A 4x4 rank-2 space-time matrix unifying gravity, electromagnetism, strong, and weak interaction by integrating curvature and torsion in geometry

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Abstract
This article deducts that electromagnetic field, strong interaction, and weak interaction are actually torsion tensors. Since we know gravity field is actually a curvature tensor, we can use geometry to unite gravity, electromagnetism, strong, and weak interaction. Perfect fluid combines with Faraday tensor with geometrized unit can give a 4x4 rank2 spacetime matrix to include the above force field. In addition, strong interaction mediating gluons can acquire mass based on Higgs mechanism. This article also provides evidence of Yang-Mills theory existence and solves the mass gap problem in strong interaction.

Curvature and Torsion are two major basic components of differential geometry which stands for gravity and electromagnetism, respectively. In fact, physic is geometry.
Torsion tensor is：

$$T(x,y) = \nabla_x Y - \nabla_y X - [x,y]$$

If it is holonomic, the torsion tensor becomes:

$$T(x,y) = \nabla_x Y - \nabla_y X$$

And torsion satisfies

$$\Theta = d\theta + \omega \wedge \theta$$

Curvature tensor is:

$$R(x,y)z = \nabla_x \nabla_y Z - \nabla_y \nabla_x Z - \nabla_{[x,y]} Z$$

If it is holonomic, the torsion tensor becomes:

$$R(x,y)z = \nabla_x \nabla_y Z - \nabla_y \nabla_x Z$$

And curvature satisfies

$$\Omega = d\omega + \omega \wedge \omega$$

Holonomic means the Lie bracket vanishes, and it means gravity and electromagnetism are conservative force.

We can use curvature form and torsion form to restore curvature and torsion. If there is a point u in F_xM, then

$$T(x,y) = u[2\Theta(\pi^{-1}(x), \pi^{-1}(y))]$$

$$R(x,y)z = u[2\Omega(\pi^{-1}(x), \pi^{-1}(y))][u^{-1}(z)]$$

And, Bianchi identities link torsion and curvature in geometry. If cyclic sum is G, then

$$G(R(x,y)z) = R(x,y)z + R(y,z)x + R(z,x)y$$

The First Bianchi identity：

$$G(R(x,y)z) = G[T(T(x,y),z) + (\nabla_x T)(y,z)]$$

The Second Bianchi identity：

$$G[(\nabla_x R)(y,z) + R(T(x,y),z)] = 0$$

Thus, torsion tensor and curvature tensor can be linked together.

In my previous article, I proposed that radiation pressure (universal lightity) is the best candidate of dark energy which causes universe expansion. Because gravity, electromagnetism, and lightity are all mediated by space-time, the three fundamental forces can be united in one equation. I call this a Grand-unified Field Theory.

In electroweak theory, electromagnetic radiation is a U(1) group. The electromagnetic radiation can also be expressed by a rank-2 tensor:

$$Tuv = \frac{\delta Au}{\delta Xv} - \frac{\delta Av}{\delta Xu}$$

(Au and Av are electro-vector potentials)

In summary, we can list the three major tensors by definition:

Gravitospinity tensor by Einstein:

$$Guv = K * Euv = Ruv - \frac{1}{2} g_{uv} R$$

Faraday(electromagnetic) tensor

$$Fuv = \frac{\delta Au}{\delta Xv} - \frac{\delta Av}{\delta Xu}$$

by U(1) Abelian group

Why is torsion tensor equal to electromagnetic tensor (Faraday tensor)? We can also derive it from the definition of torsion tensor and Faraday tensor.[1] By definition, torsion tensor is:

$$T^k_{uv} = A^k_{uv} - A^k_{vu} - \gamma^k_{uv}$$

If the basis is holonomic, then the Lie bracket vanishes. It means that ϒ^k_uv=0. Because Coulomb electromagnetic force is conservative force, the force is path independent and is only associated with the beginning state and end state of a charge. Thus, Coulomb electromagnetic force is holonomic. Then, the torsion tensor becomes:

$$\mathrm{T}^{\mathrm{k}}_{\mathrm{uv}} = \mathrm{A}^{\mathrm{k}}_{\mathrm{uv}} - \mathrm{A}^{\mathrm{k}}_{\mathrm{vu}}$$

