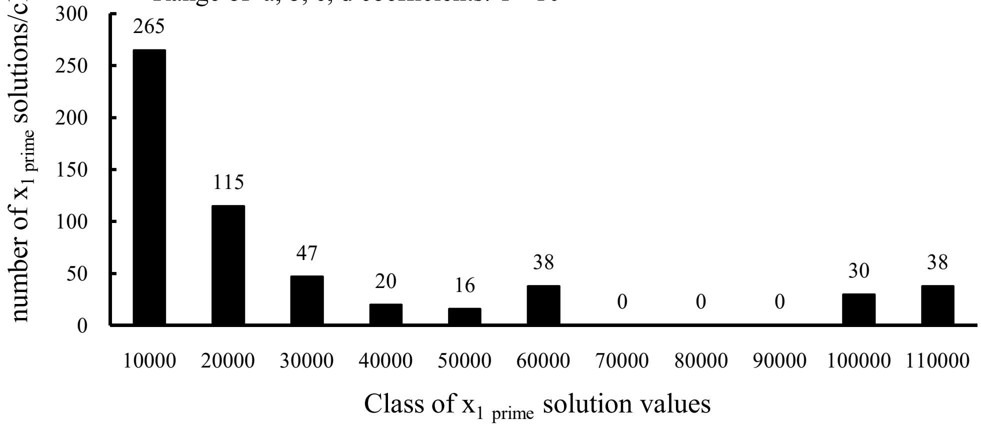


Figure n° 23: histogram of prime solutions $x_{1\ prime}$ over a total of 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns
Range of b_1 and b_2 terms: coefficients: 1000100 -1000300
Range of a, b, c, d coefficie

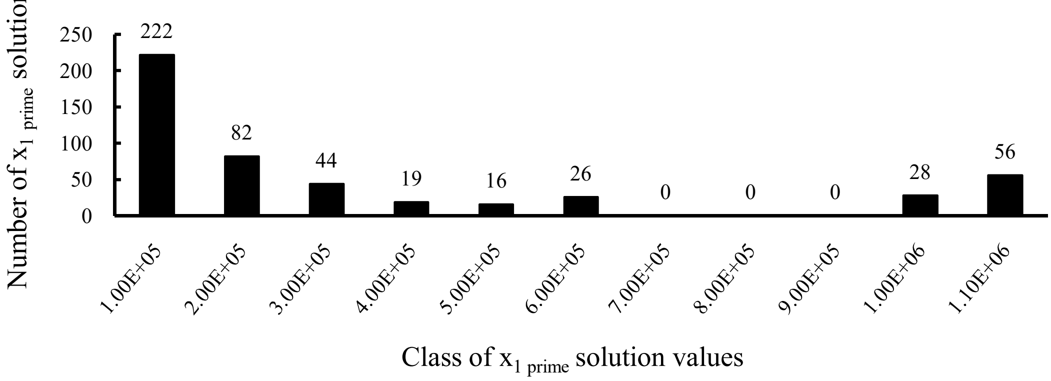


Figure n° 24: histogram of $x_{1\ prime}$ solutions over 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns .
Range of b_1 and b_2 terms: 10000100 - 10000300
Range of a, b, c, d coefficients: 1 - 10

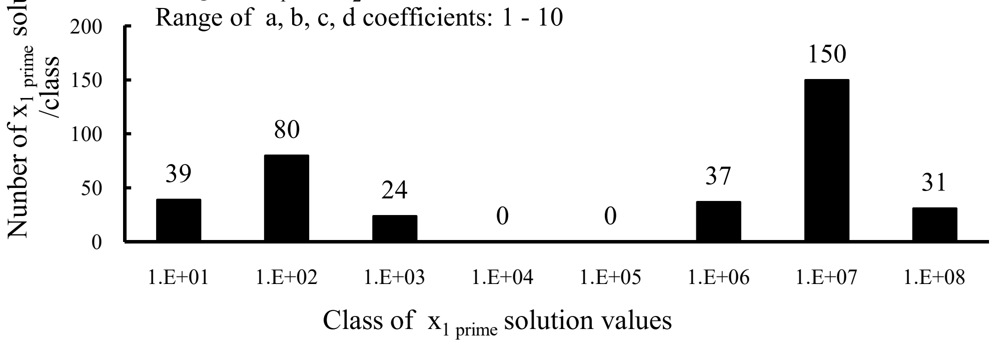


Figure n° 25: histogram of $x_{1\ prime}$ solutions over a total of 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns.
Range of b_1,b_2 terms: 100000100 - 100000300
Range of a, b, c, d coefficients: 1 - 10

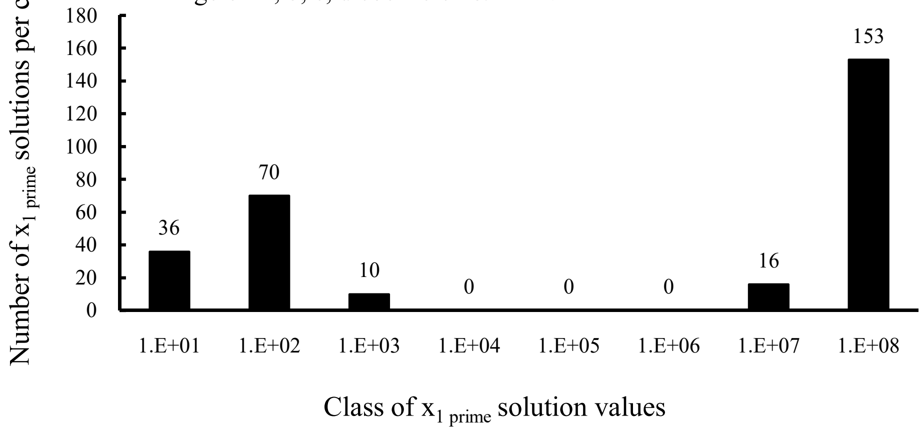


Figure n° 26: Histogram of prime solutions $x_{2\ prime}$ over a total of 100 runs, each of theh containing $2*10^4$ systems of 2 linear diophantine equations with 2 unknown.
Range o f b_1 and b_2 terms: 100 - 300
Range of a, b, c, d coefficients: 1 - 10

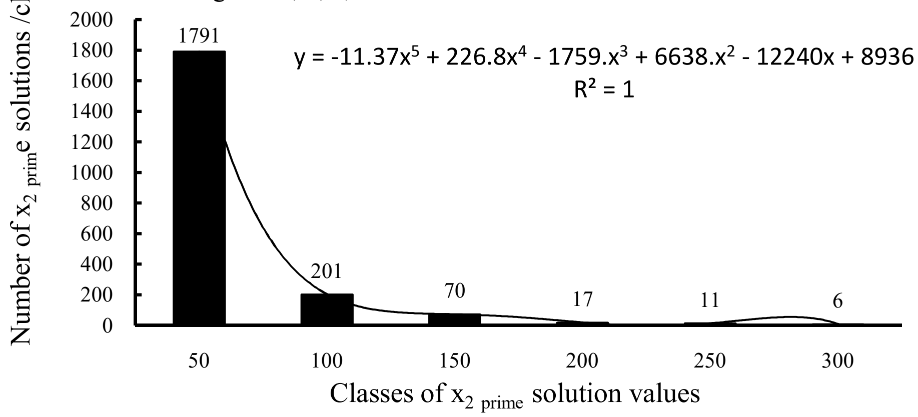


Figure n° 27: histogram of prime solutions $x_{2\ prime}$ over a otal of 100 runs of $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns.
Range of b_1 and b_2 terms: 1100 - 1300
Range of a, b, c, d coefficients: 1 - 10

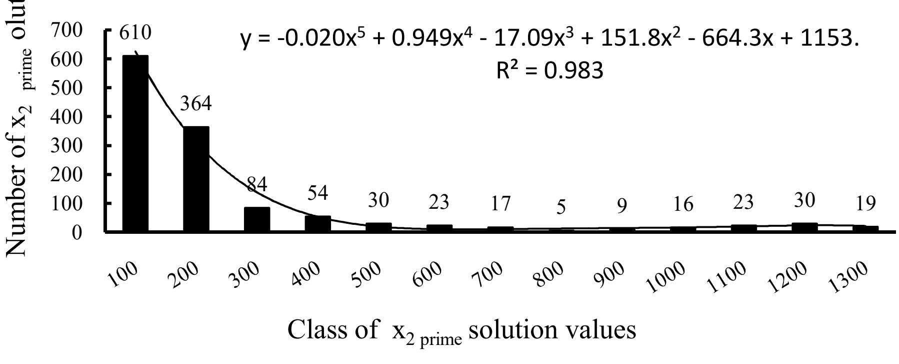


Figure n° 28: histogram of prime solutions $x_{2\ prime}$ over a total of 100 runs of $2*10^4$ systems of 2 linear diophantine equations with 2 unknons.
Range of b_1 and b_2 terms: 10100 -10300
Range of a, b, c, d coefficients: 1 - 10

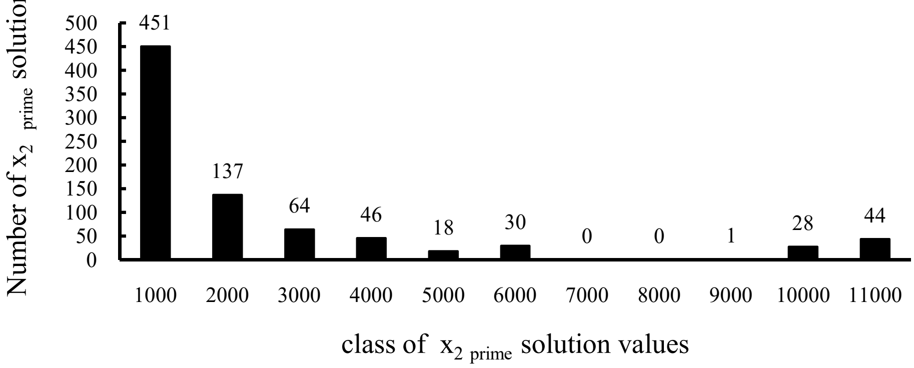


Figure n° 29: histogram of prime solutions $x_{2\ prime}$ over a total of 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns.
Range of b_1 and b_2 terms: 100100 - 100300
Range of a, b, c, d coefficients: 1 - 10

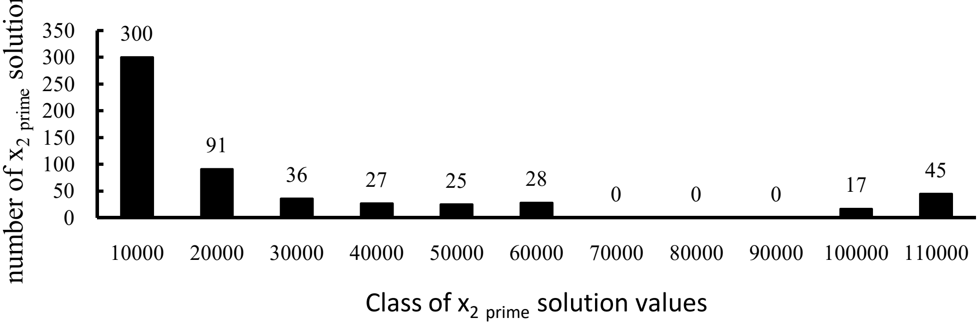
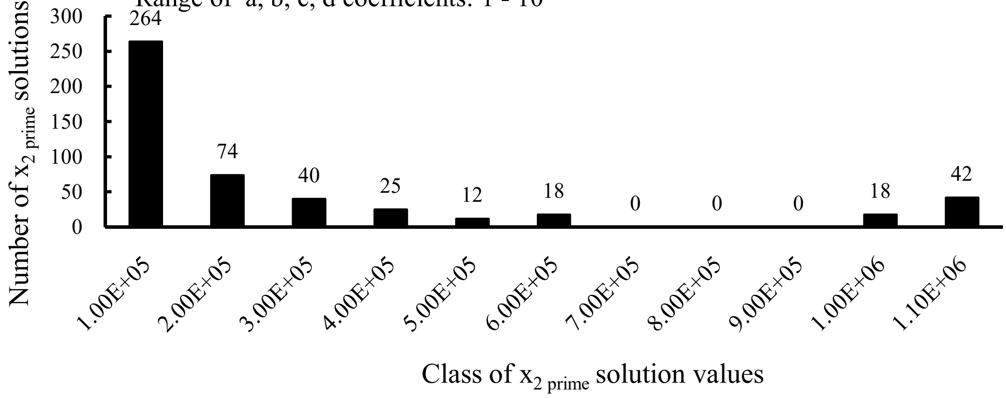


Figure n° 30: histogram of prime solutions $x_{2\ prime}$, over a total of 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns.
Range of b_1 and b_2 terms: 1000100 -1000300
Range of a, b, c, d coefficients: 1 - 10



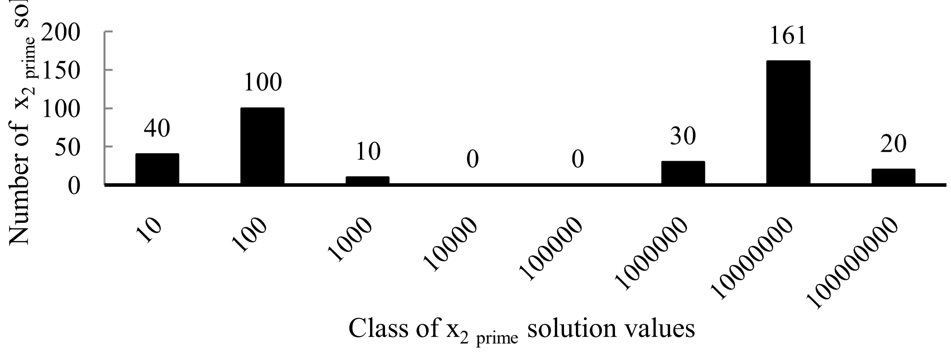


Figure n° 31: histogram of $x_{2\ prime}$ solutions over a total of 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns.
Range of b_1 and b_2 terms: 10000100-10000300
Range of a, b, c, d coefficients: 1 - 10

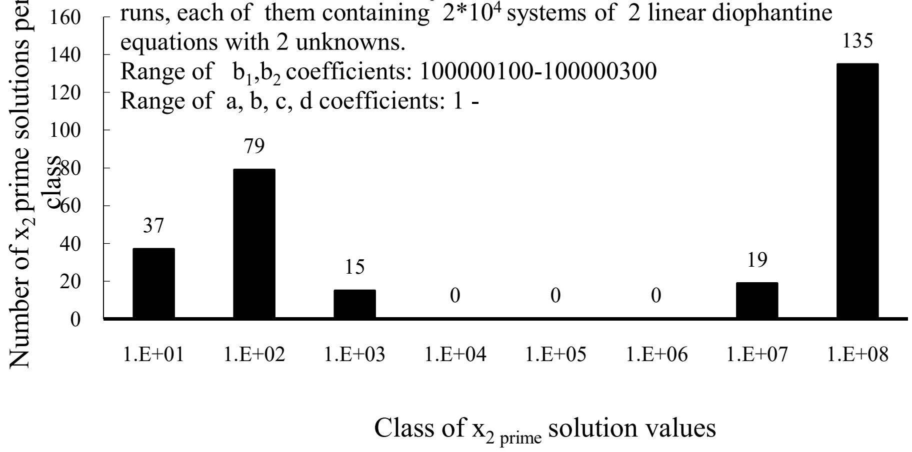
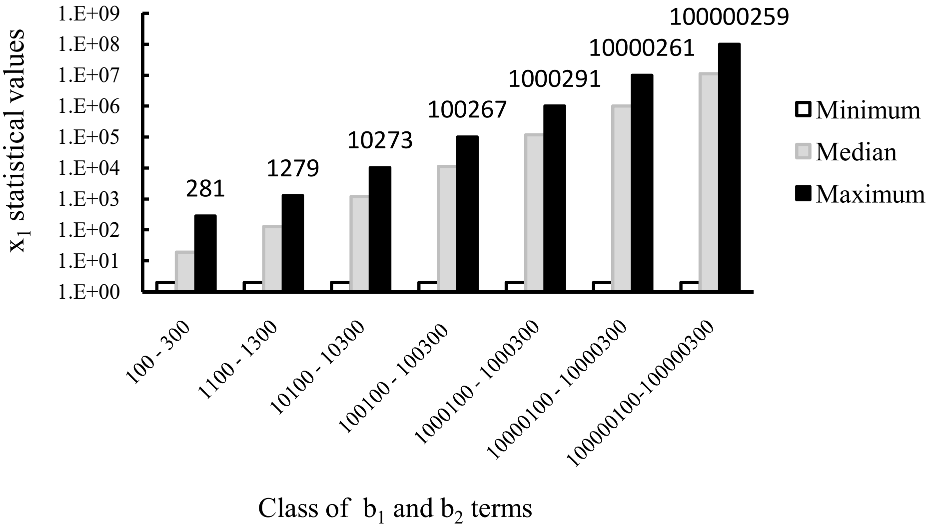


Figure n° 32: histogram of $x_{2\ prime}$ solutions over a total of 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns.
Range of b_1,b_2 coefficients: 100000100-100000300
Range of a, b, c, d coefficients: 1 -



Graph n°: 33: statistics on x_1 prime solutions over a total of 100 runs of $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns for the different classes of b_1 and b_2 terms.

