DARK GRAVITATION THEORY AT THE DISK AND HALO IN M31 GALAXY

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1.ABSTRACT

This paper is an extension of [1]Abarca,M2019. The basis of the theory is introduced in such paper, consequently it is needed to know it before to read this one. In this paper Dark gravitation theory is applied in the outskirt of disk region, whereas in the previous paper the theory was applied only for radius bigger than 40 kpc, where it is supposed that baryonic density is negligible regarding dark gravitation density.

The Differential Equation for Gravitational Field, hereafter DEfGF, inside the halo region is a Bernoulli equation which has analytical solution, whereas inside the disk region where baryonic density is not negligible the differential equation has to be solved numerically. This differential equation is a Chinis type one.

To sum up, when DEfGF is studied only inside the halo region it is got a Bernoulli differential equation whose solution is analytical. When the dominion of DEfGF is extended to the disk and halo then it is got a type Chinis differential equation whose solution is numerical.

In this paper will be introduced and solved numerically by Matlab software the differential equation for field in disk region for radius 26 kpc up to 200, 30 kpc up to 200 , 35 kpc up to 200. These results will be compared with the analytical solution of field got in previous paper.

In order to do a good comparison, firstly it will be compared the solutions of Bernoulli differential equation inside the halo dominion 40 to 200 kpc: The analytical and the numerical solution got by Runge-Kutta method. The results show that relative difference is about 2/10000. Which is a magnificent agreement.

Secondly it will be compared the Bernoulli differential equation and the Chinis diff. equation in the halo region 40 to 200 kpc. The relative difference is about 2/1000. As it was expected the difference is negligible because it was known that for radius bigger than 40 kpc the baryonic density is negligible.

Afterwards it will be solved the Chinis diff. Equation at different initial condition, 26, 30, 35 and 40 kpc and these results will be compared with results of Chinis diff. Equation from 40 to 200 kpc. It will be shown that field got throughout the dominion is always lower that field got at 40 kpc as initial condition. Furthermore, it will be show that the lower is the radius as initial condition the bigger will be the difference between both solutions.

Finally it will be shown the two possibilities to increase the field solution of Chinis equation at radius 35, 30 and 26 kpc to get the same value of field that Chinis equation at 40 kpc. It will be justified that a light increase in the factor responsible of dark gravitation matter is the most plausible solution.

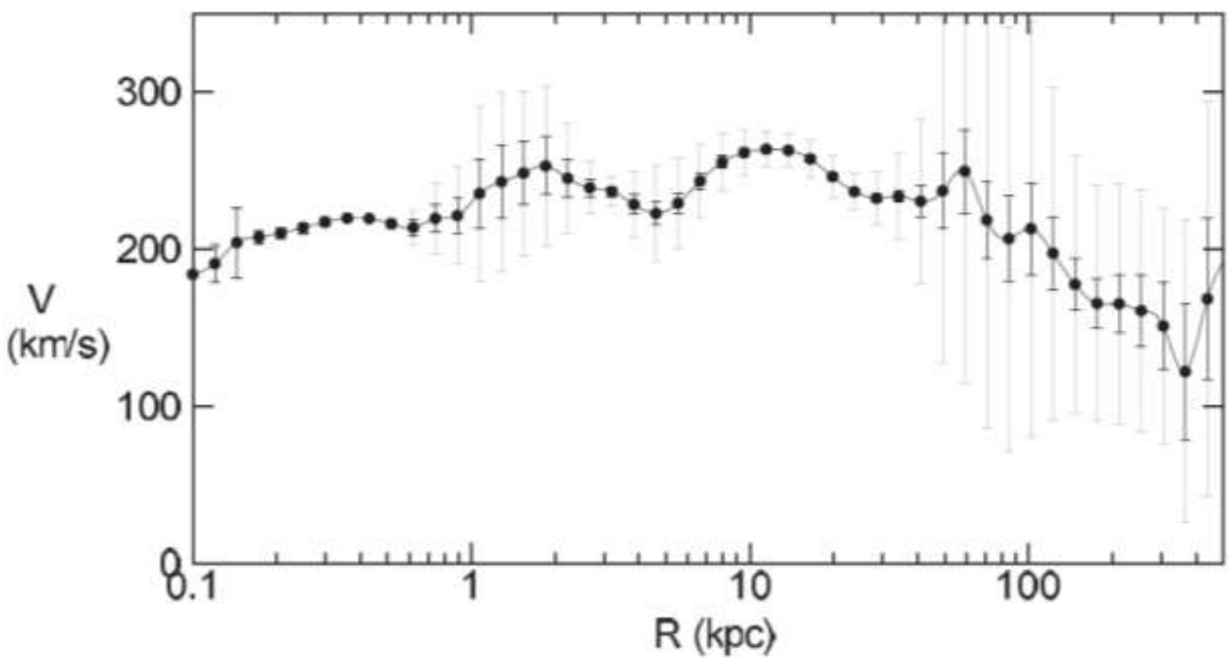
2. INTRODUCTION

Before to read this paper it is needed to read [1]Abarca,M.2019, especially chapters 1 to 9. In these chapters it is introduced the dark gravitation theory of dark matter, taking as star point data of M31 published by [2]Sofue,Y.2015.

