Beal's Conjecture is Tenable

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Abstract

In this article, first classify A, B and C according to their odevity, and thereby get rid of two kinds of $A^X+B^Y\neq C^Z$. Then, affirm that there are $A^X+B^Y=C^Z$ in which case A, B and C have a common prime factor by several concrete equalities. After that, prove that there are $A^X+B^Y\neq C^Z$ where A, B and C have not a common prime factor by the mathematical induction with the aid of interrelations of 3 integers relating to symmetry after divide $A^X+B^Y=C^Z$ in four. Finally, reach the conclusion that Beal's conjecture is tenable via the comparison between $A^X+B^Y=C^Z$ and $A^X+B^Y\neq C^Z$ under the given requirements.

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Contents

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1. Introduction

The Beal's conjecture states that if $A^X+B^Y=C^Z$, where A, B, C, X, Y and Z are positive integers, and X, Y and Z are all greater than 2, then A, B and C must have a common prime factor.

The conjecture was discovered by Andrew Beal in 1993. Later, the conjecture was announced in December 1997 issue of the Notices of the American Mathematical Society, [1]. Yet it is still both unproved and un-negated a conjecture hitherto.

Let us regard limits of values of A, B, C, X, Y and Z in the indefinite equation $A^X+B^Y=C^Z$ as given requirements for indefinite equations and inequalities concerned after this.

2. Choices for Combinations of Values of A, B and C

First, classify A, B and C according to their respective odevity, and thereby exclude following two kinds of $A^X+B^Y\neq C^Z$:

1) A, B and C, all are positive odd numbers.

2) A, B and C are two positive even numbers and a positive odd number.

After that, merely continue to have following two kinds which contain $A^X+B^Y=C^Z$ under the given requirements:

1) A, B and C, all are positive even numbers.

2) A, B and C are two positive odd numbers and a positive even number.

3. Exemplify $A^X+B^Y=C^Z$ in which case A, B and C have a Common Prime Factor

For the indefinite equation $A^X+B^Y=C^Z$ which satisfies aforesaid either qualification, in fact, it has many sets of the solution with A, B and C as positive integers, and illustrate with examples as follows respectively.

When A, B and C all are positive even numbers, let A=B=C=2, X=Y≥3, and Z=X+1, then $A^X+B^Y=C^Z$ are changed into $2^X+2^X=2^{X+1}$.So $A^X+B^Y=C^Z$ at here have a set of the solution with A, B and C as integers 2, 2 and 2, and that A, B and C have common prime factor 2.

In addition, let A=B=162, C=54, X=Y=3 and Z=4, then $A^X+B^Y=C^Z$ are changed into $162^3+162^3=54^4$. So $A^X+B^Y=C^Z$ at here have a set of the solution with A, B and C as integers 162, 162 and 54, and that A, B and C have common prime factors 2 and 3.

When A, B and C are two positive odd numbers and a positive even number, let A=C=3, B=6, X=Y=3 and Z=5, then $A^X+B^Y=C^Z$ are changed into $3^3+6^3=3^5$. So $A^X+B^Y=C^Z$ at here have a set of the solution with A, B and C as integers 3, 6 and 3, and that A, B and C have common prime factor 3.

In addition, let A=B=7, C=98, X=6, Y=7 and Z=3, then $A^X+B^Y=C^Z$ are changed into $7^6+7^7=98^3$. So $A^X+B^Y=C^Z$ at here has a set of the solution with A, B and C as integers 7, 7 and 98, and that A, B and C have common prime factor 7.

Thus it can be seen, that the indefinite equation $A^X+B^Y=C^Z$ under the given requirements plus aforementioned either qualification is able to hold water, but A, B and C must have at least a common prime factor.

4. Divide $A^X+B^Y\neq C^Z$ in Four where A, B and C have not a Common Prime Factor

As mentioned above, if can prove that there are $A^X+B^Y\neq C^Z$ under the given requirements plus the qualification that A, B and C have not a common prime factor, then the conjecture is tenable doubtlessly.

Since A, B and C have the common prime factor 2 where A, B and C all are positive even numbers, then these circumstances that A, B and C have not a common prime factor can only occur in which case A, B and C are two positive odd numbers and a positive even number.

If A, B and C have not a common prime factor, then any two of them have not a common prime factor either, because in case any two have a common prime factor, yet another has not, surely lead up to $A^X+B^Y\neq C^Z$ according to the unique factorization theorem of natural number.

