On the roots of simplexes

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1 Introduction

In this paper I present a general method for computation of rough estimates for “roots” of simplexes. This method gives a hope of potentially refining the algorithm to compute the exact roots. The method grows slowly in errors as the “exponent” grows.

2 General roots of simplexes

Using the rising powers notation

$$x^{\overline{n}} = \prod_{j=0}^{n-1}(x+j) = x*(x+1)*\cdots*(x+n-1) = \frac{(x+n-1)!}{(x-1)!} = \frac{1}{n!}\binom{x+n-1}{n}$$

And call x the “base” and n the dimension (“exponent”)

$\alpha =$ the scalar to find n^{th} root for i.e. $\alpha = \frac{x^{\overline{b}}}{b!}$

$b =$ the dimension (“exponent”), $n = b$

Given α and b, the algorithm is to computer the root $x = c_2$

2.1 Step one:

$\alpha * b! = y_1$

2.2 Step two:

$$\frac{\sqrt[b]{y_1} * 2b - (b-1)^{\underline{2}}}{2b} = c_1$$

$$\sqrt[b]{\alpha * b!} = \sqrt[b]{y_1} = x_1$$

2.3 Step three:

$$(c_1)^{\overline{b}} = y_2$$

2.4 Step four:

$$\frac{\sqrt[b]{2y_1 - y_2} * 2b - (b-1)^{\underline{2}}}{2b} = c_2$$

$$\sqrt[b]{2y_1 - y_2} = x_2$$

2.5 Step five:

$$\frac{(c_2)^{\overline{b}}}{b!} = \frac{y_3}{b!} = \alpha$$

The value c_2 is the b^{th} root of α in dimension b; the errors grow for higher values of b; the actually root gets smaller gradually as the "exponent" b increases. The method is exact for *b=2* (triangular numbers).

3 Conclusion

This is a general method that gives rough estimates but it is an inspiration to work on the method for the exact roots of the simplexes. Maybe the method can be refined, maybe the exact method is completely different. The method for the exact roots for simplexes remains an open problem but one where we have hope that it probably exists.

This method is exact for triangular numbers (2^{nd} exponents *i.e. b=2*). The method is also exact for perfect simplexes i.e. the terms in Pascal's triangle when we apply ceiling function with the result $\lceil c_2 \rceil$.

4 References