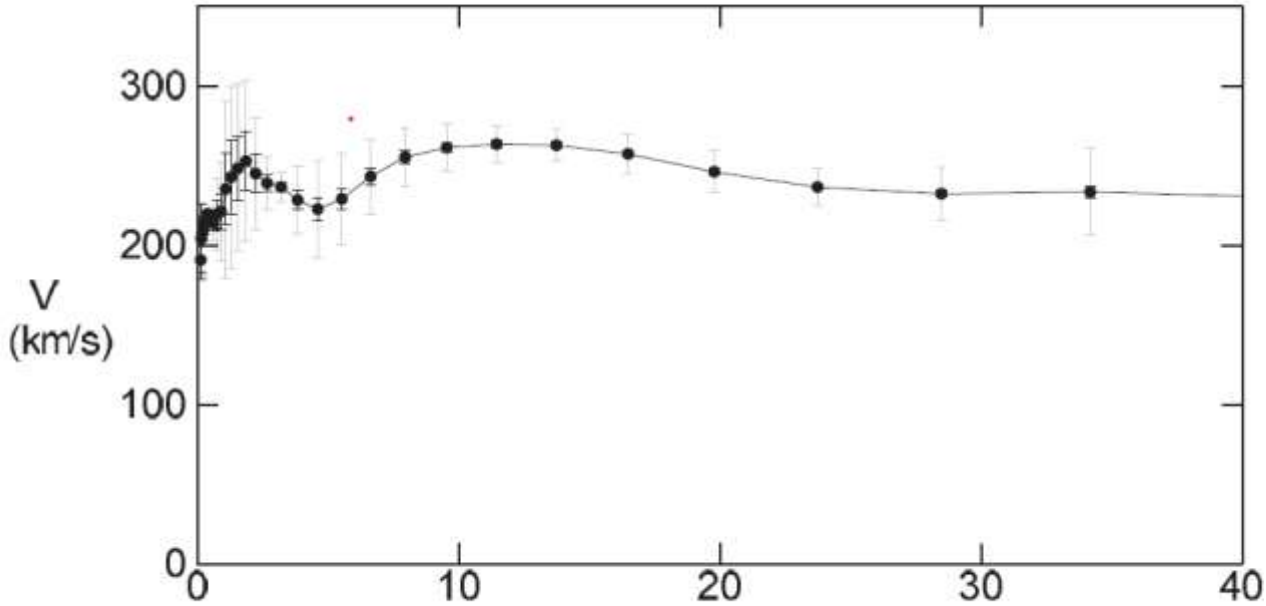
In my previous paper it is studied the rotation curve of M31 from 40 to 300 kpc, i.e. the halo region where it is supposed that baryonic density is negligible regarding DM density. According dark gravitation theory it is got a differential equation for field which is a Bernoulli type with analytical solution.

3. ROTATION CURVE OF M31 GALAXY

Below there is the rotation curve of M31 by [2]Sofue,Y.2015. See[1]Abarca,M.2019. where it is developed carefully the process to get the direct D.M density and the DM density as power of gravitational field.



Below there is the rotation curve up to 40 kpc with linear scale instead logarithm scale.



Radius kpc	Velocity km/s	Field E m/s^2
40	230	$4.29*10^{-11}$
35	231.4	$4.96*10^{-11}$
30	231.4	$5.78*10^{-11}$
26	234	$6.825*10^{-11}$

Formula used to calculate the field is $E = \frac{v^2}{r}\cdot$ being 1kpc= $3.0857*10^{19}$ m. These values will be used forward as initial condition of the differential equation of gravitational field.

4. BERNOULLI DIFFERENTIAL EQUATION FOR GRAVITATIONAL FIELD IN M31 HALO

This chapter is developed carefully in the chapter 7 of [1]Abarca,M.

4.1 INTRODUCTION

This formula $D_{DM} = \frac{a^2\cdot(2b+1)}{4\pi G}\cdot r^{2b-2}$ is a local formula because it has been got by differentiation. However E, which represents a local magnitude $E = \frac{G\cdot M(<r)}{r^2} = \frac{a^2\cdot r^{2b}}{r} = a^2\cdot r^{2b-1}$ has been got by $v = a\cdot r^b$ whose parameters a & b were got by a regression process on the whole dominion of rotation speed curve. Therefore, D_{DM} formula has a character more local than E formula because the former was got by a differentiation process whereas the later involves M(<r) which is the mass enclosed by the sphere of radius r.

In other words, the process of getting D_{DM} involves a derivative whereas the process to get E(r) involves M(r) which is a global magnitude. This is a not suitable situation because the formula $D_{DM} = A\cdot E^B$ involves two local magnitudes. Therefore it is needed to develop a new process with a more local nature or character.

It is clear that a differential equation for E is the best method to study locally such magnitude.

4.2 A DIFFERENTIAL BERNOULLI EQUATION FOR GRAVITATIONAL FIELD IN A GALACTIC HALO

As it is known in this formula $\vec{E} = -G\frac{M(r)}{r^2}\hat{r}$, M(r) represents mass enclosed by a sphere with radius r. If it is considered a region where does not exit any baryonic matter, such as any galactic halo, then the derivative of M(r) depend on dark matter density essentially and therefore $M'(r) = 4\pi r^2 \varphi_{DM}(r)$.