Unquestionably, following two inequalities add together, be able to replace fully $A^X+B^Y\neq C^Z$ under the given requirements plus the qualification that A, B and C are two odd numbers and an even number without a common prime factor.

1).$A^X+B^Y\neq(2W)^Z$ i.e. $A^X+B^Y\neq 2^Z W^Z$; **2).**$A^X+(2W)^Y\neq C^Z$ i.e. $A^X+2^Y W^Y\neq C^Z$.

In two such inequalities, A, B and C are positive odd numbers; W is a positive whole number; three terms in each inequality have not a common prime factor; each of X, Y or Z is a greatest common divisor of exponents of distinct prime factors of base number under itself, and X, Y and Z ≥ 3.

Once more divide $A^X+B^Y\neq 2^Z W^Z$ in **(1)** $A^X+B^Y\neq 2^Z$ and **(2)** $A^X+B^Y\neq 2^Z O^Z$.

Once more divide $A^X+2^Y W^Y\neq C^Z$ in **(3)** $A^X+2^Y\neq C^Z$ and **(4)** $A^X+2^Y O^Y\neq C^Z$.

In listed above four inequalities, A, B, C and O are positive odd numbers; three terms in each inequality have not a common prime factor; each of X, Y and Z is a greatest common divisor of exponents of distinct prime factors of base number under itself, and X, Y and Z ≥ 3.

Again regard above these qualifications or a part of them as known requirements for inequalities or indefinite equations concerned after this.

By this token, the proof of $A^X+B^Y\neq C^Z$ under the given requirements plus the qualification that A, B and C have not a common prime factor can be changed to prove above four inequalities under the known requirements.

5. Mainstays that Prove Preceding Four Inequalities

Before proofs begin, it is necessary to expound some of basic conception, so as to regard them as mainstays that prove preceding four inequalities.

What the author first expounds is that at positive half line of the number axis, regard any even point as a symmetric center, then odd points on the left side of the symmetric center and odd points concerned on the right side are one-to-one symmetries. Like that, in the sequence of natural numbers, regard any even number as a symmetric center, then odd numbers on the lest sides of the symmetric center and odd numbers concerned on the right side are one-to-one symmetries too, [2].

Regard any one of $2^{H-1}W^V$ as a symmetric center, then, two distances from the symmetric center to each other's symmetric odd numbers are two equilong line segments at the number axis or two same differences in the sequence of natural numbers, where W is an integer$\geq$1, H$\geq$3 and V$\geq$1. The above-mentioned symmetric relation indicates that the sum of bilateral symmetric two odd numbers is equal to the double of even number as the symmetric center. Yet, over the left, a sum of two non-symmetric positive odd numbers is unequal to the double of even number as the symmetric center surely.

In addition to this, if the sum of two positive odd numbers is equal to the double of an even number, then two such odd numbers are symmetric from each other surely whereby the even number to act as symmetric center. The author regards aforesaid conclusions on both sides derived from such a symmetric relation as interrelations of 3 integers relating to symmetry.

Besides, for any positive odd number, it is able to be expressed as one of O^V where O is an odd number, and when V=1 or 2, write O^V to $O^{1\sim2}$.

Additionally, the author stipulates that the exponent of any integer uniformly is directed to the greatest common divisor of exponents of distinct prime factors of the integer, in this article.

Thereinafter, the author will prove aforementioned four inequalities, one by one, and apply these mainstays therein.

6. Proving $A^X+B^Y\neq2^Z$ under the Known Requirements

Regard 2^{Z-1} as symmetric center of odd numbers concerned to prove $A^X+B^Y\neq2^Z$ under the known requirements by the mathematical induction.

(1) When Z-1=2, 3, 4, 5 and 6, bilateral symmetric odd numbers on two sides of symmetric centers 2^{Z-1} are listed below successively.

1^6, 3, (2^2), 5, 7, (2^3), 9, 11, 13, 15, (2^4), 17, 19, 21, 23, 25, 3^3, 29, 31, (2^5), 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, (2^6), 65, 67, 69, 71, 73, 75, 77, 79, 3^4, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101, 103, 105, 107, 109, 111, 113, 115, 117, 119, 121, 123, 5^3, 127

As listed above, there are not two of O^V with V≥3 on two places of every pair of bilateral symmetric odd numbers whereby 2^{Z-1} to act as symmetric center where Z-1=2, 3, 4, 5 and 6. Namely, there are $A^X+B^Y\neq 2^3$, $A^X+B^Y\neq 2^4$, $A^X+B^Y\neq 2^5$, $A^X+B^Y\neq 2^6$ and $A^X+B^Y\neq 2^7$ under the known requirements.

