Analogy Between Special Relativity and Finite Mathematics

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Abstract

In our publications we have proposed an approach called finite quantum theory (FQT) when quantum theory is based not on complex numbers but on finite mathematics. We have proved that FQT is more general than standard quantum theory because the latter is a special degenerate case of the former in the formal limit $p \to \infty$ where p is the characteristic of the ring or field in finite mathematics. Moreover, finite mathematics itself is more general than classical mathematics (involving the notions of infinitely small/large and continuity) because the latter is a special degenerate case of the former in the same limit. **As a consequence, mathematics describing nature at the most fundamental level involves only a finite number of numbers while the notions of limit and infinitely small/large and the notions constructed from them (e.g. continuity, derivative and integral) are needed only in calculations describing nature approximately**. However, physicists typically are reluctant to accept those results although they are natural and simple. We argue without formulas that there is a simple analogy between the above facts and the fact that special relativity is more general than nonrelativistic mechanics because the latter is a special degenerate case of the former in the formal limit $c \to \infty$.

Keywords: finite quantum theory, finite mathematics, special relativity

In our publications [1] we have proposed an approach called finite quantum theory (FQT) when quantum theory is based not on complex numbers but on finite mathematics involving a ring or field of characteristic p. As argued in Refs. [2, 3] and others, FQT is more natural than standard quantum theory not only from mathematical but also from obvious physical considerations.

Indeed, the notions of infinitely small, continuity etc. were proposed by Newton and Leibniz more than 300 years ago. At that times people did not know about atoms and elementary particles. On the basis of everyday experience they believed that any macroscopic object can be divided into arbitrarily large number of arbitrarily small parts. However, from the point of view of the present knowledge those notions are problematic. For example, a glass of water contains approximately 10^{25} molecules. We can divide this water by ten, million, etc. but when we reach the level of atoms and elementary particles the division operation loses its usual meaning and we cannot obtain arbitrarily small parts.

The discovery of atoms and elementary particles indicates that at the very basic level nature is discrete. As a consequence, any description of macroscopic phenomena using continuity and differentiability can be only approximate. For example, in macroscopic physics it is assumed that spatial coordinates and time are continuous measurable variables. However, this is obviously an approximation because coordinates cannot be *directly* measured with the accuracy better than atomic sizes and time cannot be measured with the accuracy better than $10^{-18}s$, which is of the order of atomic size over c.

As a consequence, distances less than atomic ones do not have a direct physical meaning and in real life there are no continuous lines and surfaces. As an example, water in the ocean can be described by differential equations of hydrodynamics but this is only an approximation since matter is discrete. Another example is that if we draw a line on a sheet of paper and look at this line by a microscope then we will see that the line is strongly discontinuous because it consists of atoms.

Note that even the name "quantum theory" reflects a belief that nature is quantized, i.e. discrete. Nevertheless, when quantum theory was created it was based on classical mathematics developed mainly in the 19th century. One of the greatest successes of the early quantum theory was the discovery that energy levels of the hydrogen atom can be described in the framework of classical mathematics because the Schrödinger differential operator has a discrete spectrum. This and many other successes of quantum theory were treated as indications that all problems of the theory can be solved by using classical mathematics.

As a consequence, even after 90+ years of the existence of quantum theory it is still based on classical mathematics. Although the theory contains divergences and other inconsistencies, physicists persistently try to resolve them in the framework of classical mathematics. This situation is not natural but it is probably a consequence of historical reasons. The founders of quantum theory were highly educated scientists but they used only classical mathematics, and even now discrete and finite mathematics is not a part of standard mathematical education at physics departments. Note that even regardless of applications in physics, classical mathematics has its own foundational problems which cannot be resolved (as follows, in particular, from Gödel's incompleteness theorems) and therefore the ultimate physical theory cannot be based on that mathematics.

In Refs. [4, 5] we have proved mathematically that FQT is more general than standard quantum theory because the the latter is a special degenerate case of the former in the formal limit $p \to \infty$ where p is the characteristic of the ring or field in finite mathematics. Moreover, finite mathematics itself is more general than classical one (involving the notions of infinitely small/large and continuity) because the latter is a special degenerate case of the former in the same limit. **As a consequence, mathematics describing nature at the most fundamental level involves only a finite number of numbers while the notions of limit and infinitely small/large and the notions constructed from them (e.g. continuity, derivative and integral) are needed only in calculations describing nature approximately**.

Although the proofs of those facts are rather simple, in my discussions with physicists they gave different objections against the approach when quantum theory is based on finite mathematics. I believe that before discussing those objections it would be instructive to consider typical objections against special relativity (SR). Although SR is now accepted by majority of physicists, some of them still have doubts that (if tahyons are not considered) speed of light c is the maximum possible speed for any particle. Typical objections are as follows.

A_1) It seems unnatural that velocity $0.999c$ is allowed while velocity $1.001c$ is not. This objection arises as a consequence of the fact that it is not consistent to extrapolate the everyday experience with velocities much less than c to the area where they are comparable to c.