If $E = G\frac{M(r)}{r^2}$, vector modulus, is differentiated then it is got $E'(r) = G\frac{M'(r)\cdot r^2 - 2rM(r)}{r^4}$

If $M'(r) = 4\pi r^2 \varphi_{DM}(r)$ is replaced above then it is got $E'(r) = 4\pi G\varphi_{DM}(r) - 2G\frac{M(r)}{r^3}$ As

$\varphi_{DM}(r) = A\cdot E^B(r)$ it is right to get $E'(r) = 4\pi\cdot G\cdot A\cdot E^B(r) - 2\frac{E(r)}{r}$ which is a Bernoulli differential equation.

$E'(r) = J\cdot E^B(r) - 2\frac{E(r)}{r}$ being $J = 4\pi\cdot G\cdot A$. As $A = \frac{a^{\frac{-4}{3}}}{8\pi G}$ a = 4,727513*10^{10} and B = 5/3 see chapter 9 [1] Abarca,M. then $E'(r) = J\cdot E^{5/3}(r) - 2\frac{E(r)}{r}$ being $J = \frac{a^{\frac{-4}{3}}}{2}$ =2.925*10^{-15}. . As it was widely explained in [1]Abarca,M. this differential equation works only in the halo region where baryonic density is negligible. It is a good approximation to consider radius bigger than 40 kpc.

The solution for this Bernoulli equation is $E(r) = \left(Cr^{\frac{4}{3}} + Dr\right)^{\frac{-3}{2}}$ being $D = a^{\frac{-4}{3}}$ = 5,85*10^{-15} and being

$C = \frac{E_0^{\frac{-2}{3}} - D\cdot R_0}{R_0^{\frac{4}{3}}}$ the initial condition. At 40 kpc the rotation velocity is 230 km/s which produced a E_0 = 4.29*10^{-11} .

By substitution in the formula of C, the value of Eo, D and Ro gives C = 7.1*10^{-23}.

5. SOLUTION OF BERNOULLI DIFF. EQUATION BY RUNGE-KUTTA METHOD

In this chapter will be tested the accuracy of Runge-Kutta method solving the Bernoulli diff. equation and comparing with its analytical solution.

As in further chapters the differential equation for field will be solved by runge-kutta method, it is highly interesting to compare both methods in this case, when the analytical solution is possible.

The runge-kutta algorithm has been calculated by Matlab software. Below is the code of Matlab program. The key of this program is the “ode45”which is a specific algorithm of Matlab to do runge-kutta method.

The left side of table shows the analytical solution and the right side shows the numerical solution.

Solucion Bernoulli c= 7.1E-23 d=5.85E-15		Rung-kutta method	
Radius kpc	*1E-10 m/s^2	Kpc	m/s^2
40.0000	0.4290	40	4.29e-11
44.0000	0.3698	44	3.6975e-11
48.0000	0.3228	48	3.2284e-11
52.0000	0.2849	52	2.8492e-11
56.0000	0.2537	56	2.5371e-11
60.0000	0.2278	60	2.2775e-11
64.0000	0.2059	64	2.0587e-11
68.0000	0.1872	68	1.8722e-11
72.0000	0.1712	72	1.7116e-11
76.0000	0.1573	76	1.5724e-11
80.0000	0.1451	80	1.4508e-11
84.0000	0.1344	84	1.3437e-11
88.0000	0.1249	88	1.249e-11
92.0000	0.1165	92	1.1646e-11
96.0000	0.1089	96	1.0892e-11
100.0000	0.1022	100	1.0214e-11
104.0000	0.0960	104	9.6016e-12
108.0000	0.0905	108	9.047e-12
112.0000	0.0854	112	8.5427e-12
116.0000	0.0808	116	8.0826e-12
120.0000	0.0766	120	7.6613e-12
124.0000	0.0728	124	7.2746e-12
128.0000	0.0692	128	6.9187e-12
132.0000	0.0659	132	6.5902e-12
136.0000	0.0629	136	6.2863e-12
140.0000	0.0601	140	6.0045e-12
144.0000	0.0574	144	5.7426e-12
148.0000	0.0550	148	5.4988e-12
152.0000	0.0527	152	5.2713e-12
156.0000	0.0506	156	5.0587e-12
160.0000	0.0486	160	4.8597e-12
164.0000	0.0467	164	4.6731e-12
168.0000	0.0450	168	4.4978e-12
172.0000	0.0433	172	4.3329e-12
176.0000	0.0418	176	4.1776e-12
180.0000	0.0403	180	4.0311e-12
184.0000	0.0389	184	3.8928e-12
188.0000	0.0376	188	3.762e-12
192.0000	0.0364	192	3.6381e-12
196.0000	0.0352	196	3.5208e-12
200.0000	0.0341	200	3.4094e-12

Below is written the Matlab program.

```
J=2.925e-15;kpc=3.0857e19;
ro=40;
ro=ro*kpc;
rf=200;
rf=rf*kpc;
yo=4.29E-11;% Initial condition for field.
Dominion = [ro rf];
funcionBerni=@(r,y)J*y^(5/3)-2*y/r;
[r,y]=ode45(funcionBerni,Dominion,yo);
r=r/kpc;
table(r,y)
```

The relative difference is 3/10000 which is a magnificent agreement.

