Calculation of particle decay times in the Standard Model

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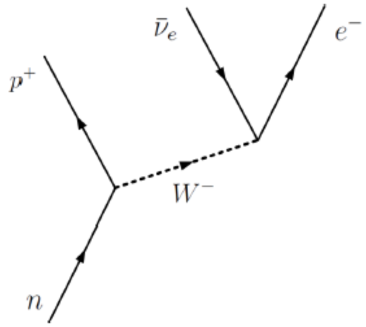
Abstract
This paper presents traditional and new methods and results for calculation of decay times of particles in the Standard Model.
In chapters 1 and 2 the phenomenological and the theoretical knowledge of the decays is presented, based on the literature.
In chapters 4 and 5 the interaction energy (mass-energy m_X of the mediating boson in the Feynman diagram) is introduced and a characterization of the decays, based on particle type, isospin, and interaction energy, is presented.
In chapter 6 the calculation model and the calculation results of selected typical decays are discussed.

1 Selected particle decays with theoretical background

1.1 Neutron

The free neutron decays into a proton, electron, and antineutrino: [29]

$n \rightarrow p + e^{-} + \overline{\nu}_e$



Neutron decay [36]

The rest energy $(m_n - m_p - m_e)c^2 = 782\ \mathrm{keV}$ is carried away by e and ν

The transition matrix of the decay is [29, 35]

$$\mathcal{M} = \left(G_V \overline{p} \gamma^{\mu} n - G_A \overline{p} \gamma^{\mu} \gamma_5 n\right)\left(\overline{e} \gamma_{\mu} (1-\gamma_5) \nu\right) \delta(E_n - E_p - E_e - E_{\nu})$$

from the interaction Hamiltonian [35]

$$H_{\text{int}} = G_F V_{ud} \left(\overline{p} \gamma^{\mu} \left(1 - \frac{G_A}{G_V} \gamma_5\right) n \right) \left(\overline{e} \gamma_{\mu} (1-\gamma_5) \nu\right)$$

with $G_A / G_V = 1.255 \pm 0.005$

$$E(G_V) = \frac{1}{\sqrt{G_V}} = 296.7 GeV$$

$G_F = 1.1663787(6) \times 10^{-5}\ \mathrm{GeV}^{-2}$ is the Fermi weak coupling constant, $V_{ud} = 0.97417(21)$ and the weak V-constant

$G_V = G_F V_{ud} = 1.135 * 10^{-5} GeV^{-2}$ (V is the CKM-matrix), $G_A = G_F V_{ud} \lambda$ and λ is the hadronic strong interaction correction.

We compute the neutron decay probability per unit time using Fermi’s golden rule:

$$dW = \frac{|M(k_1, k_2, k_3, k_4)|^2}{2m_1} \frac{d^3 k_2}{(2\pi)^3 2E_2} \frac{d^3 k_3}{(2\pi)^3 2E_3} \frac{d^3 k_4}{(2\pi)^3 2E_4} (2\pi)^4 \delta^4 (k_1 - k_2 - k_3 - k_4)$$

where $k_1 = p_n$, $k_2 = p_p$, $k_3 = p_e$, $k_4 = p_\nu$, $m_1 = m_n$ with the (dimensionless) transition matrix $M(k_1,k_2,k_3,k_4) = \langle f|H_{\text{int}}|i\rangle \delta(E_f - E_i)$ of the interaction Hamiltonian H_{int} .

Here E_e, p_e, E_n, and p_n are the electron and antineutrino total energy and momentum Δ is the neutron-proton mass difference $\Delta = 1.29333205(51)$ MeV
Integration over the antineutrino and electron momenta gives the beta electron energy spectrum

$$\frac{dW}{dE_e} = \frac{G_V^2 + 3G_A^2}{2\pi^3} E_e |p_e| (\Delta - E_e)^2 ,$$

Additional integration over electron energy yields

$$W = \left(G_V{}^2 + 3G_A{}^2\right)\frac{m_e{}^5}{2\pi^3} f_R$$

Here f_R is the phase-space term, i.e. the value of the integral over the Fermi energy spectrum, including Coulomb, recoil order, and radiative corrections
The bandwidth of the decay becomes

$$\Gamma(n \to pe\nu_e) = \left(G_V{}^2 + 3G_A{}^2\right)\frac{m_e{}^5}{2\pi^3} f_R = G_F{}^2 V_{ud}{}^2 \left(1+3\lambda^2\right)\frac{m_e{}^5}{2\pi^3} f_R$$ [29] $G_V = G_F V_{ud}$ $G_A = G_F V_{ud}\lambda$ $\lambda=1.255$ $V_{ud}=0.974$ $G_F=1.166\ 10^{-5}\ GeV^{-1}$

with the phase-space term [36] $f^R = \frac{1}{60}[2\xi^4 - 9\xi^2 - 8](\xi^2-1)^{1/2} + \frac{1}{4}\xi \ln[\xi + (\xi^2-1)^{1/2}]$ $\xi \equiv \frac{M_n - M_p}{m_e}$ $f_R = 1.6332$

here the transition probability per unit time is $W = \Gamma(n \to pe\nu_e)/\hbar$

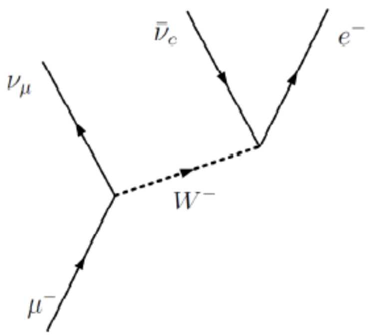
The neutron lifetime τ_n becomes

$$\tau_n = 1/\Gamma(n \to pe\nu_e) = \frac{2\pi^3}{\left(G_V{}^2 + 3G_A{}^2\right) m_e{}^5 f_R} = 881.5\text{s}$$

1.2 Muon

The muon decays into an electron, an electron-antineutrino and a muon-neutrino

$$\mu \to e^- + \bar{\nu}_e + \nu_\mu$$



Muon decay [36]

For the muon decay we derive the formula for the bandwidth Γ [11, 31]

The interaction Hamiltonian is the current-current interaction

$$H_{\text{int}} = \frac{g^2}{2M_W^{\ 2}} \bar{u}_2(k_2,s_2)\left(\gamma^\mu \frac{1-\gamma^5}{2}\right) u_1(k_1,s_1)\bar{u}_4(k_4,s_4)\left(\gamma_\mu \frac{1-\gamma^5}{2}\right) u_3(k_3,s_3)$$

From the transition matrix element

$$\mathcal{M} = \frac{g^2}{2M_W^{\ 2}} \bar{u}_2(k_2,s_2)\left(\gamma^\mu \frac{1-\gamma^5}{2}\right) u_1(k_1,s_1)\bar{u}_4(k_4,s_4)\left(\gamma_\mu \frac{1-\gamma^5}{2}\right) u_3(k_3,s_3)\delta(E_1-E_2-E_3-E_4)$$

with $\frac{G}{\sqrt{2}} = \frac{g^2}{8M_W^{\ 2}}$ and G=G_F , g is the weak dimensionless interaction constant,

we get after some γ-algebra averaging over the spins and trace-manipulation

$\overline{|\mathcal{M}|^2} = 64G^2(k_1\cdot k_3)(k_2\cdot k_4)$ and for the decay rate we have Fermi's golden rule

$$d\Gamma = \frac{|M(k_1,k_2,k_3,k_4)|^2}{2m}\frac{d^3k_2}{(2\pi)^3 2E_2}\frac{d^3k_3}{(2\pi)^3 2E_3}\frac{d^3k_4}{(2\pi)^3 2E_4}(2\pi)^4\delta^4(k_1-k_2-k_3-k_4)$$

$$d\Gamma = \frac{1}{2m}\left(64G^2(k_1\cdot k_3)(k_2\cdot k_4)\right)\frac{d^3k_2}{(2\pi)^3 2E_{k2}}\frac{d^3k_3}{(2\pi)^3 2E_{k3}}\frac{d^3k_4}{(2\pi)^3 2E_{k4}}(2\pi)^4\delta^4(k_1-k_2-k_3-k_4)$$

In the muon rest frame $k_1 = (m,0,0,0)$. and $k_1\cdot k_3 = mE_3$, and with $k_1 = k_2+k_3+k_4$,

$$d\Gamma = \frac{G^2}{8m\pi^5}\left((k_2\cdot k_4)mE_3\right)\frac{d^3k_2}{|\vec{k}_2|}\frac{d^3k_3}{|\vec{k}_3|}\frac{d^3k_4}{|\vec{k}_4|}\delta(m-|\vec{k}_2|-|\vec{k}_3|-|\vec{k}_4|)\delta^3(\vec{k}_2+\vec{k}_3+\vec{k}_4)$$

in spherical coordinates

$$d\Gamma = \frac{mG^2|\vec{k}_3|^2}{8\pi^4}\left(m-2|\vec{k}_3|\right)\frac{sin(\theta)d|\vec{k}_3|d\theta d^3k_4}{\left(|\vec{k}_3|^2+|\vec{k}_4|^2+2|\vec{k}_3||\vec{k}_4|cos(\theta)\right)|\vec{k}_4|}\delta(m-|\vec{k}_3+\vec{k}_4|-|\vec{k}_3|-|\vec{k}_4|)$$

with variable $u^2 = |\vec{k}_3|^2+|\vec{k}_4|^2+2|\vec{k}_3||\vec{k}_4|cos(\theta)$

$$d\Gamma = \frac{mG^2|\vec{k}_3|}{8\pi^4}\left(m-2|\vec{k}_3|\right)\frac{d|\vec{k}_3|d^3k_4}{|\vec{k}_4|^2}\int du\delta(m-u^2-|\vec{k}_3|-|\vec{k}_4|)$$

, and with E=k_4

we get

$$\frac{d\Gamma}{dE} = \frac{mG^2}{2\pi^3}E^2\left(\frac{m}{2} - \frac{2E}{3}\right)$$

$$\Gamma = \frac{m^2G^2}{4\pi^3}\int_0^{\frac{m}{2}} E^2\left(1 - \frac{4E}{3m}\right)dE$$, $$\Gamma = \frac{m^5G^2}{192\pi^3}$$ and the decay time $\tau = \Gamma^{-1}$

where $m = 0.1056584 GeV$

$$\tau = \frac{192\pi^3}{(.1056584\ \mathrm{GeV})^5(1.17\times10^{-5}\ \mathrm{GeV}^{-2})^2} = 3.30\times10^{18}\mathrm{GeV}^{-1}$$, or in seconds, multiplied by $\hbar = 6.58\times10^{-25}$ Gev s,

we get the lifetime $\tau = 2.17\mu s$

1.3 Tauon

The decay modes of the tauon are

$$\tau \to \mu + \overline{\nu}_\mu + \nu_\tau$$

$$\tau \to e + \overline{\nu}_e + \nu_\tau$$

$$\tau \to d + \overline{u} + \nu_\tau \ \text{(3 colors)}$$

$$\tau \to s + \overline{u} + \nu_\tau \ \text{(3 colors)}$$

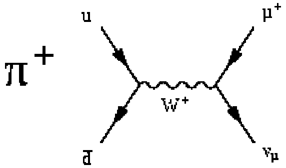
The leptonic modes give a factor 2, the hadronic modes a factor $3|V_{ud}|^2 + 3|V_{us}|^2 = 2.99$,.
and $m_\tau = 16.82\ m_\mu$

$$\mathrm{lifetime}_\tau = \frac{\mathrm{lifetime}_\mu}{4.99(16.82)^5} = 3.23\times10^{-13}\ \mathrm{s}$$

1.4 Pions

Particle name	Particle symbol	Antiparticle symbol	Quark content[11]	Rest mass (MeV/c^2)	I^G	J^{PC}	S	C	B'	Mean lifetime (s)	Commonly decays to (>5% of decays)
Pion[8]	π^+	π^-	$u\bar{d}$	139.570 18 ± 0.000 35	1^-	0^-	0	0	0	$2.6033 \pm 0.0005 \times 10^{-8}$	$\mu^+ + \nu_\mu$
Pion[10]	π^0	Self	$\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$ [a]	134.976 6 ± 0.000 6	1^-	0^{-+}	0	0	0	$8.4 \pm 0.6 \times 10^{-17}$	$\gamma + \gamma$

Charged pion decays



Feynman diagram of the dominant leptonic pion decay[32]
The $\pi^{\pm}$ mesons have a mass of 139.6 MeV/c^2 and a mean lifetime of 2.6033×10^{-8} s. They decay due to the weak interaction. The primary decay mode of a pion, with a branching fraction of 0.999877, is a leptonic decay into a muon and a muon neutrino:

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

The second most common decay mode of a pion, with a branching fraction of 0.000123, is also a leptonic decay into an electron and the corresponding electron antineutrino. This "electronic mode" was discovered at CERN in 1958

$$\pi^+ \rightarrow e^+ + \nu_e$$

$$\pi^- \rightarrow e^- + \bar{\nu}_e$$

The suppression of the electronic decay mode with respect to the muonic one is given approximately (up to a few percent effect of the radiative corrections) by the ratio of the half-widths of the pion–electron and the pion–muon decay reactions:

$$R_\pi = (m_e/m_\mu)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2 = 1.283 \times 10^{-4}$$

and is a spin effect known as helicity suppression.
Also observed, for charged pions only, is the very rare "pion beta decay" (with branching fraction of about 10^{-8}) into a neutral pion, an electron and an electron antineutrino (or for positive pions, a neutral pion, a positron, and electron neutrino).

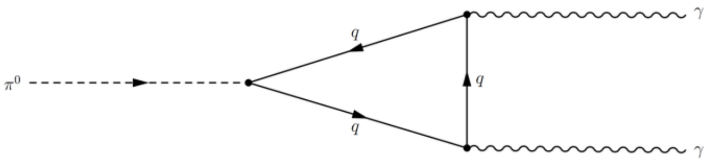
$$\pi^- \rightarrow \pi^0 + e^- + \bar{\nu}_e$$

$$\pi^+ \rightarrow \pi^0 + e^+ + \nu_e$$

The rate at which pions decay is a prominent quantity in many sub-fields of particle physics, such as chiral perturbation theory. This rate is parametrized by the pion decay constant (f_π), related to the wave function overlap of the quark and antiquark, which is about 130 MeV.

Neutral pion decays

The π^0 meson has a mass of 135.0 MeV/c^2 and a mean lifetime of 8.4×10^{-17} s. It decays via the electromagnetic force, which explains why its mean lifetime is much smaller than that of the charged pion (which can only decay via the weak force).



Anomaly-induced neutral pion decay [32]

The dominant π^0 decay mode (anomaly-induced neutral pion decay), with a branching ratio of BR=0.98823, is into two photons:

$\pi^0 \to 2\gamma.$

The second largest π^0 decay mode (BR=0.01174) is the Dalitz decay (named after Richard Dalitz), which is a two-photon decay with an internal photon conversion resulting a photon and an electron-positron pair in the final state:

$\pi^0 \to \gamma + e^- + e^+.$

The third largest established decay mode (BR=3.34×10^{-5}) is the double Dalitz decay, with both photons undergoing internal conversion which leads to further suppression of the rate:

$\pi^0 \to e^- + e^+ + e^- + e^+.$

The fourth largest established decay mode is the loop-induced and therefore suppressed (and additionally helicity-suppressed) leptonic decay mode (BR=6.46×10^{-8}):

$\pi^0 \to e^- + e^+.$

1.5 Pion-nucleon interaction and decays

Lagrangian [11]

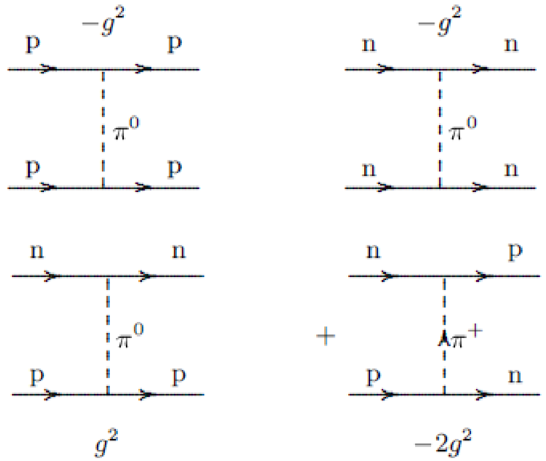
$$\mathcal{L}_{\text{int}} = \mathrm{i}g\,\overline{\psi}(x)\gamma_5\boldsymbol{\tau}\psi(x)\cdot\boldsymbol{\phi}(x)$$

with pion *φ(x)* and nucleon *ψ(x)*

explicitly

$$\begin{aligned}\mathcal{L}_{\text{int}} &= \mathrm{i}g\,(\overline{\psi}_{\mathrm{p}} \quad \overline{\psi}_{\mathrm{n}})\,\gamma_5 \begin{pmatrix}\phi_3 & \phi_1 - \mathrm{i}\phi_2\\ \phi_1 + \mathrm{i}\phi_2 & -\phi_3\end{pmatrix}\begin{pmatrix}\psi_{\mathrm{p}}\\ \psi_{\mathrm{n}}\end{pmatrix}\\ &= \mathrm{i}g\sqrt{2}\,(\overline{\psi}_{\mathrm{p}}\gamma_5\psi_{\mathrm{n}}\varphi + \overline{\psi}_{\mathrm{n}}\gamma_5\psi_{\mathrm{p}}\varphi^\dagger) + \mathrm{i}g(\overline{\psi}_{\mathrm{p}}\gamma_5\psi_{\mathrm{p}} - \overline{\psi}_{\mathrm{n}}\gamma_5\psi_{\mathrm{n}})\phi_3\end{aligned}$$

with Feynman diagrams



with the corresponding hadronic transformations

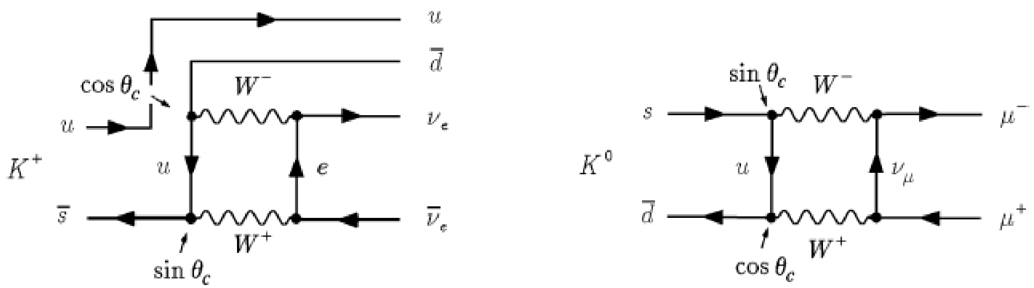
$$\nu\,\mathrm{p} \to \nu\,\mathrm{p}\,\pi^0\,, \quad \nu\,\mathrm{n} \to \nu\,\mathrm{n}\,\pi^0\,,$$
$$\nu\,\mathrm{n} \to \nu\,\mathrm{p}\,\pi^-\,, \quad \nu\,\mathrm{p} \to \nu\,\mathrm{n}\,\pi^+\,.$$