Figure n° 34: statistics on x_2 prime solutions over 100 runs of $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns for different classes of b_1 and b_2 terms.

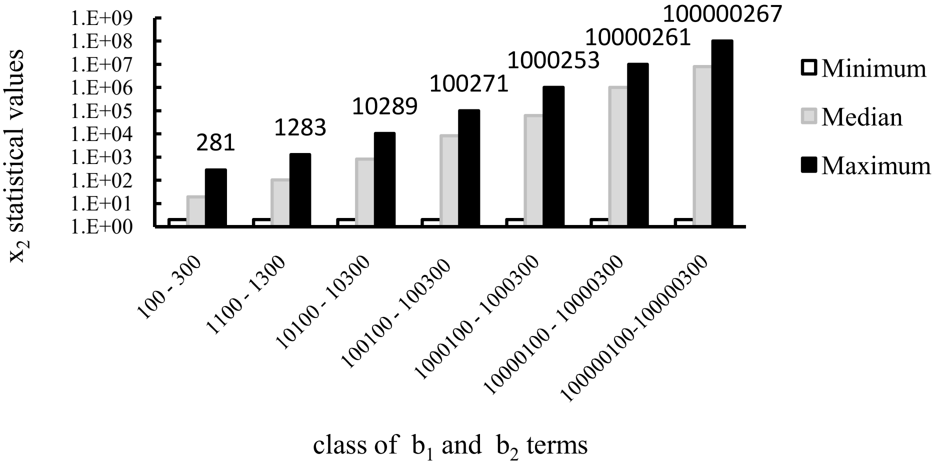


Figure n° 35: $x_{1\ prime}$ solutions from 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns. Range of a, b, c, d coefficients: 1 - 10

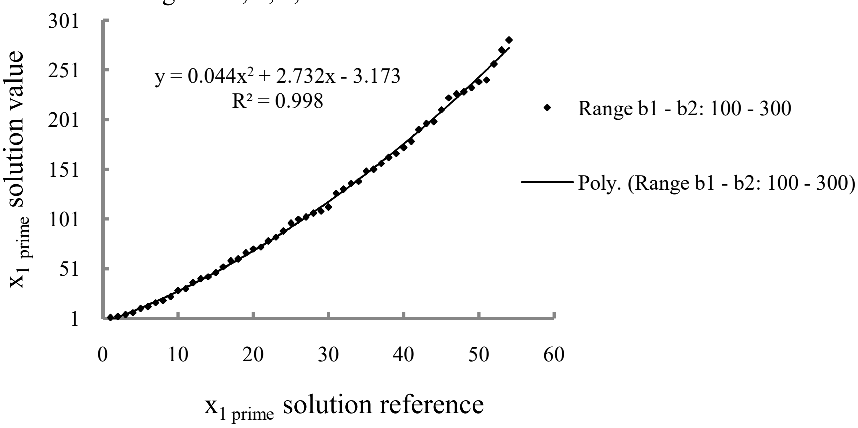


Figure n° 36: $x_{1\ prime}$ solutions over 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns. Range of a, b, c, d coefficients: 1 - 10

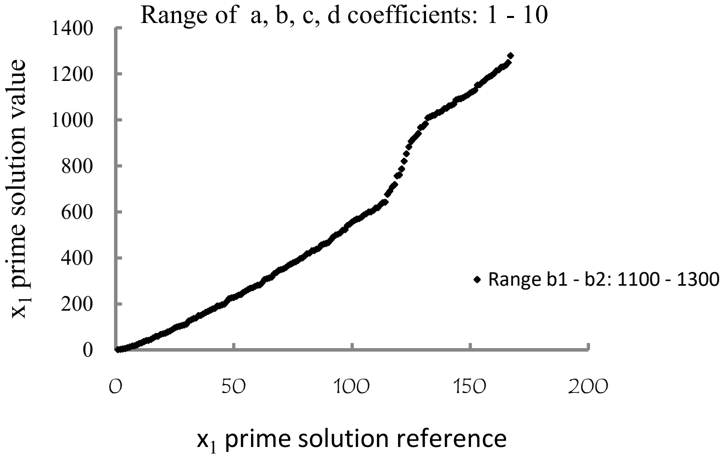


Figure n° 37: $x_{1\ prime}$ solutions over 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns.
Range of a, b, c, d coefficients: 1 - 10

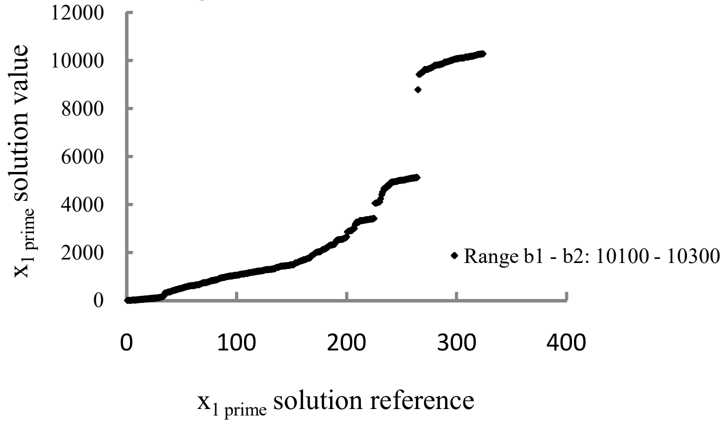


Figure n° 38: $x_{1\ prime}$ solutions over 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns.
Range of a, b, c, d coefficients: 1 - 10

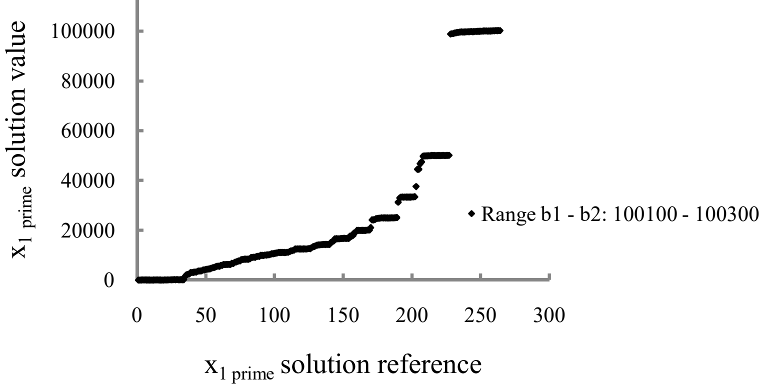


Figure n° 39: $x_{1\ prime}$ solutions over 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns.
Range of a, b, c, d coefficients: 1 - 10

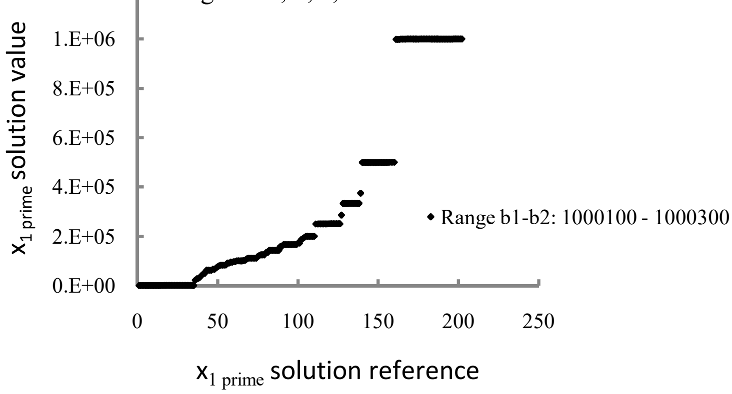


Figure n° 40: $x_{1\ prime}$ solutions over 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns. Range of a, b, c, d coefficients: 1 - 10)

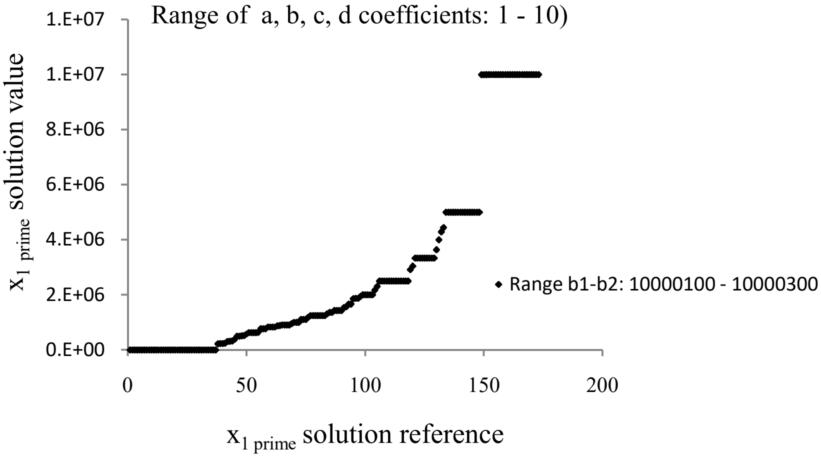


Figure n° 41: $x_{1\ prime}$ solutions over 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns. Range of a, b, c, d coefficients: 1 - 10)

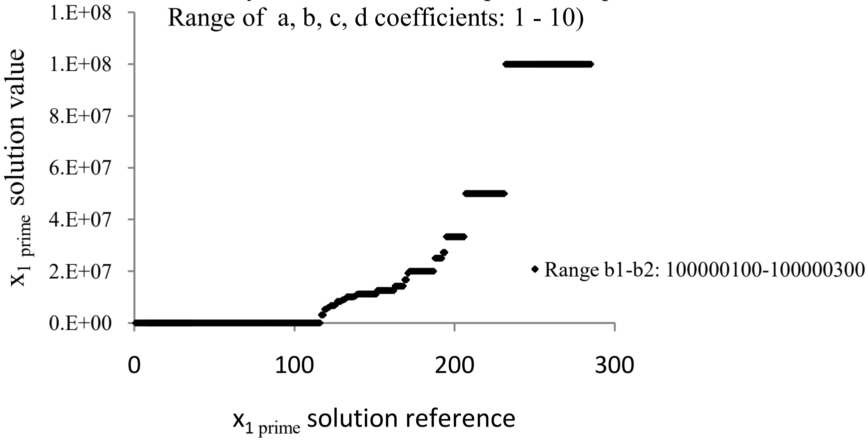


Figure n° 42: $x_{2\ prime}$ solutions from 100 runs , each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns,
Ra nge of a, b, c, d coefficienst: 1 - 10

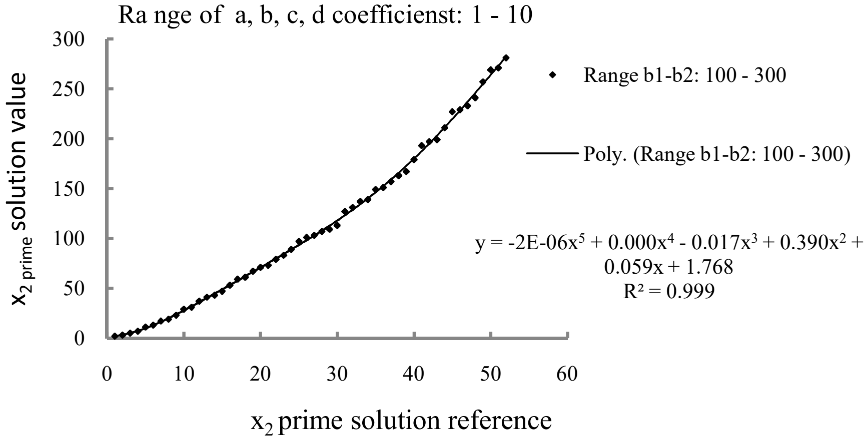


Figure n° 43: $x_{2\ prime}$ solutions over 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns, Range of a, b, c, d coefficients: 1 - 10)

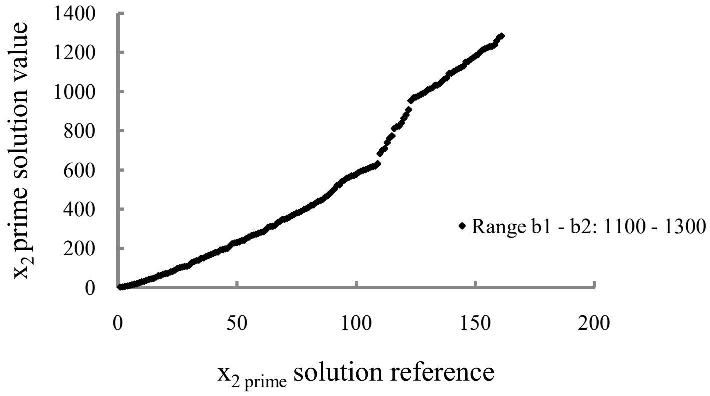


Figure n° 44: $x_{2\ prime}$ solutions over 100 runs, each of them containing $2*10^4$ systems of 2 linear equations with 2 unkowns, Range of a, b, c, d coefficients: 1 - 10

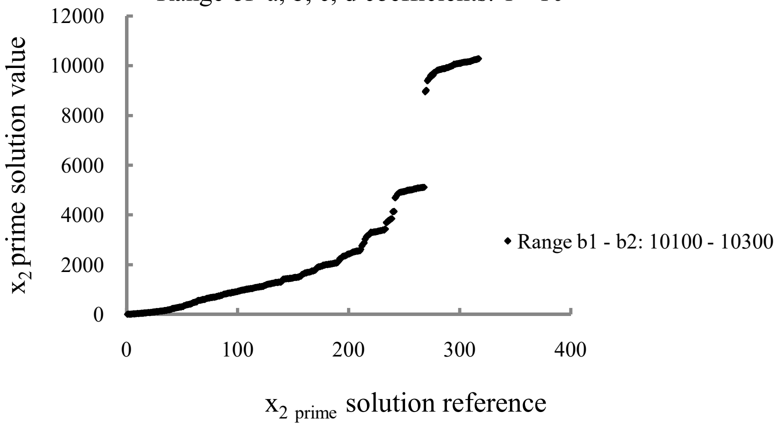


Figure n° 45: $x_{2\ prime}$ solutions over 100 runs, each of them containg $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns, Range of a, b, c, d coefficients: 1 -10