6. SURFACE BARYONIC DENSITY AND VOLUME BARYONIC DENSITY FOR BULGE AND DISK

Bellow are the surface density for bulge and disk published by [2]Sofue,Y.2015

Parameters for baryonic density in bulge and disk for M31

Component	Parameter	M31
Bulge	a_b (kpc)	1.35 ± 0.02
	$M_b(10^{11} M_\odot)$	0.35 ± 0.004
Disk	a_d (kpc)	5.28 ± 0.25
	$M_d(10^{11} M_\odot)$	1.26 ± 0.08

<table>
<tr><th>Baryonic surface bulge density</th><th>Baryonic surface disk density</th></tr>
<tr><td>

3.1. Bulge

The bulge is assumed to have the de Vaucouleurs (1958) profile for the surface mass density as

$$\Sigma_{\rm b}(r) = \Sigma_{\rm be}\exp\left[-\kappa\left\{(r/a_{\rm b})^{1/4} - 1\right\}\right], \quad (6)$$

where $\kappa = 7.6695$, $\Sigma_{\rm be}$ is the surface mass density at the half-mass scale radius $R = a_{\rm b}$. The total mass is calculated by

$$M_{\rm b} = 2\pi\int_0^\infty r\Sigma_{\rm b}(r)dr = \eta a_{\rm b}^2\Sigma_{\rm be}, \quad (7)$$

with $\eta = 22.665$ being a dimensionless constant. The circular rotation velocity is then given by

$$V_{\rm b}(R) = \sqrt{GM_{\rm b}(R)/R}. \quad (8)$$

In the fitting procedure, $M_{\rm b}$ and $a_{\rm b}$ are taken as the two free parameters. The bulge of our Galaxy was shown to be composed of multiple bulges with exponential density profiles, whereas the de Vaucouleurs law rather fails to reproduce the innermost rotation curve (Sofue 2013). Hence, the present analysis will be not accurate enough for the discussion of the bulge in the Milky Way.

</td><td>

3.2. Disk

The galactic disk is approximated by an exponential disk, whose surface mass density is expressed as

$$\Sigma_{\rm d}(R) = \Sigma_0\exp(-R/a_{\rm d}), \quad (9)$$

where Σ_0 is the central value and $a_{\rm d}$ is the scale radius. The total mass of the exponential disk is given by

$$M_{\rm d} = \int_0^\infty 2\pi r\Sigma_{\rm d}dr = 2\pi\Sigma_0 a_{\rm d}^2. \quad (10)$$

The rotation curve for a thin exponential disk is expressed by

$$V_{\rm d}(R) = \sqrt{GM_{\rm d}/a_{\rm d}}\mathscr{P}(X), \quad (11)$$

where $X = R/a_{\rm d}$, and $\mathscr{P}(X)$ is the expression obtained by Freeman (1970) for a flat exponential disk. As the two free parameters we chose $M_{\rm d}$ and $a_{\rm d}$.

</td></tr>
</table>

Baryonic volume density for bulge and disk

As previous baryonic density for bulge and disk are surface mass density, it is needed to developed a new volume density for each one. The criterion followed to get a volume extension from previous density is that the total mass enclosed by a circle should be equivalent to total mass enclosed by a sphere. Then it is right to get.

$$\rho_B = \frac{\Sigma_B}{2\cdot r} \quad \text{Where} \quad \Sigma_B = \Sigma_{BO}\exp\left\{-k\cdot(r/a_b)^{1/4} + k\right\} \quad \text{Where} \quad \Sigma_{BO} = \frac{M_B}{\eta\cdot {a_B}^2}$$

$$\rho_D = \frac{\Sigma_D}{2\cdot r} \quad \text{Where} \quad \Sigma_D = \Sigma_{DO}\exp\left\{-r/a_D\right\} \quad \text{Where} \quad \Sigma_{DO} = \frac{M_D}{2\pi\cdot {a_D}^2}$$

All parameters needed for volume density formulas are found in previous tables. See [2] Sofue,Y.

It is right to check that $\int_0^\infty 4\pi r^2\cdot\rho_B dr = M_B$ and $\int_0^\infty 4\pi r^2\cdot\rho_D dr = M_D$ which means that volume density functions are perfectly defined.

7. THE DIFFERENTIAL EQUATION CHINIS TYPE FOR GRAVITATIONAL FIELD

The field derivative is $E'(r) = G\frac{M'(r)\cdot r^2 - 2rM(r)}{r^4} = \frac{GM'}{r^2} - \frac{2E}{r}$ As this time it will be studied the field inside the region disk then disk and bulge density are not negligible, therefore $M'(r) = 4\pi r^2[\rho_{DM} + \rho_B + \rho_D]$ and by substitution in the above formula $\frac{GM'}{r^2} = 4\pi G[\rho_{DM} + \rho_B + \rho_D] = \alpha + \beta + \gamma$

Where $\alpha = J\cdot E^{5/3}$ being $J = \frac{a^{-4/3}}{2}$ =2.925*10^{-15}

The term α is the dark gravitational one, which is the same inside the disk or inside halo.