1.6 Kaons

Particle name	Particle symbol	Antiparticle symbol	Quark content	Rest mass (MeV/c^2)	I^G	J^{PC}	S	C	B'	Mean lifetime (s)	Commonly decays to (>5% of decays)
Kaon[1]	K^+	K^-	$u\bar{s}$	493.677 ±0.016	½	0^-	1	0	0	(1.2380 ±0.0021) × 10^{-8}	$\mu^+ + \nu_\mu$ or $\pi^+ + \pi^0$ or $\pi^+ + \pi^+ + \pi^-$ or $\pi^0 + e^+ + \nu_e$
Kaon[2]	K^0	$\bar{K}^0$	$d\bar{s}$	497.611 ±0.013	½	0^-	1	0	0	[a]	[a]
K-Short[3]	K^0_S	Self	$\frac{d\bar{s} - s\bar{d}}{\sqrt{2}}$ [b]	497.611 ±0.013[c]	½	0^-	(*)	0	0	(8.954 ±0.004) × 10^{-11}	$\pi^+ + \pi^-$ or $\pi^0 + \pi^0$
K-Long[4]	K^0_L	Self	$\frac{d\bar{s} + s\bar{d}}{\sqrt{2}}$ [b]	497.611 ±0.013[c]	½	0^-	(*)	0	0	(5.116 ±0.021) × 10^{-8}	$\pi^\pm + e^\mp + \nu_e$ or $\pi^\pm + \mu^\mp + \nu_\mu$ or $\pi^0 + \pi^0 + \pi^0$ or $\pi^+ + \pi^0 + \pi^-$

Main decay modes for K^+ [33]

Results	Mode	Branching ratio
$\mu^+ \nu_\mu$	leptonic	63.55 ± 0.11%
$\pi^+ \pi^0$	hadronic	20.66 ± 0.08%
$\pi^+ \pi^+ \pi^-$	hadronic	5.59 ± 0.04%
$\pi^+ \pi^0 \pi^0$	hadronic	1.761 ± 0.022%
$\pi^0 e^+ \nu_e$	semileptonic	5.07 ± 0.04%
$\pi^0 \mu^+ \nu_\mu$	semileptonic	3.353 ± 0.034%



[35]

Quark diagrams for K_+ and K_0 decays involving strangeness changing neutral currents.

K0 decay and CP-violation

$$K_L = \frac{1}{\sqrt{1+|\bar{\epsilon}|^2}}\left(\frac{K^0+\overline{K}^0}{\sqrt{2}} + \bar{\epsilon}\,\frac{K^0-\overline{K}^0}{\sqrt{2}}\right) \equiv \frac{K_2^0+\bar{\epsilon}K_1^0}{\sqrt{1+|\bar{\epsilon}|^2}}\,,$$

$$K_S = \frac{1}{\sqrt{1+|\bar{\epsilon}|^2}}\left(\frac{K^0-\overline{K}^0}{\sqrt{2}} + \bar{\epsilon}\,\frac{K^0+\overline{K}^0}{\sqrt{2}}\right) \equiv \frac{K_1^0+\bar{\epsilon}K_2^0}{\sqrt{1+|\bar{\epsilon}|^2}}\,.$$

$\bar{\varepsilon} = 2{,}25 * 10^{-3}$

1.7 The kaon-pion decay detailed theory

In [43] a semi-empirical formula for the transition matrix element A_{++} in $\Gamma(K^\pm \to \pi^\pm \pi^+ \pi^-) =$ $K^+(k) \;\; \to \;\; \pi^+(p_1)\pi^+(p_2)\pi^-(p_3)$ is derived.
First, the kinematic momentum variables s_0, s_1, s_2, s_3 are introduced

$$s_1 = (k - p_1)^2 \quad s_2 = (k - p_2)^2 \quad s_3 = (k - p_3)^2 \quad s_0 = \left(m^2 + m_1{}^2 + m_2{}^2 + m_3{}^2\right)/3$$

then, the Dalitz plot variables $x=\frac{(s_2-s_1)}{(m_1{}^2+m_2{}^2)/2}$ $y=\frac{(s_3-s_0)}{m_3{}^2}$

We get for $A_{++}(x,y)$ the expression

$$A_{++-} = (-2\alpha_1+\alpha_3)+\left(-\beta_1+\frac{1}{2}\beta_3-\sqrt{3}\gamma_3\right)y \quad +(-2\zeta_1-2\zeta_3)\left(y^2+\frac{1}{3}x^2\right)+(\xi_1+\xi_3-\xi_3')\left(y^2-\frac{1}{3}x^2\right)$$

with the constants in units 10^{-8} :

α_1	91.71 ± 0.32
α_3	-7.36 ± 0.47
β_1	-25.68 ± 0.27
β_3	-2.43 ± 0.41
γ_3	2.26 ± 0.23
ζ_1	-0.47 ± 0.15
ζ_3	-0.21 ± 0.08
ξ_1	-1.51 ± 0.30
ξ_3	-0.12 ± 0.17
ξ_3'	-0.21 ± 0.51

and $m_2=m_3=m_1$ $s_3=(p_1+p_2)^2=(E_1+E_2)^2-(\vec{p}_1+\vec{p}_2)^2$

We get the following expression for the differential transition width from Fermi's golden rule:

$$d\Gamma=\frac{|M(k,p_1,p_2,p_3)|^2}{2m}\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}(2\pi)^4\delta^4(k-p_1-p_2-p_3)$$ or, with Dalitz variables

$$d\Gamma=\frac{|M(k,p_1,p_2,p_3)|^2}{2m}\frac{d^3p_1}{(2\pi)^3 2\sqrt{m_1{}^2+p_1{}^2}}\frac{d^3p_2}{(2\pi)^3 2\sqrt{m_2{}^2+p_2{}^2}}\frac{d^3p_3}{(2\pi)^3 2\sqrt{m_3{}^2+p_3{}^2}}(2\pi)^4\delta^4(k-p_1-p_2-p_3)$$

we choose $\vec{k}=0$ $k^0=m$, i.e. $\vec{p}_3=-(\vec{p}_1+\vec{p}_2)$

$$s_1=(k-p_1)^2=(m-E_1)^2-\vec{p}_1{}^2=(m-E_1)^2-(E_1{}^2-m_1{}^2)=m^2+m_1{}^2-2mE_1$$

$$s_2=(m-E_2)^2-\vec{p}_2{}^2=m^2+m_1{}^2-2mE_2$$

$$\vec{p}_1{}^2=E_1{}^2-m_1{}^2 \quad \vec{p}_2{}^2=E_2{}^2-m_1{}^2$$

$$\vec{p}_3{}^2=\vec{p}_1{}^2+\vec{p}_2{}^2+2\vec{p}_1\vec{p}_2\cos\theta_{12}=E_1{}^2+E_2{}^2-2m_1{}^2+2\sqrt{E_1{}^2-m_1{}^2}\sqrt{E_2{}^2-m_1{}^2}\cos\theta_{12} \quad m=E_1+E_2+E_3$$

$$s_3=(p_1+p_2)^2=(m-(E_1+E_2))^2-(\vec{p}_1+\vec{p}_2)^2=(m-(E_1+E_2))^2-(E_1{}^2+E_2{}^2-2m_1{}^2+2\sqrt{E_1{}^2-m_1{}^2}\sqrt{E_2{}^2-m_1{}^2}\cos\theta_{12})$$

$$s_3=(p_1+p_2)^2=2E_1E_2-2m(E_1+E_2)+m^2+2m_1{}^2-2\sqrt{E_1{}^2-m_1{}^2}\sqrt{E_2{}^2-m_1{}^2}\cos\theta_{12}$$

$$E_1=\sqrt{m_1{}^2+\vec{p}_1{}^2} \quad E_2=\sqrt{m_1{}^2+\vec{p}_2{}^2} \quad E_3=\sqrt{m_1{}^2+(\vec{p}_1+\vec{p}_2)^2}=\sqrt{E_1{}^2+E_2{}^2-m_1{}^2+2\sqrt{E_1{}^2-m_1{}^2}\sqrt{E_2{}^2-m_1{}^2}\cos\theta_{12}}$$

$$x = \frac{(s_2 - s_1)}{m_1^{\ 2}} = \frac{2m(E_1 - E_2)}{m_1^{\ 2}}$$

$$y = \frac{(s_3 - s_0)}{m_1^{\ 2}} = \frac{2E_1E_2 - 2m(E_1 + E_2) + 2m^2/3 + m_1^{\ 2} - 2\sqrt{E_1^{\ 2} - m_1^{\ 2}}\sqrt{E_2^{\ 2} - m_1^{\ 2}}\cos\theta_{12}}{m_1^{\ 2}}$$

We insert $\left|M(k, p_1, p_2, p_3)\right|^2 = \frac{1}{8\pi}\left|A_{++-}(x, y)\right|^2$

and calculate Γ as an integral over $|p_1| = |\vec{p}_1|$, $|p_2| = |\vec{p}_2|$ θ_{12}, integration $d^3p_3\delta^3(\vec{p}_3 + \vec{p}_1 + \vec{p}_2)$ cancels out

$$d\Gamma = \frac{\left|A_{++-}(x,y)\right|^2}{2m}\frac{4\pi(E_1^{\ 2} - m_1^{\ 2})d|p_1|}{(2\pi)^3 2E_1}\frac{2\pi(E_2^{\ 2} - m_1^{\ 2})d|p_2|}{(2\pi)^3 2E_2}\frac{1}{(2\pi)^3 2E_3}(2\pi)^4\delta(m - E_1 - E_2 - E_3)$$

and changing to E_1, E_2, θ_{12}: $d|p_1| = \frac{E_1 dE_1}{\sqrt{E_1^{\ 2} - m_1^{\ 2}}}$ $d|p_2| = \frac{E_2 dE_2}{\sqrt{E_2^{\ 2} - m_1^{\ 2}}}$ $|p_1| = \sqrt{E_1^{\ 2} - m_1^{\ 2}}$ $|p_2| = \sqrt{E_2^{\ 2} - m_1^{\ 2}}$

$$d\Gamma = \frac{\left|A_{++-}(x,y)\right|^2}{8m(2\pi)^3}\sqrt{E_1^{\ 2} - m_1^{\ 2}}dE_1\sqrt{E_2^{\ 2} - m_1^{\ 2}}dE_2\frac{\sin\theta_{12}d\theta_{12}}{\sqrt{E_1^{\ 2} + E_2^{\ 2} - m_1^{\ 2} + 2\sqrt{E_1^{\ 2} - m_1^{\ 2}}\sqrt{E_2^{\ 2} - m_1^{\ 2}}\cos\theta_{12}}}\delta(m - E_1 - E_2 - \sqrt{E_1^{\ 2} + E_2^{\ 2} - m_1^{\ 2} + 2\sqrt{E_1^{\ 2} - m_1^{\ 2}}\sqrt{E_2^{\ 2} - m_1^{\ 2}}\cos\theta_{12}})$$

we solve $\delta()$ for E_2:

E_2=

$$\frac{-2\,\mathrm{E1}^2\,\mathrm{m} - \mathrm{m}^3 - \mathrm{m}\,\mathrm{m1}^2 + \mathrm{E1}\,(3\,\mathrm{m}^2 + \mathrm{m1}^2) + \sqrt{(\mathrm{E1}^2 - \mathrm{m1}^2)\,\mathrm{Cos}[\theta 12]^2\,((\mathrm{m}^2 - \mathrm{m1}^2)\,(4\,\mathrm{E1}^2 - 4\,\mathrm{E1}\,\mathrm{m} + \mathrm{m}^2 - \mathrm{m1}^2) + 4\,\mathrm{m1}^2\,(\mathrm{E1}^2 - \mathrm{m1}^2)\,\mathrm{Cos}[\theta 12]^2)}}{-2\,(\mathrm{E1} - \mathrm{m})^2 + 2\,(\mathrm{E1}^2 - \mathrm{m1}^2)\,\mathrm{Cos}[\theta 12]^2}$$

and $m - E_1 - E_2 = \sqrt{E_1^{\ 2} + E_2^{\ 2} - m_1^{\ 2} + 2\sqrt{E_1^{\ 2} - m_1^{\ 2}}\sqrt{E_2^{\ 2} - m_1^{\ 2}}\cos\theta_{12}}$

$$\frac{(m - (E_1 + E_2))^2 - E_1^{\ 2} - E_2^{\ 2} + m_1^{\ 2}}{2\sqrt{E_1^{\ 2} - m_1^{\ 2}}\cos\theta_{12}} = \sqrt{E_2^{\ 2} - m_1^{\ 2}}$$

now we carry out the integration over E_2 with the delta-function:

$$\Gamma = \int\frac{\left|A_{++-}(x,y)\right|^2}{8m(2\pi)^3}dE_1\frac{(m - (E_1 + E_2))^2 - E_1^{\ 2} - E_2^{\ 2} + m_1^{\ 2}}{2\cos\theta_{12}}\frac{\sin\theta_{12}d\theta_{12}}{m - (E_1 + E_2)}$$

and after simplification

$$\Gamma = \int\frac{\left|A_{++-}(x,y)\right|^2}{8m(2\pi)^3}\frac{\sqrt{E_2^{\ 2} - m_1^{\ 2}}\sqrt{E_1^{\ 2} - m_1^{\ 2}}}{(m - (E_1 + E_2))}\sin\theta_{12}\,d\theta_{12}dE_1$$

The integration boundary in E_1 is $m_1 \le E_1 \le (1 + e1b1(m, m_1, \theta_{12}))\,m_1$, in θ_{12} $0 \le \theta_{12} \le \pi$

where $e1b1(m,m_1,\theta_{12})=\frac{|p_1|}{m_1}$ is the relative momentum, at which E_2 becomes complex

$$\frac{2 m^3 m1 - 4 m^2 m1^2 - 2 m\, m1^3 + 4 m1^4 - 4 m1^4 \mathrm{Cos}[th12]^2 - \sqrt{2}\sqrt{m^4 m1^4 + 2 m^2 m1^6 - m^4 m1^4 \mathrm{Cos}[2\,th12] + 6 m^2 m1^6 \mathrm{Cos}[2\,th12] - m1^8 \mathrm{Cos}[2\,th12] + m1^8 \mathrm{Cos}[4\,th12]}}{4\left(m^2 m1^2 - m1^4 + m1^4 \mathrm{Cos}[th12]^2\right)}$$

Numerical integration yields for $m_1=m(\pi^+)=0.139GeV$, $m=m(K^+)=0.493GeV$ [44]
$\Gamma(m_1,m)$ = $0.033\ 10^{-16}GeV$, the measured decay width is $0.0297\ 10^{-16}GeV$ (see below).

