Derivation of a Relativistic Compton Wave

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February 27, 2020

Abstract

In 1923, Arthur Holly Compton introduced what today is known as the Compton wave. Even if the Compton scattering derivation by Compton is relativistic in the sense that it takes into account the momentum of photons traveling at the speed of light, the original Compton derivation indirectly assumes that the electron is stationary at the moment it is scattered by electrons, but not after it has been hit by photons. Here, we extend this to derive Compton scattering for the case when the electron is initially moving at a velocity v. This gives us a relativistic Compton wave, something we remarkably have not seen published before.

Keywords: Compton wavelength; Compton scattering; moving electron

1 Introduction

In 1923, Arthur Holly Compton introduced Compton scattering [1] and, indirectly in his formulation, the Compton wavelength of the electron. The Compton length of the electron plays a central part in several areas of physics. It can be used to find the rest mass of an electron as described by Prasannakumar et al. [2]. It can indirectly be found in the relativistic wave equation of Klein and Gordon and in the Dirac equation [3] as well as in the non-relativistic Schödinger equation [4]. Haug has recently suggested that the Compton wave is the true matter wave, and that the de Broglie [5, 6] wave is merely a mathematical derivative of the Compton wave. We will not discuss or make any conclusion about that suggestion here, but it is worth noticing that the de Broglie wave is always identical to the Compton wave multiplied by $\frac{c}{v}$, and naturally the Compton wave is then always equal to the de Broglie wave times $\frac{v}{c}$.

Here we will shortly repeat how to find the Compton wave from a rest-mass electron as Compton did in 1923, and next extend our derivation to also take into account Compton scattering done with an initially moving electron.

2 Compton scattering and the Compton wave

We have the following two equations:

$$p_1c + mc^2 = p_2c + mc^2\gamma_a \tag{1}$$

where p_1 and p_2 are the photon momentum in the incoming and outgoing wave, respectively, m is the electron rest mass, and v_a is the velocity of the electron after hit by the photon. Further, we have

$$\left(mc^2\gamma_a\right)^2 = p_1^2c^2 + p_2^2c^2 - 2p_1p_2c\cos\theta \tag{2}$$

where θ is the angle between the incoming and outgoing photon, and $\gamma_a = \frac{1}{\sqrt{1-\frac{v_a^2}{c^2}}}$, where v_a is the velocity of the electron after the impact of the photon. Be aware that here we, like Compton, assume the electron is at rest immediately before impact. This gives

$$
\begin{aligned}
p_1c + mc^2 &= p_2c + mc^2\gamma_a \\
(p_1 - p_2 + mc)^2 &= p_1^2 + p_2^2 - 2p_1p_2\cos\theta \\
p_1^2 - 2p_1p_2 + p_2^2 + 2mcp_1 - 2mcp_2 &= p_1^2 + p_2^2 - 2p_1p_2\cos\theta \\
mcp_2 - mcp_1 &= p_1p_2(1-\cos\theta) \\
\frac{1}{p_1} - \frac{1}{p_2} &= \frac{h}{mc}(1-\cos\theta) \\
\frac{h}{p_1} - \frac{h}{p_2} &= \frac{h}{mc}(1-\cos\theta) \\
\lambda_1 - \lambda_2 &= \frac{h}{mc}(1-\cos\theta).
\end{aligned}
\tag{3}
$$

This is the well-known Compton scattering formula, where $\frac{\hbar}{mc}$ is what today is known as the Compton wavelength. This means the rest wavelength of the Compton wavelength can be found directly from the incoming and outgoing wavelength of the photon plus a measurement given the angle θ, so we have

$$
\begin{aligned}
\lambda_1 - \lambda_2 &= \lambda(1-\cos\theta) \\
\lambda &= \frac{\lambda_1 - \lambda_2}{1-\cos\theta}
\end{aligned}
\tag{4}
$$

where λ is the Compton wavelength of the particle at rest (the electron). The Compton wavelength is therefore indirectly measured by watching the change in wavelength in the photon used to scatter the electrons. However, Compton's formula, even if relativistic, assumes the electron that is scattered by the photon is itself at rest before the scattering.

Compton scattering, if the electron is also moving initially, at velocity v relative to the laboratory frame (and $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$), is given by

$$
\begin{aligned}
p_1c + mc^2\gamma &= p_2c + mc^2\gamma_a \\
(p_1 - p_2 + mc\gamma)^2 &= p_1^2 + p_2^2 - 2p_1p_2\cos\theta \\
p_1^2 - 2p_1p_2 + p_2^2 + 2mc\gamma p_1 - 2mc\gamma p_2 &= p_1^2 + p_2^2 - 2p_1p_2\cos\theta \\
mc\gamma p_2 - mc\gamma p_1 &= p_1p_2(1-\cos\theta) \\
\frac{1}{p_1} - \frac{1}{p_2} &= \frac{h}{mc\gamma}(1-\cos\theta) \\
\frac{\hbar}{p_1} - \frac{\hbar}{p_2} &= \frac{h}{mc\gamma}(1-\cos\theta) \\
\lambda_1 - \lambda_2 &= \frac{h}{mc\gamma}(1-\cos\theta) \\
\lambda_1 - \lambda_2 &= \lambda\sqrt{1-\frac{v^2}{c^2}}(1-\cos\theta).
\end{aligned}
\tag{5}
$$

The relativistic Compton wavelength is therefore given by

$$
\lambda\sqrt{1-\frac{v^2}{c^2}} = \frac{\lambda_1 - \lambda_2}{(1-\cos\theta).}
\tag{6}
$$

In other words, we only need to measure the photon wavelength before or after the scattering and the angle θ to know the relativistic Compton wavelength. This means the relativistic Compton wavelength can also be written as

$$
\lambda_r = \frac{h}{mc\gamma} = \lambda\sqrt{1-\frac{v^2}{c^2}}.
\tag{7}
$$

3 Conclusion

We have shown how Compton scattering for the case where an electron is moving initially can be derived. This gives us the relativistic Compton wave rather than the Compton wave for an electron at rest, as was achieved by Compton himself. The relativistic Compton wave, for example, plays a central role in the recent quantum gravity theory presented by Haug [7]. Moreover, when we also have a relativistic Compton wave formula, then we see that the de Broglie wave is always the Compton wave multiplied by $\frac{c}{v}$. While the de Broglie wave is infinite (or undefined, but it is anyway convergent towards infinity, as there are no limitations in standard physics of how close v can be to zero.) for a rest-mass particle, the Compton wave is more well-defined and has been measured for rest-mass particles. Even if the derivation in this paper is trivial, we think it could be important for the physics community to be aware of the difference between the Compton wave and the relativistic Compton wave.

References

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