Yukawa Potential and Extended Klein-Gordon Equation in Rindler Space-Time

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ABSTRACT

Yukawa potential satisfy Proca equation or Klein-Gordon equation. If we represent Yukawa potential in Rindler space-time, this Yukawa potential satisfy the extended Klein-Gordon equation in Rindler space-time. We understand Yukawa force in Rindler space-time.

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1. Introduction

Atom’s nucleus force understand by Yukawa potential. We study Yukawa potential in Rindler Space-time time.

At first, Yukawa potential V describes nucleus’s combine force in semi-classical method.

$$V = -\frac{g^2}{r}\exp(-\frac{m_\pi rc}{\hbar})$$

g is real number, m_π is the meson’s mass (1)

Klein-Gordon equation is satisfied by Yukawa potential V.

$$-\partial_i\partial^i V + \frac{m^2c^2}{\hbar^2}V = -\nabla^2 V + \frac{m_\pi{}^2c^2}{\hbar^2}V = 0$$

$$V = -\frac{g^2}{r}\exp(-\frac{m_\pi rc}{\hbar}), i = 1, 2, 3 \qquad (2)$$

2. Yukawa potential from Extended Klein-Gordon Equation in Rindler-Space-Time

Rindler coordinates are

$$ct = (\frac{c^2}{a_0} + \xi^1)\sinh(\frac{a_0}{c}\xi^0) \quad , \quad x = (\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0}{c}\xi^0) - \frac{c^2}{a_0}$$

$$y = \xi^2, z = \xi^3 \qquad (3)$$

If we write Yukawa potential V in inertial frame,

$$V = -\frac{g^2}{r}\exp(-\frac{m_\pi r c}{\hbar} \qquad (4)$$

If we rewrite Yukawa potential V_ξ in Rindler space-time,

$$V_\xi = -\frac{g^2}{\sqrt{x^2 + y^2 + z^2}}\exp(-\frac{m_\pi c}{\hbar}\sqrt{x^2 + y^2 + z^2})$$

$$= -\frac{g^2}{\sqrt{\{(\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0}{c}\xi^0) - \frac{c^2}{a_0}\}^2 + (\xi^2)^2 + (\xi^3)^2}}\exp[-\frac{m_\pi c}{\hbar}\sqrt{\{(\frac{c^2}{a_0} + \xi^1)\cosh(\frac{a_0}{c}\xi^0) - \frac{c^2}{a_0}\}^2 + (\xi^2)^2 + (\xi^3)^2}]$$

(5)

This Yukawa potential satisfy the extended Klein-Gordon equation. At first, energy and momentum are in Rindler space-time[1],

$$E_\xi = i\hbar\frac{1}{(1 + \frac{a_0}{c^2}\xi^1)}\frac{\partial}{\partial\xi^0}, \vec{p}_\xi = -i\hbar\vec{\nabla}_\xi \qquad (6)$$

Energy-Momentum equation is in Rindler space-time[1],

$$E_\xi{}^2 = \vec{p}_\xi c \cdot \vec{p}_\xi c + m^2 c^4 \tag{7}$$

Hence, normal Klein-Gordon equation is in Rindler-spacetime,

$$\begin{aligned} &\frac{m^2c^2}{\hbar^2} V_\xi + \frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} \frac{\partial^2}{(\partial\xi^0)^2} V_\xi - \nabla_\xi{}^2 V_\xi \\ &= \frac{m_\pi{}^2 c^2}{\hbar^2} V_\xi + \frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} \frac{\partial^2}{(\partial\xi^0)^2} V_\xi - \nabla_\xi{}^2 V_\xi = 0 \end{aligned} \tag{8}$$

In this time, we focus the gauge Λ equation in Rindler space-time[1],

$$\frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} \frac{\partial^2}{(\partial\xi^0)^2} \Lambda - \nabla_\xi{}^2 \Lambda - \frac{\partial\Lambda}{\partial\xi^1} \frac{a_0}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} = 0 \tag{9}$$

Hence, Eq(8) change extended Klein-Gordon equation in Rindler space-time.

Extended Klein-Gordon Equation is in Rindler space-time,

$$\begin{aligned} &\frac{m^2c^2}{\hbar^2} V_\xi + \frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} \frac{\partial^2}{(\partial\xi^0)^2} V_\xi - \nabla_\xi{}^2 V_\xi - \frac{\partial V_\xi}{\partial\xi^1} \frac{a_0}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} \\ &= \frac{m_\pi{}^2 c^2}{\hbar^2} V_\xi + \frac{1}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)^2} \frac{\partial^2}{(\partial\xi^0)^2} V_\xi - \nabla_\xi{}^2 V_\xi - \frac{\partial V_\xi}{\partial\xi^1} \frac{a_0}{c^2} \frac{1}{(1+\frac{a_0}{c^2}\xi^1)} = 0 \end{aligned} \tag{10}$$

Eq(5) ,Yukawa potential V_ξ satisfy Eq(10), extended Klein-Gordon equation in Rindler space-time.

Yukawa force $\vec{f}$ is

$$\vec{f} = -\vec{\nabla} V = -\frac{g^2}{r^3}[\exp(-\frac{m_\pi rc}{\hbar})](1+\frac{m_\pi rc}{\hbar})\vec{r} \tag{11}$$

In this time, Yukawa force $\vec{f}_\xi$ is Rindler space-time,

$$\vec{f}_\xi = -\vec{\nabla}_\xi V_\xi = -\frac{g^2}{r^3}[\exp(-\frac{m_\pi rc}{\hbar})](1+\frac{m_\pi rc}{\hbar})(x\cosh(\frac{a_0\xi^0}{c}), \xi^2, \xi^3) \tag{12}$$

Hence, according to Yukawa force $\vec{f}_\xi$ in Rindler space-time, the nuclear force strongly act in accelerated frame rather than inertial frame in x-axis.

3. Conclusion

We found Yukawa potential mechanism in Rindler Space-time. We understand nuclear force in Rindler space-time.

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