1.8 The general 3-body decay

We use the momentum denominations $\Gamma(P_0(k,m)\to P(p_1,m_1)P(p_2,m_2)P(p_3,m_3)$

We start again with Fermi's golden rule:

$$d\Gamma=\frac{|M(k,p_1,p_2,p_3)|^2}{2m}\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}(2\pi)^4\delta^4(k-p_1-p_2-p_3)$$

we choose $\vec{k}=0\quad k^0=m$, i.e. $\vec{p}_3=-(\vec{p}_1+\vec{p}_2)$

$$\vec{p}_1^{\,2}=E_1^{\,2}-m_1^{\,2}\quad \vec{p}_2^{\,2}=E_2^{\,2}-m_2^{\,2}$$

$$\vec{p}_3^{\,2}=\vec{p}_1^{\,2}+\vec{p}_2^{\,2}+2\vec{p}_1\vec{p}_2\cos\theta_{12}=E_1^{\,2}+E_2^{\,2}-m_1^{\,2}-m_2^{\,2}+2\sqrt{E_1^{\,2}-m_1^{\,2}}\sqrt{E_2^{\,2}-m_2^{\,2}}\cos\theta_{12}\qquad m=E_1+E_2+E_3$$

$$E_1=\sqrt{m_1^{\,2}+\vec{p}_1^{\,2}}\quad E_2=\sqrt{m_2^{\,2}+\vec{p}_2^{\,2}}\quad E_3=\sqrt{m_3^{\,2}+(\vec{p}_1+\vec{p}_2)^2}=\sqrt{E_1^{\,2}+E_2^{\,2}+m_3^{\,2}-m_1^{\,2}-m_2^{\,2}+2\sqrt{E_1^{\,2}-m_1^{\,2}}\sqrt{E_2^{\,2}-m_2^{\,2}}\cos\theta_{12}}$$

now we calculate Γ as an integral over $|p_1|=|\vec{p}_1|$, $|p_2|=|\vec{p}_2|$ θ_{12}, integration $d^3p_3\delta^3(\vec{p}_3+\vec{p}_1+\vec{p}_2)$ cancels out

$$d\Gamma=\frac{|M(k,p_1,p_2,p_3)|^2}{2m}\frac{4\pi(E_1^{\,2}-m_1^{\,2})d|p_1|}{(2\pi)^3 2E_1}\frac{2\pi(E_2^{\,2}-m_2^{\,2})d|p_2|}{(2\pi)^3 2E_2}\frac{1}{(2\pi)^3 2E_3}(2\pi)^4\delta(m-E_1-E_2-E_3)$$

and changing to E_1, E_2, θ_{12}: $d|p_1|=\frac{E_1dE_1}{\sqrt{E_1^{\,2}-m_1^{\,2}}}\quad d|p_2|=\frac{E_2dE_2}{\sqrt{E_2^{\,2}-m_2^{\,2}}}\quad |p_1|=\sqrt{E_1^{\,2}-m_1^{\,2}}\quad |p_2|=\sqrt{E_2^{\,2}-m_2^{\,2}}$

$$d\Gamma=\frac{|M(k,p_1,p_2,p_3)|^2}{8m(2\pi)^3}\sqrt{E_1^{\,2}-m_1^{\,2}}dE_1\sqrt{E_2^{\,2}-m_2^{\,2}}dE_2\frac{\sin\theta_{12}d\theta_{12}}{\sqrt{E_1^{\,2}+E_2^{\,2}+m_3^{\,2}-m_1^{\,2}-m_2^{\,2}+2\sqrt{E_1^{\,2}-m_1^{\,2}}\sqrt{E_2^{\,2}-m_2^{\,2}}\cos\theta_{12}}}\delta(m-E_1-E_2-\sqrt{E_1^{\,2}+E_2^{\,2}+m_3^{\,2}-m_1^{\,2}-m_2^{\,2}+2\sqrt{E_1^{\,2}-m_1^{\,2}}\sqrt{E_2^{\,2}-m_2^{\,2}}\cos\theta_{12}})$$

we solve $\delta()$ for E_2:

E_2=

$$\frac{1}{-2(E1-m)^2+2(E1^2-m1^2)\mathrm{Cos}[th12]^2}\Big(-2E1^2 m+3E1\,m^2-m^3+E1\,m1^2-m\,m1^2+E1\,m2^2-m\,m2^2-E1\,m3^2+m\,m3^2+\sqrt{\big((E1^2-m1^2)\mathrm{Cos}[th12]^2}$$
$$\left(m^4+2m^2 m1^2+m1^4-2m^2 m2^2+m2^4+E1^2(4m^2-2m2^2)-2m^2 m3^2-2m1^2 m3^2-2m2^2 m3^2+m3^4-4E1\,m(m^2+m1^2-m2^2-m3^2)+2(E1^2-m1^2)m2^2\mathrm{Cos}[2\,th12]\right)\big)\Big)$$

and $m - E_1 - E_2 = \sqrt{E_1^{\,2} + E_2^{\,2} + m_3^{\,2} - m_1^{\,2} - m_2^{\,2} + 2\sqrt{E_1^{\,2} - m_1^{\,2}}\sqrt{E_2^{\,2} - m_2^{\,2}}\cos\theta_{12}}$

$$\frac{(m-(E_1+E_2))^2 - E_1^{\,2} - E_2^{\,2} - m_3^{\,2} + m_1^{\,2} + m_2^{\,2}}{2\sqrt{E_1^{\,2} - m_1^{\,2}}\cos\theta_{12}} = \sqrt{E_2^{\,2} - m_2^{\,2}}$$

now we carry out the integration over E_2 with the delta-function:

$$\Gamma = \int \frac{|M(k,p_1,p_2,p_3)|^2}{8m(2\pi)^3} dE_1 \frac{(m-(E_1+E_2))^2 - E_1^{\,2} - E_2^{\,2} - m_3^{\,2} + m_1^{\,2} + m_2^{\,2}}{2\cos\theta_{12}} \frac{\sin\theta_{12} d\theta_{12}}{m-(E_1+E_2)}$$

and after simplification

$$\Gamma = \int \frac{|M(k,p_1,p_2,p_3)|^2}{8m(2\pi)^3} \frac{\sqrt{E_2^{\,2} - m_1^{\,2}}\sqrt{E_1^{\,2} - m_2^{\,2}}}{(m-(E_1+E_2))} \sin\theta_{12}\, d\theta_{12} dE_1$$

setting $|M(k,p_1,p_2,p_3)| = 1$ we get the *partial kinematic factor* for 3-body decay $I_{\Gamma 3}(\frac{m_1}{m},\frac{m_2}{m},\frac{m_3}{m})$

$$\Gamma = \int \frac{1}{8m(2\pi)^3} \frac{\sqrt{E_2^{\,2} - m_1^{\,2}}\sqrt{E_1^{\,2} - m_2^{\,2}}}{(m-(E_1+E_2))} \sin\theta_{12}\, d\theta_{12} dE_1 = m\, I_{\Gamma 3}(m,m_1,m_2,m_3)$$

The integration boundary in E_1 is $m_1 \le E_1 \le (1 + e1b1(m,m_1,m_2,m_3,\theta_{12}))\, m_1$, in θ_{12} $0 \le \theta_{12} \le \pi$

where $e1b1(m,m_1,m_2,m_3,\theta_{12}) = \dfrac{|p_1|}{m_1}$ is the relative momentum, at which E_2 becomes complex

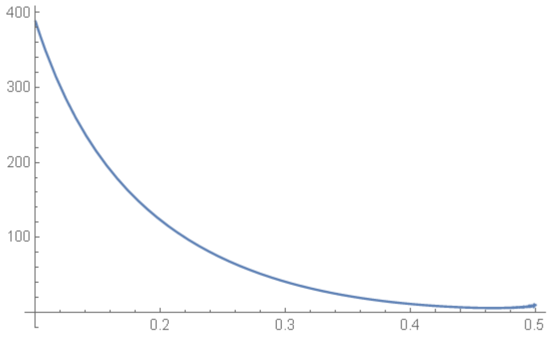
```
(4 m^3 m1 - 8 m^2 m1^2 + 4 m m1^3 - 4 m m1 m2^2 + 4 m1^2 m2^2 - 4 m m1 m3^2 - 4 m1^2 m2^2 Cos[2 th12] -
   √(-4 (m^4 - 4 m^3 m1 + 6 m^2 m1^2 - 4 m m1^3 + m1^4 - 2 m^2 m2^2 + 4 m m1 m2^2 - 2 m1^2 m2^2 + m2^4 - 2 m^2 m3^2 + 4 m m1 m3^2 - 2 m1^2 m3^2 - 2 m2^2 m3^2 + m3^4)
        (4 m^2 m1^2 - 2 m1^2 m2^2 + 2 m1^2 m2^2 Cos[2 th12]) +
       (-4 m^3 m1 + 8 m^2 m1^2 - 4 m m1^3 + 4 m m1 m2^2 - 4 m1^2 m2^2 + 4 m m1 m3^2 + 4 m1^2 m2^2 Cos[2 th12])^2)) / (2 (4 m^2 m1^2 - 2 m1^2 m2^2 + 2 m1^2 m2^2 Cos[2 th12]))
```

The kinematic factor $I_{\Gamma 3}(m,m_1,m_2,m_3)$ can be calculated numerically.

The *total kinematic factor* results from $I_{\Gamma 3}(m,m_1,m_2,m_3)$ by symmetrization over all 6 index permutations

$$I_{\Gamma 3s}(m,m_1,m_2,m_3) = (I_{\Gamma 3}(m,m_1,m_2,m_3) + I_{\Gamma 3}(m,m_1,m_3,m_2) + ..)/6$$

Here is the plot of $I_{\Gamma 3s}(1,m_1,0.1,0.1)10^6$



Example: $\Gamma(\mu \to e\bar{\nu}_e \nu_\mu)$ with kinematic factor: $I_{\Gamma 3}(m(\mu), m(e), 0, 0)$ =0.3835

1.9 The general 2-body decay

We use the momentum denominations $\Gamma(P_0(k,m) \to P(p_1,m_1)\,P(p_2,m_2)$

We start again with Fermi's golden rule for 2-body decay [45]:

$$d\Gamma = \frac{|M(k,p_1,p_2)|^2}{2m}\frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(k-p_1-p_2)$$

we choose $\vec{k}=0$ $k^0=m$, i.e. $\vec{p}_2=-\vec{p}_1$

$\vec{p}_1^{\,2}=E_1^{\,2}-m_1^{\,2}$ $\vec{p}_2^{\,2}=E_2^{\,2}-m_2^{\,2}$

$m=E_1+E_2$

$E_1=\sqrt{m_1^{\,2}+\vec{p}_1^{\,2}}$, $E_2=\sqrt{m_2^{\,2}+\vec{p}_1^{\,2}}=\sqrt{E_1^{\,2}+m_2^{\,2}-m_1^{\,2}}=m-E_1$, $\delta^4(k-p_1-p_2)=\delta(m-E_1-E_2)\delta^3(\vec{p}_1+\vec{p}_2)$

now we calculate Γ as an integral over $|p_1|=|\vec{p}_1|$, integration $d^3p_2\delta^3(\vec{p}_1+\vec{p}_2)$ cancels out

and changing to E_1: $d|p_1|=\frac{E_1 dE_1}{\sqrt{E_1^{\,2}-m_1^{\,2}}}$ $|p_1|=\sqrt{E_1^{\,2}-m_1^{\,2}}$

$$d\Gamma = \frac{|M(k,p_1,p_2)|^2}{8m(2\pi)^2}\frac{4\pi\sqrt{(E_1^{\,2}-m_1^{\,2})}dE_1}{\sqrt{E_1^{\,2}+m_2^{\,2}-m_1^{\,2}}}\delta(m-E_1-E_2)$$

from $\sqrt{E_1^{\,2}+m_2^{\,2}-m_1^{\,2}}=m-E_1$ we get the solution for E_{10}= $\frac{m^2+m1^2-m2^2}{2\,m}$, $\sqrt{(E_{10}^{\,2}-m_1^{\,2})}=\sqrt{m^4-2\,m^2\,m1^2+m1^4-2\,m^2\,m2^2-2\,m1^2\,m2^2+m2^4}$ /(2m)

$\sqrt{(E_{10}^{\,2}-m_1^{\,2}+m_2^{\,2})}=\sqrt{m^4-2\,m^2\,m1^2+m1^4+2\,m^2\,m2^2-2\,m1^2\,m2^2+m2^4}$ /(2m)

the integration $dE_1\delta(m-E_1-E_2)$ cancels out and we get setting $|M(k,p_1,p_2,p_3)|=1$

$$\Gamma = \frac{1}{4m(2\pi)} \frac{\sqrt{E_{10}{}^2 - m_1{}^2}}{\sqrt{E_{10}{}^2 + m_2{}^2 - m_1{}^2}} = \frac{1}{8m\pi} \frac{\sqrt{m^4 + m_1{}^4 + m_2{}^4 - 2m^2 m_1{}^2 - 2m^2 m_2{}^2 - 2m_1{}^2 m_2{}^2}}{\sqrt{m^4 + m_1{}^4 + m_2{}^4 - 2m^2 m_1{}^2 + 2m^2 m_2{}^2 - 2m_1{}^2 m_2{}^2}}$$

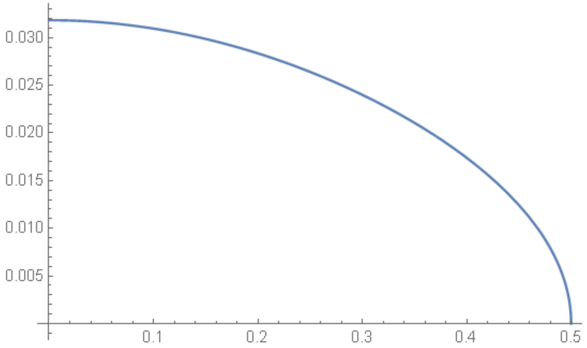
we get the *kinematic factor* for 2-body decay $I_{\Gamma 2}(m, m_1)$

$$I_{\Gamma 2}(m, m_1, m_2) = m\Gamma = \frac{1}{8\pi} \frac{\sqrt{m^4 + m_1{}^4 + m_2{}^4 - 2m^2 m_1{}^2 - 2m^2 m_2{}^2 - 2m_1{}^2 m_2{}^2}}{\sqrt{m^4 + m_1{}^4 + m_2{}^4 - 2m^2 m_1{}^2 + 2m^2 m_2{}^2 - 2m_1{}^2 m_2{}^2}}$$

The *total kinematic factor* for 2-body decay results from the symmetrized $I_{\Gamma 2}(m, m_1)$

$$I_{\Gamma 2s}(m, m_1, m_2) = \frac{I_{\Gamma 2}(m, m_1) + I_{\Gamma 2}(m, m_2)}{2}$$

As an example, here is the plot $I_{\Gamma 2s}(1, m_1, 0.5)$



Example: $\Gamma(\pi \to \mu\nu)$, with kinematic factor : $I_{\Gamma 2s}(m(\pi), m(\mu), 0)$ =0.0251

2 The theoretical background and the phenomenological decay formula

2.1 The phenomenological decay formula

The phenomenological formula for the decay width is [34]

$\Gamma = \tilde{G}^2 m_i^{\,k} \left|P_l^{\,m}(x)\right|^2 = \dfrac{G^2}{C_1} m_i^{\,k} \left|P_l^{\,m}(x)\right|^2$, where $P_l^{\,m}(x)$ Legendre polynomial $m=l$ or $m=l+1$, l=isospin I , $x = \dfrac{m_f}{m_i}$ mass ratio, $C_1 = 4\pi N$ or $C_1 = N$, where N is an integer or a simple fraction, and m_i is the initial mass , $\tilde{G} = \dfrac{G}{\sqrt{C_1}}$ is the *interaction constant* .

The constants are: for K $g_1^{\,2} = 2.06*10^{-14}$, for π $g_0^{\,2} = 2.18*10^{-14}$, for leptonic $A' \to Ae^-\bar{\nu}_e (\Delta S = 0)$,

Fermi-constant $G_F = G_0 = 1.02*10^{-5}\left(\dfrac{1}{m_p^{\,2}}\right) = 1.16*10^{-5} GeV^{-2}$, for hyperons $g_h = 6.2*10^{-7}\left(\dfrac{m_p}{m_i}\right)$

The power $k=1$ for a dimensionless $\tilde{G}$, like in pion decay $\Gamma(\pi \to \mu\ \nu_\mu) = \tilde{G}^2 m_i\, x^2\left(1-x^2\right)^2 = \dfrac{G^2 m_i}{4\pi} x^2\left(1-x^2\right)^2$, $G^2 = g_0^{\,2} = 2.18*10^{-14}$ or

power $k=5$ for a dimensional $\tilde{G} = const * G_F$, $\left[\tilde{G}\right] = GeV^{-2}$, like in muon decay $\Gamma(\mu \to e\,\nu_e\,\nu_\mu) = \tilde{G}^2 m_i^{\,5}\left(1-x^2\right)^4 = \dfrac{G_F^{\,2} m_i^{\,5}}{192\pi^3}\left(1-x^2\right)^4$

power k=3 for a dimensional $\tilde{G}$, $\left[\tilde{G}\right] = GeV^{-1}$, like in π^0 decay $\Gamma\left(\pi^0 \to \gamma\,\gamma\right) = \tilde{G}^2 m_i^{\,3}\left(1-x^2\right)^3$

The *extended isospin* I includes higher generation quarks, I(s)= I(c)=1/2 and $I(l) = 1$ for leptons l as well as $I(\gamma) = 1$ for photon.

The extended isospin has the following values:

$I(\Lambda) = 1/2$

$I(\Sigma) = 1/2$

$I(\Xi) = 1/2$

$I(K) = 1$

$I(\gamma) = 1$

$I(\pi) = 1$

$I(l) = 1$ for lepton l

$I(p) = 1/2 \quad I(n) = 1/2$

but $I(ud) = I(dd) = I(uu) = 1$

$I(\eta) = 1$

The angular momentum in decay width: $l = |\Delta I| = |I_i \pm I_f|$ is the difference or sum of the initial and final isospin.

The *interaction energy* m_X is the (excitation) energy of the mediating virtual exchange boson (for pure weak decays: W or Z-boson).