By differentiation, form covariant terms of second rank:[1]

$$\mathrm{Auv} = \frac{\delta \mathrm{Au}}{\delta \mathrm{Xv}} - \{\mathrm{uv}, \mathrm{t}\}\mathrm{At}$$

Auv is extension (covariant derivative) of the tensor Au

The second term in the above equation is symmetrical in the indices u and v. Thus,

$$\mathrm{Tuv} = \mathrm{Auv} - \mathrm{Avu} = \frac{\delta \mathrm{Au}}{\delta \mathrm{Xv}} - \frac{\delta \mathrm{Av}}{\delta \mathrm{Xu}}$$

By the definition of Faraday tensor, electromagnetic tensor F is equal to:

$$\mathrm{Fuv} = \frac{\delta \mathrm{Au}}{\delta \mathrm{Xv}} - \frac{\delta \mathrm{Av}}{\delta \mathrm{Xu}}$$

Thus, Fuv=Tuv. Faraday tensor is torsion tensor. It is worth noting that this torsion tensor can be reduced as a two rank covariant tensor(vector valued 2-form). If we say whirlpool line is mataching torsion tensor, it is logically intact. Every torsion tensor component (6 components) has each Faraday tensor component(6 components: Ex Ey Ez Bx By Bz) to match. Thus, charge can recognize spacetime structure in XY, YZ, and ZX planes and causes a 3D vortex structure.

From the above equations, we can get electromagnetic field tensor (F). It is also because vector Cu represents the generalized electromagnetic potential. If we say electromagnetism causes space-time structure spiral formation, then it is very reasonable to use torsion tensor to describe electromagnetic tensor since vortex lines are aligned with torsion tensor. We can find similarity because both Faraday tensor and torsion tensor are anti-symmetric tensors. Although one may argue that torsion tensor can have 24 components, I think each torsion tensor(6 components) matches each Faraday tensor(6 components; Ex,Ey,Ez,Bx,By,Bz) since charge can recognize the space-time structure orientation and give one 3D spiral structure in xy, yz, and xz planes.

By definition, A^u=(ϕ/c,A) and A_u=(ϕ/c,-A). And,

$$\mathrm{E} = -\frac{\partial \mathrm{A}}{\partial \mathrm{t}} - \nabla \varphi$$

$$\mathrm{B} = \nabla \mathrm{x} \mathrm{A}$$

$$\text{if Fuv} = \partial \mathrm{uAv} - \partial \mathrm{vAu}$$

$$\mathrm{F}_{10} = \partial_1 \mathrm{A}_0 - \partial_0 \mathrm{A}_1 = \mathrm{Ex/c}$$

$$\mathrm{F}_{12} = \partial_1 \mathrm{A}_2 - \partial_2 \mathrm{A}_1 = \mathrm{Bz}$$

And so on! We can get Ey, Ez, Bx, By are all torsion tensors. Thus, both electric field (E/c) and magnetic field (B) are torsion tensors which can cause space-time torsion!

Because Faraday tensor is an anti-symmetric tensor, F^T=-F. If positive charge stands for F, then negative charge stands for -F (F^T=-F). Positive and negative charges are transposes to each other. And, every anti-symmetric tensor can undergo Cayley transform.

Transformed positive charge F is:

$$\mathrm{R} = (I - F)(I + F)^{-1}$$

Transformed negative charge -F is:

$$\mathrm{R}' = (I + F)(I - F)^{-1}$$

We can get R*R'=I. By definition, positive charge and negative charge are matching to two rotation matrices with opposing rotation direction. It can help to prove the concept of charge relativity.

If we use geometrized unit with c=1, then the electromagnetic tensor becomes:

$$\mathrm{Fuv} = \begin{bmatrix} 0 & Ex & Ey & Ez \\ -Ex & 0 & -Bz & By \\ -Ey & Bz & 0 & -Bx \\ -Ez & -By & Bx & 0 \end{bmatrix}$$