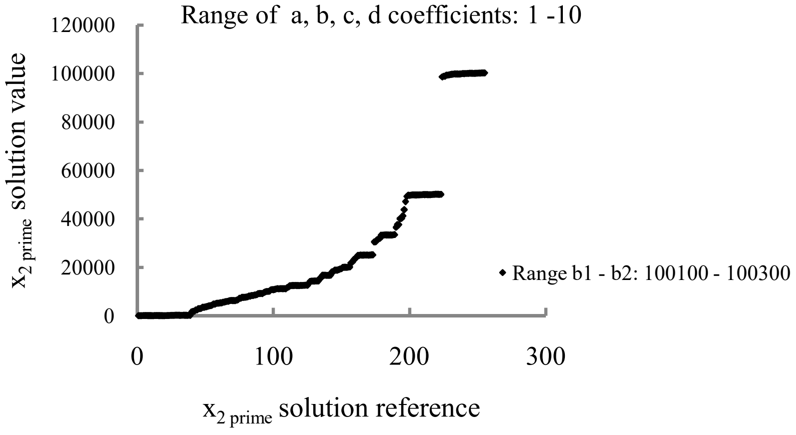


Figure n° 46: $x_{2\ prime}$ solutions over 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns, Range of a, b, c, d coefficients: 1 - 10

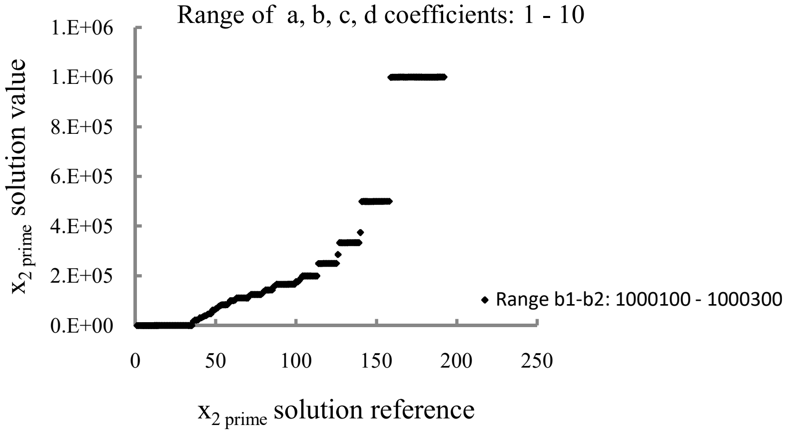


Figure n°47: $x_{2\ prime}$ solutions over 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns. Range of a, b, c, d coefficients: 1 -10

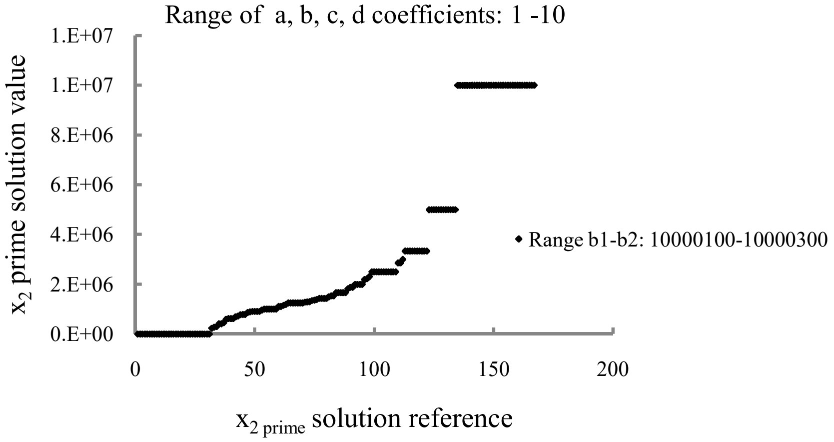
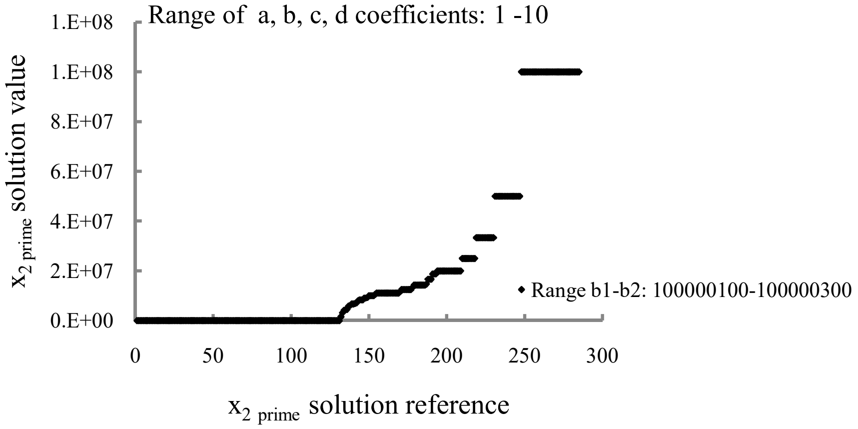


Figure n° 48: $x_{2\ prime}$ solutions over 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns. Range of a, b, c, d coefficients: 1 -10



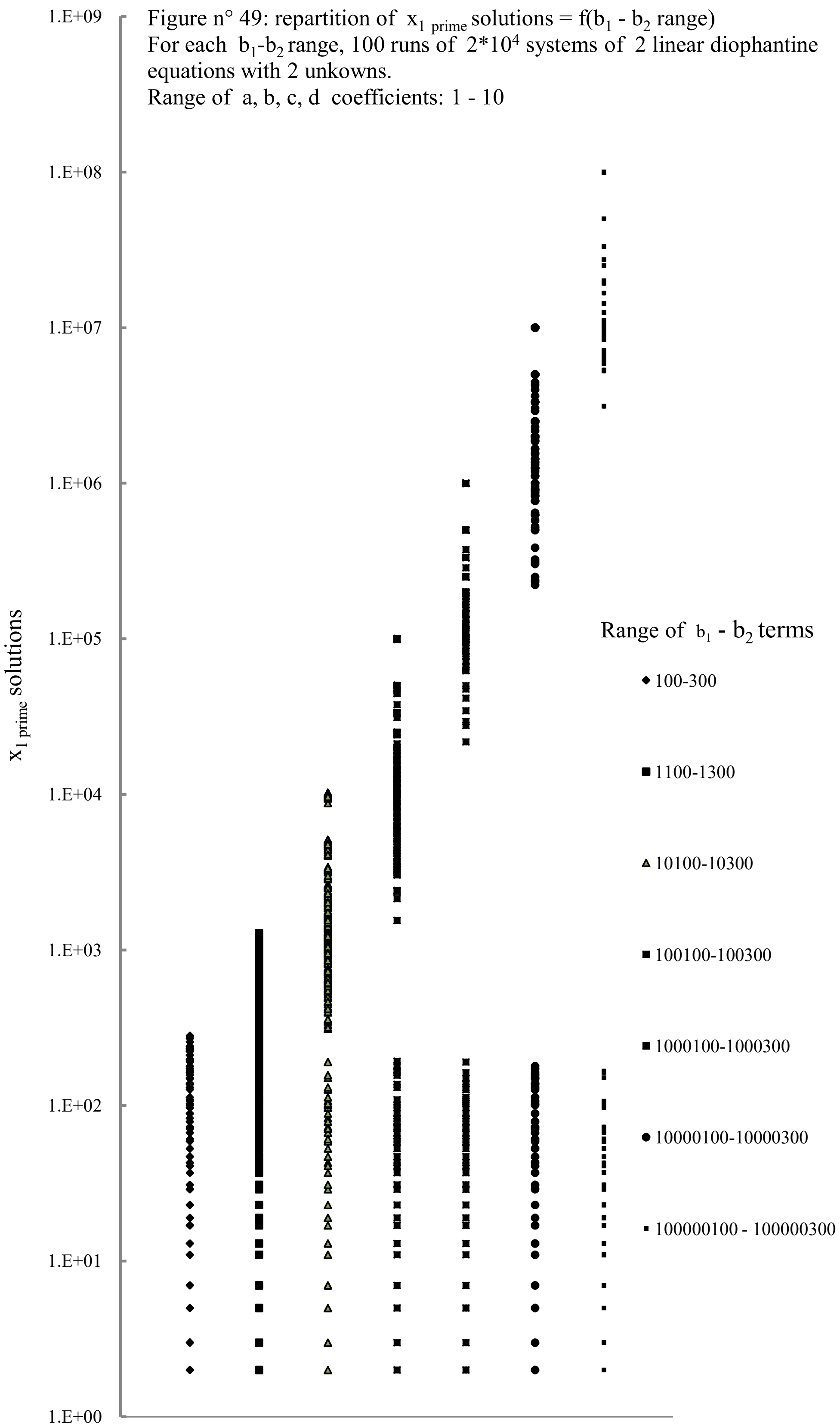


Figure n° 49: repartition of $x_{1\ prime}$ solutions = f(b_1 - b_2 range)
For each b_1-b_2 range, 100 runs of $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns.
Range of a, b, c, d coefficients: 1 - 10
1.E+09
1.E+08
1.E+07
1.E+06
1.E+05
1.E+04
1.E+03
1.E+02
1.E+01
1.E+00
$x_{1\ prime}$ solutions
Range of b_1 - b_2 terms
100-300
1100-1300
10100-10300
100100-100300
1000100-1000300
10000100-10000300
100000100 - 100000300

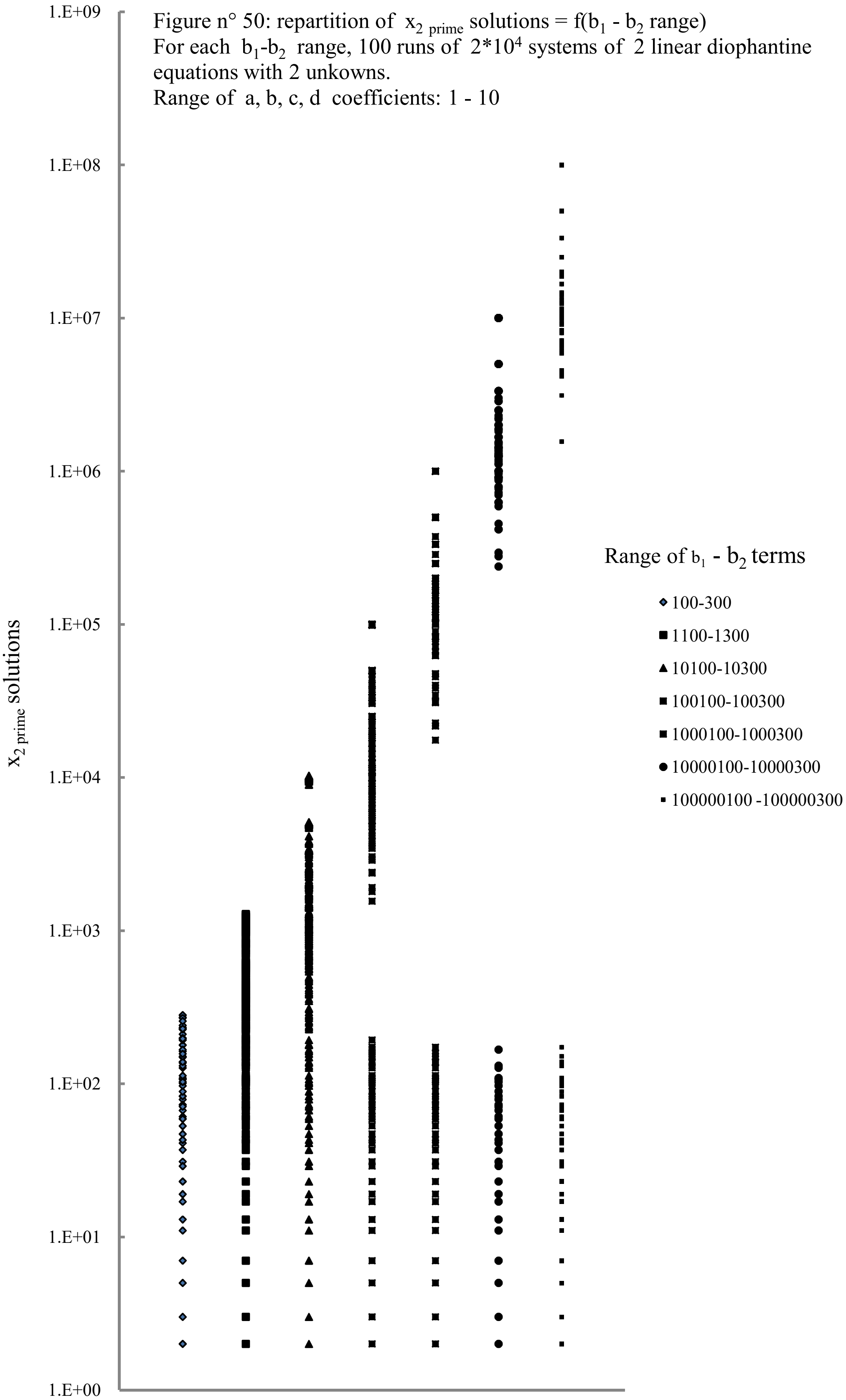


Figure n° 50: repartition of $x_{2\ prime}$ solutions = f(b_1 - b_2 range)
For each b_1-b_2 range, 100 runs of $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns.
Range of a, b, c, d coefficients: 1 - 10
1.E+09
1.E+08
1.E+07
1.E+06
1.E+05
1.E+04
1.E+03
1.E+02
1.E+01
1.E+00
$x_{2\ prime}$ solutions
Range of b_1 - b_2 terms
100-300
1100-1300
10100-10300
100100-100300
1000100-1000300
10000100-10000300
100000100 -100000300

Figure n° 51: gaps in prime solutions
For each b_1-b_2 range, 100 runs of $2*10^4$ systems of 2 linear equations with 2 unkowns.
Range of a, b ,c, d coefficients: 1 - 10

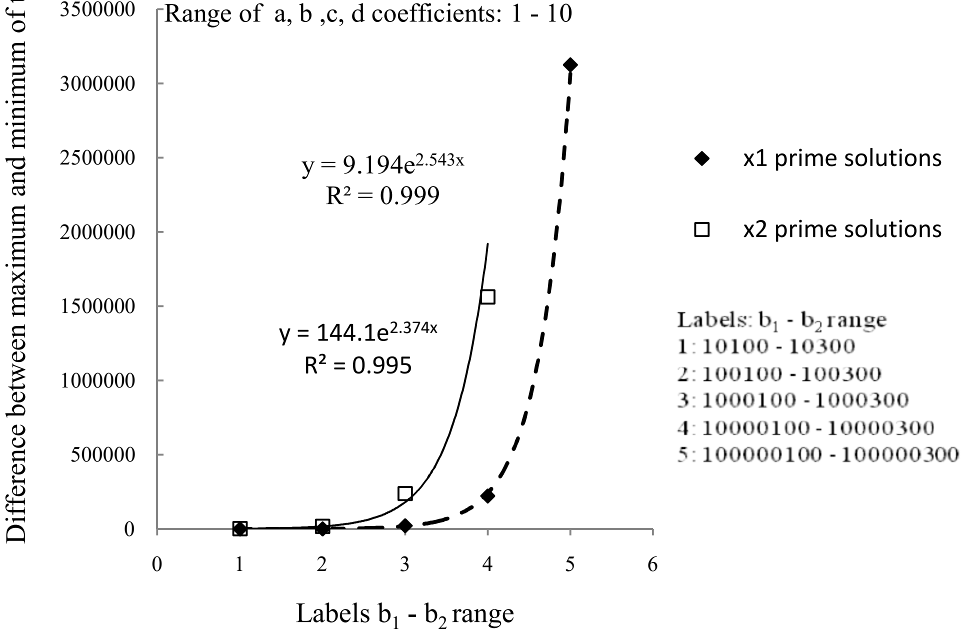


Figure n° 52: gaps in prime solutions
For each b_1-b_2 range, 100 runs of $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns.
Range of a, b, c, d coefficients: 1 - 10