The decay width with the matrix element $M = \frac{g^2}{8{m_X}^2}$ and the kinematic I_Γ factor is

$$\Gamma = |M|^2 I_\Gamma m_i = \left(\frac{g^2 {m_i}^2}{8{m_X}^2}\right)^2 I_\Gamma m_i \text{ or in general } \frac{m_X}{m_i} = f_I \left(\frac{m_i}{\Gamma}\right)^{1/4}, \text{ where } f_I = \left(\frac{{I_\Gamma}^{1/4} g}{2\sqrt{2}}\right)$$

From this formula we can derive a general formula for the interaction energy m_X setting *g=1*

for k=1 $\left(\frac{{m_i}^4}{{m_X}^4}\right) = \frac{64\tilde{G}^2}{I_\Gamma}|P_l^m(x)|^2$ with the phenomenological formula $\Gamma = \tilde{G}^2 m_i |P_l^m(x)|^2$

for k=5 $\left(\frac{1}{{m_X}^4}\right) = \frac{64\tilde{G}^2}{I_\Gamma}|P_l^m(x)|^2$

for k=3 $\left(\frac{1}{{m_X}^2}\right) = \frac{64\tilde{G}^2}{I_\Gamma}|P_l^m(x)|^2$

2.2 Derivation of angular momentum dependence in the phenomenological formula

Static Schrödinger equation in momentum representation reads [34]

$$\frac{1}{p^2}[\frac{\partial}{\partial p}(p^2\frac{\partial\psi}{\partial p}) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial\psi}{\partial\theta}) + \frac{1}{\sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2}] + 2\tau T\psi = 0\,.$$

solution $\psi = R(p)\Theta(\theta)\Phi(\varphi)$

for rigid rotator

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\frac{\partial\psi}{\partial\theta}) + \frac{1}{\sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2} + 8\pi^2 IT\psi = 0$$

with kinetic energy $T = \frac{1}{8\pi^2 I}l(l+1)$

where *l* is the angular momentum

solution $\psi_{l,m} = N_{lm}P_l^{|m|}(\cos\theta)e^{im\varphi}$ $\quad |\psi_{l,m}|^2 = \frac{1}{4\pi}\frac{(l-|m|)!(2l+1)}{(l+|m|)!}|P_l^{|m|}(\cos\theta)|^2$

and decay rate $\Gamma = A|\psi_{l,m}|^2 = \frac{A}{4\pi}\frac{(l-|m|)!(2l+1)}{(l+|m|)!}|P_l^{|m|}(\cos\theta)|^2$

with $x = \cos\theta = p_z/\vec{p}$ and $x = \frac{\sum m_f}{m_i}$, *m=l* or *m=l-1*

Legendre functions $P_\lambda^\mu(z) = \frac{1}{\Gamma(1-\mu)} \left[\frac{1+z}{1-z}\right]^{\mu/2} {}_2F_1\left(-\lambda, \lambda+1; 1-\mu; \frac{1-z}{2}\right)$

with hypergeometric function ${}_2F_1$

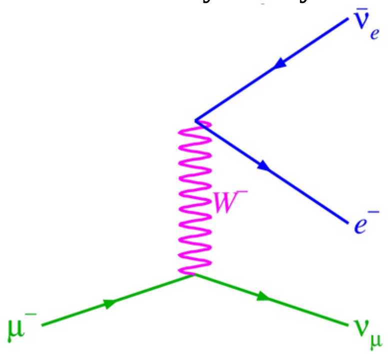
associated Legendre polynomials $P_l^m(x) = (-1)^m \cdot 2^l \cdot (1-x^2)^{m/2} \cdot \sum_{k=m}^{l} \frac{k!}{(k-m)!} \cdot x^{k-m} \cdot \binom{l}{k} \binom{\frac{l+k-1}{2}}{l}$

$P_{\ell+1}^{\ell}(x) = x(2\ell+1)P_\ell^\ell(x)$ $\quad P_\ell^\ell(x) = (-1)^\ell (2\ell-1)!!(1-x^2)^{(\ell/2)}$ $\quad P_{\ell+1}^{\ell+1}(x) = -(2\ell+1)\sqrt{1-x^2}P_\ell^\ell(x)$

$$P_0^0(x) = 1$$
$$P_1^{-1}(x) = -\tfrac{1}{2} P_1^1(x)$$
$$P_1^0(x) = x$$
$$P_1^1(x) = -(1-x^2)^{1/2}$$
$$P_2^0(x) = \tfrac{1}{2}(3x^2-1)$$
$$P_2^1(x) = -3x(1-x^2)^{1/2}$$
$$P_2^2(x) = 3(1-x^2)$$
$$P_3^0(x) = \tfrac{1}{2}(5x^3-3x)$$
$$P_3^1(x) = -\tfrac{3}{2}(5x^2-1)(1-x^2)^{1/2}$$
$$P_3^2(x) = 15x(1-x^2)$$
$$P_3^3(x) = -15(1-x^2)^{3/2}$$
$$P_4^0(x) = \tfrac{1}{8}(35x^4-30x^2+3)$$
$$P_4^1(x) = -\tfrac{5}{2}(7x^3-3x)(1-x^2)^{1/2}$$
$$P_4^2(x) = \tfrac{15}{2}(7x^2-1)(1-x^2)$$
$$P_4^3(x) = -105x(1-x^2)^{3/2}$$
$$P_4^4(x) = 105(1-x^2)^2$$

and for $u = x^2$ $\quad \Gamma = Cu^{\alpha-1}(1-u)^l$; α=1,2

2.3 Muon decay theory



The analytical formula from the Feynman diagram is

$\Gamma(\mu \to e\,\nu_e\,\nu_\mu) = \frac{G_F{}^2 m_\mu{}^5}{192\pi^3}$ [11] , G_F Fermi-constant

exact formula with corrections [35]

$$\Gamma(\mu \to e\,\nu_e\,\nu_\mu) = \frac{G_\mu^2 m_\mu^5}{192\pi^3} f\left(\frac{m_e^2}{m_\mu^2}\right)(1+\text{R.C.})\left(1+\frac{3}{5}\frac{m_\mu^2}{m_W^2}+\ldots\right)$$

$$\begin{aligned}\text{R.C.} &= \frac{\alpha}{2\pi}\left(\frac{25}{4}-\pi^2\right)\left[1+\frac{\alpha}{\pi}\left(\frac{2}{3}\log\frac{m_\mu}{m_e}-3.7\right)\right.\\ &+ \left.\left(\frac{\alpha}{\pi}\right)^2\left(\frac{4}{9}\log^2\frac{m_\mu}{m_e}-2.0\log\frac{m_\mu}{m_e}+C\right)+\ldots\right]\end{aligned}$$

and the phenomenological formula

$$\Gamma(\mu \to e\,\nu_e\,\nu_\mu) = \frac{G_F^{\,2} m_\mu^{\,5}}{192\pi^3}\left(1-\frac{m_e^{\,2}}{m_\mu^{\,2}}\right)^4 = \frac{G_F^{\,2} m_\mu^{\,5}}{192\pi^3}\left(1-x^2\right)^4$$ [34, *l*=4]

This is the general formula for a *leptonic weak 3-body decay*, setting initial mass $m_i = m_\mu$.

The (charged) weak interaction in the Feynman-Gell-Mann form reads

$$\mathcal{H}_{\text{weak}} = \frac{G_F}{\sqrt{2}} J'^\mu J'^\dagger_\mu \,.$$

[11,(9.1)] , where J'_μ is the charged leptonic-hadronic current

$$J'_\mu = L_\mu + H_\mu$$ and

$L_\mu(x) = 2\,\overline{e}_L(x)\gamma_\mu \nu_{eL}(x)$ +... $= \overline{e}(x)\gamma_\mu(1-\gamma_5)\nu_e(x)$ +... is the leptonic current, H_μ is the analogous hadronic current.

In the standard model, the (charged) weak interaction is mediated by the massive W-boson W_μ with mass M_W for the charged current, with the Lagrangian

$$\mathcal{L}_{\text{weak}} = -\frac{g}{2\sqrt{2}}(J'^\mu W_\mu^\dagger + J'^{\mu\dagger} W_\mu)$$

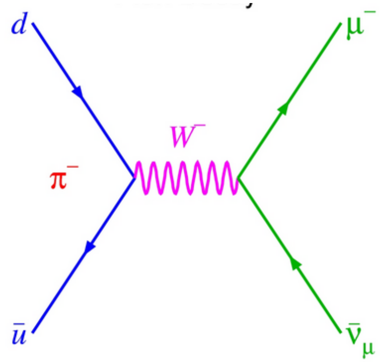
, where $G_F/\sqrt{2} = g^2/(8M_W^2)$

We can use the total current and use an *excited intermediate W-boson*, which includes the hadronic part, with the total mass $m_X > M_W$ and calculate it from the effective measured coupling constant G instead of G_F , setting $g=1$:

$$G^2 = \frac{1}{32 m_X^{\,4}}$$

The isospin numbers are $l = \Delta I = I_f + I_i = 3 + 1 = 4$ and $m = l = 4$

2.4 Pion decay theory



The analytical formula from the Feynman diagram is

$$\Gamma(\pi \to \mu\ \nu_\mu) = \frac{G_F^{\,2} m_\pi}{8\pi} f_\pi^{\,2} V_{ud}^{\,2} m_\mu^{\,2} \left(1 - \frac{m_\mu^{\,2}}{m_\pi^{\,2}}\right)^2 \approx \frac{G_F^{\,2} m_\pi^{\,5}}{8\pi} \frac{m_\mu^{\,2}}{m_\pi^{\,2}} \left(1 - \frac{m_\mu^{\,2}}{m_\pi^{\,2}}\right)^2$$ [11, (13.26)] , G_F Fermi-constant

and the phenomenological formula

$$\Gamma(\pi \to \mu\ \nu_\mu) = \frac{G^2 m_\pi}{4\pi} \frac{m_\mu^{\,2}}{m_\pi^{\,2}} \left(1 - \frac{m_\mu^{\,2}}{m_\pi^{\,2}}\right)^2 = \frac{G^2 m_\pi}{4\pi} x^2 \left(1 - x^2\right)^2$$ [34]

The (charged) weak interaction has the form

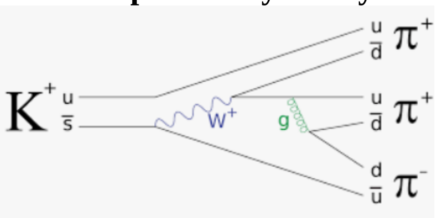
$H_{weak} = \frac{G_F}{\sqrt{2}} J^\lambda(\mu, \nu_\mu) J_\lambda(u, d)$ with the leptonic current $J^\lambda(\mu, \nu_\mu) = \bar{\mu}\gamma^\lambda(1 - \gamma_5)\nu$ and the hadronic current $J^\lambda(u, d) = \bar{u}\gamma^\lambda d$ for $\pi^- = \bar{u}d$ [11, (13.6)]

Using the same procedure as above with the excited intermediate W-boson , we calculate M_X from the above two formulas:

$\frac{G_F^{\,2}}{8\pi}\left(\frac{m_\pi}{m_X}\right)^4 = \frac{G^2}{4\pi}$ and $\frac{G_F}{\sqrt{2}} = \frac{g_F^{\,2}}{8 M_X^{\,2}}$ and setting $g_F=1$ and initial mass $m_i = m_\pi$: $\left(\frac{m_i}{m_X}\right)^4 = 64 G^2$

The isospin numbers are $l = \Delta I = I_f + I_i = 2 + 1 = 3$ and $m = l - 1 = 2$

2.5 Kaon pion decay theory



The generalized and isospin-adapted 3-body semi-leptonic formula (from the muon) decay is

$$\Gamma(K^\pm \to \pi^\pm\ \pi^+ \pi^-) = \frac{g^4 m_i}{32 * 192\pi^3} \left(\frac{m_i^{\,4}}{m_X^{\,4}}\right) \left(1 - \frac{m_f^{\,2}}{m_i^{\,2}}\right)^4$$

and the phenomenological formula

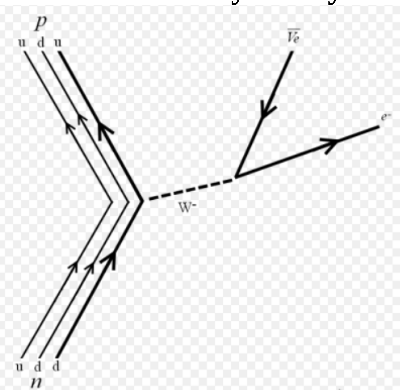
$$\Gamma(K^{\pm} \to \pi^{\pm}\,\pi^{+}\pi^{-}) = \frac{G^2 m_{K+}}{4\pi}\left(1-\frac{m_{\pi}{}^{2}}{m_{K+}{}^{2}}\right)^{2}$$ [34] , where $G = 2g_1\sqrt{\alpha}$

From these two formulas setting $g=1$ we get for m_x :

$$\left(\frac{m_i{}^4}{m_X{}^4}\right) = 8*192\pi^2 G^2 = 32*192\pi^2 g_1{}^2\alpha$$

The interaction is mediated by W-boson and a gluon: it is a weak-hadronic transformation.
The isospin numbers are $l = \Delta I = I_f - I_i = 3-1 = 2$ and $m = l = 2$

2.6 Neutron decay theory



The analytical formula from the Feynman diagram is

$$\Gamma(n \to pe\nu_e) = \left(G_V{}^2 + 3G_A{}^2\right)\frac{m_e{}^5}{2\pi^3} f_R = G_F{}^2 V_{ud}{}^2\left(1+3\lambda^2\right)\frac{m_e{}^5}{2\pi^3} f_R$$ [29] $G_V = G_F V_{ud}$ $G_A = G_F V_{ud}\lambda$ $\lambda=1.255$ $V_{ud}=0.974$ $G_F=1.166\ 10^{-5}\ GeV^{-1}$

with the phase-space term [36] $f^R = \frac{1}{60}[2\xi^4 - 9\xi^2 - 8](\xi^2-1)^{1/2} + \frac{1}{4}\xi\ln[\xi + (\xi^2-1)^{1/2}]$ $\xi \equiv \frac{M_n - M_p}{m_e}$ $f_R = 1.6332$

and the phenomenological formula

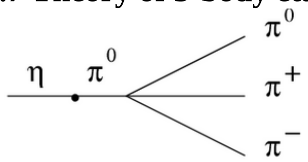
$$\Gamma(n \to pe\nu_e) = G^2 m_i^5\left(1-\frac{m_f{}^2}{m_i{}^2}\right)^4$$ [34, l=4] with initial mass $m_i = m_n$ and final mass $m_f = m_p + m_e$ and get with the same ansatz as for $\Gamma(\mu \to e\,\nu_e\,\nu_\mu)$:

$$G^2 = \frac{1}{192\pi^3\, 32 M_X{}^4}$$

The neutron decay involves in fact only 2 quarks $\Gamma(n \to pe\nu_e) = \Gamma(dd \to ud\ e\nu_e)$

so the isospin numbers are $l = \Delta I = I_f + I_i = 3 + 1 = 4$ and $m = l = 4$ with $I_f = I(ud) + I(e) + I(\nu) = 1 + 1 + 1 = 3$ and $I_i = I(dd) = 1$

2.7 Theory of 3-body eta-pion decay



The generalized and isospin-adapted 3-body semi-leptonic formula (from the muon) decay is

$$\Gamma(\eta \to \pi^0 \pi^+ \pi^-) = \frac{g^4 m_i}{32 * 192 \pi^3} \left(\frac{m_i^{\ 4}}{m_X^{\ 4}} \right) \left(1 - \frac{m_f^{\ 2}}{m_i^{\ 2}} \right)^4$$

and the phenomenological formula

$$\Gamma(\eta \to \pi^0 \pi^+ \pi^-) = \frac{G^2}{4\pi} m_i \left(1 - \frac{m_f^{\ 2}}{m_i^{\ 2}} \right)^4 \quad [34]$$

From this setting *g=1* we get for m_x :

$$\left(\frac{m_i^{\ 4}}{m_X^{\ 4}} \right) = 8 * 192 \pi^2 G^2$$

The decay is mainly hadronic, but the kinematics is one of a 3-body decay, so we can use the generalized 3-body semi-leptonic formula from above.
The intermediate boson here is π^0 , so $m_X \propto m(\pi^0)$ and the isospin numbers are $l = \Delta I = I_f + I_i = 3 + 1 = 4$ and $m = l = 4$

2.8 Theory of 2-photon meson decay

The formula for the radiative 2-photon meson decay is:

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{\alpha^2 m_i^{\ 3}}{64 \pi^3 m_X^{\ 2}}$$ where $m_i = m(\pi^0)$ and $m_X = F_\pi$ $F_\pi = \frac{2\, m_u}{Z_\pi^{1/2}}$, pseudoscalar weak decay constant [42]

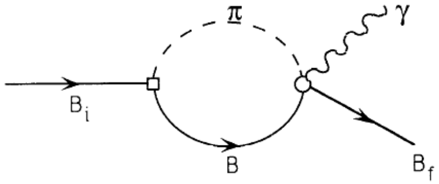
the phenomenological formula is

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{G^2 m_i^{\ 3}}{4\pi} \left(1 - \frac{m_f^{\ 2}}{m_i^{\ 2}} \right)^3$$

From this we get for m_x : $m_X = \frac{4\pi\alpha}{G}$

The intermediate boson here is the strongly excited π^0 , so $m_X \propto 10 m(\pi^0)$ and the isospin numbers are $l = \Delta I = I_f + I_i = 2 + 1 = 3$ and $m = l = 3$

2.9 Theory of 1-photon hyperon decay



The interaction becomes for the transition $s \to d\,\gamma$

$H_{sd\gamma} = \bar{d}\sigma_{\mu\nu}(a + b\gamma_5)s\, q^{\mu} A^{\nu}\, \alpha$, where A^{ν} q^{μ} are the photon and its momentum .