(2) When Z-1=K with K≥6, suppose that there are not two of O^V with V≥3 on two places of every pair of bilateral symmetric odd numbers whereby 2^K to act as symmetric center. Namely, suppose that there are $A^X+B^Y\neq 2^{K+1}$ under the known requirements.

(3) When Z-1=K+1, prove that there are not two of O^V with V≥3 on two places of every pair of bilateral symmetric odd numbers whereby 2^{K+1} to act as symmetric center. Namely, prove that there are $A^X+B^Y\neq 2^{K+2}$ under the known requirements.

Proof· Supposition that A^X and B^Y are a pair of bilateral symmetric odd numbers whereby 2^K to act as symmetric center, then there are $A^X+B^Y=2^{K+1}$ according to interrelations of 3 integers relating to symmetry.

While, there are $A^X+B^Y\neq 2^{K+1}$ under the known requirements, additionally, there are not two of O^V with V≥3 on two places of every pair of bilateral symmetric odd numbers whereby 2^K to act as symmetric center, pursuant to the supposition of №2 step of the mathematical induction.

So, first, the author tentatively regard A^X as one of O^V with V≥3, and regard B^Y as one of $O^{1\sim 2}$, i.e. let X≥3 and Y=1 or 2.

To sum up, the existence of the equality $A^X+B^Y=2^{K+1}$ must possess two requirements integrally, namely on the one hand, A^X and B^Y must be two bilateral symmetric odd numbers whereby 2^K to act as symmetric center; on the other hand, at least one of Y and X is equal to 1 or 2. If you change either such requirement, even though it is a little bit, also lead to $A^X+B^Y\neq 2^{K+1}$ inevitably. So long as there are $A^X+B^Y\neq 2^{K+1}$, A^X and B^Y are not a pair of bilateral symmetric odd numbers whereby 2^K to act as symmetric center. Thereupon, there are surely $A^X+B^Y=2^{K+1}$ under the known requirements except for Y, and Y=1 or 2.

As thus, there are $A^X+(A^X+2B^Y)=2^{K+2}$ under the known requirements except for Y, and Y=1 or 2, and that A^X and A^X+2B^Y are two bilateral symmetric odd numbers whereby 2^{K+1} to act as symmetric center; and there are $A^X+(A^X+2B^Y)\neq 2^{K+2}$ under the known requirements, yet A^X and A^X+2B^Y are not two symmetric odd numbers whereby 2^{K+1} to act as symmetric center, according to interrelations of 3 integers relating to symmetry.

In any case, the sum of A^X+2B^Y is an odd number, so let $A^X+2B^Y=O^E$, unquestionably O at here is yet an odd number, and E is an exponent.

After the substitution, on the one hand, there are $A^X+(A^X+2B^Y)=A^X+O^E= 2^{K+2}$ under the known requirements except for Y, and Y=1 or 2, and that A^X and O^E are two bilateral symmetric odd numbers whereby 2^{K+1} to act as symmetric center. On the other hand, there are $A^X+(A^X+2B^Y)=A^X+O^E\neq 2^{K+2}$ under the known requirements, and that A^X and O^E are not two bilateral symmetric odd numbers whereby 2^{K+1} to act as symmetric center. Since it is so, no matter what integer which E equals, including E≥3, all are able to satisfy $A^X+O^E\neq 2^{K+2}$.

Although two of O^E derive itself from A^X+2B^Y, but since limits of values of Y in two expressions are entirely different from each other, namely they are $Y\geq 3$ in $A^X+(A^X+2B^Y)=A^X+O^E\neq 2^{K+2}$ and Y=1 or 2 in $A^X+(A^X+2B^Y)=A^X+O^E=2^{K+2}$, therefore A^X+2B^Y within $A^X+(A^X+2B^Y)=A^X+O^E\neq 2^{K+2}$ are greater than A^X+2B^Y within $A^X+(A^X+2B^Y)=A^X+O^E=2^{K+2}$. That is to say, O^E within $A^X+O^E\neq 2^{K+2}$ are greater than O^E within $A^X+O^E=2^{K+2}$.

Then, when A^X within $A^X+O^E\neq 2^{K+2}$ and A^X within $A^X+O^E=2^{K+2}$ are one and the same, and O within $A^X+O^E\neq 2^{K+2}$ be equal to O within $A^X+O^E=2^{K+2}$, E within $A^X+O^E\neq 2^{K+2}$ be greater than E within $A^X+O^E=2^{K+2}$ undoubtedly.