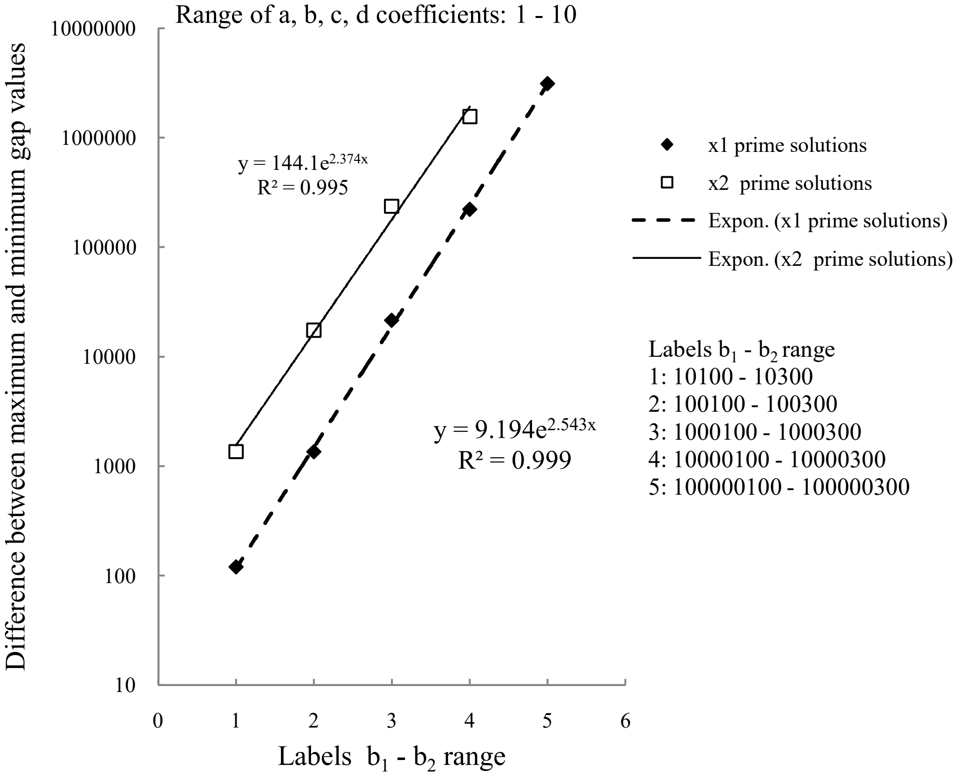


Figure n° 53: x_2 prime = f($x_{1\ prime}$)
100 runs of $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns.
Range of b_1 and b_2 terms: 100 - 300
Range of a, b, c, d coefficients: 1 - 10

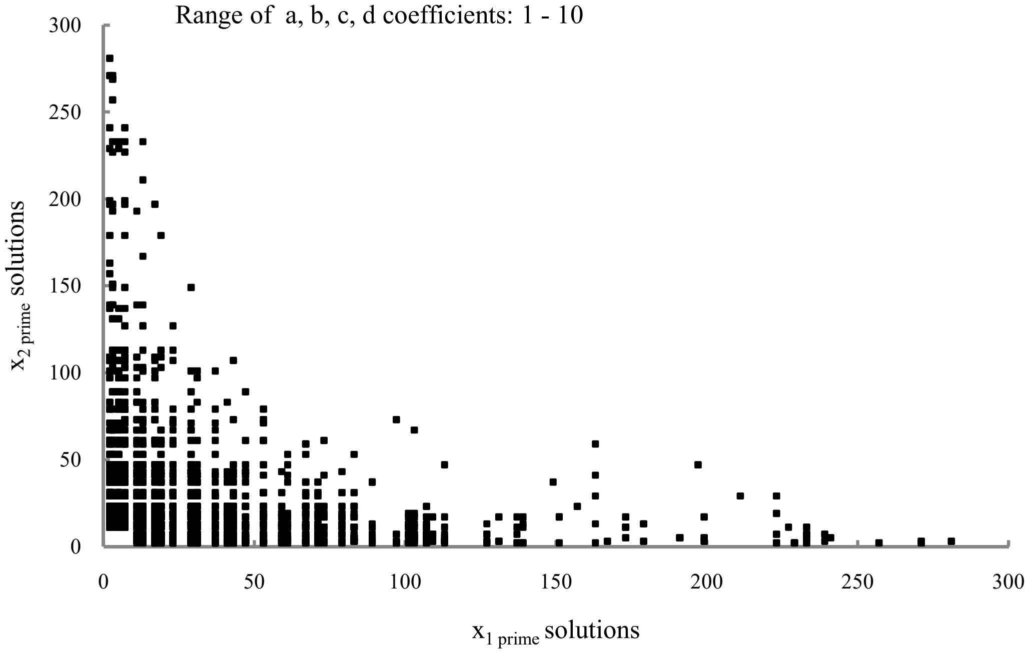
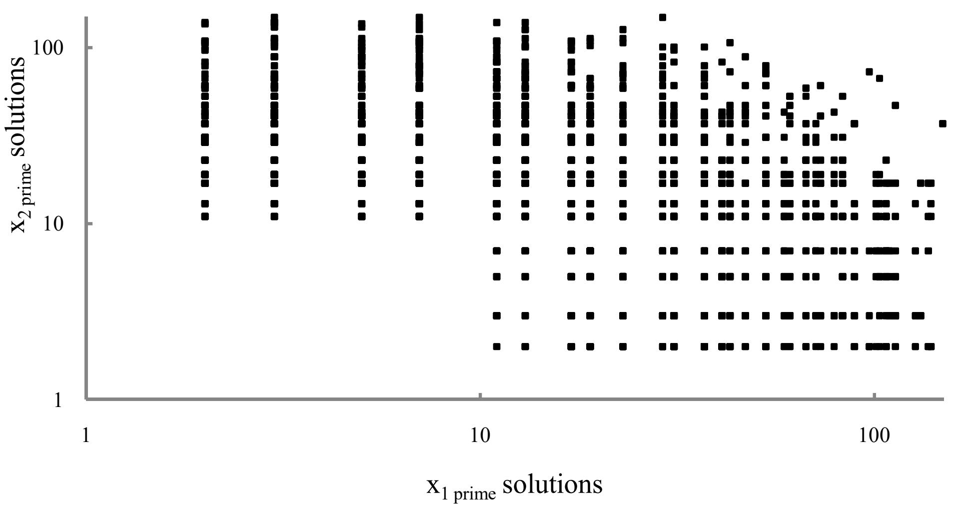


Figure n° 54: $x_{2\ prime}$ f($x_{1\ prime}$)
100 runs of $2*10^4$ systems of 2 linear diophantine equations with
2 unkowns,
Range of b_1 and b_2 terms: 100 - 300
Range of a, b, c, d coefficients: 1 - 10
zoom-1



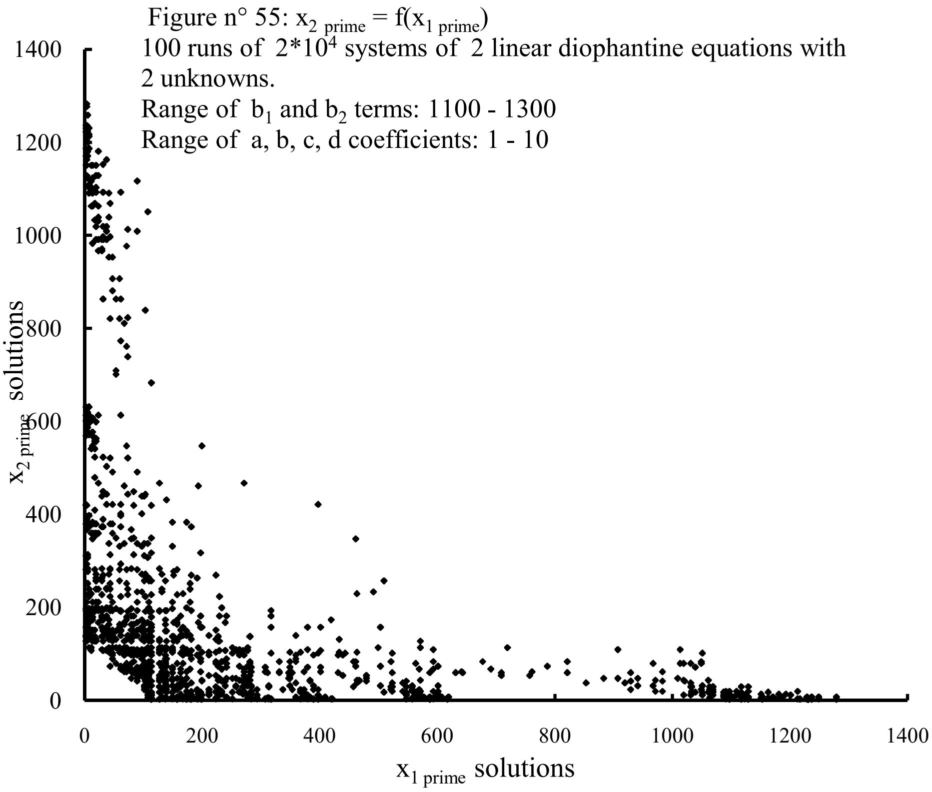


Figure n° 55: x2 prime = f(x1 prime)
100 runs of 2*10^4 systems of 2 linear diophantine equations with 2 unknowns.
Range of b1 and b2 terms: 1100 - 1300
Range of a, b, c, d coefficients: 1 - 10
1400
1200
1000
800
600
400
200
0
x2 prime solutions
0
200
400
600
800
1000
1200
1400
x1 prime solutions

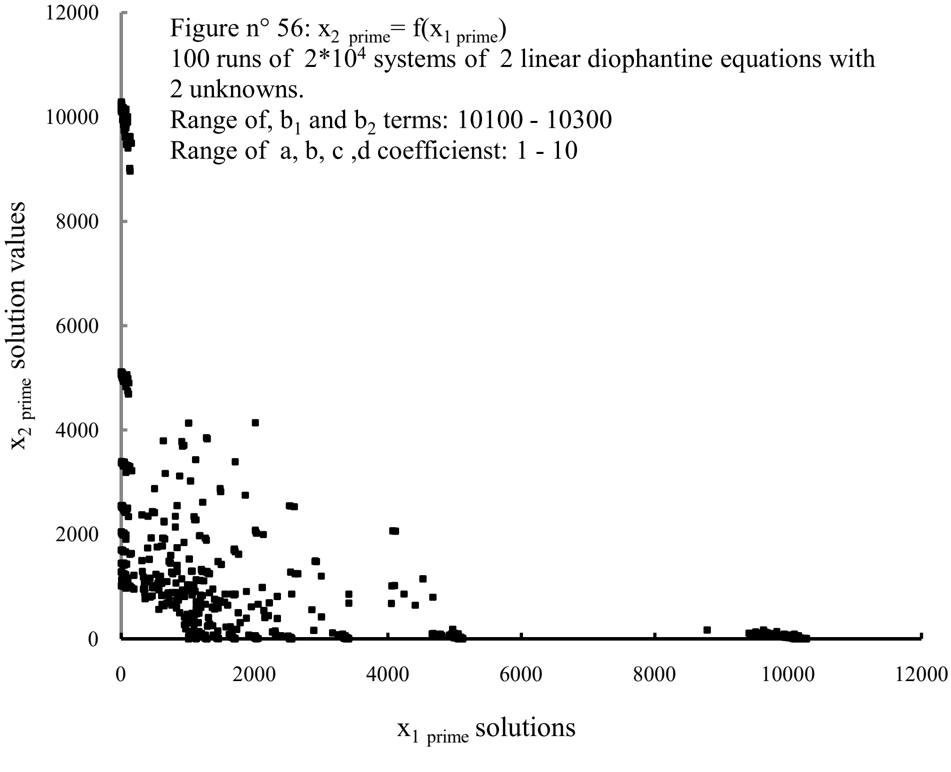


Figure n° 56: x2 prime= f(x1 prime)
100 runs of 2*10^4 systems of 2 linear diophantine equations with 2 unknowns.
Range of, b1 and b2 terms: 10100 - 10300
Range of a, b, c ,d coefficienst: 1 - 10
12000
10000
8000
6000
4000
2000
0
x2 prime solution values
0
2000
4000
6000
8000
10000
12000
x1 prime solutions

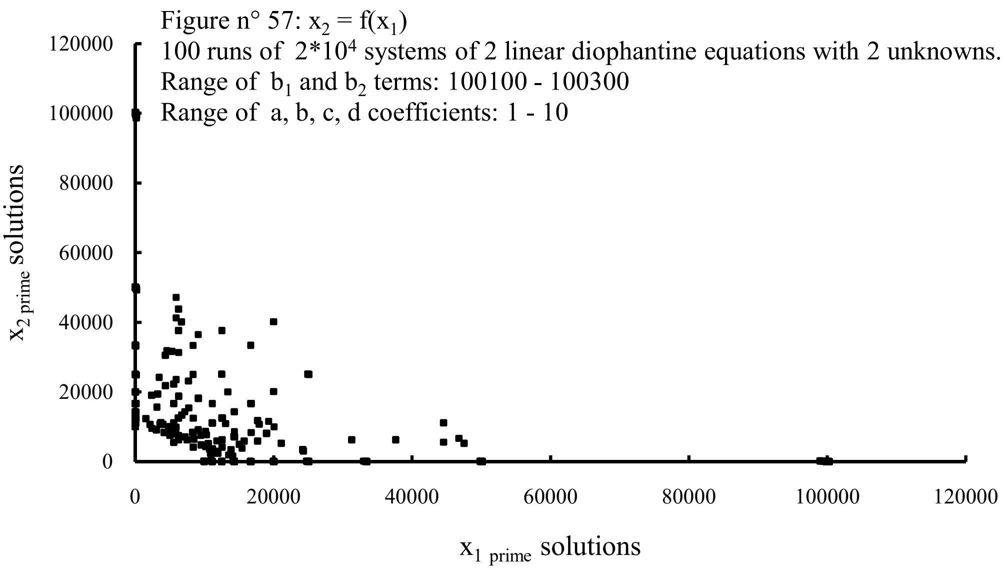


Figure n° 57: $x_2 = f(x_1)$
100 runs of $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns.
Range of b_1 and b_2 terms: 100100 - 100300
Range of a, b, c, d coefficients: 1 - 10
0
20000
40000
60000
80000
100000
120000
$x_{2\ prime}$ solutions
$x_{1\ prime}$ solutions