For the analytical formula we can use the generalized isospin-adapted expression from the pion decay (here $\Gamma(\Sigma^{+} \to p\gamma)$)

$$\frac{G_F{}^2 m_\pi{}^5}{8\pi}\frac{m_\mu{}^2}{m_\pi{}^2}\left(1 - \frac{m_\mu{}^2}{m_\pi{}^2}\right)^2$$

$\Gamma(\Sigma^{+} \to p\gamma) = f_\pi \dfrac{G_F{}^2 \alpha^2 m_i{}^5}{8\pi}\left(1 - \dfrac{m_f{}^2}{m_i{}^2}\right)^2$ where $m_i = m(\Sigma^{+})$ and $m_f = m(p)$, f_π is the hadronic correction factor

the phenomenological formula is

$$\Gamma(\Sigma^{+} \to p\gamma) = \frac{G^2 m_i{}^5}{4\pi}\left(1 - \frac{m_f{}^2}{m_i{}^2}\right)^2$$

From this we get for m_X : $m_X{}^4 = \dfrac{\alpha^2}{64G^2}$ or $m_X = \sqrt{\dfrac{\alpha}{8G}}$

and the isospin numbers are $l = \Delta I = I_f + I_i = (1/2+1) + 1/2 = 2$ and $m = l = 2$

2.10 The generalized decay formula

We have seen in 2.3 for the muon decay that the decay interaction has the Feynman-Gell-Mann form

$H_{weak} = \dfrac{G_F}{\sqrt{2}}\bar{J}^{\mu}J_{\mu}$ where $G_F/\sqrt{2} = g^2/(8M_W^2)$

or in generalized form (in natural units)

$H_{\text{int}} = \dfrac{g^2}{8m_X{}^2}\bar{J}_1{}^{\mu}(J_2)_\mu$, where g is the (dimensionless) interaction constant, m_X is the interaction energy (excitation energy of the intermediate boson), J_1 and J_2 are the currents involved, e.g. the lepton current $(J_2)_\mu = \bar{e}(x)\gamma_\mu(1-\gamma_5)\nu_e(x)$

The current has dimension *length*-^{3} , so the formula in cgs units reads

$H_{\text{int}} = (\hbar c)^3 \dfrac{g^2}{8m_X{}^2}\bar{J}_1{}^{\mu}(J_2)_\mu$, so H_{int} has dimension *energy/length*^{3} , i.e. energy density, as it should be.

The decay width (energy) becomes then
$$\Gamma = \int H_{\text{int}}(x) d^3 x$$

3 Particle data

name	mass[GeV]	e-charge	color-charge	chirality	spin	isospin
e	0.000511	-1	0	0	1/2	1
nue	3.*10^-13	0	0	1	1/2	1
u	0.0023	2/3	3	0	1/2	1/2
d	0.0048	-1/3	3	0	1/2	1/2
mu	0.106	-1	0	0	1/2	1
numu	1.1*10^-11	0	0	1	1/2	1
c	1.34	2/3	3	0	1/2	1/2
s	0.106	-1/3	3	0	1/2	1/2
tau	**1.78**	-1	0	0	1/2	1
nutau	9.8*10^-11	0	0	1	1/2	1
t	171.	2/3	3	0	1/2	1/2
b	4.2	-(1/3)	3	0	1/2	1/2
W-	80.4	-1	0	1	1	1
Z	91.2	0	0	0	1	1
gamma	0.	0	0	0	1	1
g	0.	0	8	0	1	0
H	125.1	0	0	0	0	0
p	0.93827	1	3	0	1/2	1/2
n	0.93956	0	3	0	1/2	1/2
Lambda	1.1157	0	3	0	1/2	1/2
Sigma+	1.1894	1	3	0	1/2	1
Sigma0	1.1926	0	3	0	1/2	1
Sigma-	1.19745	-1	3	0	1/2	1
Xi0	1.31486	0	3	0	1/2	1/2
Xi-	1.3217	-1	3	0	1/2	1/2
rho+	0.7751	1	3	0	1	1
rho0	0.77526	0	3	0	1	1
omega	0.78265	0	3	0	1	0
phi	1.01946	0	3	0	1	0
K*+	0.89166	1	3	0	1	1
K*0	0.89581	0	3	0	1	1
pi+	0.13957	1	3	0	0	1
pi0	0.134977	0	3	0	0	1
eta	0.54786	0	3	0	0	1
eta'	0.95778	0	3	0	0	1
K+	0.49368	1	3	0	0	1

K0	0.49761	0	3	0	0	1
KS0	0.49761	0	3	0	0	1
KL0	0.49761	0	3	0	0	1

,.,..

4 Decay width and interaction energy for different types of decays

4.1 Strange hyperon decays with pions and kaon-pion decay

Here we have $|\Delta S| = 1$ $\Gamma = C|\psi_{1,1}|^2 = \frac{G^2}{4\pi} m_i (1 - x^2)$ *l= ΔI =1 m=1* $x = \frac{m_f}{m_i}$

with $g_h = 6.2 * 10^{-7} \left(\frac{m_p}{m_i} \right)$

$\Lambda \to p\pi^-, \to n\pi^0$ $G = g_h$ resp. $G = g_h / \sqrt{2}$

$\Sigma^+ \to p\pi^0, \to n\pi^+$; $\Sigma^- \to n\pi^-$; $G = g_h$

$\Xi^0 \to \Lambda\pi^0$ $\Xi^- \to \Lambda\pi^-$ $G = g_h$ resp. $G = g_h \sqrt{2}$

$K^\pm \to \pi^\pm \pi^+ \pi^- \gamma$ $G = g_1 \alpha \sqrt{2}$

decay	width_calc[GeV]	width_obs[GeV]	rel. Δwidth	I_i	I_f	$l = \Delta I$	G	x	m_X	G formula
Λ ->p pi-	1.61032*10^-15	1.599*10^-15	0.00813008	1/2	3/2	1	5.21401*10^-7	0.966066	352.232	$G = g_h$
Λ ->n pi0	8.74088*10^-16	8.96*10^-16	0.0145089	1/2	3/2	1	3.68686*10^-7	0.963106	423.206	$G = g_h / \sqrt{2}$
Σ+ ->p pi0	4.20623*10^-15	4.233*10^-15	0.00590598	1/2	3/2	1	4.89093*10^-7	0.902343	426.379	$G = g_h$
Σ+ ->n pi+	4.00357*10^-15	3.966*10^-15	0.00630358	1/2	3/2	1	4.89093*10^-7	0.907289	424.288	$G = g_h$
Σ– ->n pi-	4.22472*10^-15	4.444*10^-15	0.00562556	1/2	3/2	1	4.85805*10^-7	0.90119	430.76	$G = g_h$
Xi0 -> Λ pi0	1.95069*10^-15	2.259*10^-15	0.0110668	1/2	3/2	1	4.42425*10^-7	0.951186	471.39	$G = g_h$
Xi– -> Λ pi-	3.99332*10^-15	4.011*10^-15	0.00623286	1/2	3/2	1	6.22446*10^-7	0.949739	399.863	$G = g_h \sqrt{2}$
K+ ->pi+ pi+ pi- γ	6.05074*10^-21	5.53*10^-21	0.385172	1	0	1	7.40795*10^-10	0.84814	3574.06	$G = g_1 \alpha \sqrt{2}$

The 4-body kaon decay K+ ->pi+ pi+ pi- γ is similar to K+ ->pi0 pi+ γ and has a similar energy, but a different isospin value, therefore is included here.
The decays can be roughly ordered according to the interaction energy
lambda into nucleon pion $m_X \approx 400 GeV$
sigma into nucleon pion $m_X \approx 400 GeV$

kaon into 3 pion photon $m_X \approx 3500 GeV$

4.2 Two-body non-strange decays of mesons ΔS=0

$$\Gamma = C|\psi_{3,2}|^2 = \frac{G^2}{4\pi} m_i x^2 (1 - x^2) \quad l = \Delta I = 3 \; m = 2 \quad x = \frac{m_f}{m_i}$$

$\pi^\pm \to l\nu$ G=g_0

$K^\pm \to l\nu$ G=g_1

$K_S^0 \to \pi^+\pi^-, \to \pi^0\pi^0$ $G = 2g_1\alpha 443$ $G = 2g_1\alpha 443/\sqrt{2}$

$K^\pm \to \pi^\pm\pi^0$ $G = 2g_1\alpha\sqrt{443}$

$K_L^0 \to \pi^+\pi^-, \to \pi^0\pi^0$ $G = 2g_1\alpha$ $G = 2g_1\alpha/\sqrt{2}$

decay	width_calc[GeV]	width_obs[GeV]	rel. Δwidth	I_i	I_f	$l = \Delta I$	G	x	m_X	G formula
pi+->mu numu	2.50122*10^-17	2.528*10^-17	0.000791139	1	2	3	1.47648*10^-7	0.759476	96.3675	$G = g_0$
pi+->e nue	3.24552*10^-21	3.11*10^-21	0.0186495	1	2	3	1.47648*10^-7	0.00366125	107.988	$G = g_0$
K+->mu numu	3.3949*10^-17	3.372*10^-17	0.00266904	1	2	3	1.43527*10^-7	0.214714	383.074	$G = g_1$
K+->e nue	8.67067*10^-22	8.238*10^-22	0.0581452	1	2	3	1.43527*10^-7	0.00103508	387.415	$G = g_1$
KS0->pi+ pi-	5.0423*10^-15	5.084*10^-15	0.00354052	1	2	3	9.28211*10^-7	0.560961	146.47	$G = 2g_1\alpha 443$
KS0->pi0 pi0	2.5002*10^-15	2.255*10^-15	0.00798226	1	2	3	6.56344*10^-7	0.542501	174.823	$G = 2g_1\alpha 443/\sqrt{2}$
K+->pi+ pi0	1.12741*10^-17	1.112*10^-17	0.00719424	1	2	3	4.41006*10^-8	0.556123	667.317	$G = 2g_1\alpha\sqrt{443}$
KL0->pi+ pi-	2.56934*10^-20	2.543*10^-20	0.0247739	1	2	3	2.09528*10^-9	0.560961	3082.85	$G = 2g_1\alpha$
KL0->pi0 pi0	1.27399*10^-20	1.119*10^-20	0.204647	1	2	3	1.48159*10^-9	0.542501	3679.59	$G = 2g_1\alpha/\sqrt{2}$

The decays can be roughly ordered according to the interaction energy
pion-lepton $m_X \approx 100 GeV$,
kaon-lepton: $m_X \approx 400 GeV$,
kaon-pion: $m_X \approx 600 GeV$,
short-lived K$_{s0}$-pion: $m_X \approx 150 GeV$,
long-lived K$_{L0}$-pion: $m_X \approx 3200 GeV$

4.3 Three-body decays of strange mesons and hyperons ΔS=1

$\Gamma = C|\psi_{2,2}|^2 = \frac{G^2}{4\pi} m_i (1-x^2)^2$ $l = \Delta I = 2$ $m=2$ $x = \frac{m_f}{m_i}$

with $g_h' = 1.2806 * 10^{-8} \left(\frac{m_p}{m_i} \right)^{3/2}$

$K^\pm \to \pi^0 l \nu$ $G = g_1 \sqrt{\alpha} / \sqrt{2}$

$K_L^0 \to \pi^\pm l \nu$ $G = g_1 \sqrt{\alpha}$

$K^\pm \to \pi^\pm \pi^+ \pi^-$, $G = 2.53 g_1 \sqrt{\alpha}$

$K^\pm \to \pi^0 \pi^0 \pi^\pm$ $G = 2.53 g_1 \sqrt{\alpha}$ /2

$K_L^0 \to 3\pi^0$ $G = 2 g_1 \sqrt{\alpha}$

$K_L^0 \to \pi^+ \pi^- \pi^0$ $G = 2 g_1 \sqrt{\alpha} / \sqrt{3/2}$

$K^\pm \to \pi^\pm \pi^\mp l^\pm \nu, \to \pi^0 \pi^0 l^\pm \nu$ N=2 $G = g_1 \alpha / \pi$

$K^\pm \to \pi^0 \pi^\pm \gamma$ $G = g_1 \left(\frac{\alpha}{\sqrt{2}} \right)$

$\Lambda \to p l \nu$ $G = g_h' \sqrt{3}$

$\Sigma^- \to n l \nu$ $G = g_h' 2$

decay	width_calc[GeV]	width_obs[GeV]	rel. Δwidth	I_i	I_f	$l = \Delta I$	G	x	m_X	G formula
K+->pi0 e ve	2.52543*10^-18	2.565*10^-18	0.0105263	1	3	2	8.67078*10^-9	0.274445	2168.89	$G = g_1 \sqrt{\alpha} / \sqrt{2}$
K+->pi0 μ vμ	1.7138*10^-18	1.764*10^-18	0.0272109	1	3	2	8.67078*10^-9	0.488124		$G = g_1 \sqrt{\alpha} / \sqrt{2}$

									1899.19	
KL0->pi+ e ve	5.04792*10^-18	5.217*10^-18	0.0120759	1	3	2	1.22623*10^-8	0.281508	1830.36	$G = g_1\sqrt{\alpha}$
KL0->pi+ μ vμ	3.40719*10^-18	3.478*10^-18	0.00920069	1	3	2	1.22623*10^-8	0.493499	1602.52	$G = g_1\sqrt{\alpha}$
K+->pi+ pi+ pi-	2.97836*10^-18	2.971*10^-18	0.00538539	1	3	2	3.10237*10^-8	0.84814	127.367	$G = 2.53g_1\sqrt{\alpha}$
K+->pi0 pi0 pi+	9.19439*10^-19	9.34*10^-19	0.0289079	1	3	2	1.55119*10^-8	0.829533	191.076	$G = 2.53g_1\sqrt{\alpha}/2$
KL0->pi0 pi0 pi0	2.71785*10^-18	2.518*10^-18	0.0353455	1	3	2	2.45247*10^-8	0.813752	159.959	$G = 2g_1\sqrt{\alpha}$
KL0->pi+ pi- pi0	1.50061*10^-18	1.617*10^-18	0.0185529	1	3	2	2.00243*10^-8	0.832212	167.813	$G = 2g_1\sqrt{\alpha}/\sqrt{3/2}$
K+->pi+ pi- e+ ve	2.01491*10^-21	2.174*10^-21	0.0367985	1	3	2	3.33475*10^-10	0.566462	8453.24	$G = g_1\alpha/\pi$
K+->pi+ pi- μ+ vμ	6.69205*10^-22	7.44*10^-22	0.642473	1	3	2	3.33475*10^-10	0.780141	6435.05	$G = g_1\alpha/\pi$
K+->pi0 pi0 e+ ve	1.06991*10^-21	1.169*10^-21	0.182207	1	3	2	2.35802*10^-10	0.547855	10293.3	$G = g_1\alpha/(\pi\sqrt{2})$
K+->pi0 pi0 μ+ vμ	3.8545*10^-22	4.2*10^-22	0.5	1	3	2	2.35802*10^-10	0.761534	7981.24	$G = g_1\alpha/(\pi\sqrt{2})$
KL0->pi0 pi+ e ve	2.12372*10^-21	2.764*10^-21	0.0209841	1	3	2	3.33475*10^-10	0.552758	8671.14	$G = g_1\alpha/\pi$
KL0->pi0 pi+ μ vμ	7.58984*10^-22	8.*10^-22	0.0725	1	3	2	3.33475*10^-10	0.76475	6715.43	$G = g_1\alpha/\pi$
Λ->p e ve	2.21509*10^-18	2.081*10^-18	0.0168188	1/2	5/2	2	1.71059*10^-8	0.841428	1504.53	$G = g_h'\sqrt{3}$
Λ ->p μ vμ	3.99111*10^-19	3.93*10^-19	0.223919	1/2	5/2	2	1.71059*10^-8	0.935977	1037.67	$G = g_h'\sqrt{3}$
Σ- ->n e ve	4.42673*10^-18	4.526*10^-18	0.0393283	1/2	5/2	2	1.77643*10^-8	0.785061	1879.7	$G = g_h'2$

									1	
Σ− ->n μ νμ	1.69761*10^-18	2.003*10^-18	0.0888667	1/2	5/2	2	1.77643*10^-8	0.873155	1546.45	$G = g_h'2$
K+ ->pi0 pi+ γ	1.37146*10^-20	1.462*10^-20	0.0581395	1	1	2	8.55396*10^-10	0.556123	5707.21	$G = g_1\alpha / \sqrt{3/2}$

The decays can be roughly ordered according to the interaction energy
K+, KL0 into pi 2lepton $m_X \approx 1.8TeV$
K+, KL0 into 3 pi $m_X \approx 150GeV$
K+, KL0 into 2 pi 2lepton $m_X \approx 7...10TeV$
Λ into pi 2lepton $m_X \approx 1.2TeV$
Σ into pi 2lepton $m_X \approx 1.7TeV$
K+ into 2pi photon $m_X \approx 5.7TeV$

4.4 Non-strange leptonic three-body decays

$A' \to Ae\nu(\Delta S = 0)$

$\Gamma = C|\psi_{4,4}|^2 = G^2 m_i^5 (1 - x^2)^4$ *l= ΔI =4 m=4*

$\pi^{\pm} \to \pi^0 e\nu$ $G = G_0 / \sqrt{192 * 50\pi^3}$

$n \to pe\nu$ $G = G_0 / \sqrt{192 * 175\pi^3}$

$\Sigma^{\pm} \to \Lambda e^{\pm}\nu$ $G = G_0 / \sqrt{192 * 65\pi^3}$

$\mu \to e\overline{\nu}\nu_{\mu}$ $\tau \to e\overline{\nu}\nu_{\tau}, \to \mu\overline{\nu}_{\mu}\nu_{\tau}$ $G = G_0 / \sqrt{192\pi^3}$

decay	width_calc[GeV]	width_obs[GeV]	rel. Δwidth	I_i	I_f	*l= ΔI*	G	x	m_X	G formula
μ ->e νe νμ	3.01736*10^-19	2.954*10^-19	0.014218	1	3	4	1.50165*10^-7	0.00482075	717.027	$G = G_0 / \sqrt{192\pi^3}$
tau ->e νe ντ	4.02938*10^-13	4.041*10^-13	0.00296956	1	3	4	1.50165*10^-7	0.000287079	717.105	$G = G_0 / \sqrt{192\pi^3}$
tau ->μ νμ ντ	3.97253*10^-13	3.932*10^-13	0.00305188	1	3	4	1.50165*10^-7	0.0595506	695.878	$G = G_0 / \sqrt{192\pi^3}$
pi+ ->pi0 e+ νe	2.63622*10^-25	2.619*10^-25	0.067583	1	3	4	2.12366*10^-8	0.970753	468.38	$G = G_0 / \sqrt{192 * 50\pi^3}$

n ->p e ve	7.12154*10^-28	7.239*10^-28	0.000897914	1	3	4	1.13514*10^-8	0.999171	204.69	$G = G_0 / \sqrt{192*175\pi^3}$
Σ+ ->Λ e+ ve	1.67174*10^-19	1.642*10^-19	0.249695	1	3	4	1.86257*10^-8	0.938466	703.24	$G = G_0 / \sqrt{192*65\pi^3}$
Σ- ->Λ e ve	2.5218*10^-19	2.55*10^-19	0.0470588	1	3	4	1.86257*10^-8	0.932157	740.33	$G = G_0 / \sqrt{192*65\pi^3}$