$\beta = J_B \cdot \frac{\exp\{-k_B(r/a_B)^{1/4} + k_B\}}{r}$ and $J_B = \frac{2\pi GM_B}{\eta\cdot a_B{}^2}$ = 7.425*10^{-10}

The term β is the field generated by baryonic bulge density.

$\gamma = J_D \cdot \frac{\exp(-r/a_d)}{r}$ and $J_D = \frac{GM_D}{a^2{}_D}$ = 6.3*10^{-10}

The term γ is the field generated by baryonic disk density.

The full expression of field derivative remain.

$E'(r) = JE^{5/3} - \frac{2E}{r} + J_B \cdot \frac{\exp\{-k_B(r/a_B)^{1/4} + k_B\}}{r} + J_D \frac{\exp(-r/a_d)}{r}$ whose parameters are bellow.

Parameters halo region	Bulge	Disk
J=2.925*10^{-15} I.S. units.	J_B=7.425*10^{-10} m/s² a_B =1.35 kpc = 4.1657*10^{19} m K_B= 7.6695 dimensionless	J_D=6.3*10^{-10} m/s² a_D =5.28 kpc=1.629*10^{20} m

7.1 MATLAB ALGORITHM TO SOLVE NUMERICALLY THE CHINIS EQUATION

By this algorithm it is possible to calculate the Chinis equation at different initial conditions. The initial conditions are ro(kpc) as radius and yo as field (m/s²).

Different initial conditions	
Radius kpc	Field E m/s²
40	4.29*10^{-11}
35	4.96*10^{-11}
30	5.78*10^{-11}
26	6.825*10^{-11}

```
J=2.925E-15;%dark gravitational constant
  kpc=3.0857e19;
JD=6.3E-10;%disck constant
ad=1.629E20;% disk parameter
ab=4.166e19;% bulge parameter
kb=7.6695; % bulge parameter
JB=7.425e-10;%bulge constant
ro=40;% kpc                          ro is the initial condition for radius.
ro=ro*kpc;
rf=200;% kpc
rf=rf*kpc;%200 kpc
yo=4.29e-11;%                         yo is the initial condition for field.
Dominion=[ro rf];
funcionDisk_Bulge=@(r,y)J*y^(5/3)-2*y/r+JD*exp(-r/ad)/r+JB*exp(-kb*(r/ab)^0.25+kb)/r;
[r,y]=ode45(funcionDisk_Bulge,Dominion,yo);
rkpc=r/kpc;
table(rkpc,y)
```

The key of this algorithm is the especial command of Matlab **ode45,** which is a specific subroutine to do the Runge-Kutta method for any Ordinary Differential Equation.
Below are tabulated the four solution at different radius as initial condition.

Numerical solution for Chinis diff. Equation at different initial condition.							
Kpc	E m/s^2	Kpc	E m/s^2	Kpc	E m/s^2	Kpc	E m/s^2
40	4.29e-11	35	4.96e-11	30	5.78e-11	26	6.825e-11
44	3.6997e-11	39.125	4.1675e-11	34.25	4.6954e-11	30.35	5.3643e-11
48	3.2314e-11	43.25	3.5614e-11	38.5	3.9023e-11	34.7	4.3389e-11
52	2.8523e-11	47.375	3.0853e-11	42.75	3.3001e-11	39.05	3.5794e-11
56	2.5402e-11	51.5	2.7032e-11	47	2.8314e-11	43.4	3.0051e-11
60	2.2804e-11	55.625	2.393e-11	51.25	2.464e-11	47.75	2.5745e-11
64	2.0614e-11	59.75	2.1369e-11	55.5	2.1678e-11	52.1	2.2349e-11
68	1.8748e-11	63.875	1.9224e-11	59.75	1.925e-11	56.45	1.9616e-11
72	1.7141e-11	68	1.7405e-11	64	1.7228e-11	60.8	1.7378e-11
76	1.5747e-11	72.125	1.5851e-11	68.25	1.5528e-11	65.15	1.5523e-11
80	1.4529e-11	76.25	1.451e-11	72.5	1.4083e-11	69.5	1.3967e-11
84	1.3457e-11	80.375	1.3343e-11	76.75	1.2843e-11	73.85	1.2646e-11
88	1.2509e-11	84.5	1.2321e-11	81	1.1768e-11	78.2	1.1513e-11
92	1.1664e-11	88.625	1.142e-11	85.25	1.0831e-11	82.55	1.0534e-11
96	1.0909e-11	92.75	1.062e-11	89.5	1.0008e-11	86.9	9.6823e-12
100	1.023e-11	96.875	9.9078e-12	93.75	9.2812e-12	91.25	8.935e-12
104	9.6169e-12	101	9.2694e-12	98	8.6353e-12	95.6	8.2756e-12
108	9.0616e-12	105.13	8.6949e-12	102.25	8.0585e-12	99.95	7.6905e-12
112	8.5566e-12	109.25	8.1759e-12	106.5	7.541e-12	104.3	7.1688e-12
116	8.0959e-12	113.38	7.7051e-12	110.75	7.0748e-12	108.65	6.7012e-12
120	7.6741e-12	117.5	7.2765e-12	115	6.653e-12	113	6.2804e-12
124	7.2869e-12	121.63	6.8851e-12	119.25	6.2701e-12	117.35	5.9001e-12
128	6.9304e-12	125.75	6.5267e-12	123.5	5.9213e-12	121.7	5.5552e-12
132	6.6015e-12	129.88	6.1974e-12	127.75	5.6025e-12	126.05	5.2414e-12
136	6.2971e-12	134	5.894e-12	132	5.3103e-12	130.4	4.9548e-12
140	6.0149e-12	138.13	5.614e-12	136.25	5.0417e-12	134.75	4.6923e-12
144	5.7527e-12	142.25	5.3547e-12	140.5	4.7942e-12	139.1	4.4513e-12
148	5.5085e-12	146.38	5.1142e-12	144.75	4.5655e-12	143.45	4.2294e-12
152	5.2807e-12	150.5	4.8907e-12	149	4.3538e-12	147.8	4.0245e-12
156	5.0678e-12	154.63	4.6825e-12	153.25	4.1573e-12	152.15	3.835e-12
160	4.8685e-12	158.75	4.4882e-12	157.5	3.9746e-12	156.5	3.6593e-12
164	4.6815e-12	162.88	4.3065e-12	161.75	3.8044e-12	160.85	3.496e-12
168	4.506e-12	167	4.1364e-12	166	3.6455e-12	165.2	3.3439e-12
172	4.3409e-12	171.13	3.9769e-12	170.25	3.497e-12	169.55	3.2022e-12
176	4.1853e-12	175.25	3.827e-12	174.5	3.3579e-12	173.9	3.0697e-12
180	4.0386e-12	179.38	3.686e-12	178.75	3.2274e-12	178.25	2.9457e-12
184	3.9001e-12	183.5	3.5532e-12	183	3.1048e-12	182.6	2.8294e-12
188	3.7691e-12	187.63	3.428e-12	187.25	2.9895e-12	186.95	2.7203e-12
192	3.645e-12	191.75	3.3096e-12	191.5	2.8808e-12	191.3	2.6177e-12
196	3.5275e-12	195.88	3.1977e-12	195.75	2.7783e-12	195.65	2.5211e-12
200	3.416e-12	200	3.0918e-12	200	2.6815e-12	200	2.43e-12