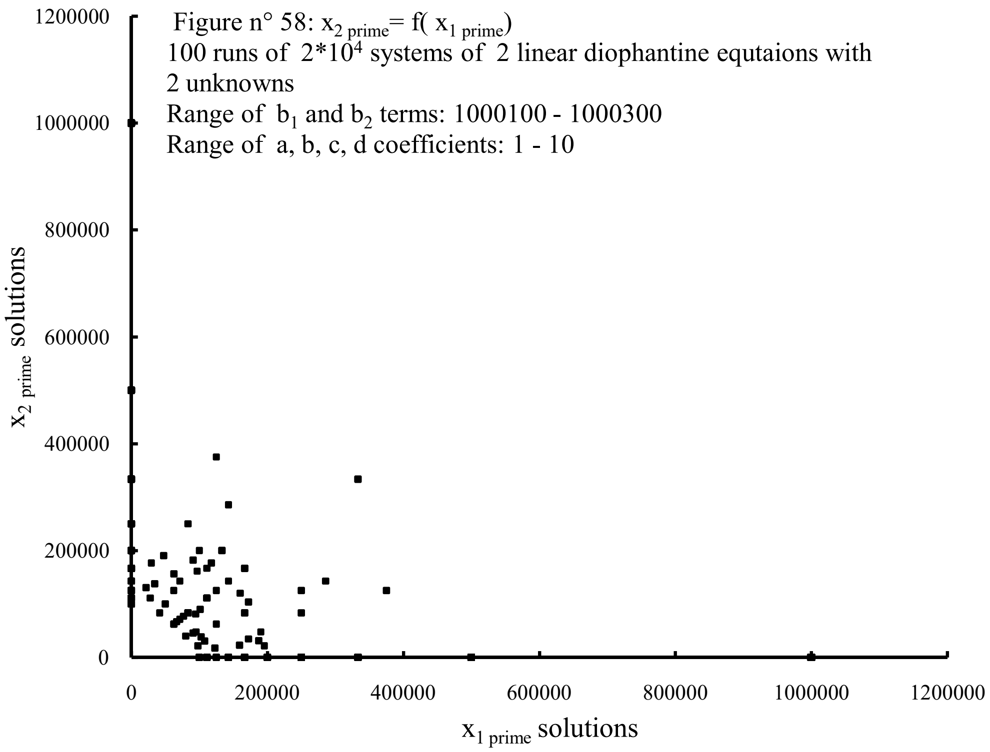


Figure n° 58: $x_{2\ prime} = f(x_{1\ prime})$
100 runs of $2*10^4$ systems of 2 linear diophantine equtaions with 2 unknowns
Range of b_1 and b_2 terms: 1000100 - 1000300
Range of a, b, c, d coefficients: 1 - 10
0
200000
400000
600000
800000
1000000
1200000
$x_{2\ prime}$ solutions
$x_{1\ prime}$ solutions

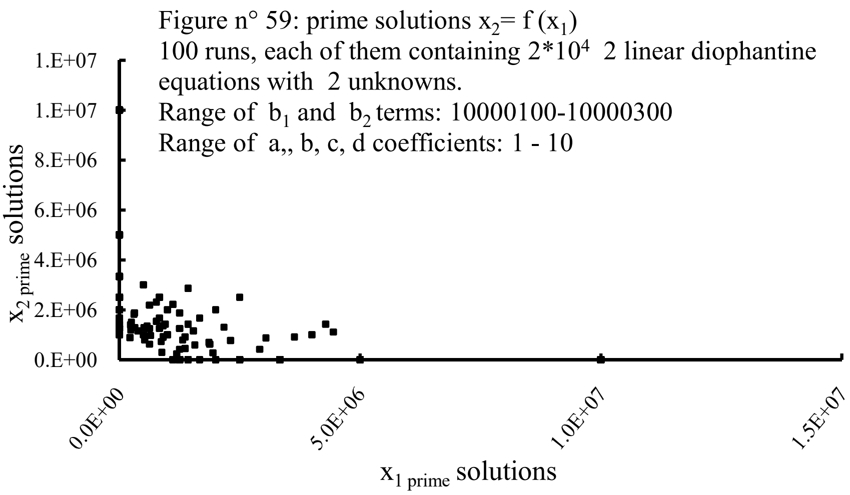


Figure n° 59: prime solutions $x_2 = f(x_1)$
100 runs, each of them containing $2*10^4$ 2 linear diophantine equations with 2 unknowns.
Range of b_1 and b_2 terms: 10000100-10000300
Range of a,, b, c, d coefficients: 1 - 10
1.E+07
1.E+07
8.E+06
6.E+06
4.E+06
2.E+06
0.E+00
$x_{2\ prime}$ solutions
0.0E+00
5.0E+06
1.0E+07
1.5E+07
$x_{1\ prime}$ solutions

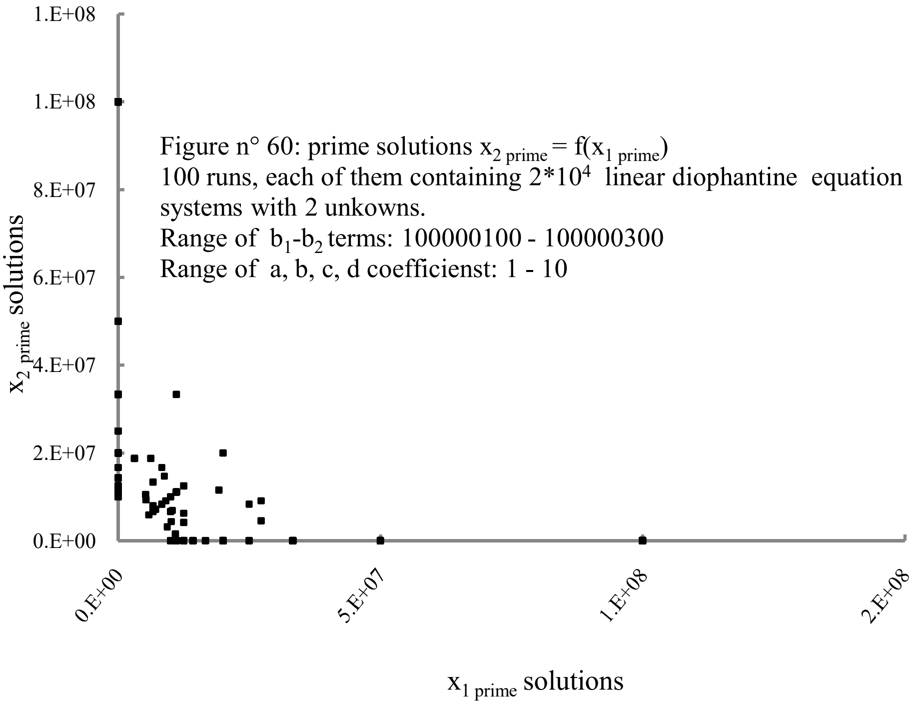


Figure n° 60: prime solutions $x_{2\ prime} = f(x_{1\ prime})$
100 runs, each of them containing $2*10^4$ linear diophantine equation systems with 2 unkowns.
Range of b_1-b_2 terms: 100000100 - 100000300
Range of a, b, c, d coefficienst: 1 - 10
1.E+08
1.E+08
8.E+07
6.E+07
4.E+07
2.E+07
0.E+00
$x_{2\ prime}$ solutions
0.E+00
5.E+07
1.E+08
2.E+08
$x_{1\ prime}$ solutions

Figure n° 61: prime solutions $x_{2\ prime} = f(x_{1\ prime})$
100 runs, each of them containing $2*10^4$ 2 linear diophantine equation systems with 2 unkowns.
Range of b_1-b_2 terms: 100000100 - 100000300
Range of a, b, c, d coefficients: 1 - 10
zoom-1

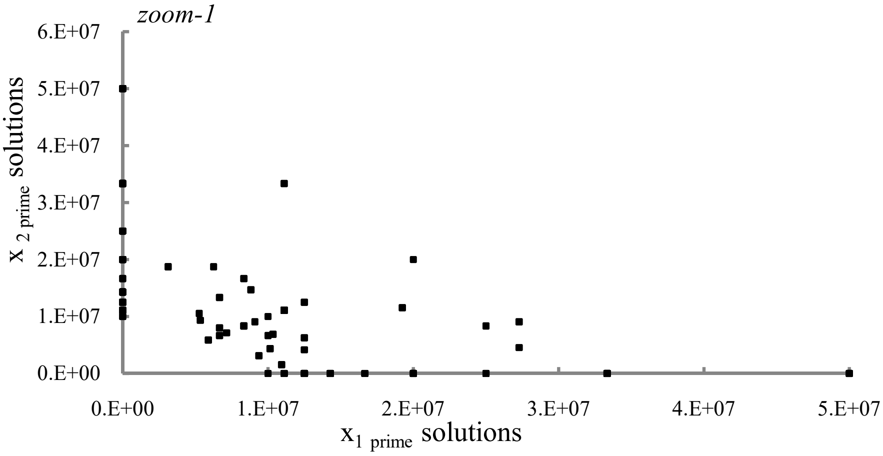


Figure n° 62: all solutions
$x_{2\text{-}all}=f\ (x_{1\text{-}all})$
2096 $x_{1\text{-}all}$ and 2096 $x_{2\text{-}all}$ values
Range of b_1-b_2 terms: 100 - 300
Range of a, b, c, d coefficients: 1 - 10

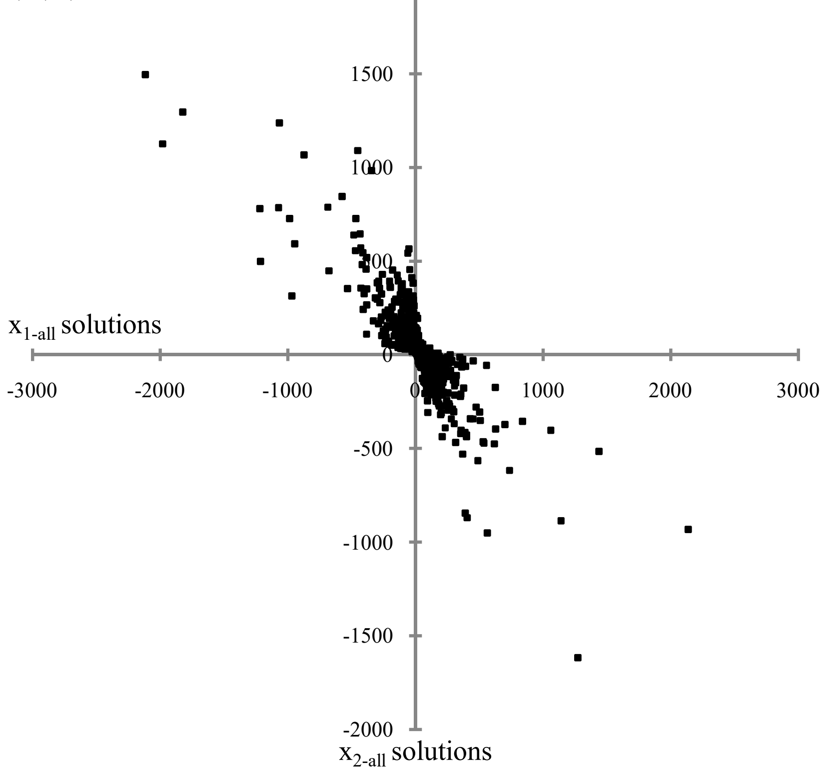


Figure n° 63: all solutions,
$x_{2\text{-all}} = f\,(x_{1\text{-all}})$
2096 $x_{1\text{-all}}$ and 2096 $x_{2\text{-all}}$ values
Range of b_1 - b_2 terms: 100 - 300
Range of a, b, c, d coefficients: 1 - 10
zoom-1

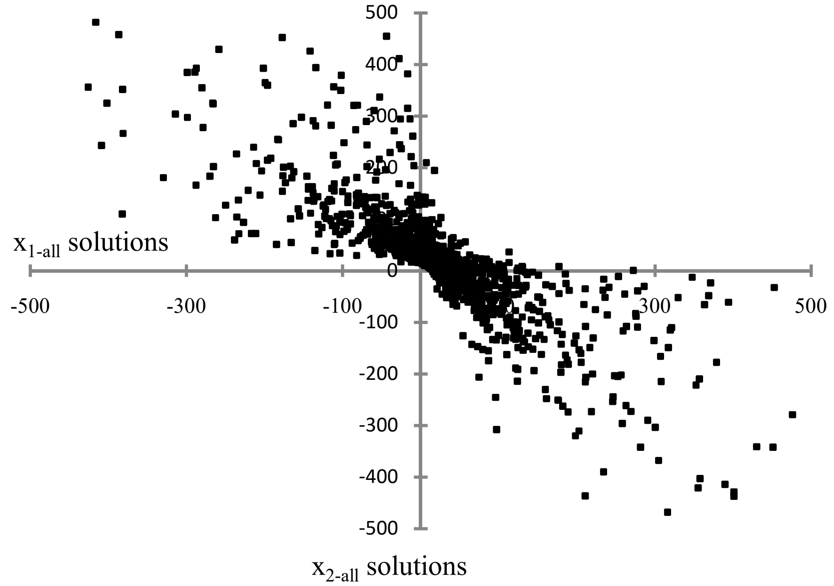
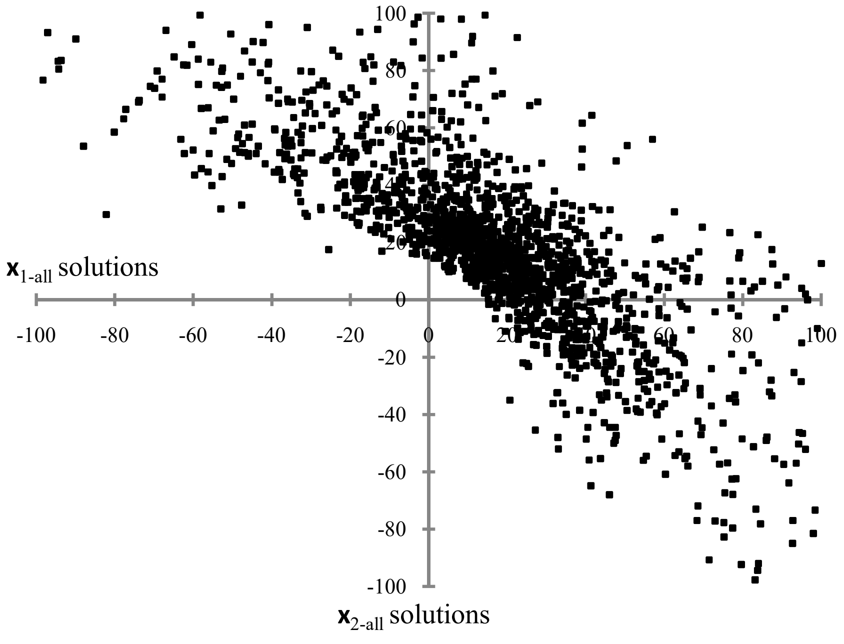


Figure n° 64: all solutions
$x_{2\text{-all}} = f\,(x_{1\text{-all}})$
2096 $x_{1\text{-all}}$ and 2096 x_2 values
Range of b_1 - b_2 terms: 100 - 300
Range of a, b, c, d coefficients: 1 - 10
zoom-2