The eigen values of the above matrix is:

$$(iE, -iE, iB, -iB)$$

$$E^2 = E_x^2 + E_y^2 + E_z^2$$

$$B^2 = B_x^2 + B_y^2 + B_z^2$$

Similar to Einstein's universe field equation (-Guv=Ruv-1/2g_uvR), I propose:

$$\mathrm{Tuv} = \mathrm{Guv} + \mathrm{Fuv}$$

$$\mathrm{Tuv} = \begin{bmatrix} \rho & Ex & Ey & Ez \\ -Ex & -Px & -Bz & By \\ -Ey & Bz & -Py & -Bx \\ -Ez & -By & Bx & -Pz \end{bmatrix}$$

In addition, we can get contracovariant tensor:

$$T^{uv} = \begin{bmatrix} \rho & -Ex & -Ey & -Ez \\ Ex & -Px & -Bz & By \\ Ey & Bz & -Py & -Bx \\ Ez & -By & Bx & -Pz \end{bmatrix}$$

Here,ρ is mass-energy density, P is light pressure, E is electric field, B is magnetic field, and xyz are standing for the vector direction of xyz axis. The above entitities are using Geometrized Unit System: c=8πG=k=e=1 to let each unit to be $L^{-2}$and using the concept of perfect fluid in symmetric part with Faraday tensor in anti-symmetric part:

$$\mathrm{Guv} = \left(\rho_m - \frac{P}{c^2}\right) UuUv - Pg_{uv}$$

Uv=(c,0,0,0) and g_uv=(-1,1,1,1), $\rho \quad = \rho_m c\hat{}2$

In a rank-2 4x4 tensor, we know:

$$T^{uv} * T_{uv} = I$$

The covariant and contracovariant tensors are inverse of each other. And, we solve the equation and get:

$$\rho = -P_x = -P_y = -P_z$$

This solves why energy density is minus-equal to pressure based on current observation. And,

$$E_x^2 = B_x^2$$
$$E_y^2 = B_y^2$$
$$E_z^2 = B_z^2$$

The inner product of the above two tensors is:

$$T^{uv}T_{uv} = \rho^2 + P^2 = 4\rho^2$$
$$P^2 = P_x^2 + P_y^2 + P_z^2$$

The determinant of the above tensor is:

$$Det(T) = \rho^4 + E^4$$

The trace of the above tensor is:

$$Tr(T) = 4\rho$$

The trace decides the fate of our universe. Based on Friedmann equation, the universe fate can be decided by the following:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P)$$

And, the universe equation state constant is the ratio of energy density and pressure:

$$\mathrm{P} = \omega\rho$$

When ω<-1, phantom energy will cause big rip; when ω>-1/3, it will cause big crunch. Only when the constant between the above values, the universe will become an accelerating expansion quintessence which is our universe.

And the eigen values of the above matrix is:

$$(\rho + Ee^{i(\pi/4)}, \rho - Ee^{i(\pi/4)}, \rho + Ee^{i(-\pi/4)}, \rho - Ee^{i(-\pi/4)})$$

We can compare the above eigen values to annihilation and creation operators of matter and anti-matter in the quantum field theory. Annihilation and creation operators are complex conjugate. And, anti-matter and matter have same mass and opposite charge.

It is worth nothing that the original Einstein field equation is derived from Bianchi identity with vanishing torsion. Since the Fuv and Tuv cancel each other, the net effect of torsion is vanished. Einstein provided the relativity of Energy, INertia and SpaceTime. Einstein's field equation is only valid with vanished torsion. Thus, Einstein's deduction is reliable and correct.

Due to Einstein's deduction in his book: the meaning of general relativity:

R=g^uvRuv and g_uv*g^uv=4 in four dimensional space-time

Introduce this, we can get:

$$\mathrm{Tuv} = \mathrm{Guv} + \mathrm{Fuv}$$

$$\mathrm{Ruv} = \mathrm{Guv} = \mathrm{K} * \mathrm{Euv} = \left(\frac{\mu}{\mathrm{c}^2}\right)\mathrm{Euv}$$

(Ruv=Ricci-Riemann curvature tensor; means sphere surface)

Guv is "G"ravitospinity tensor, Fuv is "U"(1) "F"araday electromagnetic tensor, Tuv is u(1) "T"emperature-lightity tensor. It is worth noting that Einstein originally introduced a cosmological constant in his Einstein field equation because he assumed that our universe is a static universe. However, Hubble observed that our universe is actually accelerated expansion. Thus, the item of cosmological constant should be omitted. Professor Friedmann omitted this cosmological constant in Einstein field equation, and he found the solutions of this equation means our universe is either expanding or contracting. This fits the observation of our expanding universe.

