Quantum-mechanical derivation of the formula of mass–energy equivalence of A. Einstein.

Bezverkhniy Volodymyr Dmytrovych, Bezverkhniy Vitaliy Volodymyrovich.
Ukraine, e-mail: bezvold@ukr.net

Abstract: Using energy quantization according to Planck and de Broglie wave-particle duality, the Einstein mass and energy equivalence formula is theoretically rigorously derived. The conclusion of this formula is also given on the basis of the concepts of classical physics and the principle of mass equivalence. It is shown that the equivalence of invariant and gravitational masses in the microworld manifests itself as the duality of the wave and particle, which expresses the equivalence of kinetic and potential energies. But, since the equivalence of the particle and wave properties of microparticles has long been proven experimentally, therefore, the principle of mass equivalence can be considered strictly proved.

Keywords: Planck energy quantization, Einstein's formula of mass–energy equivalence, de Broglie wave-particle duality, principle of mass equivalence, invariant mass, gravitational mass.

INTRODUCTION.

Using the M. Planck formula (E = h*γ) [1] and the Louis de Broglie formula (λ = h/(m*Vg)) [2], one can analytically rigorously derive A. Einstein's mass-energy equivalence formula [3, 4]

$$E = m * c^2$$

That is, Einstein's formula can be obtained on the basis of the fundamental foundations of quantum mechanics (the hypothesis of quanta, and particle-wave duality of microparticles). First we give this conclusion, and then continue the discussion. We also provide the theoretical conclusion of this formula on the basis of the concepts of classical physics, and the principle of equivalence of invariant and gravitational masses, and show what conclusions this leads to.

RESULTS AND DISCUSSION.

So, we will write the famous Max Planck formula, which expresses the quantization of energy, and in fact with which the era of quantum mechanics began:

$$E = h * \gamma$$

And now we write the frequency of the radiation through the speed and wavelength.

$$Vf = \gamma * \lambda$$

where Vf - phase velocity of a wave.

This implies:

$$\gamma = Vf / \lambda$$

And, therefore, according to the Planck formula, we get:

$$E = h * \gamma = h * Vf / \lambda$$

$$E * \lambda = h * Vf$$

The resulting formula is key

$$E * \lambda = h * Vf$$

since substituting the wavelength into it from the de Broglie formula, we strictly obtain the Einstein formula.

Recall the de Broglie formula:

$$\lambda = h / (m * Vg)$$

where m - mass of the microparticles,

Vg - group velocity of a wave, or in other words the speed of a microparticle.

Substituting into the formula

$$E * \lambda = h * Vf$$

de Broglie wavelength we get:

$$E * \lambda = h * Vf$$

$$(E * h) / (m * Vg) = h * Vf$$

$$E / (m * Vg) = Vf$$

$$E = m * Vg * Vf$$

That is, we got the Einstein mass-energy equivalence formula in the form:

$$E = m * Vg * Vf$$

where m - mass of the microparticles,

Vg - group velocity of a wave, or microparticle velocity,

Vf - phase velocity of a wave.

But, consider two facts:

1. The speed of light in vacuum is the maximum (limiting) speed for microparticles, that is, for the group velocity of a wave.
2. If a wave carries energy, then its phase velocity cannot be greater than the speed of light in a vacuum.

Therefore, in the extreme case, the formula obtained by us

$$E = m * Vg * Vf$$

goes into the usual Einstein formula:

$$E = m * Vg * Vf = m * c^2$$

$$E = m * c^2$$

Once again, we emphasize that the above conclusion is strict at every stage, which confirms the correctness of Einstein's formula. In the historical aspect, events developed a little differently:

1. First, in 1900, Planck proposed his formula for quantizing energy (E = h * γ) [1].
2. Further, in 1905 - 1907, Einstein proposed his formula for mass-energy equivalence in the modern interpretation (E = m * c^2) [3, 4].
3. And only in 1923 - 1924, Louis de Broglie proposed his formula for wave-particle duality (λ = h / (m * Vg)) [2]. And, using Einstein's formula (E = m * c^2) in his reasoning, de Broglie obtained that Vg * Vf = c^2, that is, the phase velocity of de Broglie waves will always be greater than the speed of light in vacuum. But, this is not surprising, since de Broglie waves do not transfer energy (in the classical sense).

But, as you can see, Einstein’s formula and de Broglie’s formula are “interchangeable” (if I may say so): if de Broglie’s formula were first proposed, then Einstein’s formula would have come out of it automatically. Naturally, this would also take years of research and reflection by scientists, but on paper everything is fast.

Thus, the conclusion clearly shows that the Einstein formula (E = m * c^2) is a formula that expresses the fundamental foundations of quantum mechanics. This is the very essence of quantum mechanics. But, all the more surprising, since it can be obtained on the basis of the concepts of classical physics, and the principle of equivalence of invariant and gravitational masses. For further reasoning and conclusions, we give another conclusion of Einstein's formula.

We give the second conclusion of the Einstein's formula.

Let us have a body of mass m, and it go a distance ΔL in a time Δt. Moreover, at the initial point of the path, its speed is zero, and at the end point of the path its speed is V.