Taking as reference the field at 200 kpc in the first column, ro=40 kpc, it is right to calculate the relative difference at r = 200 kpc for the other ones.

kpc	Relt. difference
35	-9.4 %
30	-21.5 %
26	-28.8 %

There are two possible reasons to explain this relative difference:

a) There is a lack of matter. The terms $\beta+\gamma$, which are baryonic disk and bulge density should be bigger.

b) The factor J of term $\alpha = J \cdot E^{5/3}$, which is responsible of dark matter should be bigger.

In the following chapter will be explored both possibilities.

8 INCREASING THE FIELD BY INCREASING THE BARYONIC DISK AND BULGE DENSITY

In chapter 7 has been shown that $E'(r) = \frac{GM'}{r^2} - \frac{2E}{r} = 4\pi G[\rho_{DM} + \rho_B + \rho_D] - \frac{2E}{r} = \alpha + \beta + \gamma - \frac{2E}{r}$

There are two ways to increase the solution of field throughout the dominion: increasing $\alpha = J \cdot E^{5/3}$ or increasing $\beta + \gamma$. In this chapter will be explored the last option. As $\beta + \gamma$ are proportional to parameters Jd and Jb, it is right to increase these parameters up to get that field achieve a similar value to the first column.

At radius ro=35 kpc and ro= 30 kpc are needed factors 22*Jd and 22*Jb. At radius ro=26 factors 16*Jd and 16*Jb. In the Matlab algorithm is enough to change Jd by 22*Jd and Jb by 22*Jb.

It is clear that such changes mean that baryonic matter of disk and bulge are 22 times bigger, which is totally unacceptable. Therefore this option has to be rejected.