Thus it can be seen, that on the one hand, values of E within $A^X+O^E\neq 2^{K+2}$ contain $E\geq 3$; on the other, E within $A^X+O^E\neq 2^{K+2}$ be greater than E within $A^X+O^E=2^{K+2}$, then E within $A^X+O^E=2^{K+2}$ can only be equal to 1 or 2 surely.

For $A^X+O^E\neq 2^{K+2}$ and $A^X+O^E=2^{K+2}$, substitute B for O, since B and O express any positive odd number; also substitute Y for E, but two of E express entirely different integers, and the expression of Y follow E.

After such substitutions, get $A^X+B^Y\neq 2^{K+2}$ under the known requirements, and $A^X+B^Y=2^{K+2}$ under the known requirements except for Y, and Y=1 or 2.

In this proof, if B^Y is one of O^V with $V\geq 3$, then A^X is surely one of $O^{1\sim 2}$. Then, a conclusion concluded from this is one and the same with $A^X+B^Y\neq 2^{K+2}$ under the known requirements.

If A^X and B^Y are bilateral symmetric two of $O^{1\sim 2}$, though they in pairs are two symmetric odd numbers whereby 2^K to act as symmetric center, but no matter what odd number which A^X+2B^Y or B^Y+2A^X equal, it is unable to satisfy bilateral symmetric two of O^V with $V\geq3$, since A^X or B^Y in two addends is not one of O^V with $V\geq3$ originally.

So much for, the author has proven that when $Z-1=K+1$ with $K\geq6$, there are $A^X+B^Y\neq2^{K+2}$ under the known requirements.

In other words, there are not two of O^V with $V\geq3$ on two places of every pair of bilateral symmetric odd numbers whereby 2^{K+1} to act as symmetric center.

By the preceding way of doing thing, can continue to prove that when $Z-1=K+2$, $K+3$…up to each and every integer ≥3, there are all $A^X+B^Y\neq2^{K+3}$, $A^X+B^Y\neq 2^{K+4}$ … up to $A^X+B^Y\neq2^Z$ under the known requirements.

7. Proving $A^X+B^Y\neq2^ZO^Z$ under the Known Requirements

Regard $2^{Z-1}O^Z$ as symmetric center of odd numbers concerned to prove $A^X+B^Y\neq2^ZO^Z$ under the known requirements by the mathematical induction successively, and point out $O\geq3$ emphatically.

(1) When $O=1$, $2^{Z-1}O^Z$ i.e. 2^{Z-1}. You have seen surely that there are $A^X+B^Y\neq2^Z$ under the known requirements, and there are $A^X+B^Y=2^Z$ under the known requirements except for Y, and Y=1 or 2, in №6 section. Namely, there are not two of O^V with $V\geq3$ on two places of every pair of bilateral symmetric odd numbers whereby 2^{Z-1} with $Z\geq3$ to act as symmetric center.

(2) When $O=J$ and $J\geq1$, $2^{Z-1}O^Z$ i.e. $2^{Z-1}J^Z$. Suppose that there are $A^X+B^Y\neq2^ZJ^Z$ under the known requirements. Namely, suppose that there are not two of O^V with $V\geq3$ on two places of every pair of bilateral symmetric odd numbers whereby $2^{Z-1}J^Z$ to act as symmetric center.

(3) When O=K and K=J+2, $2^{Z-1}O^Z$ i.e. $2^{Z-1}K^Z$. Prove that there are $A^X+B^Y\neq 2^ZK^Z$ under the known requirements. Namely, prove that there are not two of O^V with $V\geq 3$ on two places of every pair of bilateral symmetric odd numbers whereby $2^{Z-1}K^Z$ to act as symmetric center.

Proof· Suppose that A^X and B^Y are a pair of bilateral symmetric odd numbers whereby $2^{Z-1}J^Z$ to act as symmetric center, then there are $A^X+B^Y= 2^ZJ^Z$ according to interrelations of 3 integers relating to symmetry.

And yet, there are $A^X+B^Y\neq 2^ZJ^Z$ under the known requirements, pursuant to the supposition of №2 step of the mathematical induction, and thereby there are $A^X+B^Y=2^ZJ^Z$ under the known requirements except for Y, and Y=1 or 2.