Ricci flow is a concept used by Grigori Perelman to prove Poincare conjecture. In fact, the formula of Ricci flow is identical to Einstein field equation:

$$\text{Ricci flow} = \text{Ruv} - \frac{1}{2}g_{uv}\text{R}$$

By the characteristics of Ricci flow, positive Ricci curvature can contract to a point and negative Ricci curvature can expand in terms of Einstein manifold (Ricci=constant*metric g_uv).

Since

$$\text{Ruv} - \frac{1}{2}g_{uv}\text{R} = \text{K} * \text{Euv}$$

Energy-momentum tensor

$$\text{Euv} = \rho\text{UuUv} - \text{p}g_{uv}$$

(ρ=mass density, Uu or Uv=velocity, p=pressure=radiation pressure, g_uv=metric tensor)

Since radiation has both characteristics of mass($E=mc^2$) and charge(EM radiation), it should has one tensor included in mass-like energy-momentum tensor and the other tensor included in charge-like temperature-lightity tensor.

Thus,

$$\text{Ricci flow} = \text{Ruv} = \text{Guv} = \text{K} * \text{Euv} = \text{K} * (\rho\text{UuUv} - \text{p}g_{uv})$$

When mass is dominant than radiation, ρUuUv is greater(>>) than pg_uv. Then, the positive Ricci curvature will contract due to Ricci flow. When radiation is dominant than mass, ρUuUv is less(<<) than pg_uv. Then, the negative Ricci curvature will expand to its maximum. This formula can prove our universe is radiation dominant expanding universe.

Radiation pressure P is $\sigma T^4/c$ which is in proportion to the fourth power of acceleration. Our universe is accelerated expanding!

G mediates space-time curvature and F mediates space-time torsion. G and F cause matter to gather together. It is a new universe field equation.

Currently, standard model is a successful theory to describe fundamental forces and related particles. It successfully predicts the masses of Z and W particles. It united electromagnetism, weak force, and strong force in the equation: U(1)xSU(2)xSU(3). Currently, U(1) is quantum electrodynamics(QED) mediated by photon transfer. However, I proposed that QED is wrong and photons are the actual mediators of radiation pressure. Thus, U(1) should be radiation pressure(universal lightity) which can explain the Casimir force. Thus, the new U(1)xSU(2)xSU(3) unites dark energy, weak force, and strong force. The basic structure of the unification is not changed. U(1) is still a gauge theory mediated by one dimensional photon boson. SU(2) is also a gauge theory mediated by W+, W-, and Z bosons(3 dimensional=2^2-1). SU(3) is also a gauge theory mediated by 8 gluon bosons(8 dimensional=3^2-1). Thus, the new standard model equation is given by:

Lagrangian

$$\mathrm{L} = \int \left(\frac{1}{\mathrm{y}^2}\right) \mathrm{Y}_{\mathrm{uv}}\mathrm{Y}^{\mathrm{uv}} + \left(\frac{1}{\mathrm{w}^2}\right) \mathrm{trW}_{\mathrm{uv}}\mathrm{W}^{\mathrm{uv}} + \left(\frac{1}{\mathrm{g}^2}\right) \mathrm{trG}_{\mathrm{uv}}\mathrm{G}^{\mathrm{uv}}$$

1. With A U(1) gauge field Y with coupling y (weak hypercharge or weak U(1))
2. An SU(2) gauge field W with coupling w (weak SU(2) or weak isospin)
3. An SU(3) gauge field G with coupling g (gluons or strong color)

In the above equation, photon is y, W/Z particles are w, and gluons are g. The traces (tr) are over the SU(2) and SU(3) indices hidden in W and G respectively. The two-index objects are the field strengths derived from W and G the vector fields.