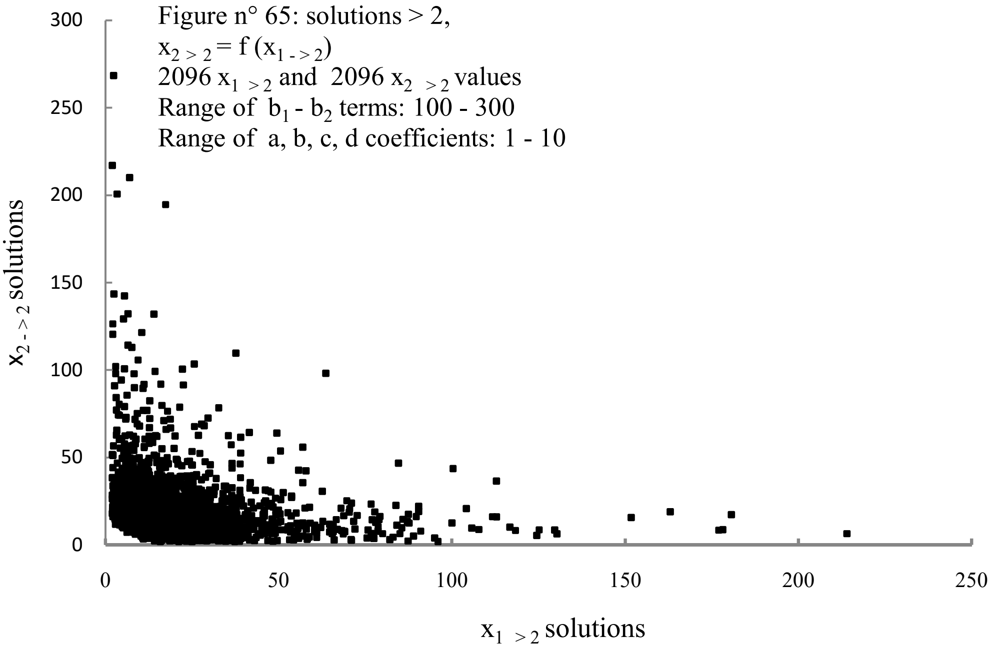


Figure n° 65: solutions > 2,
$x_{2>2} = f(x_{1->2})$
2096 $x_{1\ >2}$ and 2096 $x_{2\ >2}$ values
Range of b_1 - b_2 terms: 100 - 300
Range of a, b, c, d coefficients: 1 - 10

Figure n° 66: solutions > 2
$x_{2\ >2}= f(x_{1>2})$
2096 $x_{1>2}$ and 2096 $x_{2>2}$ values
Range of b_1 - b_2 terms: 100 - 300
Range of a, b, c, d coefficients: 1 - 10
zoom-1

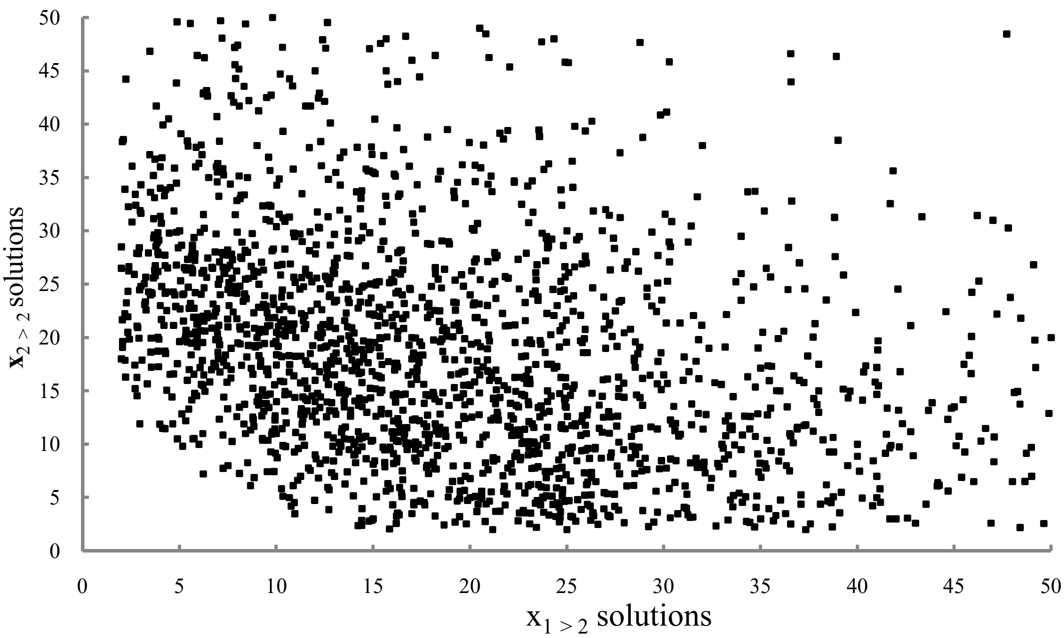
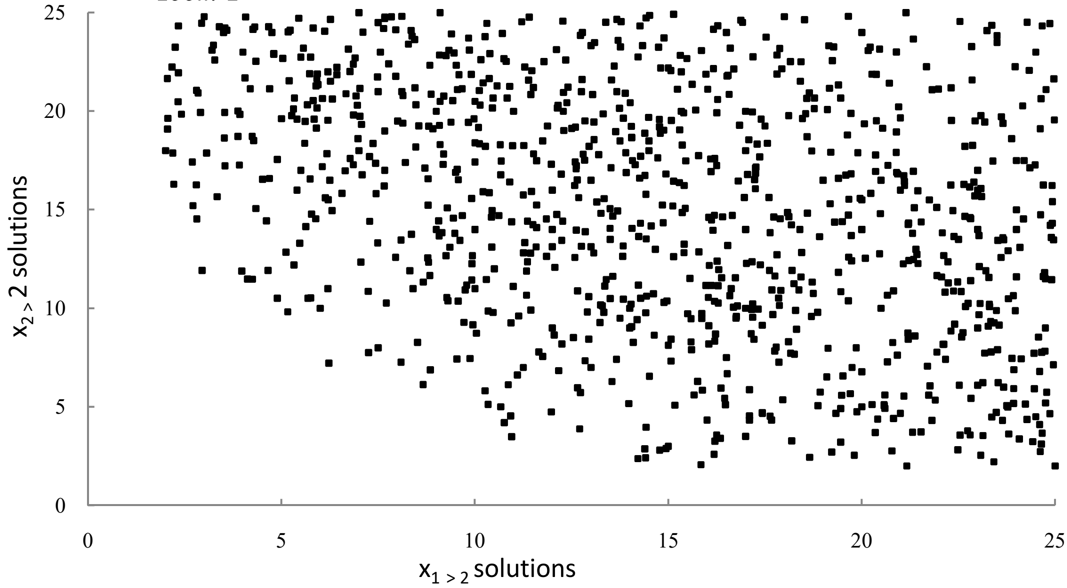


Figure n° 67: solutions > 2, $x_{2>2} = f(x1 > 2)$
2096 $x_{1>2}$ and 2096 $x_{2>2}$ values
Range of b_1 - b_2 terms: 100 - 300
Rnage of a, b, c, d coefficients: 1 - 10
zoom-2



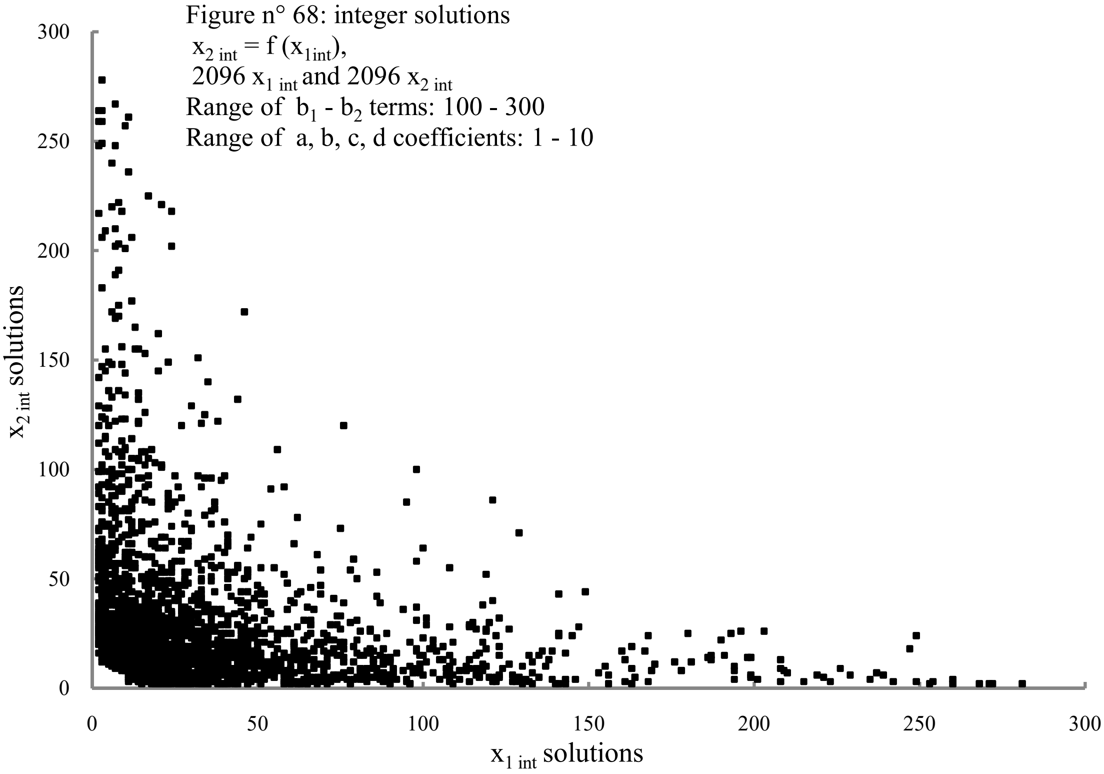


Figure n° 68: integer solutions
$x_{2\ int} = f(x_{1int})$,
2096 $x_{1\ int}$ and 2096 $x_{2\ int}$
Range of b_1 - b_2 terms: 100 - 300
Range of a, b, c, d coefficients: 1 - 10

Figure n° 69: integer solutions
$x_{2\ int} = f(x_{1\ int})$,
2096 $x_{1\ int}$ and 2096 $x_{2\ int}$
Range of b_1 - b_2 terms, 100 - 300
Range of a, b, c, d coefficients: 1 - 10
zoom-1

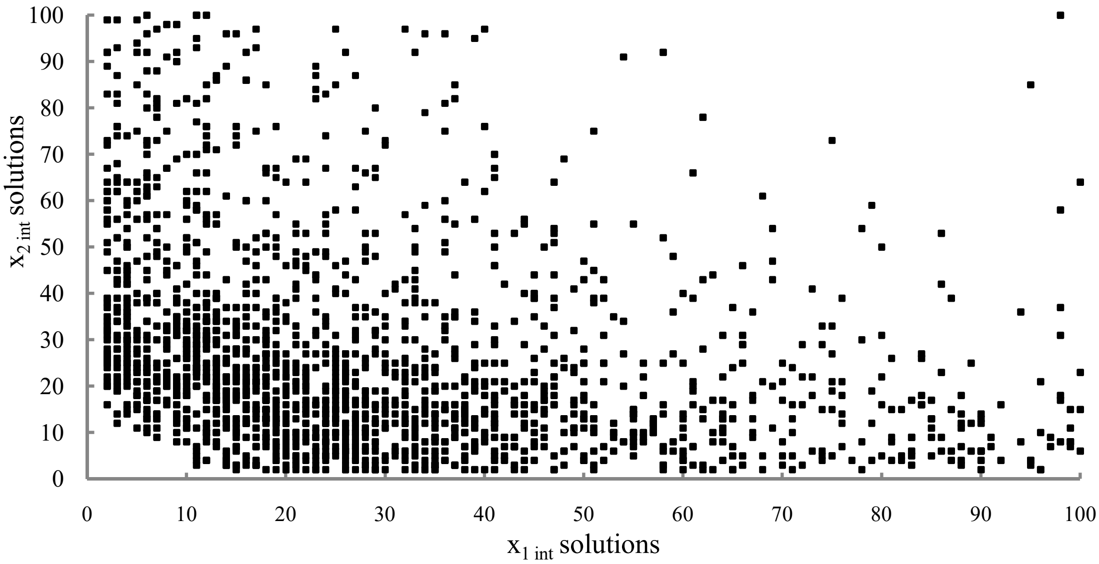


Figure n° 70: integer solutions,
$x_{2\ int} = f(x_{1\ int})$
2096 $x_{1\ int}$ and 2096 $x_{2\ int}$
Range of b_1 - b_2 terms: 100 - 300
Range of a, b, c, d coefficients: 1 - 10
zoom-2

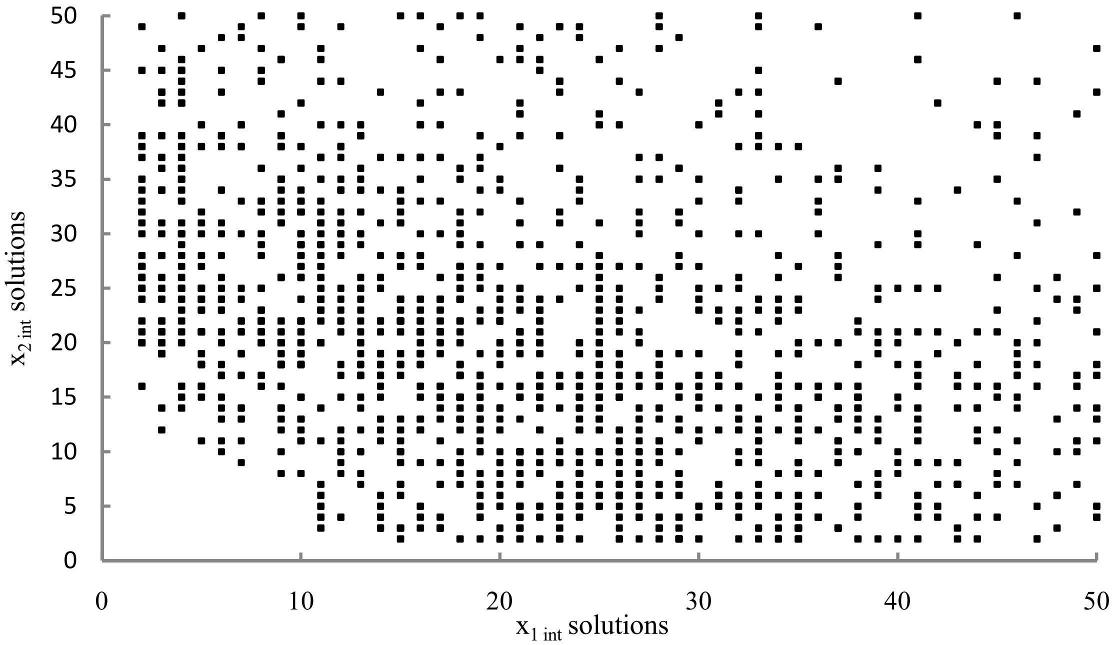


Figure n° 71: prime solutions
$x_{2\ prime}=f\ (x_{1prime})$
2096 $x_{1\ prime}$ and 2096 $x_{2\ prime}$
Range of b_1 - b_2 terms: 100 - 300
Range of a, b, c, d coefficients: 1 - 10

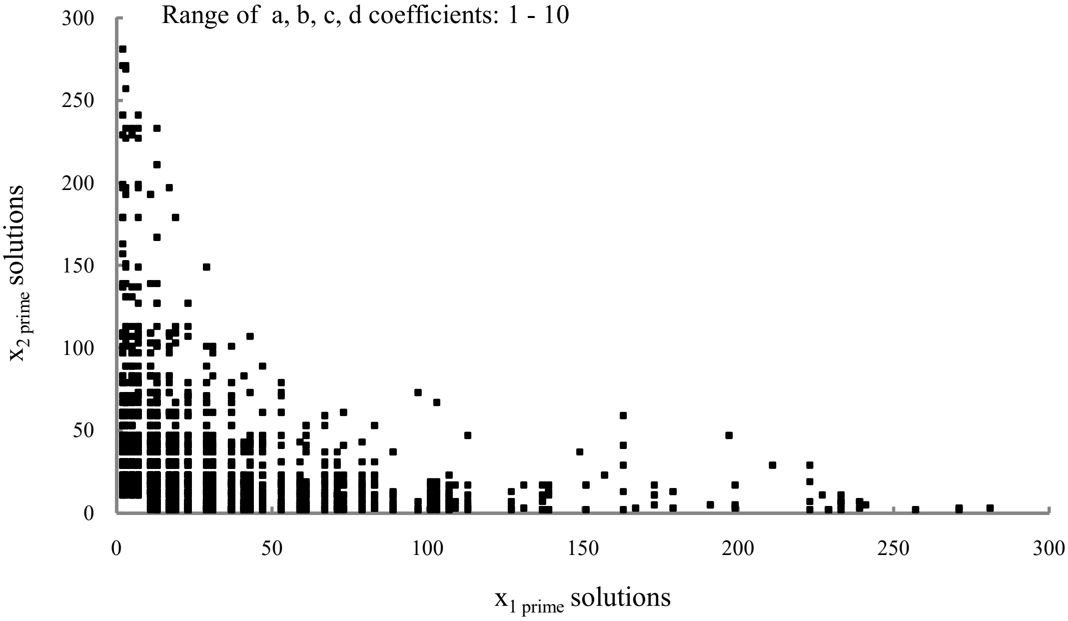


Figure 72: prime solutions,
$x_{2\ prime} = f\ (x_{1\ prime})$,
2096 $x_{1\ prime}$ and 2096 $x_{2\ prime}$
Range of b_1 - b_2 terms: 100 - 300
Range of a, b, c, d coefficients: 1 - 10
zoom-1