Next, there are $A^X+[B^Y+2^Z(K^Z-J^Z)]=(A^X+B^Y)+2^ZK^Z-2^ZJ^Z=2^ZK^Z$ under the known requirements except for Y, and Y=1 or 2. So that A^X and $B^Y+2^Z(K^Z-J^Z)$ are a pair of bilateral symmetric odd numbers whereby $2^{Z-1}K^Z$ to act as symmetric center, and the pursuant reason is as above.

As stated, there are $A^X+B^Y\neq 2^ZJ^Z$ under the known requirements, then hereby conclude $A^X+[B^Y+2^Z(K^Z-J^Z)]=(A^X+B^Y)+2^ZK^Z-2^ZJ^Z\neq 2^ZK^Z$, so that A^X and $B^Y+2^Z(K^Z-J^Z)$ under the known requirements are not two bilateral symmetric odd numbers whereby $2^{Z-1}K^Z$ to act as symmetric center, and the pursuant reason is as above too.

Such being the case, let the odd number $B^Y+2^Z(K^Z-J^Z)$ is equal to D^E where D expresses an odd number, and E expresses an exponent still.

By this token, on the one hand, there are $A^X+[B^Y+2^Z(K^Z-J^Z)]=A^X+D^E\neq 2^ZK^Z$ under the known requirements. On the other, there are $A^X+[B^Y+2^Z(K^Z-J^Z)]=$ $A^X+D^E=2^ZK^Z$ under the known requirements except for Y, and Y=1 or 2.

Although two of A^X+D^E derive themselves from $A^X+[B^Y+2^Z(K^Z-J^Z)]$, but since limits of values of Y in two expressions are entirely different from each other, namely they are $Y\geq 3$ in $A^X+[B^Y+2^Z(K^Z-J^Z)]=A^X+D^E\neq 2^HK^Z$ and Y=1 or 2 in $A^X+[B^Y+2^Z(K^Z-J^Z)]=A^X+D^E=2^ZK^Z$, therefore $B^Y+2^Z(K^Z-J^Z)$ in $A^X+[B^Y+2^Z(K^Z-J^Z)]=A^X+D^E\neq 2^ZK^Z$ are greater than $B^Y+2^Z(K^Z-J^Z)$ in $A^X+[B^Y+2^Z(K^Z-J^Z)]=A^X+D^E=2^ZK^Z$. That is to say, D^E in $A^X+D^E\neq 2^ZK^Z$ be greater than D^E in $A^X+D^E=2^ZK^Z$.

Thus, when D in $A^X+D^E\neq 2^ZK^Z$ be equal to D in $A^X+D^E=2^ZK^Z$, E in $A^X+D^E\neq 2^ZK^Z$ be greater than E in $A^X+D^E=2^ZK^Z$ surely.

According to interrelations of 3 integers relating to symmetry and regard $2^{Z-1}K^Z$ as symmetric center, in $A^X+D^E=2^ZK^Z$, A^X and D^E must be a pair of bilateral symmetric odd numbers; while in $A^X+D^E\neq 2^ZK^Z$, A^X and D^E are not two bilateral symmetric odd numbers, and that no matter what integer which E equals, including $E\geq 3$, all are able to satisfy $A^X+D^E\neq 2^ZK^Z$.

Now that E in $A^X+D^E\neq 2^ZK^Z$ be greater than E in $A^X+D^E=2^ZK^Z$, additionally E in $A^X+D^E\neq 2^ZK^Z$ contain integers ≥ 3, so that E in $A^X+D^E=2^ZK^Z$ can only be equal to 1 and 2.

To sum up, this has shown that there are $A^X+D^E=2^ZK^Z$ under the known requirements except for E, and E=1 or 2; also, there are $A^X+D^E\neq 2^ZK^Z$ under the known requirements.

For $A^X+D^E=2^ZK^Z$, substitute B for D, since B and D express every odd number, and substitute Y for E where E=1 or 2, then get $A^X+B^Y=2^ZK^Z$ under the known requirements except for E, and E=1 or 2.

Also, for $A^X+D^E\neq 2^ZK^Z$, substitute B for D, since B and D express every odd number, and substitute Y for E where E≥3, and Y≥3, then get $A^X+B^Y\neq 2^ZK^Z$ under the known requirements.

In this proof, if B^Y is one of O^V with V≥3, then A^X is one of $O^{1\sim2}$ surely.

A conclusion reached from this is one and the same with $A^X+B^Y\neq 2^ZK^Z$ under the known requirements.