B_1) Let the reference frame K is moving relative to the observer O along the positive direction of the x axis with the speed $V = 0.6c$. Suppose also that, in this reference frame a particle is moving in the same direction with the speed $v_K = 0.6c$. Then, from the point of view of standard experience, one might think that the particle is moving relative to O with the speed $v = V + v_K = 1.2c$ while the formula of SR for addition of velocities gives $v \approx 0.882c$. Analogously, if $V = 0.99c$ and $v_K = 0.99c$ then SR gives not $v = 1.98c$ but $v \approx 0.9999495c$. The reason of the fictitious inconsistency of SR is again that it is not correct to extrapolate our experience with velocities much less than c to the area where they are comparable to c.

C_1) It is not clear why $c \approx 3 \cdot 10^8 m/s$ and not say $c = 10^9 m/s$. However, as explained e.g. in Ref. [5], relativistic quantum theory itself does not need the value of c at all. The value of c in m/s arises because people's choice is to measure velocities in such units, and the question why c is as is does not arise. In the modern system of units it is postulated that the value of c in m/s does not change with time. However, as discussed in Ref. [4], this postulate is not based on rigorous physical principles.

As shown in Refs. [4, 5], SR is more general than classical mechanics (CM) for the following reasons. The latter is a special degenerate case of the former in the formal limit $c \to \infty$. This means that SR can reproduce any result of CM if c is chosen to be sufficiently large. On the other hand, since c is finite, in SR there are phenomena which cannot be reproduced by CM, i.e. they cannot be reproduced when the limit $c \to \infty$ is already taken.

Consider now the relationship between finite mathematics and classical one (involving the notions of infinitely small/large and continuity). In FQT all physical quantities are elements of a ring or field with characteristic p. This means that all operations are modulo p and no physical quantity can be greater or equal than p. So there is an analogy with SR where speed of any particle cannot be greater than c. If we accept that FQT describes nature, a problem arises what the value of p is. In Ref. [4] we have derived the approximate expression for the gravitational constant which depends on p as $1/lnp$. By comparing this expression with the experimental value we get that lnp is of the order of 10^{80} or more, i.e. p is a huge number of the order of $exp(10^{80})$ or more. However, since lnp is "only" of the order of 10^{80} or more, the existence of p is observable while in the formal limit $p \to \infty$ gravity disappears.

In my discussions with physicists, they gave objections against FQT analogous to those in $A_1) - C_1)$. Those objections are as follows.

A_2) By analogy with A_1) one might think that it is unnatural that a value of a quantity can be $p-1$ but cannot be $p+1$. However, it is not consistent to extrapolate our experience with numbers much less than p to the area where the numbers are comparable to p.

B_2) If $n_1 = n_2 = p/2 + 10$ then one might think that $(n_1 + n_2)$ exceeds p. However, in finite rings or fields of characteristic p, all operations are modulo p, and therefore the result of any operation cannot exceed p.

C_2) By analogy with C_1) one might pose a question why p is as is. According to FQT, nature is described by finite mathematics with some value of p. A problem whether p is a fundamental quantity which is always the same during the whole history of the Universe or this quantity can be different at different periods of the history. The first possibility seems unrealistic because it is not clear what was the reason for nature to prefer a particular value of p. Note that the problem of time is one of the most fundamental problems in quantum theory. In Ref. [3] we discussed a conjecture that not p changes with time because time is only a classical notion but vice versa the manifestation of time is a consequence of the fact that p changes.

The facts that FQT is more general than standard quantum theory and finite mathematics is more general than classical one have been proved in Refs. [4, 5]. Finite mathematics can reproduce description of phenomena obtained in classical mathematics because p is huge and for those phenomena it is not important whether p is finite or infinite. On the other hand, there are phenomena where it is important that p is finite. Those phenomena are discussed in Refs. [4], and, as noted above, one of those phenomena is gravity.

In summary, the facts that FQT is more general than standard quantum theory and finite mathematics is more general than classical one are analogous to the fact that SR is more general than CM.

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References

[1] F. Lev, *Modular Representations as a Possible Basis of Finite Physics*. J. Math. Phys. **30**, 1985–1998 (1989); *Finiteness of Physics and its Possible Consequences*. J. Math. Phys. **34**, 490–527. (1993).

[2] F. Lev, *de Sitter Symmetry and Quantum Theory*. Phys. Rev. **D85**, 065003 (2012).

[3] F. Lev, *Finite Mathematics, Finite Quantum Theory and a Conjecture on the Nature of Time*. Phys. Part. Nuclei , **50**, 443–469. (2019).

[4] F. Lev, *Finite Mathematics, Finite Quantum Theory and Applications to Gravity and Particle Theory*. arXiv:1104.4647. (2019).

[5] F. Lev, *Symmetries in Foundation of Quantum Theory and Mathematics*. Symmetry **12**(3), 409 (2020).