This equation is standard model with new meaning. After this modification, the advantage is preserved and the disadvantage is avoided in the new standard model. W and Z particles' masses can still be accurately predicted. But, the detailed equations and deductions need further efforts. Fundamental forces mediated by particles can be beautifully presented in one equation: U(1)xSU(2)xSU(3).
For SU(2) and SU(3):
Standard model is according to Yang-Mill theory:

$$\mathrm{Fuv} = \partial\mathrm{uAv} - \partial\mathrm{vAu} - [\mathrm{Au}, \mathrm{Av}]$$

It is worth noting that the Lie bracket of weak interaction and strong interaction is not vanished. Thus, they are called non-abelian gauge fields. It means that strong and weak interaction are non-holonomic and strong and weak interaction are non-conservative force. This will explain why path integral methods can be used in strong and weak interaction.

From above article, I also united the fundamental forces mediated by space-time. Those are radiation pressure (dark energy), electromagnetism, and gravity. Electromagnetism causes space-time vortex (torsion tensor) and gravity causes space-time curvature (curvature tensor). In addition, light pressure (dark energy) is time arrow and causes space expansion. This equation is given by:

G+F=T

G mediates space-time curvature (gravity and radiation pressure), F mediates space-time spiral shape (electromagnetism), and T mediates space-time spiral expansion (radiation field strength).

In previous research, Professor Weinberg proposed an electroweak interaction to predict the masses of W and Z particles. His theory is very successful. However, it is actually the interaction of photon and W/Z bosons. So, it is Weak-Light Interaction. It is the interaction between weak force and light. Here, I propose an interaction between strong force and light. Thus, it can solve the problem of gluon mass.

Based on Yang-Mills theory of standard model, we know the Yang-Mills equation is:

$$\mathrm{Fuv} = \partial \mathrm{uAv} - \partial \mathrm{vAu} - [\mathrm{Au}, \mathrm{Av}]$$

In addition, the QCD formula is:

$$\mathrm{U(SU(3))} = \exp\,[\mathrm{ig} \sum_{\mathrm{j}=1}^{8} \mathrm{F_j}\,\mathrm{G_j(x)}]$$

Thus, the covariant derivative is:

$$\partial^{\mu} = \partial^{\mu} + \mathrm{igF} * \mathrm{G(x)}$$

Besides, F=1/2λ, and λ is Gell-Mann matrix:

$$\lambda_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\lambda_2 = \begin{bmatrix} 0 & -\mathrm{i} & 0 \\ \mathrm{i} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\lambda_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\lambda_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\lambda_5 = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix}$$

$$\lambda_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\lambda_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix}$$

$$\lambda_8 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

For photon, there is another matrix:

$$\lambda_9 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We let r or |r>=(1,0,0), b or |b>=(0,1,0), and g or |g>=(0,0,1). Then, the whole matrix is:

$$\begin{bmatrix} r\bar{r} & b\bar{r} & g\bar{r} \\ r\bar{b} & b\bar{b} & g\bar{b} \\ r\bar{g} & b\bar{g} & g\bar{g} \end{bmatrix}$$

In addition, each matrix has its corresponding gluons and photon:

$$G_1 = \frac{1}{\sqrt{2}}\left(r\bar{b} + b\bar{r}\right)$$

$$G_2 = \frac{i}{\sqrt{2}}\left(r\bar{b} - b\bar{r}\right)$$

$$G_3 = \frac{1}{\sqrt{2}}\left(r\bar{r} - b\bar{b}\right)$$

$$G_4 = \frac{1}{\sqrt{2}}\left(r\bar{g} + g\bar{r}\right)$$

$$G_5 = \frac{i}{\sqrt{2}}\left(r\bar{g} - g\bar{r}\right)$$

$$G_6 = \frac{1}{\sqrt{2}}\left(g\bar{b} + b\bar{g}\right)$$

$$G_7 = \frac{i}{\sqrt{2}}\left(b\bar{g} - g\bar{b}\right)$$

$$G_8 = \frac{1}{\sqrt{6}}\left(r\bar{r} + b\bar{b} - 2g\bar{g}\right)$$

Besides, the photon boson is:

$$B = G_9 = \frac{1}{\sqrt{3}}\left(r\bar{r} + b\bar{b} + g\bar{g}\right)$$

Thus, there are totally 9 bosons(8 gluons plus 1 photon) for the whole 3X3 matrix, and they are acquiring masses due to the interaction with Higgs bosons. In order to maximize the total gluons, we need to use complex scalar field here to include 6 Higgs bosons. We predict that six of the bosons will interact with Higgs field, and three gluons will have no mass. The Higgs field is:

$$\varphi(x) \equiv \frac{1}{\sqrt{2}}\begin{pmatrix}\varphi 1 + i\varphi 2\\ \varphi 3 + i\varphi 4\\ \varphi 5 + i\varphi 6\end{pmatrix}$$