Here pure-leptonic transitions are bi-quark transitions

$n \to pe\bar{\nu}$ becomes $du \to uu\, e\bar{\nu}$

$\Sigma^+ \to \Lambda\bar{e}\,\nu$ becomes $uu \to ud\,\bar{e}\,\nu$

The decays can be roughly ordered according to the interaction energy

lepton into lepton 2 neutrino $m_X \approx 700 GeV$

pi into pi 2 lepton $m_X \approx 300 GeV$

neutron decay n ->p e ve $m_X \approx 100 GeV$

Σ into Λ 2lepton $m_X \approx 800 GeV$

4.5 Three-body decays eta-pions

$\Gamma = C|\psi_{4,4}|^2 = \frac{G^2 m_i}{4\pi}(1-x^2)^4$ *l= ΔI =4 m=4*

$\eta \to 3\pi^0 \quad \eta \to \pi^+\pi^-\pi^0 \quad G = 0.0145$

$\eta \to \pi^+\pi^-\gamma \quad G = 0.00213$

decay	width_calc[GeV]	width_obs[GeV]	rel. Δwidth	I_i	I_f	*l= ΔI*	G	x	m_X	G formula
eta ->pi0 pi0 pi0	3.88428*10^-7	4.226*10^-7	0.00851869	1	3	4	0.0145	0.739114	0.26643	$G = 0.0145$
eta ->pi+ pi- pi0	3.09444*10^-7	2.951*10^-7	0.0176211	1	3	4	0.0145	0.755881	0.25729	$G = 0.0145$
eta ->pi+ pi- γ	5.94407*10^-8	6.097*10^-8	0.021978	1	3	4	0.00213	0.50951	7.4388	$G = 0.00213$

The decays can be roughly ordered according to the interaction energy

eta into 3 pion $m_X \approx 0.3 GeV$

eta into 2 pion photon $m_X \approx 1\, GeV$

4.6 Photon-radiative decays

$\Gamma = C\left|\psi_{3,3}\right|^2 = \frac{G^2 m_i^3}{4\pi}\left(1-x^2\right)^3$ *l= ΔI =3 m=3*

with $\alpha = \frac{e^2}{4\pi}$ $g_{ph}' = 0.138$

pseudoscalar mesons $P \to \gamma\gamma$ P= π^0, η, η' theory $\Gamma(P \to \gamma\gamma) = \frac{e^4 g_P^2}{64\pi} m_P^3$ x=0 $G = 2\pi\alpha\, C_1 g_{ph}'$ $C_1 = 1, \sqrt{5/4}, \sqrt{5/3}$

$\Gamma = C\left|\psi_{2,2}\right|^2 = \frac{G^2 m_i^5}{4\pi}\left(1-x^2\right)^2$ *l= ΔI =2 m=2*

$g_{ph} = 9.769 * 10^{-9}$ $G = C_1 g_{ph}$

hyperons $\Lambda \to n\gamma$, $\Sigma^+ \to p\gamma$, $\Xi^0 \to \Lambda\gamma, \Sigma^0\gamma$ $\Xi^- \to \Sigma^-\gamma$ $C_1 = \sqrt{7/2}, 2, 1, \sqrt{8}, 1/\sqrt{2}$

decay	width_calc[GeV]	width_obs[GeV]	rel. Δwidth	I_i	I_f	*l= ΔI*	G	x	m_X	G formula
pi0 ->γ γ	7.86*10^-9	7.84*10^-9	0.0687023	1	2	3	0.00632905	0.	23.8527	$G = 2\pi\alpha\, g_{ph}'$
eta ->γ γ	6.4*10^-7	6.55*10^-7	0.21875	1	2	3	0.00707609	0.	21.334	$G = 2\pi\alpha\, g_{ph}'\sqrt{5/4}$
eta' ->γ γ	4.57*10^-6	4.67*10^-6	0.0547046	1	2	3	0.0081707	0.	18.476	$G = 2\pi\alpha\, g_{ph}'\sqrt{5/3}$
Λ ->n γ	3.88647*10^-18	3.96*10^-18	0.0368098	1/2	3/2	2	1.82761*10^-8	0.842126	164.32	$G = g_{ph}\sqrt{7/2}$
Σ+ ->p γ	1.03154*10^-17	9.81*10^-18	0.0487805	1/2	3/2	2	1.9538*10^-8	0.78886	161.01	$G = 2g_{ph}$
Xi0 -> Λ γ	2.33984*10^-18	2.34*10^-18	0.150943	1/2	3/2	2	9.769*10^-9	0.848531	224.40	$G = g_{ph}$
Xi0 -> Σ0 γ	7.50752*10^-18	7.87*10^-18	0.120787	1/2	3/2	2	2.76309*10^-8	0.907017	131.51	$G = g_{ph}\sqrt{8}$
Xi- -> Σ- γ	4.91693*10^-19	4.99*10^-19	0.179688	1/2	3/2	2	6.90773*10^-9	0.905992	263.09	$G = g_{ph}/\sqrt{2}$

The decays can be roughly ordered according to the interaction energy

pi , eta into 2 photon $m_X \approx 20 GeV$

Λ , Σ into nucleon photon $m_X \approx 130 GeV$

Xi into Λ photon $m_X \approx 180 GeV$

Xi into Σ photon $m_X \approx 100,...200 GeV$

5 Characterization and calculation of different types of decays based on interaction energy

5.1 Table of decays based on interaction energy

decay	P_{in}	P_i	m1[GeV]	m2[GeV]	m3[GeV]	m4[GeV]	mX[GeV]	scheme
Λ -> n π	uds	udd/(uu'-dd')	1.1157	0.93956	0.134977	0.	442.27	sd‘(2h)→Z→π0(2h)
Σ -> n π	uds	udd/(uu'-dd')	1.1894	0.93956	0.134977	0.	480.94	sd‘(2h)→Z→π0(2h)
Ξ ->Λ π	uss	uds/(uu'-dd')	1.31486	1.1157	0.134977	0.	497.84	sd‘(2h)→Z→π0(2h)
π+ -> l v	ud'	2rL-	0.13957	0.106	1.1*10^-11	0.	107.98	ud‘(1h)→W→W
K+->l v	us'	2rL-	0.49368	0.106	1.1*10^-11	0.	387.41	us‘(2h)→W→W
K+-> π+ π0	us'	ud'/(uu'-dd')	0.49368	0.13957	0.134977	0.	668.18	sd‘(4h)→Z→2π0(3h)
KS0-> π π	(ds'+sd')	2(uu'-dd')	0.49761	0.13957	0.13957	0.	160.64	ds‘(1h)→Z→2π0
K+-> π+ π π	us'	ds'/2(uu'-dd')	0.49368	0.13957	0.134977	0.134977	159.22	us‘(1h)→W→ π+ 2π0
KL0->π0 π π	ds'	(uu'-dd')/2(uu'-dd')	0.49761	0.134977	0.134977	0.134977	163.88	ds‘(1h)→Z→ π0 2π0
KL0-> π π	(ds'-sd')	2(uu'-dd')	0.49761	0.13957	0.13957	0.	3381.2	ds‘(12h)→Z→2π0(4h)
K+->π0 l v	us'	(uu'-dd')/W	0.49368	0.134977	0.106	1.1*10^-11	2034.04	us‘(6h)→W→ π0 W(6h)
KL0->π+ l v	ds'	ud‘/W	0.49761	0.13957	0.106	1.1*10^-11	1716.44	ds‘(6h)→Z→ π+ W(2h)
K+->π+ π- l v	us'	2(uu'-dd')/W	0.49368	0.134977	0.134977	0.106	7444.14	us‘(15h)→W→2π0 W(6h)
K+->π0 π0 l v	us'	2(uu'-dd')/W	0.49368	0.134977	0.134977	0.106	9137.27	us‘(15h)→W→2π0 W(15h)
KL0->π+ π0 l v	us'	ud'/(uu'-dd')/W-	0.49761	0.13957	0.134977	0.106	7693.29	us‘(15h)→W→2π0 W(6h)
K+-> π+ π+ π- γ	us'	ud'/(ud'+u'd)	0.49368	0.13957	0.27914	0.	3574.06	us‘(12h)→W→ π+π-π+ γ(6h)
K+->π0 π+ γ	us'	(uu'-dd')/ud'	0.49368	0.134977	0.13957	0.	5707.21	us‘(12h)→→W→π0π+γ(12h)
Λ -> p *l* v	uds	uud/ W	1.1157	0.93827	0.106	1.1*10^-11	1271.1	su‘(6h)→W→W(1h)
Σ- -> n *l* v	dds	udd/ W	1.1197	0.93956	0.106	1.1*10^-11	1713.08	su‘(6h)→W→W(2h)
μ/τ -> e ve v	*l*	*l*	1.78	0.000511	3.*10^-13	1.1*10^-11	717.06	*l v‘* (4h)→W→W
τ -> μ vμ vμ	*l*	*l*	1.78	0.106	1.1*10^-11	9.8*10^-11	695.878	*l v‘* (4h)→W→W

π+ -> π0 *l* v	ud'	(uu'-dd')/ W	0.13957	0.134977	0.106	1.1*10^-11	468.38	d'u(2h)→W→W
n->p e ve	udd	uud/ W	0.93956	0.93827	0.000511	3.*10^-13	204.69	du'(1h)→W→W
Σ+ -> Λ *l* v	uus	uds/ W	1.1894	1.1157	0.106	1.1*10^-11	721.78	ud'(4h)→W→W
η -> π0 π0 π0	(uu'+dd'-2ss')	3(uu'-dd')	0.54786	0.134977	0.134977	0.134977	0.26186	sd'(3g)→π0→3π0
η -> π0 π0 γ	(uu'+dd'-2ss')	2(uu'-dd')	0.54786	0.134977	0.134977	0.	7.4388	sd'(6g)→π0→2π0 γ
π0/η -> γ γ	(uu'-dd')		0.134977	0.	0.	0.	21.221	uu'(8g)→π0→π0 2γ
Λ/Σ -> n γ	uds	udd	1.1157	0.93956	0.	0.	184.80	sd'(1h)→Z→Z γ
Σ0 -> Λ γ	uss	uds	1.31486	1.1157	0.	0.	256.98	sd'(2h)→Z→Z γ
Ξ0 -> Σ0 γ	uss	uds	131486	11926	0.	0.	152.80	sd'(1h)→Z→Z γ
Ξ- -> Σ- γ	dss	dds	1.3217	1.19745	0.	0.	305.61	sd'(2h)→Z→Z γ (2h)

In the above table, the decays are grouped according to type and interaction energy m_X .
Consider the general decay

$$P_{in} \rightarrow P_1 + P_2 + P_3 + P_4$$

In the table above, row P_{in} contains the structure of the original particle, the row P_i contains the structures of the outgoing particles, separated by slash, the rows m_1,...,m_4 and m_X contain the respective mass.
The configuration is described either by quarks (like *Λ=uds*) or by *l* (lepton) or by Z , *W* .
The scheme in the last column describes the QHCD/QCD model of the interaction energy with number of active hc bosons, e.g. sd'(2h)→Z→π0(2h) for the decay Ξ ->Λ π.
E.g. the generic decay Λ/Σ -> n π has the incoming configuration P_{in} = uds and the outgoing generic configuration P_{12} = (n=udd)/(π0=(uu'-dd')) , with the interaction energy m_X=400GeV, and the decay scheme sd'(2h)→Z→π0 , where the significant incoming current is $s\overline{d}$ interacting via 2 hc-bosons , the intermediate boson is the Z-boson, and the outgoing current is $\pi^0 = (u\overline{u} - d\overline{d})/\sqrt{2}$. The number of active hc-bosons (or active gluons, in the pion-mediated decays) determines roughly the energy level.
The table reveals a simple principle for the scheme:
$q_1\overline{q}_2 \rightarrow b \rightarrow p$ or $\overline{q}_1 q_2 \rightarrow b \rightarrow p$, where q_1, q_2 are quarks in the incoming quark-current, *b* is the mediating boson $b = W, Z, \pi^0$, *p* are the outgoing particles , $p = \pi^0, \pi, W, \gamma$, where p can be represented as one or more quark-currents except for the photon γ, which is itself the electromagnetic current.
The resulting interaction energy m_X in the table above is not distributed uniformly, but accumulates around certain values, the energy classes.

$E_{h1} \approx 150 GeV$ for 1 hc-boson

$E_{h2} \approx 400 GeV$ for 2 hc-bosons

$E_{h4} \approx 700GeV$ for 4 hc-bosons

$E_{h6} \approx 1500GeV$ for 6 hc-bosons

$E_{h12} \approx 3500GeV$ for non-diagonal 12 hc-bosons outgoing W(1hcb)

$E_{h12,3h} \approx 5700GeV$ for non-diagonal 12 hc-bosons outgoing W(3hcb)

$E_{h15} \approx 7500GeV$ for all 15 hc-bosons outgoing W(3hcb)

$E_{h15},3h \approx 9000GeV$ for all 15 hc-bosons outgoing W(6hcb)

$E_{c1} \approx 0.3GeV$ for 3 gluons (color interaction, factor 1000 weaker than hc-interaction)

$E_{c6} \approx 7GeV$ for 6 non-diagonal gluons; $E_{c8} \approx 20GeV$ for all 8 gluons

For weak decays the energy span in m_X is roughly: $\left(\frac{E_{h15}}{E_{h1}}\right) = \frac{9000GeV}{150GeV} = 60$,

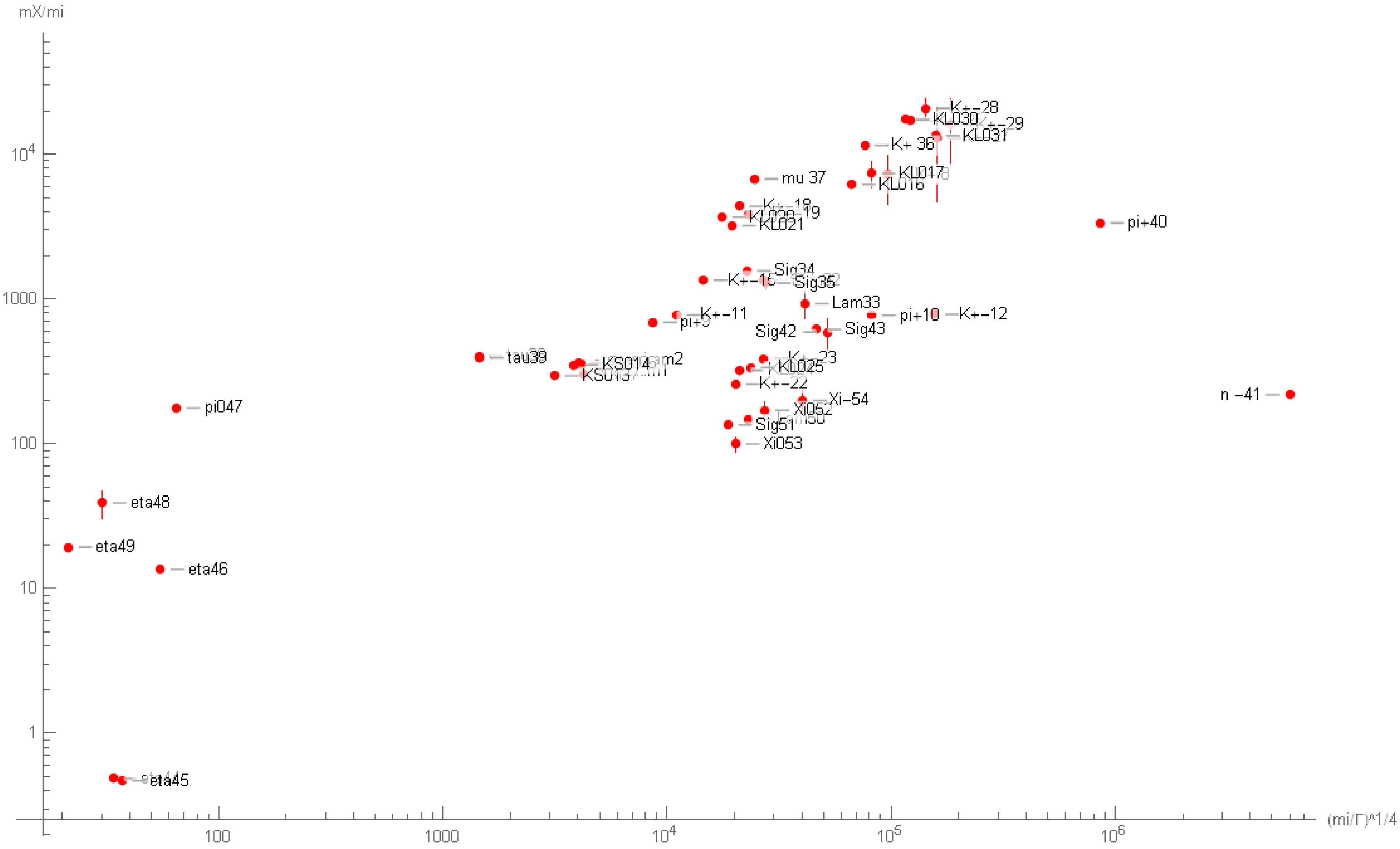
so the energy span scales like $\left(\frac{E_{h15}}{E_{h1}}\right) = 60 \approx (n_h)^{3/2}$

5.2 The interaction energy and the bandwidth

In 2.1 a general relationship between the interaction energy m_X and the decay bandwidth Γ was derived:

$$\frac{m_X}{m_i} = f_I\left(\frac{m_i}{\Gamma}\right)^{1/4}, \text{ where } f_I = \left(\frac{I_\Gamma{}^{1/4}}{2\sqrt{2}}\right)$$

The following plot depicts this relationship for all 54 decays of the quarks u, d, s and all leptons, dealt with in this chapter



The x-axis is $x = \left(\frac{m_i}{\Gamma}\right)^{1/4}$, the y-axis is $y = \frac{m_X}{m_i}$, the labels consist of the first 3 characters of the name of the corresponding decay, followed by the number in the total decay table, e.g. pi0 ->γ γ has the number 47, and the label "pi047" .
One sees immediately, that the decays separate in two large groups: those with x>1000 are weak, i.e. hypercolor decays, those with x<60 are strong (pure color) decays.
If there are 1 or 2 photons on the right side, then the electromagnetic Lagrangian component is activated in the calculation in chapter 6.
In the pure-color decays only the color SU(3)-Lagrangian is activated, in the weak decays both the SU(3) and the SU(4)-Lagrangian is activated.