V=0, 1----------------------ΔL--------------------------------2, V

That is, according to Newton's second law, a constant force F = m*a acts on a body of mass m. And, passing beyond Δt, the path ΔL, the body acquires a speed V.

$$F = m * a$$

where a - acceleration of a body of mass m, a = const.

$$a = \Delta V/ \Delta t = (V - 0)/ \Delta t = V / \Delta t$$

$$a = V / \Delta t$$

$$F = m * a = (m*V) / \Delta t$$

$$F = (m*V) / \Delta t$$

That is, we can argue that on the path ΔL, the force F does the work of A.

$$A = F * \Delta L$$

We substitute the expression for the force F = (m * V) / Δt, and we obtain the formula:

$$A = F * \Delta L = (m * V * \Delta L) / \Delta t$$

Now we take into account the fact that the expression ΔL / Δt is the average velocity of the body (Va) along the path ΔL.

From here:

$$A = (m * V * \Delta L) / \Delta t$$

$$Va = \Delta L / \Delta t$$

$$A = m * V * Va$$

And given that mechanical work is the energy spent on moving the body, we actually get the formula:

$$E = m * V * Va$$

where V — is the maximum speed of the body at the end of the path,

Va – average speed of the body,

m – body weight.

E — energy that must be given to the body in order to overcome the inertia of this body, that is, to accelerate the body to a certain speed (give the body a certain impulse). That is, in this case, we consider the invariant mass, and the energy of the body is the kinetic energy of the body.

But, since the speed of light in vacuum is maximum, and the body cannot overcome it (let's say that the body has reached the speed of light in vacuum), starting from a certain point in time, the speed of the body will be equal to the speed of light, that is, the speed of the body will be constant and will be equal to the speed of light in vacuum (V = const, V = c). And this means that the average speed will also be equal to the speed of light in a vacuum.

$$V = c$$

$$Va = c$$

That is, from the formula E = m * V * Va, we get the usual Einstein formula (V = c, Va = c) for the invariant mass, in fact for the limiting kinetic energy of the body:

$$E = m * V * Va = m * c * c$$

$$E = m * c^2$$

It is absolutely obvious that if the body has the speed of light in a vacuum, then its inertia will be completely overcome. But, according to the principle of mass equivalence [5], invariant mass (i.e., inertial mass) is completely equivalent to gravitational mass. And this means that the gravitational mass also contains a maximum of energy (already potential energy), which is determined by the well-known Einstein's formula

$$E = m * c^2$$

Since both the maximum potential energy of the body (gravitational mass) and the maximum kinetic energy of the body (invariant mass) are described by Einstein's formula:

$$E = m * c^2$$

then, obviously, at the speed of light in vacuum (as well as in quantum interactions), the difference between the kinetic and potential energy disappears.

Gravitational mass is the mass that determines the gravitational field of this body, therefore, by definition, the gravitational mass determines the potential energy of the body. The fact that the gravitational mass determines the maximum potential energy of the body, and the invariant mass determines the maximum kinetic energy of the body, follows from simple considerations. We give them.

Potential energy is determined by the placement of bodies in space. Therefore, this is energy, which depends on the coordinates of space. It can be argued that potential energy is “coordinate energy” (“energy of space”). The gravitational mass (corpuscle) in space is characterized precisely by coordinates. Therefore, the gravitational mass determines the potential energy of the body.

Kinetic energy is the energy of motion, that is, it is energy that is determined by the momentum of the body (or its speed). Kinetic energy is the "energy of momentum" ("energy of speed"). The main characteristic of an invariant mass is precisely its velocity. Therefore, the energy of the invariant mass is kinetic energy, that is, energy that is determined by the speed of the body (more precisely, the momentum of the body).

CONCLUSION.

Now let's move on to the microworld.

In the microworld, by the impulse of body (or speed) determine the wave properties. Naturally, the wave is determined by the speed of motion (impulse). Therefore, in the microworld, the equivalence of invariant and gravitational masses manifests itself as wave-particle duality. In essence, this expresses equivalence in quantum mechanics of potential and kinetic energies.

So, in quantum mechanics, the corpuscle determines the potential energy (in the classical world it is the gravitational mass).

The wave properties of microparticles determine the kinetic energy (momentum) (in the classical world this is an invariant mass).

But, since according to the principle of mass equivalence, the invariant mass is completely equivalent to the gravitational mass, therefore, the wave properties of microparticles are completely equivalent to the corpuscular properties of microparticles. That is, proceeding from the principle of mass equivalence, we come to the wave-particle duality of microparticles. But, the equivalence of the particle and wave properties of microparticles has long been proven experimentally in quantum mechanics, therefore this proves the equivalence of invariant and gravitational masses. That is, the principle of mass equivalence, introduced into physics by Einstein, can be considered strictly proved. Moreover, quantum mechanics (particle-wave dualism) proved it!

REFERENCES.

1. Planck M. On the Law of Distribution of Energy in the Normal Spectrum. Annalen der Physik, vol. 4, p. 553 (1901).

2. Louis de Broglie. Selected Works. Volume 1. The formation of quantum physics: the work of 1921-1934. Moscow, Logos, 2010. (Lochak G. Prince in science. Chapter 4. Works of 1923. Dissertation. Wave Mechanics. P. 60 - 67).

3. Einstein A. Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen. Jahrbuch der Radioaktivität. 1907. Vol. 4. P. 411—462.

4. Einstein A. Berichtigung zu der Arbeit: «Uber das Relativitätsprinzip und die aus demselben gezogenen Folgerungen». Jahrbuch der Radioaktivität. 1907. Vol. 5. P. 98—99.

5. Einstein A. On the special and general theory of relativity (accessible presentation). Einstein A. Collection of scientific papers in 4 volumes. Moscow, Nauka, 1965. Volume 1, p. 563.