Chinis diff. Equation at different radius with bigger baryonic disk and bulge density							
40 kpc		35 kpc		30 kpc		26 kpc	
Parameters J /Jd / Jb		Parameters J/ 22*Jd/ 22*Jb		Parameters J/ 22*Jd/ 22*Jb		Parameters J/ 16*Jd/ 16*Jb	
40	4.29e-11	35	4.96e-11	30	5.78e-11	26	6.825e-11
44	3.6997e-11	39.125	4.3048e-11	34.25	5.1023e-11	30.35	6.074e-11
48	3.2314e-11	43.25	3.7369e-11	38.5	4.4029e-11	34.7	5.1812e-11
52	2.8523e-11	47.375	3.2645e-11	42.75	3.7931e-11	39.05	4.3769e-11
56	2.5402e-11	51.5	2.8774e-11	47	3.2999e-11	43.4	3.7444e-11
60	2.2804e-11	55.625	2.5559e-11	51.25	2.8945e-11	47.75	3.2441e-11
64	2.0614e-11	59.75	2.2877e-11	55.5	2.5608e-11	52.1	2.8388e-11
68	1.8748e-11	63.875	2.0618e-11	59.75	2.284e-11	56.45	2.5078e-11
72	1.7141e-11	68	1.8698e-11	64	2.0521e-11	60.8	2.2344e-11
76	1.5747e-11	72.125	1.7052e-11	68.25	1.856e-11	65.15	2.006e-11
80	1.4529e-11	76.25	1.5629e-11	72.5	1.6885e-11	69.5	1.8133e-11
84	1.3457e-11	80.375	1.439e-11	76.75	1.5443e-11	73.85	1.649e-11
88	1.2509e-11	84.5	1.3302e-11	81	1.4191e-11	78.2	1.5075e-11
92	1.1664e-11	88.625	1.2343e-11	85.25	1.3095e-11	82.55	1.3847e-11
96	1.0909e-11	92.75	1.1491e-11	89.5	1.2131e-11	86.9	1.2775e-11
100	1.023e-11	96.875	1.073e-11	93.75	1.1277e-11	91.25	1.1831e-11
104	9.6169e-12	101	1.0048e-11	98	1.0517e-11	95.6	1.0996e-11
108	9.0616e-12	105.13	9.4342e-12	102.25	9.8367e-12	99.95	1.0252e-11
112	8.5566e-12	109.25	8.8788e-12	106.5	9.2251e-12	104.3	9.5871e-12
116	8.0959e-12	113.38	8.3747e-12	110.75	8.673e-12	108.65	8.9895e-12
120	7.6741e-12	117.5	7.9155e-12	115	8.1726e-12	113	8.4501e-12
124	7.2869e-12	121.63	7.4958e-12	119.25	7.7175e-12	117.35	7.9614e-12
128	6.9304e-12	125.75	7.1111e-12	123.5	7.3021e-12	121.7	7.5171e-12
132	6.6015e-12	129.88	6.7574e-12	127.75	6.9219e-12	126.05	7.1116e-12
136	6.2971e-12	134	6.4315e-12	132	6.5728e-12	130.4	6.7406e-12
140	6.0149e-12	138.13	6.1303e-12	136.25	6.2513e-12	134.75	6.3999e-12
144	5.7527e-12	142.25	5.8513e-12	140.5	5.9547e-12	139.1	6.0864e-12
148	5.5085e-12	146.38	5.5924e-12	144.75	5.6802e-12	143.45	5.797e-12
152	5.2807e-12	150.5	5.3515e-12	149	5.4256e-12	147.8	5.5294e-12
156	5.0678e-12	154.63	5.1271e-12	153.25	5.1891e-12	152.15	5.2812e-12
160	4.8685e-12	158.75	4.9175e-12	157.5	4.9688e-12	156.5	5.0506e-12
164	4.6815e-12	162.88	4.7215e-12	161.75	4.7633e-12	160.85	4.8359e-12
168	4.506e-12	167	4.5378e-12	166	4.5713e-12	165.2	4.6356e-12
172	4.3409e-12	171.13	4.3654e-12	170.25	4.3914e-12	169.55	4.4484e-12
176	4.1853e-12	175.25	4.2034e-12	174.5	4.2228e-12	173.9	4.2732e-12
180	4.0386e-12	179.38	4.0509e-12	178.75	4.0644e-12	178.25	4.1089e-12
184	3.9001e-12	183.5	3.9072e-12	183	3.9154e-12	182.6	3.9546e-12
188	3.7691e-12	187.63	3.7716e-12	187.25	3.775e-12	186.95	3.8095e-12
192	3.645e-12	191.75	3.6434e-12	191.5	3.6427e-12	191.3	3.6728e-12
196	3.5275e-12	195.88	3.5221e-12	195.75	3.5176e-12	195.65	3.5439e-12
200	3.416e-12	200	3.4072e-12	200	3.3994e-12	200	3.4221e-12

9 INCREASING THE FIELD BY INCREASING THE PARAMETER OF DARK MATTER TERM

As $E'(r)=\frac{GM'}{r^2}-\frac{2E}{r}=4\pi G[\rho_{DM}+\rho_B+\rho_D]-\frac{2E}{r}=\alpha+\beta+\gamma-\frac{2E}{r}$ also it is possible to increase the field throughout the radius dominion increasing $\alpha = J\cdot E^{5/3}$. In table below is shown that at ro=35 is enough a factor 1.1*J to achieve the same field that field in the first column. For ro=30 kpc factor is 1.24*J and for ro=26 factor is 1.31*J.

It is clear that this second possibility is quite reasonable. According dark gravitation theory, the parameter J may be lightly different for different gravitational system. It is the power of field E, 5/3, the value unchangeable because this value was got by the Buckingham theorem.