6 Numerical calculation: method and results

6.1The interaction model and the Lagrangian in two examples

The basic idea of the Fermi model of weak 3-body decay in the Feynman picture mediated by the weak boson W is explained at the example of the neutron decay $n \to p\, e\, \overline{\nu}$ with the decay scheme $d\overline{u}(1h,3g) \to W \to W(1h)$.

The incoming Lagrangian is $L(d\overline{u}) = L_{QHCD}(x^{\mu}, \{u_1, u_2\}, \{Ag_4\})$ with the quark wavefunctions $u_1 = d = r^- q^+$ $u_2 = \overline{u} = r^- q^-$ in the hypercolor-SU(4)-preon model, and one hc-boson Ag_4 corresponding to the SU(4) generator matrix λ_4 and the SU(4) index pair {1,3} and the interaction $r_L^- \leftrightarrow r_R^-$ in the hc-charge-quadruple $(r_L^-, r_L^+, r_R^-, r_R^+)$.

We recall that both L_{QHCD} and L_{QCD} have the generic form

Dirac part $L_D = \overline{\psi}\left(i\hbar D_{\mu}\gamma^{\mu} - mc\right)\psi$, covariant derivative $D_{\mu} = \partial_{\mu} - igA^a{}_{\mu}\lambda_a$

field part $L_{gf} = -\frac{1}{4} F^{a\mu\nu} F_{a\mu\nu}$, field tensor $F^a{}_{\mu\nu} = \partial_{\mu} A^a{}_{\nu} - \partial_{\nu} A^a{}_{\mu} + g f^{abc} A^b{}_{\mu} A^c{}_{\nu}$

with the structure constants f^{abc} of the respective Lie algebra (SU(3) or SU(4)) and λ_a are the generators of the algebra .

From the preon composition results the following form of the SU(4) quadruple wavefunction

$$u_{11} = (r_L^- + q_L^+)/\sqrt{2} \quad u_{12} = (r_R^- + q_R^+)/\sqrt{2} \quad d = \left(\begin{pmatrix} u_{11} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} u_{12} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right)$$

$$u_{21} = (r_L^- + q_L^-)/\sqrt{2} \quad u_{22} = (r_R^- + q_R^-)/\sqrt{2} \quad \overline{u} = \left(\begin{pmatrix} u_{21} \\ u_{21} \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} u_{21} \\ u_{21} \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right) / \sqrt{2}$$

The outgoing Lagrangian is $L(W) = L_{QHCD}(x^{\mu}, \{u_3, u_3\}, \{Ag'_4\})$ with the weak boson W $u_3 = r_L^- r_R^-$ and another hc-boson Ag'_4 .

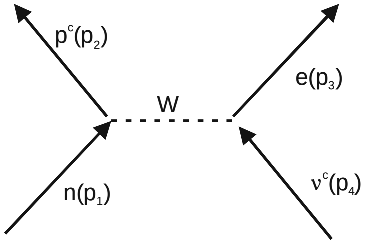
$$u_{31} = r_L^- \quad u_{32} = r_R^- \quad W = \left(\begin{pmatrix} u_{31} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} u_{31} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right)$$

The interaction Lagrangian is the Fermi current-current interaction with the mediating exchange boson with the energy $m_X = E(W) = E(u_3) + m_W + E(Ag_4)$

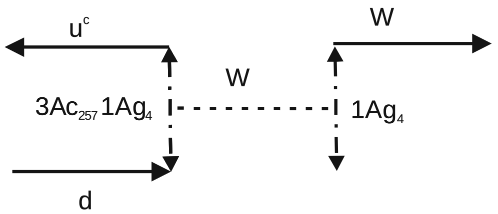
$$L_{JJ}(J(d\overline{u}), J(W)) = \frac{\left(u_1^+ \gamma^{\mu} u_2\right)\left(u_3^+ \gamma_{\mu} u_3\right)}{m_X^2}$$, with the notation Dirac-conjugate u_1^+

So we have in total two particle configurations, the incoming $n\overline{p}$ and the outgoing $e\overline{\nu}$, each with an interaction Lagrangian, coupled by the Fermi current-current interaction, and mediated by the corresponding W-boson $W = r_L^- r_R^-$, In the incoming system $d\bar{u}$ we have to take into account the color interaction of the quarks $L_C(d\bar{u}) = L_{QCD}(x^\mu, \{u_1, u_2\}, \{Ac_2, Ac_5, Ac_7\})$ in the basic gluon configuration with 3 rgb-gluons .

Feynman diagram of the decay $n \to p\, e\, \bar{\nu}$, with the notation of the antiparticle $\bar{p} = p^c$ (conjugate)

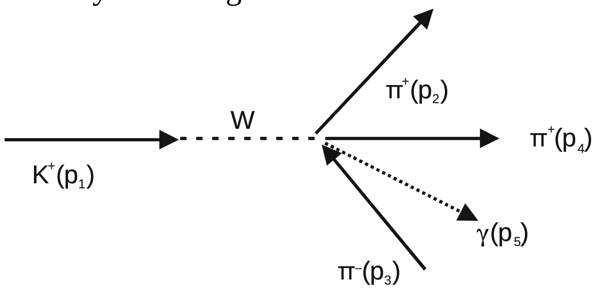


and the decay-scheme (quark decay) $d\bar{u}(1h,3g) \to W \to W(1h)$, where the mediating boson $W = r_L^{\,-} r_R^{\,-}$ acts via the current-current-interaction L_{JJ}

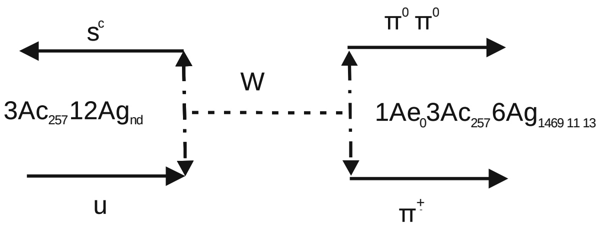


In the decay-scheme the weak (SU(4)) interaction is carried on the left side by 1h= 1 hypercolor SU(4) boson Ag_4 , and the color (SU(3)) interaction by 3g= 3 (anticoupler) gluons Ac_2 Ac_5 Ac_7 . On the right side, the weak (SU(4)) interaction is carried by 1h= 1 hypercolor SU(4) boson Ag_4 and there is no color interaction, as the mediating boson W has only a weak charge, no color charge.

We illustrate the calculation ansatz in more detail in the more complicated and computationally much more challenging example of the 4-body kaon-pion photonic decay $K^+ \to \pi^+ \pi^+ \pi^- \gamma$ with the decay scheme $u\bar{s}(12h,3g) \to W \to \pi^+ \pi^+ \pi^- (6h,3g,1\gamma)$
The Feynman diagram is



with the corresponding decay-scheme



The incoming Lagrangian is $L(u\bar{s}) = L_{QHCD}(x^{\mu},\{u_1,u_2\},\{Ag_{nd}\})$, with $K^+ = u\bar{s}$, with the quark wavefunctions $u = r^+q^+ \quad \bar{s} = r^+q^-$,

$$u_{11} = (r_L{}^+ + q_L{}^+)/\sqrt{2} \quad u_{12} = (r_R{}^+ + q_R{}^+)/\sqrt{2} \quad u = \left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}u_{11}\\u_{11}\end{pmatrix},\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}u_{12}\\u_{12}\end{pmatrix}\right)/\sqrt{2}$$

$$u_{21} = (r_L{}^+ + q_L{}^-)/\sqrt{2} \quad u_{22} = (r_R{}^+ + q_R{}^-)/\sqrt{2} \quad \bar{s} = \left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}0\\u_{21}\end{pmatrix},\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}u_{21}\\u_{21}\end{pmatrix}\right)$$

and 12 non-diagonal hc-bosons Ag_{nd} corresponding to the non-diagonal SU(4) generator matrices λ_i and the SU(4) indices {1,2,4,5,6,7,9,10,11,12,13,14} .
The outgoing Lagrangian is $L(\pi^+\pi^-\pi^+) = L_{QHCD}(x^{\mu},\{u_3,u_4,u_5\},\{Ag'_{14691113}\})$ with the pions and their corresponding wavefunctions

$$\pi^+\pi^- = u\bar{d}d\bar{u} = r_L{}^- + r_R{}^+ + q_L{}^- + q_R{}^+$$

$$u_{31} = u_{41} = (r_L{}^- + q_L{}^-)/\sqrt{2} \quad u_{32} = u_{42} = (r_R{}^+ + q_R{}^+)/\sqrt{2} \quad \pi^+\pi^- = \left(\begin{pmatrix}u_{31}-u_{41}\\u_{31}-u_{41}\end{pmatrix},\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}u_{32}-u_{42}\\u_{32}-u_{42}\end{pmatrix}\right)/2$$

$$\pi^+ = u\bar{d} = r_L{}^+ + r_R{}^+ + q_L{}^- + q_R{}^+ \quad u_{51} = (r_L{}^+ + q_L{}^-)/\sqrt{2} \quad u_{52} = (r_R{}^+ + q_R{}^-)/\sqrt{2} \quad \pi^+ = \left(\begin{pmatrix}0\\u_{51}\end{pmatrix},\begin{pmatrix}0\\u_{51}\end{pmatrix},\begin{pmatrix}0\\u_{52}\end{pmatrix},\begin{pmatrix}0\\u_{52}\end{pmatrix}\right)/\sqrt{2}$$

and the 6 hc-boson $Ag'_{14691113}$, which are the 6 couplers of SU(4).
The interaction Lagrangian is the Fermi current-current interaction with the mediating exchange boson with the energy

$$m_X = E(W) = E(u_1) + E(u_2) + m_W + E(Ag_{nd})$$

$$L_{JJ}(J(u\bar{s}), J(\pi^+\pi^-\pi^+)) = \frac{\left(u_1{}^+\gamma^{\mu}u_2\right)\left(u_3{}^+\gamma_{\mu}u_3 + u_4{}^+\gamma_{\mu}u_4 + u_5{}^+\gamma_{\mu}u_5\right)}{m_X{}^2}$$, with the notation Dirac-conjugate $u_1{}^+$

So we have in total two particle configurations, the incoming $K^+ = u\bar{s}$ and the outgoing $\pi^+\pi^-\pi^+$, each with an interaction Lagrangian, coupled by the Fermi current-current interaction, and mediated by the corresponding W-boson $W = r_L{}^- r_R{}^-$,
In the incoming system $K^+ = u\bar{s}$ and the outgoing $\pi^+\pi^-\pi^+$ we have to take into account the color interaction of the quarks $L_C(u\bar{s}) = L_{QCD}(x^{\mu},\{u_1,u_2\},\{Ac_2,Ac_5,Ac_7\})$ and $L_C(\pi^+\pi^-\pi^+) = L_{QCD}(x^{\mu},\{u_3,u_4,u_5\},\{Ac'_2, Ac'_5, Ac'_7\})$ in the basic gluon configuration with 3 rgb-gluons .
The outgoing photon is active in the additional third electromagnetic Lagrangian $L_e(\pi^+\pi^-\pi^+) = L_e(x^{\mu},\{u_3,u_4,u_5\},\{Ae_0\})$

6.2 The calculation method

Now we minimize the action $S=\int L(x^{\mu},u_i,Ag_i)\,r^2\sin\theta\,dt\,dr\,d\theta\,d\varphi$ for the total Lagrangian $L(x^{\mu},u_i,Ag_i)=L(d\bar{u})+L_{JJ}(W)+L(J(d\bar{u}),J(W))+L_C(d\bar{u})$ under the constraint of energy conservation $E(d\bar{u})=E(W)$, as required in the Feynman diagram of the process.

We have for the particle wavefunctions $\{u_1,u_2,u_3\}$ the normalization condition $\int |u_i|^2 d^3x=1$ and for the field bosons we set up a boundary condition for $r=r_0$ $Ag_i(r_0)=0$ and $Ac_i(r_0)=0$ and the Lorenz-gauge-condition $\partial_{\mu}(Ag_i)^{\mu}=0$ and $\partial_{\mu}(Ac_i)^{\mu}=0$.

The energy, length, and time are made dimensionless by using the units: E($E_0=\dfrac{\hbar c}{1am}=0.196 TeV$) , $r(fm)$, $t(am/c)$ $am=10^{-18}m$. We can assume axial symmetry, so we can set $\varphi=0$ and use the spherical coordinates (t, r, θ) .

We choose the equidistant lattice for the intervals $(t,r,\theta)\in[0,1]\times[0,1]\times[0,\pi]$ with 21x21x11 points and, for the minimization n_{sub} in parallel, n_{sub} random sublattices of length l_{sub} , where n_{sub} =8 or 16, and l_{sub} = 25 or 50 or 100 according to the complexity of the corresponding Lagrangian.

$l[ix,j]=\{\{(t_{i1},r_{i2},t_{i3})\,|(i1,i2,i3)=random(lattice,\, j=1...l_{sub})\}\,|\,ix=1,...,n_{sub}\}$.

For the Ritz-Galerkin expansion we use the 12 functions $f_k(r,\theta)=\{bfunc(r,r_0,dr_0)\,r^{k_1},k_1=0,...,n_r\}\times\{(\cos^{k_2}\theta,\cos^{k_2}\theta\sin\theta),k_2=0,...,n_{\theta}\}$

The action $S=\int L(x^{\mu},u_i,Ag_i)\,r^2\sin\theta\,dt\,dr\,d\theta\,d\varphi$ becomes a mean-value on the sublattice $l[ix]$

$\tilde{S}[ix]=\dfrac{1}{N(l[ix])}\sum_{x\in l[ix]_{sub}}L(x,u_i,Ag_i)2\pi V_{tr\theta}$,where $V_{tr\theta}=\pi$ the (t,r,θ)-volume and $l_{sub}=N(l[ix])$ is the number of points. We impose the boundary condition for $Ag_i(r=r_0)=0$ via penalty-function (imposing exact conditions is possible, but slows down the minimization process enormously).

$\tilde{S}$ is minimized n_{sub} x in parallel with the Mathematica-minimization method "simulated annealing" .

The proper parameters of the particles u_i and the hc-bosons Ag_i are:

$par(p_i)=\{Eu_i,a_i,ru_i,\theta u_i,dru_i\}$, $par(Ag_i)=\{EA_i,aA_i\}$ $par(Ac_i)=\{EAc_i,aAc_i\}$

The complexities and execution times (on a 2.7GHz Xeon E5 work-station) differ greatly for different decays.

For the neutron decay $n\to p\,e\,\bar{\nu}$ with the scheme $d\bar{u}(1h)\to W\to W(1h)$ (1hc-boson on both sides) and color interaction $L(d\bar{u},3g)$ with basic 3 gluons complexity(Lagrangian)=(3.7+4.8)*10^6 terms, minimization time t(minimization)=111s .

The mathematical details of the calculation, and the results can be studied in depth in the corresponding Mathematica programs [46].