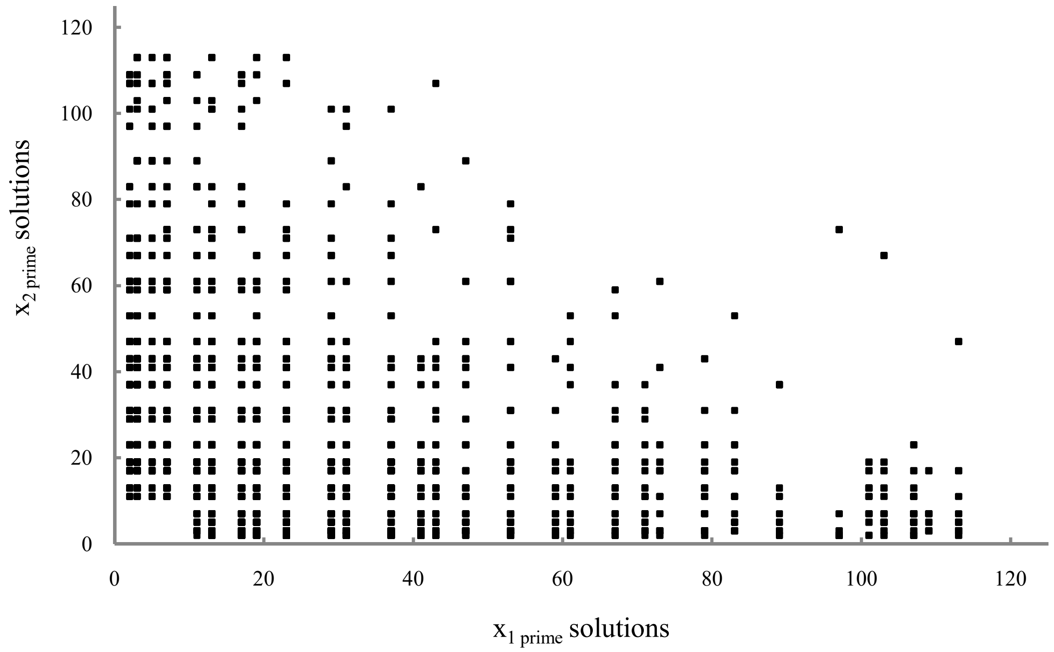
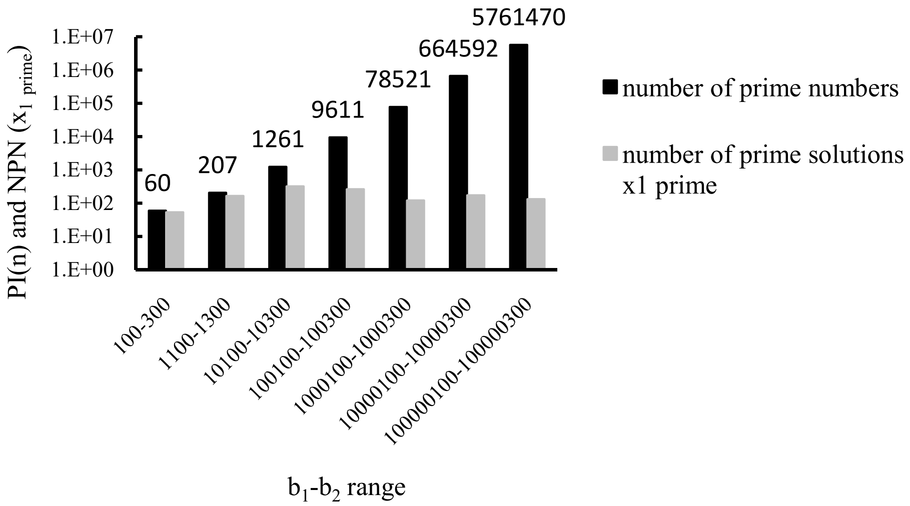
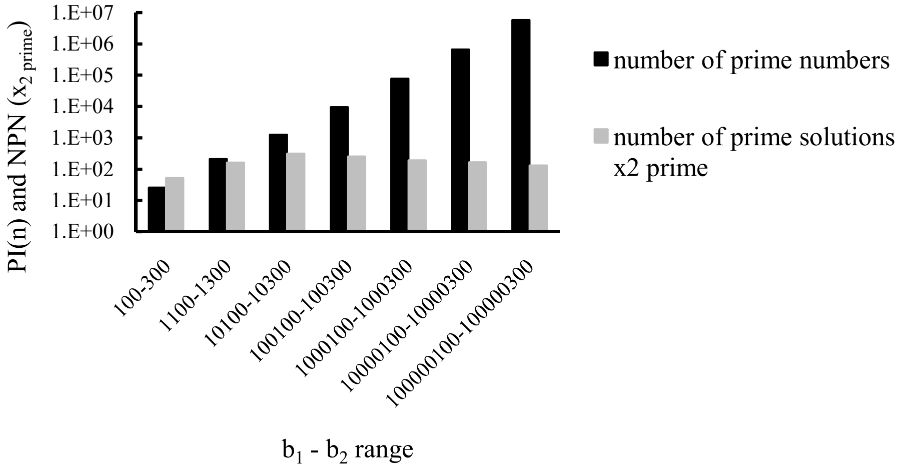


Figure n° 73: number of prime numbers and number of prime solutions $x_{1\ prime} = f\ (b_1\text{-}b_2 \text{ range})$
For each b_1-b_2 range, 100 runs of $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns.
Range of a, b, c, d coefficients: 1 - 10



Graph n° 74: number of prime numbers and number of prime solutions $x_{2\ prime} = f\ (b_1\text{-}b_2 \text{ range})$
For each b_1-b_2 range, 100 runs of $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns.
Range of a, b, c, d coefficienst: 1-10



Graph n° 75: ratio number of prime numbers / number of prime solutions = f (b_1-b_2 range)
For each b_1-b_2 range, 100 runs of $2*10^4$ systems of 2 linear diophantine equations with 2 unkowns.
Range of a, b, c, d coefficients: 1 - 10

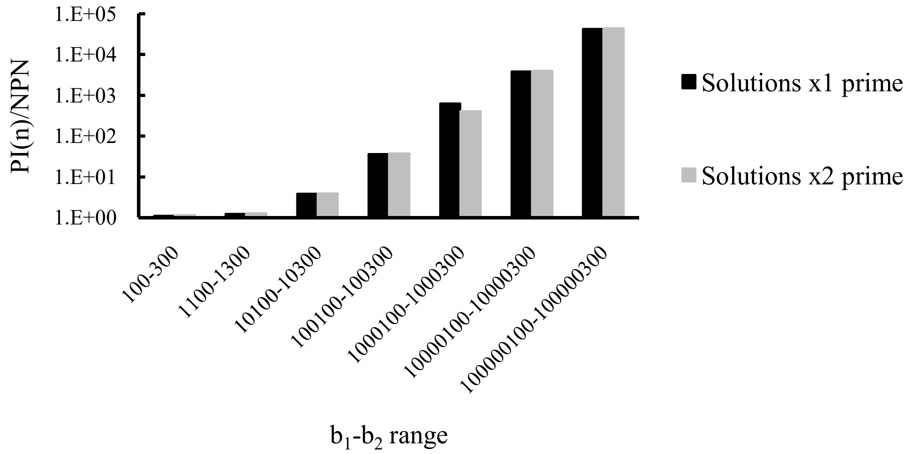


Figure n° 76: histogram of the number of runs over a total of 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns.
Range of b_1 and b_2 terms: 1000000 - 3000000
Range of a, b, c, d coefficients: 1 - 10

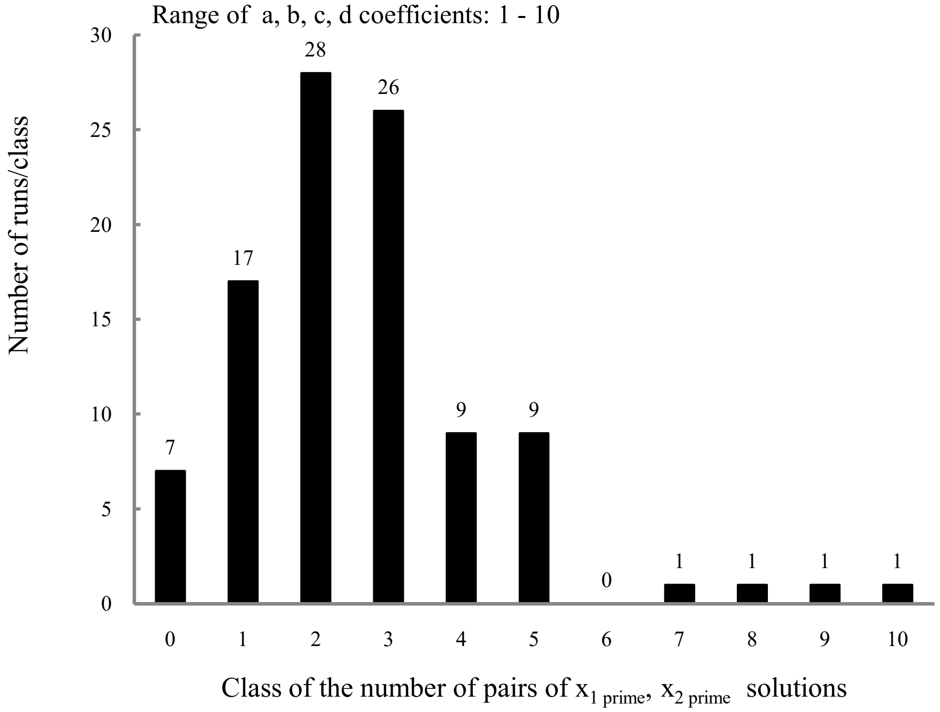


Figure n° 77: histogram of $x_{1\ prime}$ solutions over 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns.
Range of b_1 and b_2 terms: 1000000 - 3000000
Range of a, b, c, d coefficients: 1 - 10

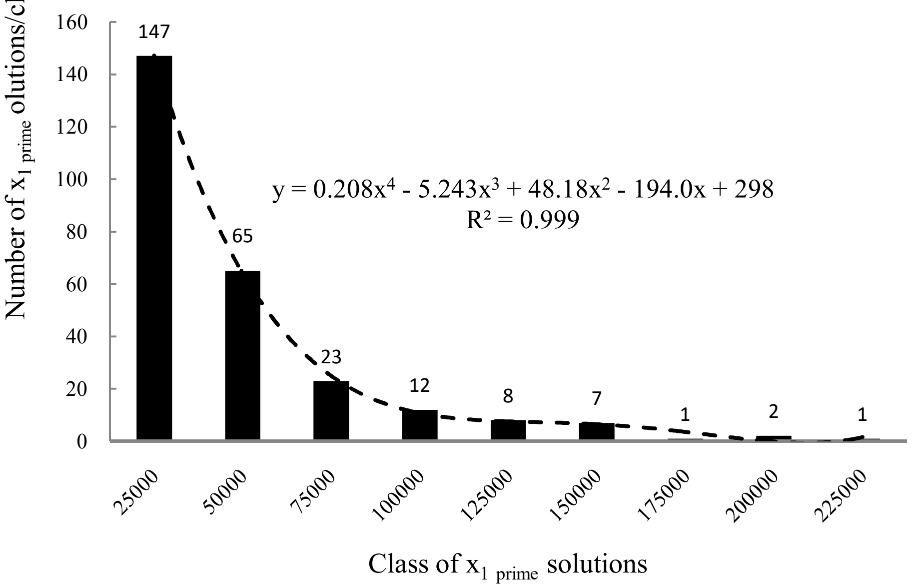


Figure n° 78: histogram of $x_{2\ prime}$ solutions over 100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns.
Range of b_1 and b_2 terms: 1000000 - 3000000
Range of a, b, c, d coefficients: 1 - 10

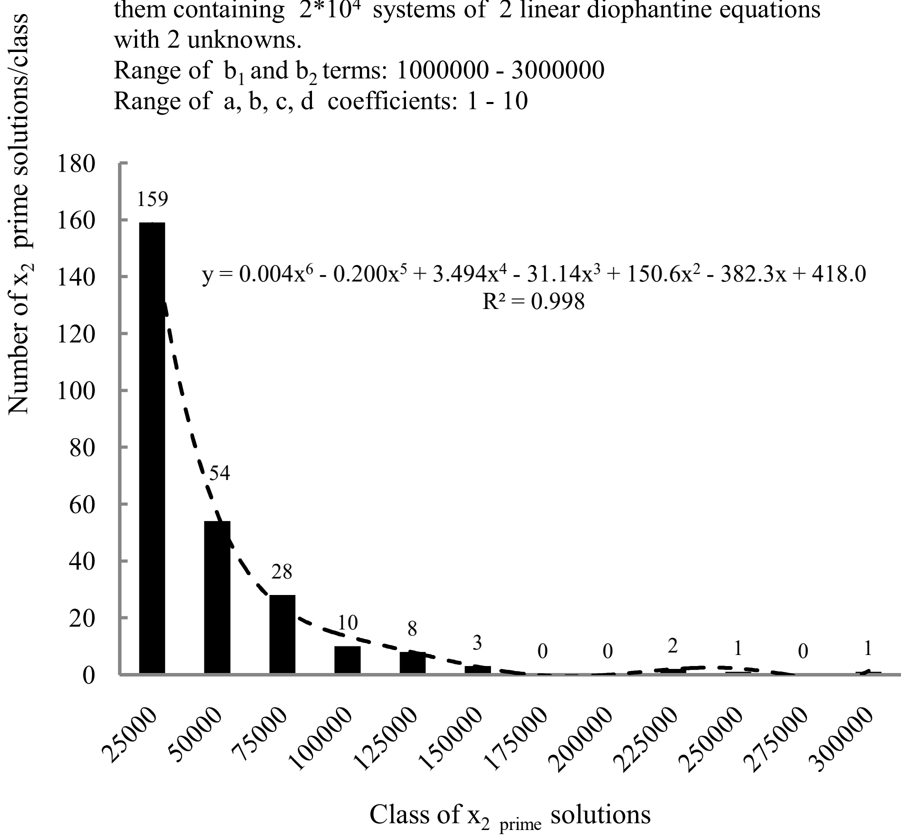


Figure n° 79: prime solutions $x_{2\ prime} = f(x_{1\ prime})$
100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns.
Range of b_1 and b_2 terms: 1000000 - 3000000
Range of a, b, c, d coefficients: 1 - 10