And, we let$\varphi 1 = \varphi 2 = \varphi 3 = \varphi 4 = \varphi 6 = 0$ and $\varphi 5 = v$. Thus, the Higgs field should be (0,0,V/$\sqrt{2}$)

The lagrangian for the complex scalar field is:

$$L(\varphi) = (\partial_v \varphi)(\partial^v \varphi) - \mu^2(\varphi(x))^2 - \lambda(\varphi(x))^4$$

Then, we introduce the covariant derivative of QCD and Gell-Mann matrixinto the lagrangian. It becomes:

$$\frac{1}{4}|\left[\left(ig\lambda G(x)\right) * \varphi(x)\right]^{+}\left[\left(ig\lambda G(x)\right) * \varphi(x)\right]| =$$

$$\frac{1}{8}\left(\frac{1}{\sqrt{2}}gv(G_4 - iG_5\right), \frac{1}{\sqrt{2}}gv(G_6 - iG_7), v\left(\frac{1}{\sqrt{3}}kB - \frac{\sqrt{2}}{\sqrt{3}}gG_8\right)) \times \left(\frac{1}{\sqrt{2}}gv(G_4 +$$

$$iG_5\right), \frac{1}{\sqrt{2}}gv(G_6 + iG_7), v\left(\frac{1}{\sqrt{3}}kB - \frac{\sqrt{2}}{\sqrt{3}}gG_8\right))$$

We let G^4=1/$\sqrt{2}$(G_4+iG_5), G^5=1/$\sqrt{2}$(G_4-iG_5) and so for G^6 and G^7. And we let $\sqrt{2}/\sqrt{3}$g =g'' and 1/$\sqrt{3}$k =g'. Then,

We let G^{8u}=$(g'B^u - g''G_8^u)/\sqrt{(g'^2 + g''^2)}$ and

A^u=$(g''G_8^u + g'B^u)/\sqrt{(g'^2 + g''^2)}$

Similar to electroweak theory, we get the mass of G^8

$$m\, G^8 = \frac{1}{\sqrt{2}}v\sqrt{g'^2 + g''^2}$$

And the mass of photon A^u is still zero. Similar to electroweak theory, we get G^8 field and photon field:

$$G^8 = \frac{g'}{\sqrt{g''^2 + g'^2}}B - \frac{g'}{\sqrt{g''^2 + g'^2}}G_8 = B\sin\theta - G_8\cos\theta$$

$$A = \frac{g'}{\sqrt{g''^2 + g'^2}}G_8 + \frac{g''}{\sqrt{g''^2 + g'^2}}B = G_8\sin\theta \quad + B\cos\theta$$

In addition, the mass of the new gluons G^1, G^2, and G^3 is still zero after the Higgs mechanism. Besides, the mass of gluons G^4, G^5, G^6, and G^7 is 1/2vg. The G^8 gluon becomes g$\underline{\text{g}}$ after the Higgs mechanism. Besides, we know

$\frac{1}{\sqrt{2}}(\mathrm{G}_1 - \mathrm{i}\mathrm{G}_2) = \mathrm{r}\bar{\mathrm{b}}$, and $\frac{1}{\sqrt{2}}(\mathrm{G}_1 + \mathrm{i}\mathrm{G}_2) = \mathrm{b}\bar{\mathrm{r}}$_etc