6.3 Discussion of calculated decays

The table of decays with calculation results for m_X is as follows

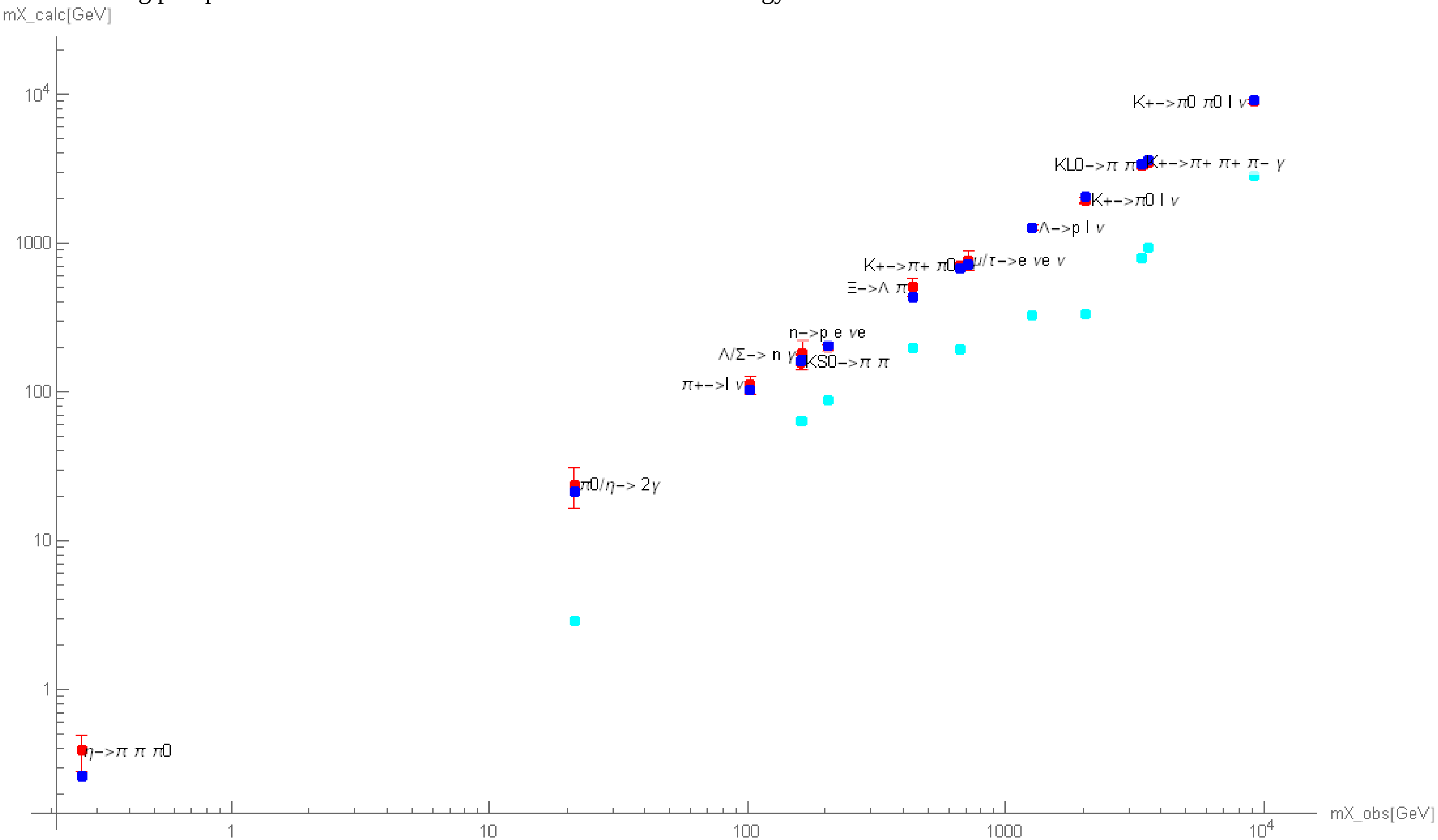
decay	P_{in}	P_{out}	Agi[am^{-1}] Aci Aei	<r12>	<dr12>	Ecol Eem	mX(er)cal	mX exp	scheme
Λ -> n π	uds	udd/(uu'-dd')						387.719	sd‘(2h)→Z→π0(2h)
Σ -> n π	uds	udd/(uu'-dd')						427.142	sd‘(2h)→Z→π0(2h)
Ξ ->Λ π	uss	uds/(uu'-dd')	{0.271,1.043}{0.258,0.348}	0.472	0.461	195(3g,3g)	505(71)	435.627	sd‘(2h)→Z→π0(2h)
π+ -> l ν	ud'	2rL-	{0.171,0.323}	0.195	0.559	0(0g)	112(16)	102.178	ud‘(1h)→W→W
K+->l ν	us'	2rL-						385.244	us‘(2h)→W→W
K+-> π+ π0	us'	ud'/(uu'-dd')	{0.314,0.219}{0.245,0.213}	1.13	1.34	194(3g,3g)	705(34)	667.317	us‘(4h)→W→π+π0(1h)
KS0-> π π	(ds'+sd')	2(uu'-dd')	{0.804,0.122} {0.225}	0.300	0.207	64(3g)	159(19)	160.647	ds‘(1h)→Z→2π0
K+-> π+ π π	us'	ds'/2(uu'-dd')						159.222	us‘(1h)→W→ π+ 2π0
KL0->π0 π π	ds'	3(uu'-dd')						163.886	ds‘(1h)→Z→ π0 2π0
KL0-> π π	(ds'-sd')	2(uu'-dd')						3381.22	ds‘(12h)→Z→2π0(4h)
K+->π0 l ν	us'	(uu'-dd')/W	{0.894,0.351} {0.249,0}	0.505	0.535	333(3g)	1940(89)	2034.04	us‘(6h)→W→ π0 W(6h)
KL0->π+ l ν	ds'	ud‘/W						1716.44	ds‘(6h)→Z→ π+ W(2h)
K+->π+ π- l ν	us'	2(uu'-dd')/W						7444.14	us‘(15h)→W→2π0 W(6h)
K+->π0 π0 l ν	us'	2(uu'-dd')/W	{0.889,0.365}{0.267,0.250}	0.449	0.555	2810(8g,8g)	8880(280)	9137.27	us‘(15h)→W→2π0 W(15h
KL0->π+ π0 l ν	us'	ud'/(uu'-dd')/W						7693.2	us‘(15h)→W→π+π0W(6h

								9	
K+-> π+ π+ π- γ	us'	ud'/(ud'+u'd)	{0.920,0.284}{0.278}0.036	0.987	1.59	930(3g,3g),640	3470(170)	3574.06	us‘(12h)→W→π+π-π+ γ(6
K+->π0 π+ γ	us'	(uu'-dd')/ud'						5707.21	us‘(12h)→W→π0π+γ(12h
Λ -> p *l* ν	uds	uud/ W	{0.205,0.135}{0.338,0}	0.245	0.513	328(3g)	1270(53)	1271.1	su‘(6h)→W→W
Σ- -> n *l* ν	dds	udd/ W						1713.08	su‘(6h)→W→W(2h)
μ/τ -> e νe ν	*l*	*l*	{0.857,0.122}	0.461	0.374		768(117)	717.06	*l ν*‘ (4h)→W→W
τ -> μ νμ νμ	*l*	*l*						695.878	*l ν*‘ (4h)→W→W
π+ -> π0 *l* ν	ud'	(uu'-dd')/ W						468.38	ud‘(2h)→W→W
n->p e νe	udd	uud/ W	{0.275,0.250} {0.221}	0.341	0.199	87(3g)	197(9.3)	204.69	du‘(1h)→W→W
Σ+ -> Λ *l* ν	uus	uds/ W						721.78	ud‘(4h)→W→W
η -> π0 π0 π0	(uu'+dd'-2ss')	3(uu'-dd')	{0.200,0.164}	0.270	0.384	(3g,3g)	.388(.109)	0.26186	sd‘(3g)→π0→3π0
η -> π0 π0 γ	(uu'+dd'-2ss')	2(uu'-dd')						7.4388	sd‘(6g)→π0→2π0 γ
π0/η -> γ γ	(uu'-dd')		{0,0}{0.153,0.156}0.065	0.284	0.250	(8g,8g)2.9	23.8(7.2)	21.221	uu‘(8g)→π0→π0 2γ
Λ/Σ -> n γ	uds	udd	{0.294,0.580}{0.757}0.667	0.680	2.742	64(3g) 9.7	181(41)	162.669	sd‘(1h)→Z→Z γ
Σ0 -> Λ γ	uss	uds						224.404	sd‘(2h)→Z→Z γ
Ξ0 -> Σ0 γ	uss	uds						131.511	sd‘(1h)→Z→Z γ
Ξ- -> Σ- γ	dss	dds						263.089	sd‘(2h)→Z→Z γ (2h)

The scheme column describes the model of the decay, on which the calculation is based, where the denomination *q’* is used for the antiparticle $\overline{q}$.
Here the calculation result (m_{Xcal}) and the value from decay time (m_{Xexp}) are given in GeV.
$m_X(er)$ is the calculated m_X-value with uncertainty *er* in GeV .
E_{col} specifies the calculated color interaction energy and the number of active gluons, e.g. 250(3g) , E_{em} is the electromagnetic energy of the involved photons, if any.

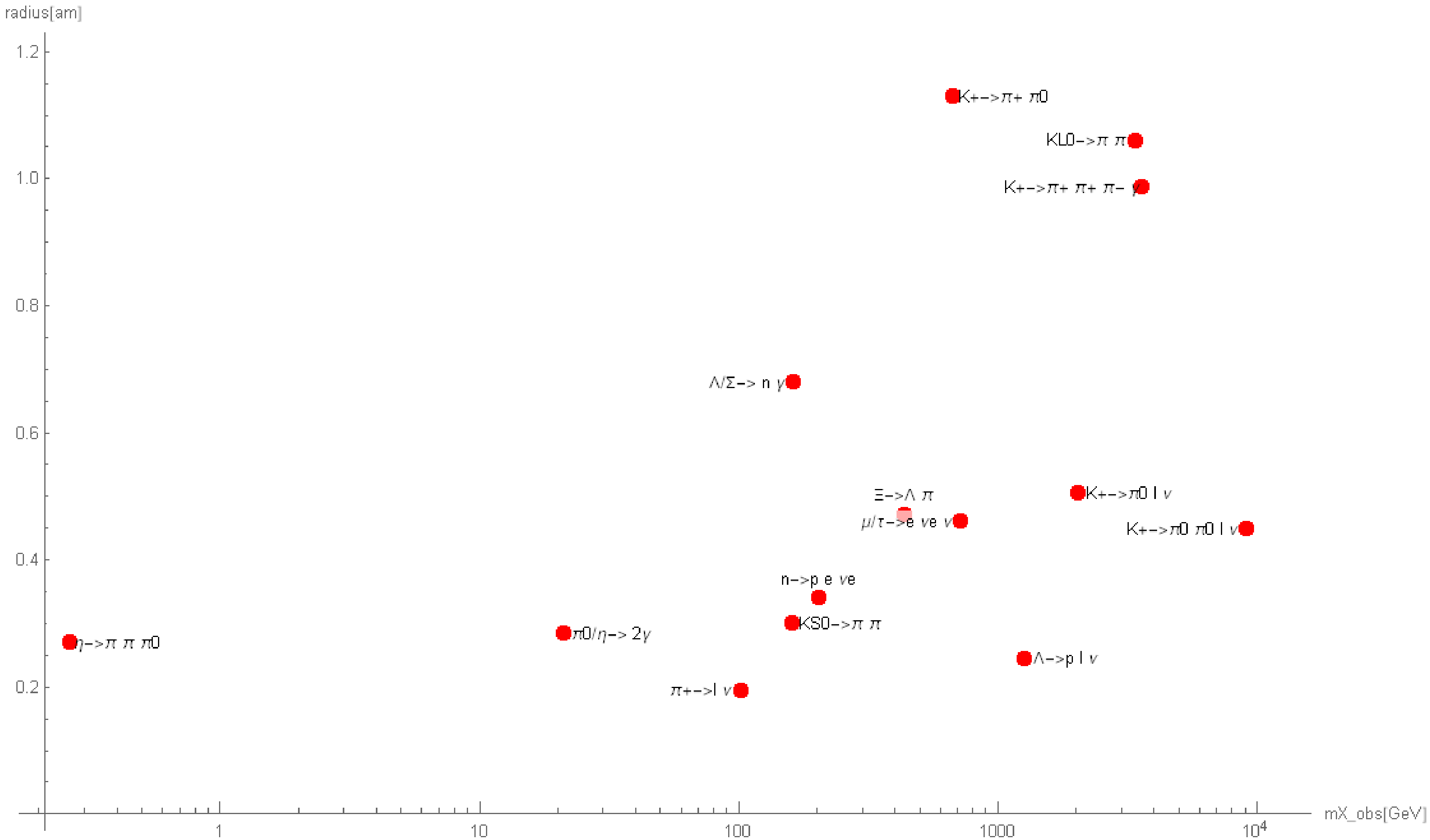
<r12> and *<dr12>* are the mean radius in *am*-units and its quantum “smear-out” in the left-side (incoming) part of the scheme.
The mean boson amplitude (hypercolor, color, electromagnetic) of the incoming and outgoing system *Agi Aci Aei* expressed in units am^{-1} is given in column four.

The following plot presents the measured and calculated interaction energy described in the above table



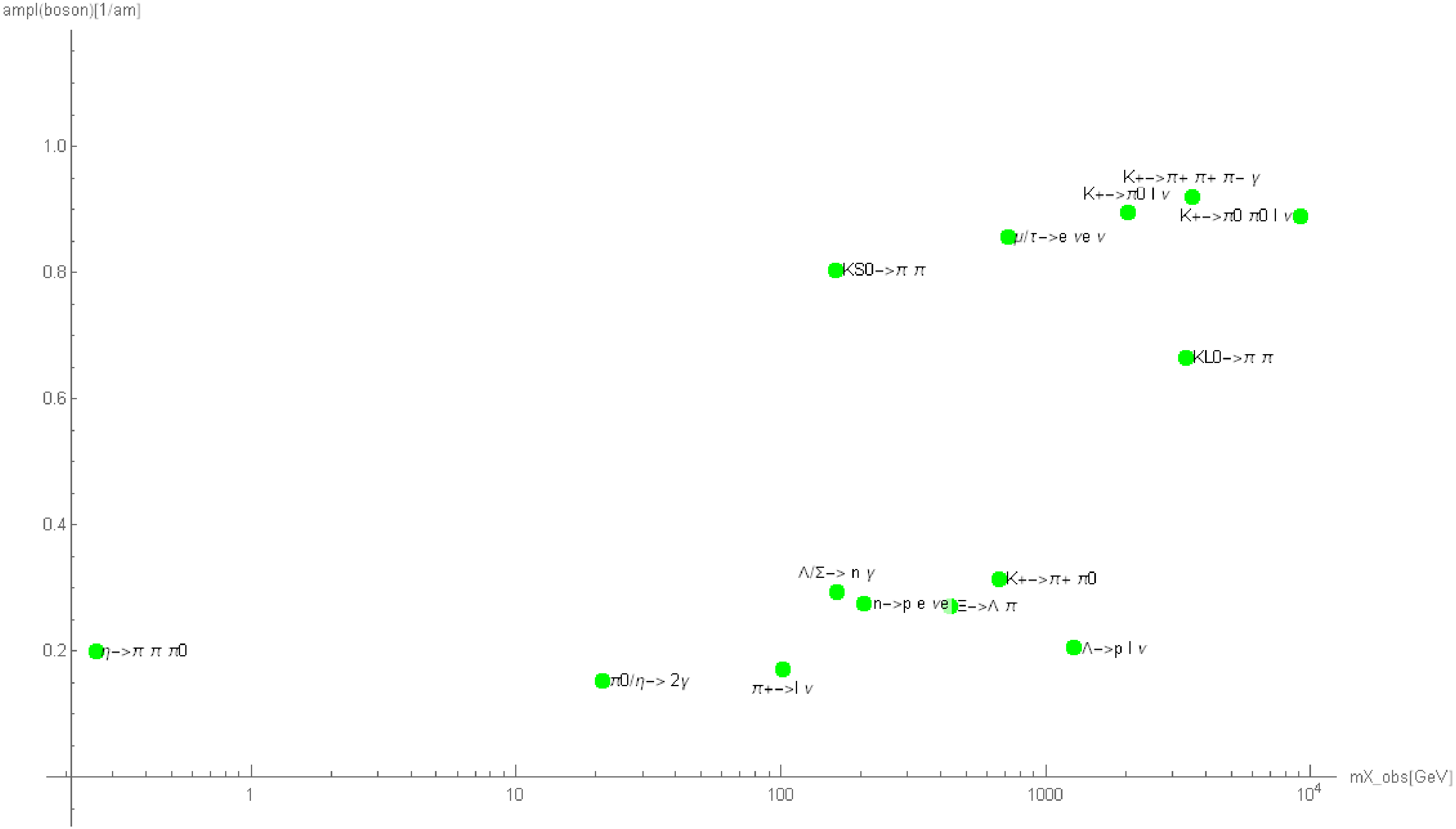
The decays above 110GeV are weak (hypercolor) decays, those below 12GeV are strong (color) decays, the observed m_X is dark-blue, the calculated m_X is red (with calculation error bar), the color energy for weak decays, respectively electromagnetic energy for strong decays is cyan.

Another interesting decay parameter is the mean radius *<r12>* of the incoming system on the left side of the scheme, e.g. for the neutron decay *n->p e ve*, the inoming system is $n\bar{p}$, in the decay scheme it is represented by $d\bar{u}$.



It is interesting to see, that the mean radius separates basically into two groups: high-energy non-leptonic kaon-pion decays with *<r12>>0.9am* and the remaining decays with *<r12> <0.5am*, apart from the photonic *Λ/Σ -> n γ* .

The other important decay parameter is the mean (hypercolor, color, electromagnetic) boson amplitude *Agi* for the weak decays, *Aci* for the color decays, of the incoming system, expressed in units am^{-1} .



Again, the amplitude separates into two groups, amplitude>=0.6 for the kaon-pion decays an pure leptonic decays, and the remaining with amplitude<=0.3, with the outlier *K+-> π+ π0*.

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