Numerical solution for Chinis diff. Equation at different initial condition.							
		Factor 1.1*J		Factor 1.24*J		Factor 1.31*J	
Kpc	E m/s^2	Kpc	E m/s^2	Kpc	E m/s^2	Kpc	E m/s^2
40	4.29e-11	35	4.96e-11	30	5.78e-11	26	6.825e-11
44	3.6997e-11	39.125	4.1877e-11	34.25	4.7576e-11	30.35	5.4685e-11
48	3.2314e-11	43.25	3.5947e-11	38.5	4.0017e-11	34.7	4.5014e-11
52	2.8523e-11	47.375	3.1271e-11	42.75	3.4225e-11	39.05	3.7774e-11
56	2.5402e-11	51.5	2.7506e-11	47	2.9678e-11	43.4	3.2227e-11
60	2.2804e-11	55.625	2.444e-11	51.25	2.6078e-11	47.75	2.7988e-11
64	2.0614e-11	59.75	2.19e-11	55.5	2.3152e-11	52.1	2.4608e-11
68	1.8748e-11	63.875	1.9767e-11	59.75	2.0736e-11	56.45	2.186e-11
72	1.7141e-11	68	1.7954e-11	64	1.871e-11	60.8	1.9586e-11
76	1.5747e-11	72.125	1.6399e-11	68.25	1.6995e-11	65.15	1.7685e-11
80	1.4529e-11	76.25	1.5055e-11	72.5	1.5528e-11	69.5	1.6076e-11
84	1.3457e-11	80.375	1.3883e-11	76.75	1.426e-11	73.85	1.4698e-11
88	1.2509e-11	84.5	1.2853e-11	81	1.3156e-11	78.2	1.3507e-11
92	1.1664e-11	88.625	1.1944e-11	85.25	1.2188e-11	82.55	1.247e-11
96	1.0909e-11	92.75	1.1135e-11	89.5	1.1333e-11	86.9	1.1561e-11
100	1.023e-11	96.875	1.0413e-11	93.75	1.0574e-11	91.25	1.0758e-11
104	9.6169e-12	101	9.7649e-12	98	9.8964e-12	95.6	1.0044e-11
108	9.0616e-12	105.13	9.1803e-12	102.25	9.288e-12	99.95	9.4064e-12
112	8.5566e-12	109.25	8.6512e-12	106.5	8.7397e-12	104.3	8.8343e-12
116	8.0959e-12	113.38	8.1703e-12	110.75	8.2434e-12	108.65	8.3183e-12
120	7.6741e-12	117.5	7.7318e-12	115	7.7923e-12	113	7.8511e-12
124	7.2869e-12	121.63	7.3306e-12	119.25	7.3811e-12	117.35	7.4265e-12
128	6.9304e-12	125.75	6.9625e-12	123.5	7.0048e-12	121.7	7.0392e-12
132	6.6015e-12	129.88	6.6239e-12	127.75	6.6596e-12	126.05	6.6847e-12
136	6.2971e-12	134	6.3114e-12	132	6.3418e-12	130.4	6.3593e-12
140	6.0149e-12	138.13	6.0224e-12	136.25	6.0486e-12	134.75	6.0597e-12
144	5.7527e-12	142.25	5.7545e-12	140.5	5.7774e-12	139.1	5.7831e-12
148	5.5085e-12	146.38	5.5056e-12	144.75	5.5258e-12	143.45	5.5272e-12
152	5.2807e-12	150.5	5.2739e-12	149	5.2921e-12	147.8	5.2898e-12
156	5.0678e-12	154.63	5.0578e-12	153.25	5.0744e-12	152.15	5.0691e-12
160	4.8685e-12	158.75	4.8558e-12	157.5	4.8713e-12	156.5	4.8634e-12
164	4.6815e-12	162.88	4.6668e-12	161.75	4.6814e-12	160.85	4.6715e-12
168	4.506e-12	167	4.4895e-12	166	4.5035e-12	165.2	4.492e-12
172	4.3409e-12	171.13	4.323e-12	170.25	4.3367e-12	169.55	4.3238e-12
176	4.1853e-12	175.25	4.1663e-12	174.5	4.1799e-12	173.9	4.166e-12
180	4.0386e-12	179.38	4.0188e-12	178.75	4.0324e-12	178.25	4.0177e-12
184	3.9001e-12	183.5	3.8796e-12	183	3.8933e-12	182.6	3.878e-12
188	3.7691e-12	187.63	3.7482e-12	187.25	3.7621e-12	186.95	3.7464e-12
192	3.645e-12	191.75	3.6239e-12	191.5	3.6382e-12	191.3	3.6221e-12
196	3.5275e-12	195.88	3.5062e-12	195.75	3.5209e-12	195.65	3.5046e-12
200	3.416e-12	200	3.3947e-12	200	3.4098e-12	200	3.3934e-12

10. CONCLUSION

Thanks to Matlalb algorithm has been possible to solve the differential equation of gravitational field when the radius as initial condition is under 40 kpc. i.e. where baryonic density is not negligible versus dark matter density. The new diff. Equation is a Chinis type whereas the diff. Equation inside the halo region is a Bernoulli type.

The solution of Chinis equation for field has been a surprise because it has shown that field throughout the dominion was lower that solution of field when the initial condition was ro= 40 kpc. This results arise a problem which may be solved by two ways: increasing the baryonic matter or increasing the dark matter.

The last two chapter has been dedicated to study both options. Surprisingly, calculus has shown that it is needed to increase 22 times the density of baryonic density to equalise the field calculated at ro=40 kpc whereas the same goal is is achieved by increasing a 10% or 20% the factor J, which is term responsible of dark gravitation matter.

It is clear that the first option has to be rejected, whereas it is quite reasonable the second option because according dark gravitation theory the parameter J may be lightly different for different gravitational systems.

11. BIBLIOGRAPHY

[1]Abarca,M.2019. https://vixra.org/abs/1909.0594

Dark Gravitation theory for M31 and Milky Way got by Buckingham theorem.V6.

[2] Sofue, Y.2015. arXiv:1504.05368v1

Dark halos of M31 and the Milky Way.