If A^X and B^Y are bilateral symmetric two of $O^{1\sim2}$ whereby $2^{Z-1}J^Z$ to act as symmetric center, then whether $A^X+2^Z(K^Z-J^Z)$ and B^Y, or $B^Y+2^Z(K^Z-J^Z)$ and A^X, though they in pairs are two symmetric odd numbers whereby $2^{Z-1}K^Z$ to act as symmetric center, but no matter $A^X+2^Z(K^Z-J^Z)$ or $B^Y+2^Z(K^Z-J^Z)$ equal what odd number, all are unable to satisfy bilateral symmetric two of O^V with V≥3, since B^Y or A^X in two addends is one of $O^{1\sim2}$ originally.

On balance, the author has proven $A^X+B^Y\neq 2^ZK^Z$ with K=J+2 under the known requirements. In other words, when O=J+2, there are not two of O^V with V≥3 on two places of every pair of bilateral symmetric odd numbers whereby $2^{Z-1}(J+2)^Z$ to act as symmetric center.

By the preceding way of doing thing, can continue to prove that when O=J+4, J+6…up to each and every positive odd number, there are all $A^X+B^Y\neq 2^Z(J+4)^Z$,

$A^X+B^Y \neq 2^Z(J+6)^Z$…up to $A^X+B^Y \neq 2^Z O^Z$ under the known requirements.

8. Proving $A^X+2^Y \neq C^Z$ under the Known Requirements

By now, set about proving $A^X+2^Y \neq C^Z$ under the known requirements by the mathematical induction, according to certain of conclusions got above.

(1) When Y=3, 4, 5, 6 and 7, bilateral symmetric odd numbers on two sides of symmetric centers 2^3, 2^4, 2^5, 2^6 and 2^7 are listed below successively.

1^7, 3, 5, 7, (2^3), 9, 11, 13, 15, (2^4), 17, 19, 21, 23, 25, 3^3, 29, 31, (2^5), 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, (2^6), 65, 67, 69, 71, 73, 75, 77, 79, 3^4, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101, 103, 105, 107, 109, 111, 113, 115, 117, 119, 121, 123, 5^3, 127, (2^7), 129, 131, 133, 135, 137, 139, 141, 143, 145, 147, 149, 151, 153, 155, 157, 159, 161, 163, 165, 167, 169, 171, 173, 175, 177, 179, 181, 183, 185, 187, 189, 191, 193, 195, 197, 199, 201, 203, 205, 207, 209, 211, 213, 215, 217, 219, 221, 223, 225, 227, 229, 231, 233, 235, 237, 239, 241, 3^5, 245, 247, 249, 251, 253, 255.

As listed above, there is only the high power 1^7 on the left side of symmetric center 2^3;
There is only the high power 1^7 on the left side of symmetric center 2^4;
There are altogether high powers 1^7 and 3^3 on the left side of symmetric center 2^5;
There are altogether high powers $1^7$and 3^3 on the left side of symmetric center 2^6;
There are altogether high powers 1^7, 3^3, 3^4 and 5^3 on the left side of symmetric center 2^7.

Clearly, it is observed that there are $1^7+2^3 \neq C^Z$; $1^7+2^4 \neq C^Z$; $1^7+2^5 \neq C^Z$, $3^3+2^5 \neq C^Z$; $1^7+2^6 \neq C^Z$, $3^3+2^6 \neq C^Z$; $1^7+2^7 \neq C^Z$, $3^3+2^7 \neq C^Z$, $3^4+2^7 \neq C^Z$ and $5^3+2^7 \neq C^Z$.

After regard 2^Y as symmetric center, where Y=3, 4, 5, 6 and 7, 2^Y lies between A^X and C^Z, then there are $A^X+2^3\neq C^Z$, $A^X+2^4\neq C^Z$, $A^X+2^5\neq C^Z$, $A^X+2^6\neq C^Z$ and $A^X+2^7\neq C^Z$ under the known requirements.

(2) When Y=N with N≥7, suppose that there are $A^X+2^N\neq C^Z$ under the known requirements, where $A^X < 2^N < C^Z$.

(3) When Y=N+1, prove that there are $A^X+2^{N+1}\neq C^Z$ under the known requirements, where $A^X < 2^{N+1} < C^Z$.

Proof· Since there are $(2^{N+1}+A^X)+(2^{N+1}-A^X)=2^{N+2}$, so $2^{N+1}+A^X$ and $2^{N+1}-A^X$ are two bilateral symmetric odd numbers whereby 2^{N+1} to act as symmetric center, according to interrelations of 3 integers relating to symmetry.