G_8 and photon Higgs interaction is in the right and bottom most position of the matrix, and we get a final g$\underline{\text{g}}$ gluon. Thus, we can get the new sets of the eight gluons $G^{1\text{-}8}$: r$\underline{\text{b}}$,b$\underline{\text{r}}$, r$\underline{\text{r}}$/b$\underline{\text{b}}$, b$\underline{\text{g}}$,g$\underline{\text{b}}$,g$\underline{\text{r}}$,r$\underline{\text{g}}$, and g$\underline{\text{g}}$. The mass term for colored gluon is $(1/2)M^2V_uV^u$, so mass of b$\underline{\text{g}}$,g$\underline{\text{b}}$,g$\underline{\text{r}}$,r$\underline{\text{g}}$ is 1/2vg, respectively. G^8 mass term is $1/2*(1/2)M^2G^{8u}G_{8u}$, so g$\underline{\text{g}}$ mass is $1/\sqrt{2}vg$. The form of r$\underline{\text{r}}$/b$\underline{\text{b}}$ is (which is similar to neutral pion):

$$\frac{1}{\sqrt{2}}(\mathrm{r}\bar{\mathrm{r}} - \mathrm{b}\bar{\mathrm{b}})$$

I don't know the exact coupling constant ratio between photon and strong force. However, if the alpha ratio is 1/2 which is similar to the color force, we can get:

$$\sin\theta = \frac{1}{\sqrt{3}}$$

$$\cos\theta = \frac{\sqrt{2}}{\sqrt{3}}$$

Thus, we will get the results of G^8-A interaction (mixing).

$$\mathrm{G}^8 = \mathrm{g}\bar{\mathrm{g}}$$

$$\mathrm{A} = \frac{1}{\sqrt{2}}(\mathrm{r}\bar{\mathrm{r}} + \mathrm{b}\bar{\mathrm{b}})$$

Green-related gluons have masses, and non-green gluons have no mass. This solves Yang-Mills mass gap problem for gluons. That's why neutron/proton has more mass than its quarks. From above, we know alpha decay is related to meson and beta decay is related to W boson. Both are SU(2). According to stronglight unification, we can get five green gluons with mass: b$\underline{\text{g}}$, g$\underline{\text{b}}$, g$\underline{\text{r}}$, rg, and g$\underline{\text{g}}$. Besides, we have four massless bosons: λ1, λ2, λ3,& A. The later four can again interact with Higgs boson $(0,V/\sqrt{2})$ to get r$\underline{\text{b}}$ and b$\underline{\text{r}}$ with mass vg/2 and b$\underline{\text{b}}$ with mass $vg/\sqrt{2}$, and massless r$\underline{\text{r}}$. Thus, there are total eight gluons to mediate short range strong force. The above A boson is the B0 boson in the electroweak theory. The four bosons can also interact with Higgs boson in the electroweak interaction and generate massive W+, W-, Z, and massless γ. Thus, we can unite strong force, weak force, and light (electromagnetism).

The above deduction also explains some phenomenon in strong interaction. It says that strong interaction also has chiral symmetric breaking to let quarks and gluons to acquire mass to explain the Yang-Mills mass gap problem. Quark confinement can also be explained because each gluon carries a pair of color and anti-color. It is like a magnet which always has a north pole and a south pole. You cannot find a magnetic monopole. Thus, we cannot either find out an isolated single colored quark or gluon. Only white colored mesons or hadrons exist. Besides, gluons and W/Z bosons have masses. Based on Yukawa potential, that means that strong and weak interaction are short range interaction compared to long range interaction of gravity and electromagnetism.

Finally, we know Yang-Mills theory in the standard model:

$$\mathrm{Fuv} = \partial_u A_v - \partial_v A_u - [A_u, A_v]$$

And, torsion tensor is:

$$\mathrm{T}(X, Y) = D_x Y - D_y X - [X, Y]$$

[X,Y] is the Lie bracket

$$[X, Y](f) = X\big(Y(f)\big) - Y\big(X(f)\big)$$
$$[X, Y] = \mathrm{XY} - \mathrm{YX}$$

Because Yang-Mills theory describes strong and weak interaction, we then know Yang-Mills field theory is also a torsion tensor like electromagnetic fields. However, Yang-Mills theory vanishes Lie bracket. In geometry, Yang-Mills theory has the existence. Thus, we can use geometry to unite electromagnetism, strong force, and weak force. Since they are anti-symmetric torsion tensors, we can use Yukawa potential for massive strong and weak force to replace the electromagnetic field in the above 4x4 spacetime matrix. And, we know there are repulsive and attractive forces in strong and weak force. For examples, same color charges in strong interaction repel each other, and different color charges in strong interaction attract each other. And, gravity field is a curvature tensor. In grand unified theories, we can use geometry to unite gravity (curvature) and electromagnetism/strong/weak force (torsion).