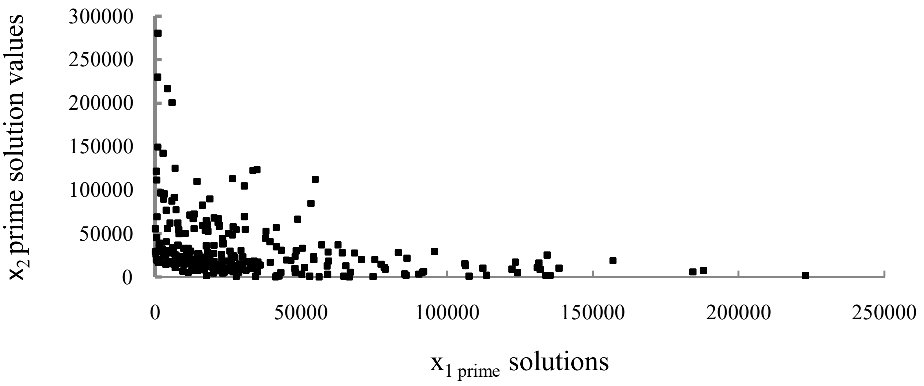


Figure n° 80: prime solutions $x_{2\ prime} = f(x_{1\ prime})$
100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns.
Range of b_1 and b_2 terms: 1000000 - 3000000
Range of a, b, c, d coefficients: 1 - 10
zoom of graph n°

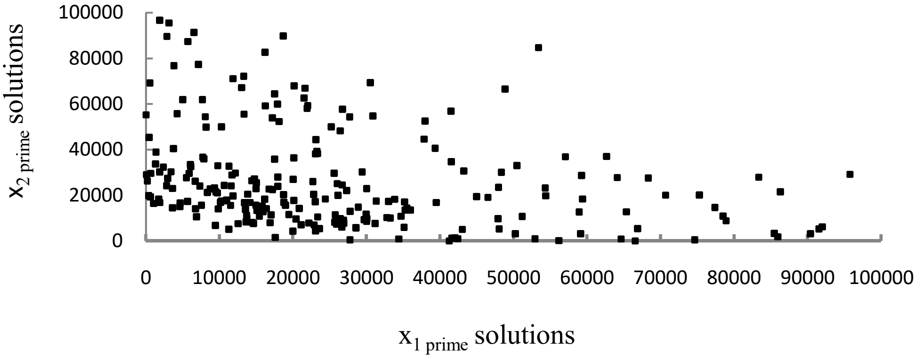


Figure n° 81: number of prime numbers per class

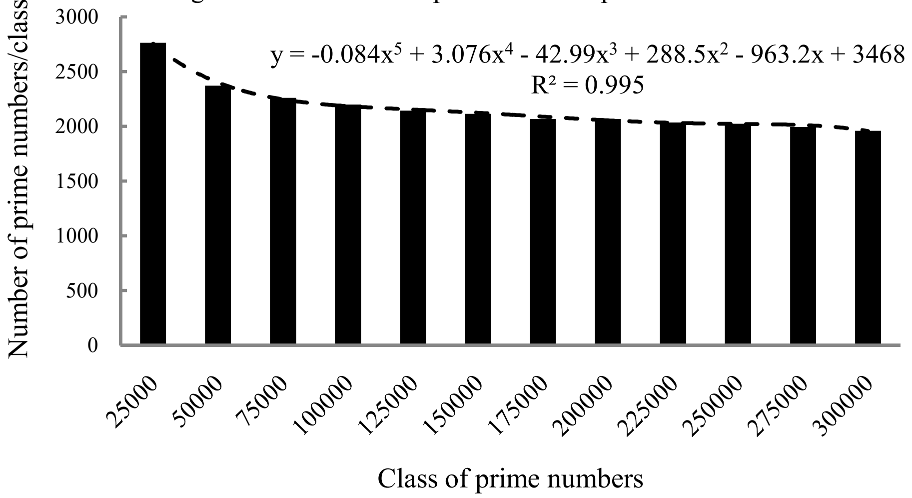


Figure n° 82 correlation number of $x_{1\ prime}$ solutions = f (number of prime numbers)
100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns.
Range of b_1 and b_2 terms: 1000000 - 3000000

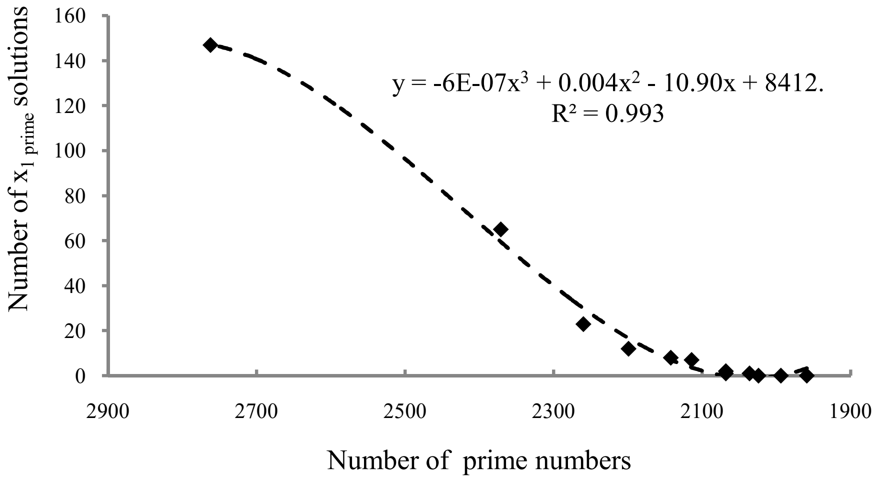


Figure n° 83: correlation number of $x_{2\ prime}$ solutions = f (number of prime numbers)
100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns.
Range of b_1 and b_2 terms: 1000000 - 3000000
Range of a, b, c, d coeffi

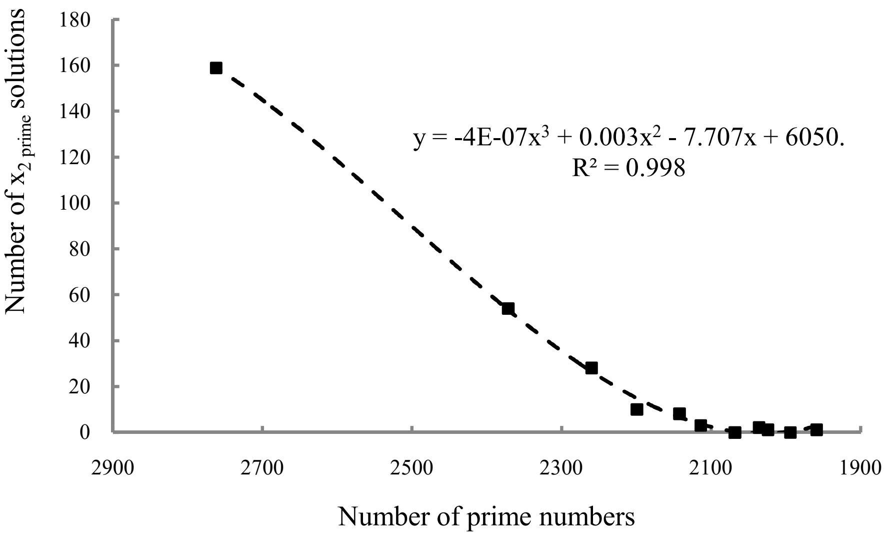


Figure n° 84: correlation % of $x_{1\ prime}$ solutions = f (% of prime numbers)
100 runs, each of them containing $2x10^4$ systems of 2 linear diophantine equations with 2 unknowns.
Range of b_1 and b_2 terms: 1000000 - 3000000
Range of a, b, c, d coefficients: 1

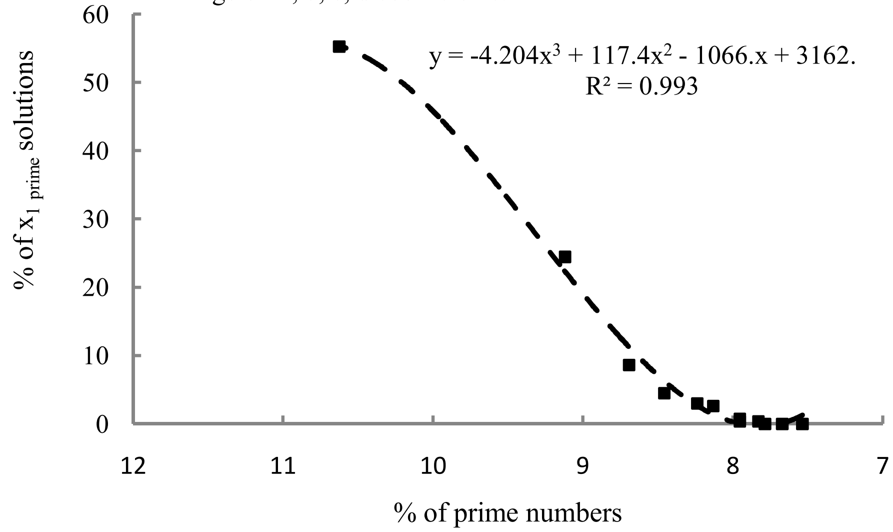


Figure n° 85: correlation % of $x_{2\ prime}$ solutions = f (number of prime numbers)
100 runs, each of them containing $2*10^4$ systems of 2 linear diophantine equations with 2 unknowns.
Range of b_1 and b_2 terms: 1000000 - 3000000
Range of a, b, c, d coefficient

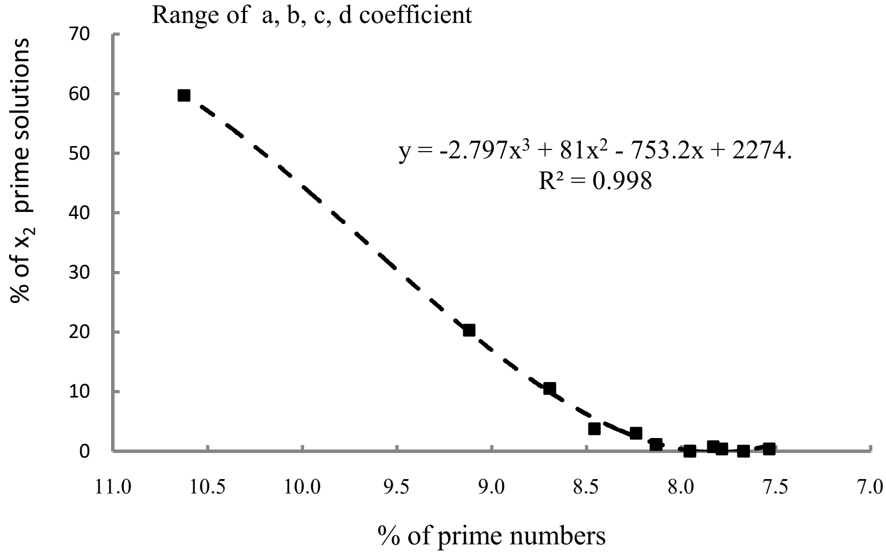
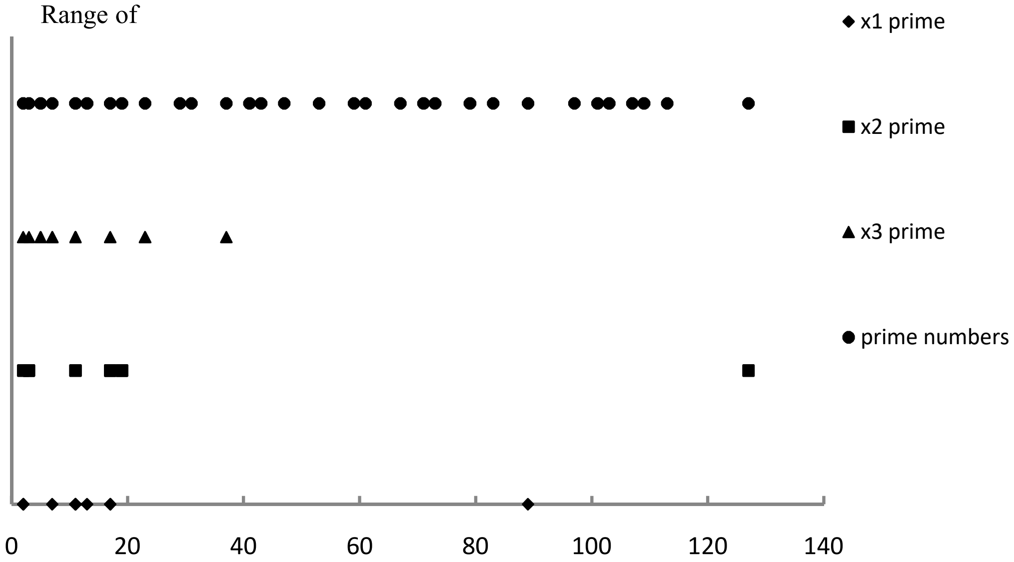


Figure n°: 86 Repartition of $x_{1\ prime}$, $x_{2\ prime}$, $x_{3\ prime}$ solutions and prime numbers
100 runs, each of them containing $2*10^4$ systems of 3 linear diophantine equations with 3 unkowns.
Range of a, b, c, d , e , f , g, h, i coefficients: 10 - 10
Range of



Annex n°3 : tables

Number of runs	25	50	75	100
Range of b_1 and b_2 terms				
100-300	539	1105	1592	2096
1100-1300	332	649	992	1283
10100-10300	192	405	596	819
100100-100300	148	287	430	569
1000100-1000300	142	250	362	493
10000100-10000300	83	182	262	361
100000100-100000300	63	125	220	285

Table n°1: number of pairs of prime solutions of $2x10^6$ 2 linear equation systems with 2 unknowns and for each range of b_1-b_2 terms.

b_1 and b_2 range	Minimum	Median	Mode	Maximum
100 - 300	11	20	17	40
1100 - 1300	4	13	14	23
10100 - 10300	3	8	9	16
100100 - 100300	1	6	7	30
1000100 - 1000300	0	5	5	12
10000100 - 10000300	0	3	3	8
100000100-100000300	0	3	3	8

Table n° 2 : statistics on the number of prime solutions per run

Number of runs	25	50	75	100
Range of b_1 and b_2 terms				
100-300	0,001078	0,001105	0,001061	0,001048
1100-1300	0,000664	0,000649	0,000661	0,000642
10100-10300	0,000384	0,000405	0,000397	0,000410
100100-100300	0,000296	0,000287	0,000287	0,000285
1000100-1000300	0,000284	0,000250	0,000241	0,000247
10000100-10000300	0,000166	0,000182	0,000175	0,000181
100000100-100000300	0,000126	0,000125	0,000147	0,000140

Table n°3: frequency of pairs of prime solutions of $2x10^6$ 2 linear equation systems with 2 unknowns and for each range of b_1-b_2 terms.

b_1 and b_2 range	Minimum	Median	Maximum
100 - 300	2	19	281
1100 - 1300	2	127	1279
10100 - 10300	2	1187	10273
100100 - 100300	2	11131	100267
1000100 - 1000300	2	117659	1000291
10000100 - 10000300	2	1000003	10000261
100000100-100000300	2	100000259	100000259

Table n° 4: Statistics of $x_{1\ prime}$ solutions

b_1 and b_2 range	Minimum	Median	Maximum
100 - 300	2	19	281
1100 - 1300	2	103	1283
10100 - 10300	2	827	10289
100100 - 100300	2	8329	100271
1000100 - 1000300	2	62467	1000253
10000100 - 10000300	2	1000033	10000261
100000100-100000300	2	8000017	100000267

Table n° 5: Statistics of $x_{2\ prime}$ solutions