In addition, $2^{N+1}-A^X\neq O^V$ i.e. $A^X+O^V\neq 2^{N+1}$ with V≥3 and proven $A^X+B^Y\neq 2^Z$ under the known requirements are just the same, thus get that the difference of $2^{N+1}-A^X$ can only be one of $O^{1\sim2}$.

Now that $2^{N+1}-A^X$ can only be one of $O^{1\sim2}$, then on the contrary, $2^{N+1}-A^{1\sim2}$ are either one of $O^{1\sim2}$ or any one of O^V with V≥3 under 2^{N+1}.

Next, there are $(2^{N+1}+A^{1\sim2})+(2^{N+1}-A^{1\sim2})=2^{N+2}$, then $2^{N+1}+A^{1\sim2}$ and $2^{N+1}-A^{1\sim2}$ are two bilateral symmetric odd numbers whereby 2^{N+1} to act as symmetric center, and the pursuant reason is as above. So that $2^{N+1}+A^{1\sim2}$ are either one of $O^{1\sim2}$ or any one of O^V with V≥3 under 2^{N+1}.

As mentioned above, $2^{N+1}-A^X$ within $(2^{N+1}+A^X)+(2^{N+1}-A^X)=2^{N+2}$ are one of $O^{1\sim2}$, then $2^{N+1}+A^X$ are either one of $O^{1\sim2}$ or any one of O^V with V≥3 under 2^{N+2}, since has proved that there are not two of O^V with V≥3 on two places of every pair of bilateral symmetric odd numbers whereby 2^Z with Z≥3 to act as symmetric center in №6 section.

Such being the case, if $2^{N+1}+A^{1\sim2}$ are any one of O^V with V≥3 under 2^{N+2}, then $2^{N+1}+A^X$, i.e. A^X+2^{N+1} can only be one of $O^{1\sim2}$.

If $2^{N+1}+A^X$ are any one of O^V with V≥3 under 2^{N+2}, then $2^{N+1}+A^{1\sim2}$ can only be one of $O^{1\sim2}$. It is obvious that this result is inconsistent with the conclusion got that $2^{N+1}+A^{1\sim2}$ are either one of $O^{1\sim2}$ or any one of O^V with V≥3 under 2^{N+1}, hence must abandon this result and return to the right judgment that $2^{N+1}+A^X$, i.e. A^X+2^{N+1} can only be one of $O^{1\sim2}$.

By this token, on the one hand, proved that A^X+2^{N+1} can only be one of $O^{1\sim2}$; on the other, C^Z under the known requirements is one of O^V with V≥3. Consequently, there are $A^X+2^{N+1} \neq C^Z$ under the known requirements.

By the preceding way of doing thing, can continue to prove that when Y=N+2, N+3…up to each and every integer≥3, there are all $A^X+2^{N+2}\neq C^Z$, $A^X+2^{N+3}\neq C^Z$…up to $A^X+2^Y\neq C^Z$ under the known requirements.

9. Proving $A^X+2^YO^Y\neq C^Z$ under the Known Requirements

Finally, let us prove $A^X+2^YO^Y\neq C^Z$ under the known requirements by the mathematical induction, according to certain of conclusions got above.

(1) When O=1, 2^YO^Y is exactly 2^Y, and the author has proved that there are $A^X+2^Y\neq C^Z$ under the known requirements in №8 section.

(2) When O=J and J≥1, 2^YO^Y is exactly 2^YJ^Y, and suppose that there are $A^X+2^YJ^Y\neq C^Z$ under the known requirements, where $A^X<2^YJ^Y< C^Z$.

(3) When O=K with K=J+2, 2^YO^Y is exactly 2^YK^Y, and prove that there are

$A^X+2^YK^Y \neq C^Z$ under the known requirements, where $A^X < 2^YK^Y < C^Z$.

Proof· Since there are $(2^YK^Y+A^X)+(2^YK^Y-A^X)=2^{Y+1}K^Y$, then $2^YK^Y+A^X$ and $2^YK^Y-A^X$ are two symmetric odd numbers whereby 2^YK^Y to act as symmetric center, according to interrelations of 3 integers relating to symmetry.

In addition, $2^YK^Y-A^X \neq O^V$ i.e. $A^X+O^V \neq 2^YK^Y$ where $V \geq 3$ and proven $A^X+B^Y \neq 2^ZO^Z$ under the known requirements are just the same, thus get that the difference of $2^YK^Y-A^X$ can only be one of $O^{1\sim2}$.

Now that $2^YK^Y-A^X$ can only be one of $O^{1\sim2}$, then on the contrary, $2^YK^Y-A^{1\sim2}$ are either one of $O^{1\sim2}$ or any one of O^V with $V \geq 3$ under 2^YK^Y.

Next, there are $(2^YK^Y+A^{1\sim2})+(2^YK^Y-A^{1\sim2})=2^{Y+1}K^Y$, then $2^YK^Y+A^{1\sim2}$ and $2^YK^Y-A^{1\sim2}$ are two bilateral symmetric odd numbers whereby 2^YK^Y to act as symmetric center, and the pursuant reason is as above. So that $2^YK^Y+A^{1\sim2}$ are either one of $O^{1\sim2}$ or any one of O^V with $V \geq 3$ under 2^YK^Y too.

As mentioned above, $2^YK^Y-A^X$ within $(2^YK^Y+A^X)+(2^YK^Y-A^X)=2^{Y+1}K^Y$ are one of $O^{1\sim2}$, then $2^YK^Y+A^X$ are either one of $O^{1\sim2}$ or any one of O^V with $V \geq 3$ under $2^{Y+1}K^Y$, since the author has proved that there are not two of O^V with $V \geq 3$ on two places of every pair of bilateral symmetric odd numbers whereby $2^{Z-1}K^Z$ with $Z \geq 3$ to act as symmetric center in №7 section.

Such being the case, if $2^YK^Y+A^{1\sim2}$ are any one of O^V with $V \geq 3$ under $2^{Y+1}K^Y$, then $2^YK^Y+A^X$, i.e. $A^X+2^YK^Y$ can only be one of $O^{1\sim2}$.

If $2^YK^Y+A^X$ are any one of O^V with $V \geq 3$ under 2^YK^Y, then $2^YK^Y+A^{1\sim2}$ can only be one of $O^{1\sim2}$. It is obvious that this result is inconsistent with the conclusion got that $2^Y K^Y + A^{1\sim2}$ are either one of $O^{1\sim2}$ or any one of O^V with $V \geq 3$ under $2^Y K^Y$, hence must abandon the result and return to the right judgment that $2^Y K^Y + A^X$, i.e. $A^X + 2^Y K^Y$ can only be one of $O^{1\sim2}$.

By this token, on the one hand, $A^X + 2^Y K^Y$ can only be one of $O^{1\sim2}$, yet on the other, C^Z under the known requirements is one of O^V with $V \geq 3$.

Consequently, there are $A^X + 2^Y K^Y \neq C^Z$, i.e. $A^X + 2^Y (J+2)^Y \neq C^Z$ under the known requirements.

By the preceding way of doing thing, can continue to prove that when $K = J+4$, $J+6$…up to each and every odd number≥ 1, there are all $A^X + 2^Y (J+4)^Y \neq C^Z$, $A^X + 2^Y (J+6)^Y \neq C^Z$… up to $A^X + 2^Y O^Y \neq C^Z$ under the known requirements.

10. Make a Summary and Reach the Conclusion

To sum up, the author has proven every kind of $A^X + B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C have not a common prime factor in №6, №7, №8 and №9 sections. In addition to this, he has proven $A^X + B^Y = C^Z$ under the given requirements plus the qualification that A, B and C have at least a common prime factor in №3 section.

Such being the case, so long as make a comparison between $A^X + B^Y = C^Z$ and $A^X + B^Y \neq C^Z$ under the given requirements, at once reach inevitably such a conclusion that an indispensable prerequisite of existence of $A^X + B^Y = C^Z$ under the given requirements is exactly that A, B and C must have a common prime factor.

The proof was thus brought to a close. As a consequence, the Beal's conjecture is tenable.

11. Proving Fermat's last theorem From Beal's Conjecture

Fermat's last theorem is a special case of the Beal's conjecture, [3]. If Beal's conjecture is proved to hold water, then let X=Y=Z, so $A^X+B^Y=C^Z$ are changed into $A^X+B^X=C^X$.

Furthermore, divide three terms of $A^X+B^X=C^X$ by greatest common divisor of the three terms, then get a set of solution of positive integers without common prime factor. Obviously the set of conclusion is in contradiction with proven Beal's conjecture. As thus, we have proved Fermat's last theorem by reduction to absurdity as easy as pie.

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